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**MASTER IN
ACTUARIAL SCIENCE**

**MASTER'S FINAL WORK
INTERNSHIP REPORT**

**LOSS RESERVING: AN INFLATION-ADJUSTED MODEL FOR
CLAIMS PROVISION IN GENERAL INSURANCE**

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1. ABSTRACT

This internship report will provide an extensive summary of my time spent working as an intern actuarial analyst in the Financial Services team at KPMG Portugal, with the focus on "Loss Reserving: An Inflation-Adjusted Model for Claims Provisions in General Insurance."

KPMG Portugal is an expert company in offering services related to audit, tax, and consulting. The branch of Financial Services is dedicated to risk management, regulatory compliance, and actuarial services. The profession of actuarial uses actuarial techniques comprised of statistical and mathematical models, actuarial analysts forecast and estimate future liabilities to ensure clients' financial stability and regulatory compliance, being used in many decisions, giving its clients the power to mitigate risks, and achieve long-term financial sustainability.

Due to its importance and real-world relevance in the insurance sector, a non-life insurance/general insurance related project was the focus of my project. This model was focused on claims reserving, using statistical methodologies to obtain claims provisions, showing the expected future payments of past claims and their financial impact. Insurers must comprehend how inflation affects future payments of past claims to properly set aside reserves, mitigate risks, and establish effective pricing tactics.

Claims provisions are obtained using sophisticated computations that take into account five distinct approaches and use the three inputs, those methodologies are Chain Ladder, variants of the Link-Ratio deterministic, Grossing Up Factors, and Grossing Up Worst Factors. Additionally, the model incorporates inflation, yielding a more precise measurement.

This model improves the accuracy and dependability of actuarial forecasts, demonstrating the role that precise claims provision calculations play in risk management and regulatory compliance. By looking at claims provision with an inflation-adjusted viewpoint, we can not only fill a significant gap in current actuarial methods but also support the overall aim of enhancing financial forecasting precision in the insurance sector.

The model was intensely tested for statistical significance during the process. The model was later fitted with an inflation adjustment, taking into consideration past and future inflation for more efficient and practical results, after careful considerations and testing. The future inflation was obtained using ARIMA forecasting and Exponential smoothing forecasting, with the help of R-Studio and Microsoft Excel using time series analysis.

In addition, the report mentions some of the nature of actuarial work at KPMG, emphasizing the importance of teamwork and communication with other teams, such as finance, compliance, and data analytics. Additionally, the report discusses the link between aligning actuarial assumptions with dynamic market conditions and the importance of maintaining data integrity.

Keywords: Actuarial Techniques, Actuarial Methodologies, Inflation-Adjusted, Inflation Forecast, Statistical Significance, Sensitivity Test, Loss Reserving, Claims Provisions, Claims Reserves, Chain Ladder, Grossing Up, Worst Factors, Average Factors, Average T-Factors, Link Ratio Deterministic, General Insurance.

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1.2. GLOSSARY

| Term | Definition |
|----------------|--|
| CV | Coefficient of Variation |
| CI | Confidence Interval |
| Qu | Quartile |
| ADF | Augmented Dickey-Fuller test |
| KPSS | Kwiatkowski-Phillips-Schmidt-Shin test |
| PP | Phillips-Perron test |
| P-value | A statistical measure that determines the significance of the assumptions in hypothesis testing |
| Tibble | A modern reimaging form of the data that allows for a cleaner interface, and prevents data handling issues |
| ARIMA(p,d,q) | Autoregressive integrated moving average of orders p,d,q |
| AR | Autoregressive |
| MA | Moving Average |
| p | Number of AR terms |
| q | Number of MA terms |
| d | Number of differences to achieve stationarity |
| SARIMA | Seasonal Arima model |
| AIC | The Akaike Information Criterion |
| AICC | Corrected AIC |
| BIC | Bayesian Information Criterion |
| Auto-ARIMA | A function that automatically chooses the ARIMA model with the best orders of (p,d,q) that minimize AIC, AICC, and BIC |
| Bias | Mean Error |
| RMSE | Root Mean Squared Error |
| MAE | Mean of the absolute error |
| MPE | Mean percentage errors |
| MAPE | Mean absolute percentage errors |
| MASE | Mean absolute squared error |
| MAE naive | Mean absolute error produced by a naive forecast |
| Naive forecast | Simple forecasting method which uses the most recent observation as the predicted value for the following time period |
| ACF1 | Auto-correlation function at lag 1 |

2. INTRODUCTION

Over the period of the three months as an Actuarial Analyst intern at KPMG Portugal, I was privileged to understand some of the functions of an Actuarial Analyst within the Financial Service team and to master the mentioned topic as the project of my internship.

This internship report aims to provide a capture my contributions throughout the internship at KPMG Portugal. A key aspect that will be emphasized in the information sharing process is the practical use of actuarial techniques within the actual financial institutions with emphasis on projects conducted in the firm. Additionally, the study investigates how the mentioned actuarial procedures are used in risk management to preserve the stability of the clients' financial organizations, notably in the insurance and pension fund sectors. Through detailed analysis, it will demonstrate the role and importance of actuaries in the field by providing information regarding the day-to-day tasks and duties of these specialists within the KPMG Financial Services department.

In this internship, my role included contributing to clients' projects, mostly related to pension funds, life, and non-life insurance. Some of my responsibilities were attending trainings and education programs, contributing to projects providing calculations in compliance with actuarial works and strategies, preparing actuarial reports, and analyzing large datasets to ensure financial efficiency and regulatory compliance conducting actuarial valuations. Specific methods as cash flow modeling, mortality and morbidity analysis, reserve calculations, and risk assessment were followed in my internship.

As a starting point of my internship, I have completed the essential risk management trainings offered by KPMG. Professional service firms like KPMG must have strong risk management to navigate complex and highly regulated environments. To make sure everyone follows the rules and maintains a high level of professionalism, KPMG requires all staff to participate in thorough training programs such as Global Independence, Integrity, Data Privacy, Information Protection, and Cloud Confidentiality. These courses are carefully planned to provide employees with the necessary knowledge and skills to effectively handle risks and follow the company's ethical standards. Through implementing thorough risk management training, companies such as KPMG can improve their capacity to navigate intricate regulatory environments, safeguard confidential data, and uphold the confidence of their clients.

Moreover, I had a thorough learning process about Insurance guidelines and IFRS 17 and Solvency regulations. KPMG's insurance guides are crucial in helping to promote ongoing learning and sharing of knowledge in the insurance sector. These guides provide information on challenges specific to certain industries, new regulations, and upcoming trends, helping professionals stay updated and improve insurance portfolio management, strengthen business resilience, and helps making informed decisions.

KPMG professionals can ensure compliance and transparency only by mastering IFRS 17, which helps managing the complexities of financial reporting for insurance contracts. Businesses handling the detailed accounting of insurance must be familiar with IFRS 17 and all of its elements. Through proficiency in aggregate, onerous contract recognition, General Measurement Model (GMM), Variable Fee Approach (VFA), Premium Allocation Approach (PAA), Contractual Service Margin (CSM), and Loss Component

(LC), and other standard components, we may guarantee adherence, boost financial transparency, and offer clients with significant perceptions into their financial performance. Similarly, to maintain their financial stability, comply with legal obligations, and foster confidence among stakeholders, insurers need to understand Solvency standards.

The focus of my Master's Final Work is to create a new KPMG template for general insurance that calculates quarterly claims provision by simply inputting the clients' historical data of number of claims, amounts paid, and claim costs. Sophisticated calculations using those 3 inputs obtain claims provisions, while accounting for 5 different methodologies. The model also has the allows for inflation inclusion, providing a more accurate measure. Having the inputs of other claims data such as claims numbers and claims costs, the model can be extended to predict the expect future values of those inputs.

The decision to prioritize general insurance's claims provision, particularly by factoring in inflation for future predictions, is supported by its vital importance and real-world relevance in the insurance sector. Claims provision is a vital component of actuarial practice, necessary for guaranteeing the financial stability and adherence to regulations of insurance firms. By including inflation in these predictions, the project tackles an important element that greatly influences the worth of upcoming claims, thus improving the accuracy and dependability of actuarial forecasts.

In today's economic environment, with inflation rates becoming more unpredictable, traditional actuarial techniques might not be able to accurately forecast future obligations. This project seeks to address this deficiency by offering a more holistic method that considers the economic factors impacting claims. Insurers must comprehend how inflation affects future payments of past claims to properly set aside reserves, mitigate risks, and establish effective pricing tactics. By looking at claims provision with an inflation-adjusted viewpoint, we can not only fill a significant gap in current actuarial methods but also support the overall aim of enhancing financial forecasting precision in the insurance sector. This highlights the selected subject's significant relevance and importance for my internship report, demonstrating the crucial role of actuarial work in the current rapidly changing economic landscape.

In addition, participating in other projects will be mentioned in the appendix due to their irrelevance to the report's topic. Those diverse projects included complex calculations for pension funds and life annuities, and the preparation of actuarial-specific reports, with the use of diverse methodologies. Comprehensive evaluations of pension fund and life annuity's financial health and sustainability have been conducted. Statistical analysis were used for historical data interpretation and determining reserve amounts, ensuring regulatory compliance and financial stability for insurance companies, enhancing my comprehensive understanding of the complex role of actuarial analysis.

Furthermore, some of my tasks and responsibilities throughout the internship included preparing and revising detailed reports, attending team meetings, extracting and evaluating data, and continuously updating actuarial models based on the latest data and regulatory changes.

2.1. Research Questions

1. What are the advantages of completing an internship in actuarial analysis from a professional and academic standpoint, and how does recording this experience advance knowledge of actuarial science's practical applications?
This topic highlights the usefulness of internship reports for both professional and personal development, emphasizing the role that practical experience plays in bridging the theoretical knowledge gap and its practical implementation in the field.
2. Value of Claims Provisions Calculations: What part do precise claims provision calculations play in risk management and regulatory compliance, and how do they improve the financial stability of insurance companies?
This issue highlights the significance of accuracy in these computations by addressing the crucial role that claims provisions play in preserving solvency and meeting commitments to policyholders.
3. Finding Claims Provisions: What techniques may be used to fill in the spaces below the triangle in actuarial calculations in order to ascertain claims provisions through the collection and decumulation of data?
The methods utilized in actuarial analysis to efficiently estimate claims provisions are the subject of this inquiry, which also emphasizes the need of data visualization in triangular tables for comprehending liabilities.
4. Appropriateness or Worth of the Applied Methodologies: How do the methodologies employed in this internship for claims reserving compare to industry norms and are they adequate and valuable in offering precise estimates?
This inquiry assesses the suitability and efficacy of the chosen approaches, encouraging a dialogue on their dependability and conformity to industry best practices.
5. To what extent does the inclusion of inflation adjustments in claims provision models follow from statistical tests and analyses, and how does this affect the precision of liability estimations in the future?
The purpose of this inquiry is to determine whether or not inflation should be taken into account calculating claims provisions using advanced statistical analysis, question explores the impact of inflation adjustments on the provisioning procedure and overall financial health of insurance firms.
6. Past or Future Inflation in the Model: Does the inclusion of past and future inflation rates in claims provision models provide more accurate and thorough forecasts or is it adequate to simply apply future inflation rates?
In order to comprehend their effect on the dependability of claims provisions, this question invites investigation into the advantages and disadvantages of employing both historical and projected inflation rates.
7. Forecasting Inflation: What statistical methods and models are most effective for reliably forecasting inflation, and how can these forecasts be integrated into actuarial practices?
This question highlights how important it is to have trustworthy inflation

forecasting methods in order to create precise financial models for risk management and insurance.

8. How may statistical significance testing affect risk management decision-making and what effect does it have on the validity of claims provision models in actuarial practice?

The significance of statistical techniques in actuarial model validation is emphasized in this question, which is essential to guaranteeing the robustness and dependability of the models used to estimate claims provisions.

9. Can the model be extended for other purposes? This question aims to explore the possibility of acquiring other future results such as claim numbers and claim costs using the same model.

These research questions seek to thoroughly examine actuarial techniques, their efficiency, and the obstacles encountered within my experience, all while taking into account the wider consequences on financial stability and regulatory adherence.

Identification of the Gap: Despite focusing on claims provisions and including inflation considerations for future forecasting in my internship project, there is still a lack of understanding regarding how much this practice improves the accuracy and reliability of claim projections. Current research frequently highlights conventional actuarial techniques for setting aside funds for claims but does not focus on how inflation affects this process. Hence, the issue exists in assessing how well incorporating inflation into predictions for claims provision can enhance readiness for future payments in the insurance industry.

2.2. Theoretical and managerial relevance

2.2.1. Theoretical Relevance

The study's theoretical significance stems from its added insights into actuarial science and financial forecasting within existing literature. Conventional actuarial techniques frequently fail to consider the ever-changing characteristics of economic factors like inflation, which can result in possible errors in claims reserves. This study improves the theoretical framework in actuarial analysis by including inflation in forecasting models, resulting in a more thorough and dependable approach to predicting future amounts to be paid in regards to past claims, referencing courses from the Masters in actuarial science at ISEG, such as PRVS - Loss Reserving, GAP-CA - Asset-Liability Management, MSOLV - Solvency Models and MP-CA - Time Series. This improvement in approach may result in enhanced models that more accurately represent actual conditions, thus pushing forward the field of actuarial science.

2.2.2. Managerial Relevance

From a management point of view, the findings of this research hold great importance for insurance firms and financial organizations. Ensuring precise claims provisioning is essential for upholding financial stability, adhering to regulations, and managing risks effectively. By factoring in inflation when projecting claims, managers can make more

accurate predictions of future debts, impacting strategic planning, pricing tactics, and reserve handling. This enhanced precision enables insurers to predict financial requirements more accurately, distribute resources more efficiently, and improve overall financial strategizing. Moreover, recognizing how inflation affects claims can assist in creating proactive plans to reduce potential financial risks, ultimately resulting in more sustainable business practices and enhanced financial performance.

3. Methodological Approach

3.1. Claims provisioning

Insurers are required to maintain reserves, also known as claims provisions, to guarantee protection against potential obligations arising from past insurance claims. Those requirements are key to ensure that an insurer can meet its obligations towards policyholders in the cases of claims. Claims provisions consist of Case Reserves, Incurred But Not Reported Reserves (IBNR), and Incurred But Not Enough Reported Reserves (IBNER). Accurately estimating and managing claims provisions can prevent potential solvency issues.

For a number of reasons, the accurate estimation of claims provision is quite important. First, it has an immediate effect on the solvency and financial stability of an insurer. Underestimating liabilities as a result of inadequate provisioning puts the insurer's capacity to pay obligations from past claims at jeopardy. Under frameworks such as Solvency II, insurers are required to maintain a best estimate of claims provisions, which is based on projected future cash flows from claims discounted to present value (Dreksler et al., 2015). Additionally, since insurers must pay for both present and future payments of past and present claims, claims provisions have an impact on premium pricing. Accurate estimation also guarantees transparency and adherence to legal frameworks, especially when it comes to financial reporting (Cazzari and Moreira, 2022).

Numerous deterministic models and procedures are used in the process of computing claims provisions across various business lines. In order to keep sufficient reserves, these models are used to estimate the amounts and expenses of past claims. Typical models comprise the Chain Ladder Method, and variants of the Link Ratio Deterministic such as Average Factors, Average T Factors, and the Worst Factor. To estimate the future, these models use historical information such as total paid amounts and the percentage of claims that were recorded. Every approach has advantages and disadvantages, and the best one is chosen depending on the particulars of the data and the industry being studied.

Expert opinion integration into stochastic models for the chain ladder methodology is explored by Verrall and England (2002), who show how important deterministic models and cutting-edge statistical methods are to calculate and validate claims provisions, maintaining actuarial compliance and financial stability.

3.1.1. Deterministic Models for Claims Provisions

The most common deterministic methods that can be used in the models for claims provisions are Chain Ladder, some variants of the Link Ratio Deterministic, and also Grossing Up Factor and Grossing Up Worst Factor. First, the underlying conditions must be suitable to use any of the methodologies, hence the statistical significance must be confirmed. Meeting all underlying assumptions and requirements is key to obtain reliable and accurate results.

Examining the Adequacy of the Statistical Indicators

To make sure that the selected models appropriately represent potential future payments

due to incurred liabilities, intensive testing and validation are necessary when analyzing the validity of statistical indicators. These indicators are used by actuaries to evaluate the adequacy of the reserves set aside and make any required adjustments to their models and assumptions. The goodness-of-fit tests, confidence intervals, and standard error of estimates are examples of key indicators.

Merz and Wüthrich (2015) highlight the significance of these statistical indicators by talking about the prediction inaccuracy of the chain ladder reserving method used on correlated run-off triangles. Their study sheds light on the validity and precision of the chain ladder approach and emphasizes the necessity of thorough statistical analysis in claims reserving.

Recognizing and Evaluating Techniques

It is essential to comprehend and evaluate every technique utilized in the initial computations to accurately estimate reserves. This entails a thorough examination of the employed methodologies, which include sophisticated statistical procedures and software programs like RStudio for bootstrap techniques and the Thomas Mack model. These techniques take into consideration the variability and uncertainty in the data, which strengthens the robustness of reserve estimates.

In the discussion of the Mack chain ladder model's application of one-year and ultimate reserve risk, Szatkowski and Delong (2021) further highlight the usefulness of sophisticated modeling tools in actuarial practice (Cambridge University Press & Assessment).

3.1.1.1. Chain Ladder

An established method in actuarial science for determining reserves in general insurance is the Chain Ladder method. This method's basic premise is to forecast future payments owing to past claims by applying development factors derived from previous claims experience to historical claims data. Actuaries can more accurately forecast outstanding liabilities by evaluating the way claims evolve over time through the analysis of run-off triangles (England & Verrall, 2002; Mack, 1993). When determining claims provisions, many actuaries prefer the Chain Ladder technique because of its simplicity and clarity, as it involves fewer assumptions than more complex stochastic models (Pinheiro et al., 2003). The method's value in reserve estimate is further supported by the fact that it can be tailored to diverse data patterns and has demonstrated performance across a range of business domains. The Chain Ladder linear model was represented in state space by Verrall (1989), who also supplied a mathematical framework that highlights the method's assumptions and improves its suitability for use in actuarial practice. In order to guarantee correct reserve estimates, this study emphasized how crucial it is to validate these assumptions.

A starting point is always confirming the validity of the Chain ladder. Being a base for all the other methodologies, its solidness also reflects the legitimacy of the others.

Key assumptions of the chain ladder are homogeneity, independence, consistency of development factors, and sufficiency of data.

Homogeneity: The claims in each development period should be comparable in character, which means that the data utilized should be homogeneous. To preserve uniformity, any

notable modifications to the policy's terms, claims handling procedures, or other elements should be taken into consideration.

To guarantee the accuracy of actuarial calculations, homogeneity in development factors is essential. The premise of homogeneity is supported when the mean of the development components significantly surpasses their standard deviation, indicating a stable average and a lack of variability (Carrato, 2019). Furthermore, for modeling to be effective, development factors must be consistent over time. This is because consistent factors guarantee that the factors will not change over time, resulting in more accurate forecasts (Actuarial Standards Board, 2013).

