MASTER IN

ACTUARIAL SCIENCE

FINAL MASTER WORK

INTERNSHIP REPORT / PROJECT REPORT / DISSERTATION

PROJECTING THE EFFECT OF INFLATION ON SPANISH MOTOR VEHICLE BODILY INJURY CLAIMS

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SUPERVISOR:

WALTHER NEUHAUS (ISEG)
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OCTOBER - 2023

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I would like to express my heartfelt gratitude and appreciation to all those who have contributed to the successful completion of my internship report, which also serves as my Master's final work, conducted at Liberty Seguros Company during my tenure as a student at ISEG College.

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This internship has been a significant milestone in my academic and professional journey, and I am deeply thankful to everyone who has been a part of this experience. Their support and guidance have been instrumental in my growth, and I look forward to carrying the lessons and experiences gained during this internship into my future endeavours.

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Abstract

This report describes my 5-month internship experience in the Actuarial Non-Life Reserving team at Liberty Seguros, a leading insurance company in Portugal. Liberty Seguros Portugal is a branch of Liberty Seguros Spain, and together with Spain and Ireland operates as the Western Europe Market ("WEM") business unit for Liberty Mutual Group. During my internship, I was tasked with assisting the team of actuaries in analysing the company's reserve levels and developing financial models to estimate future liabilities.

Throughout the internship, I gained valuable experience in using statistical software packages and actuarial models to conduct data analysis, estimate reserve levels, and analyse loss trends. I also participated in various team meetings and had the opportunity to collaborate with senior actuaries and other members of the reserving team on several projects.

The report provides an overview of the specific project I worked on during the internship, developing a reserve model for the motor line of business in Spain and analysing the impact of continuous and step change inflation. It also highlights the skills and knowledge I acquired, including an understanding of actuarial reserving methodologies, improved data analysis skills, and experience working with statistical software packages.

Additionally, the report includes recommendations for Liberty Seguros on how to improve its reserving processes and procedures, based on my observations and experiences during the internship.

For reserve modelling, we use the traditional Chain Ladder method or the Bornhuetter-Ferguson ("BF") method to estimate the ultimate claim cost. A key assumption of the traditional chain ladder method is that the observed historical inflation will remain constant in the future, which may not be realistic in the real world. This research paper focuses majorly on incorporating future inflation in the reserving model.

Overall, the internship at Liberty Seguros provided me with valuable insights into the operations of an actuarial reserving department in the insurance industry, and I gained practical experience that will be useful in my future career as an actuary.

Key words: Loss Reserving, Chain Ladder, Bodily Injury, Baremo, Inflation, Regression.

Glossary

Term	Definition
BI	Bodily Injury
Baremo	It represents the "Baremo de indemnizaciones por accidents de tráfico," which can be translated as the "Compensation Scale for Traffic Accidents"
AIC	Akaike Information Criterion
Adj_Inf_Rate	Inflation index adjusted to base year 2015
HICP	Harmonized Index of Consumer Prices
RPI	Retail Price Index
CPI	Consumer Price Index of Spain, adjusted to base year 2015
Avg_Wage_Index	Wage index of Spain, adjusted to base year 2015
GDP_Index	Gross domestic product index of Spain, adjusted to base year 2015
Unit_Labour_Cost	Index of cost associated to per unit labour in Spain, adjusted to base year 2015, used to analyze and compare the cost of labor in producing a single unit of output

Labour_Compensation	Labour Compensation Index of Spain, adjusted to base year 2015, used to track changes in the cost of labor over time
Health_Spending_Index	Health Spending Index of Spain, adjusted to base year 2015, measure the changes in healthcare expenditures over time
HICP_Hospital_Services	HICP Hospital Service Index of Spain, adjusted to base year 2015, that tracks and measures changes in the prices of hospital services over time
HICP_Medical_Prod	HICP category "Medical Products, Appliances, and Equipment in Spain, adjusted to base year 2015, used to measure the changes in prices of various medical goods and devices purchased by consumers
HICP_Med_ParaMed_Service	The Harmonized Index of Consumer Prices category "Medical and Paramedical Services adjusted to base year 2015, is a classification of services that includes consultations of physicians in general or specialist practice
20XXQY	Represent the Yth Quarter of calendar year 20XX
IBNR	Incurred But Not Reported

Voll v	Voll v denotes the collection of claim values from the vth previous accident year within a specified development period
QY	Represents the Yth Quarter of a given calander year.

Introduction

3.1 Motivation and Goals

Inflation in General Insurance claim reserving is a key question. “Why?” people ask, with the common misperception that **“If there even is such a thing as claims inflation, it will emerge over a very long time. There is little need to worry now**”. However, even if future inflation is difficult to predict, it affects the balance sheet today. Under-reserving can result in the need to strengthen booked reserves and can consume a company’s capital. It can also result in underpricing and the writing of unprofitable business. But what is meant by “inflation” in the context of claim reserving? It is the increase in the amount it would cost to settle an insurance claim over time.

Past inflation affects the shape of the data and the assumptions for future inflation have a significant impact on the final value to be set on the reserve. Many methods automatically project the level of past inflation into the future, which makes sense under stable economic conditions. On an average cost per claim, apart from the economic inflation, social influences, such as court awards, attitudes in society towards compensation of accident victims and legislation, including that of the EU, can also give rise to inflation. So, claim inflation is the sum of economic and excess inflation. Here economic inflation is the changes in claim cost as captured through published economic indices relevant to the insurer's mix of business, whereas excess inflation is the changes in claim cost beyond what is captured in economic indices.

As mentioned further in the report, a standard chain ladder method assumes that future inflation will equal past inflation. However, we know that inflation in the current economic environment is very high and this follows a long period of low, and reasonably stable inflation, meaning this assumption does not hold.

3.2 Context

In this report, we will focus on the Spain motor line of business for Bodily Injury ("BI") coverage with the main objective of incorporating explicit inflation assumptions into the reserving models.

Bodily Injury is a coverage within Third-Party Liability Motor policies that pays out when you are at fault for an accident that has resulted in a third party suffering an injury or fatality. This coverage will pay for their medical needs, their lost wages, and their pain and suffering. The amount awarded for BI claims in Spain is impacted by the Baremo tables and is a formula-driven approach for calculating the value of the award. The Baremo tables are updated each year to allow for inflation.

The term "Baremo" stands from "Baremo de indemnizaciones por accidents de tráfico." This translates to "Scale of compensation for traffic accidents". It is the first compulsory compensation system for fatality or injuries in traffic accidents, that was implemented in Spain in 1995. More than 20 years later, a panel of experts appointed by the Insurance Authority carried out an in-depth review of the system. In September 2015 the Spanish Parliament approved the new "Baremo" that came into force on 1 January 2016. Within the first two years, it appears the reform had achieved what it set out to do: adjust compensation levels for fatality and severe injury cases and reduce the level of fraud in frequent claims, such as whiplash. In terms of compensation, the most noticeable changes affect prejudiced parties in case of fatality and third-party assistance, and loss of earnings in case of injuries. Prejudiced parties in case of death have been extended to close relatives, and the level of compensation for third-party assistance and loss of earnings has been reviewed in line with the principle of full reparation of damage. In this report, our primary emphasis will not be on forecasting inflation indices; instead, we will work with specific assumptions and make adjustments to account for inflation in the reserves.

Data

There are different segments for BI in Spain Motor Line of business based on the distribution channel. For simplicity, the data in these channels are combined and analysed at a total BI level. The data is also split based on claim size. We use claims capped at €90,000. The excess of this threshold goes to another segment which we didn´t incorporate in this report. The purpose of using €90,000 cap data is to have more reliability on statistics, as very large claim amounts would result in spurious statistics and increased volatility.

Although the estimation procedures can be applied both to incurred claims (paid claims and aggregate case estimates combined) or paid claims, it is usually better to use the paid claims, since negative values are less likely to appear. Case estimates are set individually and often tend to be conservative, resulting in over-estimation in the aggregate. This leads to negative incurred claims amounts in the later stages of development. Typically these negative values are the result of the market practice, which is to set case reserves conservatively, meaning they typically develop down.

In Figure 3.1, we display the incremental paid claim amounts for Bodily Injury Coverage in Spain's Motor line of business. The table columns represent accident years, while the rows indicate the development years, presented in months (e.g., 24m denotes the second year). Figure 3.2 showcases the Estimated Ultimate Claim Numbers for the same coverage across various accident years, determined using the company's specific software.

