



Lisbon School
of Economics
& Management
Universidade de Lisboa

MASTER
MATHEMATICAL FINANCE

MASTER'S FINAL WORK
DISSERTATION

**LEAD LAG RELATIONSHIP BETWEEN
DIFFERENT FINANCIAL ASSETS**

DANIELA SOFIA DÔRO DOMINGOS

JUNE - 2025



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SUPERVISION:
ANA SOFIA MELO MONTEIRO

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ACRONYMS

AIC - Akaike Information Criterion.

AU – Australia.

CAC 40 – Cotation Assistée en Continu 40.

DAX 40 – Deutscher Aktienindex 40.

DE – Germany.

EMF – Efficient Market Hypothesis.

ES – Spain.

FR – France.

FTSE 250 – Financial Times Stock Exchange 250 Index.

FTSE MIB – Financial Times Stock Exchange Milano Indice di Borsa.

IBEX 35 – Índice Bursátil Español 35.

IT – Italy.

JP – Japan.

LM – Lagrange Multiplier.

OLS – Ordinary Least Squares.

S&P 500 – Standard & Poor's 500 Index.

S&P/ASX 200 – Standard & Poor's/Australian Securities Exchange 200 Index.

TOPIX – Tokyo Stock Price Index.

UK – United Kingdom.

US – United States.

VAR – Vector Autoregressive.

VEC – Vector Error Correction.

ABSTRACT

This study investigates the causal relationship and feedback mechanisms between equity indexes and fixed-income securities across eight developed economies: the United States, Germany, Japan, Spain, France, the UK, Italy and Australia. We employed bivariate vector autoregressive (VAR) models, pairwise Granger causality tests and Geweke's feedback measures to both daily and weekly data over the period from December 31, 1987 to April 23, 2025. Results reveal that the US equity market plays a leading role in global return and volatility transmission, particularly among equity indexes. European indexes also show notable influence, while Asia-Pacific instruments reveal limited impact beyond their domestic markets. Geweke feedback measures indicate a predominance of contemporaneous feedback over lagged feedback, suggesting that information is mostly transmitted within the same time interval rather than with delay. Interestingly, major indexes such as the S&P 500 and DAX 40 exhibit a stronger lagged influence from other indexes than the influence they exert on them. The results also reveal stronger information transmission within the same asset class. A comparison of the period before and after the 2008 financial crisis show increased equity market integration and weakened relationships between major equity indexes and bonds.

KEYWORDS: Equity Indexes; Government Bonds; Granger Causality; Geweke Feedback Measures; International Markets.

JEL CODES: C12; C32; G11; G15; F36.

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1. INTRODUCTION

While the dominant view in financial economics supports the idea that information flows efficiently across markets, several authors and theories highlight imperfections, biases and challenges. Fama (1970) was one of the authors that defended the role of information in financial markets. By formalizing the “Efficient Market Hypothesis” (EMH), Fama stated that financial markets efficiently incorporate all available information into asset prices. Later, subsequent literature has challenged and extended Fama’s conclusions. For instance, Grossman & Stiglitz (1980) argued that if markets were perfectly efficient, no one would have the incentive to obtain costly information, thus perfect efficiency is paradoxical. Shiller (1981) found that stock prices are more volatile than fundamentals, suggesting irrational behavior and challenging EMH. De Bondt & Thaler (1985) found evidence of overreaction in stock markets, introducing behavioral biases. More recently, researchers have explored anomalies and predictable patterns in asset returns, systematically challenging EMH.

These inefficiencies and behavioral biases also have implications on how information flows across financial markets. Rather than all assets adjusting simultaneously to new information, empirical studies suggest that markets do not influence each other the same way, with certain asset classes reacting quicker than others. This phenomenon is particularly evident in today’s financial system where information spreads quickly. Thus, to understand how information flows across markets, it is important to investigate how financial assets are related to each other. One way to explore these relations is by examining lead-lag relationships, that is, situations in which one asset’s movements have predictive power over another.

The concept of lead-lag relationships is thus grounded in the idea that information and market reactions do not always occur simultaneously across all financial assets. Based on this concept, this study explores the relationship between stock market indexes and government bonds by considering contemporaneous and lead-lag effects across eight countries.

This thesis contributes to the literature by analyzing daily and weekly stocks and bonds returns over an extended period that includes several major episodes of economic turmoil, such as the 2008 subprime mortgage crisis, the 2012 sovereign debt crisis, and

the COVID-19 pandemic. Different from current literature, this analysis spans multiple continents and focuses on some of the world's largest and most influential economies, which naturally attract significant investor attention due to their global relevance. In addition, this research evaluates the causality and strength of the interrelationships between markets and instruments using Geweke's feedback measures. To capture potential structural shifts in market dynamics, the analysis is also conducted separately for two sub-periods: before and after the 2008 financial crisis. Thus, this research is crucial to understand how these key economies interact in response to changes in bonds and stocks returns.

The findings carry significant implications for risk management and policy coordination, while also offering insights for investors seeking to exploit or hedge against these relationships.

According to our results, the US equity market plays a leading role in return and volatility transmission across markets, especially within equity indexes. European indexes also exhibit notable influence, whereas Asia-Pacific instruments remain regionally contained. Geweke feedback results reveal a stronger contribution of contemporaneous feedback to the total feedback, suggesting that most information is transmitted within the same interval rather than with delay. Additionally, it is also notable that major indexes such as the S&P 500 and the DAX 40, exhibit a stronger lagged influence from other indexes than the influence they exert on them. The results also highlight a stronger information flow within assets of the same class. The analysis of the periods before and after the 2008 financial crisis reveal an increase in equity market integration, while the connections between the S&P 500 and the DAX 40 indexes and government bonds weakened.

The remainder of this thesis is organized as follows. Section 2 reviews the theoretical and empirical literature on lead-lag relationships and information flow across financial assets. Section 3 describes the data used and presents descriptive statistics. Section 4 presents the methodological framework. Section 5 discusses empirical results. Section 6 concludes with a summary of findings and directions for future research.

2. LITERATURE REVIEW

The lead-lag relationship between financial assets plays a crucial role in financial markets, as it provides insights into how asset returns are interconnected across markets, countries, regions, and industries, for example.

Several influential studies have looked into these relationships. For instance, Lo and MacKinlay (1990) focused its analysis on firm size and argued that large-cap stocks tend to lead small-cap stocks due to faster information processing. Brennan et al. (1993), on the other hand, investigated the role of analyst coverage, arguing that firms with a higher number of analysts following them experience quicker price adjustments, as information spreads more rapidly. As a result, returns on portfolios of firms followed by many analysts tend to lead those of firms with fewer analysts.

Copeland and Copeland (1998) analyzed the daily return index for a specific region, country, or industry by performing ordinary least squares (OLS) regressions for up to four lags with the returns from the corresponding comparison region, country, or industry. Focusing on 3 regions (Americas, the Pacific, and Europe), 29 countries, and 121 industry groups, Copeland and Copeland (1998) concluded that the United States exhibits statistically significant one-day lead effects over European and Asian markets, with changes in foreign interest rates playing a role in strengthening the connections between these markets. As for industries, Copeland and Copeland (1998) argued that those classified as "global" demonstrate significantly greater sensitivity to market leads compared to those categorized as "local".

Berben and Jansen (2003) examined correlations in aggregate and sectorial returns across Germany, Japan, the UK and the US from 1980 to 2000. By introducing a multivariate GARCH model with smoothly time-varying correlations and employing a Lagrange Multiplier (LM) test, it was found that correlations among the Germany, UK and the US equity markets doubled over the period, while the Japanese market correlations remained mainly unchanged. The findings highlight that the rise in comovement was broadly reflected across industries in Europe and the US, but not in Japan.

Hou (2007) studied the lead lag effect in stock returns by decomposing the unconditional lead-lag effect into two components: inter-industry and intra-industry.

Using weekly returns from July 1963 to December 2001, Hou (2007) estimated cross-autocorrelations and conducted a vector autoregressive (VAR) test to conclude that the intra-industry component accounts for the majority of the observed lead-lag effect, as firms within the same industry often react more slowly to negative information, which results in the observed lead-lag patterns. Furthermore, Hou (2007) found that within the same industry, the returns of larger firms tend to lead the returns of smaller firms.

Concerning connections among bond markets, Jeon et al. (2012) conducted a study on international linkages between Japanese government bond yields and the government bond yields of the US, the UK and Germany. The findings, based on a VAR and vector error correction (VEC) models applied to monthly data during a period from January 1980 to December 2004, revealed that the Japanese bond market is rarely impacted by the bond market of the US, the UK and Germany.

Cambon (2017) carried out two types of predictive regressions estimated through an OLS estimation – one focusing on market and industry returns, and the other examining economic activity and industry returns – with the aim of evaluating the gradual information diffusion theory proposed by Hong et al. (2007), which suggests that industries possessing valuable fundamental economic information tend to lead both the equity market and broader economic activity. Contrary to the findings of Hong et al. (2007), and focusing on Spain and other European core countries, the study revealed that industries that lead the market are not the same as those leading economic activity.

Gruener and Finke (2017) investigated lead-lag relationships in stock portfolios categorized based on both size and analyst coverage as well as institutional ownership across seven developed market: Canada, France, Germany, Italy, Japan, the United Kingdom and the United States. The findings were aligned with Lo and MacKinlay (1990) and Brennan et al. (1993), confirming the prevalence of lead-lag relationships in portfolios sorted by firm size and analyst coverage in the majority of markets. However, regarding institutional ownership, the findings provide limited support for the existence of such relationships.

Camilleri (2019) conducted a study addressing connections between stock prices and key macroeconomic indicators, such as inflation, industrial production, interest rates, money supply and two moderating variables to account for the anticipated interactions

between specific macroeconomic variables: money supply with inflation and interest rates with money supply. The findings, based on an analysis of five European countries (Belgium, France, Germany, Netherlands and Portugal) through a series of ordinary least squares (OLS) regressions followed by vector autoregressions (VAR's) to test for Granger causality, revealed that stock prices consistently led inflation across all countries studied and industrial production in four of them, with the relationship being predominantly positive. It was also proved that the interaction between interest rates and money supply emerged as a leading indicator of stock prices, particularly in France, Germany, and Portugal, and that there were no significant links between interest rates and stock indexes.

More recently, Monteiro et al. (2023) studied international financial market interdependencies by examining the linear relationships between 11 industries in the United States and six other countries (Canada, France, Germany, Japan, China, and the UK). With weekly data from January 1, 1973 until May 17, 2021, Monteiro et al. (2023) estimated a bivariate VAR model of order one for the 11 industries and 7 countries followed by pairwise Granger causality tests and feedback measures. The findings reveal significant causal relationships between US industry returns and those of other countries, with the exception of China, where trading constraints may impede such linkages.

3. DATA DESCRIPTION

This study exploits historical data on equity indexes and fixed-income securities returns for eight countries: the US, Germany, Japan, Spain, France, the UK, Italy and Australia. Daily closing values were obtained from the Thompson Reuters DataStream database from December 31, 1987, to April 23, 2025. Local-currency values were converted to U.S. dollars using spot foreign exchange rates obtained from Banco de Portugal on April 29th. The use of daily data is particularly effective in precisely identifying lead-lag relationships, a notion supported by Lo and MacKinley (1990), as the lead-lag effects tend to increase with data frequency. However, non-synchronous trading problems due to trading schedule differences tend to arise with daily data (Burns et al. 1998). Thus, this research comprises both daily and weekly frequencies to provide a more comprehensive understanding of these international financial links. The conversion from daily to weekly frequency was based on Wednesday-to-Wednesday intervals.

The equity indexes include the S&P 500 (US), DAX 40 (Germany), TOPIX (Japan), IBEX 35 (Spain), CAC 40 (France), FTSE 250 (UK), FTSE MIB (Italy), and S&P/ASX 200 (Australia). The fixed-income securities data comprises the 10-year government bond yield for each of the analyzed countries. An overview of the instruments included in this research is provided in Table 1. The indexes were modelled as log returns, r_t :

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

Where t is the time, P_t is the index price at time t and P_{t-1} is the index price at time $t - 1$. The bonds were transformed into first differences, denoted as ΔP :

$$\Delta P = P_t - P_{t-1}$$

Table 1 Overview of Instruments

Country	Country Code	Index	Bond	Currency
United States	US	S&P 500	10-year Government Bond	USD
Germany	DE	DAX 40	10-year Government Bond	EUR
Japan	JP	TOPIX	10-year Government Bond	JPY
Spain	ES	IBEX 35	10-year Government Bond	EUR
France	FR	CAC 40	10-year Government Bond	EUR
United Kingdom	UK	FTSE 250	10-year Government Bond	GBP
Italy	IT	FTSE MIB	10-year Government Bond	EUR
Australia	AU	S&P/ASX 200	10-year Government Bond	AUD

The time series plots in Figure 1 provide a visual representation of the daily returns of two major equity indexes, S&P 500 (US) and DAX 40 (Germany), and their respective 10-year government bond yields. Periods of market turmoil are clearly identified in both asset classes plots, with the most notable episodes being the Global Financial Crisis of 2008-2009 and the COVID-19 Pandemic in early 2020, manifested as periods of sharply increased volatility. For reasons of parsimony, the time series plots for the remaining countries and their respective instruments can be found in Appendix A.

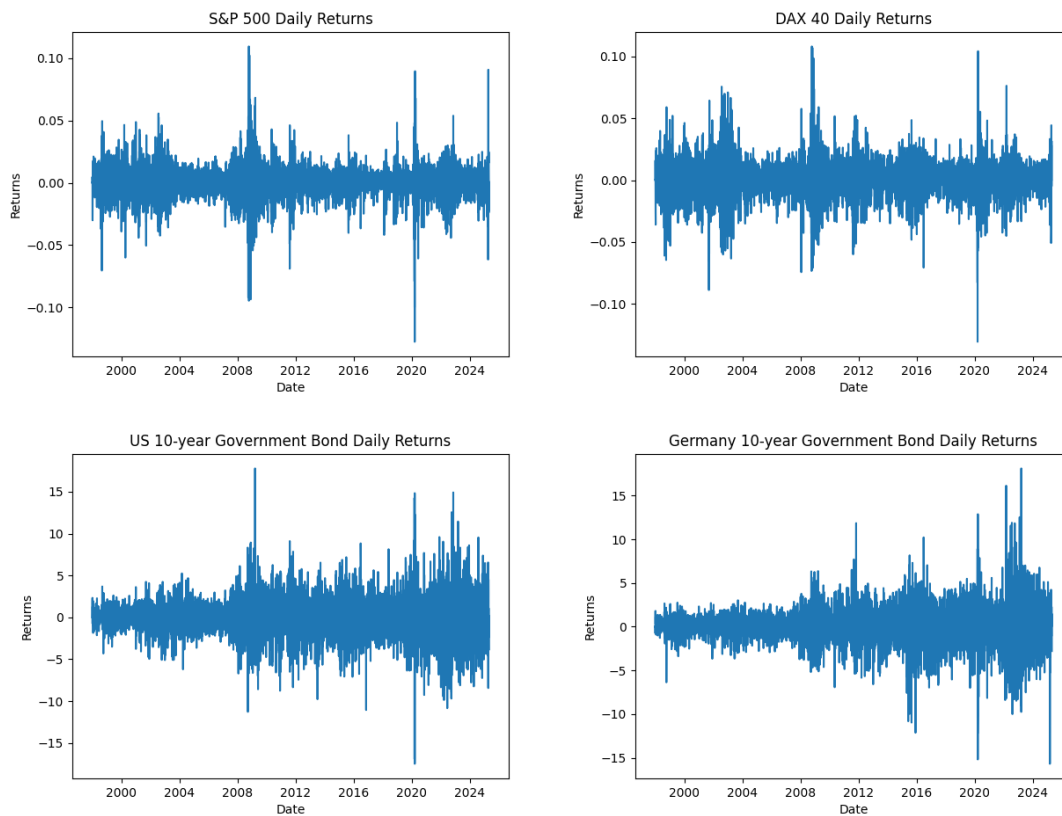


Figure 1 Time series of daily returns of the US and Germany's instruments. This figure displays the daily returns of the S&P 500 and DAX 40 equity indexes as well as the 10-year government bond yield for the US and Germany. The sample covers December 31, 1987, to April 23, 2025. Equity returns are expressed in percentages, whereas bond yield returns are measured in basis points.

Table 2 and Table 3 provide general descriptive statistics for the daily and weekly returns, respectively, across the eight major economies. As expected, the mean and standard deviation values for both stock returns and bond returns are smaller for daily data than for weekly data. Regarding the evaluation between asset classes, government bonds exhibit the highest mean and standard deviation for both daily and weekly data.

Moreover, Italy and Spain government bonds returns stand out with the highest mean (a daily mean of around 0.14 and a weekly mean of around 0.71 for both countries) and the highest standard deviation (a daily standard deviation of 5.04 and 4.44, respectively and weekly standard deviation of 11.66 and 10.45, respectively).