Consistency of Development Factors: It relies on the idea that claims evolve in a predictable way over time, from one period to the next. This suggests that the ratios of claims paid (or reported) are constant and future-projectable from one development era to the next.

Independence: The approach centers on the idea that each period's claim development occurs independently of the others.

The Chi-square test compares observed and expected frequencies across categories, which is a popular method for demonstrating the independence of variables. According to statistical methods such as those described by Embrechts, Paul, et al. (1997) in *Categorical Data Analysis*, the variables can be regarded as independent if the computed Chi-square statistic is less than the critical value at a particular significance level.

Sufficiency of Data: To accurately estimate the development factors, there must be a sufficient amount of previous data. Typically, this requires access to claims data spanning several years in order to accurately observe the development pattern.

Hiabu and Nielsen (2016) have shown the method's adaptability and resilience in various reserving scenarios, demonstrating the statistical significance of the model in real-world applications and supporting its fundamental assumptions. Conditions being met, all other models may be used accordingly.

3.1.1.2. Other models (Variants of the Link Ratio Deterministic):

The link ratio deterministic approach provides the basis for some of the methodologies that will be used in this study, such as the Worst Factors, Average Factors, Average T-Factors. Other methods such as Grossing Up Factor and Grossing Up Worst Factor are also often employed by KPMG. Actuarial scientists frequently utilize the mentioned methods to obtain claims provisions using historical data. Establishing development factors from the ratios of cumulative claims between subsequent periods is the basic idea behind the link ratio method, which aids in the estimation of liabilities. The Link Ratio Deterministic, as mentioned by England and Verrall (2002), enables actuaries to recognize patterns and forecast claims development with confidence. The link ratio approach's adaptability to varied situations and data characteristics is due to its flexibility, which allows it to handle multiple tactics like pessimism, grossing up and averaging.

The worst factor method is a conservative method that assumes homogeneity being based on the worst-case scenario, it is useful situations where negative developments in claims

may raise worries. It uses the highest development factor in the historical data as a base in reserves estimation, as proven by Peremans et al. (2018).

Similarly, the Grossing Up Worst factors method bases the worst observed scaling factor when calculating the reserves. These 2 methods assume homogeneity and extremism.

Extremism is usually seen as an inherent assumption in approaches such as the worst factor model, especially when working with extreme value theory in the insurance industry. Grossing Up Worst Factors acts as a safety net against underestimating liabilities in situations where claims experience is highly unclear or volatile. Since that these techniques are based on the idea that extreme values, or worst-case scenarios, predominate in the modeling approach, there is no need for empirical validation. According to well-established theoretical frameworks like those covered in publications like Modelling Extremal Events for Insurance and Finance (Embrechts et al., 1997), this assumption concentrates on outlier risk.

The Average Factors method assumes a constant development factor for the claims data which is the average of the development factors of the provided set of data, offering a more balanced perspective. Its foundation is the idea that historical trends will hold true, mitigating the impact of anomalies or outlier data that might distort outcomes. The validity of this method was extensively studied by Mack (1993).

The Average T-Factors work in the same way, but it allows for time adjustment, in attempt to limit data fluctuations and provide a more reliable and adjustable estimate.

Grossing Up Factors method scales reserve based on an assumed percentage, as per England and Verrall (2002). Using the Grossing Up Factors method, reported claims are adjusted by a factor to reflect the anticipated development of future payments. This technique is especially helpful for long-tail claims since it guarantees that all prospective future obligations are recorded. Actuaries seek to increase the precision of their reserve estimates by grossing up the claim's amounts.

Average Factors, Average T-Factors, and Grossing Up Factors assume consistency, stability of averages or time, proportional scaling and homogeneity. The coefficient of variation (CV) in statistical analysis can be used to evaluate how stable averages are over time. In general, minimal variability and steady data are indicated when the CV is less than 10%. This has been used in a number of areas, including economic modeling, quality control, and clinical trials. For example, (Shechtman, 2013) point out that a reduced CV, especially one below 10% is frequently a sign of consistency and stability in quantitative tests and can be utilized to demonstrate the long-term trustworthiness of data.

3.1.2. Claims Provisions calculation: Extending triangles, Accumulation and Deaccumulation for missing years

Having knowledge acquired from our master's course "PRVS - Loss Reserving" and "GAP-CA - Asset-Liability Management" has been an advantage, allowing me to understand the concept of triangles, knowing the methods and importance of its validation.

In actuarial science, data triangles are essential instruments, notably for claims reserving. They arrange data according to developmental stages, which facilitates the identification

of patterns and trends across time. Ensuring the precision and dependability of actuarial models designed to project future claim liabilities requires validating these data sets. This subsection examines into several sophisticated methods for verifying data triangles, stressing their significance in upholding strong actuarial procedures.

In several publications, the significance of validating data triangles in actuarial science is emphasized, and numerous sophisticated methods for guaranteeing the precision and dependability of claims reserving models are covered.

Use of statistical diagnostic tests is one of the main techniques for validating data triangles. A thorough analysis of stochastic models for claims reserving is given by (England and Verrall, 2006), who emphasize the significance of residual diagnostics in confirming the hypotheses that underlie these models. Actuaries can identify abnormalities and departures from model assumptions with the aid of residual diagnostics, which include residual plot analysis and standardized residual analysis. This helps to guarantee that the data triangles appropriately capture the underlying claims processes. Actuaries can improve the accuracy of their reserving estimations by using these diagnostic tests to find and fix any biases or inconsistencies in the data.

(Mack, 1993) presents a stochastic chain-ladder model that enables actuaries to compare projected and actual outcomes over time to assess the accuracy of their reserve predictions. This back-testing strategy sheds light on the dependability of various reserving techniques and aids in locating any regular biases in the data sets. Actuaries can enhance the accuracy of their claims reserving procedures and their models by regularly validating data triangles through back-testing.

With the goal to improve the validation process, (Wüthrich and Merz, 2015) investigate the integration of machine learning algorithms with conventional actuarial procedures. Actuaries can find links and patterns in complex data by using machine learning models, which may not be seen using other techniques. This hybrid technique offers a more dynamic and adaptable framework for claims reserving in addition to increasing the accuracy of data triangle validation.

The processes of accumulation and decumulation are essential for controlling future cashflows in non-life insurance. This requires performing quarterly adjustments for missing years and predicting future claim payments. Inflation effects must be taken into account in order for these estimates to be accurate. The future cash outflows needed to settle claims are impacted by inflation adjustments, which take into account how money changes in value over time.

Mack (1993) emphasizes the necessity of taking economic considerations into account in order to preserve the accuracy of future cashflow projections in his discussion of the importance of inflation adjustments in claims reserving. The works of Wüthrich and Merz (2015), expound on dynamic modeling approaches that account for inflation and other economic factors, contain the precise methodologies for integrating inflation effects.

In actuarial practice, development triangles are essential for estimating ultimate claims and evaluating the sufficiency of reserves. To assure the validity of the finalised triangles, the original estimates must be adjusted and corrected. Revision of historical data, application of suitable adjustment factors, and verification that the triangles appropriately depict the claims evolution over time are all part of this procedure.

As per Green and Iarkowski (2021); for accurate claims estimation, we must extend the triangles, the space below them must be filled in using suitable methodologies. In order to accurately reserve, this extension aids in forecasting future payments of past claims and calculating the ultimate estimate. To assess how variations in inflation impact insurance reserves, further sensitivity analysis is required.

Quarterly data based Model

For run-off triangles, using quarterly data rather than annual data can yield substantial advantages, especially in general insurance (non-life insurance), where claims tend to show more regular patterns of emergence and settlement. More responsiveness is possible with quarterly data, which can also give actuaries early warning of trends or changes in claims patterns. In short-tail business lines, where claims can develop and settle more quickly, this is very helpful. England and Verrall (2002) point out that regular, detailed data can help refine projections and adjustments for future obligations by seeing patterns earlier and increasing the accuracy of reserves. England and Verrall (2002) has also pointed out that as long as the data is appropriately modified for the period-specific properties, stochastic techniques like the Chain Ladder can be applied successfully to any time aggregation, including quarterly periods.

Quarterly statistics for general insurance is particularly helpful in identifying seasonal or cyclical differences in claims, which may be more noticeable in certain industries such as travel and motor insurance. While there may be some seasonality in the motor insurance market, it is usually not as significant as in other insurance markets, like agriculture or travel, where there are pronounced spikes in claims at particular periods of the year.

According to Pinheiro et al. (2003), forecasting models for claims provisions can be improved by using higher frequency data, such as quarterly data which can reveal seasonal patterns more successfully than annual data. This enables insurers to respond to shifts in the market, in claims, or in regulatory settings faster. On the other hand, there are other opinions disagreeing with the use of quarterly data in seasonal lines of business such as agriculture and travel insurance.

To conclude, these established techniques may be adapted to work with quarterly data for our model which is mainly designated to fit for motor insurance without sacrificing their robustness, and they frequently gain from the greater granularity that comes with more regular observations.

3.2. Inflation Adjusted Model fitting

Forecasts of inflation can have a big impact on insurance firms' reserves. Increased inflation drives up the cost of claims, particularly in industries like construction and energy that significantly rely on commodity pricing. The significance of these projections in financial reserve planning is underscored by the Monetary Policy Committee's (MPC) emphasis on reining in inflation (Pettinger, 2021).

(Sepp, 2022) indicated that inflation impacts both property damage and business disruption losses, insurers must take this into account when forecasting reserves for future payments of incurred claims.

For the efficiency of claims provision models and to guarantee that reserves are adequate to pay future obligations linked to past claims, inflation must be taken into account. In past years, inflation has had a big impact on insurance products. This is especially true for non-life insurance, as shifting economic conditions can cause claims to rise significantly over time. Actuaries can find patterns that could affect future expectations by analyzing past inflation rates, which provide insightful information about historical trends (Deloitte, 2020). The Society of Actuaries (2021) states that under-reserving significantly due to inflation can put an insurer's financial viability and regulatory compliance in jeopardy.

To appropriately predict future liabilities, incorporating projections for both historical and future inflation into claims provision models is a must. Actuaries can spot patterns that could affect future payments by using historical inflation rates, which provide insightful information about previous economic situations. Comprehending these past patterns can improve risk evaluation and sufficiency of reserves. On the other hand, future inflation forecasts are essential because they enable actuaries to make proactive model adjustments, guaranteeing that reserves will still be adequate to pay anticipated claims in an inflationary environment.

According to research, under-reserving can result from neglecting to account for inflation, especially in non-life insurance where long-term commitments are subject to influences from inflation. The necessity for insurers to implement more detailed inflation adjustments based on past and projected future patterns has been highlighted by the recent increase in inflation, which has forced a reevaluation of conventional reserving procedures (Giuffre and Borselli, 2023).

The importance of using future inflation projections in actuarial models is emphasized by research by EIOPA (2023), pointing out that these adjustments have the potential to significantly increase the accuracy of liability calculations. Accurate inflation forecasts are also necessary to keep pricing competitive and reduce the hazards brought on by unstable economies. In the tightly controlled insurance sector, inflation-accounting models can show an active approach to financial management and adhere to the industry's regulations. Furthermore, the Financial Stability Board (2024) has emphasized the importance of considering inflation when setting insurance prices to maintain long-term stability and ensure companies can remain profitable and flexible in response to market changes.

According to EIOPA (2023), the insurance sector sees a significant enhancement in the precision of reserves and claims provisions with the application of an inflation-adjusted model. Actuaries can make more precise predictions of future financial obligations by considering fluctuations in purchasing power and the value of money with time in their models.

It is imperative that reserve calculations take inflation into account. Sensitivity analyses are useful in deciding whether inflation should be applied retroactively to past claims or simply to predicted future payments. By examining the sufficiency and accuracy of reserves, these evaluations make sure that the real economic costs driven by inflation trends are reflected in them.

3.2.1. Testing the possibility of fitting an Inflation-Adjusted Model

In actuarial science, analysis is a vital technique, particularly when attempting to comprehend how inflation affects claims forecasting.

This subtopic examines the use of analysis by actuaries the effects of past claims in forecasting future payments and accounting for inflation. In order to estimate future payments, analyzing the models is useful for assessing the link between historical inflation rates and claim costs. Actuaries may anticipate future payments that are adjusted for inflation by using previous data, which guarantees precise financial planning and risk management.

Detailed models that illustrate how inflation rates affect claims expenses over time are frequently included in Actuarial reports. Setting premiums, calculating reserves, and guaranteeing regulatory compliance all depend on this information.

Accurate claims projection requires examining historical inflation rates and projecting future inflation. Actuaries apply econometric models and historical inflation trends to forecast future rates. To guarantee that reserves are sufficient to meet potential obligations, this aids in modifying claims estimates to match the predicted economic conditions.

Poufinas et al. (2023) talk about forecasting motor insurance claims using machine learning techniques, such as regression models. They discovered that forecast accuracy is much increased by include variables like weather and car sales. By adding inflation as a predictive variable, such methods can be modified for general insurance, improving the accuracy of claims projections.

Actuaries can gain insights into the possible financial impact by performing sensitivity analysis to determine the relationship between inflation rates and reserve needs. In order to help insurers prepare for different economic scenarios and ensure appropriate reserve levels to cover future payments in regard to ongoing claims, this approach involves changing inflation rates inside actuarial models to see how reserve estimations fluctuate.

IFOA (1989) indicates the essentiality of sensitivity analysis in assessing how various inflation scenarios impact reserve estimates, guaranteeing the accuracy and dependability of the reserves. Sensitivity analysis serves as a decisive tool for analyzing the robustness of forecasting models, particularly in evaluating the impact of various inflation rates on future payments forecasts. A deeper grasp of the potential risk and uncertainty related to the forecasts is made possible by the ability to observe how variations in inflation affect the output through the introduction of random inflation scenarios into the model.

Sensitivity analysis not only helps determine how much claims cost fluctuation results from variations in inflation, but it also strengthens the model's credibility by showcasing the model's adaptability to outside economic variables. According to studies (Saltelli et al., 2000; (Tolk and Rainey, 2014), sensitivity analysis is crucial to financial modeling because it sheds light on the correlations between variables and facilitates decision-making in the face of uncertainty. This methodology verifies the possibility of incorporating inflation into the model and that it is always possible to effectively modify the model's predictions to account for various inflationary scenarios.

LMA (2022) have also justified the applicability of adjusting future payments estimates by using economic forecasts and applying inflation indices to historical claims data.

3.3. Forecasting Inflation

Before performing a forecast, analyzing the data is a must, to be able to know which model can be fitted. The inflation data being used is the yearly inflation data for Portugal between 1999 and 2022 by Statista (2024) as a reference, since that 1999 is the year of introduction of the Euros as a currency in Portugal.

The data also includes a forecast of the inflation in Portugal until year 2028. This forecast was done by Aaron O'Neill, to avoid referencing the past few years' extremely high inflation rates to our forecast. The forecasted inflation rates by (O'Neill, 2024) bring stability to the desired forecast, being based on economic assumptions.

As per (O'Neill, 2024), Portugal's average inflation rate was predicted to drop by 0.2% between 2023 and 2028. Notably, this general decline does not continue in 2026 and 2027. In 2028, the predicted rate of inflation is 2.04%. This indicator, according to the International Monetary Fund, is a gauge of inflation based on changes in the average consumer price index from year to year. The latter represents the average price level of a nation using a standard basket of products and services for consumers. The figures displayed here are the % change in this index metric from year to year.

3.3.1. Preliminary Analysis of the Inflation Data set

Regression analysis and time series analysis are necessary to determine whether inflation can be forecasted from the provided dataset. Time series models enable us to look for patterns in the inflation data, such as seasonality or trends, whereas regression analysis is useful in determining the relationship between past inflation rates and other variables. We can choose a suitable forecasting technique if the dataset passes these tests.

3.3.1.1. Regression Analysis

In terms of regression models, we have one independent variable, which is time and one dependent variable, which is inflation therefore we are looking for simple linearity.

Simple linear regression assumes homogeneity, when there is homogeneity of variance, also known as homoscedasticity, the amount of the prediction error is relatively constant across the range of values for the independent variable.

The first assumption is normality: The distribution of the data is normal, another assumption is independence of observations: There are no unobserved associations between the observations in the dataset, which were gathered using statistically sound sampling techniques, and the final assumption is linearity, the line of greatest fit through the data points is a straight line (rather than a curve or some other type of grouping factor), indicating a linear relationship between the independent and dependent variables (Evans,2020).

The following step after meeting the regression requirements is to verify that the model appropriately describes the data by looking at the residual plots and goodness-of-fit

statistics and seek to identify any trends in the residuals. If trends are seen, it may be necessary to use a more sophisticated model.

In the case of absence of linearity, a regression model cannot be a good fit. Other models need to be explored.

3.3.1.2. Absence of regression, Alternative model (Time Series)

In case of failure to fit a regression model, time series analysis can be used in attempt to fit a time series forecasting model.

Starting with time series analysis, we should verify Stationarity, verifying that the residuals are steady even if you choose to use a straightforward linear regression model. The reasoning for that is that non-stationary residuals may indicate that some patterns in the data have not been fully captured by the model.

Hyndman and Athanasopoulos (2018) advise to use linear regression models for data with linear connections, and to use time series models such as ARIMA and exponential smoothing in data with no linear patterns, such as trends and seasonality.

Time series forecasting is highly dependent on stationarity since forecasts made using non-stationary data may be erroneous. The paper emphasizes the importance of conducting stationarity tests, including as the Augmented Dickey-Fuller (ADF) test, the Ljung-Box test, the Phillips-Perron (PP) test, and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test, to determine whether a dataset is stationary (Monigatti, 2023).

The ADF test's stationarity is assessed by looking for unit roots, investigating various statistical techniques, and confirming the assumptions of time series forecasting models such as ARIMA. Testing whether the data behaves randomly is essential for establishing stationarity. The Ljung-Box test looks for autocorrelation over several lags in a time series. In the meantime, the KPSS test (Kwiatkowski-Phillips-Schmidt-Shin) rejects stationarity when a trend is found in the time series by analyzing if the data has a consistent trend or variance over time.

3.3.2. Choosing the Model for forecasting Inflation

In order to increase the accuracy of inflation forecasts, a number of techniques have been used in economic planning and policymaking. Due to its simplicity and statistical resilience, ARIMA (Auto-Regressive Integrated Moving Average) and exponential smoothing stand out among them.

3.3.2.1. Stationarity: ARIMA (Auto-Regressive Integrated Moving Average)

Gaining the knowledge from the master's course "MP-CA - Time Series", it can be confirmed that ARIMA models can capture a variety of data patterns, including trends and seasonality, they are frequently employed for time series forecasting.

Defining ARIMA, it means auto regressive moving averages. The term AR (Auto-Regressive) describes a technique for predicting future values of a series based on past values. It shows the degree to which the present value is influenced by prior values. By adding the mistake of earlier forecasts into the model, the MA (Moving Average) component evaluates the impact of prior forecast errors on the current value. To

accurately forecast time series data, AR and MA work together to help identify underlying trends in the data.

ARIMA is a widely used time series model for forecasting stationary data. In the discussion of ARIMA models' use in forecasting, Nokeri (2021) highlights how well these models handle linear trends in financial data. This study examines ARIMA and its seasonal variation, SARIMA, showing how useful they are for forecasting future values based on historical data. This is especially helpful for inflation forecasting, as past inflation rates can reveal patterns for the future.

