Lisbon School of Economics & Management
Universidade de Lisboa

MASTERS IN
FINANCE

MASTER´S FINAL WORK
PROJECT

MSCI EMERGING MARKETS INDEX (PREISINDEX) EXPRESS

JOSÉ TORRES BRANCO FRAZÃO SARDINHA

JUNE-2024

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Supervisor:
Professor João Luís Correia Duque, Ph.D.

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Resumo:

Este projeto incide sobre o pricing e valorização de produtos estruturados multi-correlacionados, com ênfase no impacto da correlação neste tipo específico de produto. Este estudo baseia a sua análise no produto estruturado Express Certificate linked to MSCI Emerging Markets (Price Index), emitido pela Deutsche Bank AG. Apresenta a descrição do produto, destacando os vários riscos incorridos pelos emitentes e investidores. Em seguida, passa pela metodologia utilizada, incluindo o uso da simulação de Monte Carlo e modelo binomial, para desenvolver modelos de precificação.

Palavras-chave: Produtos Estruturados; Simulação de Monte Carlo; Modelo Binomial; Teste de Stress; Cobertura de Delta-Gamma

Abstract:

This project focuses on the pricing and valuation of multi-correlated structured products, with an emphasis on the impact of correlation on this specific type of product. This study based its analysis through the Express Certificate linked to MSCI Emerging Markets (Price Index) structured product issued by Deutsche Bank AG. It features the product's description, highlighting the various risks incurred by issuers and investors. Then goes through the methodology employed, including the use of Monte Carlo simulation, and binomial model, to develop pricing models.

Keywords: Structured Products; Monte Carlo Simulation; Binomial Model; Stress Testing; Delta-Gamma Hedging

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1. Introduction

In this work, I will analyze the product “MSCI Emerging Markets Index (Preisindex) Express” issued by Deutsche Bank AG, Frankfurt., with maturity on 2026/5/15 – ISIN: XS0459851647. I shall decompose the product and analyze the risk and possible advantages and disadvantages associated with it. I shall also go over some theoretical concepts, like the 3 Valuation Models used in this work - Black Sholes Model, Binomial Model and Monte Carlo Simulation and other pieces of data used, like the risk rates, volatility, dividend yield and others. Afterwards, I shall propose a theoretical value for the product using the three mentioned valuations models mentioned and further explore this product by going over a stress testing and delta-gamma analysis.

2. Literature Review

This literature review examines key contributions from seminal texts, offering a comprehensive overview of the methodologies and models utilized in valuing structured products. The selected books cover derivatives, mathematical finance, stochastic calculus, financial markets, simulation techniques, and dynamic hedging, providing a multi-faceted perspective on the valuation process.

For the core concepts and mathematical foundations, Wilmott's (1998) work is foundational in understanding the basic principles and mathematical frameworks underpinning derivatives. In addition, Joshi (2008) thoroughly explores mathematical finance, highlighting key models such as Black-Scholes and binomial trees.
Also, Shreve’s (2005) text is a cornerstone in stochastic calculus, focusing on the binomial asset pricing model.

For advanced financial models and applications, Cvitanic and Zapatero (2004) offer a comprehensive integration of economic theories with mathematical models.
In addition, Paul Wilmott's (1995) book is a practical introduction to the mathematical techniques used in derivative pricing.
Also, Hull’s (2003) text is a widely recognized introduction to futures and options markets

For numerical methods and simulation techniques, Glasserman's (2003) work is essential for those interested in the numerical methods used in financial engineering.
In addition, Ross’s (2006) book on simulation techniques offers a broad overview of methods used to model and analyze complex systems.

For risk management and hedging strategies, Taleb’s (1997) book focuses on the practical aspects of hedging derivatives, with a strong emphasis on risk management.

3. Product description

3.1. Explanation of the payoffs of the product:

This product is a structured product based in Equity, and under the German Law Government

Certificate with an Underlying of MSCI Emerging Markets.

It has a minimum investment of EUR 100 with a five-year maturity, starting on 18/05/2021 and maturing on 20/05/2026.

Its composition includes a zero-coupon bond and four auto-call options with different maturity dates.

The product will terminate before the maturity date if, the reference level is at or above the relevant autocall barrier level on any autocall observation dates. If the condition is met on any of the observation dates, the investors will receive a one-time financial transaction equal to the respective autocall payment on the respective payment date. The observation and payment dates, autocall barriers and respective payments are shown in the table 1 in the Attachments section.

If the product does not terminate early and reaches the maturity date, the following scenarios will come into place:

- If the final reference level is equal or higher than the reference level 1,128.409, there will be a cash payment of 129.75 USD.

- If the final reference level is equal or higher than the reference level 862.901 and below the reference level 1,128.409, there will be a cash payment of 100 USD.

- If the final reference level is below the reference level 862.901, there will be a one-time financial transaction connected to the underlying`s performance. The transaction will equal the product notional amount multiplied by the final reference level divided by 1,327.54. You can see this calculation in the formula below:

$$100 \times \frac{UND_T}{1327.54}$$

The decomposition of the structured product is divided between the product and the autocall payments:

3.1.1. Product:

The product consists of a Zero-Coupon Bond and a series of Barrier type options, such as, a Put and two Binary Options. If, at the maturity date, the product was not terminated before, then if the reference level is below 862.901, three components come into play: a Put, a Binary Cash-or-Nothing Put valued at $35, and a zero-coupon bond valued at $100. These components collectively determine the cash payment based on the underlying asset's performance. The correct way to calculate this payment is to subtract the combined value of the Put and the Binary Cash-or-Nothing Put from the $100 value of the Zero-Coupon Bond. The value of the Put option at maturity depends on the reference level and can range between $0 and $65. When the reference level is between 862.901 and below 1,128.409, only the Zero-Coupon Bond valued at $100 is effective. Finally, if the reference level exceeds 1,128.409, the product uses the $100 Zero-Coupon Bond valued along with a $29.75 Binary Cash-or-Nothing Call.

3.1.2. AutoCall Payments:

The Auto-Call Payments can be structured into 4 barrier type derivatives.

3.2. Description of the risks associated with the issuer and with the product:

Investors in this product are exposed to several primary risks:

- Market Risk:
This risk stems from fluctuations in the performance of the underlying index. The structured product’s value is directly tied to the performance of this index, exposing investors to market volatility and potential losses.

- Credit Risk:
Investors are exposed to the issuer's creditworthiness, which in this case, is Deutsche Bank. If the issuer defaults or faces financial distress, investors may suffer losses, including potential loss of their initial investment.

- Inflation Risk:

Inflation erodes the value of the product over time. Investors should be mindful of inflationary pressures, as they can erode the real value of returns generated by the structured product.

- Liquidity Risk:

Structured products, due to their complexity, often have a narrower secondary market compared to more traditional investments. As a result, investors may encounter challenges exiting their positions in the product, especially during times of market stress.

- Interest Rate Risk:

Changes in interest rates can signal shifts in inflation, growth expectations, and overall market conditions. These changes may impact the performance of companies within the underlying index, affecting the structured product's value.

- Currency Risk:

European investors participating in USD-denominated structured products tied to a USD-denominated index face currency risk. Changes in exchange currency rates can impact the value of earnings denominated in USD when converted back to Euros, potentially affecting overall returns.

