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Master's Final Work
Internship Report

Maximizing Profitability in Savings Life Insurance

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INTERNSHIP REPORT

MAXIMIZING PROFITABILITY IN SAVINGS LIFE INSURANCE

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ABSTRACT

This study presents a methodology for assessing the financial viability of a savings life insurance product. It involves the creation of a reference investment portfolio and the implementation of a profit testing model to evaluate the product's profitability. The primary aim is to determine the optimal allocation of assets within the portfolio to ensure the product can meet its future obligations while maximizing returns, all while considering associated risk costs. The research framework adheres to the principles outlined in the Solvency II framework.

The findings indicate that employing an optimization strategy facilitates the identification of a reference investment portfolio that can potentially make the insurance product profitable. However, it is crucial to recognize the susceptibility of outcomes to variations in model inputs and prevailing conditions within the bond market. Moreover, this study highlights the limitations of static models, particularly in contexts where fluctuations in interest rates require periodic adjustments to the portfolio. Informed by existing literature, the study underscores the importance of diversification across sectors as a prudent risk management approach to mitigate concentration risk. Nevertheless, the analysis also reveals a correlation between the extent of bond allocation and resultant profitability. This suggests that increasing the upper limit for bond weight may lead to higher returns.

KEYWORDS: Life insurance profitability; Profit testing; Portfolio Selection; Solvency II; Market risk; Cost of Capital.

Resumo

Este estudo apresenta uma metodologia destinada a avaliar a viabilidade financeira de um seguro de capitalização. A abordagem proposta implica a construção de uma carteira de investimentos e a aplicação de um modelo de *Profit Testing* para avaliar a rentabilidade do produto. O principal objetivo consiste em determinar a alocação ótima de ativos na carteira, visando assegurar que o produto possa cumprir as suas obrigações futuras enquanto maximiza os retornos, considerando igualmente o custo de capital associado. O enquadramento desta pesquisa está em conformidade com os princípios delineados no regime de Solvência II.

Os resultados obtidos indicam que a adoção de uma estratégia de otimização facilita a identificação de uma carteira de investimentos capaz de tornar o produto rentável. Contudo, é fundamental reconhecer a sensibilidade dos resultados à variação dos *inputs* do modelo e às obrigações existentes no mercado no período em análise. Além disso, este estudo realça as limitações dos modelos estáticos, especialmente em contextos nos quais as flutuações nas taxas de juro exigem ajustes periódicos na composição da carteira. Com base na literatura existente, destaca-se a importância da diversificação entre diferentes setores como uma abordagem prudente de gestão de risco para mitigar o risco de concentração. No entanto, a análise também revela uma correlação entre a exposição da alocação de obrigações e a rentabilidade resultante. Isto sugere que aumentar o limite superior para a exposição das obrigações pode resultar em retornos mais altos.

PALAVRAS-CHAVE: Rentabilidade de Seguros de Vida; *Profit testing*; Constituição de uma carteira de investimento; Solvência II; Risco de Mercado; Custo de Capital.

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Abbreviations

BSCR Basic Solvency Capital Requirement

CF Cash Flow

CoC Cost of Capital

CQS Credit Quality Step

ECAI External Credit Assessment Institutions

EEA European Economic Area

EIOPA European Insurance and Occupational Pensions Authority

ISIN International Securities Identification Number

MV Market Value

MVT Mean-Variance Theory

NPV Net Present Value

OP Operational Profitability

PM Profit Margin

PV Present Value

SCR Solvency Capital Requirement

SP Single Premium

S&P Standard & Poor's

VaR Value-at-Risk

YTM Yield to Maturity

1 INTRODUCTION

The insurance industry serves a pivotal role in managing risks by providing financial protection against unexpected events (International Monetary Fund, 2016).

Savings life insurance combines life insurance coverage with an investment component. This study focuses on a particular real-life product issued by Lusitania Vida, with a maturity of 5 years. This product is characterized by a Single Premium (SP) paid upfront, which is then invested in a portfolio. Policyholders have the discretion to select the SP amount within the range of $1{,}000$ to $500{,}000$ euros. Upon surviving until the end of the contract year, policyholders become eligible for a payout calculated as the product of the initial premium and the annual return rate. At maturity, policyholders receive this payout along with an amount equivalent to the initial premium. In the event of the policyholder's death before maturity, the beneficiary receives an amount equivalent to the initial premium. Additionally, Lusitania Vida announces the liquid return rate for the upcoming year annually in November.

Such insurance products aim to facilitate wealth accumulation over time, as invested premiums grow throughout the policy's duration. From the perspective of insurers, this involves managing financial risks, since they must carefully invest the premiums to ensure they can meet future obligations to policyholders, including benefits and guaranteed values. Given the inherent volatility of this product, heavily influenced by market conditions, it becomes imperative to evaluate its profitability from the insurer's viewpoint. Profit testing in savings life insurance involves evaluating the gains and costs associated with the product, aiming to determine whether the insurance company is generating a profit or incurring losses by offering this type of product. Nevertheless, developing such models is challenging due to the multitude of variables involved.

Furthermore, Lusitania Vida currently employs a simplified model based on the assumption that all premiums received are allocated to a single reference asset. Therefore, the purpose of this report is to propose a more comprehensive approach by constructing a reference investment portfolio and then evaluating the profit/loss of the product through the necessary steps of the profit testing model. This approach aims to strengthen the insurance product's ability to meet future liabilities while optimizing the company's profitability.

This report is divided in five chapters. Chapter 2 provides the literature review. Chapter 3 presents the profit test basis and details the approach used for this investigation. Chapter 4 presents and discusses the results obtained. Finally, Chapter 5 summarizes the main achievements and offers recommendations for future work.

2 Literature Review

This chapter comprises a literature review on several key concepts in the fields of financial management and actuarial science. It begins by examining portfolio selection and theoretical frameworks for constructing efficient investment portfolios. Next, it discusses the Solvency II framework and profit testing. The chapter then investigates the cost of capital and optimization techniques. Finally, it concludes with an analysis of stress testing.

2.1 Portfolio Selection

As it will become clear the analysis focuses on bond portfolios. According to Fabozzi (2007), a bond is a debt instrument that requires the issuer to repay the lender the principal amount borrowed, along with interest, over a specified period of time.

Portfolio selection, as delineated by Markowitz (1952), involves two stages: forecasting future security performance based on observations and experiences, then using these forecasts to construct portfolios. Mean-Variance Theory (MVT) guides this process, emphasizing a balance between maximizing returns and minimizing risk.

Additionally, Markowitz (1952) advocates for diversification and the importance of prudent asset selection. Effective diversification transcends mere quantity; it involves the dispersion of investments across diverse sectors to minimize variance. Moreover, it is important to recognize that Markowitz's theoretical contributions overlook taxes and transaction costs. Real-world investment scenarios incur these expenses which can substantially influence optimal portfolio selection (Mangram, 2013).

Fabozzi (2007) accentuates that life insurance companies prioritize fulfilling policy obligations while ensuring profitability, with premiums contingent upon expected interest rates on investments. To generate profit, returns must surpass guaranteed interest paid to policyholders. In the bond sector, strategies like immunization, Cash Flow (CF) matching, or horizon matching are used for managing future liabilities, ensuring financial stability. Immunization, as defined by Rohan Chandrasekhar (2009), involves shielding portfolios from interest rate risk. Duration Matching involves aligning the durations of assets and liabilities to achieve immunization. However, it assumes only parallel shifts in the yield curve and a fixed investment horizon, making it an imperfect measure. Despite its limitations, it remains widely used by insurance companies because it is easy to implement (Iyengar and Ma, 2009). An alternative approach is CF matching, elucidated by Kocherlakota, Rosenbloom, and Shiu (1990), wherein the portfolio's CFs are matched precisely with the stream of liabilities, offering true immunization to the change of interest rates. However, this approach does not have a solution if liabilities possess longer time horizons compared to bond maturities available in the market (Iyengar and Ma, 2009).

2.2 Profit Testing

Carriço (1997) states that demographic shifts, interest rate changes, and insurance market liberalization drove insurers to adopt profit testing in order to seek preemptive control over financial outcomes. Dickson, Hardy, and Waters (2019) introduce profit testing in two stages: initially focusing on policy-generated CFs and then incorporating reserves. Both deterministic and stochastic profit tests are covered. Additionally, Carriço (1997) presents several metrics utilized in profitability assessment through profit testing, including the (i) Net Present Value (NPV) and the (ii) Profit Margin (PM). The NPV is calculated as the sum of annual net balances discounted to the present, derived from the difference between positive and negative cash flows each year. The PM, representing the average profit percentage relative to the premium charged, is widely used in the life insurance sector due to its simplicity in profit calculation from model projections. However, it fails to account for the timing of profits (Abkemeier and Vodrazka, 2002). Smart (1977) underscores the importance of linking profitability to the timing of emerging profits, advocating for profit testing models that assess both the level and timing of profits to enhance financial control in life insurance companies.

Moreover, regulatory requirements, such as Solvency II in the European Union, mandate that insurers hold specific amounts of risk capital. These requirements impact the amount of shareholder capital that must be retained and its duration, influencing the cost of capital (Rödel, Graf, Kling, and Reuß, 2021). Hancock, Huber, and Koch (2001) introduce an economic value approach that considers the present value of expected future CFs, including the cost of risk.

2.3 Solvency II

As outlined by the European Insurance and Occupational Pensions Authority (EIOPA), Solvency II is the prudential regime for insurance and reinsurance undertakings within the European Union, enacted in January 2016. As described in Egídio dos Reis, Gaspar, and Vicente (2010), it marks a significant advancement by acknowledging insurers' pivotal role in global financial markets and integrating associated market risks. Moreover, it acknowledges inherent risks in insurance, including underwriting and operational risks.

