Lisbon School of Economics & Management
Universidade de Lisboa

MASTER OF SCIENCE IN

FINANCE

MASTER'S FINAL WORK

PROJECT

PRIIP ANALYSIS:

9.2% P.A. BARRIER REVERSE CONVERTIBLE IN EUR ON LUFTHANSA

MARTIM FAUSTINO BOTELHO DE GRINÉ SEVERINO

JUNE – 2026

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PROFESSOR MARIA KOSOLAPOVA

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Abstract

This project analyses the *9.2% p.a. Barrier Reverse Convertible in EUR on Lufthansa*, a callable yield-enhancement structured product issued by Leonteq Securities AG and linked to Deutsche Lufthansa AG shares. The objective is to estimate its theoretical value and evaluate its risk-return profile.

The product is decomposed into a zero-coupon bond, a down-and-in put option, an issuer call option and a stream of fixed coupons. Its path-dependent payoff is valued through a Monte Carlo simulation of 100.000 Lufthansa share-price paths. The analysis also examines possible investment outcomes, sensitivity to market variables and contractual features, time decay, Greeks, tail risk and embedded costs.

Under a risk-neutral framework, the pricing model estimates a default-free theoretical value of 967,41€ and a credit-risk/issuer-funding-adjusted value of 959,89€. Relative to the 1.000€ issue price, these estimates imply embedded margins of 3,26% and 4,01%, respectively. Under real-world assumptions, the estimated annualized return is 2,70%, corresponding to an expected excess return of only 0,31 percentage points above the risk-free rate.

The primary risk is a substantial capital loss if the barrier is breached and Lufthansa's final share price is below the strike, in which case the denomination is replaced by the delivery of Lufthansa shares. Although approximately 81% of the simulated outcomes generate a positive return, the 95% Value-at-Risk is 264,87€ and the corresponding Conditional Expected Shortfall is 341,30€. The product performs most favourably under neutral-to-moderately bullish market conditions.

Overall, the estimated excess return does not adequately compensate for the downside and tail risks assumed. The product is therefore assessed as unsuitable for most investors and is recommended only for sophisticated, risk-tolerant investors with a specific market view.

JEL classification: C63; G11; G12; G13; G17; G23

Keywords: Structured products; Barrier reverse convertible; Monte Carlo simulation; Derivative pricing; Risk analysis; Lufthansa

Resumo

Este projeto analisa o *Reverse Convertible com Barreira de 9,2% a.a. em EUR sobre a Lufthansa*, um produto estruturado de reforço de rendimento, resgatável antecipadamente pelo emitente, emitido pela Leonteq Securities AG e ligado às ações da Deutsche Lufthansa AG. O objetivo é estimar o seu valor teórico e avaliar o seu perfil de risco-retorno.

O produto é decomposto numa obrigação de cupão zero, numa opção *put down-and-in*, numa opção de resgate antecipado pelo emitente e num fluxo de cupões fixos. O seu *payoff*, dependente da trajetória, é avaliado através de uma simulação de Monte Carlo de 100.000 trajetórias do preço das ações da Lufthansa. A análise examina também os possíveis resultados do investimento, a sensibilidade às variáveis de mercado e às características contratuais, a erosão temporal, as gregas, o risco de cauda e os custos incorporados.

Num enquadramento de neutralidade face ao risco, o modelo de avaliação estima um valor teórico sem risco de incumprimento de 967,41€ e um valor ajustado ao risco de crédito/custo de financiamento do emitente de 959,89€. Relativamente ao preço de emissão de 1.000€, estas estimativas implicam margens incorporadas de 3,26% e 4,01%, respetivamente. Com base em pressupostos do mundo real, a rendibilidade anualizada estimada é de 2,70%, correspondendo a uma rendibilidade em excesso esperada de apenas 0,31 pontos percentuais acima da taxa isenta de risco.