The distribution of equity indexes is left-skewed for both daily and weekly data. As for bonds, returns are also left-skewed except for Spain and UK in the daily data analysis and Japan and Spain in the weekly data analysis. Nevertheless, the indicator points to a balanced distribution overall.

Regarding Kurtosis, all instruments of all countries exhibit excess kurtosis, except for the weekly returns of US Bonds. The analysis also reveals higher kurtosis for some government bond series than that observed in equity indexes, particularly for Italy and Spain, with a daily kurtosis of 22.25 and 11.21, respectively, for bond returns, compared to a daily kurtosis of 8.46 and 7.82, respectively, for equity index returns. This suggests that during certain periods, these bond markets experienced tail risk more severe than that observed in equity markets, challenging the conventional view of government bonds as a safer asset class. Overall, kurtosis is higher for daily data than for weekly data.

Table 2 Descriptive statistics of daily returns

	Mean	Std	Min	Max	Skew	Kurt
<i>Indexes</i>						
S&P 500	0.0002	0.0121	-0.1277	0.1096	-0.3475	10.4716
DAX 40	0.0002	0.0142	-0.1305	0.1080	-0.1951	5.7685
TOPIX	0.0001	0.0129	-0.1305	0.1286	-0.4019	7.8271
IBEX 35	0.0001	0.0141	-0.1515	0.1348	-0.3345	7.8217
CAC 40	0.0001	0.0138	-0.1310	0.1059	-0.2152	6.3122
FTSE 250	0.0002	0.0103	-0.0982	0.0804	-0.4866	6.9652
FTSE MIB	0.0001	0.0149	-0.1854	0.1087	-0.5573	8.4625
S&P/ASX 200	0.0002	0.0097	-0.1020	0.0677	-0.6766	8.2404
<i>Bonds</i>						
US	0.0546	2.2705	-17.4810	17.777	-0.0108	4.6418
Germany	0.0583	2.0241	-15.6902	18.1172	-0.0242	6.3849
Japan	0.0001	0.0041	-0.0347	0.0378	-0.3478	8.7395
Spain	0.1434	4.4447	-32.4642	45.9776	0.4093	11.2079
France	0.0819	2.5459	-17.1084	20.5055	-0.0303	6.2283
UK	0.1061	4.0223	-33.4564	39.7358	0.2604	8.2221
Italy	0.1416	5.0380	-78.8558	66.9654	-0.2854	22.5886
Australia	0.0886	3.0370	-31.9904	29.2955	-0.2018	7.7735

This table presents the mean, standard deviation (Std), minimum (Min), maximum (Max), skewness (Skew), and kurtosis (Kurt) of daily logarithmic returns of indexes (S&P 500, DAX 40, TOPIX, IBEX 35, CAC 40, FTSE 250, FTSE MIB and S&P/ASX 200) and daily first differences of 10-year government bond yields for the US, Germany, Japan, Spain, France, the UK, Italy and Australia, covering a period from December 31, 1987 to April 23, 2025.

Table 3 Descriptive statistics of weekly returns

	Mean	Std	Min	Max	Skew	Kurt
<i>Indexes</i>						
S&P 500	0.0012	0.0238	-0.1645	0.1072	-0.8284	4.9897
DAX 40	0.0012	0.0314	-0.2123	0.1715	-0.7999	5.3488
TOPIX	0.0006	0.0284	-0.2028	0.1523	-0.5124	3.7937
IBEX 35	0.0004	0.0309	-0.1808	0.1391	-0.5110	3.1729
CAC 40	0.0006	0.0299	-0.2052	0.1663	-0.5567	5.7229
FTSE 250	0.0010	0.0250	-0.2874	0.1304	-1.3724	14.5653
FTSE MIB	0.0003	0.0324	-0.2022	0.1314	-0.6482	3.5789
S&P/ASX 200	0.0008	0.0202	-0.1450	0.1202	-0.7465	4.8200
<i>Bonds</i>						
US	0.2727	4.9142	-21.6070	27.5990	0.2514	2.6736
Germany	0.2913	4.6218	-40.4788	28.2915	-0.5981	6.9889
Japan	0.0006	0.0090	-0.0709	0.0411	-0.9418	7.1215
Spain	0.7165	10.4504	-139.7037	60.6283	-1.5432	26.062
France	0.4095	5.8267	-57.3813	30.3989	-1.0331	9.6817
UK	0.5301	8.9518	-63.9003	50.5319	-0.3063	5.6062
Italy	0.7076	11.6634	-143.3157	98.6107	-1.0448	23.0161
Australia	0.4430	6.4677	-48.3530	32.2921	-0.7574	6.0581

This table presents the mean, standard deviation (Std), minimum (Min), maximum (Max), skewness (Skew), and kurtosis (Kurt) of weekly logarithmic returns of indexes (S&P 500, DAX 40, TOPIX, IBEX 35, CAC 40, FTSE 250, FTSE MIB and S&P/ASX 200) and weekly first differences of 10-year government bond yields for the US, Germany, Japan, Spain, France, the UK, Italy and Australia, covering a period from December 31, 1987 to April 23, 2025.

Correlation patterns among the analyzed financial instruments for each country, at daily and weekly intervals, are visualized in Figure 2. Looking at the upper-left quadrant of each matrix, the correlation analysis reveals a highly integrated global equity market, with strong positive correlations, particularly across the US and the European economies. Japan and Australia's indexes are notable for their positive but relatively low correlations with the other indexes. A comparable trend is observed in government bonds returns, shown in the lower-right quadrant of each matrix, displaying similar cross-country patterns but with diminished correlation strength.

Conversely, a significant negative correlation is observed between the equity indexes and the government bonds, visualized in the upper-right and lower-left quadrants, highlighting their role as diversifiers in a portfolio.

When comparing the two matrices, it becomes evident that weekly data exhibit stronger and more clearly defined correlations, especially among assets within the same asset class.

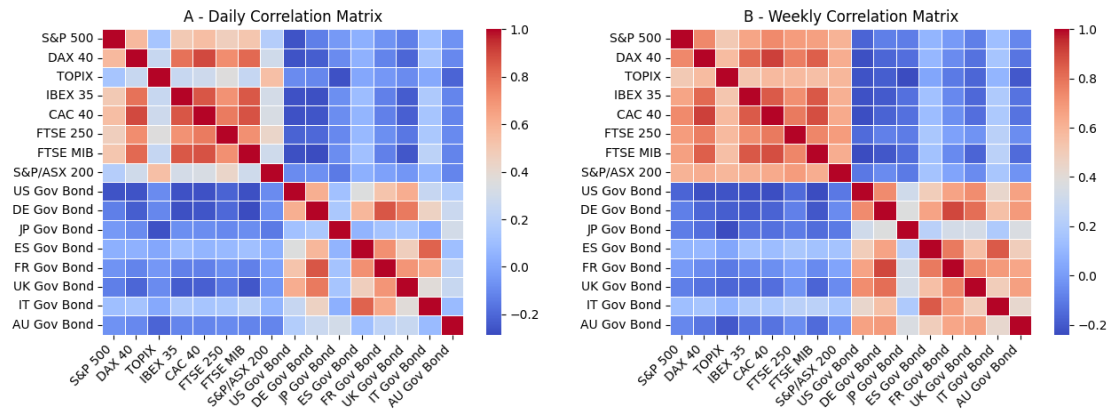


Figure 2 Correlation maps. This figure shows the correlation heat maps between the equity index returns and government bonds returns of eight countries for daily data (A) and weekly data (B). The countries are the US, Germany, Japan, Spain, France, the UK, Italy and Australia. The equity indexes are the S&P 500, DAX 40, TOPIX, IBEX 35, CAC 40, FTSE 250, FTSE MIB, S&P/ASX 200 and the government bonds are the 10-year government bond yield for each of the eight countries. The correlation scales, displayed on the right side of each panel, map colors to values, with dark blue denoting a correlation of -0.2 and dark red denoting a correlation of 1.

4. METHODOLOGY

In this section, we will present the methodologies chosen with the purpose of analyzing the lead-lag effects between each financial instrument for each country. The analysis will begin with the estimation of a bivariate VAR model of order five, VAR(5), for daily data, and of order one, VAR(1), for weekly data. Additionally, the analysis is also performed separately for the periods before and after the 2008 financial crisis. For this purpose, VAR models of order two, VAR(2), are estimated for the pre-crisis period and VAR models of order one, VAR(1), are estimated for the post-crisis period. The lag order selection was proposed by the Akaike Information Criterion (AIC). With the estimated bivariate VAR models, pairwise Granger causality and feedback measures are computed for both returns and volatility.

4.1. Granger Causality

Granger causality tests (Granger, 1969) were initially used to identify lead-lag relationships. Considering two time series of returns x_t and y_t with dynamics following a bivariate VAR(1),

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{x,t} \\ \varepsilon_{y,t} \end{bmatrix} \quad (1)$$

Where $\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$ is the coefficient matrix.

In the presented bivariate VAR, y does not Granger-cause x if $\phi_{12} = 0$, meaning the coefficient matrix Φ is lower triangular. In the same way, x does not Granger-cause y if $\phi_{21} = 0$, meaning the coefficient matrix Φ is upper triangular. The coefficient matrix Φ becomes diagonal when there is no Granger-causality in either direction, reducing VAR(1) to

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} \phi_{11} & 0 \\ 0 & \phi_{22} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \xi_{x,t} \\ \xi_{y,t} \end{bmatrix} \quad (2)$$

In this situation, to evaluate if y Granger-causes x , it is necessary to compute the residual sum of squares of the regression of x_t on x_{t-1} , RSS_0 , the residual sum of squares of x_t on x_{t-1} and y_{t-1} , RSS_1 , to then be able to compute

$$S = \frac{(RSS_1 - RSS_0)/p}{RSS_0/(T - 2p - 1)} \quad (3)$$

Which results in

$$S = \frac{(RSS_1 - RSS_0)}{RSS_0/(T - 3)} \quad (4)$$

With p being the lag length, equal to 1 in this case. Hence, S is a test that follows an F distribution with 1 and $T - 3$ degrees of freedom, $F(1, T - 3)$. The Granger-causality from x to y was assessed using the same approach.

4.2. Geweke Measures of Feedback

Using the residuals of standard Granger causality tests, Geweke (1982) proposed a measure to capture instantaneous feedback. That is, considering the previous time series, linear feedback between x and y , as well as between y and x , can be estimated using the variance-covariance matrix of residuals from the VAR(1) estimation:

$$\Omega = Cov \begin{bmatrix} \varepsilon_{x,t} \\ \varepsilon_{y,t} \end{bmatrix} = \begin{bmatrix} \sigma_x^2 & \sigma_{x,y}^2 \\ \sigma_{x,y}^2 & \sigma_y^2 \end{bmatrix} \quad (5)$$

Hence, the measure of lagged feedback from x to y is given by:

$$F_{x \rightarrow y} = \ln \left(\frac{\sigma_{\xi_y}^2}{\sigma_{\varepsilon_y}^2} \right) \quad (6)$$

Measure of lagged feedback from y to x :

$$F_{y \rightarrow x} = \ln \left(\frac{\sigma_{\xi_x}^2}{\sigma_{\varepsilon_x}^2} \right) \quad (7)$$

Measure of contemporaneous feedback between x and y :

$$F_{x \leftrightarrow y} = \ln \left(\frac{\sigma_{\varepsilon_x}^2 \sigma_{\varepsilon_y}^2}{|\Omega|} \right) \quad (8)$$

Measure of total feedback (total linear dependence) between x and y :

$$F_{x,y} = \ln \left(\frac{\sigma_{\xi_x}^2 \sigma_{\xi_y}^2}{|\Omega|} \right) \quad (9)$$

$F_{x,y}$ also equals $F_{x \rightarrow y} + F_{y \rightarrow x} + F_{x \leftrightarrow y}$ and T is the number of observations.

Under the null hypothesis,

$$T\hat{F}_{x \rightarrow y} \sim \chi'^2(1, TF_{x \rightarrow y}), \quad (10)$$

$$T\hat{F}_{y \rightarrow x} \sim \chi'^2(1, TF_{y \rightarrow x}), \quad (11)$$

$$T\hat{F}_{x \leftrightarrow y} \sim \chi'^2(1, TF_{x \leftrightarrow y}), \quad (12)$$

$$T\hat{F}_{x,y} \sim \chi'^2(3, TF_{x,y}). \quad (13)$$

With $F_{x,y} = F_{x \rightarrow y} + F_{y \rightarrow x} + F_{x \leftrightarrow y}$.

That is, multiplying these measures by the total number of observations T results in asymptotically independent values that follow chi-squared distributions with degrees of freedom 1, 1, 1, and 3, respectively, with the lag-length p equal to 1. Because the feedback measures are only log-likelihood ratio statistics under the null hypotheses, their asymptotic distributions are well defined.

5. RESULTS

This section reports findings on causality and feedback relationships between returns of equity indexes and government bonds across eight major economies – the US, Germany, Japan, Spain, France, the UK, Italy and Australia – over the period from December 31, 1987 to April 23, 2025. Through bivariate VAR models, the analysis includes tests for causality and feedback measures in the mean, conducted using daily and weekly data, and in volatility, conducted using weekly data only. In addition, the analysis is also conducted for two subperiods, before and after the 2008 financial crisis. For simplicity, the split is set on January 1, 2008, with the pre-crisis period covering December 31, 1987, to December 31, 2007, and the post-crisis period covering January 1, 2008, to April 23, 2025.

5.1. Granger Causality in the Mean

This subsection examines the causal relationships between the returns of stock indexes and government bonds across the eight countries in the sample, using both daily and weekly data frequencies. To better understand the directional relationships, the analysis is divided into two parts: the first focusing on equity indexes and the second on government bonds.

5.1.1. Equity Index's Influence Across Markets

Table 4 reports the estimated coefficients and corresponding significance levels from the VAR(5) models for daily returns (Panel A) and the VAR(1) models for weekly returns (Panel B), focusing on the analyzed equity indexes. As expected, a higher number of statistically significant coefficients are observed in the daily return models presented in Panel A, highlighting the effectiveness of a higher frequency in detecting these relationships.

Doing an index-to-index analysis on Panel A, the S&P 500 and DAX 40 stand out as the most influential indexes, exhibiting highly significant Granger causality toward all other indexes in the sample. In every case, the null hypothesis of no causality is rejected at the 1% significance level, highlighting their leading role in information transmission across international markets. The CAC 40 and FTSE MIB indexes also reveal strong causality relationships toward seven out of the eight analyzed indexes. Conversely, the TOPIX equity index shows a weaker influence on the remaining indexes, revealing

significant causality with respect to only two of them: FTSE 250 and S&P/ASX 200 indexes. This pattern, although less pronounced, is also evident in the weekly index-to-index analysis presented in Panel B. In particular, the S&P 500 index stands out as the most influential index, while the TOPIX and S&P/ASX 200 indexes show no significant Granger causality toward any of the other indexes in the sample.

Turning to a stock-to-bond perspective on Panel A, it is notable that all indexes, except for TOPIX and S&P/ASX 200, reveal a significant causality toward the majority of the analyzed government bonds. In contrast, the TOPIX and S&P/ASX 200 indexes show significant causality only with the government bonds of Germany, Japan and Australia. The weekly data, presented in Panel B, suggests a consistent pattern, with, as expected, fewer significant relationships overall.

Overall, the results suggest a leading role of the US equity index, followed by an also significant influence of most of the European indexes, particularly regarding index-to-index relationships. In contrast, the Asia-Pacific indexes seem to have limited impact beyond their domestic markets.

5.1.2. Government Bonds' Influence Across Markets

Turning to the causal influence of government bonds, Table 5 reports the estimated coefficients and corresponding significance levels from the VAR(5) models for daily returns (Panel A) and the VAR(1) models for weekly returns (Panel B).

The results of the daily analysis, presented in Panel A, indicate that the government bonds of the US and Spain show causality to all indexes except the S&P 500 in the case of the US and the DAX 40 in the case of Spain. Meanwhile, the Italy bond show Granger causality toward all eight indexes, with six of these relationships significant at the 1% level. In terms of relationships between bonds, the US, Spain, France, the UK and Italy exhibit significant Granger causality with all other bonds in the sample. The only exception is the UK-Italy pair, where the Italian bond Granger causes the UK bond but not vice versa.

Interestingly, the bonds of the US, Germany, Japan and France do not Granger cause their respective equity indexes in either the daily or weekly analysis. This pattern, however, is only observed in the case of Italy from the perspective of the equity market,

where the Italian index FTSE MIB does not Granger cause the Italian bond in either frequency analysis.

The Japanese government bond exhibits the least influence on the other instruments in the sample. In the weekly analysis, presented in Panel B, it shows no Granger causality with any other bond or equity index. In the daily analysis, displayed in Panel A, its influence remains limited, showing Granger causality only with the bonds of Spain, Italy and Australia, and none with any equity index.

