MASTER
MATHEMATICAL FINANCE

MASTER´S FINAL WORK
INTERNSHIP REPORT

OPTIMIZED PORTFOLIOS FOR VARYING RISK PROFILES

CAROLINA ALVES

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JUNE - 2026

Mãe, Pai

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Tios, Tias

Sebastião

Marta, Mariana

Tiago

Mafalda

Inês

Margarida

Beatriz

Not being able to name everyone,

To all who crossed my path and I somehow liked.

Glossary

1/N – Homogenous portfolio

EM EQUITY – Emerging markets equity

EUR EQUITY – European equity

EUT – Expected utility theory

MVO – Mean-variance optimization

MV – Minimum variance portfolio

RRA – Coefficient of relative risk aversion

SR – Sharpe ratio

T – Tangent portfolio

UCITS – Undertaking for collective investment in transferable securities

ABSTRACT, KEYWORDS AND JEL CODES

This report documents a brief overview of my curricular internship at the Capital Markets Division of Banco Atlântico Europa. I pursued this opportunity due to my eagerness to academically deepen my knowledge of financial markets and instruments. This experience allowed me to understand the practical dynamics of the investment management industry, within a professional setting.

Realizing the gap between financial theory and the real-life practices of financial institutions, I was motivated to further research these divergences. Hence, this study focuses on the application of theoretical principles of mean-variance theory and expected utility theory to analyze asset allocation choices for varying risk profiles, contrasting mathematically optimized portfolios against static benchmarks. Additionally, a comparative analysis is developed, particularly, in terms of efficiency, of the inclusion of alternative assets on portfolios suggested by main asset management companies.

It is concluded that more conservative investors prefer investing in bonds while more aggressive profiles prioritize equity assets. Alternative assets play a more relevant role for moderate to aggressive profiles. Moreover, even though all the assessed ad-hoc portfolios are inefficient, some present themselves as satisfactory proxies for optimized portfolios.

KEYWORDS: Asset Allocation; Mean-Variance Theory; Expected Utility Theory; Risk Aversion, Alternative Assets.

JEL CODES: C61, D53, D81, G11.

RESUMO, PALAVRAS-CHAVE E CÓDIGOS JEL

O presente relatório apresenta uma breve descrição do meu estágio curricular na Direção de Mercados de Capitais do Banco Atlântico Europa. Esta experiência decorreu da minha vontade em aprofundar academicamente o meu conhecimento sobre mercados e instrumentos financeiros. Esta oportunidade permitiu-me compreender as diversas dinâmicas da indústria de gestão de ativos, num contexto profissional.

Ao constatar o desfasamento claro entre a teoria financeira e práticas reais das variadas instituições financeiras, surgiu a motivação para analisar essas divergências mais detalhadamente. Deste modo, este estudo foca-se na aplicação dos princípios teóricos da Teoria da média variância e da Teoria da utilidade esperada para conduzir uma análise às estratégias de alocação de ativos para diversos perfis de risco, contrastando carteiras matematicamente otimizadas com *benchmarks* de alocação fixa. Adicionalmente, é desenvolvida uma análise comparativa do impacto, em termos de eficiência, da integração de ativos alternativos nas carteiras, em concordância com as recomendações das principais sociedades gestoras de ativos.

Os resultados demonstram que os investidores mais conservadores preferem investir em obrigações enquanto perfis mais agressivos priorizam ativos mais voláteis, como ações. Por sua vez, os ativos alternativos desempenham um papel fundamental para perfis moderados a agressivos. Por fim, demonstra-se que as carteiras *ad-hoc* são ineficientes à luz da fronteira eficiente, embora algumas sejam *proxies* razoáveis para carteiras otimizadas.

PALAVRAS-CHAVE: Alocação de Ativos; Teoria da Média Variância; Teoria da Utilidade Esperada; Aversão ao Risco, Ativos Alternativos.

CÓDIGOS JEL: C61, D53, D81, G11.

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A kind word to Inês, my supervising colleague, with whom I shared doubts, helping me navigate this stressful process.

To my family,

To those friendships that remain strong since the 5th grade,

To those that ISEG introduced me to and are now friends for life,

To the one with infinite patience who survived all my drama, tears, and non-stop complaining,

(You all know who you are!)

(You might not be in any of these categories but if you also played an important part in my life, know that I am thinking of you too!)

This has been a challenging road and without you it wouldn't have been as interesting. Thank you all for your support, for the adventures and for all the fun!

1. INTRODUCTION

The internship inherent to this report was carried out within the Capital Markets Division of Banco Atlântico Europa, a private bank headquartered in Lisbon that focuses on international private, corporate, and institutional clients. It offers various financial services and products to support foreign investment in Portugal. The division's main mission is to present investment solutions such as alternative investments, risk hedging solutions, financial advisory, and portfolio construction to clients. Furthermore, the division also focuses on delivering research on the current financial markets and overall macroeconomic news and conditions to provide the client management and sales team with the most current and accurate information. This internship opportunity allowed me to navigate the daily operations of a financial institution and observe some theoretical concepts being adapted and implemented by professionals. I was entrusted with some autonomous tasks: my primary responsibility was the preparation of weekly training presentations that featured relevant market news and updates, macroeconomics indicators, geopolitical events, performance analysis of various assets with market data extracted from Bloomberg, and investment strategies. Simultaneously, I accompanied the team's activities, assisting them on on-going projects, attending meetings with partners and even some external events.

This environment enabled me to significantly steepen my learning curve regarding financial markets, asset allocation and the diversity of investor profiles. It broadened my perspective on the variety of available assets beyond bonds and stocks, introducing me to truly multi-asset approaches, and how change is constantly requiring investors to adapt to new investment paradigms. I easily observed that every client has unique needs and is highly selective about where they allocate its capital. Thus, it rapidly became obvious that managing a portfolio needs to be a process extremely personalized to each client – it is crucial the application of an adequate asset allocation strategy so that distinct risk-return profiles can all achieve their investment goals despite their limitations and preferences.

While I gained plentiful insights on the practical methods utilized in portfolio construction and management by professionals, I realized that they do not entirely rely on the mathematical rigor characteristic of the principles of financial theory. In fact, asset allocation choices are not made based on raw data and rigorous and exact performance metrics and optimization methods as Modern Portfolio Theory implies. Moreover, investors' risk aversion is not precisely quantified, and it is not directly incorporated in the asset allocation models.

Witnessing the divergence between academic theory and real-world asset allocation and portfolio construction practices, I was motivated to conduct a thorough comparative analysis between both frameworks.

Hence, this study focuses on the application of theoretical principles of mean-variance theory and expected utility theory to analyze asset allocation choices for investors that show varying risk aversion levels. These risk preferences are strictly described by the relative risk aversion coefficient. It is detailed the role of each asset in each portfolio. These optimized portfolios are contrasted with static benchmarks portfolios studied by asset management professionals. Additionally, a comparative analysis is developed, particularly in terms of efficiency, of the inclusion of alternative assets on portfolios.

The remaining text is organized as follows: Section 2 presents review of the existing literature that proved to be relevant to this study; Section 3 provides a detailed description of the data used and Section 4 explains the employed methodology; Section 5 discusses the results obtained; and, finally, Section 6 summarizes the main conclusions and, considering the research's limitations, provides suggestions for future research on the topic.

2. Literature Review

A definitive consensus has yet to emerge regarding the construction of the most efficient and perfectly adequate portfolio for each individual investor: what and how many assets to incorporate and in what precise proportion. Brinson et al. (1986), by studying a large range of pension plans, sought to distinguish the effects of investment policies such as asset allocation and active management strategies like market timing and security selection. Their findings, ultimately, concluded that asset allocation accounted for 93.6% of the variation in portfolio returns, overriding the importance of other investment decisions. Recognizing the relevance of these conclusions, this study focuses on how different asset allocations can accommodate the preferences of investors with distinct profiles. Bodie et al. (2018), while supporting the notion that asset allocation acts as the main determinant of a portfolio's performance and risk profile, describes an approach to portfolio construction dictating that a macro-level allocation decision across a broader asset classes should precede the picking of individual securities, avoiding the attempt of optimizing a portfolio in a sub-optimal asset universe. Thus, the first step of this research is the establishment of three main asset classes – fixed income, equity and alternatives – and only then, considering the insights and projections of main asset management institutions, detailing the choice of individual securities.