O principal risco é uma perda substancial de capital caso a barreira seja atingida e o preço final das ações da Lufthansa seja inferior ao preço de exercício, caso em que o valor nominal é substituído pela entrega de ações da Lufthansa. Embora aproximadamente 81% dos resultados simulados gerem uma rendibilidade positiva, o Valor em Risco a 95% é de 264,87 € e a correspondente Perda Esperada Condicional é de 341,30€. O produto apresenta o desempenho mais favorável em condições de mercado neutras a moderadamente otimistas.

No geral, a rendibilidade em excesso estimada não compensa adequadamente os riscos de perda e de cauda assumidos. O produto é, por conseguinte, considerado inadequado para a maioria dos investidores e é recomendado apenas para investidores sofisticados, tolerantes ao risco e com uma perspetiva específica sobre o mercado.

Classificação JEL: C63; G11; G12; G13; G17; G23

Palavras-chave: Produtos estruturados; *Reverse Convertible* com Barreira; Simulação de Monte Carlo; Avaliação de derivados; Análise de risco; Lufthansa

Acknowledgments

I would like to express my sincere gratitude to my supervisor, Professor Maria Kosolapova, for her guidance, support, and the knowledge she shared throughout the development of this project.

I would also like to thank my family and friends for their support and encouragement, which helped me remain motivated throughout this journey.

Abbreviations

PRIIP - Packaged Retail and Insurance-based Investment Product

BRC – Barrier Reverse Convertible

IFC – Issuer's Funding Cost

ZCB – Zero-Coupon Bond

DIP – Down-and-In Put

GBM – Geometric Brownian Motion

MRP – Market Risk Premium

ETR – Expected Total Return

EAR – Expected Annualized Return

ER – Expected Return

EER – Expected Excess Return

VaR – Value at Risk

CES – Conditional Expected Shortfall

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Executive Summary

The *9.2% p.a. Barrier Reverse Convertible in EUR on Lufthansa* is a callable reverse convertible issued by Leonteq Securities AG and linked to Deutsche Lufthansa AG shares.

The product aims to provide enhanced income through fixed coupon payments in a stable or moderately rising market. In exchange, the investor accepts limited upside participation and exposure to losses if Lufthansa's share price declines significantly.

The valuation indicates that the product was issued above its estimated theoretical value and includes economically relevant embedded costs. Its main risk is a loss of capital if the barrier is breached and Lufthansa's final share price is below the strike, in which case the investor receives shares instead of full repayment of the denomination. The product performs best in neutral-to-moderately bullish market conditions. Its payoff is asymmetric because gains are limited by the fixed coupons and the issuer's early redemption right, while losses may be substantial in adverse market scenarios.

The product is not recommended for the general investor because its expected return does not adequately compensate for its downside risk, complexity and embedded costs. It may only be suitable for sophisticated, risk-tolerant investors with a specific neutral-to-moderately bullish view of Lufthansa.

1 Introduction

Structured products developed alongside modern derivatives markets, particularly from the 1980s onwards, as financial institutions began combining bonds and derivatives to create customised payoff structures. Once mainly directed at institutional investors, they later became widely available to private investors. Today, they represent an established segment of financial markets, offering exposure to different assets and market views, but their complexity can make their valuation, risks and costs difficult to assess.

This project analyses the *9.2% p.a. Barrier Reverse Convertible in EUR on Lufthansa*, issued by Leonteq Securities AG. The product is used as a case study of a yield-enhancement structured product that combines fixed coupon payments with exposure to the performance of an underlying equity. Its detailed characteristics and payoff structure are presented in a later section.

The main objectives of the project are to understand the economic structure of the product, estimate its theoretical value and evaluate its risk-return profile. More specifically, the analysis aims to identify the product's embedded financial components, compare its estimated value with its issue price, assess the probability and magnitude of different investment outcomes, measure its sensitivity to changes in relevant market variables and determine whether the expected return adequately compensates the investor for the risks assumed.