- Counterparty Risk:

In addition to credit risk, investors should also consider counterparty risk, which refers to the risk of default or financial instability of any third parties involved in the structured product's transactions, such as clearinghouses or counterparties to derivative contracts.

- Regulatory Risk:

Changes in regulatory requirements or legislation, both domestically and internationally, may impact the operation or profitability of the structured product, affecting investor returns and overall performance.

3.3. Advantages and Disadvantages

In addition, Investors considering this product are presented with various Advantages and Disadvantages.

Investors consider that this product offers several advantages, such as the potential for higher returns compared to a traditional ETF, allowing investors to receive improved returns even if the auto-call feature triggers early payments. It also offers conditional capital protection, which, through the auto-call feature, gives investors the opportunity to receive predefined payments as well as a fixed payment at maturity. However, this capital protection is subject to the performance of the underlying index.

However, this product also shows some disadvantages, including the complexity of its structure compared to a straightforward ETF, which potentially makes it difficult for investors to understand its intricacies and evaluate its risks. There is also the absence of dividends, which, unlike an ETF, does not offer dividends from the underlying index, potentially impacting investors who seek regular income streams from their investments. Additionally, the reduced liquidity of this structured product compared to widely traded ETFs can be challenging. Considering that the sale of this product before maturity can incur additional costs and be difficult due to limited market activity, it adds to the complexity. Market conditions' uncertainty can make the product's performance unpredictable, potentially exposing investors to higher levels of risk compared to more stable investment options. Lastly, the risk of losses remains, where despite conditional capital protection, investors risk losing part or all of their investment if the underlying index performs poorly, especially if the auto-call feature is not triggered.

4. Valuation Methods Description

In this section. I will go over with explaining the three Valuation Methods more commonly used for Valuation analysis.

4.1. Black-Scholes Merton Model

The Black-Scholes-Merton model, is a model of derivative markets from which the Black-Scholes formula can be derived. This formula allows the computation of the prices of the European call and put options. Just like it is described in the work of John C. Hull (2003), a call option gives the holder the right to buy an asset by a certain date for a certain price and a put option gives the holder the right to sell an asset by a certain date for a certain price. The price in the contract is known as the exercise price or the strike price and the date in the contract is known as the expiration date or the maturity date. A European option can be exercised only on the maturity date and an American option can be exercised at any time during its life.

As we can see in the work of Paul Wilmott (1995), this model has some assumptions associated with it:

- The underlying follows a lognormal random walk, meaning the logarithm of stock prices are normally distributed.
- The risk-free interest rate and the asset volatility are known functions of time over the life of the option.
- There are no transaction costs associated with hedging a portfolio.
- The underlying asset pays no dividends during the life of the option.
- There are no arbitrage possibilities.
- Trading of the underlying asset can take place continuously.
- Short selling is permitted and the assets are divisible.

The Black-Scholes formulas are the following:

For a European Call Option:

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2)$$ (1)

$$\text{where} \quad d_1 = \frac{\ln \frac{S_t}{K} + \left(r + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}$$ (2)

$$\text{and} \quad d_2 = d_1 - \sigma \sqrt{t}$$ (3)

For a European Put Option:

$$P = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$$ (4)

Where:

C = Call option price

P = Put Option price

S0 = Current stock price

K = Strike price

t = Time till expiration date

r = Risk-free interest rate

σ = Volatility or standard deviation of underlying asset's returns

N(d1) and N(d2) = Cumulative distribution functions for standard normally distributed random variables d1, d2

4.2. Binomial Tree model

The binomial tree model provides a method for the valuation of options, especially American options that can exercised at any time.

The binomial model divides time, between inception and expiry, into intervals/steps. This model is based on the idea that the price of the underlying asset can move into two possible prices over a period of time until the option's expiration. Either the price goes up (there is an increase in the valuation of the option) or it goes down (there is a decrease in the valuation of the option). This model works on the basis of the risk neutral approach, where investors simply don't focus on the risk, based on an assume hedging strategy that leads to discard the risk assessment. In that case there is a probability of the price of the stock going up or going down. It also assumes that there are no arbitrage opportunities, meaning the prices should be consistent with a risk-free rate of return. Arbitrage is a trading strategy that begins with zero capital and trades in the stock and money markets to make money with positive probability without any possibility of losing money (Steven E. Shreve, 2005).

The underlying asset starts with an initial value S and can either rise to a value Su or fall to a value Sd, with $0 < d < 1 < u$. The probability of a rise is p and so the probability of a fall is 1-p.

The three constants u, d and p are obtained based on the formulas below:

(6) $$p = \frac{e^{rt/n} - d}{u - d}$$ (7) $$u = e^{\sigma \sqrt{t/n}}$$ (8) $$d = e^{-\sigma \sqrt{t/n}}$$

Where:

t: Time to expiration (in years).

r: Risk-free interest rate (annualized).

σ: Volatility of the underlying asset's returns (annualized).

n: Number of time steps.

The option value is then estimated based on the discounted value of the option expected cash-flow at maturity.

4.3. Monte Carlo Simulation

The Monte Carlo Simulation is a valuation model used to calculate the possible outcomes of any
random event. It involves creating several random samples from a probability distribution and using these samples to estimate the behavior of an Option in a certain space in time. This method is based on the fact that the mathematical expectation of a random variable can be approximated by the arithmetic average of a random sample of values from its distribution (Jakša Cvitanic, 2004).

Monte Carlo techniques involves setting up a mathematical Model by identifying what is referred as the dependent variable and the independent variables, then specify the probability distributions of the independent variables and use historical data and/or define a range of values and assign probability weights for each one and run simulations repeatedly, generating random numbers of the independent variables. A random number represents the value of a random variable uniformly distributed on (0,1) (Sheldon M. Ross, 2006). This is done until, enough results are gathered to represent the infinite possible combinations.

Just like is described by Paul Glasserman (2003), valuing a derivative security by Monte Carlo typically involves simulating paths of stochastic processes used to describe the evolution of underlying asset prices, interest rates, model parameters, and other factors relevant to the security in question. Rather than simply drawing points randomly from [0, 1], we seek to sample from a space of paths. Depending on how the problem and model are formulated, the dimension of the relevant space may be large or even infinite. The dimension will ordinarily be at least as large as the number of time steps in the simulation.

The step-by-step formulas are:

Simulate Stock Prices:

(9) $$S_T^{(i)} = S_0 \times e^{(r-\frac{1}{2}\sigma^2)T+\sigma\sqrt{T}Z^{(i)}}$$

Calculate Payoffs:

- Call option:

(10) $\text{Payoff}_{\text{call}}^{(i)} = \max(S_T^{(i)} - K, 0)$

- Put option:

(11) $\text{Payoff}_{\text{put}}^{(i)} = \max(K - S_T^{(i)}, 0)$

Discount Payoffs:

- Call option:

(12) $\text{PV Payoff}_{\text{call}}^{(i)} = \text{Payoff}_{\text{call}}^{(i)} \times e^{-rT}$

- Put option:

(13) $\text{PV Payoff}_{\text{put}}^{(i)} = \text{Payoff}_{\text{put}}^{(i)} \times e^{-rT}$

Average Present Value:

- Call option

(14) $\text{Call Option Price} = \frac{1}{N} \sum_{i=1}^{N} \text{PV Payoff}_{\text{call}}^{(i)}$

- Put option

(15) $\text{Put Option Price} = \frac{1}{N} \sum_{i=1}^{N} \text{PV Payoff}_{\text{put}}^{(i)}$

Where:

S: The current price of the underlying asset.

