JAN 2013 — EN - Matemática II - Tópicos de Resolução

1. $Q(x_1, x_2, x_3) = [x_1 \ \ x_2 \ \ x_3] \begin{bmatrix} 1 & 0 & \alpha \\ 0 & 5 & 0 \\ \alpha & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $\Delta_1 = 1$, $\Delta_2 = \begin{vmatrix} 1 & 0 \\ 0 & 5 \end{vmatrix} = 5$, $\Delta_3 = \begin{vmatrix} 1 & 0 & \alpha \\ 0 & 5 & 0 \\ \alpha & 0 & 1 \end{vmatrix} = 5(1-\alpha^2)$

R: Se $\boxed{-1 < \alpha < 1}$, $\Delta_1 > 0, \Delta_2 > 0, \Delta_3 > 0 \Rightarrow$ $\boxed{Q \text{ é DP}}$

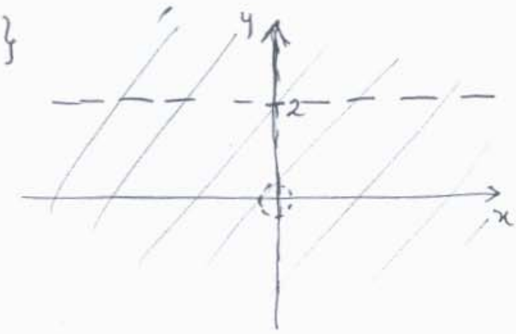
Se $\boxed{\alpha < -1 \text{ ou } \alpha > 1}$, $\Delta_1 > 0, \Delta_2 > 0, \Delta_3 < 0 \Rightarrow$ $\boxed{Q \text{ é IND}}$

Se $\boxed{\alpha = \pm 1}$, $|A - \lambda I| = 0 \Leftrightarrow \begin{vmatrix} 1-\lambda & 0 & \pm 1 \\ 0 & 5-\lambda & 0 \\ \pm 1 & 0 & 1-\lambda \end{vmatrix} = 0 \Leftrightarrow (5-\lambda)\left[(1-\lambda)^2 - 1\right] = 0 \Leftrightarrow \lambda = 5 \vee \lambda = 0 \vee \lambda = 2$

$\Downarrow$

$\boxed{Q \text{ é SDP}}$

2. a) $D_f = \{(x,y) \in \mathbb{R}^2 : (x,y) \neq (0,0) \text{ e } y \neq 2\}$



b) $fr(D_f) = \{(0,0)\} \cup \{(x,y) \in \mathbb{R}^2 : y = 2\}$

$ext(D_f) = \emptyset$

$D'_f = \mathbb{R}^2$

c) $$0 \le |f(x,y)| = \frac{x^4 y^2}{x^2+y^2} \left| \sin \frac{xy}{y-2} \right| \le \frac{x^4 y^2}{x^2+y^2} \times 1 \le \frac{x^4(x^2+y^2)}{x^2+y^2} = x^4$$

$0 \downarrow 0$, $\quad x^4 \downarrow 0$

$\therefore \ |f(x,y)| \xrightarrow[(x,y)\to(0,0)]{} 0$

Assim, f é prolongável por continuidade ao ponto (0,0) e o prolongamento é

$$\tilde{f}(x,y) = \begin{cases} f(x,y), & (x,y) \neq (0,0),\ y \neq 2 \\ 0, & (x,y) = (0,0) \end{cases}$$

3. Como f é dif em todo o seu domínio, $D_{\vec{v}} f(\bar{a}) = \nabla f(\bar{a}) \cdot \vec{v}$

$\frac{\partial f}{\partial x}(0, \pi/4, 0) = 2\cos(x+y)(-\sin(x+y))\Big|_{(0,\pi/4,0)} = 2 \times \frac{\sqrt{2}}{2} \times \left(-\frac{\sqrt{2}}{2}\right) = -1$

$\frac{\partial f}{\partial y}(0, \pi/4, 0) = 2yz + 2\cos(x+y)(-\sin(x+y))\Big|_{(0,\pi/4,0)} = 0 - 1 = -1$