Figure: 3.1 Paid Incremental amount of Bodily Injury claim

		Development Year																
		12m	24m	36m	48m	60m	72m	84m	96m	108m	120m	132m	144m	156m	168m	180m	192m	204m
	2006	35 744 893	42 910 516	9 853 715	3 658 869	2 014 618	1 309 616	432 011	150 892	237 157	127 188	64 716	-19 438	70 141	4 270	-390	31 139	2 751
	2007	38 596 139	47 898 522	11 026 501	3 661 634	2 203 107	1 714 685	959 544	257 852	287 265	240 212	29 094	127 529	12 987	24 953	-65	3 109	
A	2008	42 272 313	46 281 643	10 376 720	4 444 583	2 148 380	1 464 211	855 635	195 401	401 168	129 540	122 675	42 305	18 381	2 865	36 239		
c	2009	40 906 171	49 498 604	12 947 718	5 346 317	3 248 550	1 824 383	633 452	609 443	425 851	108 550	10 527	42 382	10 532	26 011			
c	2010	37 079 661	50 623 729	13 564 276	4 958 756	2 877 487	1 494 268	736 141	363 393	85 895	77 346	60 779	22 185	8 880				
i	2011	39 081 311	42 248 895	10 424 815	4 610 691	2 304 559	1 212 840	551 515	126 178	167 981	39 204	97 191	22 226					
d	2012	33 767 166	36 339 415	9 543 278	4 048 093	2 400 673	1 509 386	530 375	282 467	48 112	41 121	-864						
e	2013	33 909 551	39 410 672	10 197 748	4 923 489	2 163 417	913 893	546 405	364 251	140 622	66 284							
n	2014	33 412 680	35 171 779	9 647 518	4 862 844	1 710 129	952 184	490 480	655 728	30 988								
t	2015	32 895 781	31 304 727	10 475 963	4 531 010	2 511 178	887 578	557 393	175 468									
	2016	30 117 167	32 799 358	9 828 333	4 570 768	2 024 283	1 014 464	537 094										
Y	2017	30 874 051	32 308 284	8 728 317	3 346 457	2 385 021	1 355 276											
e	2018	27 625 874	32 776 104	9 852 596	4 310 141	2 884 555												
a	2019	30 843 351	31 314 886	10 052 965	4 210 926													
r	2020	20 979 571	27 181 683	6 645 948														
	2021	30 235 935	33 888 515															
	2022	29 756 562																

Ultimate Claim Number of Bodily Injury claim

Accident Year	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
Ultimate Claim Number	25 303	28 207	28 691	31 642	31 620	28 849	27 250	29 485	28 923	29 325	30 586	30 163	29 321	30 494	21 519	27 582	26 167

Figure: 3.2, represents the estimated ultimate claim number

Methodologies

5.1 Introduction

Dealing with inflation for claims reserving purposes normally embraces two aspects, either to identify the inflation element implicit in the past data on claim amounts or to set a suitable inflation assumption for the future projections of the data.

There is a general technique mentioned in Straub & Grubbs (1998) for taking inflation explicitly into account in projection.

i. If the data is presented in cumulative form, the year-by-year values can be derived by subtracting data along the rows of the development table.
ii. An inflation index that is relevant to the specific business category is selected, or a suitable assumption is made regarding claims inflation in recent years.
iii. The year-by-year values from previous years are adjusted to align with the current year by applying inflation based on the chosen index.
iv. The adjusted values are then summed along the rows of the table, resulting in the data being represented in cumulative form.
v. The reserving actuary uses their chosen method to project the adjusted data, completing the development triangle into a rectangular format.
vi. Year-by-year values are once again calculated, this time pertaining to future year projections, found in the lower-right area of the development table.
vii. Assumptions are made regarding the level of future inflation, which can be based on the previously mentioned index projection or independent considerations.

viii. The projected year-by-year values are adjusted according to the expected future inflation rate.

ix. Summing the projected inflated year-by-year values along the rows provides the estimated reserve figure for the business.

5.2 Run-off Triangle

We use standard notation, so let $i \in \{0, \ldots, n\}$ denote the rows corresponding to the accident year in the triangle, and $k \in \{0, \ldots, n\}$ denote the columns corresponding to the development year in the triangle. In addition, we assume that the claims are fully settled in development year n. The expected value of the incremental claims is Z_{ik} $(i\ k, \in\ \nabla, where\ \nabla = \{i = 0, \ldots, n;\ k = 0, \ldots, n-i\}$ refers to the upper triangle)

Year of Origin	Development Year								
	0	1	2	.	k	.	.	n-1	n
0	Z_{00}	Z_{01}	Z_{02}	.	.	.	.	Z_{0n-1}	Z_{0n}
1	Z_{10}	Z_{11}	Z_{12}	.	.	.	.	Z_{1n-1}	
2	Z_{20}	Z_{21}	Z_{22}	.	.	.	.		
.	.	.	.	.	.	.			
i	.	.	.	.	Z_{ik}				
.	.	.	.	.					
.	.	.	.						
n-1	Z_{n-10}	Z_{n-11}							
n	Z_{n0}								

Models

6.1 Separation Method

6.1.1 Separation Model

The Separation method can be used to separate the stationary claim delay distribution from the exogenous influences that are affecting the stationarity. As per Taylor (1977), If we assume that the factors influencing individual claim sizes remain consistently unchanged, then the ratio of the average amount paid in development year k to the average amount paid by the end of development year n would exhibit a stationary expected value ϑ_k. This means it remains constant and is independent of the accident year i. Furthermore, we assume that the average claim cost for a given development year is proportionate to an index related to the payment year rather than the year of origin. This assumption is appropriate for the inflation caused by Baremo as it affects the group of claim amounts paid in a specific calendar year, irrespective of which accident year the claims belong to.

Consider a run-off triangle, with incremental claim amounts Z_{ik}, standardized by the number of claims v_i in the accident year i: $X_{ik} = \frac{Z_{ik}}{v_i}$. The number of claims can be troublesome here as in the real world the number of claims in the origin year i might not be known until a much later development year period. If we were to take the claim number from the latest development year available, it will result in decreasing claim numbers and therefore an increase in average claim cost per accident period. This leads to an underestimation of the inflation index and hence of the provision of the claim outstanding. As an alternative, we use an estimate of the ultimate claim. There are several methods, such as the traditional Chain ladder method, that can be used for the ultimate claim number. However, we shall not go into detail about claim numbers and focus on claim costs. We used the estimated ultimate claim numbers for each accident year i that are already calculated in Liberty Seguros.

The assumption underlying the separation method is that, the average claim cost of development year k per claim with year of origin i is $E(X_{ik})$ = $\vartheta_k\lambda_{i+k}$ with $\sum_{k=0}^{n}\vartheta_k = 1$.

Here ϑ_k is the effect of development year k which impacts the columns of the run-off triangle and λ_{i+k} is the inflationary effect of calendar year $i + k$. The purpose of the separation method is to separate the factors for the development year from the calendar year effects.

The table below is the run-off triangle formed by these expected values.

Year of Origin	Development Year								
	0	1	2	.	k	.	.	n-1	n
0	$\vartheta_0\lambda_0$	$\vartheta_1\lambda_1$	$\vartheta_2\lambda_2$	.	.	.	.	$\vartheta_{n-1}\lambda_{n-1}$	$\vartheta_n\lambda_n$
1	$\vartheta_0\lambda_1$	$\vartheta_1\lambda_2$	$\vartheta_2\lambda_3$	.	.	.	.	$\vartheta_{n-1}\lambda_n$	
2	$\vartheta_0\lambda_2$	$\vartheta_1\lambda_3$	$\vartheta_2\lambda_4$	.	.	.	.		
.	.	.	.	.	.	.			
i	.	.	.	.	$\vartheta_i\lambda_k$				
.	.	.	.	.					
.	.	.	.						
n-1	$\vartheta_0\lambda_{n-1}$	$\vartheta_1\lambda_n$							
n	$\vartheta_0\lambda_n$								

With $\sum_{k=0}^{n}\vartheta_k = 1$, if we sum along the diagonal involving λ_n we obtain:

$$\sum_{i=0}^{n} X_{i,n-i} = \lambda_n(\vartheta_0 + \vartheta_1 + \vartheta_2 + \cdots + \vartheta_n)$$

$$\sum_{i=0}^{n} X_{i,n-i} = \lambda_n$$

Hence, λ_n is the sum of the latest diagonal.

For the next diagonal we obtain:

$$\sum_{i=0}^{n-1} X_{i,n-i-1} = \lambda_{n-1}(\vartheta_0 + \vartheta_1 + \vartheta_2 + \cdots + \vartheta_{n-1})$$

$$\hat{\lambda}_{n-1} = \frac{\sum_{i=0}^{n-1} X_{i,n-i-1}}{1 - \vartheta_n}$$

Similarly,

$$\hat{\vartheta}_n = \frac{X_{0n}}{\hat{\lambda}_n} \textit{ and } \hat{\vartheta}_{n-1} = \frac{X_{0n-1}+X_{1n-1}}{\hat{\lambda}_{n-1}+\hat{\lambda}_n}$$

Therefore, unknown parameters of the calendar year effect λ_t and the development year effect ϑ_t are estimated by for all $t = \{0,\ldots,n\}$ *and* $k = \{0,\ldots,n\}$

$$\hat{\lambda}_t = \frac{\sum_{i=0}^{t} X_{i,t-i}}{1-\sum_{k=t+1}^{n}\widehat{\vartheta_k}} \quad \text{and} \quad \hat{\vartheta}_k = \frac{\sum_{i=0}^{n-k} X_{i,k}}{\sum_{i=0}^{n-k}\hat{\lambda}_{n-i}}\text{, respectively,}$$

The calculation of the parameters is done recursively with a starting point $t = n$.

The Separation model can be extended further to include the estimation of influences which make claim size vary by year of origin as well as by year of payment. The new formula representing the average claim of origin year i and development year k is given by $\hat{\lambda}_{i+k}\hat{\vartheta}_k q_i$ with the q_i´s normalized such that $\sum_{i=1}^{n} q_i = 1$.

However, this produces more computational difficulties and reduces the degrees of freedom from $\frac{1}{2}n(n-1)\ to\ \frac{1}{2}n(n-3)$. For these reasons we are not proceeding with extended separation model. The specific claim inflation rate r_t is given by $r_t = \frac{\hat{\lambda}_t}{\hat{\lambda}_{t-1}}$ -1.