A study by Jagero, Mageto, and Mwalili (2023) used a hybrid ARIMA-ANN (Artificial Neural Network) model to predict Kenyan inflation rates. As their research showed, ARIMA models are good at describing linear relationships, but when combined with ANN, they can better handle non-linear patterns and produce forecasts that are more accurate.

Paul Goodenough covers the significance of forecasting model accuracy as well as the several metrics used to assess it, such as Mean Absolute Scaled Error (MASE) and Mean Absolute Error (MAE). It draws attention to the fact that lower values of these metrics correspond to greater model performance, supporting the project's methodology of choosing the best model in accordance with these standards (Goodenough, 2021). The debate backs the choice of ARIMA, highlighting the significance of model fit and accuracy when working with data that has been adjusted for inflation.

In the case of trends and seasonality, seasonal Auto-Regressive Integrated Moving Average (SARIMA) can be a good fit, where SARIMA is a seasonal data-handling version of the ARIMA model. To capture the recurring seasonal behavior in the data, seasonal differencing, additional seasonal autoregressive terms, and moving average terms are used. For datasets displaying periodic swings, like sales, temperatures, or other data with a definite seasonality across time, SARIMA is frequently employed in time series forecasting.

3.3.2.2. Non-Stationarity

In the case of absence of stationarity in the dataset, other models need to be explored to forecast the inflation. One option is to fit a model that does not require stationarity, and another option is to attempt to stationarize the dataset.

3.3.2.2.1. Exponential Smoothing

Even though (Nissi, Jane, et al, 2017) have concluded that ARIMA, with its compounding feature and low error values, is the best option is supported by the thorough analysis of forecast accuracy measures, this justifies the inclusion of exponential smoothing to illustrate various scenarios.

The simplicity of exponential smoothing lies within the idea that it does not require the stationarity of the data. Time series forecasting also makes heavy use of exponential smoothing techniques, such as simple, double, and triple exponential smoothing. To predict future values, these methods use weighted averages of historical observations, with higher weights assigned to more recent observations. When it comes to capturing trends and seasonal impacts in inflation data, this technique is especially helpful.

Hyndman, Rob, et al. (2008) conducted research that shows that exponential smoothing can be a very helpful technique for producing accurate inflation projections.

Makridakis et al. (1971) describes various forecasting methods and emphasizes the importance of selecting the appropriate model based on the properties of the data and the forecasting requirements.

It implies that data in which future values depend on previous values are a good fit for compounding techniques, such as ARIMA. This validates the project's decision to employ ARIMA and compounding

The article's observations on model selection procedures support the project's approach of contrasting models, among which is exponential smoothing to determine which model has the lowest forecast error.

3.3.2.2.2. ARIMA Model of Differences

The ARIMA model of differences is frequently used to stationarize a non-stationary time series modeling by differencing the data. Differences refer to this process of differencing, when seasonality or trends in the data are eliminated in order to stabilize the variance and mean over time.

Differencing is performed by deducting the prior observation from the present observation, this procedure eliminates trends and gradually stabilizes the mean. The process of differencing can be repeated for as many orders as needed until stationarity is achieved.

3.3.2.3. Stationarity and Non-Stationarity conclusion

The integration of exponential smoothing techniques with ARIMA provides a strong foundation for predicting inflation. These techniques, which are backed by empirical research, give economists and decision-makers useful tools for precisely predicting inflation patterns. After testing the significance for our forecast, a more thorough and accurate forecast results can be obtained by utilizing the advantages of each model, which is essential for efficient economic planning and decision-making.

3.3.3. Testing the forecasting Models

Important criterias for evaluating forecasting model effectiveness are the Akaike Information Criterion (AIC), corrected AIC (AICC), and Bayesian Information Criterion (BIC). In order to prevent overfitting, AIC and AICC penalize models for complexity while rewarding quality of fit. Contrarily, BIC has a higher penalty for more parameters and is especially helpful for comparing models among larger datasets (Dheer, 2024). In order to improve forecasting dependability, these factors work together to select the best model by striking a balance between complexity and accuracy.

When interpreting AIC, AICC and BIC values, lower values indicate a better-fitting model, suggesting a balance between model complexity and goodness of fit. Specifically, AICC adjusts AIC for small sample sizes, making it more reliable in those contexts. BIC generally favors simpler models more than AIC does, as it imposes a stronger penalty for additional parameters. Thus, when comparing models, select the one with the lowest AIC,

AICC, or BIC value for the best performance in terms of forecasting accuracy and simplicity (Minitab, 2024).

A statistical metric called log likelihood is used to assess how well a statistical model fits a collection of observed data. The log likelihood is widely utilized in various statistical models, including regression analysis, generalized linear models, and time series analysis. AIC, AICC, and BIC model selection criteria are crucial for these models. By maximizing the log likelihood, statisticians can determine which model best fits the data by finding a balance between fit and complexity.

When comparing and assessing the efficacy of different forecasting methods, a variety of additional accuracy metrics are commonly included in forecasting model evaluation. These measures shed light on a number of forecast quality parameters, including bias, accuracy, and the capacity to identify underlying patterns in the data. A model's tendency to consistently overestimate or underestimate the real values is indicated by the mean forecast error, sometimes referred to as the Bias. (Singh, 2021) The root mean squared error (RMSE), which gives larger errors a higher weight than smaller errors, and the mean absolute error (MAE) are used to measure how accurate the forecasts are overall. A scale-independent measure of accuracy is provided by the mean percentage error (MPE) and mean absolute percentage error (MAPE), which normalize predicted mistakes in relation to the actual values.

The mean absolute scaled error (MASE) offers information about the relative performance of the forecast by comparing its accuracy to a naive benchmark model. The autocorrelation function at lag 1 (ACF1) is another tool used to assess the presence of autocorrelation in the residuals. It shows if any patterns are still not explained by the model. Numerous research (Hyndman & Koehler, 2006; Makridakis, Wheelwright, & Hyndman, 1998) have shown that combining these indicators enables a thorough assessment of forecasting models. These comparison methods allow for a comprehensive and subtle evaluation of forecast performance, which aids in identifying the model that is best suited for practical application.

4. EMPIRICAL WORK

This chapter explores the empirical work conducted in this project. I will detail the methods and procedures adopted throughout the process, the chapter explores the critical examination and adjustment of assumptions.

Additionally, the potential option of fitting inflation-adjustment model will be presented in detail, evaluating the possible impact of fitting inflation into the model. This chapter also includes forecasting inflation with the help of multiple models, testing and comparing the forecasting models.

Finally, I will detail how to fit the chosen inflation forecast into the base model, enhancing its efficiency. The results and outcomes of the works presented in this chapter will be detailed in the results chapter.

4.1. Introduction and objective

The project's main objective is to assess and enhance the calculations for claims provisions in the general insurance sectors by applying a variety of statistical techniques and deterministic models, such as Chain Ladder, Worst Factors, Average Factors, Average T-Factors, Grossing Up Factors, and Grossing Up Worst Factors. This required analyzing data on paid amounts, claim expenses, and claim volume on a quarterly basis. The objective was to guarantee precise and sufficient provisioning through the adoption of diverse actuarial methodologies and models in reference to the help of what have been studied in the master's course "GAP-CA - Asset-Liability Management."

The first step in this topic involved calculating the non-adjusted for inflation claims provisions Model, using the strategies of KPMG for the clients and comparing our results to theirs.

Afterwards, future inflation is forecasted and tested, and a new project is created requiring new calculations for the inflation adjusted Claims Provisions model.

Lastly, the newly created proforma by me will be used by KPMG in audits as a new template for calculating claims provisions including and excluding the effect of inflation for quarterly data. For such template, the actuary is only required to add the quarterly inputs, adjust the inflation inputs for the more recent years, and if needed add the more recent inflation forecast input. The proforma then automatically outputs the expected amounts to be paid per period, expected costs and claim provisions for each methodology.

4.2. Base Claims Provision Model

First, the inputs received by the client team are imported. Those inputs include the quarterly accumulated paid amounts, number of claims, and claim costs from the first

quarter of year 2007 until the fourth quarter of year 2022 for Motor Insurance line of business.

Afterwards, a tab for the calculations of the amounts paid and a tab for the claims costs are added. Those tabs' inputs are the respective data until the date of calculations, the fourth quarter of 2022. The triangle of cumulative amounts paid of the most recent periods is shown below.

| Cumulative Amounts Paid | | | | | | | | | | | | | |
|-------------------------|---------|---------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Year | Quarter | Year 0 | | | | Year 1 | | | | Year 2 | | | |
| | | Q1 | Q2 | Q3 | Q4 | Q1 | Q2 | Q3 | Q4 | Q1 | Q2 | Q3 | Q4 |
| 2020 | Q1 | 352,404 | 1,786,236 | 2,239,612 | 2,363,222 | 2,448,343 | 2,496,361 | 2,548,302 | 2,566,748 | 2,574,196 | 2,574,196 | 2,634,882 | 2,636,011 |
| | Q2 | 141,952 | 1,172,953 | 1,448,808 | 1,529,524 | 1,596,005 | 1,610,694 | 1,661,162 | 1,662,487 | 1,668,029 | 1,666,182 | 1,668,837 | |
| | Q3 | 386,020 | 1,879,735 | 2,190,411 | 2,332,976 | 2,392,205 | 2,435,479 | 2,468,063 | 2,468,203 | 2,468,962 | 2,484,962 | | |
| | Q4 | 455,335 | 2,006,262 | 2,406,649 | 2,470,315 | 2,565,384 | 2,615,661 | 2,633,986 | 2,636,125 | 2,642,870 | | | |
| 2021 | Q1 | 369,066 | 1,523,082 | 1,720,194 | 1,814,072 | 1,862,187 | 1,895,087 | 1,910,896 | 1,913,621 | | | | |
| | Q2 | 510,820 | 2,029,175 | 2,452,245 | 2,676,927 | 2,810,196 | 2,825,714 | 2,845,473 | | | | | |
| | Q3 | 438,664 | 2,154,781 | 2,582,855 | 2,792,438 | 2,868,459 | 2,899,335 | | | | | | |
| | Q4 | 422,960 | 2,293,636 | 2,789,536 | 2,950,954 | 3,046,937 | | | | | | | |
| 2022 | Q1 | 418,086 | 1,906,888 | 2,248,971 | 2,370,384 | | | | | | | | |
| | Q2 | 443,001 | 1,928,553 | 2,490,103 | | | | | | | | | |
| | Q3 | 377,658 | 1,958,314 | | | | | | | | | | |
| | Q4 | 405,345 | | | | | | | | | | | |

Figure 4.2.1: Run-off Triangle of cumulative amounts paid

4.2.1. Data Triangles validation for Historical Claims Data

Calculating tail factors can provide a sign regarding the settlement and reporting of the claims, providing an understanding if additional costs are likely to be required (England & Verrall, 2002; Pinheiro et al., 2003).

We start by calculating the tail factors for years the last 3 years: 2020, 2021, and 2022. Within the KPMG methodologies, the tail factor is calculated by the formula below.

(4.1)

$$\text{Tail factor for a quarter} = \frac{\text{Cumulative claims cost at the end of the period}}{\text{Cumulative amount paid for the time period}}$$

- Where cumulative claims cost at the end of the period refers to the total settled claims cost of the last quarter available, which is the 4th quarter of 2022.
- The cumulative amount paid for the time period is the cumulative amount paid in the quarter for which the tail factor is being calculated.

We then find the average of tail factors for each of the 3 years, to make sure all tail factors are close to 1 as expected.

The calculated section for the average tail factors is as presented below.

| Year | 2020 | 2021 | 2022 |
|----------------------|-------------|-------------|----------|
| Average Tail Factors | 1.003486952 | 1.002439466 | 1.002009 |

Figure 4.2.1.1: Average Tail factors per year

Analyzing the table above, we observe that all averages of tail factors are close to 1, where an average of tail factors that is close to 1 indicates that the majority of claims have been settled and reported, lowering the possibility of major future developments in the claims. The validity of the data set gives an indication that fitting the desired model is a practical idea.

4.2.2. Development factors calculation per method

Although data from 2007 is available, KPMG requirements and standards advise on the referencing of more recent data, which offers a better reflection of current trends and claims development, the calculation for the development factors' starting point is 2013. This can be explained by the changes in rules, market conditions, and company practices, earlier data may not completely reflect the current claims conditions.

The development factors are then calculated for all quarters between the first quarter of 2013 and the fourth quarter of 2022, totaling to 40 development periods for the first quarter of 2023. The first development period is for quarter 1 of 2023 is 10 years from the first quarter of 2013, the second development period is 9 years and 3 quarters from Q2 of 2013, and the third entry is 9 years and 2 quarters from Q3 of 2013 and so on

4.2.2.1. Chain Ladder Development Factors

The development factors for chain ladder are calculated by the following formula:

(4.2)

$$\begin{aligned} & \text{Chain ladder development factor for development period } k \\ &= \frac{\sum_k^{n-1} \text{Cumulative amounts paid for development period } k + 1}{\sum_k^{n-1} \text{Cumulative amounts paid for development period } k} \end{aligned}$$

- Where n is the last period available for the development period k

4.2.2.2. Overall development factors (For the Link Ratio Deterministic)

The overall development factors are referenced for the variants of the link ratio deterministic. For each development period, the development factors are calculated by dividing the cumulative amounts paid of the following quarter by this quarter's cumulative amounts paid. For illustration, the formula is presented below.

(4.3)

$$\begin{aligned} & \text{Development factor for year development period } k \\ &= \frac{\text{Cumulative amount paid for development period } k + 1}{\text{Cumulative amount paid for development period } k} \end{aligned}$$

4.2.2.2.1. Worst factor

The development factor for the worst factor methodology returns the largest value of the overall development factors for each development period.

4.2.2.2.2. Average Factor

The development factor with reference to the average factor's methodology uses the average of the calculated overall development factors for each development period.

4.2.2.2.3. Average T-Factor

The first step in this method is to agree on a certain number of years to reference for this methodology, this agreed number of years will be called "T years". The development factors are then found by returning the average of the last T years' calculated overall development factors for each development period.

4.2.2.3. Additional Factors for KPMG methods

Additional factors recommended by KPMG are grossing up factors and grossing up worst factors. The theory underlying those approaches is more adaptable and made for extreme or variable settings, as previously discussed in the literature review, resulting in noticeably different components and projections.

4.2.2.3.1. Grossing Up Factors

The first step to find the grossing up development factors is to establish the grossing up adjustments $F(k)$ for all 40 development periods, where $F(1)$ is equal to the expected cumulative amounts paid for the first quarter of 2013 obtained by chain ladder.

We can then obtain the grossing up factor references for all quarters from Q1 2013 by dividing the respective development period's cumulative amounts paid by the grossing up adjustment for the first development period.

From the 2nd development period onwards, the grossing up adjustment $F(k)$ is calculated as indicated below:

(4.4)

$$\begin{aligned} \text{Grossing Up Adjustment } F(k) \\ = \frac{\text{Cumulative amount paid for development period } k}{\text{Average}(G(1), G(2), \dots, G(k-1))} \end{aligned}$$

- Where $G(k)$ is the Grossing Up factor references for development period k .

After obtaining the grossing up adjustment for period k , we can deduce the grossing up factor references by the formula below:

(4.5)

$$\begin{aligned} \text{Grossing Up factor reference } G(k) \\ = \frac{\text{Cumulative amount paid for development period } k}{F(k)} \end{aligned}$$

The triangle of grossing up factor references for all development periods can be filled accordingly. The grossing up development factors per development period is the last entry of the period's grossing up factor references.

4.2.2.3.2. Grossing Up Worst Factors

To obtain the development factors for the Grossing Up Worst factors method, we use the same methodology as the Grossing Up factors with minor changes.

The first development period's grossing up worst adjustment is equal to the cumulative amounts paid for the quarter before. The grossing up worst factor references for all quarters starting Q1 2013 are then obtained by dividing the respective period's cumulative amounts paid by the grossing up worst adjustment for the first development period.

From the 2nd development period onwards, the grossing up adjustment WF(k) is calculated as indicated below:

(4.6)

$$\begin{aligned} & \text{Grossing Up Worst Adjustment WF}(k) \\ &= \frac{\text{Cumulative amount paid for development period } k}{\min(H(1), H(2), \dots, H(k-1))} \end{aligned}$$

- Where H(k) is the Grossing Up worst Factor's reference for development period k.

After obtaining the grossing up adjustment for period k, we can deduce the grossing up factor references by the formula below:

(4.7)

$$\begin{aligned} & \text{Grossing Up Worst factor reference } H(k) \\ &= \frac{\text{Cumulative amount paid for development period } k}{WF(k)} \end{aligned}$$

The triangle of grossing up factor references for all development periods can be filled accordingly.

The grossing up worst factors per development period is the period's last entry of the triangle of grossing up worst factor references.