The risk-based nature of Solvency II ensures that capital requirements are proportionate to the specific risks undertaken by insurers. The structure of this regime comprises three pillars. Pillar I sets the quantitative requirements such as asset and liability valuation and capital requirements, including the Solvency Capital Requirement (SCR), which is a key focus of this study. Pillar II sets the qualitative requirements and Pillar III sets the supervisory reporting and public disclosure.

EIOPA (2009) emphasizes the Prudent Person Principle's role within Solvency II, stipulating that insurers must invest only in assets whose risks they can properly identify, measure, monitor, manage, control, and report. Investments must prioritize policyholders' and beneficiaries' best interests, ensuring portfolio security, quality, liquidity, and profitability.

Furthermore, the ongoing review of Solvency II, as highlighted in Petra Hielkema (2022), aims to maintain the relevance of the framework despite changing economic conditions. Key objectives include the introduction of macroprudential tools, improvements to the recovery and resolution framework, and the explicit incorporation of sustainability considerations. These updates ensure the regime remains robust by promoting adaptability in the insurance sector.

2.4 Cost of Capital

Insurers obtain capital from investors incurring a cost in return, termed the Cost of Capital (CoC). Insurers aim to maximize economic profit by ensuring that their earnings exceed the opportunity CoC employed (Kielholz, 2000). According to Floreani (2011), this cost represents the rate of return, in addition to the risk-free rate, that the insurance market participants require to exchange the insurance contract CFs.

Engsner, Lindholm, and Lindskog (2016) propose a computable CoC framework for evaluating insurance liabilities in a multi-period setting. Their approach involves two steps: first, replicating cash flows through financial instruments, and second, managing residual cash flows periodically using capital requirements. This study highlights the need for market consistency and attention to risk dynamics.

Moreover, regulatory requirements influence capital costs, often surpassing economic requisites to safeguard solvency and policyholder interests. Higher capital requirements reduce insolvency risks, but also increase frictional costs and premiums (Nirmalendran, Sherris, and Hanewald, 2012).

2.5 Optimization

Bertsekas and Tsitsiklis (1997) introduce linear programming, a method for minimizing (maximizing) a linear cost function while adhering to linear equality and inequality constraints. The authors explore various equivalent forms of this problem and illustrate its versatility through several examples across different contexts. Additionally, they devise a methodology for conducting sensitivity analysis within linear programming problems.

2.6 Stress Testing

Stress testing is a method used to evaluate how extreme situations would impact an institution, with the goal of assessing survivability and identifying weaknesses that could potentially lead to insolvency (Monteiro, 2016). Stress testing is categorized into sensitivity testing, scenario testing, and reverse stress testing, each serving specific purposes. According to Creech (2016), sensitivity testing, the most common form, involves stressing one assumption at a time to measure its individual impact.

This study contributes to the existing literature by exemplifying a practical application of optimization within a profit testing model, with the goal of maximizing profitability while concurrently constructing an optimal immunized portfolio. Moreover, adherence to the guidelines outlined by the Solvency II regime underscores a commitment to regulatory compliance and industry standards.

3 Data, Methodology and Assumptions

The primary focus of this study is the company's existing profit testing model and its underlying assumptions. This chapter also examines the constraints in constructing the reference portfolio and the strategy for maximizing profitability. Lastly, we discuss the methodology for stress testing the proposed solution.

3.1 Profit Test Basis

Profit testing is the commonly used term for cash flow analysis in life insurance. This study focuses on deterministic profit tests. Dickson et al. (2019) state that to estimate future CFs, the insurer must establish assumptions regarding the (i) expenses which will be incurred, (ii) the survival model for the policyholder, (iii) the rate of interest expected to be earned on CFs within each time period prior to profit realization, and (iv) potentially other factors such as an evaluation of the likelihood of policyholder policy surrenders. These assumptions collectively form the profit test basis. In our case we have,

- Contract term: The term of this type of contract is 5 years.
- Time step: Cash flows from this policy are projected at discrete one-year intervals throughout its term, beginning from the moment of policy issuance. Time zero is specified as March 28, 2024.
- Estimated total amount of premiums: This policy entails a single premium paid by each policyholder at contract inception. The total premiums for all anticipated policyholders subscribing to this contract are estimated at $100{,}000{,}000$ €.
- Return rate: The underlying gross technical rate for the product is constant at 3.4% throughout the five-year contract period, specifically for the purpose of profit testing. Additionally, considering management fees at 0.20%, the liquid return rate for policyholders is 3.20%.
- Surrender policy: The surrender payout is calculated by subtracting a predetermined penalty percentage from the single premium, as follows:
  - 1st policy year: 1.5%
  - 2nd policy year: 1.0%
  - 3rd policy year: 1.0%
  - 4th policy year: 0.5%
  - 5th policy year: 0.0%

- Policy underwriting profile: The estimated number of policies to be issued is $4{,}082$ with an average premium per policy of $24{,}500$ €. Gender distribution is 50% male and 50% female, with an average policyholder age of 60 years.

- Mortality table: Female policyholders use mortality table GKF80, while male policyholders use GKM80. Death probabilities are multiplied by a company-specific factor, resulting in the equation,

$$q_{60+t} = 6.820\% \times (0.50 \times GKF80 + 0.50 \times GKM80). \tag{1}$$

Figure 1 illustrates male mortality surpassing female mortality, as anticipated.

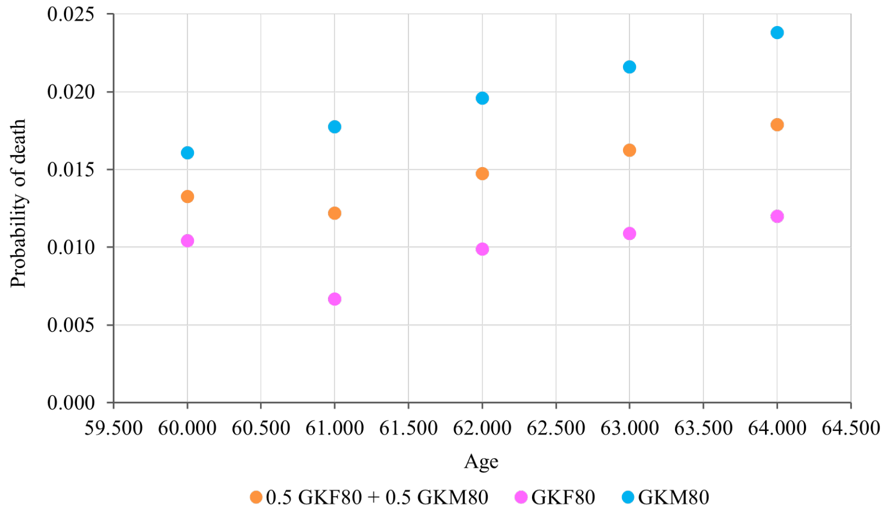

FIGURE 1: Policyholder death probabilities (q_x).

- Surrender probabilities: Surrender probabilities are estimated using historical data from similar policies. Predictions suggest that in the 1st policy year, 3.719% of all policyholders surrender, in the 2nd policy year 9.069% of the remaining policyholders, followed by 4.271% in the 3rd policy year, 9.415% in the 4th policy year and 8.615% in the 5th policy year. Detailed statistics are presented in Table I.

TABLE I: SURRENDER PROBABILITIES DESCRIPTIVE STATISTICS.

Min	Q1	Q2	Q3	Max	Range	Mean	SD	Variance	Skewness
3.720%	4.270%	8.620%	9.070%	9.410%	5.690%	7.018%	2.781%	0.077%	-0.585

- Expected Future Liabilities: A liability is a financial obligation that Lusitania Vida must settle in the future. Table II summarizes anticipated future liabilities for each contract year, considering commissions, expenses, mortality rates, surrenders, and the technical rate offered to policyholders.

TABLE II: EXPECTED FUTURE LIABILITIES.

Year	Liability (Eur)
1	11,998,945.805
2	6,863,032.566
3	10,592,493.509
4	9,029,929.271
5	68,340,475.523

- Duration: The estimated duration of the future liabilities is 3.977. To calculate this, we first determine the rate that equals the sum of the Present Value (PV) of the liability for each year to 100,000,000, which is the estimated total amount of premiums received. This rate, which we denote by r, is 1.640%. After finding this rate, we employ Equation (2).

$$D = \frac{1}{1+r} \times \frac{\sum_{i=1}^{5} i \times \frac{Liability_i}{(1+r)^i}}{100{,}000{,}000}, \quad for \quad i = 1, \ldots, 5 \text{ (contract year)} \tag{2}$$

- Risk free term structure: Cash flows (CF) are discounted using the risk-free interest rate term structures published by EIOPA on February 29, 2024 (Table III).

TABLE III: EIOPA RISK-FREE INTEREST RATE TERM STRUCTURES ON 29-02-2024.

Year	Spot rate
1	3.597%
2	3.127%
3	2.893%
4	2.763%
5	2.685%

All results presented in this report are gross of tax.

3.2 Reference investment portfolio

3.2.1 Constraints

In the current approach, Lusitania Vida uses a single bond as a reference asset, specifically an Italian government bond. The characteristics of this bond are summarized in Table IV.

TABLE IV: BOND CHARACTERISTICS.

ISIN	YTM	Duration	Coupon	Maturity	Spread Risk SCR
IT0005519787	4.380%	5.230	3.850%	15/12/2029	0.000

Now, our aim is to propose a more comprehensive approach by building a reference portfolio with a coupon rate exceeding the product's annual gross return rate, ensuring sufficient capital to meet liabilities. We achieve this by allocating funds to fixed-income assets, specifically government and corporate bonds. Using Bloomberg, we identify appropriate bonds, adhering to the following constraints:

- We restrict the currency to Euro to mitigate exposure to currency risk.
- Bonds must have a maturity of less than five years, aligning with the contract term.
- Only bonds rated as Investment Grade are considered for inclusion.
- Bonds must possess a coupon rate exceeding 3.40%.
- We only include fixed rate bonds.
- We exclusively include bullet bonds, which are non-callable bonds, meaning the principal is paid as a lump sum when the bond matures.
- The spread risk solvency capital requirement on bonds must be equal to or lower than 0.045, aligning with the spread risk solvency capital requirement for Lusitania Vida.