Contrary to the results obtained for equity indexes, where causality tests show similar significance regardless of asset class, the analysis on government bonds reveal an apparent divergence, in both daily and weekly perspectives, with more significant relationships found within bonds themselves rather than between bonds and equity indexes.

Table 4 Granger causality in the mean: coefficients from equity indexes

Panel A: Daily Returns

	S&P 500	DAX 40	TOPIX	IBEX 35	CAC 40	FTSE 250	FTSE MIB	S&P/ASX 200
<i>Indexes</i>								
S&P 500		8.00**	2.06	4.39**	5.31**	5.38**	4.70**	2.73
DAX 40	3.83**		1.65	1.81	4.19**	1.64	4.54**	1.57
TOPIX	17.35**	11.54**		9.77**	11.79**	8.49**	9.89**	15.12**
IBEX 35	3.28**	7.30**	2.72		7.88**	4.88*	1.51	6.67**
CAC 40	5.99**	52.67**	2.50	1.61		0.42	3.02*	6.25*
FTSE 250	6.81**	46.03**	7.37**	10.75**	8.75**		8.74**	8.07**
FTSE MIB	3.35**	19.15**	1.03	5.42*	0.56	0.89		1.07
S&P/ASX 200	22.47**	13.39**	9.54**	11.13**	13.14**	11.40**	10.82**	
<i>Bonds</i>								
US	4.41**	3.58	1.12	9.16**	11.25**	6.13*	12.03**	1.95
DE	13.51**	5.81**	3.16**	4.83**	7.83**	4.66**	6.78**	6.83**
JP	76.83**	60.58**	9.00**	63.90**	64.18**	28.55**	57.56**	7.13**
ES	3.14*	3.39*	0.81	21.54**	9.01**	4.41*	12.01**	1.38
FR	4.81**	7.66**	2.20	9.05**	10.40**	5.25**	8.30**	2.93
UK	3.41*	6.27**	1.33	3.43**	3.93**	7.36**	4.55**	2.37
IT	0.30	1.07	0.40	4.15*	0.77	0.61	2.41	0.69
AU	59.02**	60.70**	5.74*	64.99**	57.27**	20.79**	71.90**	3.57*

Panel B: Weekly Returns

	S&P 500	DAX 40	TOPIX	IBEX 35	CAC 40	FTSE 250	FTSE MIB	S&P/ASX 200
<i>Indexes</i>								
S&P 500		0.14	1.10	1.90	0.33	1.11	0.43	0.03
DAX 40	6.59*		0.02	0.12	0.36	0.01	0.46	0.00
TOPIX	23.29**	9.33**		2.78	9.69**	2.85	5.62*	0.03
IBEX 35	7.74**	0.21	0.23		0.00	0.23	0.18	0.00
CAC 40	5.53*	0.02	0.00	0.65		0.09	1.10	0.30
FTSE 250	22.68**	4.09*	0.70	0.09	1.73		0.24	0.09
FTSE MIB	1.36	0.94	0.35	0.45	0.68	0.37		0.00
S&P/ASX 200	65.45**	32.45**	1.19	15.91**	33.03**	29.87**	18.62**	
<i>Bonds</i>								
US	0.00	0.21	0.34	0.07	0.34	0.58	0.02	2.49
DE	9.21**	12.24**	8.48**	8.35**	14.75**	7.81**	11.60**	3.96*
JP	0.35	0.05	1.23	0.59	0.22	1.45	0.16	1.05
ES	4.34*	3.52	2.65	4.86*	5.88*	2.62	5.25*	0.92
FR	3.98*	8.20**	4.02*	4.63*	8.20**	4.16*	6.52*	0.69
UK	1.73	3.98*	2.99	1.26	3.42	0.59	2.38	0.91
IT	0.57	0.51	0.04	1.05	1.03	0.37	2.56	0.01
AU	0.03	0.18	0.50	0.12	0.44	0.03	0.12	0.01

This table displays the coefficients from the estimated VAR models and the corresponding Granger causality test significance. Panel A presents the outcomes for a bivariate VAR(5) using daily returns. Panel B presents the outcomes for a bivariate VAR(1) using weekly returns. The tests were conducted between the equity indexes (S&P 500, DAX 40, TOPIX, IBEX 35, CAC 40, FTSE 250, FTSE MIB and S&P/ASX 200) and all the assets in the sample: the equity indexes and the 10-year government bond yields for the US, Germany, Japan, Spain, France, the UK, Italy and Australia. The null hypothesis is that the equity index (column variable) does no Granger-cause the other asset (row variable). The data covers a period from December 31, 1987 to April 23, 2025. Numbers in bold indicate the rejection of the null hypothesis at the 5% level (one asterisk, “*”) or at the 1% (two asterisks, “**”).

Table 5 Granger causality in the mean: coefficients from government bonds

Panel A: Daily Returns

	US	DE	JP	ES	FR	UK	IT	AU
<i>Indexes</i>								
S&P 500	3.58	1.99	2.19	5.05**	3.68*	2.69*	4.91**	5.32**
DAX 40	11.32**	0.55	0.72	2.02	0.58	1.47	3.51*	2.75
TOPIX	2.55**	1.24**	1.52	2.72*	43.22**	1.32**	15.61**	17.59**
IBEX 35	10.71**	1.13	1.53	5.25*	1.29	1.53	3.06*	2.11
CAC 40	17.06**	0.88	1.31	4.14**	0.77	1.40	4.74**	2.93
FTSE 250	2.60*	3.68**	2.18	8.33**	2.43*	5.76**	9.60**	4.62**
FTSE MIB	19.14**	1.31	1.51	2.62*	1.87	2.98	4.77**	2.35
S&P/ASX 200	57.07**	30.43**	0.99	16.01**	8.23**	27.71**	20.45**	9.57**
<i>Bonds</i>								
US		4.22*	3.76	5.92**	5.56**	5.36**	20.97**	3.84*
DE	2.87**		1.39	9.92**	6.85**	28.05**	8.44**	1.99
JP	6.39**	3.93**		1.58**	3.12**	3.67**	1.15**	7.68**
ES	7.63**	37.22**	6.18**		26.55**	6.77**	11.33**	7.30**
FR	1.41**	1.16	1.59	33.38**		9.31**	36.56**	0.94
UK	2.24**	1.43	0.49	12.06**	4.97**		10.62**	2.37*
IT	11.28**	6.72**	5.40**	6.62**	11.76**	1.98		4.99**
AU	35.43**	23.34**	11.36**	10.45**	18.87**	20.87**	7.02**	

Panel B: Weekly Returns

	US	DE	JP	ES	FR	UK	IT	AU
<i>Indexes</i>								
S&P 500	3.80	1.27	0.83	4.05*	3.89*	3.87*	6.13*	0.07
DAX 40	0.14	2.21	1.58	0.33	0.26	0.21	0.10	0.83
TOPIX	15.23**	12.58**	3.68	4.09*	5.36*	10.20**	1.02	7.99**
IBEX 35	0.02	0.48	0.34	0.06	0.13	0.40	1.14	0.29
CAC 40	0.49	1.58	2.14	0.00	0.02	0.18	0.82	0.71
FTSE 250	0.32	0.90	0.48	2.26	0.40	0.02	0.32	0.89
FTSE MIB	0.08	1.83	1.35	0.09	0.14	0.02	0.15	0.60
S&P/ASX 200	0.40	0.20	0.81	1.47	0.02	0.39	4.68*	0.25
<i>Bonds</i>								
US		1.37	1.15	0.29	1.10	5.58*	0.44	0.10
DE	0.24		0.02	11.00**	6.16*	0.68	2.89	3.54
JP	31.77**	17.40**		13.49**	19.81**	13.73**	8.68**	4.54*
ES	0.02	3.19	0.01		0.13	0.25	0.61	0.51
FR	0.09	3.43	0.17	5.65*		0.11	7.11**	1.95
UK	0.03	1.92	1.73	0.09	0.71		1.40	14.10**
IT	0.43	3.02	0.03	0.37	0.16	0.00		0.58
AU	66.65**	51.00**	2.68	23.20**	39.10**	38.95**	14.25**	

This table displays the coefficients from the estimated VAR models and the corresponding Granger causality test significance. Panel A presents the outcomes for a bivariate VAR(5) using daily returns. Panel B presents the outcomes for a bivariate VAR(1) using weekly returns. The tests were conducted between the 10-year government bond yields for the US, Germany, Japan, Spain, France, the UK, Italy and Australia and all the assets in the sample: the equity indexes (S&P 500, DAX 40, TOPIX, IBEX 35, CAC 40, FTSE 250, FTSE MIB and S&P/ASX 200) and the 10-year government bond yields. The null hypothesis is that the government bond (column variable) does no Granger-cause the other asset (row variable). The data covers a period from December 31, 1987 to April 23, 2025. Numbers in bold indicate the rejection of the null hypothesis at the 5% level (one asterisk, “*”) or at the 1% (two asterisks, “**”).

5.2. *Granger Causality in Volatility*

This subsection examines the causal relationships between the weekly volatilities of equity indexes and government bonds across the eight analyzed countries. Due to the absence of intraday data, volatility is estimated only at the weekly level using daily returns. Specifically, volatility was calculated by applying a rolling window of returns with a 5-day window and the results were annualized by multiplying by the square root of 252, the number of trading days in a year.

Table 6 presents the estimated coefficients and corresponding significance level from the VAR(1) models for weekly volatilities. Panel A is focused on equity indexes and Panel B on government bonds.

From Panel A, the predominance of causal relationships between volatilities of equity indexes is evident, with almost all statistics showing a p-value lower than 1%. Conversely, there are relatively few significant causal relationships between the volatility of equity indexes and the one of government bonds. However, the S&P 500 and the S&P/ASX 200 indexes still stand out, showing more significant causal influence on bond market volatilities compared to the other indexes in the sample. In Panel B, the volatility of the US government bond Granger causes the volatilities of all other instruments in the sample at the 1% significance level. Meanwhile, Japan is the only country whose government bond volatility exhibits stronger causal relationships with equity indexes than with other government bonds. Notably, it does not Granger cause the volatility of any other government bond in the sample but significantly influences the volatilities of the equity indexes of the US, Japan, Spain and Italy.

Overall, the results indicate that causality in volatility is stronger between instruments within the same asset class, with the exception of Japan's government bond. US instruments, in particular, stand out by exhibiting consistently strong and significant causality in volatility across the markets in the sample. Compared to the granger causality tests in the mean, the weekly causal relationships are notably stronger in volatility.

Table 6 Granger causality in volatility

Panel A: Equity Indexes – Weekly Volatilities

	S&P 500	DAX 40	TOPIX	IBEX 35	CAC 40	FTSE 250	FTSE MIB	S&P/ASX 200
<i>Indexes</i>								
S&P 500		27.02**	24.68**	33.40**	52.24**	36.86**	42.20**	65.51**
DAX 40	1.09**		14.75**	10.38**	13.74**	2.77	3.26	42.49**
TOPIX	2.57**	2.08**		1.63**	2.19**	1.46**	1.97**	44.39**
IBEX 35	1.28**	35.84**	20.38**		29.54**	14.05**	33.49**	47.73**
CAC 40	1.54**	41.29**	17.98**	12.67**		5.67*	8.50**	50.16**
FTSE 250	1.54**	5.68*	16.59**	17.40**	12.59**		8.56**	81.69**
FTSE MIB	82.60**	18.94**	10.21**	4.40*	8.10**	9.68**		35.74**
S&P/ASX 200	4.75**	1.73**	8.20**	1.86**	2.51**	247.51**	2.24**	
<i>Bonds</i>								
US	18.20**	0.01	1.94	2.37	0.84	9.19**	8.94**	30.65**
DE	5.26*	3.34	0.69	0.88	1.15	2.15	0.44	11.17**
JP	2.27	5.35*	0.62	0.79	4.80*	0.92	0.34	0.29
ES	5.21*	0.33	0.02	0.02	0.01	2.89	7.95**	12.85**
FR	4.05*	1.88	0.00	1.16	0.82	2.62	0.98	8.75**
UK	4.00*	4.67*	1.61	0.49	1.91	1.23	0.61	11.14**
IT	0.67	1.80	0.49	3.81	1.94	0.05	0.64	4.46*
AU	36.37**	4.96*	0.49	14.84**	12.78**	30.42**	25.56**	19.13**

Panel B: Government Bonds – Weekly Volatilities

	US	DE	JP	ES	FR	UK	IT	AU
<i>Indexes</i>								
S&P 500	9.25**	2.76	8.09**	0.14	1.09	0.56	7.18**	3.83
DAX 40	7.02**	1.73	1.62	1.15	2.20	1.34	0.55	0.15
TOPIX	19.53**	5.11*	9.03**	0.00	0.00	7.67**	0.01	5.98*
IBEX 35	13.87**	0.58	6.29*	0.01	0.35	0.42	0.70	0.44
CAC 40	12.80**	1.24	2.54	0.37	1.42	1.12	0.01	0.06
FTSE 250	83.62**	12.63**	3.06	15.44**	6.88**	17.98**	16.48**	30.75**
FTSE MIB	21.53**	0.24	4.64*	1.67	0.62	1.04	2.14	6.07*
S&P/ASX 200	67.82**	17.49**	3.83	8.80**	6.67**	10.58**	15.64**	2.78
<i>Bonds</i>								
US		37.47**	0.00	59.51**	37.65**	30.42**	1.02**	36.33**
DE	1.46**		1.05	71.39**	31.37**	60.40**	94.46**	59.14**
JP	9.58**	1.80		0.32	0.62	0.00	0.62	1.03
ES	1.49**	69.11**	0.39		41.17**	82.46**	1.66**	97.49**
FR	1.14**	18.26**	0.72	44.52**		58.93**	83.61**	72.07**
UK	1.10**	26.37**	1.91	87.30**	43.26**		1.28**	63.30**
IT	73.91**	10.91**	1.15	9.56**	4.10*	15.36**		34.93**
AU	6.92**	3.30**	0.95	3.42**	2.95**	2.94**	3.47**	

This table displays the coefficients from the estimated VAR models and the corresponding Granger causality test significance. Panel A presents the outcomes for a bivariate VAR(1) using weekly volatilities between the equity indexes (S&P 500, DAX 40, TOPIX, IBEX 35, CAC 40, FTSE 250, FTSE MIB and S&P/ASX 200) and all the assets in the sample: the equity indexes and the 10-year government bond yields for the US, Germany, Japan, Spain, France, the UK, Italy and Australia. Panel B presents the same outcomes but instead of the equity indexes, the test is conducted between the 10-year government bond yields for the US, Germany, Japan, Spain, France, the UK, Italy and Australia and all the assets in the sample. The null hypothesis is that the column variable does no Granger-cause the row variable. The data covers a period from December 31, 1987 to April 23, 2025. Numbers in bold indicate the rejection of the null hypothesis at the 5% level (one asterisk, “*”) or at the 1% (two asterisks, “**”).

5.3. Geweke Feedback in the Mean

This subsection presents the lead-lag relationships between the returns of two major indexes, the S&P 500 and the DAX 40, and those of all other instruments in the sample. For reasons of parsimony, the relationships between the returns of the remaining instruments in the sample can be found in Appendix B.

Table 7 shows the estimated pairwise Geweke feedback measures obtained from the VAR(5) models for daily returns and the VAR(1) models for weekly returns. The results display, as expected, higher significant dependencies between daily returns than with weekly returns. Additionally, lagged feedback dominates in daily observations, while in weekly observations, contemporaneous feedback becomes more prominent. Nevertheless, contemporaneous feedback holds the highest weight in total feedback in both frequencies. In the daily analysis, it represents the highest portion of total feedback in 19 out of the 29 relationships, increasing to 27 out of the 29 in the weekly analysis. These findings indicate a strong degree of synchronization among financial instruments, particularly at lower frequency intervals, suggesting that a significant portion of the information transmission occurs within the same time interval rather than with delay.

The results show that the S&P 500 equity index, traditionally seen as a leader, exhibits a stronger lagged influence from other indexes than it exerts on them. This dynamic is even more pronounced for the DAX index. Notably, there is almost no lagged feedback from the weekly returns of these two major indexes to those of the other instruments.

Moreover, the DAX 40 index demonstrates high integration with European indexes, namely the IBEX 35, the CAC 40, the FTSE 250 and the FTSE MIB, where contemporaneous feedback reaches close to 100% of the total feedback in both frequencies.

In contrast, integration between equity indexes and government bonds is weaker. The S&P 500 index, for example, shows no overall significant relationship with the Australian bond on a weekly basis and both S&P 500 and DAX 40 indexes show no contemporaneous feedback between the weekly returns of the French bond. There exists significant feedback between these two major indexes and the analyzed government bonds, but these relationships are largely driven by contemporaneous rather than lagged effects.

