LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Mathematics II

Undergraduate Degrees in Economics and Management
First Exam, June 4, 2015

1. Let Ω be the domain of the function $f(x,y) = \sqrt{1-x^2-y^2} + \ln(x+y)$. Determine Ω, including a graphical representation, as well as its interior and boundary. Briefly justify that Ω is convex but not compact.

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by

$$\begin{cases} \dfrac{x^3}{x^2+y^2} & ,(x,y) \neq (0,0) \\ 0 & ,(x,y) = (0,0) \end{cases}$$

(a) Show that f is continuous in $\mathbb{R}^2$.

(b) Compute the directional derivative, $\dfrac{\partial f}{\partial \boldsymbol{v}}(0,0)$, for every nonzero vector $\boldsymbol{v} = (v_1, v_2)$.

(c) Show that in general $\dfrac{\partial f}{\partial \boldsymbol{v}}(0,0) \neq v_1 \dfrac{\partial f}{\partial x}(0,0) + v_2 \dfrac{\partial f}{\partial y}(0,0)$. Without additional calculations, what can you say about the differentiability of f at $(0,0)$?

3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a C^1 function and consider $g(x,y) = f(x^2+y^2, xy)$. Knowing that $\nabla f(2,1) = (1,1)$, compute $\nabla g(1,1)$.

4. Consider the function $f(x,y) = x^2 + y^2 + x^3$.

(a) Determine and classify all critical points of f.

(b) Justify that f attains a global maximum and minimum over $M = \{(x,y) : x^2+y^2 \leq 1\}$ and determine them.

5. Compute $\displaystyle\iint_S xy\,dx\,dy$, where $S = \{(x,y) \in \mathbb{R}^2 : x^2 \leq y \leq 2x\}$.

6. Consider the differential equation $y''(x) - y'(x) - 2y(x) = e^{-2x}$.

(a) Solve the initial value problem $y(0) = 1/4$, $y'(0) = 0$.

(b) Determine a solution that satisfies $y(0) = 0$ and $\displaystyle\lim_{x\to+\infty} \frac{y(x)}{e^{2x}} = \pi$.

Point values: **1**. 2,0 **2**.(a)2,0 (b)1,5 (c) 1,5 **3**. 1,5 **4**. (a)2,5 (b)2,5 **5**. 2,5 **6**. (a)2,5 (b)1,5