Tópicos de Resolução – Mat II – 1º sem 14/15 – EN

1 a) • $|A| = 0+4+4-0-0-8 = 0 \Leftrightarrow |A|=0 \Leftrightarrow \lambda=0$ é valor pp de A

• $\bar{u}$ é vetor pp assoc. a $\lambda=0 \Leftrightarrow A\bar{u} = 0\bar{u} \Leftrightarrow A\bar{u} = \bar{0}$

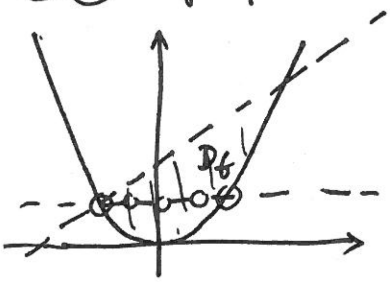
$$[A|\bar{0}] = \left[\begin{array}{ccc|c} 0 & 2 & -1 & 0 \\ 2 & 0 & -2 & 0 \\ -1 & -2 & 2 & 0 \end{array}\right] \to \cdots \to \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Leftrightarrow \begin{array}{l} u_1 = u_3 \\ u_2 = 1/2\, u_3 \end{array} \Rightarrow \bar{u} = \begin{bmatrix} u_3 \\ 1/2\, u_3 \\ u_3 \end{bmatrix}, \forall u_3 \in \mathbb{R},\ u_3 \neq 0$$

vetores pps associados a $\lambda=0$

b) • $q(x,y,z) = [x\ y\ z]\, A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 4xy - 2xz - 4yz + 2z^2$

• $q(8,0,1) = -14 < 0$
$q(0,0,1) = 2 > 0$ $\Rightarrow q$ é indefinida//

2 a) $D_f = \{(x,y) \in \mathbb{R}^2 : y \geq x^2 \wedge x+2-y > 0 \wedge y \neq 1\}$



b) $ad(D_f) = \{(x,y) : y \geq x^2 \wedge x+2-y \geq 0\}$

$D_f \neq ad(D_f) \Rightarrow D_f$ não é fechado
$\Rightarrow D_f$ não é compacto//

3 a) f é contínua em $(0,0)$ sse $\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0)$.

Ora, $0 \leq \dfrac{x^2 e^{-y}}{\sqrt{x^2+y^2}} \leq \dfrac{(x^2+y^2)\, e^{-y}}{\sqrt{x^2+y^2}} = \sqrt{x^2+y^2} \cdot e^{-y}, \ \forall (x,y) \in \mathbb{R}^2,$

e $\lim_{(x,y)\to(0,0)} 0 = \lim_{(x,y)\to(0,0)} \sqrt{x^2+y^2}\, e^{-y} = 0$, donde se conclui que

$\lim_{(x,y)\to(0,0)} \dfrac{x^2 e^{-y}}{\sqrt{x^2+y^2}} = 0$. Como $f(0,0) = 0$, fica provado q f é contínua em $(0,0)$//

b) • $\dfrac{\partial f}{\partial x}(0,0) = \lim_{t\to 0} \dfrac{f(t,0)-f(0,0)}{t} = \lim_{t\to 0} \dfrac{\frac{t^2}{\sqrt{t^2}}}{t} = \lim_{t\to 0} \dfrac{t}{|t|} = \begin{cases} 1, & \text{se } t>0 \\ -1, & \text{se } t<0 \end{cases}$

$\therefore$ não existe $\dfrac{\partial f}{\partial x}(0,0)$ (e, portanto, f não é diferenciável em $(0,0)$)

• $\dfrac{\partial f}{\partial y}(0,0) = \lim_{t\to 0} \dfrac{f(0,t)-f(0,0)}{t} = \lim_{t\to 0} \dfrac{\frac{0}{\sqrt{t^2}}}{t} = \lim_{t\to 0} \dfrac{0}{t} = \lim_{t\to 0} 0 = 0$//

4) $\begin{cases} \dfrac{\partial f}{\partial x} = 0 \\ \dfrac{\partial f}{\partial y} = 0 \end{cases} \Leftrightarrow \begin{cases} 2xy + 4y = 0 \\ x^2 - 6y + 4x = 0 \end{cases} \Leftrightarrow \begin{cases} 2y(x+2) = 0 \\ — \end{cases} \begin{cases} y = 0 \vee x = -2 \\ — \end{cases}$

Se $y = 0 \xrightarrow{eq\,2} x^2 + 4x = 0 \Leftrightarrow x = 0 \vee x = -4 \Rightarrow (0,0), (-4,0)$ são ptos críticos