3.2.2 Suitable bonds

Given the imposed restrictions, Bloomberg provides us with a list of 235 bonds that meet our criteria (as of March 26, 2024). Within this collection, 57 bonds are identified as government bonds, while the majority, comprising 178 bonds, are categorized as corporate bonds. The corporate bonds are further diversified across nine distinct sectors: Communications, Consumer Discretionary, Consumer Staples, Energy, Financials, Health Care, Industrials, Materials, and Utilities. For a visual breakdown of this distribution, see Figure 2.

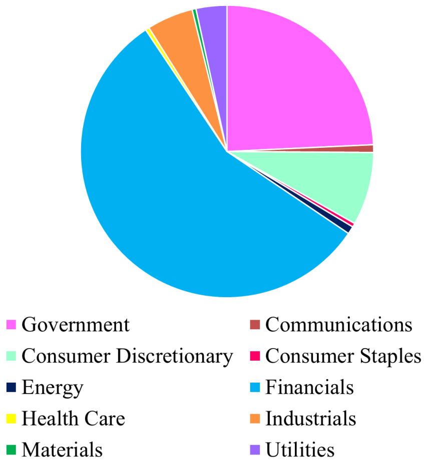

FIGURE 2: Bond categories.

Moreover, Figures 3 and 4 illustrate the bonds distribution by maturity year and Bloomberg composite credit rating[1], respectively.

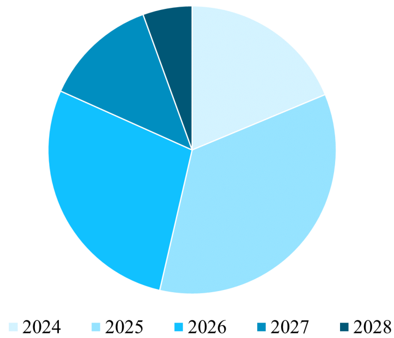

FIGURE 3: Bonds distribution by maturity year.

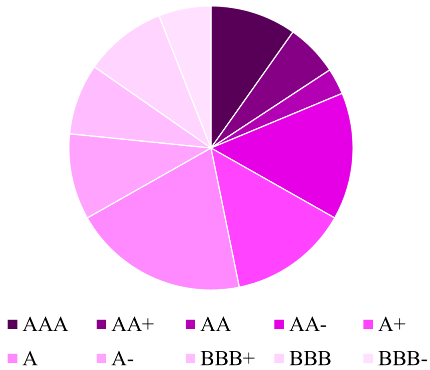

FIGURE 4: Bonds distribution by rating.

The average Yield to Maturity (YTM) across all bonds stands at 3.837%, accompanied

[1]The Bloomberg Composite Credit Ratings represent an equally weighted blend of a securitys ratings by Moodys, S&P, Fitch, and DBRS.

by an average coupon rate of 4.240%. For a deeper insight into the characteristics of these bonds, encompassing further descriptive statistics on YTM, coupon rate, and spread risk SCR, please refer to Tables V, VI, and VII, respectively.

TABLE V: YTM DESCRIPTIVE STATISTICS.

Min	Q1	Q2	Q3	Max	Range	Mean	SD	Variance	Skewness
0.263%	3.553%	3.772%	3.971%	6.448%	6.185%	3.837%	0.501%	0.251%	0.219

TABLE VI: COUPON RATE DESCRIPTIVE STATISTICS.

Min	Q1	Q2	Q3	Max	Range	Mean	SD	Variance	Skewness
3.434%	3.750%	4%	4.388%	7.550%	4.116%	4.240%	0.797%	0.635%	1.761

TABLE VII: SPREAD RISK SCR DESCRIPTIVE STATISTICS.

Min	Q1	Q2	Q3	Max	Range	Mean	SD	Variance	Skewness
0	0.014	0.024	0.033	0.045	0.045	0.023	0.013	1.690×10^{-4}	-0.182

For enhanced analysis of the optimal bonds for portfolio inclusion, Figure 5 illustrates the relationship between yield (y-axis) and bond maturity (x-axis). Similarly, Figure 6 depicts the relationship between yield (y-axis) and spread risk SCR (x-axis). In both Figures, bonds are categorized based on their Bloomberg Composite Credit Ratings: AAA bonds are denoted by pink points, AA+ to AA- by green points, A+ to A- by orange points, and BBB+ to BBB- by blue points.

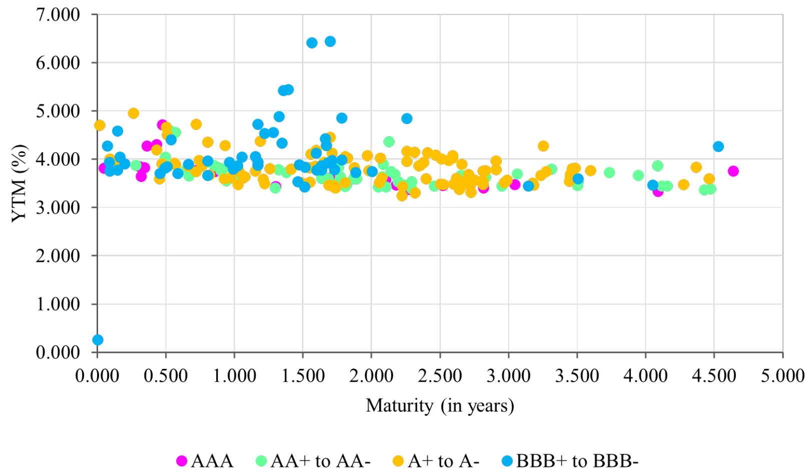

FIGURE 5: Yield versus maturity.

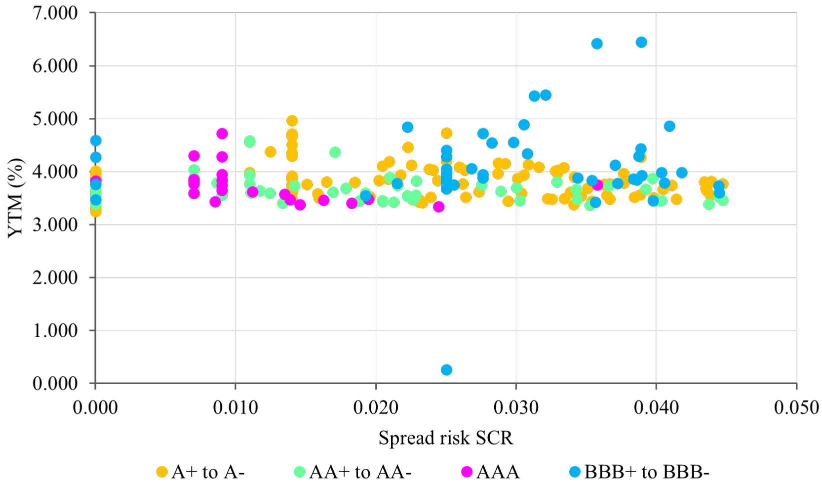

FIGURE 6: Yield versus spread risk SCR.

These visual representations guide our selection of 20 bonds (Section 4.1) that we believe strike the best balance between yield (maximal), maturity, and spread risk SCR (minimal). Aligned with the insights of Markowitz (1959) and the Mean-Variance Theory, where a portfolio comprising 20 bonds should be enough to achieve full diversification (Evans and Archer, 1968).

3.3 Profit maximization strategy

Our objective is to determine the optimal portfolio weights to maximize the insurance product's profitability while aligning the duration of assets and liabilities. To achieve this, we evaluate both the Operational Profitability (OP) of the product and its cost of capital. The estimated OP for each contract year relies on the portfolio's expected return (μ). It is computed using Equation (3), retrieving values from Table IX.

$$\begin{aligned} OP_i = P_i - Comm_i - E_{acq_i} - E_{mgmt_i} - E_{inv_i} - E_{claims_i} - Claims_i - \\ - \Delta Prov_i + Prov_{final_i} \times \mu + FI_i, \quad \text{for } i = 1, \ldots, 5 \end{aligned} \tag{3}$$

The variables in Equation (3) are defined as follows:

TABLE VIII: NOMENCLATURE FOR OPERATIONAL PROFITABILITY CALCULATION.

Variable	Description
OP_i	Operational profitability for year i.
P_i	Premiums received in year i.
$Comm_i$	Commissions paid to brokers in year i.
E_{acq_i}	Acquisition expenses in year i.
E_{mgmt_i}	Management expenses in year i.
E_{inv_i}	Investment expenses in year i.
E_{claims_i}	Claim expenses in year i.
$Claims_i$	Claims paid to policyholders in year i.
$\Delta Prov_i$	Variation in provisions in year i.
$Prov_{final_i}$	Final provisions in year i.
μ	Expected return of the portfolio.
FI_i	Free income in year i.

The variable P_i in Equation (3) represents the premiums received by Lusitania Vida, which are assumed to total $100{,}000{,}000$ € at inception. $Comm_i$ refers to commissions paid to brokers for selling policies, while E_{acq_i} captures the acquisition expenses, encompassing initial costs related to selling and issuing new policies. E_{mgmt_i} denotes management expenses, which are the ongoing costs associated with maintaining existing policies. E_{inv_i} accounts for investment expenses incurred in managing the investment portfolio that supports the insurance policies. E_{claims_i} represents claim expenses which are costs incurred when policyholders make claims, while $Claims_i$ refers to direct payments to policyholders and beneficiaries in case of death or surrender. Provision variation, $\Delta Prov_i$, refers to changes in the provisions the insurer sets aside to cover future liabilities, while $Prov_{final_i}$ are the provisions the company must hold at the end of a reporting period. Lastly, FI_i, or free income, depends on the expected return of the portfolio (μ) and is calculated as,

$$FI_i = (P_i - Comm_i - E_{acq_i} - E_{mgmt_i} - E_{inv_i} - E_{claims_i} - Claims_i - \Delta Prov_i + \\ + \; Prov_{final_i} \times \mu) \times 0.5 \times \text{Forward rate}_i, \quad \text{for } i = 1, \ldots, 5 \text{ (contract year)} \tag{4}$$

Table IX presents all relevant values, which have already been computed in Lusitania Vida profit testing model.