The remainder of the paper is organised as follows. Section 2 reviews the relevant literature. Section 3 presents the product, issuer, underlying asset and main investor risks. Section 4 describes the valuation methodology and presents the scenario and real-life analyses. Section 5 examines the sensitivity of the product to changes in the underlying asset and its contractual features, while Section 6 analyses time decay. Section 7 presents the quantitative risk analysis, Section 8 examines the product's costs, and Section 9 evaluates its investment rationale and provides the conclusion.

2 Literature Review

Structured products combine conventional financial instruments, commonly bonds, with derivatives to create more exotic and complex payoff structures that are tailored to investors' needs and market demands (Blümke, 2009). Reverse convertibles belong to the category of yield-enhancement products, which can pay coupons with a high relative yield by exposing the investor to the downside risk of a particular underlying asset. A Barrier Reverse

Convertible (BRC) is a specific type within this category. In the product studied, coupons are paid regardless of the performance of the underlying, while downside exposure at maturity arises if the barrier has been breached and the final price is below the strike. A callable BRC can be redeemed early by the issuer at specified observation dates, freeing it from future coupon and maturity obligations.

The standard valuation framework used for derivatives comes from the no-arbitrage principles developed by Black and Scholes (1973) and Merton (1973). The risk-neutral framework values a derivative by discounting its expected future payoffs under a probability measure where the underlying grows at the risk-free rate, adjusted for dividends. Rubinstein and Reiner (1991) derive closed-form formulas for binary and barrier-dependent claims, making their work relevant to the barrier- and strike-contingent maturity payoff. However, the callable feature and multiple early-redemption dates prevent these standard closed-form solutions from being applied directly to the complete product.

When valuing products whose payoff depends heavily on the path followed by the underlying asset, Monte Carlo simulation becomes particularly useful. It generates possible future price paths, computes the contractual payoffs along each path and discounts the resulting cash flows under the risk-neutral measure; their average becomes the theoretical value of the product (Boyle et al., 1997; Glasserman, 2004). Longstaff and Schwartz (2001) show how the comparison between immediate exercise and continuation can be treated in a simulation framework. This project does not implement least-squares Monte Carlo, but that methodology partly informs the simplified call mechanism used in the valuation.

Research regarding the performance and costs of structured products also raises questions about their value to retail investors. Szymanowska et al. (2009) show that ordinary and knock-in reverse convertibles are, on average, priced above their theoretical value, while Henderson and Pearson (2011) find substantial premiums over the estimated economic value of some complex structured products. Célérier and Vallée (2017) further show that higher headline rates are associated with greater complexity and risk, and that more complex products tend to generate higher markups and lower ex post performance. These results may reflect distribution expenses, hedging costs, issuer margins or the difficulty retail investors face in valuing complex payoffs independently.

This project aims to contribute to the literature by applying a risk-neutral Monte Carlo framework to a callable BRC linked to Lufthansa shares. The goal is to estimate its theoretical

value while incorporating its payoff structure, barrier mechanics and a modelled call mechanism in a methodologically sound way. Besides valuation, the analysis of expected return, payoff probabilities, sensitivities and risk using real-world measures provides a broader assessment of the product's risk-return profile.

3 Product Presentation

The product being studied is called *9.2% p.a. Barrier Reverse Convertible in EUR on Lufthansa* **(**here after, the Lufthansa BRC) and it was issued by the Guernsey branch of Leonteq Securities AG. In general terms it is an EUR denominated, callable and path dependent structured security which pays a high fixed coupon in return for downside exposure to Deutsche Lufthansa AG shares. Economically, this means that the investor is being compensated both for lending funds to Leonteq and for effectively providing protection against a decline in Lufthansa's share price. The repayment of the invested capital is not guaranteed, and the investor is also exposed to Leonteq's creditworthiness, since the future payments represent unsecured obligations of the issuer.

3.1 Product Description and Payoff

The Lufthansa BRC was issued on 10th of October 2024 with a denomination and minimum investment of 1.000€ and the initial fixing level was established at 6,34€ on 8th of October 2024. Along the initial fixing level, the strike and barrier levels are also established in relation to the first: the strike at 100% and the barrier at 69%, which result in levels of 6,34€ and 4,3746€, respectively. The barrier is monitored continuously through the lifespan of the product and is breached if the underlying's shares ever trade at or below the barrier level.