4.2.2.4. Factors per method

The table of factors per development period are presented in the figure below for reference.

| Year | Quarter | Development period k | Development factors Chain Ladder | Development factors Worst Factors | Development factors Average Factors | Development factors Average T-Factors | Grossing Up Factors | Grossing Up Worst Factors |
|------|---------|----------------------|----------------------------------|-----------------------------------|-------------------------------------|---------------------------------------|---------------------|---------------------------|
| 2013 | Q1 | 1 | 4.218 | 8.263 | 4.473 | 5.055 | 0.174 | 0.079 |
| 2013 | Q2 | 2 | 1.177 | 1.308 | 1.183 | 1.219 | 0.75 | 0.599 |
| 2013 | Q3 | 3 | 1.046 | 1.092 | 1.048 | 1.058 | 0.885 | 0.773 |
| 2013 | Q4 | 4 | 1.025 | 1.05 | 1.026 | 1.033 | 0.928 | 0.832 |
| 2014 | Q1 | 5 | 1.013 | 1.033 | 1.014 | 1.015 | 0.951 | 0.862 |
| 2014 | Q2 | 6 | 1.009 | 1.034 | 1.01 | 1.011 | 0.964 | 0.879 |
| 2014 | Q3 | 7 | 1.005 | 1.02 | 1.005 | 1.004 | 0.974 | 0.897 |
| 2014 | Q4 | 8 | 1.003 | 1.011 | 1.003 | 1.003 | 0.978 | 0.898 |
| 2015 | Q1 | 9 | 1.003 | 1.008 | 1.003 | 1.002 | 0.981 | 0.901 |
| 2015 | Q2 | 10 | 1.003 | 1.024 | 1.003 | 1.003 | 0.984 | 0.906 |
| 2015 | Q3 | 11 | 1.001 | 1.006 | 1.001 | 1.001 | 0.987 | 0.925 |
| 2015 | Q4 | 12 | 1.002 | 1.009 | 1.002 | 1.002 | 0.988 | 0.928 |
| 2016 | Q1 | 13 | 1.001 | 1.01 | 1.002 | 1.001 | 0.99 | 0.931 |
| 2016 | Q2 | 14 | 1.001 | 1.005 | 1.001 | 1.001 | 0.992 | 0.936 |
| 2016 | Q3 | 15 | 1 | 1.009 | 1 | 1 | 0.993 | 0.938 |
| 2016 | Q4 | 16 | 1.001 | 1.013 | 1.001 | 1.001 | 0.993 | 0.944 |
| 2017 | Q1 | 17 | 1.003 | 1.058 | 1.003 | 1 | 0.994 | 0.946 |
| 2017 | Q2 | 18 | 1.001 | 1.014 | 1.001 | 1.001 | 0.996 | 0.966 |
| 2017 | Q3 | 19 | 1 | 1.002 | 1 | 1 | 0.997 | 0.966 |
| 2017 | Q4 | 20 | 1 | 1.001 | 1 | 1 | 0.997 | 0.968 |
| 2018 | Q1 | 21 | 1 | 1.004 | 1 | 1 | 0.997 | 0.968 |
| 2018 | Q2 | 22 | 1.002 | 1.032 | 1.002 | 1.002 | 0.997 | 0.968 |
| 2018 | Q3 | 23 | 1 | 1.004 | 1 | 1 | 0.999 | 0.988 |
| 2018 | Q4 | 24 | 1 | 1.001 | 1 | 1 | 0.999 | 0.988 |
| 2019 | Q1 | 25 | 1 | 1 | 1 | 1 | 0.999 | 0.988 |
| 2019 | Q2 | 26 | 1 | 1 | 1 | 1 | 0.999 | 0.988 |
| 2019 | Q3 | 27 | 1 | 1.001 | 1 | 1 | 0.999 | 0.988 |
| 2019 | Q4 | 28 | 1 | 1 | 1 | 1 | 0.999 | 0.988 |
| 2020 | Q1 | 29 | 1 | 1 | 1 | 1 | 0.999 | 0.988 |
| 2020 | Q2 | 30 | 1 | 1.004 | 1 | 1 | 0.999 | 0.988 |
| 2020 | Q3 | 31 | 1 | 1 | 1 | 1 | 0.999 | 0.992 |
| 2020 | Q4 | 32 | 1.001 | 1.007 | 1.001 | 1.001 | 0.999 | 0.992 |
| 2021 | Q1 | 33 | 1 | 1 | 1 | 1 | 1 | 0.999 |
| 2021 | Q2 | 34 | 1 | 1 | 1 | 1 | 1 | 0.999 |
| 2021 | Q3 | 35 | 1 | 1.001 | 1 | 1 | 1 | 0.999 |
| 2021 | Q4 | 36 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2022 | Q1 | 37 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2022 | Q2 | 38 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2022 | Q3 | 39 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2022 | Q4 | 40 | 1 | 1 | 1 | 1 | 1 | 1 |

Figure 4.3.2.1: Factors per Method for all Development Periods

Commenting on the figure above, it is important to note that because of their underlying philosophy and calculation procedure, the grossing up factors and grossing up worst factors differ greatly from the conventional chain ladder method and link ratio factors. In summary, the growth of proportionate claims is not the same concept for the grossing up elements. Rather, they rely on modifications made to the amounts paid at each period, with the factors intended to scale the observed amounts according to specific guidelines.

4.2.3. Finding claims provisions per methodology.

Firstly, the area below the triangles is filled for each methodology. We can then obtain the data for each methodology separately and find the claims provisions per method.

The expected cumulative amounts paid per development period is then obtained with calculating for the future years, by filling the area below the triangle for each methodology separately. This can be obtained by multiplying the period's respective cumulative amounts paid or the claims costs by the period's development factor for each methodology.

The claims provisions per development period are calculated as the expected cumulative amounts paid of the respective period minus the previous period's cumulative amounts paid or claims costs for each method.

We have now obtained claims provisions per method side by side for each calculation, as it is shown below for illustration. Those results can be compared to the clients' calculation for claims provisions for further analysis, as to be discussed in the results chapter.

| Year | Quarter | Development period | Claims provision Chain Ladder | Claims provision Worst Factors | Claims provision Average Factors | Claims provision Average T-Factors | Claims provision Grossing Up Factors | Claims provision Grossing Up Worst Factors |
|------|---------|--------------------|-------------------------------|--------------------------------|----------------------------------|------------------------------------|--------------------------------------|--|
| 2013 | Q1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2013 | Q2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2013 | Q3 | 3 | -163 | 0 | -169 | -169 | -169 | 0 |
| 2013 | Q4 | 4 | -406 | 0 | -418 | -418 | -418 | 0 |
| 2014 | Q1 | 5 | -640 | 0 | -663 | -663 | -663 | 0 |
| 2014 | Q2 | 6 | -116 | 2747 | -184 | -184 | -184 | 2747 |
| 2014 | Q3 | 7 | -271 | 2873 | -354 | -354 | -355 | 2873 |
| 2014 | Q4 | 8 | -576 | 3604 | -700 | -700 | -700 | 3604 |
| 2015 | Q1 | 9 | 2703 | 32287 | 2792 | 2792 | 2768 | 32165 |
| 2015 | Q2 | 10 | 2233 | 32573 | 2280 | 2280 | 2255 | 32450 |
| 2015 | Q3 | 11 | 3429 | 47831 | 3598 | 3598 | 3558 | 47708 |
| 2015 | Q4 | 12 | 3385 | 52348 | 3545 | 3545 | 3500 | 52082 |
| 2016 | Q1 | 13 | 3086 | 50473 | 3228 | 3228 | 3184 | 50162 |
| 2016 | Q2 | 14 | 2509 | 46058 | 2664 | 2664 | 2626 | 43408 |
| 2016 | Q3 | 15 | 2362 | 44368 | 2505 | 2505 | 2469 | 40697 |
| 2016 | Q4 | 16 | 2031 | 47962 | 2216 | 2216 | 2177 | 44005 |
| 2017 | Q1 | 17 | 1918 | 46286 | 2060 | 2060 | 2025 | 39747 |
| 2017 | Q2 | 18 | 2157 | 59111 | 2237 | 2237 | 2198 | 39808 |
| 2017 | Q3 | 19 | 9096 | 170222 | 8993 | 8993 | 8776 | 111956 |
| 2017 | Q4 | 20 | 10485 | 192440 | 10348 | 10348 | 10118 | 117896 |
| 2018 | Q1 | 21 | 9387 | 170823 | 9275 | 9328 | 9074 | 102729 |
| 2018 | Q2 | 22 | 9759 | 175760 | 9649 | 9894 | 9441 | 107644 |
| 2018 | Q3 | 23 | 11592 | 210911 | 11442 | 12024 | 11228 | 102067 |
| 2018 | Q4 | 24 | 21357 | 436182 | 21388 | 13292 | 20755 | 184302 |
| 2019 | Q1 | 25 | 22215 | 452472 | 22296 | 15334 | 21692 | 178847 |
| 2019 | Q2 | 26 | 23189 | 493403 | 23360 | 15298 | 22676 | 205156 |
| 2019 | Q3 | 27 | 24210 | 462603 | 24187 | 16871 | 23575 | 193005 |
| 2019 | Q4 | 28 | 33034 | 573681 | 33319 | 24252 | 32589 | 242694 |
| 2020 | Q1 | 29 | 32280 | 491924 | 32524 | 24437 | 31889 | 203848 |
| 2020 | Q2 | 30 | 22743 | 324226 | 22852 | 17525 | 22429 | 134729 |
| 2020 | Q3 | 31 | 41147 | 552749 | 41707 | 34591 | 41022 | 256454 |
| 2020 | Q4 | 32 | 51401 | 614532 | 51750 | 42296 | 50989 | 291645 |
| 2021 | Q1 | 33 | 43060 | 469909 | 43358 | 37183 | 42785 | 216608 |
| 2021 | Q2 | 34 | 77955 | 769303 | 78200 | 66922 | 77313 | 326604 |
| 2021 | Q3 | 35 | 107002 | 908087 | 108498 | 102088 | 107489 | 398947 |
| 2021 | Q4 | 36 | 154283 | 1087986 | 156731 | 155483 | 155566 | 487237 |
| 2022 | Q1 | 37 | 181940 | 1006552 | 186120 | 202764 | 184967 | 478081 |
| 2022 | Q2 | 38 | 315526 | 1382421 | 324221 | 370939 | 322133 | 730945 |
| 2022 | Q3 | 39 | 637844 | 2024712 | 659307 | 784217 | 651269 | 1312441 |
| 2022 | Q4 | 40 | 1861422 | 6406963 | 2018233 | 2464383 | 1917825 | 4744746 |

Figure 4.2.4.1: Claims Provisions per Method for all Development Periods

4.2.4. The validity of the assumptions of the methodologies

For facilitation, the summarized requirements of all methodologies are presented in the figure below.

| Assumptions of Methods | Chain Ladder | Worst Factor | Grossing Up Worst Factors | Grossing Up Factors | Average Factors | Average T-Factors |
|------------------------------------|--------------|--------------|---------------------------|---------------------|-----------------|-------------------|
| Homogeneity | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Independence | ✓ | | | | | |
| Consistency of Development Factors | ✓ | | | ✓ | ✓ | ✓ |
| Sufficiency of Data | ✓ | | | | | |
| Extremism | | ✓ | ✓ | | | |
| Stability of Averages over Time | | | | ✓ | ✓ | ✓ |
| Proportional Scaling | | | | ✓ | ✓ | ✓ |

Figure 5.2.4.1: Assumptions of each methodology

4.2.4.1. Homogeneity and Consistency of Development Factors

Homogeneity is a requirement for all the methodologies, while consistency of development factors is a requirement for Chain Ladder.

The table below represents the means and standard deviation of the raw data of amounts paid for each development period.

| <i>Year</i> | <i>Quarter</i> | <i>Means</i> | <i>Standard</i> |
|-------------|----------------|--------------|-----------------|
| 2007 | Q1 | 0.020566 | 0.002848974 |
| | Q2 | 0.001468 | 0.001344335 |
| | Q3 | 0.004414 | 0.001435657 |
| | Q4 | 0.002148 | 0.001437846 |
| 2008 | Q1 | 0.006155 | 0.00145574 |
| | Q2 | 0.005249 | 0.00146119 |
| | Q3 | 0.004582 | 0.00144776 |
| | Q4 | -0.004439 | 0.001289403 |
| 2009 | Q1 | 0.000473 | 0.001251441 |
| | Q2 | 0.000751 | 0.001269632 |
| | Q3 | 0.009935 | 0.001086037 |
| | Q4 | 0.003159 | 0.000966927 |
| 2010 | Q1 | 0.000471 | 0.000943343 |
| | Q2 | 0.002868 | 0.000912229 |
| | Q3 | 0.001029 | 0.00091501 |
| | Q4 | 0.001504 | 0.000919993 |
| 2011 | Q1 | 5.25E-05 | 0.000930205 |
| | Q2 | 0.002543 | 0.000923953 |
| | Q3 | 0.001478 | 0.000938123 |
| | Q4 | 0.001637 | 0.000944952 |
| 2012 | Q1 | 0.001365 | 0.00094314 |
| | Q2 | 0.00015 | 0.000953472 |
| | Q3 | 0.000449 | 0.000977706 |
| | Q4 | 8.22E-05 | 0.000986283 |
| 2013 | Q1 | 0.002408 | 0.000985018 |
| | Q2 | 0.000992 | 0.001053628 |
| | Q3 | 0.002858 | 0.00106123 |
| | Q4 | 0.006662 | 0.001048727 |
| 2014 | Q1 | 0.007955 | 0.001060024 |
| | Q2 | 0.005538 | 0.001070706 |
| | Q3 | 0.005655 | 0.001089888 |
| | Q4 | 0.00554 | 0.001105461 |
| 2015 | Q1 | 0.00758 | 0.001111877 |
| | Q2 | 0.00026 | 0.001051128 |
| | Q3 | 0.001321 | 0.001066286 |
| | Q4 | 0.00396 | 0.001043051 |
| 2016 | Q1 | 0.004482 | 0.001062135 |
| | Q2 | 0.003811 | 0.000992828 |
| | Q3 | 0.001429 | 0.001007395 |
| | Q4 | 0.001217 | 0.001019965 |
| 2017 | Q1 | 0.002105 | 0.001116413 |
| | Q2 | 0.001657 | 0.001132757 |
| | Q3 | 0.001778 | 0.001150957 |
| | Q4 | 0.004014 | 0.00117943 |
| 2018 | Q1 | 0.009495 | 0.001185047 |
| | Q2 | 0.003648 | 0.001189731 |
| | Q3 | 0.007695 | 0.001213008 |
| | Q4 | 0.007444 | 0.001252784 |
| 2019 | Q1 | 0.015781 | 0.001250004 |
| | Q2 | 0.011311 | 0.00128472 |
| | Q3 | 0.008929 | 0.001333984 |
| | Q4 | 0.007673 | 0.001392497 |
| 2020 | Q1 | 0.024977 | 0.001318223 |
| | Q2 | 0.003305 | 0.00097632 |
| | Q3 | 0.001893 | 0.001020764 |
| | Q4 | 0.005259 | 0.00106607 |
| 2021 | Q1 | 0.01232 | 0.001018578 |
| | Q2 | 0.01861 | 0.000664386 |
| | Q3 | 0.004052 | 0.000626708 |
| | Q4 | 0.014579 | 0.000670678 |
| 2022 | Q1 | 0.014497 | 0.000409232 |
| | Q2 | 0.005354 | 0.000533656 |
| | Q3 | 0.032381 | 0 |
| | Q4 | 0 | 0 |

Figure 6.2.4.2: Means and standard deviation of Amounts Paid per Quarter

Looking at the table, we observe a constant pattern of low standard deviations in comparison to the means for each of the periods. This indicates that the data is homogenous, and we can also conclude that the development factors are consistent.

4.2.4.2. Independence

Independence is one of the requirements for Chain Ladder method. Testing for independence means that we need to prove the small correlation in the data set for the development factors of Chain Ladder.

We will perform the independence test using Chi-squared test. The first step is to categorize our development factors for each of the models into 3 groups: low, medium and high values. The table is presented below.

| Classification | Chain Ladder | Worst Factor | Average Factors | Average T Factors | Grossing Up Factors | Grossing Up Worst Factors |
|----------------|--------------|--------------|-----------------|-------------------|---------------------|---------------------------|
| Low | 10 | 0 | 10 | 12 | 33 | 35 |
| Medium | 28 | 38 | 28 | 26 | 7 | 5 |
| High | 2 | 2 | 2 | 2 | 0 | 0 |

Figure 7.2.4.3: Chi-square test: Data grouping

The following step is to calculate the Chi squared statistic, χ^2 . This is obtained by the following formula:

(4.8)

$$\chi^2 = \sum \frac{\text{Observed count} - \text{Expected counts}}{\text{Total number of observations}}$$

(4.9)

- Where the Expected counts is calculated as: $\frac{\text{Row total} - \text{Column total}}{\text{Total number of observations}}$

Doing so, we reach a Chi-squared statistic of 16.248, with a degree of freedom of 10, where the degree of freedom is calculated as: $(\text{Row total} - 1) \times (\text{Column total} - 1)$.

Comparing the Chi-squared statistic at 5% significance level with a critical value of 18.307, we can confirm the independence of the data.

4.2.4.3. Extremism

Extremism is an assumption for Worst Factors and Grossing Up Worst Factors methods, it is rooted in the methodology that they focus intentionally on worst-case scenarios to produce larger scales of future liabilities.

Hence, extremism does not need to be proven statistically significant for validity, it can simply be assumed.

4.2.4.4. Stability of Averages over time

This requirement is key for Grossing Up Factors, Average Factors, and Average T-Factors. This can be proved by using the coefficient of variation. The first step is finding the means and standard deviations for the claims provisions for the 3 methodologies. Then we finally find the coefficient of variation by dividing the standard deviation of all 3 means by the average of all 3 means.

The coefficient of variation was calculated as 7.4674091%, which is lower than 10%. This signifies the stability of the averages over time.

4.2.4.5. Proportional Scaling and sufficiency of data

Having a small coefficient of variation of 7.4674091% demonstrates that proportional scaling exists. A low coefficient of variation lower than 10% imply that the data is consistent over time, and that the claims amounts increase and decrease proportionally over time.

Besides, having such huge data set of quarterly data between 2013 and 2022, we can conclude the sufficiency of the data.

4.2.4.6. Meeting all assumptions

All assumptions and requirements for each methodology have been met, meaning that the environment is reasonable for fitting the desired models.

4.2.5. Testing the base model for Statistical Significance: Bootstrap test

Aiming to test the validity of the created model, a test for statistical significance is performed using bootstrap with the sampling assumption.

Due to the complexity of transferring the data and results to RStudio, a bootstrap test has been performed manually using Microsoft Excel. As a starting point, numerous simulations have been generated where each simulation involved generating random samples of the real data set of amounts paid. To be more precise, I selected random values from this dataset using the Excel formula **=INDEX(Cumulative Amounts Paid, RANDBETWEEN(1, N),t)**, where the actual dataset is in sheet Amounts paid between cells D13 and BO76, where N is the number of development periods in the dataset which is 64, and t is the quarter being calculated.

The following step was to calculate the claims provisions for each simulation using all methodologies. The mean of the samples is found by simply getting the mean of claim provisions of all simulations, and the standard deviation is found by the excel formula **=STDEV.P()**

Lastly, the 99% Confidence Interval and the 95% Confidence Interval have been constructed to evaluate the accuracy of my model.

The hypothesis of the performed tests is as below:

| |
|--|
| Null Hypothesis (H0): The claims provisions model is not significantly different from the expected values. |
| H0: The calculated claims provision falls within the range of Bootstrap estimates. |
| Alternative Hypothesis (H1): The claims provisions model is significantly different from the expected values. |
| H1: The calculated claims provision does not fall within the range of Bootstrap estimates. |

The formulas used to calculate the confidence intervals are presented below:

(4.10)

$$\text{Mean of error} = \frac{\text{standard deviation} \times \text{critical value}}{\sqrt{(\text{number of samples})}}$$

Where the critical value for 95% significance level is 1.645 and the critical value for 99% significance level is 2.575.

The confidence interval is then deduced as the following range:

$$[\text{Mean} - \text{Margin of error}, \text{Mean} + \text{Margin of error}]$$

The obtained confidence intervals are presented in the table below:

| Methodology | Lower 95 CI | Upper 95 CI | Lower 99 CI | Upper 99 CI |
|---------------------------|-------------|-------------|-------------|-------------|
| Chain Ladder | 1,319,717 | 6,464,484 | 511,253 | 7,272,948 |
| Worst Factor | -16,416,666 | 31,567,053 | -23,956,964 | 39,107,352 |
| Average Factors | -26,350,154 | 15,458,069 | -32,920,018 | 22,027,933 |
| Average T-Factors | -26,351,421 | 15,461,684 | -32,922,052 | 22,032,315 |
| Grossing Up Factors | -17,663,380 | 74,415,290 | -32,132,885 | 88,884,795 |
| Grossing Up Worst Factors | -20,578,218 | 24,726,608 | -27,697,548 | 31,845,938 |

Figure 8.2.5.1: Bootstrap Test for the Base Model: Confidence Intervals

The outcomes and findings of the performed Bootstrap test is to be presented and discussed in detail within the results chapter in section 5.2.2. Bootstrap Test for testing the Adequacy of the Model

4.3. Forecasting Inflation

For inflation forecasting, the yearly inflation data for Portugal between 1999 and 2022 by Statista (2024) will be used, this data also includes the yearly forecasted inflation until 2028. The inflation data is presented below.