TABLE IX: INPUTS FOR OPERATIONAL PROFITABILITY CALCULATION.

Year	P_i	$Comm_i$	E_{acq_i}
1	100,000,000	200,000	490,000
2	0	144,290.627	0
3	0	87,686.958	0
4	0	0	0
5	0	0	0
Year	E_{mgmt_i}	E_{inv_i}	E_{claims_i}
1	67,987.141	67,200.954	2,302.392
2	63,619.866	128,134.490	5,321.048
3	59,248.966	119,089.504	2,308.721
4	55,170.549	110,673.225	4,810.496
5	50,127.684	100,355.589	3,992.243
Year	$Claims_i$	$\Delta Prov_i$	$Prov_{final_i}$
1	7,021,099.170	96,001,363.553	96,001,363.553
2	11,657,579.774	−8,953,455.618	87,047,907.935
3	6,594,698.417	−3,967,952.437	83,079,955.498
4	10,421,839.240	−8,055,303.992	75,024,651.506
5	8,875,453.755	−6,684,175.983	68,340,475.523

According to Schoenmaker and Schramade (2023), a company's value is determined by its expected CFs and its cost of capital, which increases with risk. To calculate the CoC in Equation (5), we use the cost of capital methodology, projecting future regulatory capital requirements (SCR). This value is then multiplied by the CoC rate of 6%, as stipulated by EIOPA, and scaled by a ratio of 1.25, representing the desired Solvency Ratio that Lusitania Vida aims to maintain.

$$CoC_i = SCR_i \times 0.06 \times 1.25, \quad for \quad i = 1, \ldots, 5 \text{ (contract year)}, \tag{5}$$

Here CoC$_i$ denotes the CoC in contract year i, and SCR$_i$ signifies the Solvency Capital Requirement in contract year i.

After outlining the operational profitability and cost of capital of the product, we define the following objective function to maximize profitability.

$$Maximize\, profit = Operational\, profitability\, of\, the\, product - CoC \tag{6}$$

Subject to the constraints,

$$\sum_{i=1}^{20} w_i \times D_i = 3.977, \quad i = 1, ..., 20, \tag{7}$$

$$0.01 \leq w_i \leq 0.40, \quad i = 1, ..., 20, \tag{8}$$

$$\sum_{i=1}^{20} w_i = 1, \quad i = 1, ..., 20, \tag{9}$$

where w_i represents the weight of bond i and D_i its respective duration.

This strategy computes the optimal portfolio that maximizes profitability, by changing the bonds weights. The first constraint ensures alignment between the portfolio's duration and that of the insurance product (valued at 3.977 based on the assumptions in Section 3.1). The second constraint imposes a minimum weight of 1% for each bond in the portfolio, accounting for higher management costs associated with numerous bonds that lack substantial representation. This constraint also excludes short sales. Meanwhile, an upper limit of 40% is set. The final constraint mandates that the total weights of the portfolio sum to 100%. Additionally, we assume that upon bond maturity, the cash is utilized to cover surrenders, thus precluding any reinvestment. Consequently, there is no portfolio rebalancing, making this a static approach.

As previously mentioned, the operational profitability of the product depends on the portfolio's expected return. Thus, it is computed as a final step following the strategy's implementation. This involves deriving optimal portfolio weights using the equation above and then computing the expected return of the portfolio. Subsequently, the operational profitability is calculated using Equation (3).

The CoC is closely linked to the SCR, as shown in Equation (5). Therefore, it is crucial to outline the SCR calculation. EIOPA (2009) defines the SCR as the 1-year Value-at-Risk (VaR) of the Basic Own Funds (BOF) with a 99.5% confidence level. Thus, insurance companies are required to have sufficient Own Funds (OF) to survive an extremely negative year that statistically occurs once every 200 years (Rödel et al., 2021). OF are the financial resources available to the insurer, facilitating new business endeavors and serving as a cushion against unforeseen losses. OF include Basic Own Funds and Ancillary Own Funds. BOF encompass assets over liabilities (equity capital) and financial liabilities which satisfy the criteria to be accepted as OF.

The SCR can be computed using a standard formula calibrated at the European level, employing a modular bottom-up approach:

- Calculation of explicit capital charges per risk.
- Aggregation assumes loss distributions belong to the elliptical family (e.g. Normal) and uses linear correlations between risks (correlation matrix).
- Each module is calibrated to reflect the VaR 99.5%, 1 year. Under the assumption of elliptical distributions, the result of the aggregation corresponds also to the VaR 99.5%, 1 year.

The general formula for the SCR is given by,

$$SCR_i = BSCR_i + Adj_i + SCR_{Operational_i}, \quad for \quad i = 1, \ldots, 5 \text{ (contract year)}, \quad (10)$$

where $BSCR_i$ represents the Basic Solvency Capital Requirement (BSCR) in contract year i, Adj_i denotes the adjustment for the loss absorbing capacity of technical provisions and deferred taxes in contract year i and $SCR_{Operational_i}$ signifies the operational risk capital charge in contract year i.

For profit testing purposes, Lusitania Vida assumes $Adj_i = 0, \quad i = 1, ..., 5$. The operational risk SCR encompasses operational risks not already addressed in the module pertaining to the BSCR, with specific values for each contract year outlined in Table X. These values have been pre-calculated by the company.

TABLE X: OPERATIONAL RISK MODULE OF THE SCR.

Year	Operational Risk SCR (Eur)
1	441,696.313
2	387,701.057
3	357,889.724
4	313,070.333
5	275,767.710

Figure 7 illustrates each SCR component and the risk modules inside the BSCR.

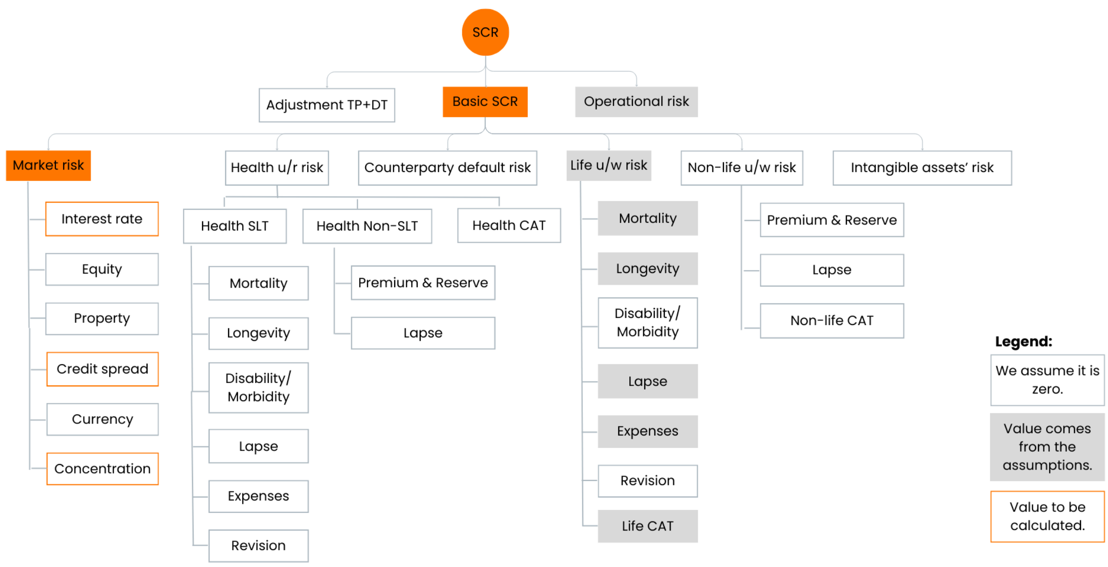

FIGURE 7: SCR MODULES.

As depicted in the figure above, the BSCR comprises six risk modules: market risk, health underwriting risk, counterparty default risk, life underwriting risk, non-life underwriting risk, and intangible assets risk. As outlined by the Solvency II framework, the general formula for calculating the BSCR is,

$$BSCR = \sqrt{\sum_{i,j} Corr_{i,j} \times SCR_i \times SCR_j} + SCR_{intangibles}, \tag{11}$$

$$for \quad i, j \in \{\text{market}, \text{counterparty\_default}, \text{life}, \text{health}, \text{non\_life}\},$$

where SCR_i represents the Solvency Capital Requirement for each risk module, while $Corr_{i,j}$ represents the correlation between each risk module i and j. These correlations, provided by EIOPA, are detailed in Table XXXVI in the Appendix.

3.3.1 Life underwriting risk

Given that this is a life insurance product, both health underwriting risk and non-life underwriting risk have zero capital charges. Additionally, we assume zero capital charges for counterparty default risk. This module addresses potential losses from unexpected defaults or credit degradation of counterparties and debtors associated with insurance and reinsurance entities. It includes various risk mitigation instruments such as reinsurance agreements, securitizations, derivative contracts, receivables from intermediaries, and other credit exposures not covered by the spread risk submodule. Intangible assets must be able to be sold independently to retain value on the insurer's balance sheet for solvency purposes. Moreover, there must be evidence of an active market where comparable intangible assets are exchanged. As the company's intangible assets do not meet these requirements, their capital charge is considered zero.