Accident Year	Diagonal sum	Column Sum	$\hat{\vartheta}_k$	$\hat{\lambda}_t$	r_t
2006	1412,6741	19910,9231	0,362495	3897	0%
2007	3064,1845	21370,6908	0,418784	3922	1%
2008	3560,9013	5313,6562	0,112796	3983	2%
2009	3441,4033	2098,5860	0,048662	3650	-8%
2010	3308,1014	1056,0205	0,026752	3412	-7%
2011	3649,6570	538,2081	0,014924	3707	9%
2012	3454,3291	234,8519	0,007258	3483	-6%
2013	3195,4798	101,6063	0,003519	3211	-8%
2014	3198,8926	46,9683	0,001830	3208	0%
2015	2994,6013	27,6914	0,001233	3000	-7%
2016	2750,4531	13,4569	0,000692	2753	-8%
2017	2786,2121	8,8072	0,000527	2788	1%
2018	2618,1175	4,1539	0,000299	2619	-6%
2019	2726,2668	1,9753	0,000175	2726	4%
2020	2581,1254	-0,0177	-0,000002	2581	-5%
2021	3000,4074	0,1102	0,000018	3001	16%
2022	2984,9904	0,1087	0,000036	2985	-1%

Table 6.1.1a; Estimate of the parameters of Sepration model.

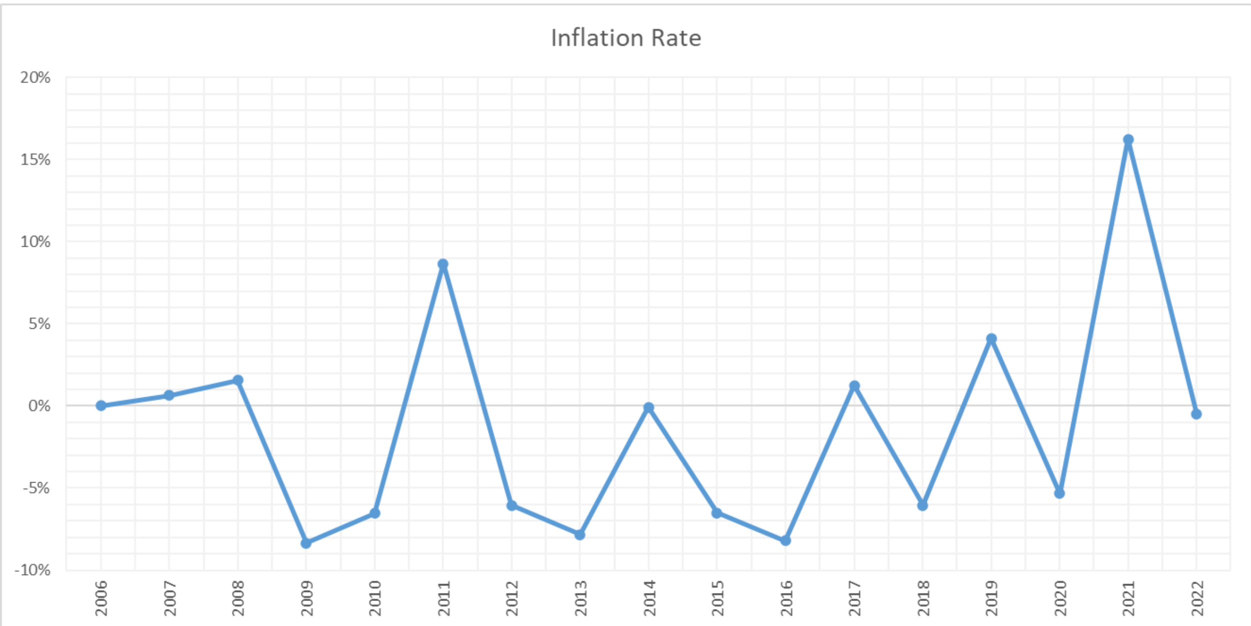


Figure 6.1.1a; Adjusted Inflation rate derived from historical data.

Using the separation method, we obtain the calendar year effects λ_t from the year 2006 to 2022 and specific claims inflation r_t. The average claim inflation in Spain Bodily Injury automobile liability insurance in the time interval 2006 to 2022 was -1.35% and total claims inflation range between -8.65% and 16.24%.

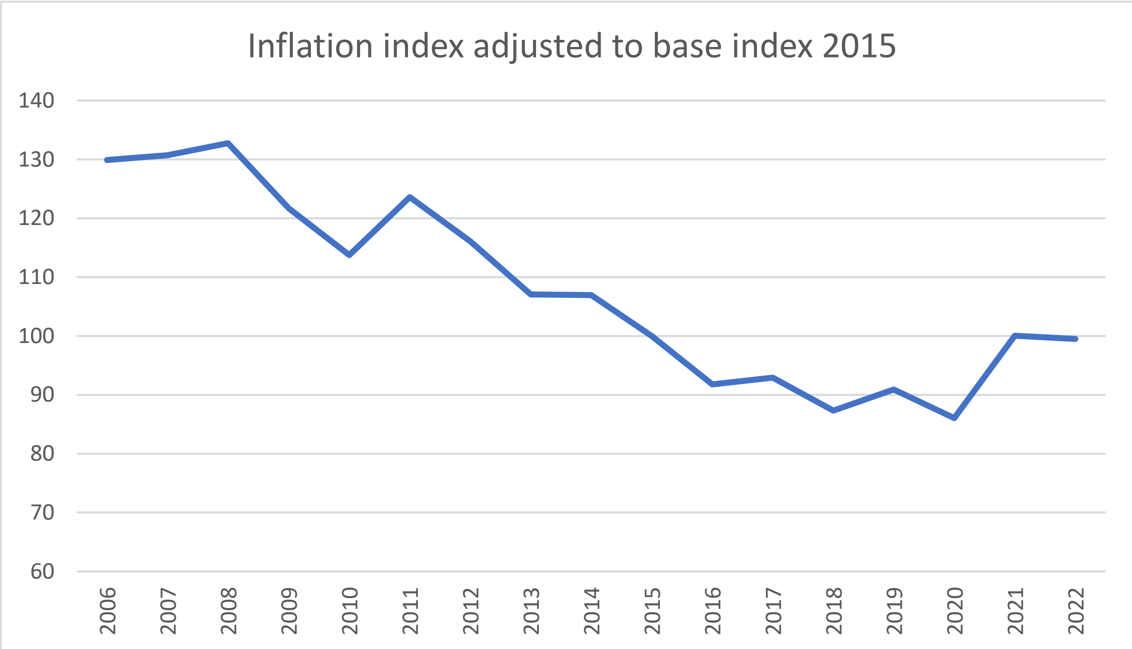


Figure 6.1.1b; Adjusted Inflation Index derived from historical data.

The inflation index adjusted to base index 2015 is given by:

$$\hat{\lambda}_t^{adj} = \frac{\hat{\lambda}_t}{\hat{\lambda}_{2015}}$$

Hence, we get the estimates of the calendar year effects $\hat{\lambda}_t$, which can be interpreted as the average claims of the different calendar years.

This calendar year effect extracted from claim data might deviate from the classical consumer price inflation index. Causes of superimposed inflation comprise legal and legislative changes that alter the average claims payments. In addition, superimposed inflation is generally caused by changing social and medical cost inflation, due to advances in medical technology. Other factors include changes in policy limits, underwriting decision, and so on.

Different economic indices are related to specific claims inflation of different lines of business, and the determination of explanatory economic indices can contribute to an improved understanding of the drivers of claims inflation and hence improve the calculation of claims reserves.

6.1.2 Regression Analysis

A stepwise regression method is selected, where at each stage, a variable may be added to the model or removed from it, to find the relevant economic "driving factors" of claim inflation and select the model that minimizes the Akaike information criterion (AIC). The selection of potential explanatory variables to be included in the stepwise multiple regression depends on the economic drivers which differ with each line of business.

Therefore, following the approach of Bohnert (2015), the multiple regression model can be written as:

$$\lambda_t = c + \sum_{j=1}^{n} \beta_j I_{t,j} + e_t$$

Where λ_t is the calendar year effect as a dependent variable at time $t \in \{1,2, \dots \dots, n\}$, c is the regression constant, β_j is the regression coefficient, $I_{t,j}$ is the value of the explanatory variable j at time $t \in \{1,2, \dots \dots, n\}$ and e_t is the error term with mean 0 and standard deviation σ.

