5 Multiple Integrals

5.1. Compute the following integrals.

a) $\int_0^1 \int_{-1}^1 \int_1^2 x\, dxdydz$ b) $\int_{-1}^1 \int_0^1 ye^{xy} dxdy$ c) $\int_0^{+\infty} \int_0^{+\infty} \int_0^{+\infty} e^{-x-y-z} dxdydz$

d) $\int_0^5 \int_0^{+\infty} (x^2 e^{-2yx} + 3ye^{-y^2}) dydx$ e) $\int_1^2 \int_1^2 (1 + x + \frac{y}{2}) dxdy$ f) $\int_0^1 \int_0^1 \int_0^{\frac{\pi}{2}} y \cos x\, dxdydz.$

Solution: a) 3 b) $e - \frac{1}{e} - 2$ c) 1 d) $\frac{55}{4}$ e) $\frac{13}{4}$ f) $\frac{1}{2}$

5.2. Compute $\iint_A f(x,y) dxdy$, where

$a) f(x,y) = \frac{2x}{y^6}$, $A = \{(x,y) \in \mathbb{R}^2 : 1 \leq y \leq 3,\ 0 \leq x \leq y^4\}$

$b) f(x,y) = e^{\frac{y}{x}}$, $A = \{(x,y) \in \mathbb{R}^2 : 1 \leq x \leq 2,\ x \leq y \leq x^3\}$

$c) f(x,y) = x^2 y^5$, $A = \{(x,y) \in \mathbb{R}^2 : 0 \leq x \leq 1,\ x^2 \leq y \leq 1\}$

$d) f(x,y) = ye^x + x^2 y$, $A = \{(x,y) \in \mathbb{R}^2 : 0 \leq y \leq 1,\ 0 \leq x \leq y^2\}$

$e) f(x,y) = xe^{-xy}$, $A = \{(x,y) \in \mathbb{R}^2 : 1 \leq x \leq 2,\ \frac{1}{x} \leq y < +\infty\}$

$f) f(x,y) = x^3 + 4y$, A is the region bounded by the lines $y = x^2$ and $y = 2x$.

Solution: a) $\frac{26}{3}$ b) $\frac{e^4}{2} - 2e$ c) $\frac{2}{45}$ d) $\frac{e}{2} - \frac{23}{24}$ e) $\frac{1}{e}$ f) $\frac{32}{3}$

5.3. Let $A = \{(x,y) \in \mathbb{R}^2 : y \leq 1 - x,\ y \leq 1 + x,\ y \geq 0\}$, and compute

a) $\iint_A (x-1)y\, dxdy$ b) $\iint_A (y - 2y^2) e^{xy}\, dxdy$ c) $\iint_A (x+y)\, dxdy.$

Solution: a) $-\frac{1}{3}$ b) 0 c) $\frac{1}{3}$

5.4. Compute $\int_{-1}^1 \int_{-1}^1 f(x,y) dxdy$, with $f(x,y) = \begin{cases} xy & x \geq 0 \ e \ y \geq 0 \\ 1 - x - y, & \text{other } (x,y) \end{cases}$.

Solution: $\frac{17}{4}$

5.5. Using a double integral compute the area of set A, where

a) $A = \{(x,y) \in \mathbb{R}^2 : -1 \leq x \leq 1 \;\wedge\; x^2 \leq y \leq 1\}$;

b) $A = \{(x,y) \in \mathbb{R}^2 : 1 - x \leq y \leq 1 \;\wedge\; x \leq 1\}$;

c) $A = \{(x,y) \in \mathbb{R}^2 : x^2 \leq y \leq 2 - x^2\}$;

d) $A = \{(x,y) \in \mathbb{R}^2 : x + y + 2 \geq 0 \;\wedge\; x + y^2 \leq 0\}$;

e) $A = \{(x,y) \in \mathbb{R}^2 : y \leq x^2 \;\wedge\; y - x \geq 0 \;\wedge\; 2y - x \leq 3 \;\wedge\; x \geq 0\}$;

f) $A = \{(x,y) \in \mathbb{R}^2 : x \geq 0 \;\wedge\; x^2 + y^2 \leq 1\}$;

g) $A = \{(x,y) \in \mathbb{R}^2 : y^2 + x \leq 2 \;\wedge\; x - y \geq 0\}$;

h) $A = \left\{(x,y) \in \mathbb{R}^2 : y^2 \leq x \;\wedge\; x \leq \frac{y^2}{4} + 3\right\}$;

Solution: a) $\frac{4}{3}$ b) $\frac{1}{2}$ c) $\frac{8}{3}$ d) $\frac{9}{2}$ e) $\frac{35}{48}$ f) $\frac{\pi}{2}$ g) $\frac{9}{2}$ h) 8

5.6. Compute $\displaystyle\int_0^4 \int_{2x}^8 \sin(y^2)\,dydx$ (sugestion: Draw the integratio region and invert the integration order)

Solution: $\frac{1-\cos 64}{4}$

5.7. Compute $\displaystyle\int\int_A g(x,y)dxdy$, where $A = [0,2] \times [0,4]$ e $g(x,y) = \begin{cases} x - y, & x \leq y \leq 2x \\ 0, & \text{otherwise} \end{cases}$.

Solution: $-\frac{4}{3}$