Furthermore, specific values for SCR_{life}, pre-calculated by the company, are referenced in Table XI.

TABLE XI: LIFE UNDERWRITING RISK MODULE OF THE SCR.

Year	Life u/w Risk SCR (Eur)
1	4,257,672.215
2	9,193,898.958
3	4,121,130.195
4	8,201,116.123
5	6,775,244.657

3.3.2 Market risk

Now, our focus lies on computing the module for market risk. The capital charge for market risk is determined as follows,

$$SCR_{market} = \sqrt{\sum_{i,j} Corr_{i,j} \times SCR_i \times SCR_j}, \tag{12}$$

where i and j refer to all sub-modules of the market risk module: interest rate risk, equity risk, property risk, spread risk, concentration risk and currency risk.

In the context of this insurance contract, we assume no equity risk due to having only bonds in our investment portfolio. As highlighted in Ostrum (2019), the only fixed income assets for which equity risk is relevant are convertible bonds, which we do not possess. Property risk evaluation incorporates the market value sensitivity of all property investments, a capital charge we assume to be zero. Additionally, currency risk is mitigated by our reference portfolio assumption, including only bonds denominated in euros. The correlations between each sub-module i and j are provided by EIOPA in Tables XXXVII and XXXVIII in the Appendix. The choice between these tables depends on the interest rate risk sub-module. Table XXXVII is referenced for the interest rate upward shock, while Table XXXVIII is consulted for the downward shock. Further explanation of both shocks is provided in Section 3.3.3.

3.3.3 Interest rate risk

Interest rate risk involves potential losses from fluctuations in the interest rate term structure, affecting the value of assets and liabilities. The capital charge SCR_{mkt_int} is determined by the variation in the insurer's BOF. This variation (ΔBOF) results from the most severe of two shocks (EIOPA, 2014):

- Int_{up}: The capital requirement for the risk of an increase in the term structure of interest rates must equal the loss in BOF from an instantaneous increase in basic risk-free interest rates at various maturities in accordance with Table XII.

TABLE XII: INCREASE IN THE TERM STRUCTURE OF INTEREST RATES.

Maturity (in years)	Increase
1	70%
2	70%
3	64%
4	59%
5	55%

In any case, this increase must be at least one percentage point, at any maturity.

- Int_{down}: The capital requirement for the risk of a decrease in the interest rate term structure must equal the loss in BOF resulting from an instantaneous decrease in basic risk-free interest rates at various maturities in accordance with Table XIII.

TABLE XIII: DECREASE IN THE TERM STRUCTURE OF INTEREST RATES.

Maturity (in years)	Increase
1	75%
2	65%
3	56%
4	50%
5	46%

For negative interest rates, the decrease shall be nil.

For each contract year, SCR_{mkt_int} represents the most severe scenario, determined by comparing the capital charges net of the loss absorbing capacity of technical provisions.

3.3.4 Spread risk

Spread risk refers to potential losses due to changes in the creditworthiness of issuers of securities held in the insurers investment portfolio, reflected in alterations in the underlying credit spreads. The capital charge SCR_{mkt_spread} is given by the sum of the capital charges for positions held in charges in (i) bonds and loans, (ii) securitizations, and (iii) credit derivatives (not used for hedging).

In this study, our portfolio comprises only bonds. For bonds, the capital charges are given by the ΔBOF after an instantaneous relative decrease in the instrument's value caused by the widening of their credit spreads. We apply the following equation,

$$SCR_{bonds} = MV_{bonds} \times (\sum_i \%MV_{bonds_i} \times stress_i - \\ -(\%MV_{bonds_{norating}} \times min\{D_{norating} \times 0.03; 1\})) + \Delta Liab_{ul}, \quad (13)$$

where $\%MV_{bonds_i}$ denotes the proportion of the portfolio assets subject to a capital requirement for spread risk with Credit Quality Step (CQS) i, $stress_i$ measures the sensitivity of the instrument to a shock on the credit spread, $\%MV_{bonds_{norating}}$ is the proportion of the portfolio assets subject to a capital requirement for spread risk for which no credit assessment is available, $D_{norating}$ denotes the duration denominated in years of the assets subject to a capital requirement for spread risk where no credit assessment is available. Finally, $\Delta Liab_{ul}$ refers to unit-linked contracts, which is not our case.

The reduction in value of asset i is given by Equation (14), where MV_i is the market value and $stress_i$ is a function of the CQS and the duration of the bond.

$$MV_i \times stress_i, \tag{14}$$

The CQS of an asset is based on the second-best rating from three External Credit Assessment Institutions (ECAI) (or more). If only one rating from an ECAI is available, it should be used. If only two ratings are available, the worst rating is used. Additionally, securitization positions with only one ECAI are considered unrated. In this study, we have chosen the ECAIs Moody's, Fitch, and Standard & Poor's (S&P). The correspondence between CQS and rating classes is as follows:

TABLE XIV: MAPPING OF CREDIT RATING SCALES.

Moody's		Fitch		S&P	
Rating	CQS	Rating	CQS	Rating	CQS
Aaa	1	AAA	1	AAA	1
Aa	1	AA	1	AA	1
A	2	A	2	BBB	3
Baa	3	BBB	3	BB	4
Ba	4	BB	4	B	5
B	5	B	5	CCC	6
Caa	6	CCC	6	CC	6
Ca	6	CC	6	R	6
C	6	C	6	SD/D	6
		RD	6		
		D	6		

Depending on the CQS and duration, a risk factor $stress_i$ is assigned to each bond, as outlined by EIOPA. For bonds with a duration up to 5 years, as is the case in our study, this stress is calculated by multiplying the bond's duration by a factor b_i. The values for b_i according to the CQS are shown in Table XV.

TABLE XV: FACTOR b_i ACCORDING TO THE CQS.

CQS	0	1	2	3	4	5 and 6
b_i	0.9%	1.1%	1.4%	2.5%	4.5%	7.5%

Additionally, it must be noted that exposures to European Economic Area (EEA) governments have a spread risk capital charge of zero.

3.3.5 Concentration risk

Finally, concentration risk refers to the risk of loss from reduced portfolio diversification (idiosyncratic risk), i.e. increased exposure to individual counterparties. The capital charge ($Conc_i$) is calculated as ΔBOF following an instantaneous decrease in the value of the exposure to counterparty i. The following equation is employed,

$$SCR_{conc} = \sqrt{\sum_i Conc_i^2}, \tag{15}$$

which can also be expressed as,

$$SCR_{conc} = \sqrt{\sum_i (g_i \times \max\{0; E_i - CT_i \times Assets\})^2}, \tag{16}$$

where i represents individual counterparties. The term E_i is the exposure at default to counterparty i, and CT_i denotes the relative concentration threshold applicable to counterparty i (Da Costa Ferreira, 2016). Thus, $XS_i = \max\{0; E_i - CT_i \times Assets\}$ is the excess exposure to counterparty i above the predefined threshold CT_i. Additionally, g_i is a risk factor that amplifies exposures on the worst CQS (Ostrum, 2019). Both CT_i and g_i depend on the rating of counterparty i. Table XVI illustrates the values of CT_i and g_i based on the CQS.

TABLE XVI: CT_i AND g_i BY CQS.

CQS	CT_i	g_i
0	3.0%	12.0%
1	3.0%	12.0%
2	3.0%	21.0%
3	1.5%	27.0%
4	1.5%	73.0%
5	1.5%	73.0%
6	1.5%	73.0%

Similar to the spread risk module, exposures to EEA governments carry a concentration risk capital charge of zero.

3.4 Stress Tests

Dickson et al. (2019) claim that profit testing enables insurers to subject assumptions to stress tests, gauging the sensitivity of the resulting profit to various scenarios.

In this study, we assess five distinct scenarios:

- A 1% increase in spot rates (parallel shift).
- A 1% decrease in spot rates (parallel shift).
- A twist in the yield curve where the initial spot rate increases by 1% and the ending spot rate decreases by 1%.
- A twist in the yield curve where the initial spot rate decreases by 1% and the ending spot rate increases by 1%.

- A 1% increase in the gross technical rate of the product, which may be more appealing to policyholders.

Additionally, we conduct sensitivity analysis on one of the constraints of our linear programming problem, as outlined by Bertsekas and Tsitsiklis (1997). Specifically, we examine the effects of varying the lower and upper limits of the bonds weights defined in Equation (8).

4 Results

To attain the goal, we start by constructing the reference investment portfolio and computing the market risk module of the SCR. Subsequently, we calculate the CoC, evaluate the profitability of the insurance contract, and conduct various stress tests.

4.1 Selected bonds

To build the reference portfolio, we use Bloomberg, which offers a selection of 235 bonds meeting our constraints (see Section 3.2.1). Subsequently, by analyzing Figures 5 and 6, we carefully curate a portfolio of 20 bonds that we believe achieves the optimal balance between yield (maximized), maturity, and spread risk SCR (minimized). The characteristics of the selected bonds are detailed in Table XVII.

TABLE XVII: BOND CHARACTERISTICS.