For the Baremo effect on Spain Bodily Injury motor liability insurance, we hypothesize that the following indices have major impact on the claim inflation:

	Index statistics			
Exogenous Variable	Average	Min	Max	Std
Adj_Inf_Rate	108	86	133	16
CPI	99	86	116	7
Avg_Wage_Index	99	94	105	3
GDP_Index	103	88	131	13
Unit_Labour_Cost	103	94	114	6
Labour_Compensation	99	82	112	8
Health_Spending_Index	103	79	148	19
HICP_Hospital_Services	102	98	108	3
HICP_Medical_Prod	102	97	108	4
HICP_Med_ParaMed_Service	99	85	110	6

Table 6.1.2a; Statistics of the indices of difference exogenous variable

The indices mentioned above are the Harmonized Index of Consumer Prices (HICP) in Spain, retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org

Both HICP and CPI are used to measure the changes in the prices over time of buying goods and services, however, there are some differences between the two indices. HICP includes the rural population and excludes owner-occupied housing from its scope while the CPI is the other way around.

```
Call:
lm(formula = Adj_Inf_Rate ~ CPI + +Avg_Wage_Index + +HICP_Hospital_Services,
    data = BI_data)

Residuals:
    Min      1Q  Median      3Q     Max
-12.398  -2.064   1.121   3.758   5.222

Coefficients:
                        Estimate Std. Error t value Pr(>|t|)
(Intercept)             116.9008    59.9502   1.950 0.073082 .
CPI                      -1.1652     0.1986  -5.867 5.53e-05 ***
Avg_Wage_Index           -2.4978     0.5650  -4.421 0.000691 ***
HICP_Hospital_Services    3.4564     0.4921   7.024 9.02e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5.199 on 13 degrees of freedom
Multiple R-squared:  0.9129,    Adjusted R-squared:  0.8928
F-statistic: 45.42 on 3 and 13 DF,  p-value: 3.767e-07
```

Figure 6.1.2a; Estimates of the multiple linear regression model.

The table above shows the result of the final multiple regression model. It can be seen that the CPI, Average Wage Index and HICP_Hospital_Service show a statistically significant relationship with the adjusted claim inflation index. This analysis shows in particular that HICP_Hospital_Services contributes highly to the observed claim inflation index, as progress in medical technology strongly affects the cost for bodily injuries and thus drives claim inflation.

The present parameterization of the multiple regression model leads to an AIC of 109, an adjusted R-Squared of 0.8928, a corresponding p-value for the F-test of 0.00000038 and variance inflation factor (VIF) that rejects multicollinearity.

Variable	CPI	Avg_Wage_index	HICP_Hospital_Services
VIF	1.192	1.4	1.63

Table 6.1.2b; Variance inflation factors for significant explanatory variables.

Hence the chosen indices contribute to observed claim inflation and are thus able to explain a large part of specific claim inflation of Spain motor Bodily Injury.

The explanatory variables have to be predicted into the future before the multiple linear regression model can be used to forecast claim inflation. We use a simplified approach and do not model the explanatory indices $I_{t,j}$ stochastically, but instead use the empirically estimated mean $\overline{I_j}$, which is constant for $t \in \{0,2,\dots\dots,n\}$ and implies less model volatility, reducing the model risk potentially arising due to the aggregation of multiple stochastic processes. Hence the multiple linear regression model used for forecasting claims inflation is given by:

$$\lambda_t = c + \sum_{j=1}^{n} \beta_j \bar{I}_j + e_t$$

Where λ_t denotes the forecasted claims inflation at future time $t \in \{n+1,2,\dots\dots,2n\}$

			Forecasted Inflation Index	
	A	B	C	D = $\hat{\lambda}_t$
AY	Baremo	Impact	Adj_inflation	Inflation
2023	8,50%	5,95%	114,1	3423
2024	4,30%	3,01%	110,9	3328
2025	3,20%	2,24%	110,1	3303
2026	2,00%	1,40%	109,2	3276
2027	1,80%	1,26%	109,1	3272
2028	1,70%	1,19%	109,0	3269
2029	1,70%	1,19%	109,0	3269
2030	1,70%	1,19%	109,0	3269
2031	1,70%	1,19%	109,0	3269
2032	1,70%	1,19%	109,0	3269
2033	1,70%	1,19%	109,0	3269
2034	1,70%	1,19%	109,0	3269
2035	1,70%	1,19%	109,0	3269
2036	1,70%	1,19%	109,0	3269
2037	1,70%	1,19%	109,0	3269
2038	1,70%	1,19%	109,0	3269
2039	1,70%	1,19%	109,0	3269

Table 6.1.2c: Represents the estimates for future inflation index.

From the regression model in Figure 6.1.2a and Table 6.1.2a, we calculate the constant estimate for the inflation index which is given by $c + \sum_{j=1}^{n} \beta_j \bar{I}_j = 107{,}7059$ and we incorporate the Baremo impact on this constant index. The following steps are used to calculate the columns of Table 6.1.2c.

$B = 70\% \times A$

The reason behind this impact in column B is, after the discussions with the Claims Department and leveraging a separate analysis which they completed we have assumed that the impact of Baremo on Spanish Motor Vehicle Bodily Injury Claim is 70% of the published amount, as not all claims are affected in same way by Baremo.

$C = (c + \sum_{j=1}^{n} \beta_j \bar{I}_j)(1 + B)$

$D = (C\hat{\lambda}_{2015})/100$

The Forecasted Increment is given by;

$Z_{ik} = \vartheta_k \lambda_{i+k}; \forall\, i = 0,1, \ldots .n;\ k = 0,1,..,n; n < i + k \leq 2n$

The expected ultimate claim cost for Accident Year i is given by $\sum_{k=0}^{n} Z_{ik}$

The ultimate calculated was then compared to the results obtained by the Company's existing Chain Ladder methodology used at the 2022Q4 reserve review – "2022Q4 Ultimate" and "2022Q4 IBNR".

Hence, we obtain the following table of results:

Accident Year	Ultimate	Paid Claim	Outstanding	IBNR	2022Q4 Ultimate	2022Q4 IBNR
2006	96 592 666	96 592 665	2110	-2 110	96 594 775	0
2007	107 046 585	107 043 067	148 707	-145 191	107 191 775	0
2008	108 797 345	108 792 057	81 530	-76 244	108 873 588	0
2009	115 644 015	115 638 492	399 746	-394 224	116 038 238	0
2010	111 977 206	111 952 794	191 998	-167 587	112 144 793	0
2011	100 938 669	100 887 405	199 821	-148 558	101 087 226	0
2012	88 605 947	88 509 222	241 144	-144 420	88 750 366	0
2013	92 808 976	92 636 332	342 832	-170 189	92 979 165	0
2014	87 223 257	86 934 329	340 053	-51 126	87 274 383	0
2015	83 811 370	83 339 098	1 153 256	-680 985	84 475 821	-16 534
2016	81 745 598	80 891 466	1 621 173	-767 042	82 476 400	-36 239
2017	80 576 612	78 997 407	2 143 595	-564 391	80 994 818	-146 184
2018	80 457 871	77 449 270	4 246 724	-1 238 124	81 304 028	-391 966
2019	82 291 743	76 422 128	7 025 322	-1 155 708	82 465 206	-982 245
2020	62 466 532	54 807 201	10 564 549	-2 905 219	63 711 941	-1 659 810
2021	84 434 004	64 124 449	21 934 950	-1 625 396	80 870 579	-5 188 820
2022	86 203 970	29 756 561	56 518 387	-70 979	78 994 184	-7 280 765
Total	**1 551 622 364**	**1 454 773 953**	**107 155 906**	**-10 307 495**	**1 546 227 296**	**-15 702 563**

Table 6.1.2d: Result from separation method

6.2 Inflation Adjusted Chain Ladder Method

The Chain Ladder method is a traditional method and is a popular way for insurance companies to estimate their required claim reserves. As per Cristian (2015) and Richard (1979), the underlying assumption of the chain ladder method is that past claims experience is a good predictor of future outcomes, which is reliable in the case of a stable environment. Adjusting for inflation in the Chain Ladder method requires the estimation of the inflation element implicit in the past data on the claim amounts and adjusting the past data with that inflation element. The future projections of data are then considered with suitable inflation assumptions. One possibility is to compare the data against some suitable index of inflation.

Of such indexes, the Retail Price Index (RPI) is the best known but will often not be the most apt one for insurance reserving. It is better to seek an index with some more direct connection with the line of business in hand. We can use Spain CPI to adjust the historical data, and our future inflation assumption is based on our knowledge that there is a linked between the Baremo adjustment and Spain CPI.

Assume a run-off triangle of incremental claims Z_{ik} relating to accident year i and delay year k $\forall$ $i, k = 0,1,2, \dots, n$ and for future years $n < i + k \leq 2n$. We start with adjusting the historical data in the run-off triangle with past inflation using the inflation adjustment factor.

Year	0	1	.	n-1	n	n+1	.	2n-1	2n
Inflation	i_0	i_1	.	i_{n-1}	i_n	i_{n+1}	.	i_{2n-1}	i_{2n}
Inflation adjustment	a_0	a_1	.	a_{n-1}	$a_n = 1$	a_{n+1}	.	a_{2n-1}	a_{2n}

$$Where\ a_t = \begin{cases} \prod_{m=t}^{n-1}(1+i_m)\,;\ t = 0,1,\dots n \\ \prod_{m=n}^{t}(1+i_m)\,;\ t = n+1, n+2, ..2n \end{cases} \quad \forall\, t = i+k$$

is an inflation adjustment factor, which adjusts the past claim amount to the present and inflates the forecasted claim amount with the assumed future inflation.

