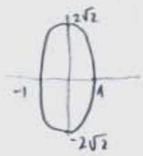
(1) • $\mathcal{L}(x,y;\lambda) = x + y + \lambda\left(x^2 + \frac{y^2}{8} - 1\right)$

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial x} = 0 \\ \frac{\partial \mathcal{L}}{\partial y} = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda} = 0 \end{cases} \Leftrightarrow \begin{cases} 1 + 2\lambda x = 0 \\ 1 + \frac{1}{4}\lambda y = 0 \\ x^2 + \frac{y^2}{8} - 1 = 0 \end{cases} \Leftrightarrow \begin{cases} x = -\frac{1}{2\lambda} \\ y = -\frac{4}{\lambda} \\ \frac{1}{4\lambda^2} + \frac{16}{8\lambda^2} = 1 \Leftrightarrow \frac{9}{4\lambda^2} = 1 \Leftrightarrow \lambda^2 = \frac{9}{4} \Leftrightarrow \lambda = \pm\frac{3}{2} \end{cases}$$

Se $\lambda = 3/2$, $(x,y) = (-1/3, -8/3)$
Se $\lambda = -3/2$, $(x,y) = (1/3, 8/3)$
> únicos candidatos a ext abs. de f na elipse

•  $fr(\mathcal{E}) = \mathcal{E}$, $int(\mathcal{E}) = \phi$ $\Rightarrow$ a elipse $\mathcal{E}$ é um cjto fechado e como $\mathcal{E} \subset B_r(0,0)$, $r > 2\sqrt{2}$, a elipse $\mathcal{E}$ tb é um cjto limitado, pelo que $\mathcal{E}$ é compacto

Então, pelo T. Weierstrass, como $f(x,y)$ é contínua em $\mathcal{E}$, f tem max e min absolutos em $\mathcal{E}$.

Assim, um dos ptos candidatos será maximizante abs e o outro min. abs. de f em $\mathcal{E}$.

• $f(-1/3, -8/3) = -3$ ⟵ MIN ABS
$f(1/3, 8/3) = 3$ ⟵ MAX ABS

(2) (a) $y' = \frac{x e^y}{e^{x^2}} \Leftrightarrow y' = \frac{x}{e^{x^2}} \times e^y$ $\Rightarrow \int e^{-y}\,dy = \int x e^{-x^2}\,dx \Rightarrow$

$\Rightarrow -e^{-y(x)} = -\frac{1}{2} e^{-x^2} + C$ // ← $y(x)$ é a fç def implicita/ pela esta eq.

(b) • $D^2 - 2D + 1 = 0 \Leftrightarrow D = 1 \Rightarrow y_h(x) = (C_1 + C_2 x)e^x, \forall C_1, C_2 \in \mathbb{R}$

• 1ª sol. ensaio: $y_p(x) = A + Bx$

$y_p'' - 2y_p' + y_p = 2x \Leftrightarrow 0 - 2B + A + Bx = 2x \Leftrightarrow \begin{cases} -2B + A = 0 \\ B = 2 \end{cases} \Leftrightarrow \begin{cases} A = 4 \\ B = 2 \end{cases}$

$\therefore y_p(x) = 4 + 2x$

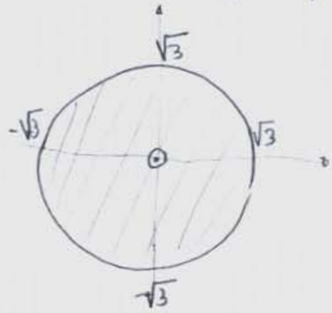
• Assim, $y_g(x) = (C_1 + C_2 x)e^x + 4 + 2x$, $\forall C_1, C_2 \in \mathbb{R}$

(3) $Q(x,y,z) = \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} a-1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$\Delta_1 = a - 1$
$\Delta_2 = a - 1$
$\Delta_3 = -1 \Rightarrow$ f.q. DN ou IND

Ora, se $a - 1 < 0$, $\Delta_1 < 0, \Delta_2 < 0, \Delta_3 < 0 \Rightarrow Q(x,y,z)$ INDEF
se $a - 1 \geq 0$, $\Delta_1 \geq 0, \Delta_2 \geq 0, \Delta_3 < 0 \Rightarrow Q(x,y,z)$ tb é IND
$\Rightarrow Q$ é INDEF $\forall a \in \mathbb{R}$

(4) (a) $D_g = D_{f_1} \cap D_{f_2} = \{(x,y) \in \mathbb{R}^2 : 3 - x^2 - y^2 \geq 0\} \cap \{(x,y) \in \mathbb{R}^2 : x^2 + 3y^2 \neq 0\}$
$= \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 3 \ \wedge \ (x,y) \neq (0,0)\}$



(b) $\text{int}(D_g) = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < 3 \ \wedge \ (x,y) \neq (0,0)\}$

$D_g \subset B_r(0,0)$, $r > 3 \implies D_g$ é limitado

(c). $\lim\limits_{(x,y) \to (0,0)} \dfrac{\sin(x^2+3y^2)}{x^2+3y^2} = 1$

- $0 \leq \left| \dfrac{x^3}{x^2+3y^2} \right| = \dfrac{|x| x^2}{x^2+3y^2} \leq \dfrac{|x|(x^2+3y^2)}{x^2+3y^2} = |x|$

$(x,y) \to (0,0)$: $0 \to 0$; $|x| \xrightarrow{(x,y)\to(0,0)} 0$

Assim, $\lim\limits_{(x,y) \to (0,0)} \dfrac{x^3}{x^2+3y^2} = 0$

- Pelas propriedades dos limites ($\lim f+g = \lim f + \lim g$, caso existam)

$$\lim_{(x,y) \to (0,0)} \frac{\sin(x^2+3y^2) + x^3}{x^2+3y^2} = \lim_{(x,y)\to(0,0)} \frac{\sin(x^2+3y^2)}{x^2+3y^2} + \lim_{(x,y)\to(0,0)} \frac{x^3}{x^2+3y^2} = 1 + 0 = 1$$