Issuer	ISIN	Dirty price (Eur)	YTM	Duration	Coupon	Maturity
Landesbank Hessen-Thueringen Girozentrale	XS0106052458	103.773	5.632%	1.417	6.080%	17/09/2024
Citigroup Inc	XS0381986453	112.832	4.930%	3.670	6.969%	28/06/2027
Bayerische Landesbank	DE000BLB3Q89	100.171	4.694%	2.283	3.750%	18/08/2025
Bayerische Landesbank	DE000BLB3QQ7	100.356	4.658%	2.248	3.730%	05/08/2025
Region of Campania Italy	XS0259658507	104.371	4.586%	2.987	4.849%	29/06/2026
Aareal Bank AG	DE000AAR0355	103.215	4.406%	2.199	4.500%	25/07/2025
Instituto de Credito Oficial	ES0200130369	109.766	4.356%	3.406	6.750%	28/12/2026
Autonomous Community of Catalonia	ES0000095606	107.338	4.324%	4.835	5.325%	05/10/2028
DZ Bank AG Deutsche Zentral-Genossenschaftsbank Frankfurt Am Main	DE000DW6CY96	100.753	4.119%	4.225	3.920%	29/10/2027
DZ Bank AG Deutsche Zentral-Genossenschaftsbank Frankfurt Am Main	DE000DJ9AHU9	98.248	4.084%	3.403	3.540%	30/09/2027
BNG Bank NV	XS0477344286	100.953	4.048%	1.758	4.125%	13/01/2025
Bayerische Landesbank	XS0105884489	112.555	3.911%	4.891	6.000%	14/11/2028
Santander Consumer Bank AG	XS2579322814	100.658	3.891%	3.611	3.893%	25/01/2027
DZ Bank AG Deutsche Zentral-Genossenschaftsbank Frankfurt Am Main	DE000DW6C4D0	98.399	3.878%	4.668	3.520%	27/04/2028
Export-Import Bank of Korea	XS0805157319	101.453	3.861%	3.972	3.600%	19/07/2027
Hong Kong Government International Bond	HK0000895893	100.858	3.845%	1.752	3.875%	11/01/2025
Bank Gospodarstwa Krajowego	XS2678204574	100.729	3.800%	3.776	4.000%	07/09/2027
Autoroutes du Sud de la France SA	FR0011276906	102.396	3.769%	1.228	3.580%	02/07/2024
Malta Government Bond	MT0000013830	101.510	3.606%	4.430	3.950%	08/08/2028
Bpifrance SACA	FR0011204007	103.413	3.600%	2.872	3.625%	25/04/2026

This portfolio includes 11 corporate bonds and 9 government bonds, with the corporate bonds exclusively sourced from two sectors: Financials and Industrials. For a visual representation of this distribution, refer to Figure 8.

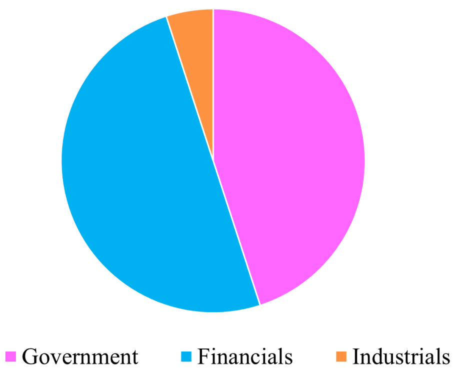

FIGURE 8: Bond categories.

4.2 *Profit maximization strategy*

4.2.1 *Optimal portfolio*

To determine the optimal portfolio weights, we implement the maximization strategy detailed in Equations (6) to (9). The objective is to achieve equality between the duration of assets and liabilities, to obtain the immunized portfolio.

Initially, we allocate equal weights to each bond (homogeneous portfolio), and then the strategy involves adjusting these weights until reaching the optimal point. Following this process, we achieve a portfolio with a Yield to Maturity (YTM) of 4.391% and a duration of 3.977, as expected. The weights and nominal values for each security within the portfolio are detailed in Table XVIII. Additionally, Figure 9 provides a visual representation of the portfolio.

TABLE XVIII: PORTFOLIO ALLOCATION.

ISIN	Weight	Nominal (Eur)
ES0000095606	0.400	37,265,000
XS0105884489	0.185	16,452,000
XS0106052458	0.152	14,633,000
DE000DW6CY96	0.062	6,190,000
DE000DW6C4D0	0.039	4,003,000
ES0200130369	0.018	1,626,000
XS0381986453	0.013	1,183,000
Other bonds ($weight_i = 0.010$)	0.130	12,822,000

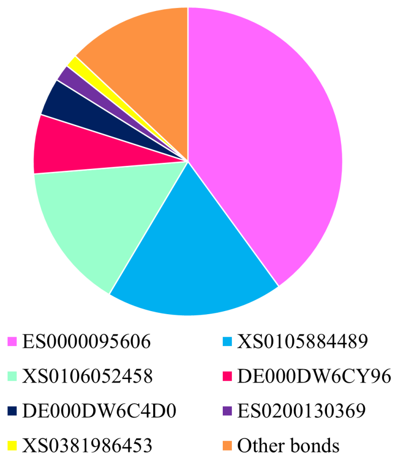

FIGURE 9: Optimal portfolio.

We can observe that the portfolio's weight is nearly evenly distributed between corporate bonds (51.215%) and government bonds (48.785%).

Compared to the initial homogeneous portfolio depicted in Figure 8 prior to optimization, there has now been a shift in the distribution by bond categories. As illustrated in Figure 10, the allocation to bonds categorized as Financials remains relatively stable. However, there is now a higher weight in Government bonds and a lower weight in Industrials.

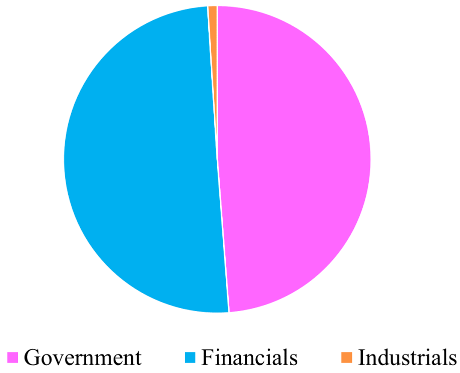

FIGURE 10: Bond categories.

Furthermore, Figure 11 illustrates the bond distribution by country of risk.

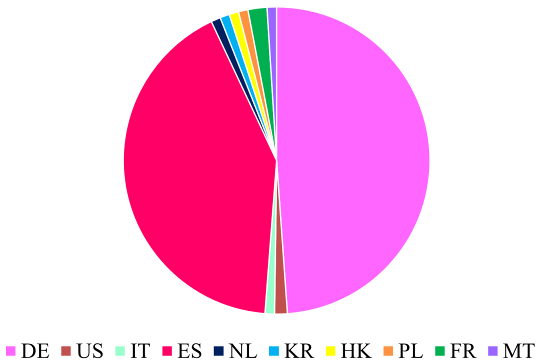

FIGURE 11: Bond distribution by country of risk.

Figure 12 shows the bond distribution by Moody's, Fitch and S&P ratings, respectively.

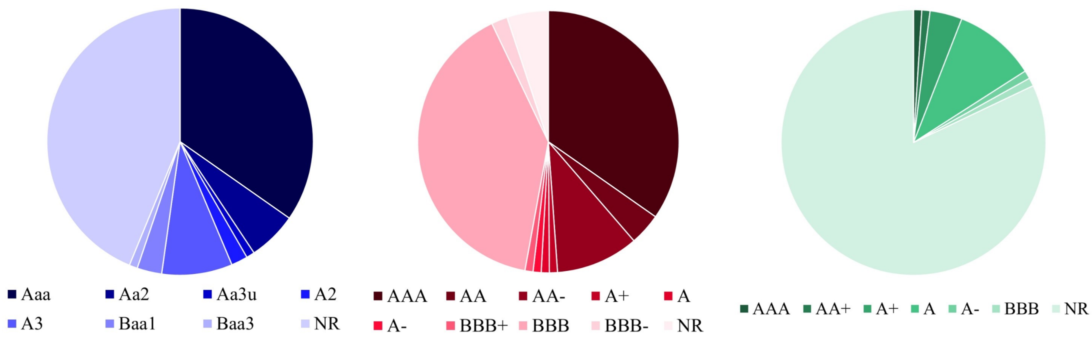

FIGURE 12: Bonds distribution by Moody's, Fitch and S&P ratings.

4.2.2 *Interest rate risk*

Given that the SCR for market risk is contingent upon the assets held, with the weights now established for each bond, we can compute the values for each sub-module within market risk.

The interest rate risk capital charge arises from the most severe of two shocks: an increase or decrease in the term structure of interest rates. Table XIX displays the values calculated as part of our analysis, reflecting the impact of these two shocks on the insurer's balance sheet for each contract year.

TABLE XIX: INTEREST RATE SHOCKS FINANCIAL PROJECTION.

	Baseline	**Upward Shock**	**Downward Shock**
Year 1			
Assets	103,181,064.151	97,876,200.360	108,056,756.712
Liabilities	92,668,092.274	90,469,258.844	95,145,765.259
A-L	10,512,971.876	7,406,941.516	12,910,991.453
ΔBOF = shock - baseline		−3,106,030.360	2,398,019.577
Year 2			
Assets	85,801,288.297	82,462,881.786	88,446,328.640
Liabilities	84,793,146.001	83,281,170.444	85,939,267.634
A-L	1,008,142.297	−818,288.658	2,507,061.006
ΔBOF = shock - baseline		−1,826,430.955	1,498,918.710
Year 3			
Assets	79,111,520.593	77,064,138.080	80,578,924.749
Liabilities	81,111,705.609	80,184,460.114	81,746,823.592
A-L	−2,000,185.016	−3,120,322.034	−1,167,898.843
ΔBOF = shock - baseline		−1,120,137.018	832,286.173
Year 4			
Assets	73,662,675.679	72,478,398.811	74,524,452.013
Liabilities	73,284,879.704	72,595,479.355	73,764,793.835
A-L	377,795.975	−117,080.544	759,658.178
ΔBOF = shock - baseline		−494,876.519	381,862.203
Year 5			
Assets	60,422,119.990	59,914,356.932	60,807,987.873
Liabilities	66,755,961.630	66,194,971.529	67,182,278.707
A-L	−6,333,841.641	−6,280,614.597	−6,374,290.835
ΔBOF = shock - baseline		53,227.043	−40,449.194

The interest rate risk charge is determined by the shock that results in a greater loss on the financial position of the insurer, in absolute value. These values are consequently summarized in Table XX.