Therefore, the inflation triangle is given by:

Year of Origin	Development Year								
	0	1	2	.	k	.	.	n-1	n
0	a_0	a_1	a_2	.	.	.	.	a_{n-1}	1
1	a_1	a_2	a_3	.	.	.	.	1	a_{n+1}
2	a_2	a_3	a_4	.	.	.	.	a_{n+1}	a_{n+2}
.	.	.	.	.	.	.	.	a_{n+2}	a_{n+3}
i	.	.	.	.	1	.	.	.	.
.	.	.	.	.	a_{n+1}	.	.		.
.	.	.	.	.	.	.			.
n-1	a_{n-1}	1	a_{n+1}	a_{n+2}	.				.
n	1	a_{n+1}	a_{n+2}	a_{n+3}	.	.	.	a_{2n-1}	a_{2n}

As noted previously, the adjustment factor of 70% was selected by leveraging an analysis completed by the Claims Department. There is a high correlation of 0,98 between CPI inflation with a year delay and Baremo rates, i.e. comparing CPI inflation from 2016 to 2022 with Baremo rates from 2017 to 2023. The Baremo came into effect in Jan 2016, and we observed an impact of -35% on claim cost in 2016. In the pre-Baremo period from 2006 to 2015, the claim amount is impacted by economic inflation, however, assuming that there are no impacts from court proceedings on development patterns. The Baremo assumption in the interval 2024-2039 is based on a forecast for Spanish CPI produce by the International Monetary Fund; https://www.imf.org/en/Countries/ESP

AY	CPI Inflation	BAREMO	BAREMO Impact	i_t	a_t
2006	3,52%	0,00%	0,00%	3,52%	0,8
2007	2,79%	0,00%	0,00%	2,79%	0,78
2008	4,08%	0,00%	0,00%	4,08%	0,75
2009	-0,29%	0,00%	0,00%	-0,29%	0,75
2010	1,80%	0,00%	0,00%	1,80%	0,74
2011	3,20%	0,00%	0,00%	3,20%	0,72
2012	2,45%	0,00%	0,00%	2,45%	0,7
2013	1,41%	0,00%	0,00%	1,41%	0,69
2014	-0,15%	0,00%	0,00%	-0,15%	0,69
2015	-0,50%	0,00%	0,00%	-0,50%	0,69
2016	-0,20%	0,00%	0,00%	-35,00%	1,07
2017	1,96%	0,25%	0,18%	0,18%	1,07
2018	1,67%	1,60%	1,12%	1,12%	1,05
2019	0,70%	1,60%	1,12%	1,12%	1,04
2020	-0,32%	0,90%	0,63%	0,63%	1,04
2021	3,09%	0,90%	0,63%	0,63%	1,03
2022	8,39%	4,13%	2,89%	2,89%	1
Projections					
2023	4,30%	8,50%	5,95%	5,95%	1,06
2024	3,20%	4,30%	3,01%	3,01%	1,09
2025	2,00%	3,20%	2,24%	2,24%	1,12
2026	1,80%	2,00%	1,40%	1,40%	1,13
2027	1,70%	1,80%	1,26%	1,26%	1,15
2028	1,70%	1,70%	1,19%	1,19%	1,16
2029	1,70%	1,70%	1,19%	1,19%	1,17
2030	1,70%	1,70%	1,19%	1,19%	1,19
2031	1,70%	1,70%	1,19%	1,19%	1,20
2032	1,70%	1,70%	1,19%	1,19%	1,22
2033	1,70%	1,70%	1,19%	1,19%	1,23
2034	1,70%	1,70%	1,19%	1,19%	1,24
2035	1,70%	1,70%	1,19%	1,19%	1,26
2036	1,70%	1,70%	1,19%	1,19%	1,27
2037	1,70%	1,70%	1,19%	1,19%	1,29
2038	1,70%	1,70%	1,19%	1,19%	1,30
2039	1,70%	1,70%	1,19%	1,19%	1,32

Table 6.2a; Inflation assumptions.

The high Baremo rate observed for 2022 and forecasted for 2023 and 2024 is a result of the current high inflation environment being experienced post-Coronavirus.

The adjusted incremental claim is then given by $Z_{ik}' = Z_{ik} a_{i+k}$

The statistical approach uses the incremental claims, but the inflation adjusted chain ladder technique is applied to the inflated cumulative claims, which are defined by:

$$C_{ik} = \sum_{j=0}^{k} Z'_{ij}$$

The chain ladder technique assumes that the cumulative claims for each accident year develop similarly by delay year, and estimates development factors as a ratio of cumulative claims with the same delay index. However, instead of using all years, sometimes it is beneficial to give credibility to the payment pattern of recent years. Since "New Baremo" came into effect in 2016, it would be expected that the development factor should incorporate the Baremo effect from 2016 to the latest year, i.e. 2022. In the table below the volume means the number of years used to calculate the development factor. The estimate of the development factor for development year k with volume v is:

$$\psi_k^v = \frac{\sum_{i=n-k-v}^{n-k} C_{ik}}{\sum_{i=n-k-v}^{n-k} C_{i\,k-1}} \quad \forall\, k = 1,3, \dots\dots, n\,;\, v = 1,2, \dots, n-k$$

From the graphs of Development and Cumulative Development Factors, it can be observed that the factors are similar for different volumes during the development period. However, given the size of the dataset and the materiality of the coverages like bodily injury, a slight variation has a large impact on the ultimate claim cost. One must do careful analysis using reserving diagnostics such as the Paid/Incurred ratio, reported claim severity and recent payment pattern to select appropriate development factors. Curves like Weibull and exponential are also used to smoothen the development factors. Sometimes manual adjustment in development factors is also needed.

Development Factor					
Development	Vol all	Vol 12	Vol 7	Vol 6	Vol 5
1 – 2	2,146	2,140	2,143	2,102	2,105
2 – 3	1,146	1,153	1,164	1,157	1,150
3 – 4	1,055	1,058	1,066	1,063	1,059
4 – 5	1,029	1,029	1,034	1,032	1,032
5 – 6	1,015	1,015	1,016	1,016	1,014
6 – 7	1,007	1,007	1,008	1,008	1,008
7 – 8	1,004	1,004	1,005	1,005	1,005
8 – 9	1,003	1,003	1,003	1,002	1,001
9 – 10	1,001	1,001	1,001	1,001	1,001
10 – 11	1,001	1,001	1,001	1,001	1,001
11 – 12	1,001	1,001	1,001	1,001	1,001
12 – 13	1,000	1,000	1,000	1,000	1,000
13 – 14	1,000	1,000	1,000	1,000	1,000
14 – 15	1,000	1,000	1,000	1,000	1,000
15 – 16	1,000	1,000	1,000	1,000	1,000
16 – 17	1,000	1,000	1,000	1,000	1,000

Table 6.2b; Represent Development Factors with different volumes.

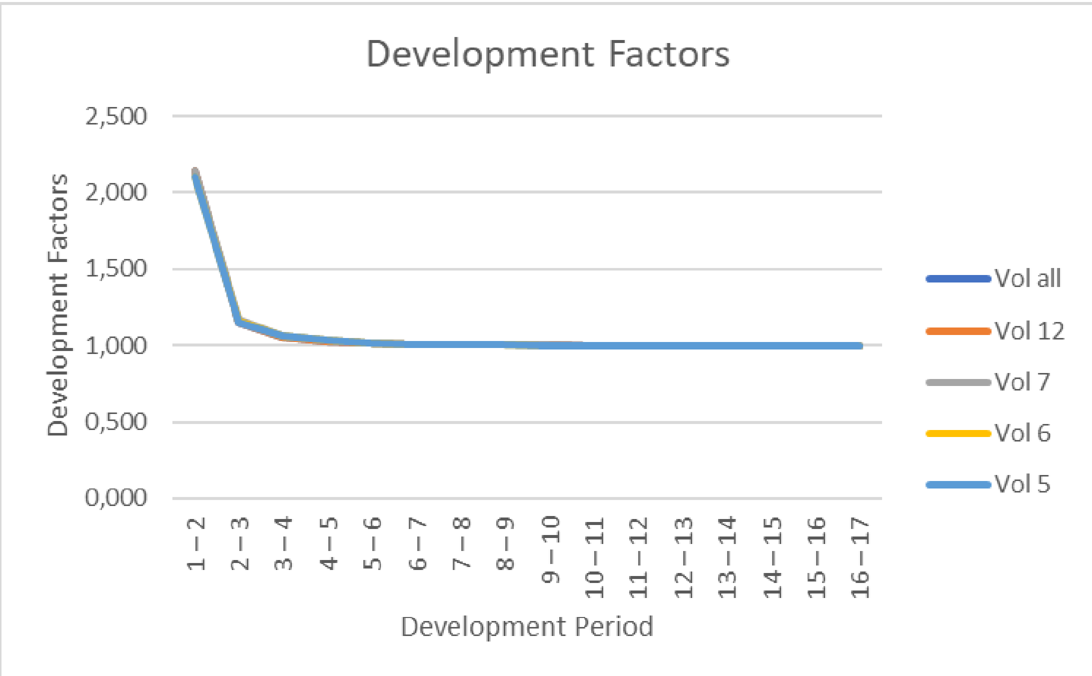


Figure 6.2a; Represent the graph of Development Factors with different volumes.

For example, if a large claim movement has distorted the development factor and is not expected to repeat. Here for simplicity, we will be using *Vol All, Vol 12, Vol 7, Vol 6* and *Vol* 5 to develop the ultimate claim cost.

Table 6.2c; Represent Cumulative Development Factors with different volumes.