Industry professionals address various concerns for the upcoming 2026 investment paradigm. Major asset management companies agree that the established and more traditional duality of equity and bonds needs to be fundamentally transformed and adapted to the new market and macroeconomic conditions. The famous 60/40 portfolio benchmark – the 60% investment in equity and 40% in bonds – is no longer a balanced and resilient strategy. Nowadays, this model is much riskier (Goldman Sachs Asset Management, 2025) since its purpose relies on the negative correlation between both asset classes and it is predicted that these will start performing more similarly during market disruptions (Fidelity International, 2025; J.P. Morgan Asset Management, 2025). Hence, the use of long-term and government bonds as protection and risk-mitigating instruments becomes less reliable (BlackRock Investment Institute, 2025; UBS Asset Management, 2025). The degree of equity market concentration has also become a vulnerability for investors (Goldman Sachs Asset Management, 2025). HSBC Asset Management (2025) enunciates the need to *"diversify the diversifiers"*, advising investors to include alternative assets to ensure their portfolios are resilient. J.P. Morgan Asset Management (2025) reinforces this idea, calling for a structural evolution regarding portfolio construction. Commodities are being taken as a critical hedge against systemic risk and inflation. Fidelity International (2025) highlights that gold could provide protection in an environment of geopolitical instability and UBS Asset Management (2025) even recommends a 5% gold allocation. Regarding other real assets, J.P. Morgan Asset Management (2025) draws attention to real estate investments whereas Goldman Sachs (2025) and BlackRock Investment Institute (2025) emphasize the importance of infrastructure, specially, regarding data centers, the energy sector and logistics, as this investment is essential to bridge the technologic gap between Europe and the United States. Geographic diversification is viewed as another strategic necessity. Due to the expected convergence with the American market, investors are advised to shift towards European, Asian and other emerging markets as these are expected to continue to offer a more favorable risk-return trade-off (Fidelity International, 2025; Goldman Sachs Asset Management, 2025; J.P. Morgan Asset Management, 2025).

Attending to these professional insights, the five assets used in this research are selected also considering that the power of diversification to mitigate idiosyncratic risk does not lie only in choosing a large range of assets but more on low or negative correlation between them, as combining them reduces the portfolio's total standard deviation without a proportional sacrifice in expected return (Bodie et al., 2018).

While identifying the exact securities within a multi-asset context is a crucial initial phase, the main challenge of determining the precise weights allocated to each one remains. Bodie et al. (2018) describes a two-step approach to portfolio construction designated by Separation Property: the first step is the solution of a mathematical risk-return optimization problem to define the optimal risky portfolio while the following step introduces human and behavioural factors to adjust optimization to individual investors. The former involves the applicability of mean-variance optimization, and the determination of the tangent portfolio by maximizing the Sharpe ratio (Markowitz, 1952; Sharpe, 1966). The latter seeks to determine allocation of between the optimal risky portfolio and the risk-free rate subject to the risk profile of the investor. This approach is adopted with a few modifications, as the allocation differences for various risk profiles are analyzed only considering the investment in risk assets.

Risk aversion (or risk tolerance) is a behavioural choice simultaneously driven by the desire to achieve a certain goal and the fear of loss, in a context of uncertainty (Kogan & Wallach, 1964). Grable et al. (2022) verified that the wealth and level of risk aversion of an investor are significant determinants of the structural asset allocation of its portfolio. Even though the stocks to bonds ratio is the most evident proof of the link between these variables, their findings also suggest that, as risk aversion decreases, there is a shift from more traditional investments and safer assets to the inclusion of alternative assets, such as commodities. Yook & Everett (2003) also concluded that the proportion of equity held in a portfolio is closely related to the investor's tolerance of risk. Barsky et al. (1997) ultimately confirmed, based on empirical data from a health and retirement study, that there is, in fact, a significant heterogeneity of profiles regarding risk preferences, which explains the coexistence of vastly different portfolio choices – investors who only care about capital preservation and hold only risk-free assets and other who hold large amounts of volatile equities. This broad spectrum of risk aversion levels raises the need to identify and quantify these distinct individual preferences, allowing a better customization of multi-asset portfolio strategies for each unique investor.

In this context, DeMiguel et al. (2009) evaluated the widespread implementation of simple, fixed asset allocations across the financial industry, rather than relying on optimization methods. They discovered that, even though these portfolios may appear to perform reasonably, they are inefficient, typically delivering lower Sharpe ratios when compared to mathematically optimized portfolios. Thus, these common ad-hoc institutional models force investors into sub-optimal investments, not allowing for the maximization of utility. A clear example of the reality of these practices, in accordance with the aforementioned transformation of asset allocation views, is the attempt to substitute the previous 60/40 standard benchmark with another static model, introducing a new 40/30/30 approach – 40% equity, 30% bonds and 30% alternatives (Mauchand, 2024).

Similarly, Gaspar & Oliveira (2024), by studying the gap between portfolio theory and asset allocations proposed by robo-advisors, concluded that these aggregate investors into some pre-determined risk categories and consequently, match a correspondent set asset allocation. They do not capture the full scope of the investor's risk profile, therefore, do not match its exact risk aversion level to the correct portfolio weights. These authors demonstrate that it is possible to quantify an investor's risk aversion level using the coefficient of relative risk aversion (RRA) to solve customized optimization problems to define the optimal portfolio weights for investors with varying risk preferences. as they analyze RRA values of $\{-1, 0, 1, 2, 3, 4, 5, 6\}$, indicating an interval between 0 and 3 to be a realistic range.

This report follows the methodology of this paper with slight modifications, as it studies an extended range: it considers, for risk averse investors RRA values of $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ as suggested by Mehra & Prescott (1985). This allows this research to examine allocation strategies across a broader spectrum of investors risk profiles.

3. Data

3.1. Asset Selection

The selection of assets for this research is centered on the insights portraited in market outlooks for 2026 published by main asset management institutions (BlackRock Investment Institute, 2025; Fidelity International, 2025; Goldman Sachs Asset Management, 2025; HSBC Asset Management, 2025; J.P. Morgan Asset Management, 2025; UBS Asset Management, 2025). Among these, there seems to be consensus that a transition from binary bonds-equity portfolios to a multi-asset investment strategy is a must to ensure a true resilience against the upcoming market, economic and geopolitical conditions. The main objective of this work is to bridge the gap between these institutional insights and practical implementation of different allocation strategies for varying investor profiles. To construct a diversified investment universe, this selection is organized into three pillars: equity, representing growth, fixed income, as a vehicle for capital preservation, and alternatives, for strength.

All assets are represented exclusively by exchange-traded funds (ETFs) due to their advantages. They allow for instant diversification, offer low management fees as they are passively managed and offer a more liquid and transparent approach as information such as price and holdings are published daily.

Considering the perspective of Portuguese investors, only accumulating (Acc) classes – dividends are automatically reinvested – are considered due to tax efficiency as there is an established 28% tax on dividends in Portuguese law.

Additionally, continuing to consider this perspective: all assets are denominated in Euros (EUR). This allows for the results' analysis to focus strictly on diversification benefits, risk-return profiles of each asset and portfolio and the different allocation approaches, rather than currency fluctuations and exchange rate risk. To also follow European regulations, only UCITS-compliant ETFs were included. UCITS (Undertaking for Collective Investment in Transferable Securities) is a set of European Union directives created to harmonize investment rules and for investor's protection.

Within this framework, Table 1 details the selected assets. Table A.1 (appendix) provides a more detailed description of each ETF.

Class	Sub-class	ETF
Fixed Income	World Treasury Bonds	Xtrackers II ESG Global Aggregate Bond UCITS ETF 5C EUR hedged
Equity	European Equity	iShares Core MSCI Europe UCITS ETF EUR (Acc)
Equity	Emerging Markets Equity	iShares Core MSCI Emerging Markets IMI UCITS ETF (Acc)
Alternatives	Infrastructure	Xtrackers S&P Global Infrastructure Swap UCITS ETF 1C
Alternatives	Commodities - Gold	iShares Physical Gold ETC

Table 1 - Classification of the selected ETFs.

3.2. Data Source and Description

The ETFs presented above are all recent: inception dates vary from 2005 to 2015 so only 10 years of common historical price data (2015-2025) is available. Due to this critical limitation, although it is a common practice in portfolio analysis, the use of historical data as inputs for the optimization would have led to biased results. Consequently, this study resorts to J.P. Morgan's 2026 *Long-term Capital Market Assumptions*. These estimates provide a 10-to-15-year horizon for expected returns, annualized volatility (standard deviation) and correlations. In line with this time horizon, the 2.8142%[1] 10-year long-term yield for German government bonds is adopted as the risk-free rate (R_f).

To ensure consistency, each ETF was mapped to its corresponding asset class within J.P. Morgan's framework. This alignment guarantees that the data utilized in the optimizations reflect the selected underlying assets. Table 2 resumes this correspondence.

[1] Organization for Economic Co-operation and Development, Interest Rates: Long-Term Government Bond Yields: 10-Year: Main (Including Benchmark) for Germany [IRLTLT01DEM156N], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/IRLTLT01DEM156N, Feb 23, 2026.