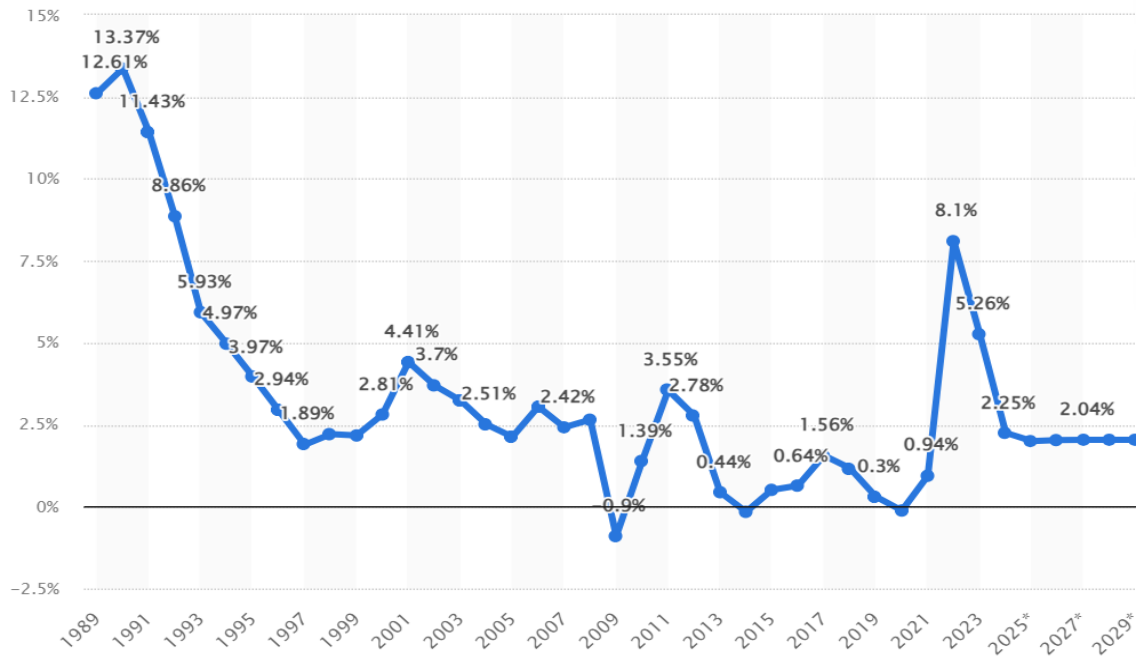


Figure 9.3.1: Yearly Inflation Rates in Portugal

This dataset is then converted to quarterly inflation rates and is then imported to RStudio for further workings.

4.3.1. Testing the inflation dataset

Firstly, exponential smoothing can be used for forecasting inflation without prerequisites and requirements, but the options of forecasting using other models should also be explored to choose the best fit. Hence, the inflation dataset should be tested using regression analysis and time series analysis to significantly choose the appropriate model for forecasting inflation.

4.3.1.1. Regression Analysis

The regression analysis will be performed with the help of RStudio libraries ggplot2, dplyr, broom, and ggpubr. This analysis is done to test the possibility of forecasting inflation with linear regression. As an overview, checking the summary of the quarterly inflation data in Portugal from 1999 until 2028 is a prerequisite. A presentation of the summary of regression analysis key statistical values is provided below.

```

> summary(inflação)
  Year                Inflation
Length:96           Min.      :-0.002258
Class :character    1st Qu.   : 0.001434
Mode  :character    Median   : 0.005332
                        Mean    : 0.005075
                        3rd Qu.  : 0.007050
                        Max.    : 0.020134

```

Figure 10.3.1.1: Regression Analysis: Summary of Quarterly Inflation Data

These statistical values represent the dataset's distribution, comprise the Minimum, First Quartile (1st Qu), Median, Mean, Third Quartile (3rd Qu), and Maximum.

After analyzing the data, we can conclude that the quarterly data set's lowest inflation rate is -0.2258%, its largest is 2.0134%, and its average is 0.05332%. 25% of the dataset is below 0.02434%, 75% of the dataset is below 0.07050%, and 50% of the dataset is below 0.05332%. The simple regression outputs among the quarters show acceptable and realistic results in comparison to the real-life inflation rates.

4.3.1.1.1. Testing Normality

A histogram is made to verify visual normality. The following is the hypothesis that is tested for normality:

Null Hypothesis (H0): The data is normally distributed.

Alternative Hypothesis (H1): The data is not normally distributed.

The histogram of the quarterly inflation rates presented below is created as a check for normality of the dataset.

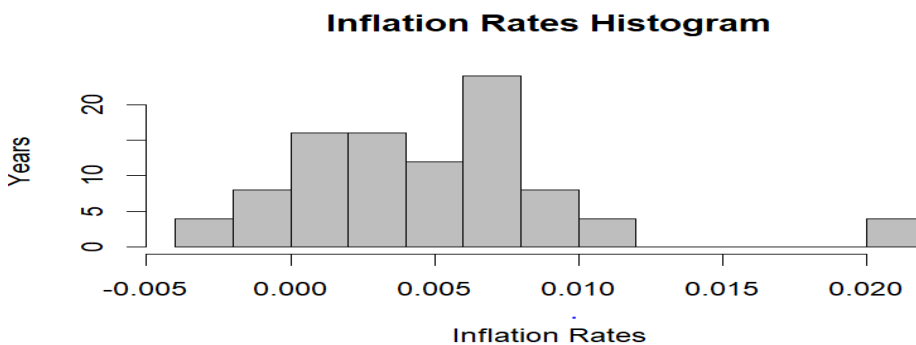


Figure 11.3.1.2: Histogram of the Quarterly Inflation Data

Looking at the histogram, we deduce that the inflation data is normally distributed, and we accept the null hypothesis. The first requirement of the simple regression analysis is

met, the histogram shows the normality of the quarterly inflation data in Portugal from 1999 until 2022.

4.3.1.1.2. Testing Linearity

In terms of linearity, plotting the historical data of inflation rates in Portugal may be sufficient to identify any linearity trends.

Below is presented a plot of inflation rates in Portugal against time, in search for linear relationship.

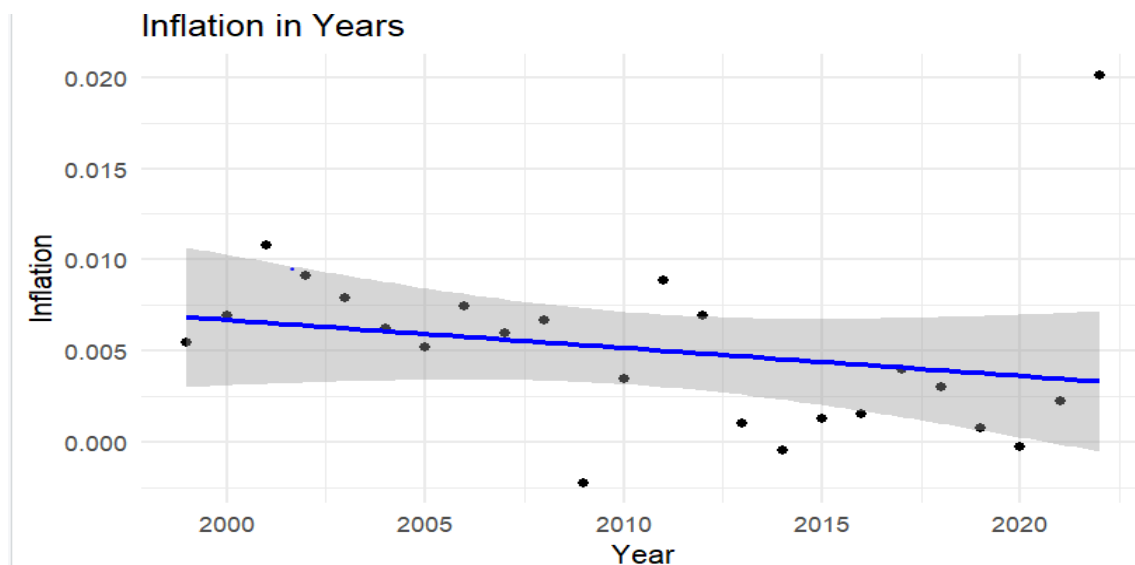


Figure 12.3.1.3: Linear plot of the Quarterly Inflation Data

The linear regression fit is represented by the blue line, which slopes slightly downward. The confidence interval, which displays fit uncertainty, is shown by the dark region surrounding the line. The black dots, or data points, are dispersed, but not too far from, the regression line. Despite some variation among the data points, a linear relationship appears to be implied by the general trend. The data points do, however, exhibit some spread around the line, suggesting that although a linear model is a fair fit, other models or factors might also need to be considered for a more precise prediction.

Hence with the help of our master’s course “MR-CA - Risk Models”, we perform additional testing using the R-squared coefficient and the coefficient of determination. This test’s hypothesis is:

Null Hypothesis (H0): There is no linear relationship between the variables (Pearson correlation coefficient is equal to 0).

Alternative Hypothesis (H1): There is a significant linear relationship between the (Pearson correlation coefficient is not equal to 0).

We proceed with running the codes below, we have reached the presented results.

```

> model <- lm(Inflation ~ Year, data = quarterlyinf)
> pearson_correlation <- cor(quarterlyinf$Year, quarterlyinf$Inflation)
> cat("Pearson correlation coefficient:", pearson_correlation, "\n")
Pearson correlation coefficient: -0.2310056
> model_summary <- summary(model)
> r_squared <- model_summary$r.squared
> cat("R-squared value:", r_squared, "\n")
R-squared value: 0.05336357

```

Figure 13.3.1.4: Pearson Correlation test on the Inflation Dataset

Looking at the results above, we observe that the acquired Pearson correlation coefficient indicate a very weak negative linear relationship between the variables time and inflation, hence we reject the null hypothesis.

4.3.1.1.3. Testing Independence

Due to the absence of linearity, there is no need to look for any hidden links between the variables, although it is fair to say that inflation and time are most likely to be independent because inflation is more linked to other factors like economic conditions.

We can conclude that the data do not fit well into the linear model. It suggests that factors other than those in the model could significantly affect inflation more than the time, therefore we do not have to test the data for homogeneity.

4.3.1.2. Time Series Analysis (Stationarity)

The first requirement for time series analysis has been met by proving the non-linearity of the data set. We now aim to prove the stationarity of the data set.

The upcoming tests will be performed on a 10% significance level, as a 10% cutoff point in the assessment of inflation data for forecasting may allow for possible fluctuations in economic indicators and acknowledge the difficulties involved in predicting inflation, particularly in a dynamic economic landscape. In addition, it can indicate that the analysis was early in nature, allowing for more research and improvement in later studies while lowering the possibility of Type I errors, where type I error is when a null hypothesis that is true is mistakenly rejected.

Checking for stationarity, the stationarity tests are performed on our dataset of quarterly inflation rates between 1999 and 2028. The testing will be done using ADF, KPSS, Ljung-Box, and PP tests, with the hypothesis at 10% significance level as follows:

Null Hypothesis (H₀): The time series is stationary (P-value < 0.1)

Alternative Hypothesis (H₁): The time series is non-stationary (P-value > 0.1)

Running the codes presented below we obtain the p-value of the ADF test.

```

> adf_test <- adf.test(quarterlyinf$Inflation)
> print(adf_test)

                Augmented Dickey-Fuller Test

data:  quarterlyinf$Inflation
Dickey-Fuller = -2.1222, Lag order = 3, p-value = 0.5257
alternative hypothesis: stationary

```

Figure 14.3.1.5: Stationarity testing: ADF Test on the Dataset

The p-value for the ADF test is big, which suggests that the dataset is not stationary, rejecting the null hypothesis. By further testing, we raise the significance level, and the data then becomes stationary. Hence, we do not exclude the idea of fitting an auto ARIMA model, subject to further testing.

```

> kpss_test <- kpss.test(quarterlyinf$Inflation)
> print(kpss_test)

                KPSS Test for Level Stationarity

data:  quarterlyinf$Inflation
KPSS Level = 0.40511, Truncation lag parameter = 2, p-value = 0.07495

> pp_test <- pp.test(quarterlyinf$Inflation)
> print(pp_test)

                Phillips-Perron Unit Root Test

data:  quarterlyinf$Inflation
Dickey-Fuller Z(alpha) = -15.851, Truncation lag parameter = 2, p-value
= 0.09878
alternative hypothesis: stationary

> Box.test(quarterlyinf$Inflation, lag=12, type="Ljung-Box")

                Box-Ljung test

data:  quarterlyinf$Inflation
X-squared = 19.384, df = 12, p-value = 0.07967

```

Figure 15.3.1.6: Stationarity testing: Other Stationarity tests on the Dataset

On the other hand, the KPSS test accepts the idea of stationarity of the original dataset, as we may deduce that the data is stationary at 10% significance level with such small p-value of 0.07495. The p-value of the PP test is also very small, which concludes stationarity of the dataset. Similarly, the Ljung-Box test indicates that the dataset is stationary with such small p-value. In summary, we accept the H_0 and deduce that the data set is stationary.

4.3.1.2.1. Stationarizing the dataset

We now try to use the differencing method to check if the data becomes stationary using the ADF test. The testing will be done using ADF test, with hypothesis at 10% significance level as follows:

Null Hypothesis (H0): The difference of the inflation is stationary (P-value < 0.1).

Alternative Hypothesis (H1): The difference of inflation is non- stationary (P-value > 0.1).

The codes below were run to execute the differencing method, and then testing it.

```
> inflation_ts <- ts(quarterlyinf$Inflation, start = c(1990, 1), frequency = 4)
> differenced_ts <- diff(inflation_ts)
> adf_test_diff <- adf.test(differenced_ts)
> print(adf_test_diff)

      Augmented Dickey-Fuller Test

data:  differenced_ts
Dickey-Fuller = -3.4046, Lag order = 3, p-value = 0.07582
alternative hypothesis: stationary
```

Figure 16.3.1.7: Stationarizing the Dataset: ADF Test on the Dataset of Difference

The result obtained for the ADF test suggests that the difference of the inflation rates model is stationary at 10% significance level.

In summary, we have sufficient proof that the dataset and the difference of the dataset are both stationary. Hence, we may use both Arima models and exponential smoothing to forecast the inflation rates from 2028 onwards, ensuring careful consideration and validations against the historical data.

4.3.1.2.2. Seasonality and trends check

Moreover, we can also explore other models that may be fitted, then we can decide which model is a better fit. We can now explore the possibility of having seasonality in the timeseries, which opens the door for a seasonal ARIMA model.

Running the codes below, a plot will be outputted which may show seasonality.

```
ts_inflation <- ts(quarterlyinf$Inflation, frequency = 4)
decomp <- decompose(ts_inflation, type = "additive")

plot(decomp)
```

Figure 17.3.1.8: Seasonality Check: R codes for plotting the Dataset

Below is the plot of the decomposition of trends in our timeseries.

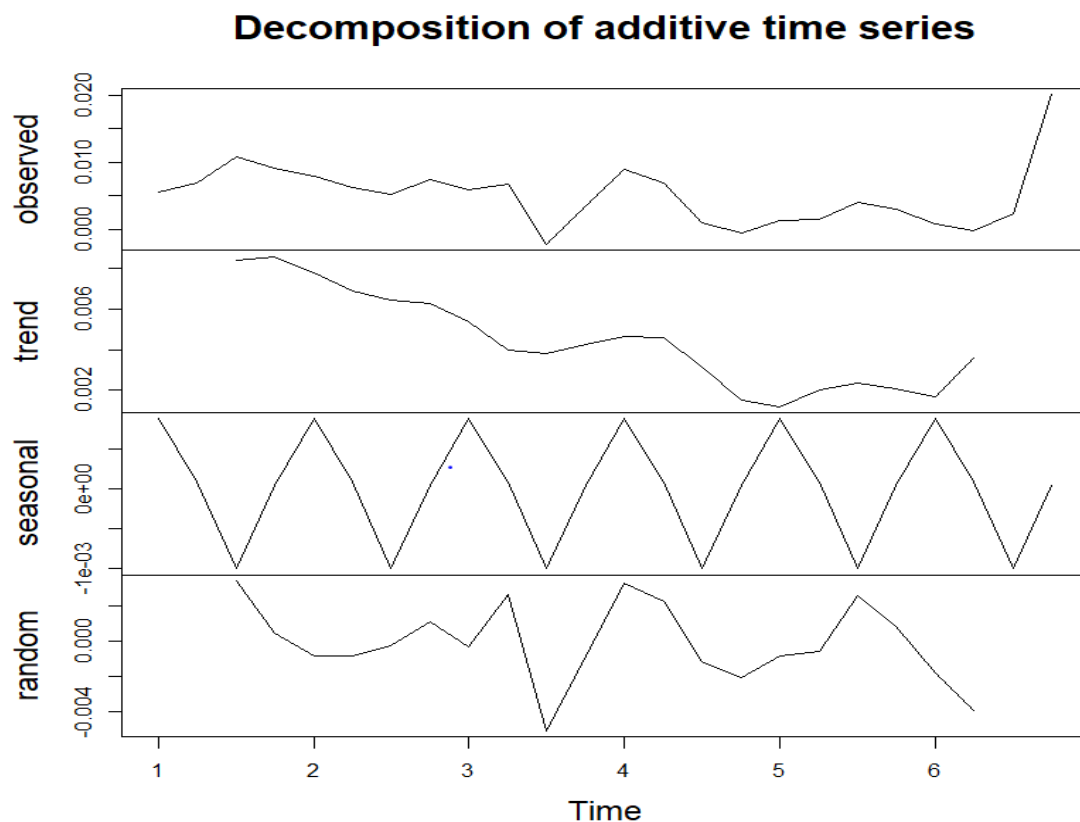


Figure 18.3.1.9: Seasonality Check: Plot of the Dataset

Looking at the graph above, we can capture trends. We can conclude that this timeseries may have seasonality, which opens the door for a seasonal Arima model to forecast inflation.

4.3.2. Forecasting Inflation

4.3.2.1. Model 1: Auto-ARIMA models

Using library forecast on RStudio, we run the codes provided below, accompanied with the results and assumptions.

The first line converts the dataset into a tibble. We then apply the auto-ARIMA to decide for us the order that suits our model best, by running the codes below.

```
> ts_inflation <- ts(inflacao$Inflation, start = c(1999, 1), frequency = 4)
> autoarima_model <- auto.arima(ts_inflation)
> print(autoarima_model)
Series: ts_inflation
ARIMA(0,1,0)

sigma^2 = 5.884e-06: log likelihood = 437.26
AIC=-872.51 AICC=-872.47 BIC=-869.96
```

Figure 19.3.2.1: Model 1: R codes for applying Auto-ARIMA function

We hence obtain the results of the inflation data. The ARIMA(0,1,0) model was chosen by the Auto ARIMA process, signifying that the number of autoregressive items is zero and the number of moving averages (MA) is zero, meaning that there is no autocorrelation, while the 1 refers to the order of differencing. This suggests that the series' stationarity was attained with only a first-order differencing ($d=1$). In time series data, differencing is frequently used to eliminate trends or stabilize variation.