Se $x = -2 \underset{eq\,2}{\Longrightarrow} -4 - 6y = 0 \Leftrightarrow y = -2/3 \Rightarrow (-2, -2/3)$ tb é pto crítico de f

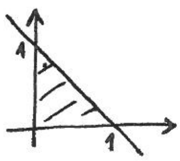
$H_f = \begin{bmatrix} 2y & 2x+4 \\ 2x+4 & -6 \end{bmatrix}$

• $H_f(0,0) = \begin{bmatrix} 0 & 4 \\ 4 & -6 \end{bmatrix}$, $\Delta_1 = 0$, $\Delta_2 = -16 \Rightarrow H_f(0,0)$ indef $\Rightarrow (0,0)$ é pto de sela //

• $H_f(-4,0) = \begin{bmatrix} 0 & -4 \\ -4 & -6 \end{bmatrix}$, $\Delta_1 = 0$, $\Delta_2 = -16 \Rightarrow (-4,0)$ é pto de sela //

• $H_f(-2, -\frac{2}{3}) = \begin{bmatrix} -4/3 & 0 \\ 0 & -6 \end{bmatrix}$, $\Delta_1 = -\frac{4}{3} < 0$, $\Delta_2 = 8 > 0 \Rightarrow$

$\Rightarrow H_f(-2, -\frac{2}{3})$ DN $\Rightarrow (-2, -\frac{2}{3})$ é max local de f //

5) 

$$\iint_A 6x e^{(1-y)^3} \, dx\, dy = \int_0^1 \left(\int_0^{1-y} 6x e^{(1-y)^3} \, dx \right) dy =$$

$$= \int_0^1 \left[3x^2 e^{(1-y)^3} \right]_{x=0}^{x=1-y} dy = \int_0^1 3(1-y)^2 e^{(1-y)^3} \, dy =$$

$$= -e^{(1-y)^3} \Big]_{y=0}^{y=1} = -1 + e \;//$$

6) a) • $D^2 - D - 6 = 0 \Leftrightarrow D = 3 \vee D = -2 \Rightarrow y_h(x) = c_1 e^{3x} + c_2 e^{-2x}, \forall c_1, c_2 \in \mathbb{R}$

• $y_p = Ax^2 + Bx + C \Rightarrow y_p'' - y_p' - 6y_p = 18x^2 \Leftrightarrow$

C.A.: $y_p' = 2Ax + B$
$y_p'' = 2A$

$\Leftrightarrow 2A - (2Ax + B) - 6(Ax^2 + Bx + C) = 18x^2$

$\Leftrightarrow -6Ax^2 + (-2A - 6B)x + 2A - B - 6C = 18x^2$

$\Leftrightarrow \begin{cases} -6A = 18 \\ -2A - 6B = 0 \\ 2A - B - 6C = 0 \end{cases} \Leftrightarrow \begin{cases} A = -3 \\ B = 1 \\ C = -7/6 \end{cases} \Rightarrow y_p = -3x^2 + x - 7/6$

$\therefore y_g(x) = c_1 e^{3x} + c_2 e^{-2x} - 3x^2 + x - 7/6 \;//$

b) $y \dfrac{dy}{dx} = \dfrac{e^{-y^2} x^3}{1+x^4} \Rightarrow \int e^{+y^2} y \, dy = \int \dfrac{x^3}{1+x^4} dx \Leftrightarrow$

$\Leftrightarrow +\frac{1}{2} e^{+y^2} = \frac{1}{4} \ln(1+x^4) + C, \forall C \in \mathbb{R}$ R: $y(x)$ é a fç definida implicitamente por esta equação //

7 $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} =$

$$= g(u,v) + x\left(\frac{\partial g}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial g}{\partial v}\cdot\frac{\partial v}{\partial x}\right) + x\left(\frac{\partial g}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial g}{\partial v}\,\frac{\partial v}{\partial y}\right)$$

$$= g(u,v) + x\left(\frac{\partial g}{\partial u}\cdot 2x e^{x^2} y + \frac{\partial g}{\partial v}\,\frac{2x}{x^2+y^2+1} + \frac{\partial g}{\partial u}\, e^{x^2} + \frac{\partial g}{\partial v}\cdot\frac{2y}{x^2+y^2+1}\right)$$

e, pq $(x,y)=(1,1) \longrightarrow (u,v) = (e^1\cdot 1\,,\ \ln(1+1+1)) = (e, \ln 3)$,

$$\frac{\partial f}{\partial x}(1,1) + \frac{\partial f}{\partial y}(1,1) = g(e,\ln 3) + \frac{\partial g}{\partial u}(e,\ln 3)\times(2+1)e + \frac{\partial g}{\partial v}(e,\ln 3)\times 2\times\frac{2}{3}$$

c.q.d.//