Table 7 Geweke feedback measures in the mean for the S&P 500 and DAX 40 indexes

X1	X2	$F_{X1 \rightarrow X2}$	$F_{X2 \rightarrow X1}$	$F_{X1 \leftrightarrow X2}$	$F_{X1, X2}$	$F_{X1 \rightarrow X2}$	$F_{X2 \rightarrow X1}$	$F_{X1 \leftrightarrow X2}$	$F_{X1, X2}$
Daily Frequency					Weekly Frequency				
S&P 500	DAX 40	0.0052** 1.0%	0.0571** 10.8%	0.4660** 88.2%	0.5284**	0.0001 0.0%	0.0046* 0.6%	0.8093** 99.4%	0.8140**
S&P 500	TOPIX	0.0011 0.4%	0.2313** 84.7%	0.0408** 14.9%	0.2732**	0.0008 0.2%	0.0162** 4.9%	0.3162** 94.9%	0.3332**
S&P 500	IBEX 35	0.0031** 0.8%	0.0481** 12.6%	0.3297** 86.6%	0.3809**	0.0013 0.2%	0.0054** 1.0%	0.5578** 98.8%	0.5645**
S&P 500	CAC 40	0.0037** 0.7%	0.0873** 16.3%	0.4437** 83.0%	0.5347**	0.0002 0.0%	0.0039* 0.5%	0.8073** 99.5%	0.8114**
S&P 500	FTSE 250	0.0038** 0.9%	0.0994** 23.3%	0.3232** 75.8%	0.4264**	0.0008 0.1%	0.0158** 2.3%	0.6578** 97.5%	0.6745**
S&P 500	FTSE MIB	0.0033** 0.8%	0.0491** 12.3%	0.3482** 86.9%	0.4005**	0.0003 0.1%	0.0010 0.2%	0.5996** 99.8%	0.6009**
S&P 500	S&P ASX 200	0.0009 0.2%	0.3241** 80.8%	0.0759** 18.9%	0.4009**	0.0000 0.0%	0.0450** 8.3%	0.5003** 91.7%	0.5453**
S&P 500	US Bond	0.0012 1.7%	0.0025** 3.4%	0.0707** 94.9%	0.0744**	0.0027 8.6%	0.0000 0.0%	0.0282** 91.4%	0.0309**
S&P 500	DE Bond	0.0006 2.6%	0.0052** 20.8%	0.0193** 76.7%	0.0252**	0.0009 5.7%	0.0065** 40.9%	0.0084** 53.4%	0.0158**
S&P 500	JP Bond	0.0012 7.9%	0.0109** 70.3%	0.0034** 21.8%	0.0156**	0.0006 5.8%	0.0002 2.4%	0.0094** 91.8%	0.0102**
S&P 500	ES Bond	0.0020* 40.8%	0.0011** 22.1%	0.0018* 37.1%	0.0049**	0.0028* 18.8%	0.0030* 20.1%	0.0093** 61.2%	0.0152**
S&P 500	FR Bond	0.0013 16.5%	0.0021** 27.5%	0.0043** 56.1%	0.0076**	0.0027* 49.4%	0.0028* 50.5%	0.0000 0.0%	0.0055*
S&P 500	UK Bond	0.0013 6.0%	0.0015** 7.1%	0.0188** 86.8%	0.0216**	0.0027* 22.7%	0.0012 10.1%	0.0080** 67.2%	0.0120**
S&P 500	IT Bond	0.0034** 22.8%	0.0003 1.7%	0.0114** 75.5%	0.0151**	0.0043* 17.6%	0.0004 1.6%	0.0198** 80.8%	0.0245**
S&P 500	AU Bond	0.0037** 19.3%	0.0090** 46.7%	0.0066** 34.0%	0.0194**	0.0000 1.0%	0.0000 0.5%	0.0043* 98.5%	0.0044
DAX 40	TOPIX	0.0003 0.1%	0.1558** 64.9%	0.0839** 35.0%	0.2401**	0.0000 0.0%	0.0065** 1.6%	0.4094** 98.4%	0.4159**
DAX 40	IBEX 35	0.0013 0.1%	0.0021** 0.2%	0.9992** 99.7%	1.0025**	0.0001 0.0%	0.0001 0.0%	1.1273** 100.0%	1.1275**
DAX 40	CAC 40	0.0029** 0.2%	0.0096** 0.6%	1.6021** 99.2%	1.6147**	0.0002 0.0%	0.0000 0.0%	1.8440** 100.0%	1.8443**
DAX 40	FTSE 250	0.0012 0.2%	0.0086** 1.1%	0.7460** 98.7%	0.7558**	0.0000 0.0%	0.0029* 0.3%	0.9090** 99.7%	0.9119**
DAX 40	FTSE MIB	0.0032** 0.3%	0.0040** 0.4%	1.0994** 99.4%	1.1065**	0.0003 0.0%	0.0007 0.1%	1.2777** 99.9%	1.2787**
DAX 40	S&P ASX 200	0.0003 0.1%	0.1877** 62.6%	0.1118** 37.3%	0.2998**	0.0000 0.0%	0.0226** 4.1%	0.5314** 95.9%	0.5539**
DAX 40	US Bond	0.0019* 2.5%	0.0008* 1.1%	0.0709** 96.3%	0.0736**	0.0001 0.2%	0.0002 0.3%	0.0532** 99.5%	0.0534**
DAX 40	DE Bond	0.0006 0.9%	0.0041** 6.7%	0.0563** 92.4%	0.0609**	0.0016 4.3%	0.0086** 23.9%	0.0257** 71.7%	0.0358**
DAX 40	JP Bond	0.0005 2.7%	0.0097** 52.1%	0.0085** 45.3%	0.0187**	0.0011 5.1%	0.0000 0.2%	0.0208** 94.8%	0.0219**
DAX 40	ES Bond	0.0014 31.7%	0.0010** 23.1%	0.0020* 45.3%	0.0045**	0.0002 1.8%	0.0025 18.7%	0.0105** 79.5%	0.0132**
DAX 40	FR Bond	0.0004 2.3%	0.0037** 20.3%	0.0140** 77.5%	0.0181**	0.0002 2.2%	0.0058** 71.0%	0.0022 26.8%	0.0081**
DAX 40	UK Bond	0.0006 1.3%	0.0026** 5.3%	0.0450** 93.4%	0.0482**	0.0001 0.8%	0.0028* 14.6%	0.0162** 84.6%	0.0191**
DAX 40	IT Bond	0.0016* 9.5%	0.0004 2.2%	0.0150** 88.2%	0.0170**	0.0001 0.2%	0.0004 1.3%	0.0275** 98.5%	0.0279**
DAX 40	AU Bond	0.0010 4.6%	0.0104** 46.1%	0.0111** 49.4%	0.0225**	0.0006 3.8%	0.0001 0.8%	0.0145** 95.3%	0.0153**

This table displays the Geweke feedback measures obtained from a bivariate VAR(5) for daily returns and a bivariate VAR(1) for weekly returns. Pairwise tests are performed between the S&P 500 index and DAX 40 index returns and those of all other instruments in the sample: the equity indexes (S&P 500, DAX 40, TOPIX, IBEX 35, CAC 40, FTSE 250, FTSE MIB and S&P/ASX 200) and the 10-year government bond yields for the US, Germany, Japan, Spain, France, the UK, Italy and Australia. The measure $F_{X1 \rightarrow X2}$ reflects

Table 7 (continued)

the lagged feedback from S&P 500/DAX 40 index (X1 variable) and the other instrument (X2 variable). The measure $F_{X2 \rightarrow X1}$ reflects the lagged feedback from each instrument (X2 variable) and the S&P 500/DAX 40 index (X1 variable). The measure $F_{X1 \leftrightarrow X2}$ reflects the contemporaneous feedback. The measure $F_{X1, X2}$ reflects the total feedback. Percentages in italic display the weight of each feedback component to the total feedback. The data covers a period from December 31, 1987 to April 23, 2025. Numbers in bold indicate the rejection of the null hypothesis at the 5% level (one asterisk, “*”) or at the 1% (two asterisks, “**”).

5.4. Geweke Feedback in Volatility

This subsection presents the lead-lag relationships between the weekly volatilities of two major indexes, S&P 500 and DAX 40, and those of all other instruments in the sample. The relationships between the volatilities of the remaining instruments in the sample can be found in Appendix C.

Table 8 presents the estimated pairwise Geweke feedback measures obtained from the VAR(1) models for weekly volatilities. Consistent with the findings based on returns, contemporaneous feedback accounts for the largest share of the total feedback in 23 out of the 29 pairs of instruments.

Contrary to the findings based on returns, the results show significant bidirectional lagged feedback between the volatility of the S&P 500 index and all other indexes. Additionally, the TOPIX and the S&P/ASX 200 indexes demonstrate stronger unidirectional volatility feedback on the S&P 500 index, reflecting external volatility spillovers from Asia-Pacific into the US market. In contrast, the relationships between the S&P 500 index and the government bonds are comparatively weaker, with low or insignificant feedback in both directions. The DAX 40 index, as concluded in the analysis between returns, displays particular strong integration between the European indexes, with high percentages of contemporaneous feedback over all the feedback.

The findings suggest that the equity markets are highly interconnected in volatility, while bond markets show weaker influence on equity volatility. These results underscore the importance of taking into account movements that occur in stock markets when formulating investments decisions across both bond and stock markets.

Table 8 Geweke feedback measures in volatility for the S&P 500 and DAX 40 indexes

X1	X2	$F_{X1 \rightarrow X2}$	$F_{X2 \rightarrow X1}$	$F_{X1 \leftrightarrow X2}$	$F_{X1, X2}$
S&P 500	DAX 40	0.0038** 1.9%	0.0153** 7.5%	0.1848** 90.7%	0.2039**
S&P 500	TOPIX	0.0035** 7.7%	0.0355** 78.5%	0.0063** 13.8%	0.0452**
S&P 500	IBEX 35	0.0047** 3.5%	0.0179** 13.3%	0.1125** 83.3%	0.1352**
S&P 500	CAC 40	0.0073** 4.1%	0.0215** 12.0%	0.1501** 83.9%	0.1789**
S&P 500	FTSE 250	0.0052** 4.4%	0.0215** 18.2%	0.0917** 77.4%	0.1184**
S&P 500	FTSE MIB	0.0059** 4.1%	0.0115** 8.0%	0.1268** 87.9%	0.1442**
S&P 500	S&P ASX 200	0.0092** 10.9%	0.0647** 76.8%	0.0104** 12.3%	0.0842**
S&P 500	US Bond	0.0013** 2.9%	0.0026** 5.7%	0.0406** 91.3%	0.0445**
S&P 500	DE Bond	0.0004 3.7%	0.0007* 7.0%	0.0095** 89.4%	0.0106**
S&P 500	JP Bond	0.0011** 48.4%	0.0003 13.6%	0.0009* 38.0%	0.0023**
S&P 500	ES Bond	0.0000 0.3%	0.0007* 10.9%	0.0060** 88.8%	0.0067**
S&P 500	FR Bond	0.0002 3.2%	0.0006* 11.8%	0.0041** 85.0%	0.0048**
S&P 500	UK Bond	0.0001 1.0%	0.0006* 6.9%	0.0075** 92.1%	0.0082**
S&P 500	IT Bond	0.0010** 10.8%	0.0001 1.0%	0.0083** 88.2%	0.0094**
S&P 500	AU Bond	0.0005 5.9%	0.0051** 55.6%	0.0035** 38.5%	0.0092**
DAX 40	TOPIX	0.0021** 4.6%	0.0289** 64.4%	0.0139** 31.0%	0.0449**
DAX 40	IBEX 35	0.0015** 0.2%	0.0050** 0.8%	0.5941** 98.9%	0.6006**
DAX 40	CAC 40	0.0019** 0.2%	0.0058** 0.5%	1.0499** 99.3%	1.0576**
DAX 40	FTSE 250	0.0004 0.1%	0.0008* 0.2%	0.3673** 99.7%	0.3685**
DAX 40	FTSE MIB	0.0005 0.1%	0.0027** 0.4%	0.6874** 99.5%	0.6905**
DAX 40	S&P ASX 200	0.0060** 12.4%	0.0240** 49.9%	0.0181** 37.7%	0.0481**
DAX 40	US Bond	0.0010** 3.3%	0.0000 0.0%	0.0285** 96.7%	0.0295**
DAX 40	DE Bond	0.0002 0.5%	0.0005 1.0%	0.0483** 98.5%	0.0490**
DAX 40	JP Bond	0.0002 10.0%	0.0008* 32.8%	0.0013** 57.2%	0.0023**
DAX 40	ES Bond	0.0002 1.2%	0.0000 0.4%	0.0127** 98.4%	0.0129**
DAX 40	FR Bond	0.0003 1.3%	0.0003 1.1%	0.0224** 97.5%	0.0230**
DAX 40	UK Bond	0.0002 0.6%	0.0007* 2.0%	0.0316** 97.4%	0.0325**
DAX 40	IT Bond	0.0001 0.4%	0.0003 1.3%	0.0191** 98.3%	0.0194**
DAX 40	AU Bond	0.0000 0.4%	0.0007* 11.3%	0.0054** 88.3%	0.0061**

This table displays the Geweke feedback measures obtained from a bivariate VAR(1) using weekly volatilities. Pairwise tests are performed between the S&P 500 index and DAX 40 index volatilities and those of all other instruments in the

Table 8 (continued)

sample: the equity indexes (S&P 500, DAX 40, TOPIX, IBEX 35, CAC 40, FTSE 250, FTSE MIB and S&P/ASX 200) and the 10-year government bond yields for the US, Germany, Japan, Spain, France, the UK, Italy and Australia. The measure $F_{X1 \rightarrow X2}$ reflects the lagged feedback from S&P 500/DAX 40 index (X1 variable) and the other instrument (X2 variable). The measure $F_{X2 \rightarrow X1}$ reflects the lagged feedback from each instrument (X2 variable) and the S&P 500/DAX 40 index (X1 variable). The measure $F_{X1 \leftrightarrow X2}$ reflects the contemporaneous feedback. The measure $F_{X1, X2}$ reflects the total feedback. Percentages in italic display the weight of each feedback component to the total feedback. The data covers a period from December 31, 1987 to April 23, 2025. Numbers in bold indicate the rejection of the null hypothesis at the 5% level (one asterisk, “*”) or at the 1% (two asterisks, “**”).

5.5. Causality and Feedback in the Mean – Analysis Before and After the 2008 Financial Crisis

This subsection presents the causal and feedback analysis done in the previous sections, now applied separately to two subperiods: before and after the 2008 financial crisis. The pre-crisis period covers December 31, 1987 to December 31, 2007 while the post-crisis period covers January 1, 2008 to April 23, 2025. The analysis is conducted at weekly frequency and is focused on returns. Table 9 and Table 10 report the estimated coefficients and corresponding significance levels from the VAR(2) models for the period before the 2008 financial crisis (Panel A) and VAR(1) models for the period after the 2008 financial crisis (Panel B) for equity indexes and government bonds, respectively.

The results of Table 9 show an increased equity market interdependence after the crisis, with an increase in significant Granger causality coefficients between the equity indexes in the sample. Notably, there was a shift in the equity market influence between the two periods, with the FTSE 250 and CAC 40 indexes showing the strongest causal relationships before the 2008 financial crisis and the S&P 500 index dominating in the period post-crisis. This suggests an equity market post-crisis mainly led by the US. The results also reveal a shift in equity-to-bond transmissions following the crisis, with the S&P 500 and DAX 40 indexes showing influence on government bonds that previously were nonexistent. In contrast, the FTSE MIB index experienced a decline in its influence on bond markets.

A notable change is observed in the case of the German government bond. Before the crisis, it was only significantly influenced by the CAC 40 and FTSE MIB indexes.

However, in the post-crisis period, it becomes significantly Granger caused by nearly all equity indexes in the sample, with the exception of the S&P/ASX 200 index.

Panel A of Table 10 shows that, prior to the 2008 financial crisis, the influence of all the government bonds remained largely contained within bond markets as there was almost no evidence that returns of government bonds influenced those of equity indexes. As for relationships between bonds, there is a leading role of the US government bond, showing significant Granger causality towards the returns of all other bonds in the sample at the 1% significance level. The period following the financial crisis, presented in Panel B, marks the emergence of significant influence from five out of the eight government bonds in the sample on the TOPIX equity index, indicating a higher integration of the Japanese equity market into global financial dynamics. In addition, the US bond no longer holds its pre-crisis leadership role, as it significantly affected only the Japanese and Australian government bonds in the post-crisis period.

Table 11 shows the estimated pairwise Geweke feedback measures obtained from the VAR(2) models for the period before the 2008 financial crisis and VAR(1) models for the period after the 2008 financial crisis. It presents the lead-lag relationships between the weekly volatilities of two major indexes, the S&P 500 and the DAX 40, and those of all other instruments in the sample for the two periods.