The product pays a coupon of 9,20% per annum through six quarterly payments of 23,00€. The coupon payments are independent from the performance of the underlying and, if the product is not early redeemed by the issuer and is held until maturity by the investor, he is guaranteed a total of 138,00€ in these payments alone.

When it comes to the redemption amount or the conditional repayment of the denomination, there are two main factors to consider: the possibility of early redemption and the breaching of the barrier.

If we assume that the product is not early redeemed at any of the possible dates, the final fixing occurs on 8th of April 2026 and the final settlement on 10th of April 2026. If the

product reaches its maturity, the final closing price is not sufficient to determine the redemption amount and there are three possible outcomes:

1. If the barrier was never breached during the observation period, the investor receives the full denomination of 1,000€, irrespective of the final share price.
2. If the barrier has been breached but Lufthansa closes at or above the strike price, the investor also receives the full denomination of 1,000€.
3. If the barrier has been breached and Lufthansa closes below the strike price, the investor receives 157,7287 shares per unit of the product, instead of the denomination. The conversion ratio is defined from the initial fixing date, and any fractional entitlements of the investor due to the rounding of the number of shares will be settled in a cash payment.

If we define N as the denomination, S_0 = K as the initial fixing level and the strike price, B as the barrier, S_T as the final fixing level, CR as the Conversion Ratio and

$$I_B = \begin{cases} 1, & \text{if } S_T \leq B \text{ at any time during the barrier observation period,} \\ 0, & otherwise. \end{cases} \quad (1)$$

as the barrier indicator, the redemption amount at maturity can then be defined as

$$R_T = N - I_B \max(N - CR \times S_T, 0), \quad (2)$$

or equivalently,

$$R_T = \begin{cases} N, & I_B = 0, \\ N, & I_B = 1 \text{ and } S_T \geq K, \\ CR \times S_T, & I_B = 1 \text{ and } S_T < K. \end{cases} \quad (3)$$

If we assume that there is no early redemption, and define C as the amount of each individual coupon payment, the final cumulative cash-equivalent proceeds of the investor are defined as

$$X_T = R_T + 6C \quad (4)$$

and the nominal profit or loss of the investor can be defined as:

$$P_T = X_T - N \quad (5)$$

Regarding early redemption, it is a right and not an obligation of the issuer, and it can happen in four pre-specified dates that always coincide with coupon payment dates (2^{nd}-5^{th}). The decision is taken by the issuer on the respective observation dates (one week prior) and, if

the early redemption is activated, the issuer must repay the full denomination amount of 1.000€ and the coupon of 23€ on the specified date. However, if the product is indeed early redeemed, the issuer frees itself from the obligation of paying all expected future coupons and the product's life effectively ends.

Table 1: Lufthansa BRC Key Dates

Event	Date
Initial Fixing	**08/10/2024**
Issue	10/10/2024
1st Coupon Payment	**15/01/2025**
1st Early Redemption Observation	08/04/2025
1st Early Redemption & 2nd Coupon Payment	**15/04/2025**
2nd Early Redemption Observation	08/07/2025
2n Early Redemption & 3rd Coupon Payment	**15/07/2025**
3rd Early Redemption Observation	08/10/2025
3rd Early Redemption & 4th Coupon Payment	**15/10/2025**
4th Early Redemption Observation	08/01/2026
4th Early Redemption & 5th Coupon Payment	**15/01/2026**
Final Fixing	**08/04/2026**
Redemption & 6th Coupon Payment	**10/04/2026**

It is very important to highlight that, contrary to want we find in the structure of many products from the same family, there is no automatic underlying price threshold for early redemption. This means that there are no contractual redemption conditions and the issuer decision may be influenced by, not only Lufthansa's share price, but also other factors such as interest rates, volatility, hedging costs as well as the issuer's assessment of the trade-off between the expected continuation value and the early redemption payments.