K: The price at which the option can be exercised.

T: The time until the option expires.

r: The risk-free interest rate.

σ: The volatility of the underlying asset's returns.

N: The number of random paths to generate.

5. Data

In order to make the appropriate Valuations, certain variables needed to be used in the calculations:

5.1. Risk-free rates

A risk-free rate represents the theoretical idea of a rate of return for an investment, where there is no risk of financial loss.

In theory, a risk-free rate from a no risk investment could be found to show the minimum rate of return necessary on riskier investments, however in practice there is no such thing as a riskless investment so certain products tend to be used has a representation of a risk-free rate. The factor to choose the product is not related with how much an investor gain from that product but how much is the product covered and how stable is the financial institution that emitted the product. For these reasons government bonds are the more reliable products to be used as a representation of the risk-free rate since the bonds are fully covered by the government and likelihood of the government to default is low.

This rate is a key component in some financial models and in Valuation Methods like CAPM, Discounted Cash Flow and Bond Pricing.
Besides these models, it also impacts certain Valuation Methods like the Black Sholes Model, as will be seen in the next chapter.

However, when several international markets are under scope, an investor needs to keep in mind the changes in inflation and exchange rates. The nominal risk-free rate is the observed rate not being impacted by the inflation rate, while the real risk-free rate adjusts for inflation. The real rate provides a clearer picture of the true purchasing power of the returns, adjusted for the real conditions in that market. On the other hand, fluctuations in the currencies can impact the effective return. Investors might use government bonds from countries with stable currencies and low default risk to approximate the risk-free rate in different currencies.

The risk-free rate is also an important economic indicator, reflecting the state of the economy, with a lower risk-free rate showing a potential decline of the economy and a high risk free rate reflecting the conditions for a strong economy, and the type of policies being implemented by the central banks, with lower risk-free rate showing a low degree of intervention by the central bank and a high risk-free rate showing a certain level of intervention.

5.2. Risky rates

These rates reflect the expected return on investments that include compensation for the risk taken by investors. Unlike the risk-free rate, which assumes no risk, risky rates account for the uncertainty and potential variability in returns due to various types of risks. The purpose of such is to enable fair comparisons between investments with different levels of risk. Between two investments with the same rate of return, the risk-adjusted return would be higher in the investment with the lowest risk

It can be decomposed between the risk-free rate, which was mentioned in the previous chapter, and the amount that will compensate the investor for accepting the extra risk in doing the investment, which is referred as Risk Premium.

Investors use this component to compare the expected returns of different investments to determine which offers the best risk-adjusted returns, diversify portfolios by allocating investments across assets with different risk levels to optimize returns for a given risk tolerance, assess the performance of investments and fund managers by comparing actual returns to risk-adjusted benchmarks, determine the appropriate discount rate for valuing projects and companies which affects investment and financing decisions, to be able detect and manage the many different types of risks related with investments and inform strategic business decisions by incorporating risk-adjusted return metrics.

To measure an investment performance that considers the amount of risk undertaken to generate a specific return, the investors have specific models available to them to calculate Rate of Rate adjusted to risk and to make a proper analysis on the viability of the investment in such conditions, for example the Sharpe Ratio, the Treynor Ratio, the Jensen`s Alpha and the Information Ratio.

5.3. Volatility

Volatility is the measure of variation of the price of an asset over a specific time and its normally computed based on the standard deviation of returns. This quantifies the amount of variation or dispersion in a set of values. The historical volatility is expressed by the following formula:

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(R_i - \bar{R})^2} \quad (16)$$

Where:

σ: the standard deviation

Ri: rate of return for each timestep

$\bar{R}$: the average return

N: is the number of observed returns

Other volatility measures that are used in financial computations are the implied volatility that is estimated from option prices and reflects the market's expectations of future volatility.

Several factors may influence the volatility of a specific product. They might come from market factors, like changes in economic indicators, changes in interest rates and geopolitical events. Also, the behavior of the investors and their approach to the market can also impact the volatility of the market. In addition, other factor of diverse nature can also impact volatility like changes in regulations and company’s financial performance.

The implications associated with volatility are many. Volatility is a key component for an Investor to be able to analyze the level of risk of a specific product (a higher volatility, tend to represent a higher chance of price variability and a higher chance of potentially damaging the investment that was made). In order to keep the volatility of a portfolio in check, investors relay on the idea of diversifying their investment by investing on several products with different volatility in order to reduce the overall risk associated with the portfolio. Traders may use volatility-based strategies, such as options trading or volatility arbitrage, to capitalize on price movements or hedge against risk. Because of the impact of this component, Volatility plays a decisive role in Investor's decision in what products they choose to buy. Volatility can be measured using various techniques, including standard deviation, like We have seen previously and variance as well.

5.4. Correlations

Correlation is a measure that describes the numerical degree from which two variables move in relation to each other. It allows an Investor to measure the strength and direction of a linear relationship between two variables. The correlation coefficient, typically denoted as r, and it ranges from -1 to +1. A Correlation with a positive value represents two variables that move in the same direction, while a Correlation with a negative value represents two variables moving in opposite directions. If a Correlation has a value of zero, the variables are independent from each other.

The most common method to calculate the Correlation variable is the Pearson Correlation method which assesses the linear relationship between two variables. Its calculated based on the formula bellow:

$$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2}} \quad (17)$$

Where:
r: correlation coefficient
xi and yi: data points
x^{-} and y^{-}: means of the data sets

Correlation has a large impact on the way a investor approaches a potential investment. It help to understand how different product relate to each other and helps the investor what products should it buy if it needs to diversify its portfolio, if it needs to reduce the overall risk of the portfolio, help to improve asset allocation and what trading strategies should the Investors take in order to reduce risk while maintaining the same potential gains.

5.5. Dividend Yield

Dividends are cash flows made by a financial institution to its shareholders, usually derived from the company's profits. Dividend yield measures the income an investor receives from a dividend related to a stock as a percentage of its current market price.

It is calculated by the formula bellow:

$$\text{Dividend Yield} = \left(\frac{\text{Annual Dividends per Share}}{\text{Price per Share}}\right) \times 100$$

(18)

The Dividend Yield is a variable with a lot of significance in the decision making of the investors in what products they invest. It’s crucial for income-focused investors who seek regular income from their investments, it allows investors to compare the income-generating potential of different stocks, Investors can choose investments that align with their income goals. Dividend yield is a component of the total return on an investment, which includes both capital gains (price appreciation) and income (dividends). Changes in dividend yield can signal shifts in market sentiment. Stable or increasing dividends often indicate a company’s strong financial health and consistent earnings, while cutting dividends may signal financial trouble.