Relative to the inflation data, the model's estimated variance of $5.884e-06$ is small, which can be explained by capturing variability effectively after differencing.

The log likelihood value of 437.26 is relatively big, making the model a better fit.

Such negative values of AIC, AICC, and BIC is common, as a better trade-off between model complexity and data fit is indicated by lower values.

We perform the forecast then using the r-formula “forecast()” to obtain the expected inflation rates of the 1st quarter of 2023 until the last quarter of 2038. The graph presented below shows the obtained results, along with the 80 and 95 Confidence Intervals.

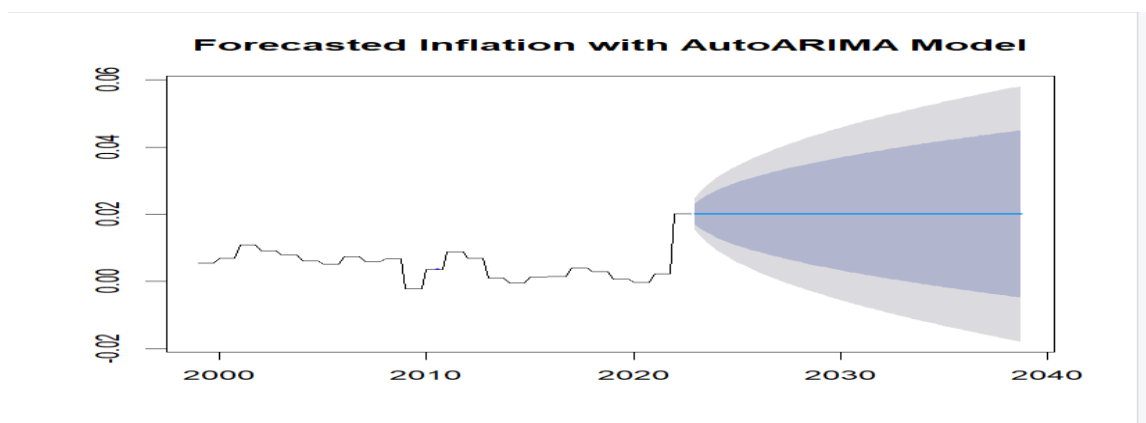


Figure 20.3.2.2: Forecast 1: Plot of the Auto-ARIMA Model forecast.

In summary, the ARIMA(0,1,0) model does the job for short-term inflation forecasting; longer-term projections and the capture of unexpected economic developments may require more complicated models, which are subject to continuous evaluation.

4.3.2.2. Model 2: Auto-ARIMA for the differences model

Following the same auto-ARIMA procedures for the model of differences, we obtain the presented below forecast, with the 80 and 95 Confidence Intervals.

Forecasted Difference of Inflation with AutoARIMA Model

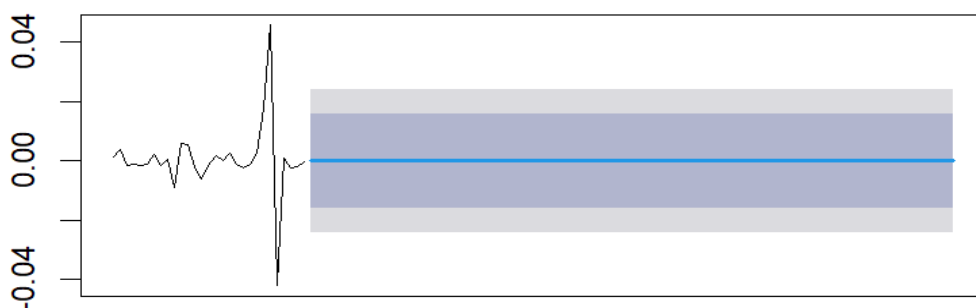


Figure 21.3.2.3: Forecast 2: Plot of the forecast of the Auto-ARIMA Model of Differences.

4.3.2.3. Model 3: Seasonal Arima (SARIMA)

Below is attached the codes ran on RStudio, accompanied by the results for the seasonal Arima model.

```
> sarima_model <- Arima(ts_inflation, order = c(1, 0, 0), seasonal = list(order = c
(1, 0, 1), period = 4))
> print(sarima_model)
Series: ts_inflation
ARIMA(1,0,0)(1,0,1)[4] with non-zero mean

Coefficients:
      ar1      sar1      sma1      mean
 0.8899 -0.5268  0.6698  0.0064
s.e.  0.0539   0.5848  0.4983  0.0023

sigma^2 = 5.788e-06:  log likelihood = 443.74
AIC=-877.48  AICc=-876.82  BIC=-864.66
```

Figure 22.3.2.4: Model 3: R Codes and Results of SARIMA Model

The ar1 parameter for the short-term autocorrelation is captured by the coefficient 0.8899, which indicates a positive correlation between the inflation value today and the inflation value one period ago. The coefficient -0.5268 of the Seasonal AR parameter, or sar1, shows a negative correlation between the same quarter's inflation in prior years (seasonal period 4).

The seasonal MA parameter, or sma1, has a coefficient of 0.6698, indicating that the moving average of errors from prior seasons has an impact on the present inflation estimate.

mean: A continuous adjustment to the series mean is shown by the mean term, which is 0.0064.

The variance is notably small with a value of 5.788e-06, indicating that the model captures the variability of the inflation data. The SARIMA model fits the data better than the Auto ARIMA model, as indicated by the higher log likelihood value of 443.74.

In comparison to the Auto ARIMA model, AIC, AICC, and BIC have negative values, with lower values suggesting a better trade-off between model complexity and data fit.

The model takes into account both seasonal and non-seasonal factors, making the model appropriate for predicting inflation over a period of several quarters.

Performing the forecast, we reach the forecast presented by the plot below.

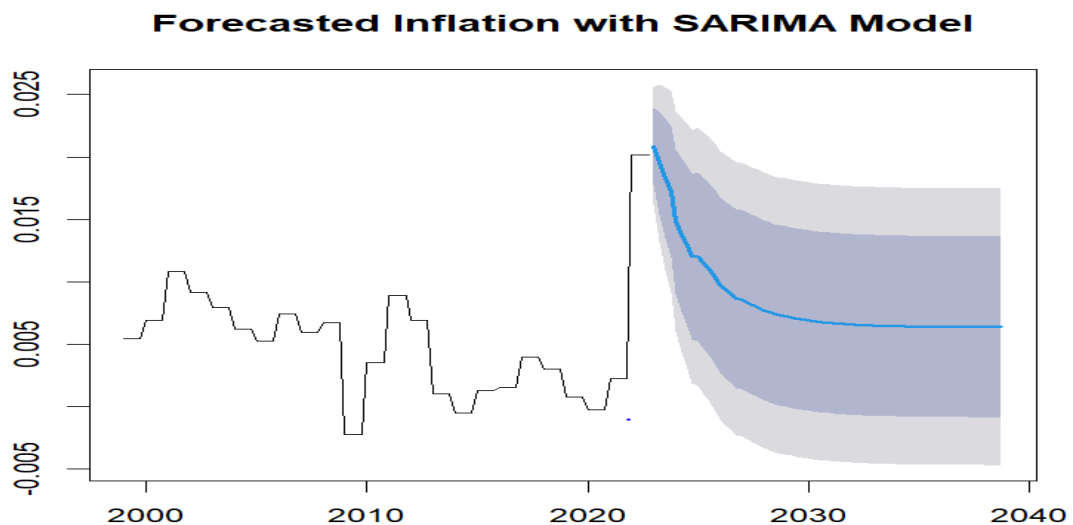


Figure 23.3.2.5: Forecast 3: Plot of SARIMA forecast

To summarize, it seems that the SARIMA(1,0,0)(1,0,1) model is a good fit for predicting inflation. Given that it incorporates seasonal components and fits the data slightly better than the AutoARIMA model, it is possible that it will produce more accurate forecasts, especially over longer forecasting horizons.

4.3.2.4. Model 4: Exponential smoothing

This forecast has been performed using Microsoft Excel's Forecast property. This forecast is done straight forward after inputting the inflation rates between 1999 and 2028. We then obtain the expected future quarterly inflation rates in Portugal between the beginning of 2023 and the end of 2038. The images attached below show the plot of inflation rates between 1999 and 2038.

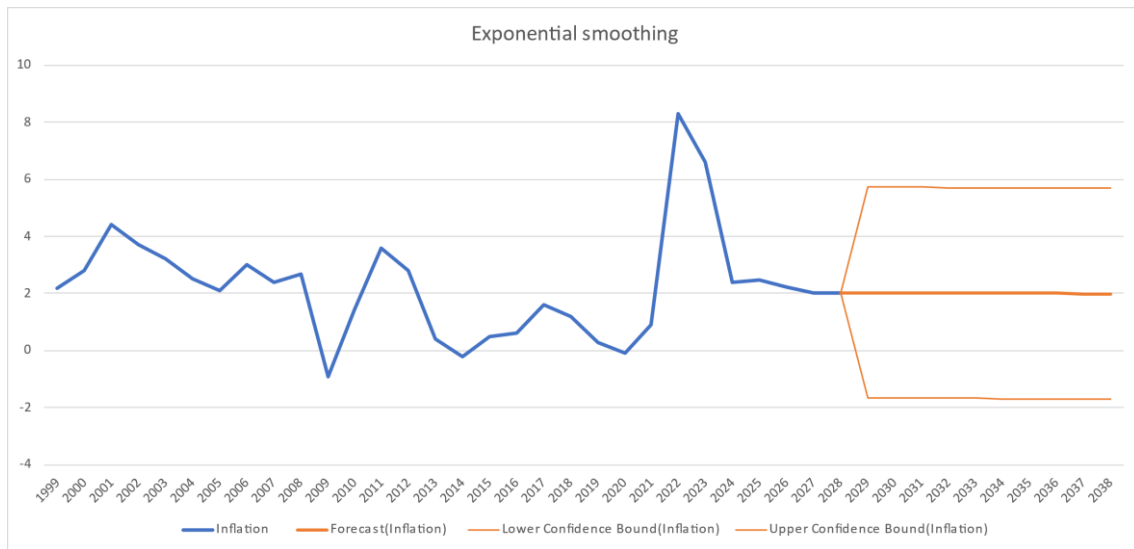


Figure 24.3.2.6: Forecast 4: Plot of Exponential Smoothing forecast

The obtained results are explained by the figure below:

| Year | Inflation | Forecast(Inflation) | Lower Confidence Bound(Inflation) | Upper Confidence Bound(Inflation) |
|------|-----------|---------------------|-----------------------------------|-----------------------------------|
| 27 | 2024 | 2.4 | | |
| 28 | 2025 | 2.48 | | |
| 29 | 2026 | 2.24 | | |
| 30 | 2027 | 2.04 | | |
| 31 | 2028 | 2.03 | 2.03 | 2.03 |
| 32 | 2029 | 2.034052663 | -1.66 | 5.73 |
| 33 | 2030 | 2.029755718 | -1.67 | 5.72 |
| 34 | 2031 | 2.025458774 | -1.67 | 5.72 |
| 35 | 2032 | 2.021161829 | -1.67 | 5.72 |
| 36 | 2033 | 2.016864885 | -1.68 | 5.71 |
| 37 | 2034 | 2.01256794 | -1.68 | 5.71 |
| 38 | 2035 | 2.008270996 | -1.69 | 5.70 |
| 39 | 2036 | 2.003974051 | -1.69 | 5.70 |
| 40 | 2037 | 1.999677107 | -1.70 | 5.70 |
| 41 | 2038 | 1.995380162 | -1.70 | 5.69 |

Figure 25.3.2.7: Results of Forecast 4 (Exponential Smoothing)

4.3.3. Choosing the adequate model for Inflation Forecasting

The following analysis helps us to choose the most adequate model, or the model that is expected to have the least errors.

The first step is to always find the error by subtracting the actual data from the respective forecasted data for each observation.

The bias, also known as the mean error, is the average of the errors.

The Root Mean Squared Error, RMSE is the square root of the average error squared.

The Mean of the absolute error, MAE is the average of the sumitions of the absolute values of the errors.

Mean percentage errors, MPE is the percentage of errors. The formula is presented below for illustration, where a^t is the t^{th} observation of actual data and f^t is the t^{th} osbervation of the forecasted values.

(4.11)

$$\text{MPE} = \frac{100\%}{n} \sum_{t=1}^n \frac{a^t - f^t}{a^t}$$

Mean Absolute Percentage Error is the sum of absolute error divided by each period separately.

Mean absolute squared error, MASE is calculated by dividing MAE by MAE naive, where MAE naive is the mean absolute error produced by a naive forecast, where a naive forecast is a simple forecasting method which uses the most recent observation as the predicted value for the next time period.

ACF1 is the autocorrelation of errors at lag one, it reflects the influence of previous values on current values in a time series.

Analyzing those values help us to reach our desired model.

The r-codes used to apply those formulas on r-studio are presented below.

```

108 calculate_metrics <- function(forecast, actual) {
109   bias <- mean(forecast - actual)
110   rmse <- sqrt(mean((forecast - actual)^2))
111   mae <- mean(abs(forecast - actual))
112   mpe <- mean((forecast - actual) / actual) * 100
113   mape <- mean(abs((forecast - actual) / actual)) * 100
114   mase <- mean(abs(forecast - actual)) / mean(abs(diff(actual)))
115   mae_naive <- mean(abs(actual[-length(actual)] - actual[-1]))
116   acf1 <- acf(forecast - actual, plot = FALSE)$acf[2]
117
118   return(list(bias = bias, rmse = rmse, mae = mae, mpe = mpe, mape = mape,
119             mase = mase, mae_naive = mae_naive, acf1 = acf1))
120 }
121
122
123 actual_inflation <- as.numeric(actual_inflation)
124
125
126 autoarima_forecast_values <- as.numeric(autoarima_forecast$mean)
127 autoarima_forecast2_values <- as.numeric(autoarima_forecast2$mean)
128 sarima_forecast_values <- as.numeric(sarima_forecast$mean)
129
130 metrics_autoarima <- calculate_metrics(autoarima_forecast_values, actual_inflation)
131 metrics_autoarima_diff <- calculate_metrics(autoarima_forecast2_values, actual_inflation)
132 metrics_sarima <- calculate_metrics(sarima_forecast_values, actual_inflation)
133

```

Figure 26.3.3.1: R Codes for testing the forecasting Models.

Calculating for all the models, the results below were obtained.

| | Auto ARIMA | ARIMA for differences | SARIMA | Exponential Smoothing |
|-------------|------------|-----------------------|----------|-----------------------|
| Bias | 0.0150588 | -0.00507 | 0.002622 | -0.022191 |
| RMSE | 0.0157459 | 0.00685 | 0.005589 | 0.0221908 |
| MAE | 0.0150588 | 0.005326 | 0.004108 | 0.0238074 |
| MPE | -61.87231 | -100 | -68.1349 | -1.154112 |
| MAPE | 1043.594 | 100 | 308.2198 | 1.2905655 |
| MASE | 19.34717 | 6.842137 | 5.277602 | 1.4068735 |
| ACFI | 0.8065906 | 0.806591 | 0.78402 | 1 |

Figure 27.3.3.2: Results of the Accuracy tests for the forecasting Models

Analyzing those results, we can conclude that exponential smoothing's values are the most adequate. With a very small bias within the acceptable range [-1,1], a very low MAE, an acceptable MPE, a good MAPE indicator of 1.29% which means that the forecast is 98.71% accurate, a poor MASE which can be overlooked by the other results, and a positive autocorrelation at lag 1.

4.4. Inflation-Adjusted Model

Before fitting the Inflation-Adjusted Model, sensitivity analysis is required to observe the claims amounts fluctuation results from variations in inflation.

4.4.1. Exploring the possibility to fit an Inflation-Adjusted Model (Sensitivity Analysis)

To know if the inflation adjusted model can be fit into the current model, sensitivity analysis is required. This is simply done by calculating using multiple random inflation rates and checking the adequacy of those random results.

In my sensitivity analysis, random quarterly inflation rates have been generated between 0.5% and 5% yearly inflation rates. Now the historical inflation is added to our original data to make sure that the model considers historical inflation data also. This section will be explained in detail in the upcoming section 4.4.2. Inflation Application into the base Model

The following step is to simply apply the randomly generated future inflation rates into our adjusted claims provision by the formula below:

(4.12)

$$\begin{aligned} \text{Sensitivity Provision} &= \text{Adjusted Claims Provision} \\ &\times (1 + \text{random inflation rates})^{\text{Time Period}} \end{aligned}$$

The results and conclusions of the sensitivity analysis will be presented in the Results chapter in details in section 5.2.3. Can we fit an Inflation Adjusted Model (Sensitivity Analysis)?

4.4.2. Inflation Application into the base Model

Applying inflation to the historical data cannot be operated with cumulative data. Hence, we start by obtaining the incremental payments triangle by subtracting each cumulative amount paid from the previous cumulative amount paid.