After analysing all possible scenarios, we can distinguish six possible cumulative payoff formulas:

Table 2: Lufthansa BRC Possible Payoff Formulas

Scenario Description		Formula	Total Payoff
Early Redemption	1st Redemption Date	2C + N	1 046 €
	2nd Redemption Date	3C + N	1 069 €
	3rd Redemption Date	4C + N	1 092 €
	4th Redemption Date	5C + N	1 115 €
Reaching Maturity	Barrier **NOT** Breached ($I_B = 0$) **OR** Barrier **IS** Breached ($I_B = 1$) **BUT** $S_T \geq K$	6C + N	1 138 €
	Barrier **IS** Breached ($I_B = 1$) **AND** $S_T < K$	6C + CR x S_T	138€ + 157,7287 x S_T

Figure 1 shows the payoff diagram of the Lufthansa BRC, assuming there is no early redemption. When the barrier is breached, the red line replaces the green one. X = 5,4651€ represents the breakeven price of the underlying at the final fixing date, given that the barrier is already breached. If the product is early redeemed, nominal profit can go from 46€ to 115€.

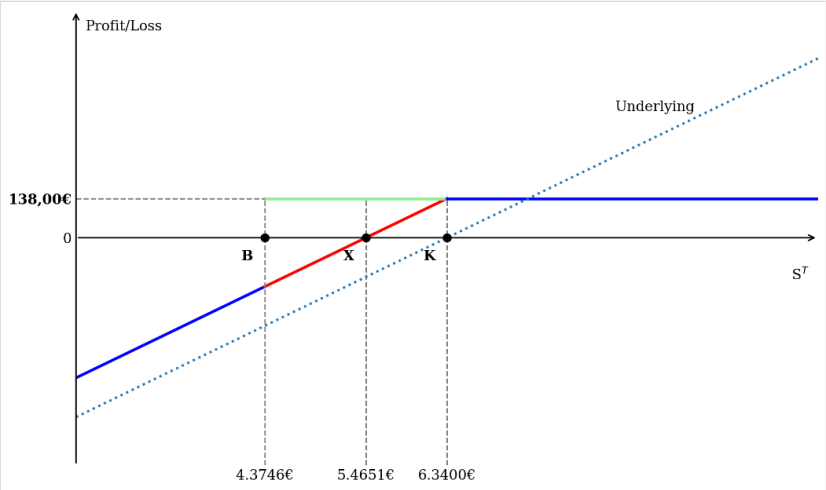

Figure 1: Lufthansa BRC Payoff Diagram

This specific product is primarily suited for an investor with a neutral to moderately bullish view on Lufthansa shares. On one hand, an increase in price can easily lead to an early redemption, while the investor doesn't benefit directly from upside movement. On the other hand, a decline in price is very dangerous because it can lead to the breaching of the barrier, which in turn opens the door for stock delivery and the worst payoff outcomes.

3.2 Issuer

The issuer of the product is the Guernsey branch of Leonteq Securities AG, located in St. Peter Port, Guernsey. Leonteq is a Swiss financial-services company founded in 2007 with its headquarters in Zurich. It operates in various markets around Europe and Asia through various subsidiaries and specializes in the design, issuance and distribution of structured products, although it also provides technology and services to other financial institutions. It is publicly listed on the SIX Swiss Exchange.

At the time of the product's issuance, in October 2024, Leonteq had a credit rating of BBB assigned by Fitch. This rating is within the investment-grade category which means that the chances of Leonteq defaulting on its obligations are considered extremely reduced. Nonetheless, this reality doesn't eliminate that possibility and should not be interpreted as a bullet-proof guarantee of the payments promised under the product.

3.3 Underlying Asset

The underlying asset of the product is the registered share of Deutsche Lufthansa AG (LHA.DE). The share is denominated in EUR and is traded through the Xetra, an electronic trading venue operated by the Frankfurt Stock Exchange (FSE).