$\frac{\partial f}{\partial z}(0, \pi/4, 0) = y^2\Big|_{(0,\pi/4,0)} = \frac{\pi^2}{16}$

R: $\boxed{D_{\overrightarrow{(-1,1,-4)}} f(0,\pi/4,0)} = \left(-1, -1, \frac{\pi^2}{16}\right)(-1, 1, -4) = 1 - 1 - \frac{\pi^2}{16} \times 4 = \boxed{-\frac{\pi^2}{4}}$

(2)

(4) $$y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = y\left(y + \underbrace{\frac{\partial f}{\partial u}}_{1}\underbrace{\frac{du}{dx}}_{2x} + \frac{\partial f}{\partial v}\times 0\right) - x\left(x + \frac{\partial f}{\partial u}\times 0 + \underbrace{\frac{\partial f}{\partial v}}_{1}\times\underbrace{\frac{dv}{dy}}_{2y}\right) =$$

$$= y\,(y+2x) - x\,(x+2y) = y^2 + 2xy - x^2 - 2xy = y^2 - x^2$$

c.q.p.

(5) • $\mathcal{L}(x,y;\lambda) = \frac{1}{2}xy + \lambda(x^2+y^2-16)$

$$\begin{cases}\frac{\partial \mathcal{L}}{\partial x}=0\\ \frac{\partial \mathcal{L}}{\partial y}=0\\ \frac{\partial \mathcal{L}}{\partial \lambda}=0\end{cases} \Leftrightarrow \begin{cases}\frac{1}{2}y + 2\lambda x = 0\\ \frac{1}{2}x + 2\lambda y = 0\\ x^2+y^2-16=0\end{cases} \Leftrightarrow \begin{cases} y = -4\lambda x\\ \frac{1}{2}x + 2\lambda(-4\lambda x) = 0\\ \text{—}\end{cases} \Leftrightarrow \begin{cases} y=-4\lambda x\\ x\left(\frac{1}{2}-8\lambda^2\right)=0\\ \text{—}\end{cases} \Leftrightarrow \begin{cases}\text{—}\\ x=0 \vee \lambda^2 = 1/16\\ \end{cases}$$

Se $x=0 \xRightarrow{1^a\, eq} y=0 \xRightarrow{3^a\, eq} 0-16=0$ $\times$ $\Rightarrow \boxed{x \neq 0}$

Se $x \neq 0 \xRightarrow{2^a\, eq} \lambda^2 = 1/16 \Leftrightarrow \lambda = \pm 1/4$

Se $\lambda = 1/4 \xRightarrow{1^a\, eq} y=-x \xRightarrow{3^a\, eq} 2x^2=16 \Leftrightarrow x = \pm 2\sqrt{2}$

Se $\lambda = -1/4 \xRightarrow{1^a\, eq} y = x \xRightarrow{3^a\, eq} x = \pm 2\sqrt{2}$

∴ As soluções do sistema são $\left.\begin{array}{l}(2\sqrt{2}, -2\sqrt{2})\\ (-2\sqrt{2}, 2\sqrt{2})\end{array}\right\}$ associados a $\lambda = 1/4$

$\left.\begin{array}{l}(2\sqrt{2}, 2\sqrt{2})\\ (-2\sqrt{2}, -2\sqrt{2})\end{array}\right\}$ associados a $\lambda = -1/4$.

• Seja $C = \{(x,y)\in\mathbb{R}^2 : x^2+y^2=16\}$.

$\left.\begin{array}{l}\text{int}(C)=\emptyset\\ \text{fr}(C)=C\end{array}\right\} \rightarrow \text{ad}(C)=C \Rightarrow C$ é fechado $\Longrightarrow C$ é compacto

C é limitado pq $C \subseteq B_r((0,0), r)$

Como $f(x,y)=\frac{1}{2}xy$ é polinómio, f é contínua em $\mathbb{R}^2$ e ptto f é contínua em C e, pelo T. Weierstrass, f tem <u>max</u> e <u>mín</u> <u>abs</u> em C

• $f(2\sqrt{2}, -2\sqrt{2}) = -4$

$f(-2\sqrt{2}, 2\sqrt{2}) = -4$

$f(2\sqrt{2}, 2\sqrt{2}) = 4$

$f(-2\sqrt{2}, -2\sqrt{2}) = 4$

R: 4 é o valor máx que f toma em C e -4 é o valor mínimo.