To apply historic quarterly inflation into our raw data, the quarterly inflation adjustment for past data is calculated to reflect the effect of all time periods' inflation rates. It is calculated by $(1 + \text{the quarter's inflation rate}) \times (1 + \text{all the following quarters' inflation rates})$, where the inflation rates being used are between the 1st quarter of 2013 and the 4th quarter of 2022. The obtained inflation rates and the inflation adjustments are shown in the table below.

| Quarter | 1+Inflation | tion adjustr |
|----------------|--------------------|---------------------|
| Q1 2013 | 1.0009985 | 1.140482 |
| Q2 2013 | 1.0009985 | 1.139345 |
| Q3 2013 | 1.0009985 | 1.138208 |
| Q4 2013 | 1.0009985 | 1.137073 |
| Q1 2014 | 0.9994996 | 1.135939 |
| Q2 2014 | 0.9994996 | 1.136507 |
| Q3 2014 | 0.9994996 | 1.137076 |
| Q4 2014 | 0.9994996 | 1.137645 |
| Q1 2015 | 1.0012477 | 1.138215 |
| Q2 2015 | 1.0012477 | 1.136797 |
| Q3 2015 | 1.0012477 | 1.13538 |
| Q4 2015 | 1.0012477 | 1.133965 |
| Q1 2016 | 1.0014966 | 1.132552 |
| Q2 2016 | 1.0014966 | 1.13086 |
| Q3 2016 | 1.0014966 | 1.12917 |
| Q4 2016 | 1.0014966 | 1.127482 |
| Q1 2017 | 1.0039762 | 1.125797 |
| Q2 2017 | 1.0039762 | 1.121339 |
| Q3 2017 | 1.0039762 | 1.116898 |
| Q4 2017 | 1.0039762 | 1.112474 |
| Q1 2018 | 1.0029866 | 1.108068 |
| Q2 2018 | 1.0029866 | 1.104769 |
| Q3 2018 | 1.0029866 | 1.101479 |
| Q4 2018 | 1.0029866 | 1.098199 |
| Q1 2019 | 1.0007492 | 1.094929 |
| Q2 2019 | 1.0007492 | 1.09411 |
| Q3 2019 | 1.0007492 | 1.093291 |
| Q4 2019 | 1.0007492 | 1.092472 |
| Q1 2020 | 0.9997499 | 1.091654 |
| Q2 2020 | 0.9997499 | 1.091927 |
| Q3 2020 | 0.9997499 | 1.0922 |
| Q4 2020 | 0.9997499 | 1.092474 |
| Q1 2021 | 1.0022424 | 1.092747 |
| Q2 2021 | 1.0022424 | 1.090302 |
| Q3 2021 | 1.0022424 | 1.087863 |
| Q4 2021 | 1.0022424 | 1.085429 |
| Q1 2022 | 1.0201337 | 1.083 |
| Q2 2022 | 1.0201337 | 1.061626 |
| Q3 2022 | 1.0201337 | 1.040673 |
| Q4 2022 | 1.0201337 | 1.020134 |

Figure 28.4.2.1: Presenting historic Inflation and Inflation Adjustments per quarter

The past inflation is then applied to the historical incremental payments by multiplying the quarter's incremental payment by its inflation adjustment. We can then find the new cumulative payments considering historic inflation, by summing up the subsequent incremental payments. The triangle of revised cumulative payments including historical inflation for the most recent years is shown below.

| Cumulative Amounts Paid (including historical inflation) | | | | | | | | | | | | | |
|--|---------|---------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Year | Quarter | Year 0 | | | | Year 1 | | | | Year 2 | | | |
| | | Q1 | Q2 | Q3 | Q4 | Q1 | Q2 | Q3 | Q4 | Q1 | Q2 | Q3 | Q4 |
| 2020 | Q1 | 384,703 | 1,950,344 | 2,445,521 | 2,580,562 | 2,673,577 | 2,725,931 | 2,782,437 | 2,802,458 | 2,810,524 | 2,810,524 | 2,873,679 | 2,874,831 |
| | Q2 | 155,002 | 1,281,061 | 1,582,425 | 1,670,628 | 1,743,112 | 1,759,092 | 1,813,871 | 1,815,306 | 1,821,189 | 1,819,267 | 1,821,976 | |
| | Q3 | 421,612 | 2,053,456 | 2,392,946 | 2,548,384 | 2,612,818 | 2,659,789 | 2,695,077 | 2,695,226 | 2,696,016 | 2,712,338 | | |
| | Q4 | 497,442 | 2,192,212 | 2,628,755 | 2,698,015 | 2,801,205 | 2,855,655 | 2,875,110 | 2,877,336 | 2,884,217 | | | |
| 2021 | Q1 | 403,295 | 1,661,522 | 1,875,952 | 1,977,850 | 2,029,959 | 2,064,886 | 2,081,339 | 2,084,118 | | | | |
| | Q2 | 556,949 | 2,208,710 | 2,667,922 | 2,911,252 | 3,052,735 | 3,068,883 | 3,089,040 | | | | | |
| | Q3 | 477,206 | 2,339,928 | 2,803,532 | 3,026,031 | 3,105,144 | 3,136,642 | | | | | | |
| | Q4 | 459,093 | 2,485,035 | 3,011,495 | 3,179,479 | 3,277,394 | | | | | | | |
| 2022 | Q1 | 452,787 | 2,033,337 | 2,389,333 | 2,513,191 | | | | | | | | |
| | Q2 | 470,301 | 2,016,275 | 2,589,131 | | | | | | | | | |
| | Q3 | 393,019 | 2,005,499 | | | | | | | | | | |
| | Q4 | 413,506 | | | | | | | | | | | |

Figure 29.4.2.2: Triangle of revised cumulative payments with past inflation

The next step is to find the development factors for each of the methodologies using the same methods employed previously in section 4.2.2. Development factors calculation per method. The new factors are presented below for illustration.

| Year | Quarter | Development period | Development factors Chain Ladder | Development factors Worst Factors | Development factors Average Factors | Development factors Average T-Factors | Grossing Up Factors | Grossing Up Worst Factors |
|------|---------|--------------------|----------------------------------|-----------------------------------|-------------------------------------|---------------------------------------|---------------------|---------------------------|
| 2013 | Q1 | 1 | 4.200022 | 8.26483 | 4.462908 | 5.039178 | 0.17514 | 0.078796 |
| 2013 | Q2 | 2 | 1.175251 | 1.307862 | 1.181975 | 1.217741 | 0.751751 | 0.605013 |
| 2013 | Q3 | 3 | 1.04596 | 1.091206 | 1.047635 | 1.057952 | 0.886512 | 0.776907 |
| 2013 | Q4 | 4 | 1.024579 | 1.048598 | 1.025557 | 1.032504 | 0.928478 | 0.834273 |
| 2014 | Q1 | 5 | 1.013107 | 1.033246 | 1.013403 | 1.01513 | 0.952125 | 0.121076 |
| 2014 | Q2 | 6 | 1.009152 | 1.033527 | 1.009585 | 1.0113 | 0.964859 | 0.88127 |
| 2014 | Q3 | 7 | 1.004758 | 1.019904 | 1.004678 | 1.00397 | 0.974076 | 0.899538 |
| 2014 | Q4 | 8 | 1.00296 | 1.010366 | 1.002983 | 1.003325 | 0.978622 | 0.900996 |
| 2015 | Q1 | 9 | 1.002814 | 1.008228 | 1.002712 | 1.002013 | 0.981531 | 0.90315 |
| 2015 | Q2 | 10 | 1.002813 | 1.022471 | 1.002977 | 1.003284 | 0.984182 | 0.908618 |
| 2015 | Q3 | 11 | 1.001346 | 1.006429 | 1.001314 | 1.00119 | 0.987093 | 0.926215 |
| 2015 | Q4 | 12 | 1.002028 | 1.008428 | 1.002032 | 1.001759 | 0.988377 | 0.929408 |
| 2016 | Q1 | 13 | 1.001463 | 1.009348 | 1.001555 | 1.00138 | 0.990371 | 0.931877 |
| 2016 | Q2 | 14 | 1.001095 | 1.004411 | 1.00103 | 1.001025 | 0.991905 | 0.936512 |
| 2016 | Q3 | 15 | 1.000174 | 1.008893 | 1.000198 | 0.999891 | 0.99293 | 0.938508 |
| 2016 | Q4 | 16 | 1.000727 | 1.013051 | 1.000749 | 1.000937 | 0.993106 | 0.945088 |
| 2017 | Q1 | 17 | 1.002605 | 1.057354 | 1.002635 | 0.999994 | 0.993846 | 0.947013 |
| 2017 | Q2 | 18 | 1.000758 | 1.013653 | 1.000744 | 1.000859 | 0.996348 | 0.967602 |
| 2017 | Q3 | 19 | 1.000121 | 1.001534 | 1.000121 | 1.000181 | 0.997084 | 0.967475 |
| 2017 | Q4 | 20 | 1.000079 | 1.000929 | 1.00008 | 1.000097 | 0.997203 | 0.968959 |
| 2018 | Q1 | 21 | 1.000247 | 1.003422 | 1.000239 | 1.000239 | 0.997283 | 0.968959 |
| 2018 | Q2 | 22 | 1.001976 | 1.030784 | 1.001917 | 1.001917 | 0.997521 | 0.968959 |
| 2018 | Q3 | 23 | 1.000086 | 1.003953 | 1.000067 | 1.000067 | 0.999384 | 0.988772 |
| 2018 | Q4 | 24 | 1.000026 | 1.000864 | 1.000018 | 1.000018 | 0.99945 | 0.989015 |
| 2019 | Q1 | 25 | 0.999867 | 1.000034 | 0.999875 | 0.999875 | 0.999468 | 0.98903 |
| 2019 | Q2 | 26 | 1.000007 | 1.000405 | 1.000007 | 1.000007 | 0.999343 | 0.989063 |
| 2019 | Q3 | 27 | 0.999958 | 1.000618 | 0.999966 | 0.999966 | 0.99935 | 0.989131 |
| 2019 | Q4 | 28 | 0.999962 | 1.000037 | 0.999959 | 0.999959 | 0.999316 | 0.989131 |
| 2020 | Q1 | 29 | 0.999922 | 1.000028 | 0.999917 | 0.999917 | 0.999275 | 0.989155 |
| 2020 | Q2 | 30 | 1.000284 | 1.003576 | 1.000312 | 1.000312 | 0.999192 | 0.989155 |
| 2020 | Q3 | 31 | 0.999883 | 1 | 0.999873 | 0.999873 | 0.9995 | 0.992692 |
| 2020 | Q4 | 32 | 1.000744 | 1.00662 | 1.000791 | 1.000791 | 0.999373 | 0.992692 |
| 2021 | Q1 | 33 | 0.999945 | 1.000017 | 0.99994 | 0.99994 | 1.000158 | 0.999236 |
| 2021 | Q2 | 34 | 0.999959 | 1 | 0.999956 | 0.999956 | 1.000098 | 0.999254 |
| 2021 | Q3 | 35 | 1.000114 | 1.000747 | 1.000101 | 1.000101 | 1.000054 | 0.999254 |
| 2021 | Q4 | 36 | 0.999943 | 1 | 0.999941 | 0.999941 | 1.000155 | 1 |
| 2022 | Q1 | 37 | 0.999952 | 1 | 0.999951 | 0.999951 | 1.000096 | 1 |
| 2022 | Q2 | 38 | 0.999955 | 1 | 0.999953 | 0.999953 | 1.000047 | 1 |
| 2022 | Q3 | 39 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2022 | Q4 | 40 | 1 | 1 | 1 | 1 | 1 | 1 |

Figure 30.4.2.3: Factors per method for the Inflation-Adjustment Model

Then, the area below the triangle is filled for each model using each of methodologies, finding the cumulative paid amounts for each model taking historical inflation into account.

Afterwards, we find the projected increments in order to apply future inflation to future payments. In other words, we find the incremental payments for the area below the triangle for each of the models by subtracting each cumulative payment from the previous cumulative payment.

Using compounding, the incremental payments for the future years are then multiplied by the relevant future inflation rate, to obtain the new incremental payments including the future inflation.

4.4.3. Inflation-Adjusted Claims Provisions

To finalize, the revised cumulative payments are calculated by adding the previous quarter's cumulative payment to the current quarter's incremental payment with future inflation for each model. Just like that, it is straightforward to get the claims provisions by deducing the respective cumulative amounts paid from the previous value of the cumulative amounts. The figure below shows the provisions of the inflation-adjusted model.

| Year | Quarter | Development period | Claims provision Chain Ladder | Claims provision Worst Factors | Claims provision Average Factors | Claims provision Average T-Factors | Claims provision Grossing Up Factors | Claims provision Grossing Up Worst Factors |
|------|---------|--------------------|-------------------------------|--------------------------------|----------------------------------|------------------------------------|--------------------------------------|--|
| 2013 | Q1 | 1 | 0 | 0 | 0 | 0 | 507881.5 | 507881.5 |
| 2013 | Q2 | 2 | 0 | 0 | 0 | 0 | 474592 | 474592 |
| 2013 | Q3 | 3 | -177.078 | 0 | -183.229 | -183.229 | 441190.9 | 441362.7 |
| 2013 | Q4 | 4 | -457.597 | 0 | -471.183 | -471.183 | 534184.5 | 534612.7 |
| 2014 | Q1 | 5 | -748.134 | 0 | -774.341 | -774.341 | 523767.7 | 524451.6 |
| 2014 | Q2 | 6 | -220.966 | 2991.883 | -297.452 | -297.452 | 452128.3 | 455137.4 |
| 2014 | Q3 | 7 | -373.728 | 3335.041 | -471.309 | -471.309 | 471022.3 | 474343.4 |
| 2014 | Q4 | 8 | -703.442 | 4454.488 | -853.694 | -853.694 | 581122.2 | 585571.8 |
| 2015 | Q1 | 9 | 2973.806 | 36492.27 | 3053.067 | 3053.067 | 545646.7 | 576180.8 |
| 2015 | Q2 | 10 | 2525.688 | 38341.5 | 2577.885 | 2577.885 | 544833.8 | 576187.8 |
| 2015 | Q3 | 11 | 4126.446 | 58224.58 | 4328.564 | 4328.564 | 541111 | 587473.7 |
| 2015 | Q4 | 12 | 4466.63 | 67593.73 | 4683.192 | 4683.192 | 582036.6 | 632982.9 |
| 2016 | Q1 | 13 | 3514.187 | 61214.8 | 3673.117 | 3673.117 | 553477.9 | 602687.6 |
| 2016 | Q2 | 14 | 3059.932 | 57690.88 | 3248.772 | 3248.772 | 471615.4 | 514264.1 |
| 2016 | Q3 | 15 | 2705.017 | 54056.12 | 2863.07 | 2863.07 | 433041.6 | 472953.8 |
| 2016 | Q4 | 16 | 2449.778 | 59882.31 | 2663.07 | 2663.07 | 457494.9 | 501088.7 |
| 2017 | Q1 | 17 | 2390.297 | 58999.55 | 2564.373 | 2564.373 | 401297.9 | 440489.1 |
| 2017 | Q2 | 18 | 2726.008 | 74434.79 | 2824.967 | 2824.967 | 380167.8 | 419222.5 |
| 2017 | Q3 | 19 | 10735.79 | 203595.7 | 10603.52 | 10603.52 | 381491.9 | 491678.2 |
| 2017 | Q4 | 20 | 13062.71 | 240209.1 | 12865.56 | 12865.56 | 389740.5 | 504455.9 |
| 2018 | Q1 | 21 | 12173.28 | 221970.8 | 11986.71 | 12046.86 | 329741.8 | 429130 |
| 2018 | Q2 | 22 | 13179.24 | 235357.2 | 12993.09 | 13276.12 | 319391.6 | 423309.2 |
| 2018 | Q3 | 23 | 14206.17 | 212895.3 | 14009.9 | 14690.59 | 296969.6 | 392791 |
| 2018 | Q4 | 24 | 25970.68 | 469072.8 | 25868.21 | 16631.33 | 329387 | 506070.8 |
| 2019 | Q1 | 25 | 27714.7 | 564911.8 | 27690.8 | 19165.75 | 306522.4 | 475759.8 |
| 2019 | Q2 | 26 | 30140.69 | 639324.4 | 30210.95 | 19785.74 | 309713.4 | 506564.2 |
| 2019 | Q3 | 27 | 31669.11 | 614230.1 | 31535.51 | 21333.93 | 282104.3 | 464390.6 |
| 2019 | Q4 | 28 | 41798.63 | 735840.1 | 41965.17 | 30977.8 | 332487.2 | 557483 |
| 2020 | Q1 | 29 | 40891.66 | 636482.6 | 41036.97 | 30889.42 | 272625 | 457173.8 |
| 2020 | Q2 | 30 | 29519 | 426437.6 | 29542.86 | 22778.12 | 176963.2 | 298282 |
| 2020 | Q3 | 31 | 53089.19 | 726600 | 53655.59 | 44143.65 | 270969.4 | 500162 |
| 2020 | Q4 | 32 | 66669.4 | 823242.4 | 66933.19 | 54380.67 | 295616.4 | 550637.1 |
| 2021 | Q1 | 33 | 55904.05 | 638572.4 | 56155.85 | 47674.55 | 216024.3 | 399507.3 |
| 2021 | Q2 | 34 | 101339 | 1055409 | 101489.7 | 86220.8 | 325778.5 | 588557.8 |
| 2021 | Q3 | 35 | 136371 | 1229476 | 137904.3 | 127385.1 | 351544.7 | 659894.2 |
| 2021 | Q4 | 36 | 193988.8 | 1469572 | 196852 | 192737 | 395251.3 | 744831 |
| 2022 | Q1 | 37 | 223585.7 | 1342830 | 228723 | 245210.4 | 336401.4 | 642046.9 |
| 2022 | Q2 | 38 | 376884.8 | 1798505 | 387844.9 | 439022.7 | 430477.7 | 842511.6 |
| 2022 | Q3 | 39 | 730604.8 | 2521609 | 757414.3 | 895734.6 | 709456.3 | 1356490 |
| 2022 | Q4 | 40 | 2073173 | 7626401 | 2256477 | 2756854 | 1955660 | 4842451 |

Figure 31.4.3.1: Claims Provision per method for the Inflation-Adjustment Model

5. Results: Effect of Inflation on claims provisions

This chapter will detail the outcomes of my internship at KPMG Portugal, covering the results and outcomes of each of the mentioned procedures, along with the conclusion of the whole experience.

My internship has been both informative and influential, focusing on the real-world implementation of forecasting techniques and how they fit into actuarial modeling. The main topic involving the calculation of claims provisions, which are an essential component of risk and insurance management, as well as the application of multiple approaches to assess the effect of inflation on these provisions.

5.1. Reasoning and planning

This project focuses on the impact of historical and future inflation on claims provisions for general insurance. By comparing multiple actuarial methodologies against the client calculated provisions, we aimed to assess the accuracy and robustness of each method. This allowed us to examine both the technical and financial implications of using different models when inflation is factored into future liabilities.

Including historical and projected future inflation in the models is a crucial component of this research, as the insurers' future obligations are significantly shaped by inflation, especially in such long-tailed insurance line where claims stay unsettled for longer periods where inflation compounds over time. Besides, the cost of repairs, medical and legal fees, and many other general-insurance specific elements are tied by inflation. Inflation inclusion also reduces the likelihood of reserve shortages. Compared to more conventional models that might not fully consider shifting economic situations, this offers a significant improvement.

5.2. Interpreting and testing the results of the base model (Not including Inflation Adjustment)

5.2.1. Base Model Results

The table presented below shows the results for the base model without inflation per each model.

Table 5.2.1.1: Results of the Base Model for Claims Provisions

| 31/12/2022 | Client calculated Provision | Total Provision Chain Ladder | Total Provision Worst Factor | Total Provision Average Factors | Total Provision Average T Factors | Total Provision Grossing up factor | Total Provision Grossing up worst factor |
|----------------------------|-----------------------------|------------------------------|------------------------------|---------------------------------|-----------------------------------|------------------------------------|--|
| Amounts Paid | 4,048,096 | 3,724,570 | 19,846,385 | 3,922,397 | 4,463,101 | 3,799,874 | 11,560,040 |
| Difference % | | -7.99% | 390.26% | -3.11% | 10.25% | -6.13% | 185.57% |
| Absolute Difference | | -323,526 | 15,798,289 | -125,699 | 415,005 | -248,223 | 7,511,943 |

Looking at the results, we observe that the Worst Factor method produced a pessimistic estimate of 390.26% higher than the client’s estimate, other methods like Average Factors and Grossing Up Factor remained relatively close to the client calculated provision. The Chain Ladder model produced provisions of €3.72M, closely aligning with the client’s, being 7.99% lower.

5.2.2. Bootstrap Test for testing the Adequacy of the Model

These results and conclusions presented above look adequate, but they do not sufficiently indicate the significance of the methodologies. In other words, the data needs to be tested for statistical significance.