ETF	J.P. Morgan Class
Xtrackers II ESG Global Aggregate Bond UCITS ETF 5C EUR hedged	World Governement Bonds hedged
iShares Core MSCI Europe UCITS ETF EUR (Acc)	European Large Cap
iShares Core MSCI Emerging Markets IMI UCITS ETF (Acc)	Emerging Markets Equity
Xtrackers S&P Global Infrastructure Swap UCITS ETF 1C	Global Core Infrastructure
iShares Physical Gold ETC	Commodities

Table 2 - Mapping of the selected ETFs to J.P. Morgan's classes.

Table 3 presents the individual risk-return profile of each asset as well as the Sharpe ratio.

	World Bonds	Eur Equity	EM Equity	Infrastructure	Gold
Performance Metrics					
Exp. Return ($\bar{R}$)	3.58%	7.43%	8.48%	6.51%	6.03%
Volatility (σ)	4.03%	14.96%	16.78%	11.48%	15.63%
Sharpe Ratio	0.1900	0.3085	0.3377	0.3219	0.2057

Table 3 - Performance metrics for the individual assets.

The Sharpe ratio (SR) is calculated for each asset, as follows in Equation (1) and it measures, per unit of risk, the excess expected return above the risk-free rate.

$$\text{Sharpe Ratio}_i = \frac{\bar{R}_i - R_f}{\sigma_i} \tag{1}$$

Table 4 represents the correlation matrix (C).

	World Bonds	Eur Equity	EM Equity	Infrastructure	Gold
World Bonds	1.00	-	-	-	-
Eur Equity	-0.12	1.00	-	-	-
EM Equity	-0.12	0.73	1.00	-	-
Infrastructure	0.11	0.28	0.31	1.00	-
Gold	0.33	-0.16	0.01	0.14	1.00

Table 4 - Correlation Matrix

Evaluating the individual metrics shown in Table 3 and 4, it is evident the differences between the selected instruments. As expected, equity presents more aggressive risk-return profiles, exhibiting both higher expected returns and volatility. Conversely, bonds have the lowest expected return and volatility, characteristic of a more conservative asset. While both alternatives have similar expected return values, they present distinct risk dynamics. Infrastructure's volatility (11.48%) is below the volatility values for both equity assets, whereas gold presents a higher risk level (15.63%) that sits between Eur and EM equity. In terms of standalone efficiency, EM equity yields the highest Sharpe ratio (0.3377), followed by Infrastructure (0.3219) that outperforms Eur equity (0.3085) despite having lower expected returns. World bonds and gold exhibit significantly lower performance on a risk-adjusted basis (0.1900 and 0.2057 Sharpe ratios, respectively).

Since maintaining a multi-asset investment approach is one of the priorities of this research, the correlation between assets plays a more important part in its selection than their individual characteristics. World bonds display an identic negative correlation with both Eur equity and EM equity (-0.12), highlighting their defensive properties particularly during equity market downturns. Infrastructure presents a low correlation with world bonds (0.11). Gold presents hedging properties against equity risk, showing a negative correlation with Eur equity (-0.16) and a near-zero correlation with EM equity (0.01). In general, apart from the higher correlation between Eur equity and EM equity (0.73), all the other correlations between the selected assets are relatively low or negative, the highest being 0.33.

4. Methodology

4.1. Mean-Variance Optimization

Mean-variance optimization (MVO), grounded in the principles of Modern Portfolio Theory (Markowitz, 1952), defines the set of optimal portfolios that offer the highest expected return for a given level of volatility – the Efficient Frontier – relying of expected returns, variance and correlation as raw inputs. Additionally, to employ the optimization process, the variance-covariance matrix (V) needs to be constructed as described in (2), completing the set of MVO inputs.

$$V = DCD \tag{2}$$

where D is a diagonal matrix of standard deviations and C is the correlation matrix.

Denoting $R_i\,(i = 1,2,\dots,n)$ as the expected return of each asset i, $\sigma_{ij}\,(i,j = 1,2,\dots,n \quad i \neq j)$ as the covariance between the returns of two distinct assets i and j and $w_i\,(i = 1,2,\dots,n)$ as the weights of each asset i in a portfolio with n risky assets, the following vector notation can be considered:

$$\bar{R} = \begin{pmatrix} \bar{R}_1 \\ \bar{R}_2 \\ \vdots \\ \bar{R}_n \end{pmatrix}, \quad W = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix}, \quad V = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_n^2 \end{pmatrix} \tag{3}$$

The expected return ($\bar{R}_p$) and volatility (σ_p) of the portfolio are given by:

$$\bar{R}_p = W'\bar{R} \tag{4}$$

$$\sigma_p = \sqrt{W'VW} \tag{5}$$

All optimization procedures present in this research are conducted using Excel Solver tool, under two constraints: no short selling (non-negative asset weights, $w_i \geq 0$) and full investment ($\sum w_i = 1$). Due to the inequality constraints, all portfolios must be evaluated numerically.

The efficient frontier (EF) is designed by the solution of a series of mean-variance optimization problems, maximizing expected return for each volatility level. Particularly, three benchmark portfolios are derived.

The minimum variance portfolio (MV) is obtained by minimizing the overall portfolio risk;

$$\min \ \sigma_p = \sqrt{W'VW} \tag{6}$$

$$s.t \begin{cases} \sum_i wi = 1 \\ \forall i, w_i \geq 0 \end{cases} .$$

The homogenous portfolio(1/N) is a naive baseline for asset allocation as it forces equal weights for all assets ($\forall i \ w_i = w_j$) .

The tangent portfolio (T) delivers the most efficient risk return trade-off, maximizing the Sharpe ratio;

$$\max \ SR_p = \frac{W'\bar{R} - R_f}{\sqrt{W'VW}} \tag{7}$$

$$s.t \begin{cases} \sum_i wi = 1 \\ \forall i, w_i \geq 0 \end{cases} .$$

4.2. Expected Utility Theory

To transform a mean-variance study into an investor-specific analysis, this research employs the Expected Utility Theory (EUT) (Von Neumann & Morgenstern, 1947) to determine the optimal allocation for investors with different risk preferences.

Under uncertainty, EUT allows for the model of financial risk-taking behavior, by introducing the investor as one that wants to maximize its satisfaction, thus its utility (U). Apart from a few cases, the non-linearity of utility functions complicates the determination of its expected values, $E[U(W)]$. It is, however, possible to approximate this function using a second-order Taylor approximation (the full mathematical proof is provided in Appendix B) and obtain the risk tolerance function f of $\bar{R}_p$ and σ_p: $E[U(W)] \approx f(\sigma_p, \bar{R}_p)$. More precisely;

$$E[U(W)] \approx \bar{R}_p - \frac{1}{2} RRA\left(\bar{R}_p^2 + \sigma_p^2\right) \quad . \tag{8}$$

The relative risk aversion coefficient (RRA) defined by Arrow (1965) and Pratt (1964) is given by (W)

$$RRA(W) = -\frac{WU''(W)}{U'(W)} \quad , \tag{9}$$

and, for the purpose of Equation (8), is evaluated at initial wealth.

Varying RRA values represent how much an investor's utility is penalized by each unit of risk. Thus, risk seeking investors present negative RRA values ($RRA < 0$), risk neutral investors exhibit $RRA = 0$ and risk averse investors are characterized by positive values ($RRA > 0$). Within the universe of the risk averse, there are distinct risk profiles the degree of risk aversion significantly varies. Naturally, an investor that presents higher risk aversion prefers more conservative investments, and one that shows more risk tolerance prefers more aggressive investments.

In fact, for the risk-averse, the coefficient can take any positive value. However, historical data and investor's behaviour studies have narrowed it to a realistic range of values. As in Mehra & Prescott (1985), this study analyzes various RRA values within that range from 1 to 10 and seeks to see how they position themselves along the efficient frontier. To determine the optimal portfolios for each investor profile, the following maximization problem is solved for each RRA value (RRA = {1,2,3,4,5,6,7,8,9,10}).

$$\max \ f(\sigma_p, \bar{R}_p) = \bar{R}_p - \frac{1}{2} RRA(\bar{R}_p^2 + \sigma_p^2) \tag{10}$$

$$s.t \begin{cases} \sum_i wi = 1 \\ \forall i, w_i \geq 0 \end{cases} .$$

Additionally, a more detailed asset allocation analysis is conducted to investigate how investors' risk profiles also limit the part that different instruments – fixed income, equity and alternative – play in their optimal portfolios.

4.3. Comparative Analysis: Restricting the Asset Universe

To study the impact of asset diversification, particularly the introduction of alternative assets, on the risk-return profile of portfolios, a comparative framework is established by introducing investment restrictions.

The unrestricted case corresponds to the one introduced earlier in Sections 3.1, 3.2 and 4.1 that considered all five of the selected assets – world bonds, Eur equity, EM equity, infrastructure and gold.