Cumulative Development Factors					
Development	**Vol all**	**Vol 12**	**Vol 7**	**Vol 6**	**Vol 5**
1–- 2	2,759	2,779	2,850	2,761	2,728
2–- 3	1,286	1,298	1,330	1,313	1,296
3–- 4	1,122	1,126	1,142	1,135	1,127
4–- 5	1,063	1,064	1,071	1,068	1,064
5–- 6	1,033	1,033	1,036	1,034	1,032
6–- 7	1,018	1,018	1,020	1,018	1,017
7–- 8	1,010	1,010	1,012	1,010	1,010
8–- 9	1,006	1,006	1,006	1,005	1,005
9–- 10	1,004	1,004	1,004	1,003	1,003
10–- 11	1,002	1,002	1,002	1,002	1,002
11–- 12	1,001	1,001	1,001	1,001	1,002
12–- 13	1,001	1,001	1,001	1,001	1,001
13–- 14	1,001	1,001	1,001	1,001	1,001
14–- 15	1,000	1,000	1,000	1,000	1,000
15–- 16	1,000	1,000	1,000	1,000	1,000
16–- 17	1,000	1,000	1,000	1,000	1,000

The Cumulative Development Factors are given by:

$$\omega_k^v = \prod_{j=k}^{n} \psi_j^v \quad \forall\, k = 1,3, \ldots., n$$

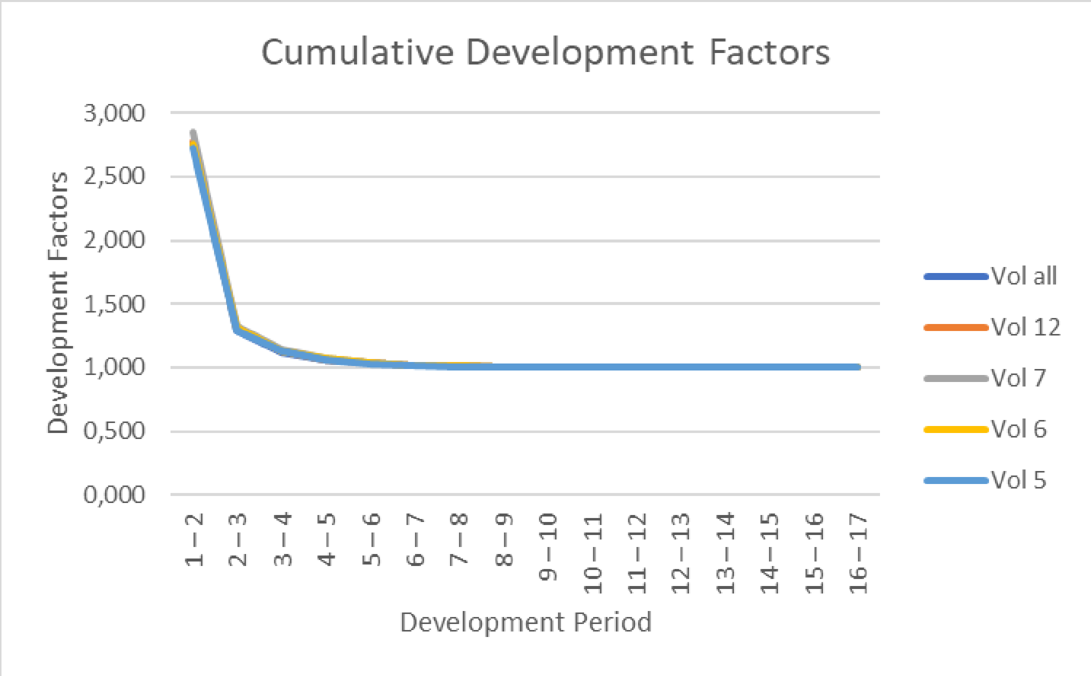


Figure 6.2b; Represent the graph of Cumulative Development Factors with different volumes.

Hence the model is given by;

$E(C_{ik} | C_{i1}, C_{i2}, \ldots\ldots\ldots\ldots, C_{i\,k-1}) = \lambda_k^v C_{i\,k-1}$; k=1,2,3,..........,n

The increment that incorporated the past inflation is given by:

$X_{ik} = C_{ik} - C_{i\,k-1}$;{ i=0,......,n; k=1,3,......,n}

The projected increments need to be adjusted with future inflation assumption.

Then Baremo adjusted forecasted increments are given by:

$$X_{ik}^{'} = X_{ik} \, . \, a_{i+k} \; ; \forall \; n < i + k \leq 2n \, ; i = 0,1, \ldots, n; k = 0,1, \ldots, n$$

Ultimate Claim Cost with different volume					
AY	Vol All	Vol 12	Vol 7	Vol 6	Vol 5
2006	96 592 666	96 592 666	96 592 666	96 592 666	96 592 666
2007	107 046 205	107 046 205	107 046 205	107 046 205	107 046 205
2008	108 814 619	108 814 619	108 814 619	108 814 619	108 814 619
2009	115 676 525	115 676 525	115 676 525	115 676 525	115 676 525
2010	112 005 375	112 005 375	112 005 375	112 005 375	112 005 375
2011	100 958 749	100 958 749	100 958 749	100 958 749	100 958 749
2012	88 607 935	88 607 935	88 607 935	88 607 935	88 617 929
2013	92 797 099	92 797 099	92 797 099	92 795 130	92 811 218
2014	87 194 584	87 194 584	87 196 542	87 171 782	87 178 337
2015	83 853 032	83 853 032	83 861 706	83 792 615	83 737 517
2016	81 864 254	81 864 254	81 976 870	81 852 769	81 788 442
2017	80 623 789	80 623 789	80 802 195	80 649 342	80 576 512
2018	80 390 036	80 390 036	80 664 064	80 478 051	80 254 362
2019	81 851 277	81 908 946	82 578 014	82 266 497	81 972 376
2020	62 287 188	62 525 105	63 539 957	63 083 269	62 588 084
2021	84 294 253	85 180 901	87 417 930	86 253 695	84 995 429
2022	86 163 831	86 824 733	89 185 326	86 304 225	85 201 571
Total	1 551 021 416	1 552 864 552	1 559 721 778	1 554 349 447	1 550 815 917

Table 6.2d; Represent the expected ultimate claim cost calculated annually with different volumes.

Where the expected ultimate claim cost with year of origin i is given by:

$$ULT_i = \begin{cases} \displaystyle\sum_{k=0}^{n} Z_{ik} \ ; i = 0 \\ \displaystyle\sum_{k=0}^{n-i} Z_{ik} + \sum_{k=n-i+1}^{n} X_{ik}' \ ; i = 1,3, \ldots\ldots, n \end{cases}$$

In Liberty Seguros, for the Spain Motor line of business, the Bodily Injury claims arise from different channels and the analysis was performed separately in these channels to come up with the final figures. Here in the Inflation Adjusted Chain Ladder approach we complete the analysis at a total level and apply the overall pattern to each individual channel. We also applied the same approach to quarterly data for each channel with the same annual inflation used before, however, assuming the uniform assumption of annual inflation for the pre-Baremo period. From the period 2016 onwards, Baremo has a step change inflation effect. For the first quarter (Q1) there is a step change for the rest of the quarters (Q2 to Q4) the inflation is zero. The reason for this is the timing of the updates made to the Baremo tables which occurs in January of each year. Also, the quarterly inflation is given by $i_{quarterly} = (1 + i_{annual})^{\frac{1}{4}} - 1$. The logic behind translating the all channels combined Annual-Annual analysis into an individual channel Quarterly-Quarterly analysis is to make the comparison of results to Liberty's current methods more appropriate, with the view of eventually incorporating this work as a new method.

Using the quarterly approach for each channel the following table of expected ultimate claim cost is obtained.

Ultimate Claim Cost with different volume					
AY	Vol All	Vol 12	Vol 7	Vol 6	Vol 5
2006	96 592 666	96 592 666	96 592 666	96 592 666	96 592 666
2007	107 047 107	107 047 107	107 047 107	107 047 107	107 047 107
2008	108 814 159	108 814 159	108 814 159	108 814 159	108 814 159
2009	115 673 852	115 673 852	115 673 852	115 673 852	115 673 852
2010	112 008 264	112 008 264	112 008 264	112 008 264	112 008 264
2011	100 962 391	100 962 391	100 962 391	100 962 391	100 961 508
2012	88 597 375	88 597 375	88 597 375	88 597 110	88 606 191
2013	92 803 619	92 803 619	92 801 566	92 794 973	92 804 475
2014	87 186 604	87 186 604	87 184 167	87 168 526	87 166 712
2015	83 815 899	83 815 899	83 837 781	83 772 061	83 736 993
2016	81 856 930	81 856 930	81 965 225	81 850 601	81 796 631
2017	80 660 031	80 660 031	80 832 761	80 676 340	80 604 687
2018	80 467 719	80 473 382	80 727 805	80 574 834	80 375 565
2019	81 900 806	81 963 538	82 547 235	82 272 008	81 971 353
2020	62 582 413	62 809 103	63 694 640	63 242 449	62 804 118
2021	84 899 665	85 650 041	87 604 551	86 523 818	85 390 634
2022	87 607 343	85 181 009	83 583 913	82 383 580	81 772 126
Total	1 553 476 843	1 552 095 970	1 554 475 457	1 550 954 740	1 548 127 040

Table 6.2e; Represent the expected ultimate claim cost calculated quarterly with different volumes.

6.3 Log Linear Model

Kremer (1982) study demonstrated that the chain ladder technique can be considered analogous to the application of a two-way analysis of variance model to the logarithmically transformed incremental claims. This allows for a direct comparison between these two approaches. The Chain Ladder technique relies on cumulative claims in its calculations, seemingly capable of accommodating negative incremental claims without issues.