6. Product Valuation and Risk Level

The determination of the theoretical value of the Certificate at inception involves consideration of auto-call payments and their respective risk-free rates. Auto-call features are designed to provide investors with predefined payments under certain conditions, typically based on the performance of the underlying assets. These payments can impact the overall value of the Certificate and need to be factored into the valuation methodologies. To review the variables that will be used in the computation of the valuations review table 2 in the Attatchments section.

In addition, we use the Balance Sheet Data to calculate the Rate of Return with Risk, (Funding rate), instead of using credit ratings or yields from traded bonds. Normally, valuation methods rely on credit ratings or yields to measure risk and determine funding rates. However, using balance sheet data offers a more detailed and intrinsic approach to understanding a company's financial health and potential risks.

Using balance sheet data offers several advantages like providing a comprehensive view of a company's financial health, including its assets, liabilities, and equity that can't be shown in a credit rating, allowing for a customized assessment of risk tailored to the specific characteristics of the company and its industry, that may not be fully captured by credit ratings, providing insights into a company's future prospects and ability to generate returns, offers a degree of independence from external rating agencies or market sentiment, provides information about the company's long-term financial stability and sustainability and allows investors to capitalize on market inefficiencies and mispricing's that may not be fully reflected in credit ratings or bond yields. To review the balance sheet data for 2021, verify the table 3 in the Attachments section.

6.1. Black-Scholes Merton Model

The Black-Scholes Merton model is widely used for valuing European-style options under certain assumptions. However, in this product there are characteristics that make the calculation of the valuation through Black-Sholes Merton model inappropriate. The product in analysis might be a European style option but the characteristics of the auto call payments creates the situation of early termination, where the option might be exercised before its time. Since early exercise can impact the option's value, the Black-Scholes model may not accurately price barrier-style options. In addition, Black Sholes assumes that the interest rate remains the same throughout the entire life of the option, but with auto call payments forces the computation of different interest rates depending on the different auto call payments. Lastly, auto call payments forces the creation of different paths where the option may follow depending on the behavior of the market, something that cannot be considered in the Black Sholes Model.

In conclusion, Black-Sholes Merton is not suited for evaluating this product, however, both the Binomial Model and Monte Carlo Simulation are suited to evaluate this product.

6.2. Binomial Tree model

One binomial tree was computed considering the different interest rates for each year and the different possible incomes from the auto calls and in the final expiry date.

The tree consists of 60 steps, with each step having a length of $\Delta t = 0.833$ year, spanning from the product's issue date to its maturity date, monthly. The parameters μ and σ were computed based on the implied volatility of the reference underlying over the last 5 years until the product's issue date. The different risk-free rates were obtained from a yield of government bonds from different years. As I mentioned before, the Rate of return with risk was obtained by using the balance sheet data and dividing the interest expense by the average interest-bearing liabilities. The dividend yield was obtained from Bloomberg. To review the data computed in this valuation model including a section of the binomial tree for the first months, review the table 4 and 5 in the Attachments section.

By working through each possible step, we can reach the value of the valuation considering the auto-call payments and their respective risk free-rate, the theoretical value will be 92.49$

6.3. Monte Carlo Simulation

A total of 10.000 simulations were conducted for each underlying stock's prices, resulting in 10.000 simulation paths for the reference underlying. For each simulation there is a random number associated with each date where I computed the possible stock price. The random numbers were generated in excel using a formula to randomly generate a number between 0 and 1 and from that number to obtain the inverse of the normal cumulative distribution. In total, 10.000 random numbers were generated for the purpose of this exercise. Each simulation implies an independent set of coupon payments, contributing to the valuation of binary options.

The funding rate of the bond was determined by dividing the interest expense value with the average interest-bearing liabilities. These values were obtained from the balance sheet from 2021. For the free risk rate, we used zero coupon bonds depending on the year the option might expire.

For each simulation, I calculated what would be the stock price for each one of the observation dates and the valuation date. Based on that, I was able to compound how many scenarios triggered the auto-call payments and how many scenarios triggered the possible outcomes in the Maturity date. With that, I can compound the probability for each payment scenario. For the present expected value, all the payment scenarios have a fixed value already established, except for the payment scenario “final reference level below 862,901” which was calculated by averaging the payment values for the scenarios that meet the necessary criteria of the reference value in the maturity date and bringing that value to the present time. Finally, I added all the present expected values, multiplied by their respective probability, to obtain the theoretical price of the certificate. To review the probabilities and the Present Expected value computed for this valuation model, review table 6 in the Attachments section.

Considering the auto-call payments and their respective risk free-rate, the theoretical value will be 93.22$

6.4. Risk Level Indicator

A risk level indicator is a tool used to assess the level of risk associated with a financial product or investment.

A typical risk level indicator operates on a scale, usually from 1 to 7, where:

- Low numbers (1 or 2) indicate a lower risk.
- Middle numbers (3 or 4) indicate a moderate risk.
- High numbers (5 or 6) indicate a higher risk.
- Highest number (5 to 7) indicates a very high risk.

For this product, the Risk level indicator is a level 5, which represents a high-risk level for a product. This is due to the high probability of losses, high volatility and a product very influenced by volatility. This product forces the investor to have a close monitoring on its performance and proper planning of hedging strategies to reduce the risk associated with it.

7. Further Product Analysis

In order to further analyze this product, I decided to do two things:

- We will start with a stress testing with certain values of this product and how it impacts everything else.

- Based on these new values, I will perform a Delta-Gamma Hedging with a product created for the purposes of this exercise and see what the best strategy is to minimize the risk of the time of the analysis.

Afterwards, in the chapter 8, I shall discuss the results computed in this chapter.

7.1. Stress Testing

Stress testing is a systematic analysis process used by financial institutions, organizations, and systems to evaluate how they can withstand and respond to extreme, unexpected conditions or scenarios. The idea is to create a plausible scenario were certain changes to the status quo occurs (changes of interest rate, financial crash, etc.), create a model where all the variables that existed in the status quo, except the ones the Investor wants to analyze, run the model and then analyze the results and based on the results, the investor takes the necessary actions in order to mitigate any potential risk that might have been uncovered in the stress testing. This type of test can go from an analysis of a single variable to an analysis of several variables. These tests tend to be done by financial institutions in order to identify potential risks and to prepare for potential future scenarios and what actions should they take in such situations. Also, regulators tend to use these tests as well to analyze the regulatory compliance of certain investments and financial institutions.

For this scenario, I am going to re-run one of the valuation methods I have shown Before, by but changing certain variables and observing what is the the impact of these changes and the theoretical value of the product.

For this, I will change 4 variables, one at a time, multiple combinations, and all at a time to obtain 16 possible scenarios, using the previous Monte Carlo Simulation we used in the previous chapter.

The 4 variables chosen to be changed were:

- Risk Free rate will be diminished in 1% for all the risk-free rates used in the valuation.
- Volatility will be increased in 5%.
- The value of the stock for all the simulations to diminished in 10% by the time of expiration compared to the values calculated in the Monte Carlo simulations with the normal values.
- Dividend yield will diminish in 1%.