The results show a decline in significant lagged feedback from the S&P 500 index towards the other instruments, indicating a reduction of its role as a leading information transmitter. On the other hand, the DAX 40 index experienced an increase in significant lagged feedback from the other instruments. Furthermore, the relationships between both the S&P 500 and DAX 40 indexes and the government bonds weakened. Notably, the relationships between the S&P 500 index and the Japanese, French and Australian bonds, as well as between the DAX 40 index and the French government bond ceased to be statistically significant in the post-crisis period. Consistent with the previous feedback analysis with the full dataset sample, the contemporaneous component dominates the feedback structure, as in the pre-crisis period its weight on total feedback ranged from 73.7% to 99.4%. Additionally, the return feedback is stronger between indexes for both periods, consistent with the findings from feedback measures in volatility using the full period.

Table 9 Granger causality in the mean: coefficients from equity indexes before and after the 2008 financial crisis

Panel A: Before the 2008 Financial Crisis - Weekly Returns

	S&P 500	DAX 40	TOPIX	IBEX 35	CAC 40	FTSE 250	FTSE MIB	S&P/ASX 200
<i>Indexes</i>								
S&P 500		1.12	1.67	4.03*	5.29**	2.02	1.14	0.12
DAX 40	3.60		0.48	1.35	3.35*	5.22*	4.05*	0.92
TOPIX	3.40	0.10		0.17	1.45	0.24	1.11	0.27
IBEX 35	2.22	3.21*	0.31		0.54	5.55*	1.01	1.93
CAC 40	0.79	0.52	0.43	2.47		4.82*	1.67	1.80
FTSE 250	2.04	0.00	1.04	0.95	0.83		0.71	0.59
FTSE MIB	0.29	3.14	0.04	4.56*	2.74	1.98		1.44
S&P/ASX 200	14.03**	6.21*	0.30	2.18	7.64**	10.46**	4.99*	
<i>Bonds</i>								
US	0.07	0.15	0.86	1.39	0.69	0.54	0.40	0.29
DE	0.66	1.66	0.99	3.14	4.31*	0.54	5.97*	1.66
JP	0.43	0.13	4.93**	0.17	0.73	0.69	0.18	0.05
ES	1.26	2.22	1.17	3.51	5.74*	1.12	6.17*	1.12
FR	0.89	1.75	0.80	3.47	4.17*	0.56	6.09*	1.34
UK	0.51	1.12	0.23	1.85	2.23	0.81	3.59	2.38
IT	1.17	1.86	1.03	4.33*	5.76*	1.05	6.34*	1.33
AU	0.24	0.54	0.39	0.16	1.26	0.30	1.02	0.13

Panel B: After the 2008 Financial Crisis – Weekly Returns

	S&P 500	DAX 40	TOPIX	IBEX 35	CAC 40	FTSE 250	FTSE MIB	S&P/ASX 200
<i>Indexes</i>								
S&P 500		2.82	0.31	0.46	1.21	0.01	0.18	0.00
DAX 40	2.41		1.46	0.39	2.38	8.11**	1.89	1.39
TOPIX	22.11**	15.82**		4.20*	10.17**	3.26	5.31*	0.00
IBEX 35	5.54*	0.01	1.30		0.01	1.26	0.15	0.05
CAC 40	4.26*	0.25	1.17	0.13		5.15*	0.21	0.09
FTSE 250	26.07**	6.17*	0.14	0.47	4.21*		0.44	0.83
FTSE MIB	3.05	0.05	0.53	0.04	0.09	2.49		0.01
S&P/ASX 200	52.47**	31.78**	0.86	14.15**	28.32**	19.59**	13.60**	
<i>Bonds</i>								
US	0.00	0.22	0.20	0.08	0.17	0.45	0.00	2.79
DE	8.45**	11.82**	7.30**	6.25*	12.15**	6.38*	7.79**	2.57
JP	1.06	0.50	0.17	1.02	2.32	1.61	0.74	1.21
ES	3.64	3.06	2.07	3.77	4.53*	2.00	3.51	0.59
FR	3.10	7.15**	3.34	2.85	5.81*	3.09	3.66	0.27
UK	1.27	3.27	2.57	0.60	2.24	0.25	1.14	0.33
IT	0.38	0.33	0.01	0.59	0.52	0.49	1.59	0.04
AU	0.01	0.27	0.69	0.08	0.41	0.02	0.03	0.03

This table displays the coefficients from the estimated VAR models and the corresponding Granger causality test significance. Panel A presents the outcomes for a bivariate VAR(2) using weekly returns between the equity indexes (S&P 500, DAX 40, TOPIX, IBEX 35, CAC 40, FTSE 250, FTSE MIB and S&P/ASX 200) and all the assets in the sample (the equity indexes and the 10-year government bond yields for the US, Germany, Japan, Spain, France, the UK, Italy and Australia) for the period before the 2008 financial crisis. Panel B presents the outcomes for a bivariate VAR(1) using weekly returns between the same instruments as Panel A but instead for the period after the 2008 financial crisis. The null hypothesis is that the equity index (column variable) does not Granger-cause the other asset (row variable). The pre-crisis period covers December 31, 1987 to December 31, 2007 while the post-crisis period covers January 1, 2008 to April 23, 2025. Numbers in bold indicate the rejection of the null hypothesis at the 5% level (one asterisk, “*”) or at the 1% (two asterisks, “**”).

Table 10 Granger causality in the mean: coefficients from government bonds before and after the 2008 financial crisis

Panel A: Before the 2008 Financial Crisis - Weekly Returns

	US	DE	JP	ES	FR	UK	IT	AU
<i>Indexes</i>								
S&P 500	2.57	2.27	1.70	3.13	4.39*	3.09	3.52	2.40
DAX 40	0.81	0.99	1.99	0.55	0.15	0.09	0.61	1.06
TOPIX	1.22	1.12	0.39	2.05	0.29	2.16	1.25	0.57
IBEX 35	0.39	1.19	0.88	1.45	2.27	3.17	2.34	0.90
CAC 40	0.23	0.03	2.46	0.13	0.47	0.53	0.05	0.60
FTSE 250	0.48	2.15	1.05	1.93	3.69	5.46*	2.43	3.57
FTSE MIB	0.04	0.19	1.17	0.37	0.77	1.26	0.27	0.63
S&P/ASX 200	0.29	0.05	2.26	0.05	0.16	0.36	0.04	0.66
<i>Bonds</i>								
US		0.36	3.65*	0.40	0.28	0.45	0.20	0.64
DE	9.20**		3.51*	3.56	0.52	0.32	0.25	6.11*
JP	15.96**	8.22**		7.49**	9.64**	3.75	6.77**	7.76**
ES	7.12**	0.76	2.81		1.30	0.20	0.37	3.79
FR	12.76**	5.46**	4.58*	7.38**		1.34	5.85**	9.45**
UK	5.35**	0.42	2.09	1.37	0.07		0.72	2.80
IT	9.45**	0.32	2.69	4.10*	0.40	0.07		5.40*
AU	19.28**	14.04**	5.97**	14.03**	9.32**	17.27**	11.50**	

Panel B: After the 2008 Financial Crisis – Weekly Returns

	US	DE	JP	ES	FR	UK	IT	AU
<i>Indexes</i>								
S&P 500	2.48	0.56	0.23	2.23	1.83	2.55	3.78	0.03
DAX 40	0.00	1.50	0.11	0.56	0.28	0.19	0.01	0.99
TOPIX	13.41**	11.20**	3.80	2.91	4.79*	8.67**	0.59	7.44**
IBEX 35	0.01	0.79	0.00	0.01	0.00	0.07	0.56	0.57
CAC 40	0.37	1.63	0.45	0.37	0.06	0.32	0.12	0.90
FTSE 250	0.29	1.22	0.40	1.48	0.54	0.01	0.06	1.43
FTSE MIB	0.08	2.12	0.40	0.14	0.32	0.01	0.11	0.90
S&P/ASX 200	0.12	0.12	0.01	1.29	0.00	0.25	4.00*	0.25
<i>Bonds</i>								
US		1.00	0.07	0.19	0.78	4.53*	0.32	0.10
DE	0.76		0.62	7.34**	4.58*	0.49	1.96	3.51
JP	23.93**	15.33**		12.31**	16.99**	13.91**	7.81**	2.51
ES	0.12	2.21	0.09		0.10	0.15	0.42	0.53
FR	0.03	2.94	1.09	3.41		0.04	4.59*	2.27
UK	0.03	1.47	4.06*	0.04	0.49		1.00	11.50**
IT	0.61	2.21	0.23	0.31	0.12	0.00		0.59
AU	43.27**	32.87**	1.28	14.30**	25.33**	24.16**	8.76**	

This table displays the coefficients from the estimated VAR models and the corresponding Granger causality test significance. Panel A presents the outcomes for a bivariate VAR(2) using weekly returns between the 10-year government bond yields for the US, Germany, Japan, Spain, France, the UK, Italy and Australia and all the assets in the sample (the equity indexes (S&P 500, DAX 40, TOPIX, IBEX 35, CAC 40, FTSE 250, FTSE MIB and S&P/ASX 200) and the 10-year government bond yields) for the period before the 2008 financial crisis. Panel B presents the outcomes for a bivariate VAR(1) using weekly returns between the same instruments as Panel A but instead for the period after the 2008 financial crisis. The null hypothesis is that the government bond (column variable) does not Granger-cause the other asset (row variable). The pre-crisis period covers December 31, 1987 to December 31, 2007 while the post-crisis period covers January 1, 2008 to April 23, 2025. Numbers in bold indicate the rejection of the null hypothesis at the 5% level (one asterisk, “*”) or at the 1% (two asterisks, “**”).

Table 11 Geweke feedback measures in the mean before and after the 2008 financial crisis

X1	X2	$F_{X1 \rightarrow X2}$	$F_{X2 \rightarrow X1}$	$F_{X1 \leftrightarrow X2}$	$F_{X1, X2}$	$F_{X1 \rightarrow X2}$	$F_{X2 \rightarrow X1}$	$F_{X1 \leftrightarrow X2}$	$F_{X1, X2}$
<i>Before the 2008 Financial Crisis</i>					<i>After the 2008 Financial Crisis</i>				
S&P 500	DAX 40	0.0043 0.5%	0.0076* 0.9%	0.8212** 98.6%	0.8331**	0.0031 0.4%	0.0027 0.3%	0.8194** 99.3%	0.8252**
S&P 500	TOPIX	0.0034 1.7%	0.0072 3.5%	0.1925** 94.8%	0.2030**	0.0003 0.1%	0.0243** 5.7%	0.4017** 94.2%	0.4264**
S&P 500	IBEX 35	0.0155* 2.7%	0.0086* 1.5%	0.5406** 95.7%	0.5647**	0.0005 0.1%	0.0061* 1.1%	0.5664** 98.8%	0.5731**
S&P 500	CAC 40	0.0203** 2.6%	0.0017 0.2%	0.7740** 97.2%	0.7961**	0.0013 0.2%	0.0047* 0.6%	0.8370** 99.3%	0.8430**
S&P 500	FTSE 250	0.0078 1.5%	0.0079* 1.5%	0.5039** 97.0%	0.5196**	0.0000 0.0%	0.0286** 3.6%	0.7679** 96.4%	0.7965**
S&P 500	FTSE MIB	0.0044 0.7%	0.0008 0.1%	0.5906** 99.1%	0.5958**	0.0002 0.0%	0.0034 0.6%	0.6111** 99.4%	0.6147**
S&P 500	S&P ASX 200	0.0005 0.1%	0.0303** 6.7%	0.4223** 93.2%	0.4532**	0.0000 0.0%	0.0567** 9.3%	0.5533** 90.7%	0.6099**
S&P 500	US Bond	0.0062 7.2%	0.0007 0.8%	0.0787** 92.0%	0.0856**	0.0027 11.3%	0.0000 0.0%	0.0216** 88.7%	0.0243**
S&P 500	DE Bond	0.0088 15.7%	0.0014 2.4%	0.0459** 81.9%	0.0561**	0.0006 3.9%	0.0093** 59.3%	0.0058* 36.7%	0.0158**
S&P 500	JP Bond	0.0066 20.1%	0.0017 5.2%	0.0244** 74.7%	0.0327**	0.0003 6.0%	0.0012 27.8%	0.0028 66.2%	0.0042
S&P 500	ES Bond	0.0085 16.9%	0.0029 5.8%	0.0388** 77.2%	0.0503**	0.0025 8.6%	0.0040 14.1%	0.0221** 77.2%	0.0286**
S&P 500	FR Bond	0.0128* 22.1%	0.0020 3.5%	0.0429** 74.3%	0.0577**	0.0020 28.7%	0.0034 48.6%	0.0016 22.7%	0.0071
S&P 500	UK Bond	0.0068 11.3%	0.0015 2.5%	0.0513** 86.1%	0.0596**	0.0028 29.9%	0.0014 14.9%	0.0052* 55.2%	0.0095*
S&P 500	IT Bond	0.0087 19.8%	0.0029 6.6%	0.0324** 73.7%	0.0440**	0.0042 9.5%	0.0004 1.0%	0.0394** 89.5%	0.0440**
S&P 500	AU Bond	0.0093 15.2%	0.0007 1.2%	0.0509** 83.6%	0.0608**	0.0000 1.8%	0.0000 0.3%	0.0017 97.8%	0.0018
DAX 40	TOPIX	0.0009 0.3%	0.0007 0.2%	0.2727** 99.4%	0.2743**	0.0016 0.3%	0.0174** 3.2%	0.5272** 96.5%	0.5463**
DAX 40	IBEX 35	0.0052 0.5%	0.0124* 1.1%	1.1081** 98.4%	1.1256**	0.0004 0.0%	0.0000 0.0%	1.1681** 100.0%	1.1686**
DAX 40	CAC 40	0.0129* 0.8%	0.0020 0.1%	1.5775** 99.1%	1.5924**	0.0026 0.1%	0.0003 0.0%	2.1625** 99.9%	2.1654**
DAX 40	FTSE 250	0.0087 1.2%	0.0001 0.0%	0.7005** 98.8%	0.7094**	0.0090** 0.8%	0.0068* 0.6%	1.1264** 98.6%	1.1422**
DAX 40	FTSE MIB	0.0080 0.7%	0.0066 0.6%	1.1831** 98.8%	1.1977**	0.0021 0.1%	0.0001 0.0%	1.4173** 99.8%	1.4194**
DAX 40	S&P ASX 200	0.0038 0.8%	0.0121* 2.5%	0.4637** 96.7%	0.4796**	0.0015 0.2%	0.0347** 5.3%	0.6170** 94.5%	0.6532**
DAX 40	US Bond	0.0031 2.7%	0.0007 0.6%	0.1142** 96.8%	0.1180**	0.0000 0.0%	0.0002 0.5%	0.0488** 99.5%	0.0490**
DAX 40	DE Bond	0.0031 3.3%	0.0040 4.2%	0.0871** 92.5%	0.0942**	0.0017 4.4%	0.0130** 34.2%	0.0235** 61.5%	0.0382**
DAX 40	JP Bond	0.0062 20.4%	0.0013 4.2%	0.0229** 75.4%	0.0303**	0.0001 0.6%	0.0006 2.9%	0.0190** 96.5%	0.0197**
DAX 40	ES Bond	0.0022 2.5%	0.0054 6.3%	0.0781** 91.1%	0.0857**	0.0006 1.6%	0.0034 8.9%	0.0340** 89.4%	0.0380**
DAX 40	FR Bond	0.0008 0.8%	0.0036 3.8%	0.0900** 95.4%	0.0944**	0.0003 3.8%	0.0079** 96.0%	0.0000 0.2%	0.0082
DAX 40	UK Bond	0.0004 0.4%	0.0028 3.0%	0.0905** 96.6%	0.0937**	0.0002 1.3%	0.0036 22.6%	0.0122** 76.1%	0.0160**
DAX 40	IT Bond	0.0016 2.1%	0.0048 6.2%	0.0706** 91.8%	0.0769**	0.0000 0.0%	0.0004 0.5%	0.0686** 99.4%	0.0690**
DAX 40	AU Bond	0.0041 4.7%	0.0021 2.4%	0.0815** 92.9%	0.0877**	0.0011 9.1%	0.0003 2.5%	0.0107** 88.4%	0.0120*

This table displays the Geweke feedback measures obtained from a bivariate VAR(2) for weekly returns before the 2008 financial crisis and a bivariate VAR(1) for weekly returns after the 2008 financial crisis. Pairwise tests are performed between the S&P 500 index and DAX 40 index returns and those of all other instruments in the sample: the equity indexes (S&P 500, DAX 40, TOPIX,

Table 11 (continued)

IBEX 35, CAC 40, FTSE 250, FTSE MIB and S&P/ASX 200) and the 10-year government bond yields for the US, Germany, Japan, Spain, France, the UK, Italy and Australia. The measure $F_{X1 \rightarrow X2}$ reflects the lagged feedback from S&P 500/DAX 40 index (X1 variable) and the other instrument (X2 variable). The measure $F_{X2 \rightarrow X1}$ reflects the lagged feedback from each instrument (X2 variable) and the S&P 500/DAX 40 index (X1 variable). The measure $F_{X1 \leftrightarrow X2}$ reflects the contemporaneous feedback. The measure $F_{X1,X2}$ reflects the total feedback. Percentages in italic display the weight of each feedback component to the total feedback. The pre-crisis period covers December 31, 1987 to December 31, 2007 while the post-crisis period covers January 1, 2008 to April 23, 2025. Numbers in bold indicate the rejection of the null hypothesis at the 5% level (one asterisk, “*”) or at the 1% (two asterisks, “**”).