To define the restricted case, the set of considered assets is limited to fixed income and equity – only the investment in world bonds, Eur equity and EM equity is allowed. Thus, the following investment constraint is implemented:

$$w_{infrastructure} = 0 \quad \text{and} \quad w_{gold} = 0 \qquad . \qquad (11)$$

The intent of this analysis is to evaluate the existent efficiency differences when transitioning from the restricted to the unrestricted asset universe. By applying the optimization methodology presented in section 3.1, the respective efficient frontier is constructed for both cases, defining a spatial relationship between the two cases.

Additionally, the tangent and the minimum variance portfolios are also derived and used as comparative metrics.

For the 5 assets unrestricted case, both optimization problems are designed and solved exactly as in (5) and (6).

For 3 assets restricted case:

Tangent:
$$\max \ SR_p \qquad (12)$$
$$s.t \begin{cases} \sum_{5} w_i = 1 \\ w_{infrastructure} = 0 \\ w_{gold} = 0 \\ \forall i \in \{1, \dots, 5\}, \quad w_i \geq 0 \end{cases} .$$

Minimum variance:
$$\min \ \sigma_p \qquad (13)$$
$$s.t \begin{cases} \sum_{5} w_i = 1 \\ w_{infrastructure} = 0 \\ w_{gold} = 0 \\ \forall i \in \{1, \dots, 5\}, \quad w_i \geq 0 \end{cases} .$$

4.4. Institutional Ad-hoc Portfolio Benchmarks

To further investigate the influence of alternative assets, in the case of this research, the influence of infrastructure and gold in portfolio efficiency, a performance analysis of pre-determined static allocations is performed. The four ad-hoc portfolios' weights are described in Table 5.

Portfolio Designation	Equity	Bonds	Alternatives
60/40	60%	40%	0%
55/35/10	55%	35%	10%
50/30/20	50%	30%	20%
40/30/30	40%	30%	30%

Table 5 - Ad-hoc portfolio weights.

This choice of portfolio weights is not arbitrary. It follows a methodology employed by J.P. Morgan Asset Management (2026) in *2026 Long Term Capital Market Assumptions* – it is studied the gradual substitution of core assets (equity and bonds) with diversified alternatives such as real estate, hedge funds, real assets, private credit and private equity. These portfolios with an expanded asset set are compared with a dual baseline, a portfolio restricted to equity and bonds, in terms of their risk-return profile.

Adapting this practice to the scope of this study, the equity weight corresponds to the sum of Eur equity and EM equity's weight, the bonds' weight is the world bonds' allocation and the alternatives' weight is the aggregated weight of infrastructure and gold. Even though the weights of the three asset classes are fixed, optimization is still necessary to determine the precise allocation of the individual assets.

Hence, to evaluate the performance of each of these portfolios, an optimization problem is solved for each one, incorporating the needed investment constraints, and maximizing the Sharpe ratio.

60/40 portfolio: $\max SR_p$ (14)

$$s.t.\begin{cases}\sum_5 w_i = 1\\ w_{Eur\ equity} + w_{EM\ equity} = 0.4\\ w_{world\ bonds} = 0.6\\ w_{infrastructure} = w_{gold} = 0\\ \forall i \in \{1, \dots, 5\}, \quad w_i \geq 0\end{cases}.$$

55/35/10 portfolio: $\max SR_p$ (15)

$$s.t.\begin{cases}\sum_5 w_i = 1\\ w_{Eur\ equity} + w_{EM\ equity} = 0.55\\ w_{world\ bonds} = 0.35\\ w_{infrastructure} + w_{gold} = 0.10\\ \forall i \in \{1, \dots, 5\}, \quad w_i \geq 0\end{cases}.$$

50/30/20 portfolio: $\max SR_p$ (16)

$$s.t.\begin{cases}\sum_5 w_i = 1\\ w_{Eur\ equity} + w_{EM\ equity} = 0.50\\ w_{world\ bonds} = 0.30\\ w_{infrastructure} + w_{gold} = 0.20\\ \forall i \in \{1, \dots, 5\}, \quad w_i \geq 0\end{cases}.$$

40/30/30 portfolio: $\max SR_p$ (17)

$$s.t.\begin{cases}\sum_5 w_i = 1\\ w_{Eur\ equity} + w_{EM\ equity} = 0.40\\ w_{world\ bonds} = 0.30\\ w_{infrastructure} + w_{gold} = 0.30\\ \forall i \in \{1, \dots, 5\}, \quad w_i \geq 0\end{cases}.$$

5. Results

5.1. Portfolio Optimization and Efficient Frontier

Following the established methodology, this section presents the empirical results of mean-variance optimization (MVO). Within the selected asset universe, Table 6 shows the performance of the homogenous portfolio (1/N), the minimum variance portfolio (MV) and the tangent portfolio (T).

	1/N	MV	T
Asset weights			
World Bonds	20.00%	87.11%	45.64%
Europe Equity	20.00%	5.96%	15.07%
EM Equity	20.00%	4.02%	13.07%
Infrastructure	20.00%	2.91%	15.19%
Gold	20.00%	0.00%	11.03%
Performance Metrics			
Exp. Return	6.41%	3.68%	5.52%
Volatility	7.40%	4.09%	5.41%
Sharpe Ratio	0.4855	0.3472	0.4986

Table 6 - Asset weights and performance metrics for the homogenous, the minimum variance and the tangent portfolio.

The homogenous portfolio generates the highest expected return (6.41%) and volatility (7.40%), but it naturally comes in second, after the tangent portfolio, considering the Sharpe ratio (0.4855).

In contrast, the MV portfolio, as expected, minimizes risk at 4.09%. It sacrifices expected return, exhibiting a low value of 3.68% and results in a lower Sharpe ratio (0.3472). It also suffers from a heavy concentration in bonds (87.11%).

Lastly, the T portfolio offers the most balanced asset allocation strategy. It is well-diversified as it maintains significant weights for all the selected assets (at least 10%). The T portfolio performed at a mid-point level with values of 5.52% expected return and 5.41% volatility. However, as predicted by mean-variance theory, it maximizes the Sharpe ratio at 0.4986, outperforming both 1/N and MV portfolios through an optimized efficient and diversified multi-asset approach.

Figure 1 allows the visualization of the numeric breakdown exposed in Table 6. It maps the three portfolios alongside all individual assets – world bonds, European equity (Eur equity), emerging markets equity (EM equity), infrastructure and gold – highlighting the efficiency gains achieved through diversification. The graph illustrates the risk-return trade-offs inherent to each allocation strategy.

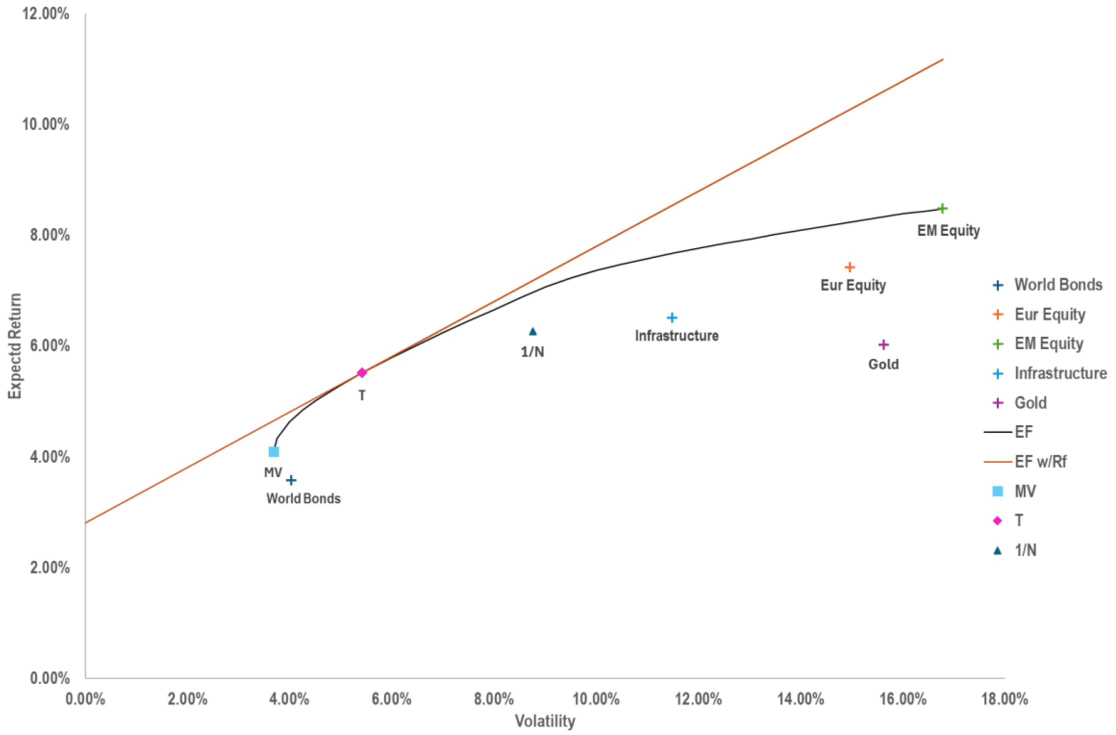

Figure 1 - Graphic representation of the efficient frontiers, individual assets, the homogenous, the minimum variance and the tangent portfolios.