After this, I obtained the value of the option for each change. The possible theoretical values for the option are the following:

Table I – Possible Theoretical Values for the option after the Stress Testing

Possible Combinations	Theoretic Value of the Certificate at Inception
The original value of the certificate computed previously, with variables unchanged	93.22
All Variables Changed - Stock price diminished by 10%, Volatility increased by 5%, Dividend yield diminished by 1% and Risk free rates diminished by 1%	90.68
1 Variable changed – Stock price diminished by 10%	89.42
1 Variable changed – Volatility increased by 5%	89.71
1 Variable changed – Risk free rates diminished by 1%	95.41
1 Variable changed – Dividend yield diminished by 1%	95.41
2 Variable changed - Stock price diminished by 10% and Volatility increased by 5%	86.55
2 Variable changed - Stock price diminished by 10% and Risk free rates diminished by 1%	91.82
2 Variable changed - Stock price diminished by 10% and Dividend yield diminished by 1%	91.82
2 Variable changed - Risk free rates diminished by 1% and Dividend yield diminished by 1%	97.31
2 Variable changed - Risk free rates diminished by 1% and Volatility increased by 5%	91.65
2 Variable changed - Volatility increased by 5% and Dividend yield diminished by 1%	91.65
3 Variable changed - Stock price diminished by 10%, Volatility increased by 5% and Risk free rates diminished by 1%	88.69
3 Variable changed - Volatility increased by 5%, Risk free rates diminished by 1% and Dividend yield diminished by 1%	93.58
3 Variable changed - Stock price diminished by 10%, Risk free rates diminished by 1% and Dividend yield diminished by 1%	94.02
3 Variable changed - Stock price diminished by 10%, Volatility increased by 5% and Dividend yield diminished by 1%	88.69

7.2. Delta-Gamma Hedging

For this second part, we will use a Delta-Gamma Hedging based on a scenario where all variables change and based on that review on what should be the strategy implemented to reduce the risk.

When a company or investor chooses to use futures products to hedge a risk, the idea is to take a position that reduces the risk as much as possible. To hedge, the company's treasurer should take a short or a long futures position that is designed to offset this risk. A short hedge involves shorting a position with future products and is an advisable action when the investor already owns an asset and expects to sell it in the future. A long hedge is a long position in a futures product and is advisable when an investor knows, it will have to purchase some assets in the future and wants to fix the price in that moment.
For more complex products the hedge can only last a short time, so the hedge has to be adjusted periodically. This is known as rebalancing.

But when the option is more complex and does not correspond to the standardized products traded by exchanges, hedging the exposure is far more difficult. To be able to reduce the risk, investors have to use alternative methods. These involve calculating measures such as the "Greek". In this work, I focused in two Greeks calculations and their respective hedges, which were the Delta and the Gamma.

Delta is a ratio of the numerical change in the price of a stock product to the change in the price of the underlying product. It shows how many shares of a stock a investor should hold for each option, to create a riskless portfolio. It is expressed as the first mathematical derivative of the product with respect to the underlying asset (Nassim Nicholas Taleb, 1997) The value of Delta can vary between -1 and +1, with call options having deltas between 0 and 1, while put options having deltas between 0 and -1, however, a Delta in a structured product can have values outside of the interval between -1 and 1.
Gamma represents the change of the delta in a specific product/portfolio when the price of the underlying asset changes by one point. The Gamma is important because it expresses how much hedging will cost in a short time interval. If the Gamma of our portfolio is positive,

then we will make money by Delta-hedging, and if negative, we will lose money. (Mark S. Joshi, 2008)

A delta hedging is a perfect elimination of risk, by exploiting the correlation between two instruments (Paul Willmott, 1998). The goal is to understand how many more stocks the investor needs to buy or to sell to reach a delta neutral position, a portfolio composition with a value of delta being zero, in another words, a portfolio with no risk.
Gamma hedging is a strategy that aims to maintain the delta in a certain level, preferably a delta neutral position, by offsetting the gamma by buying or selling stocks from a different option. The goal is to create a portfolio with a combined level of gamma close to zero, a gamma neutral position.
Combining a delta and gamma hedging is what is referred has delta-gamma hedging. This is a strategy used to reduce the risk of price changes of small scale but also of large scale. The idea is to introduce another option into the investor`s portfolio, preferably another option that is behaving in the opposite way of the investor`s portfolio, and calculate how many stocks of that option and how many stocks of the underlying does the investor needs to mitigate the total risk of the portfolio. In another words, it’s used to understand how many stocks of the underlying the investor needs to buy or short for the portfolio to be in a delta neutral level and how many new options does the investor need to buy/short for the portfolio to be in a gamma neutral level.

In this section I will present a hypothetical situation for an Investor in order to calculate the delta and gamma hedging and how an Investor should approach this situation.
Let’s say, hypothetically, that one of these options is equivalent to one stock and that an investor has a portfolio composed of one call option, and it needs to investigate the stocks one year performance between the 18º of May of 2021, the date of the Options Inception, and 18º May of 2022. In order to mitigate the risk of the portfolio, I will introduce a new put option for my consideration, in order to reduce the general risk associated with the portfolio.
Then, the investor needs to understand how much it needs to short or buy of this option and what actions should he take throughout the next twelve months

This put option has the same underlying as the call option and the following financial data:

Strike Price (K): $862

Expiration: 5 years

Initial Stock Price (S0): $1327.54

Volatility (σ): 18.20% (annualized)

Risk-free Rate (r): 0.79% (annualized)

Dividend Yield: 2.43%

Assume there is no premium cost associated with this option.

The goal is to understand the current level of risk of the portfolio and the best approach to reduce the level of risk, given the options we have available to us. Due to the complexity of this product and the risk associated with it, it will be necessary to rebalance the portfolio each month, for a total of 12 months, to make sure the level of risk is kept under control and that it be properly monitored by the investor.

The values of the stock of the underlying of the call option and of the put option for each of the 12 months and their respective delta and gamma, are in the table below:

Table II – Stock Price, delta and Gamma Values for each month

		Greek Values for the Call Option		Greek Values for the Put Option used for Hedging	
Month	Stock Price	Delta	Gamma	Delta	Gamma
0	1327.54	0.182589	0.033486	-0.126679	0.000382
1	1361.25	0.147522	0.029277	-0.113167	0.000347
2	1317.22	0.170361	0.038464	-0.127470	0.000392
3	1261.60	0.215643	0.044616	-0.148950	0.000457
4	1279.35	0.203715	0.053096	-0.139569	0.000435
5	1283.02	0.213600	0.032756	-0.136342	0.000431
6	1273.87	0.195038	0.053792	-0.138566	0.000442
7	1216.30	0.233058	0.045509	-0.164119	0.000520
8	1241.58	0.246770	0.055401	-0.149818	0.000485
9	1231.77	0.219072	0.059299	-0.152795	0.000499
10	1122.98	0.277685	0.065544	-0.214193	0.000676
11	1106.68	0.248793	0.058691	-0.223871	0.000709
12	1007.50	0.253336	0.051334	-0.302313	0.000906

For the stock of the underlying, this are real values taken from the Deutsch Bank website.
I calculated the Delta and Gamma for each one of the 12 months in order to be able to properly rebalance the portfolio. The way I obtained the delta and gamma for the Call Option was not through the usual methods. Due to the complex nature of this product, with four auto call payments being a part of it, such methods are not ideal due to the price accuracy and the path dependency that are a feature of the auto calls that are not considered.
Instead, I used a Monte Carlo to simulate several possible scenarios, to Calculate both values.
For Delta, I simulated the value of the certificate when the stock price went one point up and again, when the value when the stock price went one point down. With that, I subtracted these new certificate values and divided them by the difference between the two new stock prices.
For Gamma, I used a similar approach where I executed the necessary simulations, five to be exact, to obtain the Delta when the stock price was one point up and the Delta when the stock price is one point down.
I used the funding rate for 2021 and the implied volatility for the computation of the certificates for the possible stock prices used in the five simulations for each month. For more information regarding these computations, look for the tables 7 and 8 in the Attachments section.