6. CONCLUSION

This work investigates the relationships between equity indexes and government bonds of eight major international economies: the United States, Germany, Japan, Spain, France, the United Kingdom, Italy and Australia. By applying Granger causality tests and calculating Geweke's feedback measures within bivariate VAR frameworks, the analysis captures causality and feedback in both returns and volatilities over the period from December 31, 1987 to April 23, 2025. This period covers major economic disruptions, such as the 2008 financial crisis, the European sovereign debt crisis and the COVID-19 pandemic.

The results from the Granger causality tests in the mean reveal a leading role of the US index followed by most of the European indexes in information transmission across markets, particularly between indexes, and a limited impact of Asia-Pacific indexes beyond their domestic markets. Similar conclusions can be taken regarding government bonds, with the Italian bond standing out followed by the US and Spanish bonds and the Japanese bond showing the least influence on the remaining instruments. However, contrary to the analysis done on equity indexes, government bonds reveal the most significant relationships between themselves rather than between a different asset class. Volatility-based Granger causality tests reveal stronger interdependence within the same asset classes. Both equity and bond instruments of US display strong causal relationships, underscoring their central role in global market volatility.

The findings from the Geweke measures of feedback in the mean reveal a predominance of contemporaneous feedback over lagged feedback, suggesting a strong degree of synchronization among financial instruments, which indicates that a significant portion of the information transmission occurs within the same interval rather than with delay. Additionally, indexes traditionally considered the most influential, such as the S&P 500 and the DAX 40 indexes, exhibit a stronger lagged influence from other indexes than the influence they exert on them, as they show almost no lagged feedback towards them. It is also notable a weaker integration between indexes and government bonds. The measures of feedback in volatility reveal significant bidirectional lagged feedback and strong contemporaneous relationships between the S&P 500 and DAX 40 indexes and the remaining indexes. These results imply that equity markets are highly interconnected in

terms of volatility, whereas bond markets exhibit a weaker influence on equity market volatility, which highlight the importance of accounting for movement within stock markets when making investment decisions across both equity and bond markets.

From the analysis of causality in the mean for the periods before and after the 2008 financial crisis, it can be concluded that there was an increase in equity market interdependence, with the S&P 500 index emerging as the dominant influence, replacing the FTSE 250 and CAC 40 indexes. A shift in the bond market dynamic is also notable, with reduced dominance of the US bond and increased influence of government bonds on the TOPIX index, showing a deeper global financial integration. Regarding measures of feedback, post-crisis dynamics show reduced lagged influence from the S&P 500 and increased lagged feedback to the DAX 40 index from other instruments. Additionally, relationships with government bonds weakened, while contemporaneous feedback remained dominant.

Overall, the results highlight the US role in global return and volatility transmission, with European markets also exerting significant transmission and Asia-Pacific markets more regionally contained. Information flows are stronger within the same asset classes and the contemporaneous feedback was the major contributor to total feedback, indicating quick market reactions. Post-crisis dynamics show an increase in equity market integration, while the connections between the S&P 500 and the DAX 40 indexes and government bonds weakened.

Future research could extend the analysis to a broader range of assets and economies, such as developing countries, for example.

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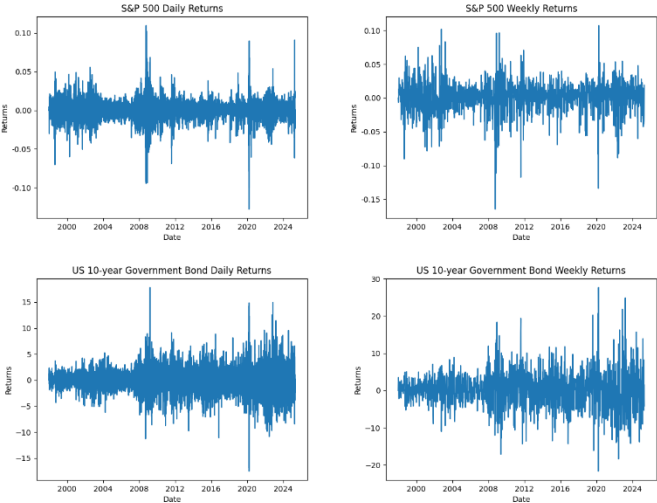
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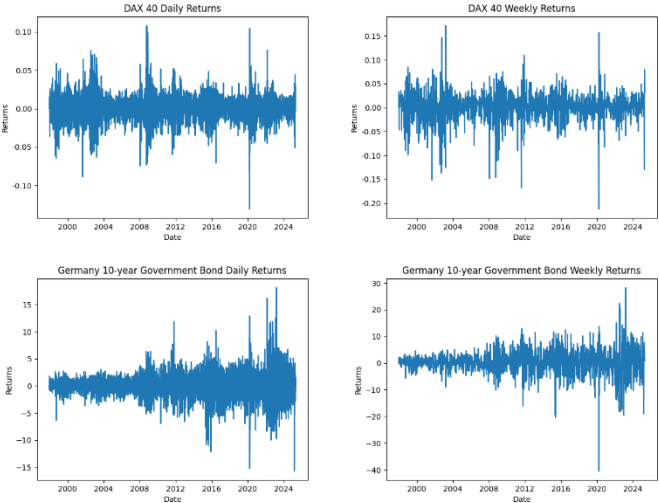
APPENDICES

Appendix A: Time Series of Daily and Weekly Returns of All Instruments

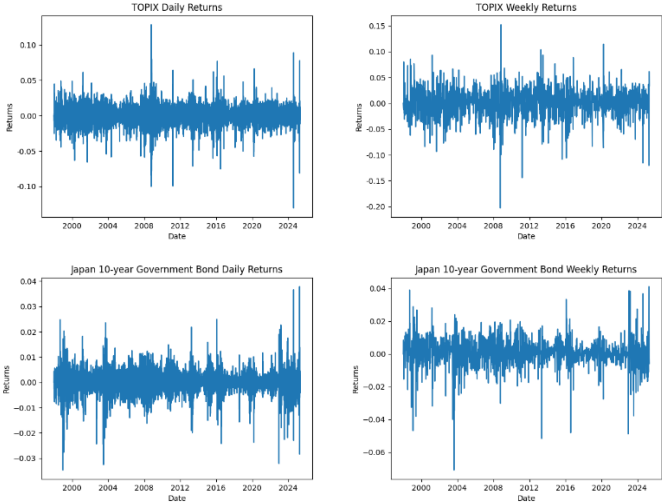
US



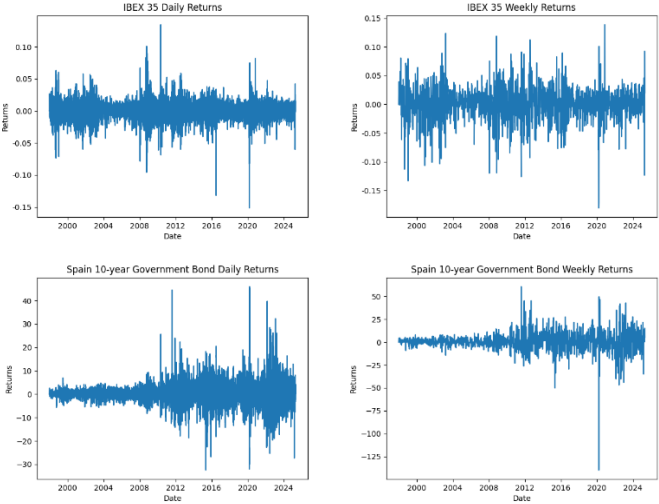
Germany



Japan

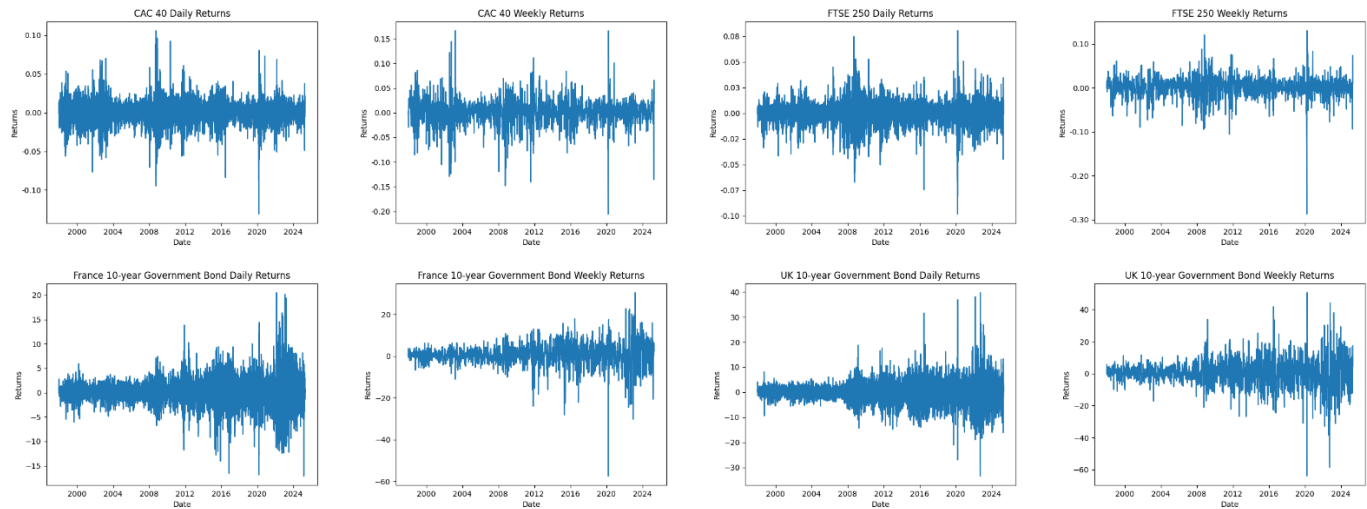


Spain



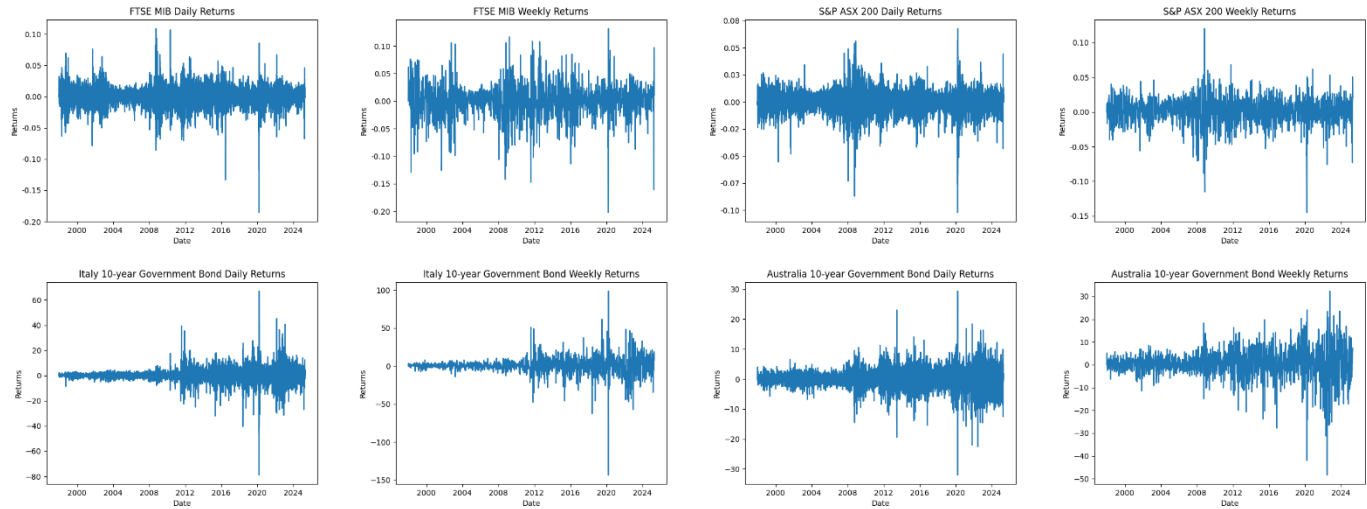
France

UK



Italy

Australia



Appendix B: Geweke Feedback Measures in the Mean

X1	X2	F _{X1→X2}	F _{X2→X1}	F _{X1↔X2}	F _{X1,X2}	F _{X1→X2}	F _{X2→X1}	F _{X1↔X2}	F _{X1,X2}
Daily Frequency					Weekly Frequency				
TOPIX	IBEX 35	0.1311** 62.4%	0.0008* 0.4%	0.0781** 37.2%	0.2100** 0.6%	0.0019 0.0%	0.0002 0.0%	0.3325** 99.4%	0.3346** 99.4%
TOPIX	CAC 40	0.1592** 60.1%	0.0007* 0.3%	0.1050** 39.6%	0.2649** 1.6%	0.0068** 0.0%	0.0000 0.0%	0.4211** 98.4%	0.4279** 98.4%
TOPIX	FTSE 250	0.1144** 47.1%	0.0025** 1.0%	0.1259** 51.9%	0.2428** 0.5%	0.0020 0.1%	0.0005 0.1%	0.3998** 99.4%	0.4023** 99.4%
TOPIX	FTSE MIB	0.1351** 62.9%	0.0007* 0.3%	0.0791** 36.8%	0.2149** 1.0%	0.0039* 0.0%	0.0002 0.1%	0.3775** 98.9%	0.3817** 98.9%
TOPIX	S&P ASX 200	0.0025** 0.7%	0.0017** 0.5%	0.3510** 98.8%	0.3551** 0.0%	0.0000 0.2%	0.0008 0.2%	0.4378** 99.8%	0.4386** 99.8%
TOPIX	US Bond	0.0367** 79.3%	0.0006* 1.3%	0.0090** 19.4%	0.0463** 17.5%	0.0107** 0.4%	0.0002 0.4%	0.0498** 82.1%	0.0607** 82.1%
TOPIX	DE Bond	0.0185** 59.0%	0.0022** 7.1%	0.0107** 34.0%	0.0314** 17.5%	0.0088** 11.8%	0.0059** 70.7%	0.0356** 70.7%	0.0504** 70.7%
TOPIX	JP Bond	0.0008 1.1%	0.0017** 2.3%	0.0728** 96.7%	0.0754** 4.0%	0.0026 1.3%	0.0009 1.3%	0.0613** 94.7%	0.0647** 94.7%
TOPIX	ES Bond	0.0019* 75.9%	0.0006* 24.0%	0.0000 0.1%	0.0025 48.5%	0.0029* 31.3%	0.0019 20.2%	0.0012 20.2%	0.0059* 20.2%
TOPIX	FR Bond	0.0068** 61.8%	0.0015** 14.1%	0.0026** 24.1%	0.0109** 22.0%	0.0038* 16.5%	0.0028* 61.6%	0.0106** 61.6%	0.0172** 61.6%
TOPIX	UK Bond	0.0187** 67.3%	0.0009* 3.3%	0.0082** 29.4%	0.0278** 19.8%	0.0071** 5.8%	0.0021 74.4%	0.0268** 74.4%	0.0361** 74.4%
TOPIX	IT Bond	0.0035** 89.2%	0.0003 6.5%	0.0002 4.3%	0.0039** 7.9%	0.0007 0.3%	0.0000 91.8%	0.0084** 91.8%	0.0091** 91.8%
TOPIX	AU Bond	0.0027** 5.3%	0.0014** 2.8%	0.0470** 91.9%	0.0512** 11.4%	0.0056** 0.7%	0.0004 87.9%	0.0432** 87.9%	0.0491** 87.9%
IBEX 35	CAC 40	0.0014 0.1%	0.0009* 0.1%	1.3669** 99.8%	1.3692** 99.8%	0.0000 0.0%	0.0005 0.0%	1.3414** 100.0%	1.3418** 100.0%
IBEX 35	FTSE 250	0.0009 0.1%	0.0018** 0.3%	0.6958** 99.6%	0.6985** 0.0%	0.0002 0.0%	0.0001 0.0%	0.7362** 100.0%	0.7365** 100.0%
IBEX 35	FTSE MIB	0.0011 0.1%	0.0010** 0.1%	1.3364** 99.8%	1.3384** 0.0%	0.0001 0.0%	0.0003 0.0%	1.3494** 100.0%	1.3498** 100.0%
IBEX 35	S&P ASX 200	0.0011 0.4%	0.1565** 57.7%	0.1136** 41.9%	0.2712** 0.0%	0.0000 2.2%	0.0111** 97.8%	0.4931** 97.8%	0.5042** 97.8%
IBEX 35	US Bond	0.0028** 3.6%	0.0015** 1.9%	0.0732** 94.5%	0.0774** 0.0%	0.0000 0.1%	0.0000 99.9%	0.0522** 99.9%	0.0523** 99.9%
IBEX 35	DE Bond	0.0008 1.0%	0.0034** 4.2%	0.0767** 94.8%	0.0809** 0.7%	0.0003 12.9%	0.0059** 86.4%	0.0393** 86.4%	0.0455** 86.4%
IBEX 35	JP Bond	0.0008 4.4%	0.0099** 57.6%	0.0065** 37.9%	0.0171** 1.2%	0.0002 2.2%	0.0004 96.6%	0.0184** 96.6%	0.0191** 96.6%
IBEX 35	ES Bond	0.0015 11.5%	0.0035** 26.5%	0.0082** 62.0%	0.0132** 0.2%	0.0000 12.8%	0.0034* 87.0%	0.0232** 87.0%	0.0267** 87.0%
IBEX 35	FR Bond	0.0009 4.3%	0.0036** 16.9%	0.0166** 78.8%	0.0210** 1.4%	0.0001 48.4%	0.0033* 50.2%	0.0034* 50.2%	0.0067* 50.2%
IBEX 35	UK Bond	0.0009 1.5%	0.0024** 4.0%	0.0572** 94.5%	0.0605** 1.1%	0.0003 3.4%	0.0009 95.5%	0.0248** 95.5%	0.0259** 95.5%
IBEX 35	IT Bond	0.0014 5.0%	0.0011** 4.0%	0.0253** 90.9%	0.0278** 2.0%	0.0008 1.8%	0.0007 96.2%	0.0387** 96.2%	0.0403** 96.2%
IBEX 35	AU Bond	0.0015 6.2%	0.0103** 43.3%	0.0120** 50.5%	0.0238** 1.3%	0.0002 0.5%	0.0001 98.2%	0.0159** 98.2%	0.0162** 98.2%
CAC 40	FTSE 250	0.0002 0.0%	0.0021** 0.2%	0.9188** 99.8%	0.9210** 0.0%	0.0001 0.1%	0.0012 99.9%	0.9748** 99.9%	0.9761** 99.9%
CAC 40	FTSE MIB	0.0021** 0.1%	0.0003 0.0%	1.4411** 99.8%	1.4434** 0.0%	0.0008 0.0%	0.0005 99.9%	1.5649** 99.9%	1.5661** 99.9%
CAC 40	S&P ASX 200	0.0009 0.3%	0.1878** 56.7%	0.1426** 43.0%	0.3312** 0.0%	0.0002 3.8%	0.0230** 96.1%	0.5743** 96.1%	0.5974** 96.1%
CAC 40	US Bond	0.0028** 3.6%	0.0016** 2.1%	0.0740** 94.3%	0.0785** 0.6%	0.0003 0.4%	0.0002 99.0%	0.0549** 99.0%	0.0555** 99.0%
CAC 40	DE Bond	0.0008 1.1%	0.0049** 6.7%	0.0670** 92.2%	0.0727** 2.8%	0.0011 26.4%	0.0103** 70.8%	0.0277** 70.8%	0.0391** 70.8%
CAC 40	JP Bond	0.0009 4.9%	0.0097** 51.5%	0.0082** 43.6%	0.0189** 7.5%	0.0015 0.8%	0.0002 91.7%	0.0184** 91.7%	0.0201** 91.7%
CAC 40	ES Bond	0.0019* 26.8%	0.0017** 24.3%	0.0035** 48.8%	0.0071** 0.0%	0.0000 18.8%	0.0041* 81.2%	0.0178** 81.2%	0.0219** 81.2%
CAC 40	FR Bond	0.0005 0.0%	0.0045** 24.3%	0.0157** 48.8%	0.0207** 0.0%	0.0000 18.8%	0.0057** 81.2%	0.0012 81.2%	0.0070* 81.2%