The efficient frontier (EF), with and without the risk-free asset appear in orange and black, respectively, represents the set of optimal portfolios that provide the maximum expected return for every volatility level. The EF without the riskless asset originates from the MV portfolio, which represents the least volatile optimized investment option, also in accordance with the data presented in Table 6.

The orange line represents the combinations of the risk-free asset with the tangent portfolio. ($\bar{R}_p = 0.0284 + 0.4986\sigma$) . Its y-axis intercept, at 2.84%, the risk-free rate.

The colored cross markers represent the individual risk-return profile of every asset. Apart from EM Equity, all assets lie below the hyperbola. This confirms that investing in individual assets is inefficient and that idiosyncratic risk can be significantly reduced through diversification. Being the most volatile asset with the highest value for expected return, EM Equity anchors at the highest risk point of the EF without the riskless asset.

5.2. Risk Aversion and Optimal Asset Allocation

While the mean-variance framework identifies the tangent portfolio as the most efficient risk-return trade-off, when the riskless asset is considered, it may not be the investment option that grants maximum satisfaction for all investors, if only risky assets are considered. Here, the principles of the expected utility theory are applied to determine the specific portfolios along the hyperbola that maximize utility of investors with varying risk profiles, across a spectrum of relative risk aversion coefficients (RRA), ranging from more aggressive ($RRA = 1$) to more conservative ($RRA = 10$). Figure 2 depicts the position of the utility-maximizing portfolios for each RRA level.

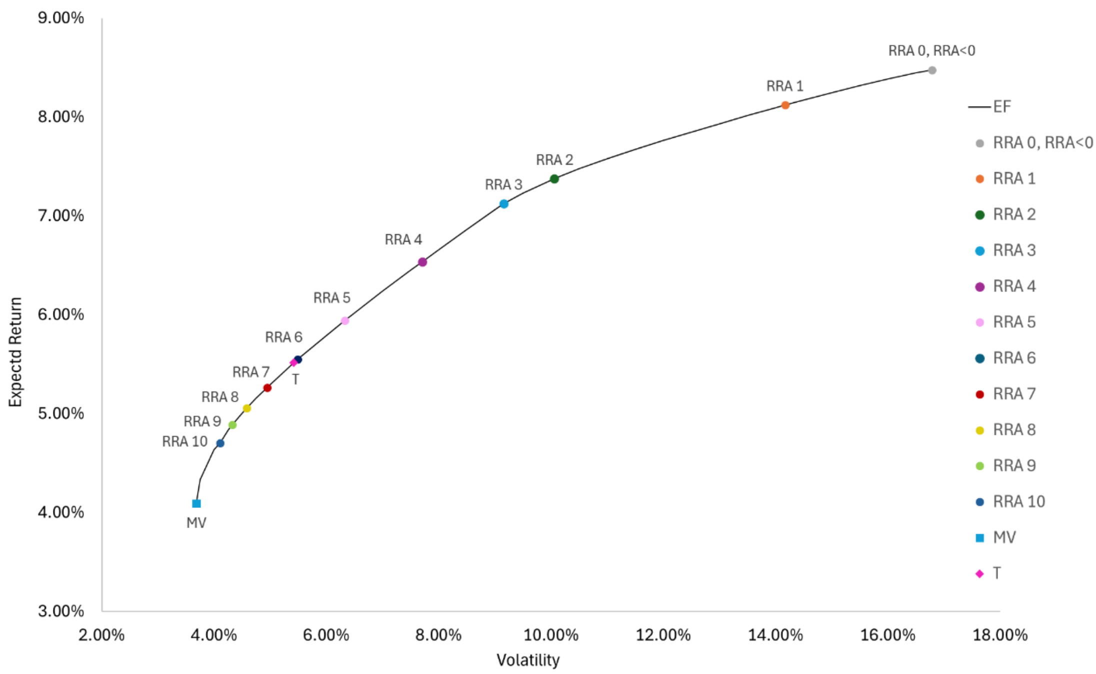

Figure 2 - Graphic representation of the efficient frontier, the minimum variance and tangent portfolio, and the optimal portfolios for relative risk aversion coefficients between zero and ten.

Risk-seeking ($RRA < 0$) and risk-neutral ($RRA = 0$) investors converge at the highest terminal point of the EF. Since they either pursue or are indifferent to risk, their utility is maximized solely by the highest expected return.

From the right to the left, risk aversion triggers a downward migration along the EF. Based on the observation of Figure 2, it becomes intuitive to aggregate investors into three profiles: aggressive ($1 \leq RRA \leq 4$), moderate for ($5 \leq RRA \leq 7$) and conservative for ($8 \leq RRA \leq 10$). Within the aggressive range, the portfolios are widely spaced, apart from the smaller gap between RRA 2 and 3, suggesting that, at lower levels of risk aversion, small incremental changes have a strong effect and result in significant allocation shifts. As the points move towards the center and the lower bound of the curve, the points begin to cluster, indicating that the addition of risk aversion loses effect as the overall risk aversion level increases. For a Moderate profile, portfolios gather around the tangent portfolio. In fact, the tangent portfolio approximately coincides with the portfolio that maximizes utility of investors with RRA 6. More exactly, the tangent portfolio corresponds to a utility-maximizing portfolio for $RRA = 6.1042$. Finally, in the conservative interval, the three portfolios appear closely positioned and as nearly equally spaced points, mid-way between the tangent portfolio and the MV portfolio.

The analysis of Figure 2 also reveals a notable distance between the portfolio corresponding to the most conservative investor (RRA=10) and the MV portfolio. This demonstrates that the RRA range of 1 to 10 suggested in the considered literature does not capture the entire EF. Since the MV corresponds to a utility-maximizing portfolio for a $RRA = 24.44$, an expanded RRA range of $[1;\ 25]$ is required to fully model the behaviour shift from aggressive return-seeking to extreme risk minimization.

Table 7 presents the compositions of the portfolios depicted in Figure 2, presenting the weight fluctuations for every asset as well as the performance metrics for each portfolio across the studied RRA range.

	RRA 0, RRA<0	RRA 1	RRA 2	RRA 3	RRA 4	RRA 5	RRA 6	RRA 7	RRA 8	RRA 9	RRA 10
Asset Weights											
World Bonds	0.00%	0.00%	0.00%	0.00%	15.62%	33.04%	44.65%	52.95%	59.17%	64.01%	69.42%
Eur Equity	0.00%	20.51%	24.00%	24.91%	21.68%	17.84%	15.28%	13.45%	12.08%	11.01%	9.77%
EM Equity	100.00%	73.67%	37.54%	25.36%	19.31%	15.69%	13.28%	11.56%	10.27%	9.27%	8.69%
Infrastructure	0.00%	0.00%	19.27%	26.33%	23.59%	18.72%	15.47%	13.15%	11.41%	10.05%	9.35%
Gold	0.00%	5.82%	19.19%	23.40%	19.81%	14.71%	11.32%	8.89%	7.07%	5.66%	2.77%
Performance Metrics											
Exp. Return	8.48%	8.12%	7.38%	7.13%	6.54%	5.94%	5.55%	5.27%	5.06%	4.89%	4.70%
Volatility	16.78%	14.17%	10.06%	9.16%	7.70%	6.33%	5.49%	4.94%	4.58%	4.32%	4.12%
Sharpe Ratio	0.3377	0.3747	0.4537	0.4709	0.4834	0.4947	0.4985	0.4920	0.4899	0.4804	0.4601

Table 7 - Asset weights and performance metrics for the utility-maximizing portfolios for risk aversion coefficients between zero and ten.

Moving from the most aggressive investor to the most conservative, both volatility and expected return steadily decrease. The Sharpe ratio rises as RRA increases up to a value of 6, peaking at 0.4985 and diminishing again for the more conservative portfolios. This is consistent with the almost perfect alignment between this portfolio and the tangent portfolio, that maximizes the Sharpe ratio. Regarding asset weights, for investors that prioritize only expected return ($RRA \leq 0$), the optimal portfolio is characterized by 100% concentration in EM Equity, asset offering the highest expected return. These risk profiles achieve the lowest Sharpe ratio, reflecting the indifference to risk or love of it. However, the introduction of the smallest degree of risk aversion triggers an immediate need for diversification, with the inclusion of the Eur equity (20.51%) and gold (5.82%), even for a highly aggressive investor ($RRA = 1$). It can be stated that by a RRA of 2, the portfolio becomes multi-asset, since four out of the five assets (excluding bonds) earn a significant weight: 24.00% Eur equity, 37.54% EM equity, 19.27% infrastructure and 19.19% gold.