For the Put Option, I used the normal formula of Delta and Gamma in the Black Sholes-Merton model. The formulas for these two variables are:

Delta for Put Options:

(19) $$\Delta_{\text{put}} = N(d_1) - 1$$

Where:

(20) $$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

(21) $$d_2 = d_1 - \sigma\sqrt{T}$$

Gamma:

(22) $$\Gamma = \frac{N'(d_1)}{S_0\sigma\sqrt{T}}$$ (23) $$N'(d) = \frac{1}{\sqrt{2\pi}}e^{-\frac{d^2}{2}}$$

Where:

S0: Current price of the underlying asset

K: Strike price of the option

r: Risk-free interest rate

σ: Volatility of the underlying asset

T: Time to maturity (in years)

$N(d)$: Cumulative distribution function

$N'(d)$: Probability density function

For more information regarding the computations of the Delta and Gamma for the put option, look for the table 9 in the Attachments section.

Based on these values, I was able to compute how many Option/stocks the Investor would need to short/buy in order be able to hedge the Portfolio successfully. Here are the values I obtained:

Table III – Nº of Put Options needed for hedging and respective cost

	Gamma Hedging			
Month	Total Nrº of Shares necessary to hedge	Nrº of Shares necessary to hedge for the current month	Action to be taken by the Investor	Cost
0	-88	-88	Short	40,830.55
1	-84	4	Buy	-5,896.29
2	-98	-14	Short	18,143.16
3	-98	0	No Action	0.00
4	-122	-24	Short	31,183.61
5	-76	46	Buy	-58,935.25
6	-122	-46	Short	58,197.27
7	-88	34	Buy	-41,672.29
8	-114	-27	Short	33,239.73
9	-119	-5	Short	5,597.09
10	-97	22	Buy	-24,566.48
11	-83	14	Buy	-15,682.71
12	-57	26	Buy	-26,291.94

Table IV – Nº of stocks of the underlying needed for hedging and respective cost

	Delta Hedging			
Month	Total Nrº of Shares necessary to hedge	Nrº of Shares necessary to hedge for the current month	Action to be taken by the Investor	Cost
0	-11	-11	Short	14,991.97
1	0	11	Buy	-14,480.32
2	-2	-2	Short	2,990.87
3	0	2	Buy	-2,157.94
4	-4	-4	Short	5,616.44
5	6	10	Buy	-12,387.50
6	-7	-13	Short	16,018.54
7	5	12	Buy	-14,492.67
8	-4	-9	Short	10,736.73
9	-1	3	Buy	-4,119.49
10	4	5	Buy	-5,975.83
11	3	-1	Short	1,642.72
12	8	5	Buy	-4,747.55

The number of stocks to hedge for gamma was obtained by dividing the gamma of the call option with the gamma of the put option. This formula is shown below:

$$n = -\frac{\Gamma_c}{\Gamma_p} \quad (24)$$

The number of stocks to hedge for Delta was obtained by adding the delta of the call option with the value of the delta of the put multiplied by the number of stocks needed to hedge for the gamma section and then multiplied all by minus one, to obtain the difference needed to reach the delta neutral position. This formula is shown below:

$$N = -(\Delta_c + n \cdot \Delta_p) \quad (25)$$

For the Gamma section, the table shows the number of options it takes to hedge for each month and if the investor should buy or short as well. When the value of the shares is negative it shows how many put options the investor needs to short to reach the Gamma neutral level and when its positive it shows how many put options the Investor needs to buy to reach the gamma neutral level.

For the Delta section, the table shows the number of underlying stocks it takes to hedge for each month and if the investor should buy or short as well. When the value of the shares is positive it shows how many stocks the portfolio is lacking and how many it needs to buy and when its negative it shows the portfolio has too many stocks by a specific number and how many options it needs to short. With this data, I was able to compute the cost of hedging for each month.

For the Gamma section, I calculate the cost of hedging with a put option using the formula bellow:

(26) premium cost per stock(stock price-strike price)*number of put options needed to hedge the portfolio = cost of hedging with a put option*

Since there is no cost of premium for this option, this variable is removed from the formula. For the Delta section, I multiplied the number of underlying stocks needed to hedge for the portfolio to reach a Delta neutral position with the stock price for each month.

Finally, with these values I obtained the costs of hedging the portfolio for each month. With the costs computed, I can compute the accumulated costs the Investor will need to spend each month in order to hedge the portfolio. They are presented in the table below:

Table V – Total Costs for each month and the accumulated cost

	Costs		
Month	Total Cost	Interest Cost	Total Cum. Cost
0	55,922.52	36.73	55,922.52
1	-20,376.61	23.37	35,582.64
2	21,134.03	37.27	56,740.05
3	-2,157.94	35.88	54,619.38
4	36,800.05	60.07	91,455.30
5	-71,322.75	13.26	20,192.63
6	74,215.81	62.02	94,421.70
7	-56,164.97	25.17	38,318.76
8	43,976.46	54.07	82,320.39
9	1,477.60	55.08	83,852.07
10	-30,542.30	35.05	53,364.84
11	-14,040.00	25.85	39,359.90
12	-31,039.49		8,346.26

The total costs shows the costs of gamma and delta hedging for each month. The interest costs represent the costs associated with an interest rate which were computed by multiplying the total accumulated cost of that month with the funding rate for 2021 and divided 12 months. With these two variables, we can compute the total accumulated cost for all the months, by adding the total cost of each month with the interest cost of the previous month and with the accumulated cost computed for the previous month.

8. Discussion of Results

For the Stress Testing section, by looking at these values we can see that stock price and volatility are the most sensitive variables in this stress test scenario, where the most significant drop happens when the volatility variable changes alone (89.71) and when the stock price changes alone (89.42).

This shows that the higher the Volatility of the product, the more the theoretical value falls, in another words, the more volatile the option is the more likely is of the investment to go wrong and the value of the certificate falls. This should not be surprising since volatility plays a critical role in determining the range and distribution of potential outcomes for financial models, particularly in the valuation of options and risk management. It impacts the spread of potential futures, the shape of the probability distribution of the simulated outcomes, it helps to assess the exposure to potential risk, it influences the simulated paths of the assets price and through its impact it helps the investor to understand best strategies to take with that in mind. Overall, this just shows that volatility is a variable that needs to be carefully watched by an investor.

The product value (e.g., stock price) is certainly important as it sets the starting point for the simulation. However, its impact is relatively straightforward compared to the dynamic nature of volatility. While the product value influences the absolute levels of future prices, it does not affect the distribution and variability of outcomes as significantly as volatility does.