TABLE XX: INTEREST RATE RISK SUB-MODULE OF THE SCR.

Year	Interest Rate Risk SCR (Eur)	Selected Shock
1	3,106,030.360	Upward
2	1,826,430.955	Upward
3	1,120,137.018	Upward
4	494,876.519	Upward
5	40,449.194	Downward

4.2.3 Spread risk

For bonds, the spread risk capital charge is determined by the ΔBOF following an instantaneous relative decrease in the value of the relevant instrument due to the widening of their credit spreads. The reduction in bond value depends on the CQS. Table XXI provides the rating for each bond according to the three chosen ECAIs – Moody's, Fitch, and S&P – and the resulting CQS, following the considerations in Section 3.3.4.

TABLE XXI: BOND RATINGS AND CQS.

ISIN	Moody's	Fitch	S&P	CQS
XS0106052458	Aaa	AAA	-	1
XS0381986453	A3	-	-	NR
DE000BLB3Q89	Baa1	BBB-	-	3
DE000BLB3QQ7	Baa1	BBB-	-	3
XS0259658507	Baa3	-	BBB	3
DE000AAR0355	Baa1	BBB+	-	3
ES0200130369	-	-	A	NR
ES0000095606	-	BBB	-	NR
DE000DW6CY96	A3	AA-	A	1
DE000DJ9AHU9	-	AA-	A	1
XS0477344286	Aaa	AAA	AAA	1
XS0105884489	Aaa	AAA	-	1
XS2579322814	A2	A	A	2
DE000DW6C4D0	Aa2	AA	A+	1
XS0805157319	Aa2	AA-	-	1
HK0000895893	Aa3u	AA-	AA+	1
XS2678204574	A2	A-	-	2
FR0011276906	A3	-	A-	2
MT0000013830	-	A+	-	NR
FR0011204007	Aa2	AA-	-	1

A special case applies to our scenario, as bonds with the following International Securities Identification Numbers (ISINs): XS0259658507, ES0200130369, ES0000095606, XS0477344286, XS2678204574, FR0011276906, MT0000013830, and FR0011204007, are issued by EEA governments and thus have a capital charge of zero.

Finally, valuing the assets at their market value and referring to Table XV, while applying Equation (13), we calculate the subsequent capital charges for spread risk for each contract year:

TABLE XXII: SPREAD RISK SUB-MODULE OF THE SCR.

Year	Spread Risk SCR (Eur)
1	3,340,301.279
2	1,926,492.599
3	1,167,484.657
4	770,839.456
5	234,564.958

4.2.4 Concentration risk

To assess the risk associated with increased exposure to individual counterparties, the initial step involves computing the weighted average of the CQS within each issuer group. These weights are determined by market value, and the resulting average CQS is rounded up. Non-rated exposures are assigned an arbitrary CQS of 5. The capital charge for concentration risk for bonds issued by EEA governments is zero. Table XXIII illustrates the weighted average CQS for each counterparty, for each contract year.

TABLE XXIII: CQS BY ISSUER AND BY CONTRACT YEAR.

Issuer	CQS Year 1	CQS Year 2	CQS Year 3	CQS Year 4	CQS Year 5
Landesbank Hessen-Thueringen Girozentrale	1	0	0	0	0
Citigroup Inc	5	5	5	5	0
Bayerische Landesbank	1	1	1	1	1
Region of Campania Italy	0	0	0	0	0
Aareal Bank AG	3	3	0	0	0
Instituto de Credito Oficial	0	0	0	0	0
Autonomous Community of Catalonia	0	0	0	0	0
DZ Bank AG Deutsche Zentral-Genossenschaftsbank Frankfurt Am Main	1	1	1	1	1
BNG Bank NV	0	0	0	0	0
Santander Consumer Bank AG	2	2	2	2	0
Export-Import Bank of Korea	1	1	1	1	0
Hong Kong Government International Bond	1	1	0	0	0

Finally, referencing Table XVI and applying Equation (15), which is equivalent to Equation (16), we calculate the subsequent capital charges for concentration risk for each contract year:

TABLE XXIV: CONCENTRATION RISK SUB-MODULE OF THE SCR.

Year	Concentration Risk SCR (Eur)
1	3,074,672.101
2	2,665,756.888
3	2,456,487.153
4	2,426,379.583
5	2,016,894.210

4.2.5 Market risk

Upon computing the SCR for each sub-module (refer to Table XXV), the aggregation process ensues to derive the capital charge for market risk across each contract year, as illustrated in Table XXVI. The computation of the market risk SCR employs Equation (12), along with data sourced from Table XXXVII in the Appendix for the initial four contract years, considering the chosen upward interest rate shock. Conversely, for the final contract year, data from Table XXXVIII in the Appendix corresponding to the downward interest rate shock is used.

TABLE XXV: MARKET RISK SUB-MODULES OF THE SCR.

Year	Interest Rate	Equity	Property	Spread	Concentration	Currency
1	3,106,030.360	0	0	3,340,301.279	3,074,672.101	0
2	1,826,430.955	0	0	1,926,492.599	2,665,756.888	0
3	1,120,137.018	0	0	1,167,484.657	2,456,487.153	0
4	494,876.519	0	0	770,839.456	2,426,379.583	0
5	40,449.194	0	0	234,564.958	2,016,894.210	0

TABLE XXVI: MARKET RISK MODULE OF THE SCR.

Year	Market Risk SCR (Eur)
1	5,500,785.922
2	3,762,111.582
3	2,941,437.828
4	2,593,533.134
5	2,033,225.780

4.2.6 Solvency Capital Requirement

Having established the market SCR, the subsequent step involves aggregating the risk modules according to Equation (11) to compute the BSCR in Table XXVII.

TABLE XXVII: BSCR.

Year	Market	Counterparty Default	Life	Health	Non-life	Intangibles	BSCR
1	5,500,785.922	0	4,257,672.215	0	0	0	7,752,205.502
2	3,762,111.582	0	9,193,898.958	0	0	0	10,769,192.100
3	2,941,437.828	0	4,121,130.195	0	0	0	5,629,990.649
4	2,593,533.134	0	8,201,116.123	0	0	0	9,198,894.117
5	2,033,225.780	0	6,775,244.657	0	0	0	7,544,915.393

Additionally, incorporating the operational risk capital charge (Table X) into the BSCR for each contract year results in the SCR, as detailed in Table XXVIII.

TABLE XXVIII: SCR.

Year	SCR (Eur)
1	8,193,901.815
2	11,156,893.157
3	5,987,880.373
4	9,511,964.450
5	7,820,683.103

4.2.7 Cost of Capital

Furthermore, with the SCR for each contract year, we can calculate the CoC, yielding the following results:

TABLE XXIX: COC AND PRESENT VALUE OF COC FOR EACH CONTRACT YEAR.

Year	CoC (Eur)	PV of CoC (Eur)
1	614,542.636	614,542.636
2	836,766.987	807,713.531
3	449,091.028	422,269.399
4	713,397.334	654,898.495
5	586,551.233	525,967.434

4.2.8 Operational Profitability

Using the weights of the optimal portfolio, we derive the expected return of the portfolio, which is then employed to compute the operational profitability showcased in Table XXX.

TABLE XXX: OP AND PRESENT VALUE OF OP FOR EACH CONTRACT YEAR.

Year	OP (Eur)	PV of OP (Eur)
1	372,198.790	359,275.645
2	787,254.997	740,236.776
3	762,233.346	699,729.936
4	766,253.833	687,108.882
5	662,964.073	580,704.186

4.3 Profit/loss assessment

Moreover, employing Equation (17), we compute the profit for each of the five years of the contract, presented in Table XXXI.

$$Profit = Operational\ profitability\ of\ the\ product - CoC \tag{17}$$

TABLE XXXI: PROFIT AND PRESENT VALUE OF PROFIT FOR EACH CONTRACT YEAR.

Year	Profit (Eur)	PV of Profit (Eur)
1	−242,343.846	−255,266.991
2	−49,511.990	−67,476.755
3	313,142.318	277,460.537
4	52,856.499	32,210.388
5	76,412.841	54,736.753

Ultimately, the profit margin for this product is presented in Table XXXII. A positive sign indicates profit, while a negative sign denotes a loss.

TABLE XXXII: FINANCIAL METRICS.

Metric	Value
OP of the product (Eur)	3,067,055.426
CoC of the product (Eur)	3,025,391.495
Profit (Eur)	41,663.931
Profit Margin (%)	0.042%

This approach allows us to achieve profit; however, it is relatively minimal when expressed as a percentage.

4.4 Stress tests

To address the sensitivity of this approach, we test different scenarios.

Firstly, a 1% increase or decrease in spot rates (parallel shift) is examined. The corresponding results are presented in Table XXXIII. As expected, there exists an inverse correlation between spot rates and the profitability of the insurance product. As spot rates increase, profitability decreases; conversely, as spot rates decrease, profitability increases.

TABLE XXXIII: COMPARISON OF PROFITABILITY ASSUMING THERE IS A PARALLEL SHIFT IN SPOT RATES.

Scenario	Spot rates up 1%	Baseline	Spot rates down 1%
OP of the product (Eur)	2,990,572.173	3,067,055.426	3,146,849.839
CoC of the product (Eur)	3,065,759.799	3,025,391.495	3,101,533.284
Profit (Eur)	−75,187.627	41,663.931	45,316.555
Profit Margin (%)	−0.075%	0.042%	0.045%

Subsequently, a twist in the yield curve is explored. Here, the initial spot rate experiences a 1% increase while the ending spot rate undergoes a 1% decline, and vice versa.

The outcomes of this analysis are displayed in Table XXXIV. These findings potentially indicate that a rise in interest rates exerts significant influence when occurring during the latter years of the contract term.

TABLE XXXIV: COMPARISON OF PROFITABILITY ASSUMING THERE IS A TWIST IN SPOT RATES.