The graphs of Figure 3, provide a detailed visual description of the fluctuations in weight of the aggregated asset classes – fixed income (world bonds), equity (Eur equity and EM equity) and alternatives (infrastructure and gold). The combined analysis of these graphs reveals the substitution between asset classes as the investor's priorities shift from gains to capital preservation.

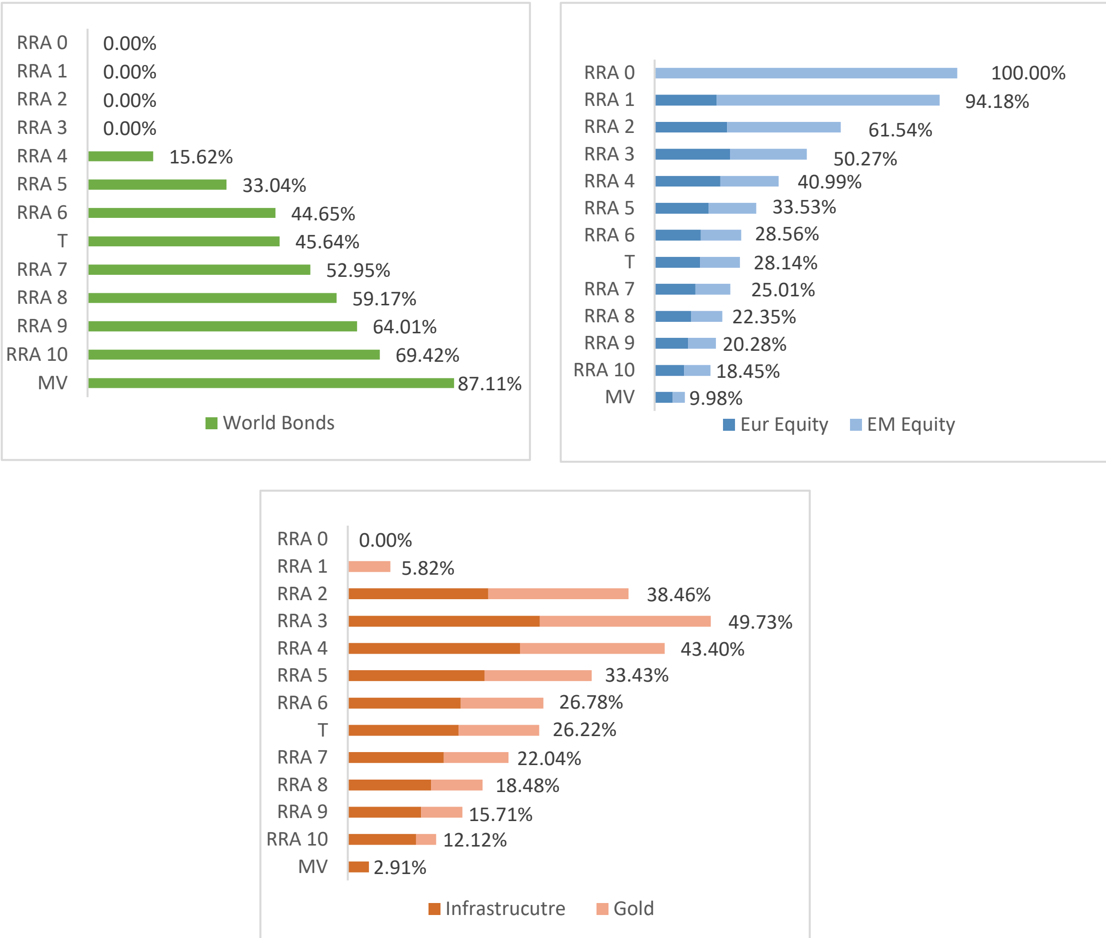

Figure 3 - Weight fluctuations for the aggregated asset classes: fixed income, equity and alternatives.

The substitution effect between fixed income and equity in portfolio construction is evident. As risk aversion rises, the weight of world bonds gradually increases while the combined weight of Eur equity and EM equity decreases.

Bonds are not incorporated in the optimized portfolios until an RRA of 4. It is only from RRA of 5 onward that do they considerably grow in weight as risk aversion intensifies, justifying the classification of these investors as moderate and conservative. It becomes clear that bonds are a mandatory tool for capital preservation: for conservative investors for which an increment of volatility significantly penalizes utility, bonds achieve a dominating presence in the portfolio, reaching 69.42% for $RRA = 10$ and topping at 87.11% for the MV portfolio.

The portfolio's exposure to equity starts at 100% for the risk-seeking and risk-neutral investors and progressively declines as the RRA increases to accommodate the dominance of bonds for more conservative profiles. Looking closer to each asset weight, EM equity is the primary driver for aggressive return seeking investors (by $RRA = 4$, only carries a 19.31% weight, down from 73.67% for $RRA = 1$). Conversely, Eur equity does not follow a linear decline: its weight increases from 20.51% ($RRA = 1$) peaking at 24.91% ($RRA = 3$), diminishing for superior RRA values. Additionally, the weight of Eur equity overrides the weight of EM equity for $RRA \geq 4$. EM equity is a high-risk asset that offers more aggressive profiles the desired higher expected returns. Eur equity still acts as a growth asset but allows moderate investors to mitigate volatility. As the investor becomes conservative ($RRA \geq 8$), both equity assets are marginalized to accommodate the dominance of bonds, collectively representing less than 20% of the portfolio.

On the contrary, the alternatives asset class does not strictly increase or decrease with risk aversion. Combined, the alternative asset weights peak at 49.73%, almost a 50% allocation for an RRA of 3 but still maintain a relevant weight (more than 20%) for RRA 2 through 7. This asset class seems to play a role of growth, as an alternative to equity, but also of some safety, as an alternative to low-return bonds. Individually, infrastructure and gold's weights reach their maximum value for an RRA of 3 (26.33% and 23.40%, respectively) but, as risk aversion goes through the moderate and conservative profiles, Gold is discarded much more rapidly when compared to infrastructure: for the most conservative investor (RRA=10), the latter maintains a resilient weight of 9.35% while the former comes down to a small weight of 2.77%. This suggests that infrastructure presents superior defensive properties, most likely due to the more favorable low correlation to bonds (0.11 against 0.33 for gold) whereas gold offers a more beneficial correlation to equity (-0.16 and 0.01 against 0.28 and 0.31 for infrastructure). Similarly to equity, regarding the MV portfolio, alternatives get overpowered by bonds, with a combined weight of merely 2.91%.

Diversification shows to be beneficial for all risk-averse investors. Nonetheless, fixed income is the preferred asset class for the most conservative, equity is the for the most aggressive, and alternatives become the prominent for the most moderate. Yet, it is remarkable that significant weights in alternative assets drive the portfolio closer to the most efficient point in the EF, the tangent portfolio.

5.3. Impact of Alternative Assets on Efficiency

In this section, Figure 4 and Table 8 show results that act as empirical evidence that expanding the investment universe beyond bonds and equity is beneficial for all risk-averse investors, regardless of the level of risk aversion. Portfolios restricted to only bonds and equity have been directly compared with portfolios that include alternatives (framework considered in the previous sections).

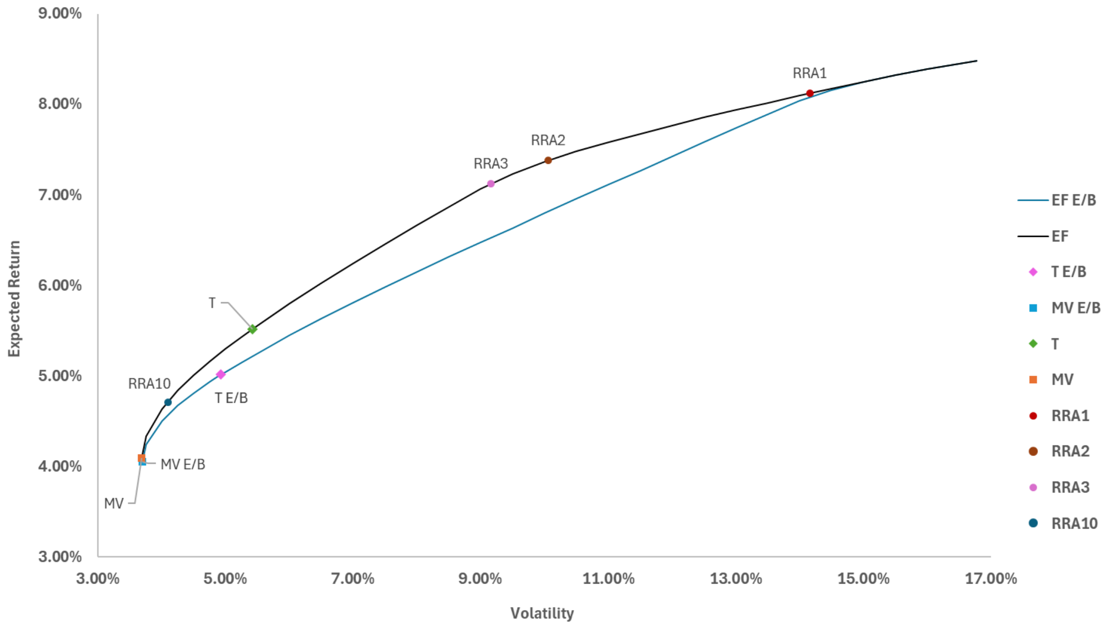

Figure 4 – Unrestricted efficient frontier and efficient frontier restricted to bonds an equity.