In addition, we can note the notable change as well when the dividend yield changes alone (95.41)

Dividend yield of an underlying asset influences the simulated price paths and the valuation of financial derivatives such as options and has significant impacts on the options performance when adjusting asset prices, in the effect of discounting, in the pricing of the option, in the hedging strategies to be used by the investor, in the valuation of the portfolio and in the expected returns.

Lastly, when all variables change simultaneously, the value drops to 90.68. By comparing with the other values calculated, it reinforces the idea that the most impactful variables are the volatility, followed by the stock price and then the dividend yield

Based on these conclusions, an Investor should monitor stock price and volatility carefully, diversify the portfolio and plan for any eventual scenarios that might come should these variables change dramatically.

For the Delta-Gamma Hedging Section, with the data computed, it is possible for the Investor to know how many options and stocks he will need to hedge each month in order to reach a delta and gamma neutral position. By the end of the 12 months, the Investor will currently have one call option, and to hedge that, the investor will have additionally 57 Put Options shorted and 8 long stocks of the underlying in his portfolio. All for an accumulated cost of 8,346.26 euros. This combination of shorting puts and holding long stock positions is designed to stabilize the portfolio’s delta across a range of underlying price movements, effectively managing the gamma exposure, protecting the portfolio from short and large movements of the prices of the underlying.

However, the large volume of Put options required to hedge just shows how great is the volatility associated with the call Option. This relates to the underlying of the Emerging Markets, which is a market that tends to be more volatile. So it can be concluded that in dealing with option connected with this market in any way, hedging strategy’s are crucial to reduce the risk to acceptable levels for a safer investment.

Conclusions

When analyzing any product, there are several factors an Investor needs to be worried about when deciding where to invest. We reviewed several of this in this work and I can conclude that there might be several measures and mechanism an Investor might use to be able to predict the possible outcomes and even then, is never certain what might happen since most of these factors are out of the control of the Investor. All an Investor can do is prepare for the worst and hope for the best. Therefore, understanding all the variables that affects an investment and taking several possible tests to be able to predict the potential risk sand gains gives a necessary hedge for an Investor to protect himself in case things go wrong and reward him when things end well. This shows in many ways that Investments are not all about the reward but also how well an Investor is protected against the risk that comes with it, and what actions he can take to mitigate it.

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Attachments

Table 1. Autocall Barrier Levels and Payments

Autocall observation dates	*Autocall barrier levels*	*Autocall payment dates*	*Autocall payments*
17 May 2022	1,327.54	20 May 2022	USD 105.95
17 May 2023	1,327.54	22 May 2023	USD 111.90
15 May 2024	1,261.163	20 May 2024	USD 117.85
15 May 2025	1,194.786	20 May 2025	USD 123.80

Table 2. Values of variables

Underlying Asset	MSCI Emerging Market Index
Initial reference	1327.54
Issue Date	5/18/2021
Valuation Date	5/15/2026
Maturity Date	5/20/2026
q - dividend yield	2.43%
Risk free rate 5 year	-0.49%
Risk free rate 4 year	-0.32%
Risk free rate 3 year	-0.43%
Risk free rate 2 year	-0.67%
Risk free rate 1 year	-0.67%
Funding Rate	0.79%
Spread	127.91
Interest Expense (2021)	5,444
Average interest-bearing liabilities (2021)	690,656
t	4.956349

Table 3. Balance sheet data

Net Interest Income

in € m. (unless stated otherwise)	2021	2020	2019	2021 increase (decrease) from 2020 in € m.	2021 increase (decrease) from 2020 in %	2020 increase (decrease) from 2019 in € m.	2020 increase (decrease) from 2019 in %
Total interest and similar income	16,599	17,806	25,208	(1,207)	(7)	(7,401)	(29)
Total interest expenses	5,444	6,280	11,458	(836)	(13)	(5,178)	(45)
Net interest income	11,155	11,526	13,749	(371)	(3)	(2,223)	(16)
Average interest-earning assets[1]	937,947	920,444	956,362	17,503	2	(35,918)	(4)
Average interest-bearing liabilities[1]	690,656	685,830	714,716	4,826	1	(28,886)	(4)
Gross interest yield[2]	1.56 %	1.82 %	2.53 %	(0.26) ppt	(14)	(0.72) ppt	(28)
Gross interest rate paid[3]	0.50 %	0.76 %	1.47 %	(0.26) ppt	(34)	(0.71) ppt	(48)
Net interest spread[4]	1.06 %	1.06 %	1.07 %	(0.00) ppt	(0)	(0.01) ppt	(1)
Net interest margin[5]	1.19 %	1.25 %	1.44 %	(0.06) ppt	(5)	(0.19) ppt	(13)

ppt – Percentage points
1 Average balances for each year are calculated in general based upon month-end balances.
2 Gross interest yield is the average interest rate earned on average interest-earning assets.
3 Gross interest rate paid is the average interest rate paid on average interest-bearing liabilities.
4 Net interest spread is the difference between the average interest rate earned on average interest-earning assets and the average interest rate paid on average interest-bearing liabilities.
5 Net interest margin is net interest income expressed as a percentage of average interest-earning assets.

Table 4. Binomial Tree Model variables

$R_{funding}$	0.79%
q	2.430%
σ	18.20%
Issue Date	5/18/2021
Valuation Date	5/15/2026
τ=T-t	4.95635
Δτ	0.083
% Rate of return	100%
Principal (NP)	100.00
U	1.053710409
D	0.949027353
P	0.463902468
1-P	0.536097532

Table 5. Binomial Tree Model first 7 possible paths

0	1	2	3	4	5	6	7
1327.54	1398.84	1473.98	1553.14	1636.56	1724.46	1817.08	1914.68
98.55	101.34	103.50	105.01	105.88	106.25	106.30	106.25
	1259.87	1327.54	1398.84	1473.98	1553.14	1636.56	1724.46
	96.03	99.36	102.09	104.14	105.45	106.09	106.25
		1195.65	1259.87	1327.54	1398.84	1473.98	1553.14
		93.06	96.88	100.22	102.90	104.79	105.85
			1134.71	1195.65	1259.87	1327.54	1398.84
			89.65	93.90	97.79	101.15	103.77
				1076.87	1134.71	1195.65	1259.87
				85.89	90.43	94.78	98.78
					1021.98	1076.87	1134.71
					81.86	86.58	91.23
						969.88	1021.98
						77.69	82.46
							920.45
							73.49

Table 6. Monte Carlo Simulation

Scenarios	Nr. Cases	Prob	Expected Value	Present Expected Value
Terminated on 1st date	3985	39.85%	105.95	105.11
Terminated on 2nd date	958	9.58%	111.90	110.15
Terminated on 3rd date	720	7.20%	117.85	115.11
Terminated on 4th date	319	3.19%	123.80	119.98
ST < 862.901	2350	23.50%	49.59	47.693
862.901 <= ST < 1128.408	1174	11.74%	100	96.169
ST >= 1128.408	494	4.94%	129.75	124.779
	10000	100%		**93.21846475**

Table 7. Computation of the values of the certificates with the change of the stock value by +1 point