Deutsche Lufthansa AG is headquartered in Germany and is the parent company of the Lufthansa Group, an aviation group whose activities include passenger airlines, logistics, maintenance, repair and overhaul, as well as additional business segments. The company's performance is mainly tied to the demand of, not only passenger traveling, but also airfreight and aircraft maintenance services.

At the time of the product's issuance, Lufthansa had recovered substantially form the extreme disruption caused by COVID-19. Although the recovery was not yet complete, the third quarter of 2024 brought a strong return of the demand for air travel, allowing Lufthansa to have all its passenger airlines operating profitably and to generate 10,7B€ in quarterly revenue. Nevertheless, delayed aircraft deliveries, higher personnel and operating costs and flight disruptions continued to affect profitability in a negative way (Lufthansa Group, 2024).

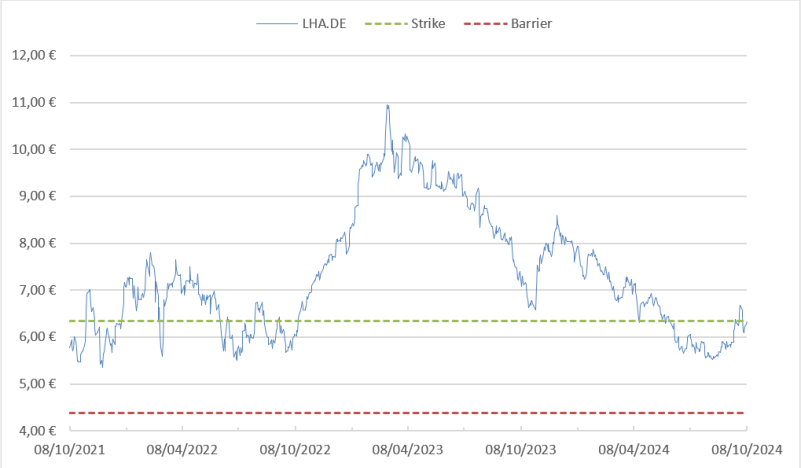

Figure 2: Lufthansa BRC 3Y Historical Price

1Y Return: -13,04%; 3Y Return: 9,90%

2Y Volatility (avoiding distortions from the COVID-19 era): 27,22%

The overview of the Lufthansa's recent history on the 8th of October 2024 gives us a neutral expectation regarding the future price of the underlying asset. While that is a favourable scenario for the investor in the Lufthansa BRC, a recent history of high volatility increases the chances of future early redemption or barrier breaching.

3.4 Investor's Advantages and Disadvantages

Main Advantages:

- Somewhat predictable and independent high-income stream of coupons
- Considerable downside buffer zone between strike level and barrier level
- Can outperform the underlying asset under sideways market conditions
- Even when receiving shares, breakeven can be achieved

Main Disadvantages:

- Invested capital is not guaranteed
- Big exposure to downside risk: maximum potential loss of (885€)
- No participation in upside potential: maximum gain caped at 138€
- Early redemption cuts gain potential and create reinvestment risk

3.5 Investor's Risks

Market Risk: The investor is exposed to this type of risk mainly through the return of Lufthansa share prices and through its volatility. The Lufthansa BRC's structure, in the perspective of the investor, is especially sensible to low returns and high volatility of the underlying. Exposure to the change in interest rates is also relevant, especially because of the fixed income like properties of the structured product in study. This risk is later quantified in the paper using Value-at-Risk and Conditional Expected Shortfall.

Credit Risk: This type of risk refers to the possibility that Leonteq might not be able to meet its obligations to the investor. This risk affects investors irrespective of market conditions because, if Leonteq becomes insolvent or experiences any kind of financial distress, coupons and repayments, as well as deliveries of shares, might be at risk. As of the initial fixing date of the studied product, Fitch assigned Leonteq a BBB credit rating (retrieved from Bloomberg terminal).

Liquidity Risk: It is related to the difficulties that the investor might experience when trying to sell the structured product on the secondary market. If the investor decides that he prefers an alternative risk-return profile in the middle of the product's life span, the secondary market for the product might not be liquid enough for him to be able to complete that operation, or if he does, it comes at either a greater cost or at a slower than desired timeline.