		2.6%	21.5%	75.9%		0.2%	82.4%	17.4%	
CAC 40	UK Bond	0.0010	0.0028**	0.0518**	0.0556**	0.0001	0.0024	0.0190**	0.0215**
		1.8%	5.0%	93.3%		0.6%	11.2%	88.3%	
CAC 40	IT Bond	0.0020*	0.0004	0.0176**	0.0200**	0.0006	0.0007	0.0364**	0.0377**
		10.1%	2.0%	87.9%		1.5%	1.9%	96.5%	
CAC 40	AU Bond	0.0015	0.0095**	0.0122**	0.0231**	0.0005	0.0003	0.0160**	0.0168**
		6.4%	41.0%	52.7%		3.0%	1.8%	95.2%	
FTSE 250	FTSE MIB	0.0018*	0.0003	0.7078**	0.7099**	0.0002	0.0003	0.8173**	0.8177**
		0.3%	0.0%	99.7%		0.0%	0.0%	99.9%	
FTSE 250	S&P ASX 200	0.0022**	0.1613**	0.1935**	0.3569**	0.0001	0.0208**	0.6522**	0.6731**
		0.6%	45.2%	54.2%		0.0%	3.1%	96.9%	
FTSE 250	US Bond	0.0018*	0.0011**	0.0507**	0.0537**	0.0002	0.0004	0.0232**	0.0238**
		3.4%	2.1%	94.5%		0.9%	1.7%	97.3%	
FTSE 250	DE Bond	0.0026**	0.0033**	0.0445**	0.0503**	0.0006	0.0055**	0.0076**	0.0137**
		5.1%	6.5%	88.4%		4.6%	39.9%	55.5%	
FTSE 250	JP Bond	0.0008	0.0050**	0.0088**	0.0146**	0.0003	0.0010	0.0106**	0.0119**
		5.3%	34.2%	60.5%		2.9%	8.6%	88.6%	
FTSE 250	ES Bond	0.0042**	0.0010**	0.0054**	0.0106**	0.0016	0.0018	0.0302**	0.0336**
		39.6%	9.2%	51.1%		4.7%	5.5%	89.8%	
FTSE 250	FR Bond	0.0017*	0.0027**	0.0072**	0.0116**	0.0003	0.0029*	0.0005	0.0037
		14.8%	23.1%	62.1%		7.6%	79.3%	13.1%	
FTSE 250	UK Bond	0.0028**	0.0025**	0.0273**	0.0325**	0.0000	0.0004	0.0009	0.0013
		8.5%	7.6%	83.9%		1.0%	31.1%	68.0%	
FTSE 250	IT Bond	0.0041**	0.0004	0.0219**	0.0264**	0.0002	0.0003	0.0553**	0.0558**
		15.5%	1.6%	82.8%		0.4%	0.5%	99.1%	
FTSE 250	AU Bond	0.0032**	0.0039**	0.0087**	0.0158**	0.0006	0.0000	0.0024	0.0031
		20.5%	24.5%	55.0%		20.3%	0.7%	79.0%	
FTSE MIB	S&P ASX 200	0.0004	0.1572**	0.1091**	0.2667**	0.0000	0.0130**	0.5075**	0.5206**
		0.1%	58.9%	40.9%		0.0%	2.5%	97.5%	
FTSE MIB	US Bond	0.0037**	0.0017**	0.0771**	0.0824**	0.0001	0.0000	0.0593**	0.0594**
		4.5%	2.0%	93.5%		0.1%	0.0%	99.9%	
FTSE MIB	DE Bond	0.0009	0.0048**	0.0847**	0.0904**	0.0013	0.0081**	0.0442**	0.0536**
		1.0%	5.3%	93.7%		2.4%	15.2%	82.4%	
FTSE MIB	JP Bond	0.0011	0.0086**	0.0082**	0.0178**	0.0009	0.0001	0.0220**	0.0230**
		6.0%	48.2%	45.9%		4.1%	0.5%	95.4%	
FTSE MIB	ES Bond	0.0014	0.0021**	0.0123**	0.0158**	0.0001	0.0037*	0.0231**	0.0268**
		8.8%	13.0%	78.2%		0.2%	13.7%	86.0%	
FTSE MIB	FR Bond	0.0011	0.0042**	0.0161**	0.0214**	0.0001	0.0046*	0.0034*	0.0080**
		5.1%	19.6%	75.3%		1.2%	56.9%	41.9%	
FTSE MIB	UK Bond	0.0013	0.0032**	0.0654**	0.0699**	0.0000	0.0017	0.0295**	0.0312**
		1.9%	4.6%	93.6%		0.1%	5.4%	94.6%	
FTSE MIB	IT Bond	0.0027**	0.0011**	0.0473**	0.0511**	0.0001	0.0018	0.0596**	0.0616**
		5.3%	2.2%	92.5%		0.2%	2.9%	96.9%	
FTSE MIB	AU Bond	0.0012	0.0122**	0.0141**	0.0275**	0.0004	0.0001	0.0229**	0.0234**
		4.3%	44.4%	51.4%		1.8%	0.3%	97.9%	
S&P ASX 200	US Bond	0.0089**	0.0014**	0.0086**	0.0189**	0.0003	0.0018	0.0125**	0.0146**
		47.4%	7.3%	45.3%		1.9%	12.0%	86.1%	
S&P ASX 200	DE Bond	0.0070**	0.0028**	0.0055**	0.0153**	0.0001	0.0028*	0.0028*	0.0058*
		45.6%	18.4%	36.0%		2.5%	48.3%	49.2%	
S&P ASX 200	JP Bond	0.0003	0.0013**	0.0214**	0.0231**	0.0006	0.0007	0.0115**	0.0128**
		1.5%	5.6%	93.0%		4.5%	5.8%	89.8%	
S&P ASX 200	ES Bond	0.0071**	0.0007*	0.0017*	0.0095**	0.0010	0.0006	0.0241**	0.0258**
		74.7%	7.3%	18.0%		4.0%	2.5%	93.5%	
S&P ASX 200	FR Bond	0.0040**	0.0012**	0.0003	0.0055**	0.0000	0.0005	0.0010	0.0015
		72.8%	22.0%	5.2%		0.9%	33.0%	66.1%	
S&P ASX 200	UK Bond	0.0059**	0.0010**	0.0030**	0.0099**	0.0003	0.0006	0.0027	0.0036
		60.0%	9.8%	30.2%		7.7%	17.8%	74.4%	
S&P ASX 200	IT Bond	0.0086**	0.0003	0.0038**	0.0127**	0.0033*	0.0000	0.0299**	0.0332**
		67.4%	2.6%	30.0%		9.9%	0.0%	90.1%	
S&P ASX 200	AU Bond	0.0029**	0.0016**	0.0116**	0.0161**	0.0002	0.0000	0.0008	0.0010
		17.8%	10.1%	72.1%		17.0%	0.7%	82.3%	
US Bond	DE Bond	0.0016*	0.0402**	0.4865**	0.5283**	0.0010	0.0002	0.7682**	0.7694**
		0.3%	7.6%	92.1%		0.1%	0.0%	99.9%	
US Bond	JP Bond	0.0004	0.0887**	0.0154**	0.1045**	0.0008	0.0221**	0.1151**	0.1380**
		0.4%	84.8%	14.8%		0.6%	16.0%	83.4%	
US Bond	ES Bond	0.0027**	0.0043**	0.1348**	0.1418**	0.0002	0.0000	0.2791**	0.2793**
		1.9%	3.0%	95.1%		0.1%	0.0%	99.9%	
US Bond	FR Bond	0.0026**	0.0204**	0.3309**	0.3539**	0.0008	0.0001	0.5654**	0.5663**
		0.7%	5.8%	93.5%		0.1%	0.0%	99.9%	

US Bond	UK Bond	0.0020* 0.4%	0.0314** 6.0%	0.4920** 93.6%	0.5254**	0.0039* 0.5%	0.0000 0.0%	0.7538** 99.5%	0.7577**
US Bond	IT Bond	0.0049** 6.4%	0.0037** 4.8%	0.0683** 88.8%	0.0769**	0.0003 0.1%	0.0003 0.1%	0.2111** 99.7%	0.2117**
US Bond	AU Bond	0.0011 0.2%	0.4475** 88.3%	0.0581** 11.5%	0.5067**	0.0001 0.0%	0.0458** 6.8%	0.6232** 93.1%	0.6691**
DE Bond	JP Bond	0.0006 0.8%	0.0556** 71.0%	0.0221** 28.2%	0.0783**	0.0000 0.0%	0.0122** 7.3%	0.1547** 92.7%	0.1668**
DE Bond	ES Bond	0.0042** 1.0%	0.0082** 1.9%	0.4102** 97.0%	0.4227**	0.0077** 1.3%	0.0022 0.4%	0.5768** 98.3%	0.5867**
DE Bond	FR Bond	0.0036** 0.3%	0.0008* 0.1%	1.3782** 99.7%	1.3826**	0.0043* 0.3%	0.0024 0.1%	1.6552** 99.6%	1.6619**
DE Bond	UK Bond	0.0043** 0.5%	0.0005 0.0%	0.9198** 99.5%	0.9246**	0.0005 0.0%	0.0013 0.1%	1.0525** 99.8%	1.0543**
DE Bond	IT Bond	0.0031** 1.3%	0.0024** 1.0%	0.2260** 97.6%	0.2315**	0.0020 0.5%	0.0021 0.6%	0.3699** 98.9%	0.3740**
DE Bond	AU Bond	0.0014 0.3%	0.3083** 76.6%	0.0927** 23.0%	0.4024**	0.0025 0.4%	0.0352** 5.1%	0.6593** 94.6%	0.6970**
JP Bond	ES Bond	0.0243** 83.2%	0.0029** 9.8%	0.0020* 7.0%	0.0292**	0.0094** 14.1%	0.0000 0.0%	0.0577** 85.9%	0.0672**
JP Bond	FR Bond	0.0450** 74.1%	0.0005* 0.9%	0.0152** 25.0%	0.0607**	0.0138** 10.5%	0.0001 0.1%	0.1173** 89.4%	0.1312**
JP Bond	UK Bond	0.0518** 73.9%	0.0003 0.5%	0.0180** 25.7%	0.0701**	0.0096** 7.2%	0.0012 0.9%	0.1220** 91.9%	0.1329**
JP Bond	IT Bond	0.0187** 86.9%	0.0017** 7.8%	0.0011 5.3%	0.0215**	0.0061** 13.2%	0.0000 0.0%	0.0402** 86.8%	0.0463**
JP Bond	AU Bond	0.0028** 2.7%	0.0018** 1.7%	0.0995** 95.6%	0.1041**	0.0032* 2.1%	0.0019 1.2%	0.1466** 96.7%	0.1517**
ES Bond	FR Bond	0.0075** 1.1%	0.0061** 0.9%	0.6939** 98.1%	0.7075**	0.0001 0.0%	0.0040* 0.4%	0.9502** 99.6%	0.9543**
ES Bond	UK Bond	0.0041** 1.6%	0.0041** 1.6%	0.2487** 96.8%	0.2570**	0.0002 0.0%	0.0001 0.0%	0.3718** 99.9%	0.3720**
ES Bond	IT Bond	0.0043** 0.4%	0.0027** 0.2%	1.1802** 99.4%	1.1872**	0.0004 0.0%	0.0003 0.0%	1.3320** 99.9%	1.3327**
ES Bond	AU Bond	0.0044** 2.8%	0.1442** 92.5%	0.0072** 4.6%	0.1558**	0.0004 0.1%	0.0162** 5.5%	0.2776** 94.4%	0.2941**
FR Bond	UK Bond	0.0019* 0.3%	0.0021** 0.3%	0.6493** 99.4%	0.6534**	0.0001 0.0%	0.0005 0.1%	0.8213** 99.9%	0.8219**
FR Bond	IT Bond	0.0061** 1.2%	0.0041** 0.8%	0.4947** 98.0%	0.5049**	0.0050** 0.8%	0.0001 0.0%	0.6584** 99.2%	0.6635**
FR Bond	AU Bond	0.0007 0.2%	0.2552** 80.8%	0.0600** 19.0%	0.3158**	0.0014 0.2%	0.0271** 4.7%	0.5501** 95.1%	0.5786**
UK Bond	IT Bond	0.0035** 2.2%	0.0014** 0.9%	0.1515** 96.9%	0.1563**	0.0010 0.4%	0.0000 0.0%	0.2781** 99.6%	0.2791**
UK Bond	AU Bond	0.0017* 0.5%	0.2772** 75.5%	0.0882** 24.0%	0.3670**	0.0099** 1.6%	0.0270** 4.3%	0.5964** 94.2%	0.6333**
IT Bond	AU Bond	0.0035** 3.2%	0.1014** 91.9%	0.0054** 4.9%	0.1103**	0.0004 0.2%	0.0100** 4.5%	0.2133** 95.4%	0.2237**