Scenario	+1% and -1%	Baseline	-1% and +1%
OP of the product (Eur)	3,095,171.813	3,067,055.426	3,040,889.251
CoC of the product (Eur)	3,022,614.732	3,025,391.495	3,190,671.578
Profit (Eur)	72,557.081	41,663.931	−149,782.327
Profit Margin (%)	0.073%	0.042%	−0.150%

EIOPA (2023) highlights that life insurers have recently reduced guaranteed rates for newly sold traditional products. Nevertheless, we test for an increase of 1% in the gross technical rate of the product, which may hold greater appeal to policyholders. The results of this investigation are presented in Table XXXV. While this product may indeed hold appeal for policyholders, its implementation could result in financial loss for the insurer.

TABLE XXXV: COMPARISON OF PROFITABILITY ASSUMING THERE IS AN INCREASE IN THE GROSS TECHNICAL RETURN RATE OF THE CONTRACT.

Scenario	Technical rate up 1%	Baseline
OP of the product (Eur)	−759,062.329	3,067,055.426
CoC of the product (Eur)	3,039,283.950	3,025,391.495
Profit (Eur)	−3,798,346.279	41,663.931
Profit Margin (%)	−3.798%	0.042%

Finally, we assess the impact of adjusting the upper and lower limits of the bonds weights on profitability.

Figure 13 illustrates the outcomes when transitioning the upper limit of the bonds weights from 0.1 to 1, while keeping the constraint for the lower limit (0.01) defined in Equation (8). It is evident that with an increase in the upper limit of the bonds weights, the expected profitability of our product rises as well. However, this increase stabilizes at an approximate bond weight upper limit of 0.7.

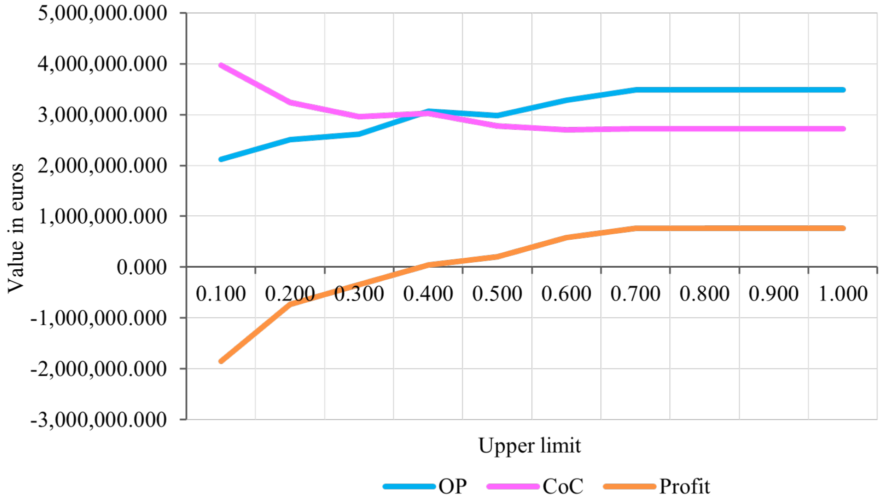

FIGURE 13: Profitability versus bond weight upper limit.

Figure 14 illustrates the results obtained by varying the lower limit of the bonds weights from 0 to 0.03, while maintaining the upper limit constraint (0.4) specified in Equation (8). We do not test for a lower limit beyond 0.03 as it would compromise our goal of aligning the duration of assets and liabilities. As depicted in the figure, increasing the lower limit of the bonds weights leads to a decrease in the expected profitability of our product.

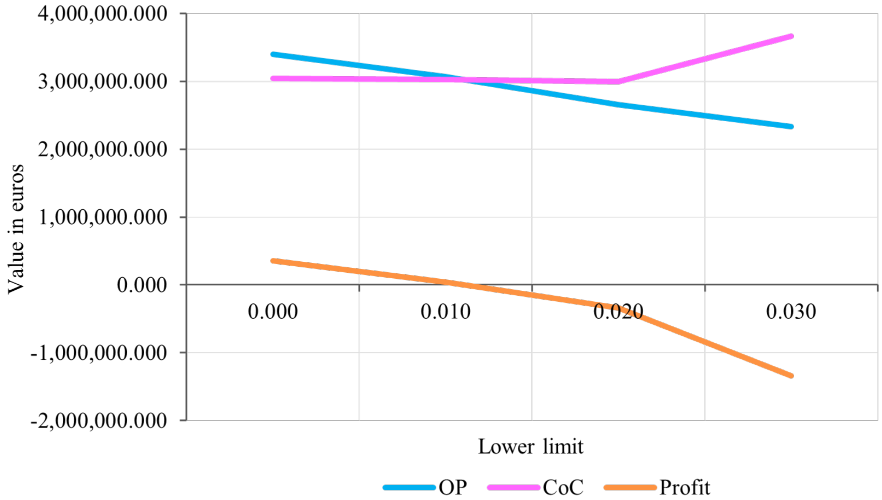

FIGURE 14: Profitability versus bond weight lower limit.

Additionally, based on our analysis, when no constraints are applied to the lower and upper limits of the bonds weights, the maximum profit achievable is $1{,}310{,}921.055$ €, corresponding to a profit margin of 1.311%. In this scenario, the lowest weight assigned to a bond within the portfolio is zero, while the highest weight reached is 0.737.

5 Conclusion

The principal aim of this report is to propose a method to maximize the expected profitability of a savings life insurance product. We employ a profit testing model based on a reference investment portfolio designed to enhance the products ability to meet future liabilities while optimizing profitability.

Our approach demonstrates the potential for achieving profitability within the defined constraints. However, the outcomes are influenced by the initial selection of bonds and subsequent choices, as well as the initial inputs to the model, as evidenced by our stress tests. Nevertheless, our method is structured to consistently yield the optimal solution under prevailing conditions.

Reducing the cost of capital was a primary focus of this study. To achieve this, we concentrated on reducing market risk. Our strategy involves constructing an immunized portfolio by matching the duration of assets with that of liabilities to decrease interest rate risk. However, this study employs a static approach. As noted by Valkenburg (2002), static models may not be satisfactory because investment portfolios are often restructured in response to changes in interest rates.

Moreover, Markowitz (1952) emphasizes the importance of diversifying investments across diverse sectors to minimize variance. Our corporate bonds are currently limited to the Industrials and Financials sectors. Future improvements should include diversifying across more sectors and reducing the amount held on a single issuer to reduce concentration risk. Additionally, to minimize spread risk in future work, investments should only focus on assets with high credit quality ratings.

While diversification is important, our study indicates that increasing the upper limit and reducing the lower limit for bond weight increases profitability. This outcome can be attributed to the strategic allocation of greater weight to bonds yielding higher returns which not only leads to increased overall returns but reduces the cost of capital, in this case.

To conclude, EIOPA (2023) highlights that market risks remain a significant concern for insurers due to ongoing global uncertainties, which are expected to lead to further losses in asset values and increased financial market volatility. Their survey identifies interest rate and equity risks as primary concerns. The rise in interest rates in 2022 has already caused substantial losses in the fixed income asset portfolios of insurers. Looking ahead, market risks are anticipated to decrease, suggesting a potentially better outcome from applying our model. Additionally, EIOPA (2023) argues that, in the medium to long term, insurers with interest rate guarantees may benefit from higher interest rates, although this also raises credit risk in investment portfolios.

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APPENDIX

TABLE XXXVI: BSCR CORRELATIONS TABLE.

	Market	Counterparty default	Life	Health	Non-life
Market	1.00	0.25	0.25	0.25	0.25
Counterparty default	0.25	1.00	0.25	0.25	0.50
Life	0.25	0.25	1.00	0.25	0.00
Health	0.25	0.25	0.25	1.00	0.00
Non-life	0.25	0.50	0.00	0.00	1.00

TABLE XXXVII: MARKET RISK MODULE CORRELATIONS TABLE WHEN Int_{up}.

	Interest	Equity	Property	Spread	Concentration	Currency
Interest	1.00	0.00	0.00	0.00	0.00	0.25
Equity	0.00	1.00	0.75	0.75	0.00	0.25
Property	0.00	0.75	1.00	0.50	0.00	0.25
Spread	0.00	0.75	0.50	1.00	0.00	0.25
Concentration	0.00	0.00	0.00	0.00	1.00	0.00
Currency	0.25	0.25	0.25	0.25	0.00	1.00

TABLE XXXVIII: MARKET RISK MODULE CORRELATIONS TABLE WHEN Int_{down}.

	Interest	Equity	Property	Spread	Concentration	Currency
Interest	1.00	0.50	0.50	0.50	0.00	0.25
Equity	0.50	1.00	0.75	0.75	0.00	0.25
Property	0.50	0.75	1.00	0.50	0.00	0.25
Spread	0.50	0.75	0.50	1.00	0.00	0.25
Concentration	0.00	0.00	0.00	0.00	1.00	0.00
Currency	0.25	0.25	0.25	0.25	0.00	1.00

DISCLAIMER

This master internship report was developed with strict adherence to the academic integrity policies and guidelines set forth by ISEG, Universidade de Lisboa. The work presented herein is the result of my own research, analysis, and writing, unless otherwise cited. In the interest of transparency, I provide the following disclosure regarding the use of artificial intelligence (AI) tools in the creation of this thesis/internship report/project:

I disclose that AI tools were employed during the development of this thesis for translation, literature review assistance, and support with LaTex software. However, all final writing, synthesis, and critical analysis are my own work.

Nonetheless, I have ensured that the use of AI tools did not compromise the originality and integrity of my work. All sources of information, whether traditional or AI-assisted, have been appropriately cited in accordance with academic standards. The ethical use of AI in research and writing has been a guiding principle throughout the preparation of this thesis.

Inês Maria Pereira Leitão, 6th June 2024