In Figure 4, two efficient frontiers are illustrated: the blue line corresponds to EF E/B that only counts with fixed income and equity assets (world bonds, Eur equity and EM equity) whereas the black line corresponds to the EF shown in Section 5.1, that considers all the selected assets (adding infrastructure and gold).

The EF sits consistently above the EF E/B: this positioning visually confirms the dominance of the use of alternative investments across the entire risk spectrum. The vertical distance between the two lines represents an "efficiency premium" offered by the inclusion of alternatives in portfolios for that volatility level.

The biggest gap occurs for investors with an RRA around 3. These investors are the ones that incorporate the heaviest weight of alternatives (49.73% as seen in Figure 3) in their portfolio. However, this gap is significant for all investors (RRA 1 through 10). In fact, the two hyperbolas converge only in both extreme cases. At the lower bound, the MV portfolios sit close together, indicating that for extremely conservative investors, the capital preservation properties of bonds override the potential benefits of the inclusion of other asset classes such as alternatives. Conversely, at the upper bound, for investors with almost no risk aversion ($RRA < 1$) and, naturally, for risk seekers and neutrals ($RRA \leq 0$), the desire for the highest returns provided by equity assets, overpowers the benefits of diversification.

It becomes explicit that alternatives are transformative when it comes to asset allocation, especially for moderate-aggressive profiles. Investors who limit themselves to bonds and stocks are "stuck" on a sub-optimal frontier while those who consider the alternatives' universe can achieve a better risk-return trade-off for their portfolios.

Table 8 allows for a more attentive look at the differences between the MV and tangent portfolio within each investment paradigm, quantifying the differences in positioning seen in Figure 4.

	MV E/B	MV	T E/B	T
Asset Weights				
World Bonds	89.07%	87.11%	61.51%	45.64%
Europe Equity	6.41%	5.96%	15.13%	15.07%
EM Equity	4.52%	4.02%	17.36%	13.07%
Infrastructure	0.00%	2.91%	0.00%	15.19%
Gold	0.00%	0.00%	0.00%	11.03%
Performance Metrics				
Exp. Return	4.05%	4.09%	5.01%	5.52%
Volatility	3.69%	3.68%	4.93%	5.41%
Sharpe Ratio	0.3342	0.3472	0.4462	0.4986

Table 8 - Asset weights and performance metrics for the minimum variance and the tangent portfolio constructed under the unrestricted framework and under the one restricted to bonds and equity.

The MV portfolio presents only a slight advantage in expected return, volatility and Sharpe ratio, compared with MV E/B. Asset weights do not significantly change – there is only the reallocation of 2.91% to infrastructure. This suggests that, for the extremely conservative, the capital preservation properties of bonds satisfy those investors.

It is regarding the tangent portfolio, that there is more evident jump in efficiency and differences in weights. The restricted tangent, curiously, approximately gives a 60/40 bond-equity ratio, a well-known financial benchmark. The unrestricted tangent portfolio puts a combined 26.33% allocation in alternatives (15.19% infrastructure and 11.03% gold), diminishing the weight of bonds by approximately 16% (from 61.51% to 45.64%). With the incorporation of these assets, the tangent portfolio achieves a higher expected return (5.52% from 5.01%) with only a small increase in volatility (0.48%). More relevant, its Sharpe ratio is 0.4985 compared to 0.4462 if one considered only bonds and equity, representing, approximately, a 12% improvement in portfolio efficiency.

5.4. Performance Assessment of Pre-determined Allocations

To further demonstrate the superiority of an investment paradigm that includes alternative assets through a more practical and "real-world" approach, Table 9 and Figure 5 show the performance of portfolios that incorporate alternatives against the 60/40 (equity/bonds) benchmark. Table 9 quantifies the efficiency improvement involved in the shift from the binary approach to the inclusion of alternative assets, gradually up to 30%.

	60/40	55/35/10	50/30/20	40/30/30
Asset weights				
World Bonds	40.00%	35.00%	30.00%	30.00%
Europe Equity	28.44%	27.58%	26.55%	21.83%
EM Equity	31.56%	27.42%	23.45%	18.17%
Infrastructure	-	3.90%	6.14%	14.75%
Gold	-	6.10%	13.86%	15.25%
Performance Metrics				
Exp. Return	6.22%	6.25%	6.27%	6.12%
Volatility	8.23%	7.73%	7.38%	6.76%
Sharpe Ratio	0.4138	0.4442	0.4684	0.4889

Table 9 - Asset weights and performance metrics for ad-hoc portfolios.

The comparative analysis of the 60/40, 55/35/10 and the 50/30/20 is rather straightforward: as the alternatives' presence grows, expected return remains rather steady while volatility decreases. Compared to the 60/40 baseline, there is a a 0.5%

reduction in volatility with 10% weight in alternatives, a 0.85% decrease for 20% weight. The 40/30/30 portfolio presents itself as more interesting case. Maintaining the 60/40 portfolio as reference, even though, its expected return falls 10 basis points (from 6.22% to 6.12%), more importantly, this portfolio enables a significant "crash" in volatility, decreasing 1.47% from 8.23% to 6.76%. In terms of efficiency, an investor that fails to consider alternatives is limited to a Sharpe ratio of 0.4138. Efficiency peaks at 0.4889, for a 30% alternatives allocation, representing an 18% improvement over the 60/40 portfolio.

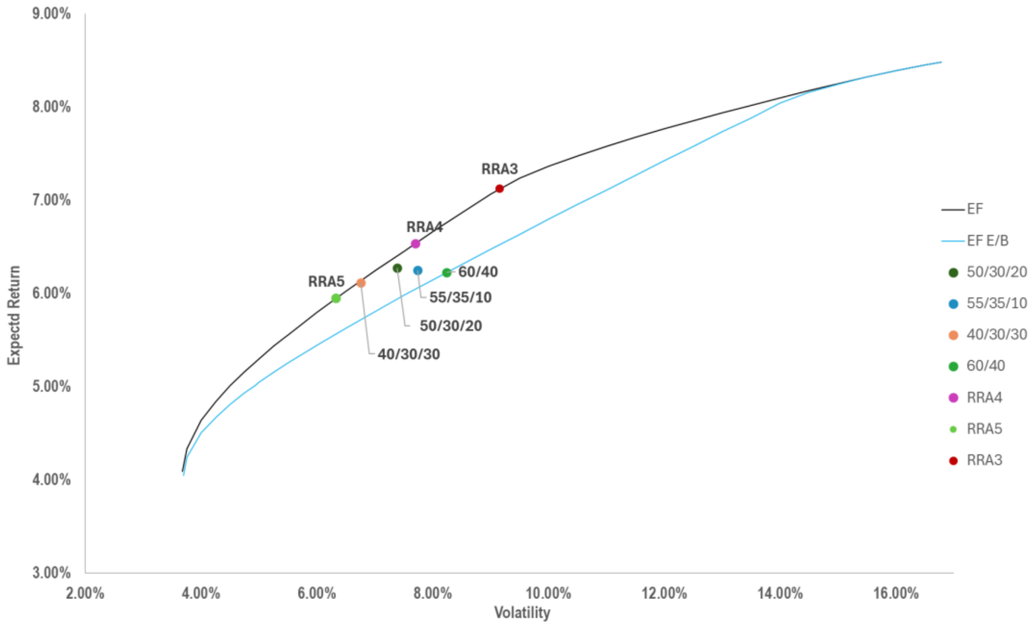

Figure 5 - Graphical Representation of ad-hoc portfolios and the efficient frontier.

Figure 5 illustrates the spatial relationship between the unrestricted efficient frontier (in black), the one restricted to bonds and equity (in blue) and the empirical performance of the studied ad-hoc portfolios.