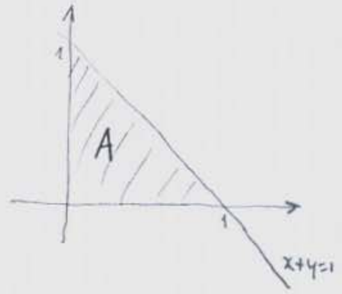
Assim, a função $f(x,y) = \begin{cases} f_2(x,y), & (x,y) \neq (0,0) \\ 1, & (x,y) = (0,0) \end{cases}$ é o prolongado por continuidade da função $f_2(x,y)$ ao ponto $(0,0)$.

(5) $f(\lambda x, \lambda y, \lambda z) = \dfrac{(\lambda x)^a (\lambda y)^2 - (\lambda y)^{2a} (\lambda z)^{b-2}}{(\lambda x)^{3a} (\lambda z)^b} = \dfrac{\lambda^{a+2} x^a y^2 - \lambda^{2a+b-2} y^{2a} z^{b-2}}{\lambda^{3a+b} x^{3a} z^b}$

Assim, para f ser homogénea, $a+2 = 2a+b-2$ e
$f(\lambda x, \lambda y, \lambda z) = \lambda^{a+2-(3a+b)} f(x,y,z)$,

donde, f é homog de grau -1 sse $\begin{cases} a+2 = 2a+b-2 \\ a+2-3a-b = -1 \end{cases} \iff \begin{cases} a = -1 \\ b = 5 \end{cases}$

(6)



(3)

$$\iint_A e^{-x-y}\,dx\,dy = \int_0^1\left(\int_0^{1-x} e^{-x-y}\,dy\right)dx = \int_0^1 \left.-e^{-x-y}\right]_{y=0}^{1-x} dx$$

$$= \int_0^1 -e^{-x-1+x} + e^{-x}\,dx = \int_0^1 -\frac{1}{e} + e^{-x}\,dx$$

$$= \left.-\frac{1}{e}x - e^{-x}\right]_{x=0}^{x=1} = -\frac{1}{e} - e^{-1} - (-\overset{0}{\widehat{\tfrac{1}{e}0}} - 1) = 1 - \frac{2}{e}$$

(7) $$\frac{\partial g}{\partial x}(0,0) = \frac{df}{du}(0)\times\frac{\partial u}{\partial x}(0,0) = \frac{df}{du}(0)\times\left.(4x-2y)\right|_{(0,0)} = 0$$

$$\frac{\partial g}{\partial y}(0,0) = \frac{df}{du}(0)\times\frac{\partial u}{\partial y}(0,0) = \frac{df}{du}(0)\times\left.(2y-2x)\right|_{(0,0)} = 0$$

$\Rightarrow (0,0)$ é pto crítico de $g(x,y)$

- $$\frac{\partial^2 g}{\partial x^2} = \frac{d^2 f}{du^2}\frac{\partial u}{\partial x}\times\frac{\partial u}{\partial x} + \frac{df}{du}\times\frac{\partial^2 u}{\partial x^2} = \frac{d^2 f}{du^2}\underbrace{\left(\frac{\partial u}{\partial x}\right)^2}_{(4x-2y)^2} + \frac{df}{du}\times\underbrace{\frac{\partial^2 u}{\partial x^2}}_{4}$$

$$\therefore \frac{\partial^2 g}{\partial x^2}(0,0) = \frac{d^2 f}{du^2}(0)\times 0 + \frac{df}{du}(0)\times 4 = 4\frac{df}{du}(0) = 4f'(0)$$

- $$\frac{\partial^2 g}{\partial y\,\partial x} = \frac{d^2 f}{du^2}\frac{\partial u}{\partial y}\times\frac{\partial u}{\partial x} + \frac{df}{du}\frac{\partial^2 u}{\partial y\,\partial x} = \frac{d^2 f}{du^2}(2y-2x)(4x-2y) + \frac{df}{du}\times(-2)$$

$$\therefore \frac{\partial^2 g}{\partial y\,\partial x}(0,0) = \frac{d^2 f}{du^2}(0)\times 0 - 2\frac{df}{du}(0) = -2\frac{df}{du}(0) = -2f'(0)$$

- $$\frac{\partial^2 g}{\partial y^2} = \frac{d^2 f}{du^2}\frac{\partial u}{\partial y}\frac{\partial u}{\partial y} + \frac{df}{du}\frac{\partial^2 u}{\partial y^2} = \frac{d^2 f}{du^2}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{df}{du}\frac{\partial^2 u}{\partial y^2} =$$

$$= \frac{d^2 f}{du^2}(2y-2x)^2 + \frac{df}{du}2 \longrightarrow \frac{\partial^2 g}{\partial y^2}(0,0) = 2\frac{df}{du}(0) = 2f'(0)$$

e como $g \in C^2$, $\frac{\partial^2 g}{\partial x\partial y} = \frac{\partial^2 g}{\partial y\partial x}$ e $H_g(0,0) = \begin{bmatrix} 4f'(0) & -2f'(0) \\ -2f'(0) & 2f'(0)\end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -2 & 2\end{bmatrix}$

Ora, $\Delta_1 = 4 > 0$, $\Delta_2 = \begin{vmatrix} 4 & -2 \\ -2 & 2\end{vmatrix} = 8-4 = 4 > 0 \Rightarrow H_g(0,0)$ DP $\Rightarrow (0,0)$ é min local de g