The table of results of the Confidence Intervals at 95% confidence level and at 99% confidence level obtained in section 4.2.5. Testing the base model for Statistical Significance: Bootstrap test is presented in the figure below, using the sampling assumption.

| Methodology | Results | Lower 95 CI | Upper 95 CI | Lower 99 CI | Upper 99 CI |
|----------------------------------|-------------|-------------|-------------|-------------|-------------|
| Chain Ladder | 3724570.219 | 1,319,717 | 6,464,484 | 511,253 | 7,272,948 |
| Worst Factor | 19846384.86 | -16,416,666 | 31,567,053 | -23,956,964 | 39,107,352 |
| Average Factors | 3922397.399 | -26,350,154 | 15,458,069 | -32,920,018 | 22,027,933 |
| Average T-Factors | 4463100.898 | -26,351,421 | 15,461,684 | -32,922,052 | 22,032,315 |
| Grossing Up Factors | 3799873.523 | -17,663,380 | 74,415,290 | -32,132,885 | 88,884,795 |
| Grossing Up Worst Factors | 11560039.54 | -20,578,218 | 24,726,608 | -27,697,548 | 31,845,938 |

Figure 5.2.2.1: Results of the Bootstrap Test for the Base Model: Confidence Intervals

Practically looking at the results we can confirm that our calculations for claims provisions fall within the range for each of the 6 methodologies. It can be deduced that

our models are statistically significant, is a representative of the true risk associated with claims provisions, and that the calculations are consistent with the real data's variability.

We can confidently say that the discrepancies between the client's calculations of claims provisions and ours cannot be the result of coincidence. Rather, they are a sign of true variations in the approaches or assumptions made by the client.

5.2.3. Can we fit an Inflation Adjusted Model (Sensitivity Analysis)?

In this subsection, the results obtained from the base claims provision model is compared to the sensitivity test results, as presented in the table below.

Table 5.2.3.1: Sensitivity Analysis: Sensitivity Results of Claims Provisions per method

| 31/12/2022 | Client calculated Provision | Total Provision chain ladder | Total Provision Worst Factor | Total Provision Average Factors | Total Provision Average T Factors | Total Provision Grossing up factor | Total Provision Grossing up worst factor |
|---------------------------------------|-----------------------------|------------------------------|------------------------------|---------------------------------|-----------------------------------|------------------------------------|--|
| Amounts Paid | 4,048,096 | 3,724,570 | 19,846,385 | 3,922,397 | 4,463,101 | 3,799,874 | 11,560,040 |
| Sensitivity check Amounts Paid | | 3,884,930 | 21,198,090 | 4,096,765 | 4,633,800 | 17,910,931 | 25,955,661 |

As per the table above, we can tell that the claims provision estimates obtained from the random inflation rates are larger than their counterparts in the first row, suggesting that inflation significantly raises the reserve requirements for all techniques. We can observe modest increases as in the estimates of Chain Ladder, Average Factors, Worst Factors, and Average T-Factors. On the other hand, there are much greater adjustments in the estimates of Grossing Up Factor and Grossing up Worst Factor, signifying that those methods are over responsive to the inflation addition. In other words, the sensitivity analysis signify that the model is responsive to the inflation effect, except for Grossing Up Factors and Grossing up Worst Factors.

The conclusion that including inflation in the model is both required and suitable can be drawn, given that the sensitivity analysis demonstrates consistent rises in all provision estimates. Because the provisions derived from the sensitivity check account for the increasing expenses linked with inflation, they better position the insurer to handle future claim payments.

5.3. Inflation-Adjusted model for Claims Provision in General Insurance

5.3.1. Inflation-Adjusted Model Results

Although the Grossing Up Factors and Grossing up Worst Factors have shown huge discrepancies in the sensitivity analysis, I have decided to include them in the model in any case, as they may show to be significant with the inclusion of the real inflation rates. In case not, the results of those methods will be disregarded in the Inflation-Adjusted Model.

The table below shows the results of the claims provisions for all models, accounting for historical and forecasted future inflation.

Table 5.3.1.1: Inflation-Adjusted Model Results: Claims Provision per method

| 31/12/2022 | Client calculated Provision | Provision chain ladder | Provision Worst Factor | Provision Average Factors | Provision Average T Factors | Provision Grossing up factor | Provision Grossing up worst factor |
|----------------------------|-----------------------------|------------------------|------------------------|---------------------------|-----------------------------|------------------------------|------------------------------------|
| Amounts Paid | 4,048,096 | 4,330,928 | 25,010,256 | 4,563,188 | 5,143,837 | 17,910,931 | 25,955,661 |
| Difference % | | 6.99% | 517.83% | 12.72% | 27.07% | 342.45% | 541.18% |
| Absolute Difference | | 282,832 | 20,962,160 | 515,092 | 1,095,741 | 13,862,835 | 21,907,565 |

Looking at the results, we observe that the Chain Ladder estimate has increased to 6.99% higher than the client's estimate. The Worst Factor method tends to overestimate in inflationary times, increasing to become 517.83% over the client calculated provision. The Grossing Up Worst Factor displayed an inflated difference of +541.18% indicating a large increase in the Grossing Up Factor as well, suggesting the exclusion of those methods in the Inflation-Adjusted Model.

5.3.2. Overall Model Conclusions

Different viewpoints on estimating claims provisions are offered by the many actuarial models which were used, including Grossing Up, Average Factors, Worst Factor, Chain Ladder, and Average T Factors. Because of this diversity, we were able to compare the accuracy of each technique with and without inflation adjustments using the client's past claims data.

In this study, the Chain Ladder technique proves to be the most precise and trustworthy model for estimating claims provisions when explicit inflation correction is performed. The balance of this model rests in its capacity to capture historical patterns of development while seamlessly accounting for inflationary impacts and avoiding exaggerating responses to extraordinary or transient events.

The accuracy of the Chain Ladder approach originates from its projection of future trends based on real claims development using robust historical data. Unlike the Worst Factor

approach, it avoids relying too much on extreme scenarios and offers more dynamic, inflation-adjusted estimates than more straightforward models like Average Factors. Because of this, it is the ideal approach for insurers who want their provisioning systems to be resilient and precise, especially in the face of inflationary pressures.

The difference between our results and the client's results can be explained by several factors. Firstly, looking at the base model without inflation adjustment, the differences can be explained by the different assumptions, data, parameter choices and development factors employed by each party. Secondly, the client calculated provision may be based on extremely basic predictions about future inflation or a less detailed treatment of inflation. Our models offer a more data-driven methodology by taking into consideration current economic developments, which inevitably results in more dynamic projections. Moreover, the worst-case situations such as Worst Factor and Grossing Up approaches produce are helpful in scenario planning, but they can cause large deviations in normal provisioning, particularly in contexts with moderate inflation.

Moreover, the deviation in comparison to the inflation adjusted model is explainable due to including real and forecasted inflation data. Because the inflation-adjusted model accounts for both past and future inflation, it yields more accurate and efficient outcomes. With the exclusion of the Grossing Up Factor and Grossing up Worst Factor methods due to the huge discrepancies in their estimates, we can predict how changes in the economy will affect future claim payments by accounting for inflation in the future using the rest of the methodologies. This makes it possible to estimate reserves more realistically, ensuring that the insurance provisions are better equipped to withstand any inflationary pressures that may arise. As a result, the model offers a more dependable foundation for keeping adequate reserves, enhancing risk control, and guaranteeing the insurance portfolio's long-term financial viability.

Also, apart from claims provision calculation, this model provides estimates of the expected future amounts paid, expected future claims number, and expected future claims costs with and without inflation's effect.

Besides, acquiring the development factors is key for each claims related information. This means that we can also obtain the development factors for claims numbers or claims costs using the same methods employed for amounts paid, and the model can then be extended to project the expected future claim costs or the expected future claims numbers respectively.

Lastly, I would like to comment with regards to the controversy regarding the unsuitability of the use of quarterly data in the existence of seasonality. As this model is created with inputs of Motor insurance claims, the model can be fitted on quarterly data due to the absence of significant seasonality. In the case of fitting the same model on other lines of general insurance, the respective data can be simply grouped into yearly rather than quarterly and the model will work with the same efficiency after minor changes on the development factors, as the inflation rates are presented in yearly and quarterly in my model.

5.4. Overview of the experience

The internship provided a priceless chance to apply actuarial ideas and methods to useful, real-world tasks. The main project in this internship involved calculating claims provisions using six different methodologies, where I have evaluated several forecasting techniques and concluded that the chain ladder method was the most accurate and dependable way to estimate future payments of past claims. The significance of inflation adjustments in actuarial models was highlighted by the integration of historical and predicted inflation, which offered crucial insights into the long-term implications on claims provisions.

Including inflation adjustment required a solid grasp of time series forecasting and its application in practical situations, in addition to a deep comprehension of the actuarial methodologies that underpin it. I learned a great deal about managing and accounting for economic factors in actuarial models through this assignment, which highlights the need of taking inflation into account when making long-term financial plans.

The necessity of choosing the appropriate model for the available data was highlighted by the successful application of exponential smoothing for inflation forecasting. The project's ultimate success depended heavily on this method's capacity to manage the inherent volatility in inflation data while producing steady and accurate estimates.

Apart from the core project, I have also contributed to other types of projects which are explained in the appendix. I have helped create an actuarial report and a Solvency II master file report, and I have contributed to calculating mathematical provisions for different lines of business, which deepened my comprehension of regulatory obligations and how they support insurance firms' financial stability.

I now have a greater understanding of the significance of precise modeling, actuarial soundness, and regulatory compliance thanks to these experiences. These studies have also improved my comprehension of the complexities associated with long-term liabilities while fortifying my capacity to evaluate and reconcile huge datasets critically. All things considered, the internship improved my technical skills, emphasized the value of accurate actuarial analysis, and equipped me for any obstacles in the future.

The knowledge acquired from this work directly relates to the issues that insurers are currently facing, as precise forecasting and financial planning are more crucial than ever. The techniques and strategies I have picked up throughout this experience will surely be a great starting point for me as I approach actuarial problems in the future.

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Appendix

In this chapter, I will mention some of the topics that are relevant to my internship experience at KPMG, but they do not add a big value to the reader.

Claims Provisions base model: Amounts paid data presentation

For reference, the triangle of cumulative amounts paid for the Motor insurance data of the latest years from the triangles used in this project calculations are presented below.

Calculations for Audits: Advanced Mathematical Provision Models for Pensions and Life Annuities: Addressing Longevity and Interest Rate Risk

To guarantee accuracy and consistency between our estimates and the client's estimates, calculation of the mathematical provisions is part of the audit process. This subtopic focuses on sophisticated mathematical provision models for life annuity and pension calculations used in Audits specifically. It explores the complexities of these models, concentrating in particular on the manner in which they can handle interest rate risk and longevity. "Mathematical Provisions", often referred to as MP is the sum that the insurer retains to meet liabilities arising from insurance or pension funds' contracts, as calculated by actuarial methodologies (The Central Bank of the Republic of Kosovo, 2016).

Empirical proof of the usefulness of stochastic modeling techniques in addressing longevity risk in pension valuations is presented in this paper (Ronkainen, 2012), indicating that more accurate estimates of future pension requirements require conducting an extensive investigation of past mortality data and future mortality projections.

To calculate the present value of future cash flows and account for variables like interest rates, mortality rates, and investment returns, these valuations require sophisticated mathematical computations (Pitacco, 2016).

With the help of our master's courses "MAFI-CA - Financial Mathematics" and "MASV - Survival Models and Life Contingencies", it has been very convenient for me to understand the concepts, ideas and applications for this project.

The goal of this project is to compute the client's Mathematical Provision, often referred to as MP using our methods, and compare it to the client's current estimated provision, using Microsoft Excel and Excel VBA for some specific company standard macros. This stage guarantees uniformity and pinpoints any inconsistencies that might require attention.

Audits for Life Annuities

As a starting point, our team classifies the members into 4 categories in accordance with the client's scheme. Individual members with payments in advance, individual with 2 heads (Joint Life Annuity) with payments in arrears, Group 105&110 with annual premium payments done in advance, and Group 112 with annual payments done in arrears.

Client's data for all members are then imported in reference to each category to the calculations Excel file, including members' Policy number, the frequency of premiums payment, the insureds' date of Birth, the fractioning for premiums, Term of the policy, the effective date of the policy, technical interest rate applied to the policy, and Premium amounts.

The first step is to calculate the member's age at date of calculation and age at date of joining service, where the 2 used mortality tables for this scheme is TV88/90 and TPRV 93. Then, we index the present value of life annuity for the member based on the age nearest, based on the respective mortality table.

Using KPMG specialized macros, the Present value of term life annuities is calculated for each category with respect to each category's timing and frequency of payments, while accounting for the number of years in which the annuity is payable. This calculation is done to double confirm that the present values used by client from the mortality tables is matching with our calculated present value of the temporary life annuities. The KPMG Mathematical Provision is then calculated by multiplying the Cashflows of premium by the term annuity present value limited by Age payable, depending on the member's Category Code. Multiplying the Age of reference limits the calculation by the retirement age or the minimum age for payments.

The total KPMG Mathematical provision is calculated for each category by summing the Mathematical Provisions for all members of the respective category.

In a separate sheet, the results are presented. Side by side, is presented the client's estimated total Mathematical Provisions for each group and ours as presented below.

| LIFE ANNUITY | | | | |
|---|-------------------------------|----------------------------|------------------|--------------|
| Mathematical Provison Estimation 31.12.2022 | | | | |
| | Mathematical Provision Client | Mathamtical Provision KPMG | Dif | Dif |
| Individual | 76,854,519 | 76,847,572 | -6,947 | -0.01% |
| Group 105 & 110 | 14,111,626 | 17,003,855 | 2,892,229 | 20.50% |
| Group 112 | 3,694,893 | 3,636,416 | -58,477 | -1.58% |
| Total | 94,661,038 | 97,487,843 | 2,826,805 | 2.99% |

| | Mathematical Provision Client's Balance sheet | Mathematical Provision Client's Inventory | Mathamtical Provision KPMG | Dif KPMG Bal |
|--------------|---|---|----------------------------|-------------------|
| Individual | 7,979,885 | 76,854,519 | 76,847,572 | 68,867,687 |
| Groups | 2,834,332 | 17,806,519 | 20,640,271 | 17,805,939 |
| Total | 10,814,217 | 94,661,038 | 97,487,843 | 86,673,626 |

Life Annuity Audit: Calculated Mathematical Provisions

Analyzing the data above, the difference between the Mathematical Provisions calculated by us and the one calculated by the client for group individual and group 112, while the differences for group 105&110 is extremely high. The differences of the values in the balance sheet also suggests that the balance sheet is not reflecting the correct mathematical provisions which are required for the liabilities of life annuities.

Upon analysis, the overall difference between ours and the client's mathematical provisions of 2.99% can be explained by the client's underestimation of future liabilities. Given the huge variances in Group 105&110, this is causing the overall discrepancy. Such differencing explains the need for a detailed review of the assumptions and calculation methodologies employed by the client.

Audits for Pension Funds

As a first step, the members are classified into 3 groups. Members with redeemable pension, members with non-redeemable pension for life, and members with temporarily non-redeemable pension. The Mortality table for all pensioners is TV 88/90, the Mortality table for deferred pensions is TD 88/90, while the Minimum Age for orphans' pensions is 26, and the Normal retirement age is 60 for both genders.

The present value of the annuity is indexed from the relevant mortality table for each group, then the present value of the revised temporary annuity is calculated using the KPMG macros only for non-minors. Both annuity values are then compared to confirm that they are equal.

First, we find the initial mathematical provision by multiplying the initial technical base factor by the initial pension, and then we calculate the actual mathematical provision by multiplying the actual pension by the actual technical base factor.

We can now easily find the reserves by subtracting the initial and actual mathematical provisions, then presenting the totals along with the clients' as below.

Pensions
Mathematical Provision Estimate

| 31/12/2022 | Mathematical Provision Balance sheet | Mathematical Provision Client | Mathematical Provision KPMG | Dif Balance sheet | | Dif BD | |
|--------------------------|--------------------------------------|-------------------------------|-----------------------------|-------------------|--------------------|----------------|------------------|
| | | | | % | abs | % | abs |
| Reconciled | 1,718,220 | 10,615,125 | 13,757,870 | 700.70% | -12,039,650 | 29.61% | -3,142,745 |
| Defined | 1,800,121 | 18,420,354 | 3,892,957 | 116.26% | -2,092,836 | -78.87% | 14,527,397 |
| Presumed | 1,038,711 | 323,503 | 2,088,838 | 101.10% | -1,050,127 | 545.69% | -1,765,335 |
| Charges with Added Value | 7,261 | 33,000 | 33,000 | 354.48% | -25,739 | 0.00% | 0 |
| Total | 4,564,313 | 29,391,982 | 19,772,665 | 333.20% | -15,208,352 | -32.73% | 9,619,317 |
| | 4,557,052 | | | | | | |

| | PM Base de Dados | Provisão Matemática KPMG | Dif % | Dif abs |
|----------------|-------------------|--------------------------|--------------|-------------------|
| Non Redeemable | 14,658,489 | 15,821,986 | 7.94% | -1,163,497 |
| Redeemable | 11,297,193 | 11,391,186 | 0.83% | -93,993 |
| Total | 25,955,682 | 27,213,172 | 4.84% | -1,257,490 |

Pension Funds Audit: Calculated Mathematical Provisions

Analyzing the balance sheet differences, we find a huge difference between the balance sheet and KPMG's calculated provisions of 333.2%. This implies that the client's balance sheet is also significantly underreporting the liability.

Comparing our calculations with the clients', we conclude that both calculations are very comparable. The small discrepancies suggest that the client's assumptions and methods may not be aligned.

Data Validation for Actuarial Report on Technical Provisions

As part of the project, I helped with the actuarial report production, paying special attention to the data processing and technical provision checks. The aim of such tasks is to maintain the actuarial calculations' integrity and accuracy as well as the actuarial report's general dependability.

The objectives of such tasks are to handle and confirm the information needed to compute technical provisions, to do extensive validations and checks to guarantee the completeness and quality of the data, to comprehend and record the actuarial calculations' methods and underlying presumptions, and to assist in the documentation of an accurate and thorough actuarial report.

This was performed by gathering and analyzing the data needed for the technical specifications from historical records, internal systems, and external reports. This data comprised details about the policyholder, claims, premiums, and other pertinent actuarial inputs. Thorough examinations were conducted to guarantee the data's accuracy, consistency, and completeness.

Creating the Master File for Solvency: Solvency II file review note

For this project, a variety of data sets, including cash flows, actuarial assumptions, market risks, and technical specifications, had to be systematically integrated. Our goal was to create an accurate and thorough master file that accurately reflected the client's current financial situation and risk profile.

The cashflow data is extracted from the client provided financial statements and historical financial records, which includes cash inflows and outflows from insurance policies to the relevant tabs interpreting the expenses, revenues and financial positions.

Solvency capital requirements data is imported from the solvency capital requirements tool and is applied to the solvency capital requirements tabs, to output the capital requirements per risk category.

Technical provisions data is imported from the internal actuarial reports and external actuarial reviews. This data contains information on the assumptions of the best estimate, and it is applied to the assumptions on the best estimate and technical provisions tabs which outputs the impact of the market movements on the financial position.

Actuarial data such as defined statistical measures, mortality rates, and lapse rates are provided by the client, and it is applied to the actuarial assumptions sheets, outputting the forecasted future claims, overall risk exposure, and policy lapses.

Market risks data is imported from the risk management reports, and the previous masters file review, which then outputs the capital requirements per risk category.