Date	Close	93.49730	5/18/2021	6/18/2021	7/19/2021
5/18/2021	1327.54	**1328.54**	93.49730	93.4973032	93.4973032
6/18/2021	1361.25	**1362.25**	95.0647779	95.0647779	95.0647779
7/19/2021	1317.22	**1318.22**	92.9831667	92.9831667	92.9831667
8/18/2021	1261.6	**1262.6**	90.2868231	90.2868231	90.2868231
9/17/2021	1279.35	**1280.35**	91.0914111	91.0914111	91.0914111
10/18/2021	1283.02	**1284.02**	91.302366	91.302366	91.302366
11/18/2021	1273.87	**1274.87**	90.8387565	90.8387565	90.8387565
12/17/2021	1216.3	**1217.3**	87.8262907	87.8262907	87.8262907
1/18/2022	1241.58	**1242.58**	89.2161598	89.2161598	89.2161598
2/18/2022	1231.77	**1232.77**	88.6249309	88.6249309	88.6249309
3/18/2022	1122.98	**1123.98**	81.8380384	81.8380384	81.8380384
4/18/2022	1106.68	**1107.68**	80.5641968	80.5641968	80.5641968
5/16/2022	1007.5	**1008.5**	73.120959	73.120959	73.120959

Table 8. Delta and Gamma extra computations of Call Option

T=0	5/18/2021				
UND_0	1325.54	1326.54	1327.54	1328.54	1329.54
Structured Product	93.07597	93.13213	93.19275	93.49730	93.44347515
Delta		0.05839	0.182589035	0.12536	
Gamma			0.033486396		

T= 1 month		**6/18/2021**			
UND_0	1359.25	1360.25	1361.25	1362.25	1363.25
Structured Product	94.73024	94.76973	94.81940	95.06478	95.02567731
Delta		0.04458	0.147521679	0.10314	
Gamma			0.029277424		
T= 2 month		**7/19/2021**			
UND_0	1315.22	1316.22	1317.22	1318.22	1319.22
Structured Product	92.60557	92.64244	92.67960	92.98317	92.90748333
Delta		0.03702	0.170361364	0.11394	
Gamma			0.038463761		
T= 3 month		**8/18/2021**			
UND_0	1259.6	1260.6	1261.6	1262.6	1263.6
Structured Product	89.80050	89.85554	89.92285	90.28682	90.22365362
Delta		0.06117	0.215643452	0.15040	
Gamma			0.044615592		
T= 4 month		**9/17/2021**			
UND_0	1277.35	1278.35	1279.35	1280.35	1281.35
Structured Product	90.62761	90.68398	90.71617	91.09141	91.01711442
Delta		0.04428	0.203715307	0.15047	
Gamma			0.053095818		
T= 5 month		**10/18/2021**			
UND_0	1281.02	1282.02	1283.02	1284.02	1285.02
Structured Product	90.80295	90.87517	90.95260	91.30237	91.23327047
Delta		0.07482	0.213600465	0.14034	
Gamma			0.032756149		
T= 6 month		**11/18/2021**			
UND_0	1271.87	1272.87	1273.87	1274.87	1275.87
Structured Product	90.40688	90.44868	90.48386	90.83876	90.77600877
Delta		0.03849	0.195038418	0.14607	
Gamma			0.053791593		
T= 7 month		**12/17/2021**			
UND_0	1214.3	1215.3	1216.3	1217.3	1218.3
Structured Product	87.29876	87.36017	87.42397	87.82629	87.73121201
Delta		0.06260	0.23305813	0.15362	
Gamma			0.045508856		
T= 8 month		**1/18/2022**			
UND_0	1239.58	1240.58	1241.58	1242.58	1243.58
Structured Product	88.67241	88.72262	88.78310	89.21616	89.1153893
Delta		0.05534	0.246770221	0.16615	

Gamma			0.055400526		
T= 9 month		**2/18/2022**			
UND_0	1229.77	1230.77	1231.77	1232.77	1233.77
Structured Product	88.14344	88.18679	88.24126	88.62493	88.5762853
Delta		0.04891	0.21907155	0.16751	
Gamma			0.059299312		
T= 10 month		**3/18/2022**			
UND_0	1120.98	1121.98	1122.98	1123.98	1124.98
Structured Product	81.22049	81.28267	81.36612	81.83804	81.77391922
Delta		0.07281	0.277684901	0.20390	
Gamma			0.065543553		
T= 11 month		**4/18/2022**			
UND_0	1104.68	1105.68	1106.68	1107.68	1108.68
Structured Product	79.99742	80.06661	80.13301	80.56420	80.50335803
Delta		0.06779	0.248792951	0.18518	
Gamma			0.058691492		
T= 12 month		**5/16/2022**			
UND_0	1005.5	1006.5	1007.5	1008.5	1009.5
Structured Product	72.53913	72.61429	72.71298	73.12096	73.09217
Delta		0.08693	0.25333624	0.18960	
Gamma			0.051334112		

Table 9. Delta and Gamma Computation for the Put Option

Month	Stock Price (S)	Time to Expiration (T)	d1	N(-d1)	N'(d1)	Delta	Gamma
0	1327.54	4.956349206	1.067413426	0.142892594	0.225682927	-0.1267	0.000382
1	1361.25	4.873743386	1.138789884	0.127395401	0.208595203	-0.1132	0.000347
2	1317.22	4.791137566	1.066012215	0.14320906	0.226020505	-0.1275	0.000392
3	1261.6	4.708531746	0.966066327	0.167005496	0.250178502	-0.1489	0.000457
4	1279.35	4.625925926	1.010308069	0.156173858	0.239476562	-0.1396	0.000435
5	1283.02	4.543320106	1.026802281	0.152256787	0.235486899	-0.1363	0.000431
6	1273.87	4.460714286	1.017617505	0.154429883	0.237708243	-0.1386	0.000442
7	1216.3	4.378108466	0.905720828	0.182541818	0.264714434	-0.1641	0.000520
8	1241.58	4.295502646	0.968882422	0.166301928	0.249497818	-0.1498	0.000485
9	1231.77	4.212896825	0.957070453	0.169265848	0.25235197	-0.1528	0.000499
10	1122.98	4.130291005	0.716612042	0.236806756	0.308601357	-0.2142	0.000676
11	1106.68	4.047685185	0.683929103	0.247009968	0.31574573	-0.2239	0.000709
12	1007.5	3.965079365	0.431945307	0.332890582	0.3634088	-0.3023	0.000906

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This master thesis/internship report/project was developed with strict adherence to the academic integrity policies and guidelines set forth by ISEG, Universidade de Lisboa. The work presented herein is the result of my own research, analysis, and writing, unless otherwise cited. In the interest of transparency, I provide the following disclosure regarding the use of artificial intelligence (AI) tools in the creation of this thesis/internship report/project:

I disclose that AI tools were employed during the development of this thesis as follows:

- AI-based research tools were used to assist in literature review and data collection.
- AI-powered software was utilized for data analysis and visualization.
- Generative AI tools were consulted for brainstorming and outlining purposes. However, all final writing, synthesis, and critical analysis are my own work. Instances where AI contributions were significant are clearly cited and acknowledged.

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I understand the importance of maintaining academic integrity and take full responsibility for the content and originality of this work.

Jose Frazão Sardinha – 30/06/2024