Consider a run-off triangle, with incremental claim amount $Z_{i,k}$ of accident year i and development year k, where $i\, k, \in\, \nabla\, = \{i\, = 0, \dots, n;\ k = 0, \dots, n - i + 1\}$ refers to the upper triangle. Here we use the same inflation assumption that we used in the "Inflation Adjusted Chain Ladder Method" and keep the same symbol that we used to represent the inflation table. We begin with adjusting the increments with historical inflation. So, the past inflation adjusted incremental claim is given by:

$$Z'_{ik} = Z_{ik} a_{i+k}$$

Following Verrall (1993), let´s define $Y_{ik} = \log(Z'_{ik})$, assuming for now that $\log(Z_{ik})$ exist, then the linear model takes the form $Y_{ik} = X_{ik}\beta + \varepsilon_{ij}$

Where β represents a vector of parameters, X_{ik} corresponds to a row from design matrix, ε_{ij} signifies an error with zero mean. Typically, ε_{ij} are presumed to be independently and identically distributed with variance σ^2, although this distributional assumption may be relaxed.

If the triangle of the data $\{Y_{ik}; i = 1,2 \ldots, n; k = 1,2, \ldots . n - i + 1\}$ is expressed as a vector, the model can be written as $Y = X\beta + \varepsilon$

Of all other models which can be cast in this form, the Chain Ladder is most widely used, which has the form: $E(Y_{ik}) = \mu + \alpha_i + \beta_k + \varepsilon_{ik}$

where μ is the overall mean, α_i is an accident year effect, and β_k is a development year effect. The usual restriction is placed on the parameters to ensure a non-singular design matrix, in this case $\alpha_1 = \beta_1 = 0$

As an example, for a 3 x 3 triangle of incremental claims:

Z'_{11}	Z'_{12}	Z'_{13}
Z'_{21}	Z'_{22}	
Z'_{31}		

The model for the triangle after taking the log of the data is:

$$\begin{bmatrix} Y_{11} \\ Y_{12} \\ Y_{21} \\ Y_{13} \\ Y_{22} \\ Y_{31} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mu \\ \alpha_2 \\ \alpha_3 \\ \beta_2 \\ \beta_3 \end{bmatrix} + \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \varepsilon_{21} \\ \varepsilon_{13} \\ \varepsilon_{22} \\ \varepsilon_{31} \end{bmatrix}$$

We start with considering the standard two parameter lognormal distribution with density function is given by:

$$f(z) = \frac{1}{(z)\sqrt{2\pi\sigma^2}} \exp\{-\frac{1}{2\sigma^2}(\log z - \mu)^2\}$$

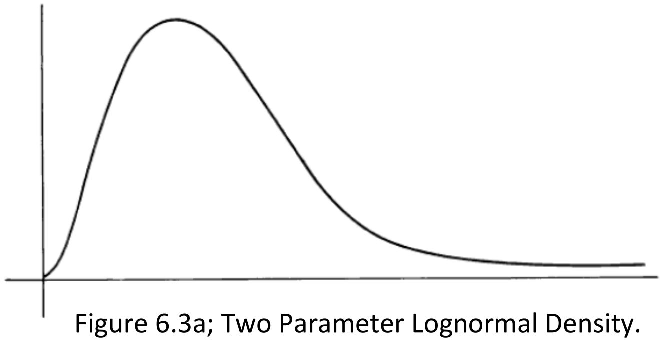


Figure 6.3a; Two Parameter Lognormal Density.

The maximum likelihood estimates of the parameters are given by:

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} \log z_i \quad and \quad \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n} (\log z_i - \hat{\mu})^2$$

It can be seen that $\hat{\mu} = \log(\prod_{i=1}^{n} z_i)^{\frac{1}{n}}$. Kremer (1982) and Verrall (1991) mentioned that, if the geometric means $(\prod_{i=1}^{n} z_i)^{\frac{1}{n}}$ is replace by arithmetic means $\frac{1}{n}\sum_{i=1}^{n} z_i$, the outcome will be similar to those obtained using the chain ladder technique. The structure of the models is identical, the only difference is the estimation technique.

Apart from this Chain Ladder linear model, other methods are also suggested which are suitable for the claim data, including a gamma curve suggested by Zehnwirth (1985) and an exponential tail suggested by Ajne (1989) in which the first few years is followed by Chain Ladder and the later years follow an exponential curve.

When data contains negative values because of overestimation of claim amounts, recoveries, etc., to avoid problems with taking logarithms of negative values, we use the following steps.

- Select a suitable large constant τ.
- Adding τ to all the incremental claims Z'_{ik}, so that the $\log Z'_{ik}$ exist.
- We apply the model to the $\log(Z'_{ik} + \tau)$ and estimate the outstanding clams.
- Subtract the τ from all the estimates and the forecasted claims.

The reformed model becomes the three-parameter lognormal model after adding the large suitable constant τ in claim amounts. This is equivalent to shifting the lognormal distribution. The density function of the three-parameter lognormal distribution is given by:

$$f(z'_{ik}) = \frac{1}{(z'_{ik+\tau})\sqrt{2\pi\sigma^2}} exp\{-\frac{1}{2\sigma^2}[\log(z'_{ik+\tau}) - X_{ik}\beta]^2\}$$

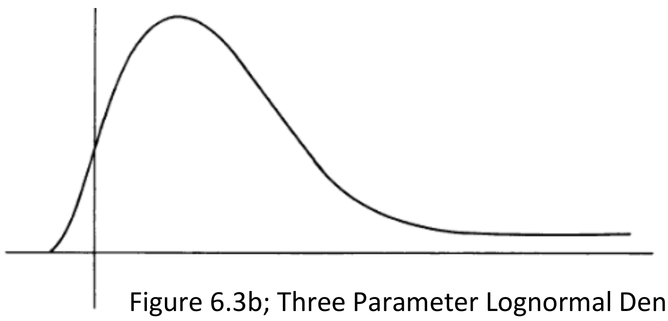


Figure 6.3b; Three Parameter Lognormal Density.

Now we redefine the $Y_{ik} = \log(Z'_{ik} + \tau)$:

The likelihood function is given by:

$$\frac{1}{(2\pi\sigma^2)^{N/2}\prod_{i,k}(Z'_{ik} + \tau)} exp\{-\frac{1}{2\sigma^2}(y - X\beta)^T(y - X\beta)\}$$

Where N =n(n+1)/2 is the number of observations in the triangle, after taking the log, the loglikelihood function is:

$$L = -\frac{N}{2}\log(2\pi\sigma^2) - \sum_{i,k}\log(Z'_{ik} + \tau) - \frac{1}{2\sigma^2}(y - X\beta)^T(y - X\beta)$$

Differentiating L with respect to β and σ^2 gives maximum likelihood estimates ostensibly in the same form as before:

$$X^T X\hat{\beta} = X^T Y$$

$$\hat{\sigma}^2 = \frac{1}{N}(Y - X\hat{\beta})^T(Y - X\hat{\beta})$$

Also, differentiating with respect to ζ gives the following likelihood equation for τ:

$$\hat{\sigma}^2 \sum_{i,k} \frac{1}{(Z'_{ik} + \hat{\tau})} - \frac{1}{\hat{\sigma}^2} \sum_{i,k} \frac{Y_{ik} - X_{ik}\hat{\beta}}{(Z'_{ik} + \hat{\tau})} = 0$$

After estimating the parameters, we got the following tables:

i	α_i	j	β_j
2	0,13	2	0,14
3	0,14	3	-1,15
4	0,22	4	-1,97
5	0,13	5	-2,54
6	0,04	6	-3,07
7	0,00	7	-3,65
8	0,11	8	-4,12
9	0,11	9	-4,39
10	0,15	10	-4,63
11	0,22	11	-4,81
12	0,23	12	-4,91
13	0,29	13	-4,98
14	0,25	14	-5,03
15	-0,08	15	-5,02
16	0,22	16	-4,96
17	0,20	17	-4,98

u	σ	τ
17,01	0,45	165668

Table 6.3a; Represent the estimates of three parameter log normal model.

Hence the incremental claim after is given by:

$$X_{ik} = \exp\{E(Y_{ik})\} - \tau;\ \forall\ i = 2,3, \ldots n; k = n - i + 2, \ldots., n$$

Now we adjust the incremental with Baremo impact inflation assumption that we assumed in Chain Ladder method, the new Baremo inflation adjusted incremental claim amount is given by:

$$X'_{ik} = a_{i+k}X_{ik}\ ;\ \forall\ i = 2,3, \ldots n; k = n - i + 2, \ldots., n$$

Hence the ultimate claim cost for accident year i is given by:

$$ULT_i = \begin{cases} \sum_{k=1}^{n-i+1} Z_{ik} & ; i = 1 \\ \sum_{k=1}^{n-i+1} Z_{ik} + \sum_{k=n-i+2}^{n} X'_{ik} & ; i = 2,3,\ldots\ldots,n \end{cases}$$

The ultimate claim cost is sensitive to parameter τ.