The four portfolios approximately stand on a horizontal line, in concordance with the previous numerical analysis: expected return remains relatively stable while volatility diminishes as allocation to infrastructure and gold grows.

The 60/40 portfolio is the point furthest to the right, the most inefficient. Yet, it sits only slightly below the corresponding efficient frontier, EF E/B. As the allocation to alternatives increases to 10% (55/35/10), 20% (50/30/20) and 30% (40/30/30), a path to the left is paved. This is a highly beneficial movement to any investor as shows the ability to mitigate risk without sacrificing expected return.

Particularly, the 40/30/30 portfolio sits remarkably close to the unrestricted EF, slightly below. While it is not an optimized portfolio, it seems to capture a significant portion of the efficiency available without the need for complex and theoretical allocation methods. It reveals that even the simplest expansion to alternatives allows the achievement of a risk-return trade-off unmatched within the binary framework.

This graphical representation provides a final confirmation that a 30% allocation to alternative assets presents the most compelling case for diversification. It may even be considered as a practical proxy for some portfolios, making optimization accessible for “real-word” implementation. It is observable that all these portfolios cluster within a range of investors with relative risk aversion coefficients of 3 to 5, so moderate-aggressive profiles. Particularly, the 40/30/30 portfolio seems to be a simple and reasonable proxy for investors with an RRA of 5.

6. Conclusion

The main purpose of this master's final work is to explore the gap between financial theory and professional approaches particularly regarding asset allocation strategies. This research is conducted for varying risk profiles due to their heterogeneity among investors. The level of risk aversion, quantified by the coefficient of relative risk aversion, is shown to be a primary determinant of how an investor chooses to allocate their capital to each asset class in their portfolio.

The first analysis confirms the expected: conservative investors with higher risk aversion prioritize capital preservation and prefer investing in lower risk assets such as bonds whereas more aggressive profiles are more interested in higher returns and prefer more volatile assets such as equities. Moreover, it is demonstrated that alternative assets do not follow a linear trend, neither increasing nor decreasing their portfolio weights with risk aversion. All risk averse investors demand diversification, and the considered alternative assets play a relevant role for these profiles, especially for the moderate-aggressive part of the spectrum.

Conducting a comprehensive study on the impact of these alternative assets on portfolio performance by comparing those that incorporate them and portfolios that are composed only by bonds and equity, it is observed that the efficient frontier for the first case is entirely above the restricted one. Consequently, for each level of risk, a portfolio that includes alternatives is more efficient as it offers a higher expected return.

However, when confronting these mathematical and rigorous optimization approaches with the industry practices, significant differences arise. Financial institutions rely on a few fixed and pre-determined allocations, categorized by risk level, to advise their clients on their portfolio composition. Here, four of these ad-hoc models used by professionals are evaluated: the 60/40 portfolio and three others that include alternative assets in varying portions. While the ones that incorporate the additional asset class are more efficient than the 60/40 benchmark, all these models are proven to always be sub-optimal when compared to optimized portfolios. Nevertheless, particularly the 40/30/30 strategy presents itself as robust proxy for an optimized portfolio, as it sits very close to the efficient frontier.

Although this research is limited to a deliberately contained asset universe, it still maintains its relevance. It primarily establishes a comparative framework between academic and professional views, concluding that the asset allocation methods currently applied by financial institutions fail to provide their clients with their service truly customized to their preferences.

In this context, a promising idea for future research lies in the development of a hybrid asset allocation approach, keeping the operational simplicity and the practicability of the current industry ad-hoc models but simultaneously incorporating the mathematical rigorous optimization methods to allow the rise in efficiency and the maximization of investors' utility, enhance the tailoring of the asset allocation strategies to each individual investor.

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APPENDICES

APPENDIX A: SUPPLEMENTARY TABLES

Table A.1 – Detailed description of the selected ETFs.

ETF	ISIN	Ticker	Index	Description
Xtrackers II ESG Global Aggregate Bond UCITS ETF 5C EUR hedged	LU0942970798	XBAE	Bloomberg MSCI Global Aggregate Sustainable and SRI Currency Neutral Index.	This index serves as a comprehensive benchmark for the global investment-grade fixed-income market, typically maintaning a rating range from AA to A. The fund is dominated by treasury bonds.
iShares Core MSCI Europe UCITS ETF EUR (Acc)	IE00B4K48X80	EUNK	MSCI Europe Index	The index is a designed to measure the equity market performance of developed markets in Europe.It captures large- and mid-capitalization companies across 15 developed countries (including the UK, Switzerland, France, and Germany, with a strong focus in financials and industrials.
iShares Core MSCI Emerging Markets IMI UCITS ETF (Acc)	IE00BKM4GZ66	IS3N	MSCI Emerging Markets Investable Market Index (IMI)	The MSCI EM IMI is a comprehensive benchmark capturing large-, mid-, and small-capitalization securities across 24 Emerging Markets (EM) countries. This fund is often used to represent the high-growth, higher-risk equity component of a global portfolio.
Xtrackers S&P Global Infrastructure Swap UCITS ETF 1C	LU0322253229	XSGI	S&P Global Infrastructure Index	This index provides exposure to 75 of the largest listed companies that are active in the infrastructure industry - utilities, transportation and energy - from both developed and emerging markets. This ETF uses Unfunded Swaps (Synthetic Replication).
iShares Physical Gold ETC	IE00B4ND3602	PPFB	LBMA Gold Price	It represents the global benchmark "spot" price for gold. As a physical asset index, it tracks the value of the metal itself.

APPENDIX B: MATHEMATICAL DERIVATION OF RISK-TOLERANCE FUNCTION

Taylor 2nd order approximation for utility, U:

$$U(W) \approx U(W_0) = U'(W_0)(W - W_0) + \frac{1}{2}U''(W_0)(W - W_0)^2 \ .$$

In the context of uncertainty, the expected value is applied:

$$E[U(W)] \approx E\left[U(W_0) + U'(W_0)(W - W_0) + \frac{1}{2}U''(W_0)(W - W_0)^2\right]$$

$$\approx E[U(W_0)] + E[U'(W_0)(W - W_0)] + E\left[\tfrac{1}{2}U''(W_0)(W - W_0)^2\right] \ .$$

Knowing that $(W - W_0) = W_0 R$,

$$E[U(W)] \approx E[U(W_0)] + E[U'(W_0)(W_0 R)] + E\left[\frac{1}{2}U''(W_0)(W_0 R)^2\right]$$

$$\approx U(W_0) + U'(W_0)W_0 E[R] + \frac{1}{2}U''(W_0)W_0^2 E[R^2] \ .$$

Knowing that $\sigma^2 = E[R^2] - E[R]^2$,

$$E[U(W)] \approx U(W_0) + U'(W_0)W_0\bar{R} + \frac{1}{2}U''(W_0)W_0^2(\bar{R}^2 + \sigma^2)$$

$$\approx U(W_0) + U'(W_0)W_0\bar{R} + \frac{1}{2}U''(W_0)W_0^2(\bar{R}^2 + \sigma^2) \ .$$

Extending the notion of *equivalent* utility functions to the risk-tolerance function (V and U are equivalent if $V(W) = a + bU(W), a \in \mathbb{R}, b \in \mathbb{R}^+$) and subtracting $U(W_0)$ and multiplying by $\frac{1}{U'(W_0)W_0}$:

$$E[U(W)] \approx \bar{R} + \frac{1}{2}\frac{U''(W_0)W_0}{U'(W_0)}(\bar{R}^2 + \sigma^2) \ .$$

Recalling the definition of the relative risk aversion coefficient, $RRA\ (W_0) = -\frac{U''(W_0)W_0}{U'(W_0)}$:

$$E[U(W)] \approx \bar{R} - \frac{1}{2}\,RRA\,(W_0)(\bar{R}^2 + \sigma^2) \ .$$

AI Disclosure

This internship report was developed with strict adherence to the academic integrity policies and guidelines set forth by ISEG, Universidade de Lisboa. The work presented herein is the result of my own research, analysis, and writing, unless otherwise cited. In the interest of transparency, I provide the following disclosure regarding the use of Artificial Intelligence (AI) tools in the creation of this internship report:

I disclose that AI tools were employed during the development of this thesis as follows:

- AI-based research tools were used to assist in the literature review.
- Generative AI tools were consulted for brainstorming and outlining purposes.
- AI tools were used for linguistic checking and refinement.

However, all final writing, synthesis, and critical analysis are my work. Instances where AI contributions were significant are cited and acknowledged.

I have ensured that the use of AI tools did not compromise the originality and integrity of my work. All sources of information have been appropriately cited in accordance with academic standards. The ethical use of AI in research and writing has been a guiding principle throughout the preparation of this thesis.

I understand the importance of maintaining academic integrity and take full responsibility for the content and originality of this work.

Carolina Alves

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