Appendix C: Geweke Feedback Measures in Volatility

X1	X2	F _{X1→X2}	F _{X2→X1}	F _{X1↔X2}	F _{X1,X2}
TOPIX	IBEX 35	0.0228** 51.3%	0.0029** 6.4%	0.0187** 42.2%	0.0444**
TOPIX	CAC 40	0.0304** 57.9%	0.0025** 4.8%	0.0196** 37.3%	0.0525**
TOPIX	FTSE 250	0.0202** 37.8%	0.0023** 4.3%	0.0310** 57.9%	0.0535**
TOPIX	FTSE MIB	0.0273** 58.0%	0.0014** 3.0%	0.0184** 39.0%	0.0471**
TOPIX	S&P ASX 200	0.0062** 3.9%	0.0012** 0.7%	0.1514** 95.4%	0.1587**
TOPIX	US Bond	0.0027** 67.7%	0.0003 6.7%	0.0010** 25.6%	0.0040**
TOPIX	DE Bond	0.0007* 49.3%	0.0001 6.7%	0.0006* 44.0%	0.0015*
TOPIX	JP Bond	0.0013** 3.9%	0.0001 0.3%	0.0309** 95.8%	0.0323**
TOPIX	ES Bond	0.0000 0.2%	0.0000 19.9%	0.0000 80.0%	0.0000
TOPIX	FR Bond	0.0000 0.1%	0.0000 0.2%	0.0002 99.8%	0.0002
TOPIX	UK Bond	0.0011** 59.4%	0.0002 12.5%	0.0005 28.1%	0.0018**
TOPIX	IT Bond	0.0000 0.3%	0.0001 19.2%	0.0003 80.5%	0.0004
TOPIX	AU Bond	0.0008* 4.9%	0.0001 0.4%	0.0162** 94.7%	0.0171**
IBEX 35	CAC 40	0.0041** 0.5%	0.0018** 0.2%	0.9130** 99.4%	0.9189**
IBEX 35	FTSE 250	0.0020** 0.6%	0.0024** 0.7%	0.3378** 98.7%	0.3422**
IBEX 35	FTSE MIB	0.0047** 0.5%	0.0006* 0.1%	0.9167** 99.4%	0.9220**
IBEX 35	S&P ASX 200	0.0067** 10.0%	0.0258** 38.5%	0.0345** 51.5%	0.0670**
IBEX 35	US Bond	0.0019** 7.0%	0.0003 1.2%	0.0254** 91.8%	0.0277**
IBEX 35	DE Bond	0.0001 0.2%	0.0001 0.2%	0.0503** 99.6%	0.0505**
IBEX 35	JP Bond	0.0009* 33.3%	0.0001 4.2%	0.0017** 62.5%	0.0026**
IBEX 35	ES Bond	0.0000 0.0%	0.0000 0.0%	0.0246** 100.0%	0.0247**
IBEX 35	FR Bond	0.0000 0.3%	0.0002 0.9%	0.0179** 98.8%	0.0181**
IBEX 35	UK Bond	0.0001 0.2%	0.0001 0.2%	0.0343** 99.6%	0.0344**
IBEX 35	IT Bond	0.0001 0.3%	0.0005 1.8%	0.0284** 97.8%	0.0290**
IBEX 35	AU Bond	0.0001 0.7%	0.0021** 23.3%	0.0068** 76.0%	0.0089**
CAC 40	FTSE 250	0.0008* 0.2%	0.0018** 0.4%	0.4896** 99.5%	0.4922**
CAC 40	FTSE MIB	0.0012** 0.1%	0.0011** 0.1%	0.9610** 99.8%	0.9633**
CAC 40	S&P ASX 200	0.0070** 9.9%	0.0347** 49.2%	0.0288** 40.8%	0.0706**
CAC 40	US Bond	0.0018** 5.0%	0.0001 0.3%	0.0339** 94.7%	0.0358**
CAC 40	DE Bond	0.0002 0.3%	0.0002 0.3%	0.0561** 99.4%	0.0565**
CAC 40	JP Bond	0.0004 15.3%	0.0007* 29.0%	0.0013** 55.7%	0.0023**
CAC 40	ES Bond	0.0001 0.3%	0.0000 0.0%	0.0199** 99.7%	0.0199**
CAC 40	FR Bond	0.0002 0.8%	0.0001 0.5%	0.0249** 98.8%	0.0252**

CAC 40	UK Bond	0.0002 0.4%	0.0003 0.6%	0.0409** 99.0%	0.0413**
CAC 40	IT Bond	0.0000 0.0%	0.0003 1.1%	0.0252** 98.9%	0.0255**
CAC 40	AU Bond	0.0000 0.1%	0.0018** 18.8%	0.0077** 81.1%	0.0095**
FTSE 250	FTSE MIB	0.0012** 0.3%	0.0014** 0.4%	0.3691** 99.3%	0.3717**
FTSE 250	S&P ASX 200	0.0114** 11.0%	0.0342** 32.9%	0.0584** 56.2%	0.1040**
FTSE 250	US Bond	0.0117** 23.3%	0.0013** 2.6%	0.0371** 74.1%	0.0501**
FTSE 250	DE Bond	0.0018** 3.2%	0.0003 0.5%	0.0530** 96.2%	0.0551**
FTSE 250	JP Bond	0.0004 13.9%	0.0001 4.2%	0.0025** 82.0%	0.0031**
FTSE 250	ES Bond	0.0022** 8.2%	0.0004 1.5%	0.0238** 90.3%	0.0264**
FTSE 250	FR Bond	0.0010** 3.7%	0.0004 1.4%	0.0248** 94.9%	0.0261**
FTSE 250	UK Bond	0.0025** 4.9%	0.0002 0.3%	0.0491** 94.8%	0.0518**
FTSE 250	IT Bond	0.0023** 7.7%	0.0000 0.0%	0.0276** 92.2%	0.0299**
FTSE 250	AU Bond	0.0043** 20.5%	0.0043** 20.3%	0.0124** 59.2%	0.0210**
FTSE MIB	S&P ASX 200	0.0050** 7.8%	0.0310** 48.3%	0.0282** 43.9%	0.0642**
FTSE MIB	US Bond	0.0030** 11.2%	0.0013** 4.7%	0.0227** 84.1%	0.0270**
FTSE MIB	DE Bond	0.0000 0.1%	0.0001 0.1%	0.0561** 99.8%	0.0562**
FTSE MIB	JP Bond	0.0007* 32.6%	0.0000 2.4%	0.0013** 65.0%	0.0020**
FTSE MIB	ES Bond	0.0002 1.4%	0.0011** 6.6%	0.0156** 92.0%	0.0169**
FTSE MIB	FR Bond	0.0001 0.5%	0.0001 0.8%	0.0170** 98.7%	0.0173**
FTSE MIB	UK Bond	0.0001 0.4%	0.0001 0.2%	0.0360** 99.4%	0.0362**
FTSE MIB	IT Bond	0.0003 0.8%	0.0001 0.2%	0.0358** 98.9%	0.0362**
FTSE MIB	AU Bond	0.0009* 9.5%	0.0036** 40.1%	0.0045** 50.4%	0.0089**
S&P ASX 200	US Bond	0.0095** 63.0%	0.0043** 28.5%	0.0013** 8.5%	0.0151**
S&P ASX 200	DE Bond	0.0025** 36.3%	0.0016** 23.2%	0.0027** 40.6%	0.0068**
S&P ASX 200	JP Bond	0.0005 9.1%	0.0000 0.7%	0.0053** 90.2%	0.0059**
S&P ASX 200	ES Bond	0.0012** 31.6%	0.0018** 46.2%	0.0009* 22.2%	0.0039**
S&P ASX 200	FR Bond	0.0009** 22.4%	0.0012** 29.4%	0.0020** 48.2%	0.0042**
S&P ASX 200	UK Bond	0.0015** 34.1%	0.0016** 35.9%	0.0013** 29.9%	0.0044**
S&P ASX 200	IT Bond	0.0022** 38.1%	0.0006* 10.9%	0.0029** 51.1%	0.0058**
S&P ASX 200	AU Bond	0.0004 1.1%	0.0027** 7.6%	0.0322** 91.3%	0.0352**
US Bond	DE Bond	0.0052** 3.0%	0.0204** 11.6%	0.1503** 85.4%	0.1759**
US Bond	JP Bond	0.0000 0.0%	0.0013** 29.6%	0.0032** 70.3%	0.0045**
US Bond	ES Bond	0.0083** 10.5%	0.0208** 26.3%	0.0500** 63.2%	0.0791**
US Bond	FR Bond	0.0053** 5.0%	0.0160** 15.0%	0.0851** 80.0%	0.1064**
US Bond	UK Bond	0.0043**	0.0154**	0.1559**	0.1755**

		2.4%	8.8%	88.8%	
US Bond	IT Bond	0.0142**	0.0103**	0.0424**	0.0670**
		21.3%	15.4%	63.3%	
US Bond	AU Bond	0.0051**	0.0928**	0.0086**	0.1065**
		4.8%	87.1%	8.1%	
DE Bond	JP Bond	0.0001	0.0003	0.0014**	0.0018**
		8.0%	13.8%	78.2%	
DE Bond	ES Bond	0.0100**	0.0097**	0.3609**	0.3805**
		2.6%	2.5%	94.8%	
DE Bond	FR Bond	0.0044**	0.0026**	0.9471**	0.9540**
		0.5%	0.3%	99.3%	
DE Bond	UK Bond	0.0085**	0.0037**	0.4689**	0.4811**
		1.8%	0.8%	97.5%	
DE Bond	IT Bond	0.0132**	0.0015**	0.2706**	0.2853**
		4.6%	0.5%	94.8%	
DE Bond	AU Bond	0.0083**	0.0453**	0.0233**	0.0769**
		10.8%	58.9%	30.3%	
JP Bond	ES Bond	0.0000	0.0001	0.0004	0.0005
		8.8%	10.9%	80.2%	
JP Bond	FR Bond	0.0001	0.0001	0.0011**	0.0013*
		6.9%	8.0%	85.0%	
JP Bond	UK Bond	0.0000	0.0003	0.0013**	0.0015*
		0.0%	17.4%	82.6%	
JP Bond	IT Bond	0.0001	0.0002	0.0003	0.0006
		14.6%	27.0%	58.4%	
JP Bond	AU Bond	0.0001	0.0001	0.0172**	0.0175**
		0.8%	0.8%	98.4%	
ES Bond	FR Bond	0.0058**	0.0062**	0.4426**	0.4546**
		1.3%	1.4%	97.4%	
ES Bond	UK Bond	0.0115**	0.0122**	0.1516**	0.1753**
		6.6%	7.0%	86.5%	
ES Bond	IT Bond	0.0231**	0.0013**	0.7691**	0.7935**
		2.9%	0.2%	96.9%	
ES Bond	AU Bond	0.0136**	0.0470**	0.0138**	0.0744**
		18.3%	63.1%	18.6%	
FR Bond	UK Bond	0.0082**	0.0061**	0.2828**	0.2971**
		2.8%	2.0%	95.2%	
FR Bond	IT Bond	0.0117**	0.0006*	0.3464**	0.3587**
		3.3%	0.2%	96.6%	
FR Bond	AU Bond	0.0101**	0.0407**	0.0135**	0.0642**
		15.7%	63.3%	21.0%	
UK Bond	IT Bond	0.0179**	0.0022**	0.1224**	0.1424**
		12.5%	1.5%	85.9%	
UK Bond	AU Bond	0.0089**	0.0406**	0.0286**	0.0780**
		11.4%	52.0%	36.6%	
IT Bond	AU Bond	0.0049**	0.0477**	0.0089**	0.0614**
		8.0%	77.6%	14.5%	

Appendix D: Granger Causality Test Code

```

import pandas as pd
import numpy as np
from statsmodels.tsa.stattools import grangercausalitytests

def granger_causality_test(data, instruments, lag):

    # empty df with instruments as columns and indexes
    granger_df = pd.DataFrame(np.zeros((len(instruments), len(instruments))),
                              columns=instruments, index=instruments)

    # loop iterating over each pair of variables (c and r)
    for c in granger_df.columns:

        for r in granger_df.index:

            # perform test
            test_result = grangercausalitytests(data[[r, c]], maxlag=lag, verbose=True)

            # p-values
            p_values = [round(test_result[i + 1][0]['ssr_ftest'][1], 4) for i in
                        range(lag)]

            # coefficients from the f test
            f_stats = [round(test_result[i + 1][0]['ssr_ftest'][0], 2) for i in
                       range(lag)]

            min_idx = int(np.argmin(p_values)) # index of the smallest p-value
            best_p = p_values[min_idx] # smallest p-value
            best_f = f_stats[min_idx] # corresponding f-statistic

            # add asterisks depending on the significance level
            if best_p < 0.01:
                stars = '**'
            elif best_p < 0.05:
                stars = '*'
            else:
                stars = ''

            granger_df.loc[r, c] = f"{best_f:.2f}{stars}"

    # populate empty df
    granger_df.columns = [var + '_x' for var in instruments]
    granger_df.index = [var + '_y' for var in instruments]

    return granger_df

```

Appendix E: Geweke Feedback Measures Code

```

import itertools
import numpy as np
import pandas as pd
from scipy.stats import chi2

def geweke_measures(data, instruments, lag):

    variable_pairs = list(itertools.combinations(instruments, 2))
    results = []

    for var1, var2 in variable_pairs:

        # time series data for var1 and var2
        x1 = data[var1].values
        x2 = data[var2].values

        # lagged matrices
        X1lag = np.column_stack([np.roll(x1, i) for i in range(1, lag + 1)])
        X2lag = np.column_stack([np.roll(x2, i) for i in range(1, lag + 1)])

        X1lag = X1lag[lag:, :]
        X2lag = X2lag[lag:, :]
        X1 = x1[lag:]
        X2 = x2[lag:]

        # unrestricted model
        T = len(X1)
        reg = np.column_stack([np.ones(T), X1lag, X2lag]) # regression matrix

        burX1 = np.linalg.lstsq(reg, X1, rcond=None)[0] # X1 regression coefficients
        EurX1 = X1 - reg @ burX1 # X1 residuals

        burX2 = np.linalg.lstsq(reg, X2, rcond=None)[0] # X2 regression coefficients
        EurX2 = X2 - reg @ burX2 # X2 residuals

        # restricted model
        reg_x1 = np.column_stack([np.ones(T), X1lag]) # restricted X1 model reg matrix
        reg_x2 = np.column_stack([np.ones(T), X2lag]) # restricted X2 model reg matrix

        brX1 = np.linalg.lstsq(reg_x1, X1, rcond=None)[0] # restricted X1 coefficients
        ErX1 = X1 - reg_x1 @ brX1 # restricted X1 residuals

        brX2 = np.linalg.lstsq(reg_x2, X2, rcond=None)[0] # restricted X2 coefficients
        ErX2 = X2 - reg_x2 @ brX2 # restricted X2 residuals

        # covariance matrices
        Vur = np.cov(EurX1, EurX2) # unrestricted model
        Vr = np.cov(ErX1, ErX2) # restricted model

        # lagged feedback from X1 to X2
        FS1 = np.log(Vr[0, 0] / Vur[0, 0])

        # lagged feedback from X2 to X1
        FS2 = np.log(Vr[1, 1] / Vur[1, 1])

        # contemporaneous feedback between X1 and X2
        FS3 = np.log((Vur[0, 0] * Vur[1, 1]) / np.linalg.det(Vur))

        # total feedback between X1 and X2:
        FS4 = np.log((Vr[0, 0] * Vr[1, 1]) / np.linalg.det(Vr))

        # relative contribution
        Rel1 = f"{FS1 / FS4 * 100:.1f}%"
        Rel2 = f"{FS2 / FS4 * 100:.1f}%"
        Rel3 = f"{FS3 / FS4 * 100:.1f}%"

        # p-values
        P1 = 1 - chi2.cdf(T * FS1, lag)

```

```

P2 = 1 - chi2.cdf(T * FS2, 1)
P3 = 1 - chi2.cdf(T * FS3, lag)
P4 = 1 - chi2.cdf(T * FS4, 2 * lag + 1)

# add asterisks depending on the significance level
def format_f_stat(F, P):
    if P < 0.01:
        return f"{F:.4f}**"
    elif P < 0.05:
        return f"{F:.4f}*"
    else:
        return f"{F:.4f}"

F1 = format_f_stat(FS1, P1)
F2 = format_f_stat(FS2, P2)
F3 = format_f_stat(FS3, P3)
F4 = format_f_stat(FS4, P4)

results.append([var1, var2, F1, F2, F3, F4, Rel1, Rel2, Rel3])

# Convert to DataFrame
geweke_df = pd.DataFrame(
    results,
    columns=["X1", "X2", "F1", "F2", "F3", "F4", "Rel1", "Rel2", "Rel3"]
)

return geweke_df

```