AY	$\tau = 20709$	$\tau = 41417$	$\tau = 82834$	$\tau = 165668$	$\tau = 2070845$
2006	96 592 666	96 592 666	96 592 666	96 592 666	96 592 666
2007	107 077 349	107 057 402	107 062 795	107 071 336	107 128 451
2008	108 871 240	108 835 150	108 843 452	108 856 185	108 941 896
2009	115 747 188	115 708 766	115 734 907	115 779 022	116 158 416
2010	112 060 732	112 011 867	112 024 068	112 048 942	112 320 198
2011	101 005 736	100 942 330	100 934 964	100 924 819	100 748 697
2012	88 553 074	88 546 647	88 538 290	88 525 443	88 101 291
2013	92 780 590	92 795 359	92 814 489	92 836 605	92 768 655
2014	87 157 011	87 176 544	87 207 599	87 254 275	87 413 941
2015	83 764 350	83 797 848	83 850 617	83 936 554	85 065 897
2016	81 687 157	81 745 731	81 843 184	82 005 856	84 257 847
2017	80 474 559	80 539 877	80 648 968	80 829 534	82 973 756
2018	80 490 582	80 581 796	80 740 597	81 015 989	84 580 368
2019	81 812 078	81 892 518	82 031 586	82 272 694	85 782 963
2020	61 860 425	61 829 407	61 752 527	61 575 770	57 127 557
2021	83 999 878	84 076 437	84 208 366	84 437 271	88 100 195
2022	85 911 439	85 980 659	86 098 066	86 298 244	89 341 695
Total	1 549 846 054	1 550 111 002	1 550 927 141	1 552 261 204	1 567 404 487

Table 6.3b; Represent the expected ultimate claim cost using different τ .

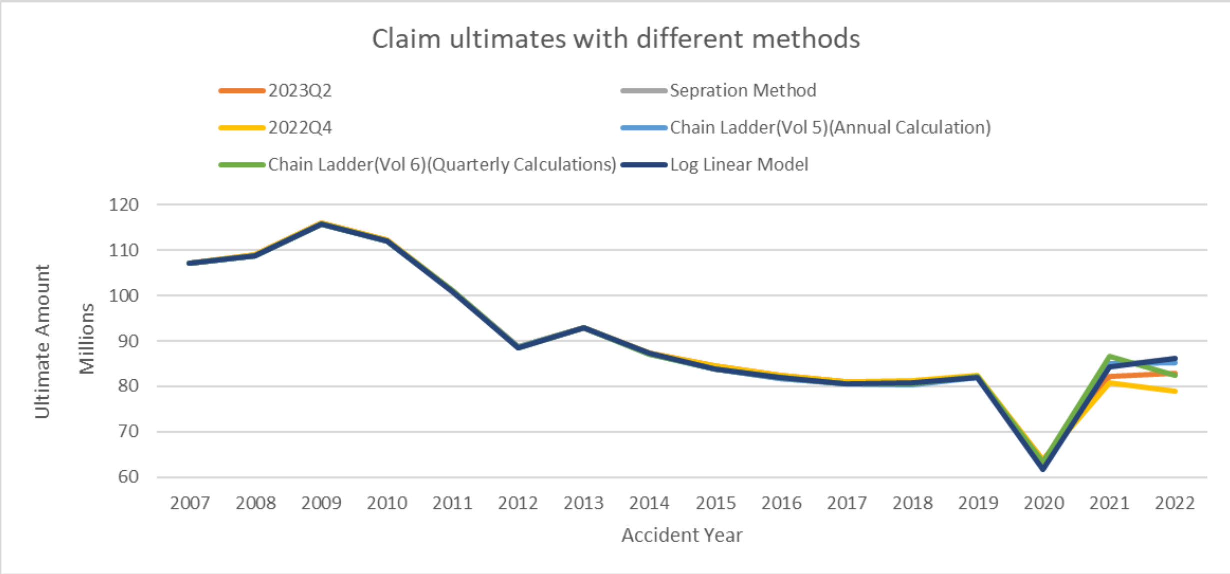
Conclusion

The credibility of ultimate claim cost estimates developed from different methods depends on whether the assumption used in the methods holds strongly or weakly in the real world conditions it is being applied to and sensitivity to the method itself. We shall compare the results obtained from different methods with the results we got from the analysis of the year-end 2022 (“2022Q4”) and the second quarter of the year 2023 (“2023Q2”) reserve reviews. Liberty Seguros use the software ResQ and follows the process for the quarterly reserve review. This process involves using diagnostic tools, such as frequency/severity analysis, Paid/Incurred ratio, and manual adjustment of the development factors. Smoothing techniques, including Weibull, and exponential curves are applied to these development factors. The final ultimate comes as the weighted average of different methods. Later, for the 2022Q4 estimate, the expected impact of the Baremo update was adjusted for in the Ultimate claim cost by adding €10 million to the estimate calculated in ResQ. This €10 million was an estimate arrived at by the Claims and Actuarial Departments in the absence of a more sophisticated reserving approach, knowing that the current methods would under-project and that there would be a very large Baremo increase in January 2023. It is important to highlight that in the 2023Q2 reserving data, the full 2023 Baremo impact is known and included in the case reserves, which should result in a much more accurate estimate when compared to 2022Q4. The final impact was much less than the initial €10 million estimate and there was no manual addition made to the 2023Q2 estimate.

Here in this internship report, we used a simplified process and tried to incorporate the inflation impact in the methodologies themselves. However, the level of flexibility decreases as the method was applied to the total annual data of different channels for Bodily Injury Coverage, assuming the claim development pattern is similar for all the channels for Bodily Injury Coverage.

Also, to compare the results we assume the Accident Year 2006 is completely developed and the following table compared the results for Accident Year 2007 to 2022.

AY	2023Q2	Sepration Method	2022Q4	Chain Ladder(Vol 5) Annual	Chain Ladder(Vol 6) Quarterly	τ=165668
2007	107 188 260	107 046 585	107 191 776	107 046 205	107 047 107	107 062 795
2008	108 867 141	108 797 345	108 873 588	108 814 619	108 814 159	108 843 452
2009	115 982 640	115 644 015	116 038 239	115 676 525	115 673 852	115 734 907
2010	112 129 391	111 977 206	112 144 793	112 005 375	112 008 264	112 024 068
2011	101 083 434	100 938 669	101 087 227	100 958 749	100 962 391	100 934 964
2012	88 748 781	88 605 947	88 750 367	88 617 929	88 597 110	88 538 290
2013	92 965 108	92 808 976	92 979 165	92 811 218	92 794 973	92 814 489
2014	87 239 375	87 223 257	87 274 383	87 178 337	87 168 526	87 207 599
2015	84 485 615	83 811 370	84 475 821	83 737 517	83 772 061	83 850 617
2016	82 378 158	81 745 598	82 476 401	81 788 442	81 850 601	81 843 184
2017	81 133 855	80 576 612	80 994 819	80 576 512	80 676 340	80 648 968
2018	81 131 285	80 457 871	81 304 029	80 254 362	80 574 834	80 740 597
2019	82 287 278	82 291 743	82 465 206	81 972 376	82 272 008	82 031 586
2020	63 826 452	62 466 532	63 711 941	62 588 084	63 242 449	61 752 527
2021	82 276 223	84 434 004	80 870 580	84 995 429	86 523 818	84 208 366
2022	82 871 381	86 203 970	78 994 185	85 201 571	82 383 580	86 098 066
Total	**1 454 594 376**	**1 455 029 700**	**1 449 632 520**	**1 454 223 250**	**1 454 362 073**	**1 454 334 475**
Difference from 2023Q2		**435 324**	**-4 961 856**	**-371 126**	**-232 303**	**-259 901**



Separation Method: The Separation Method results in an ultimate claim cost estimate which is €435 thousand more than the 2023Q2 reserve, and when comparing with 2022Q4 ultimate claim cost after adding the 10 million Baremo adjustment, the difference is 5 million.

Inflation Adjusted Chain Ladder: The Inflation adjusted Chain ladder method also gives a similar result for the development factors calculated with different volumes. Among others, the development factor calculated using Volume 5 gives a good result in the ultimate claim cost that differs from 2023Q2 by -€371 thousand. However, when the analysis was performed on quarterly data on different channels separately, the development factor calculated using Volume 6 give the ultimate claim cost that differ from 2023Q2 by -€232 thousand, which is expected from the Volume 6 development factor, i.e. from the year 2016 to the latest year, since the Spanish Baremo came into effect on January 2016, ideally our development factor should incorporate the Baremo impact from 2016 and reflect in the future projections.

Log-linear Model: In the Log-linear model, the choice of the constant τ is important in estimating the ultimate claim amount. As mentioned in Verrall (1993), the estimation of the parameters can be performed effectively in a statistical package such as GLIM (see Renshaw, 1989), or in a spreadsheet package such as SuperCalc5 (see Christofides, 1990). However, both of these packages are obsolete. Here, we choose the constant based on the negative incremental claim, so the τ is proportional to the highest absolute value of the negative incremental claim, such that the Logarithm of that value exists. However, for different values of τ , the ultimate claim costs are not dramatically different. For the choice of τ as the absolute value of most negative incremental claim, the ultimate claim cost result in €260k less that the 2023Q2.

So far, all the methods gave satisfactory results. However, among these methods, inflation adjusted chain ladder has a high potential to give a the more credible result after modifying the algorithm and allowing for more flexibility, or a weighted average of the method using a statistical approach and point estimates can be used. While it is evident that all methods yield similar annual results, it´s noteworthy that Liberty Seguros conducts quarterly reserve reviews. From a stochastic perspective, a comprehensive annual analysis at year-end may be advantageous to access whether all three models consistently produce similar results or if any of them exhibit patterns that raise substantial concerns.

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