6. $D^2 - 3D + 2 = 0 \iff D = \frac{3 \pm \sqrt{9-8}}{2} \iff D = 2$ ou $D = 1$

$\therefore y_h(x) = c_1 e^{2x} + c_2 e^{x}, \quad \forall c_1, c_2 \in \mathbb{R}$

• 1ª sol. ensaio: $y_p(x) = Axe^{2x}$

C. Aux: $y_p' = Ae^{2x} + 2Axe^{2x}$

$y_p'' = 2Ae^{2x} + 2Ae^{2x} + 4Axe^{2x}$

$y_p'' - 3y_p' + 2y_p = 2e^{2x} \iff$

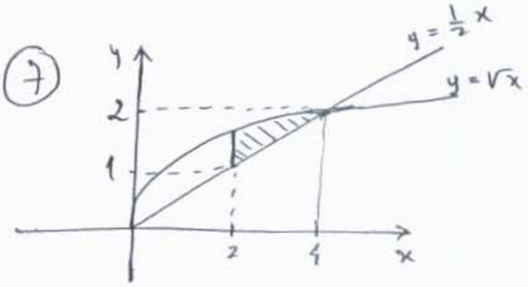
$\iff (4A + \cancel{4Ax})e^{2x} - 3(A + \cancel{2Ax})e^{2x} + \cancel{2Ax}e^{2x} = 2e^{2x}$

$\iff Ae^{2x} = 2e^{2x} \iff A = 2$

$\therefore y_p(x) = 2xe^{2x}$

R: $\boxed{y_g(x) = c_1 e^{2x} + c_2 e^{x} + 2xe^{2x}, \quad \forall c_1, c_2 \in \mathbb{R}}$

7.



$\iint_D 24xy\,dx\,dy = \int_2^4 \left(\int_{\frac{1}{2}x}^{\sqrt{x}} 24xy\,dy\right)dx =$

$= \int_2^4 \left[12xy^2\right]_{y=\frac{1}{2}x}^{y=\sqrt{x}} dx = \int_2^4 12x x - 12x\frac{1}{4}x^2\,dx =$

$= \int_2^4 12x^2 - 3x^3\,dx = \left[4x^3 - 3\frac{x^4}{4}\right]_2^4 = \underbrace{4.4^3 - 3.4^3}_{4^3} - \underbrace{4.2^3}_{32} + \underbrace{3.2^2}_{12} = 4^3 - 20 = 44$

8. $\frac{\partial}{\partial x}\left(\frac{g}{xf+yg}\right) - \frac{\partial}{\partial y}\left(\frac{f}{xf+yg}\right) =$

$= \frac{\frac{\partial g}{\partial x}(xf+yg) - g\left(\cancel{f} + x\frac{\partial f}{\partial x} + y\frac{\partial g}{\partial x}\right)}{(xf+yg)^2} - \frac{\frac{\partial f}{\partial y}(xf+yg) - f\left(x\frac{\partial f}{\partial y} + \cancel{g} + y\frac{\partial g}{\partial y}\right)}{(xf+yg)^2}$

$= \frac{f\boxed{x\frac{\partial g}{\partial x}} + \cancel{\frac{\partial g}{\partial x}yg} - g\,x\frac{\partial f}{\partial x} - \cancel{\frac{\partial g}{\partial x}yg} - \cancel{\frac{\partial f}{\partial y}xf} - g\,y\frac{\partial f}{\partial y} + \cancel{\frac{\partial f}{\partial y}xf} + f\boxed{y\frac{\partial g}{\partial y}}}{(xf+yg)^2}$

$= \frac{f\left(x\frac{\partial g}{\partial x} + y\frac{\partial g}{\partial y}\right) - g\left(x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}\right)}{(xf+yg)^2} = \frac{f\alpha g - g\alpha f}{(xf+yg)^2} = 0$ c.q.p.

f e g homog. do mesmo grau α $\Rightarrow$ id. Euler válida