Inflation Risk: It is tied to the risk that the nominal payoffs of the product lose their purchasing power during its lifespan. This increases the probability of lower than expected or even negative real returns.

Currency Risk: This risk is a very investor-specific one. The only investors exposed to this risk are the ones who have a reference currency different from the EUR. If this is the case, unless the investor employs a hedging strategy, which also comes at a cost, it is impossible to invest in this specific product without being exposed to the currency fluctuations. This means that the investor is forced to be long on the EUR alongside the structure of the product, which can both improve losses and magnify gains as well as reduce gains and magnify losses.

Pricing Risk: Most retail investors don't have the analytical tools to value structured products. Because of that, there may be implicit costs in the price that they are not able to recognise and quantify.

Reinvestment Risk: In the case that the product is early redeemed, the investor might encounter difficulties when trying to replace it with another instrument or portfolio that has the same risk-return profile, structure and maturity. This is especially difficult in the case of structured products with more specific and exotic characteristics.

Regulatory Risk: It is related to changes in regulations during the product's lifespan. Factors like taxation laws regarding coupons and investment returns can have an effect on investor's gains.

4 Product Valuation

4.1 Variable Inputs

Volatility

The volatility input was obtained using Bloomberg's option volatility surface function, as of the initial fixing level. According to the Lufthansa BRC's maturity, the target option maturity is 1,5 years. The closest available option expiry was on 19th of June 2026, and the at-the-money volatility was used because of the strike of the callable BRC being 100% of the initial fixing price. In the end, the figure chosen for the implied volatility was 28,81%.

The use of implied volatility over historical volatility is preferred because the instrument I am studying is in part composed of options. Implied volatility means it is implied by the prices of the options available in the market that have the same underlying.

Risk-free rate

The valuation of the Lufthansa BRC is going to be done under a risk-neutral framework, therefore, finding an appropriate risk-free rate is fundamental.

Because the product I am studying is EUR denominated, I decided to use German government bonds as a low credit risk proxy for the risk-free rate. I used the German yield curve that uses German federal government securities as the reference assets. The final rate of 2,39% was obtained through linear interpolation between the 1-year and 2-year sovereign yields, retrieved from the Bloomberg terminal.

Issuer's Funding Cost (IFC)

Because a Leonteq-specific funding curve was not available, its credit was checked using Fitch rating history. Its rating was BBB as of the initial fixing date, so the EUR Europe Financials BBB+/BBB/BBB- BVAL yield curve was assumed to be an appropriate proxy because of its sector and rating matching. The final rate was obtained by linear interpolation of the 1-year and the 2-year values from the BVAL curve, and it is 3,16%. It was obtained from the Bloomberg terminal.

This yield is assumed to be an approximation for the sum of the risk-free rate with Leonteq's credit risk spread and is going to be used as a credit-adjusted discount rate. That rate can in turn be used to compute credit-risk adjusted theoretical values.

Dividend Yield

Lufthansa has a publicly stated dividend policy under which it aims to distribute 20% to 40% of its net profit each year in a single annual dividend payment (Deutsche Lufthansa AG, n.d.). It is also public that it has paid the same dividend, at the same approximate date, in 2024.

Because their dividend policy is public and a substantial amount of information regarding their 2024 fiscal year was already available at the initial fixing date, I made the simplifying assumption that I would be able to accurately predict the 0,30€ amount of Lufthansa's only dividend during the product's lifespan, as well as it's ex-dividend date.

4.2 Product Decomposition

Complex derivative products, like the one I am studying, can usually be decomposed into various components.

The first component is a Zero-Coupon Bond (ZCB). Assuming there is no early redemption, the investor lends 1.000€ per unit of product to Leonteq and, at maturity, there is the repayment of the denomination.

The second component is a short Down-and-In Put option (DIP) on the Lufthansa shares. This component is different from a vanilla put option because it only becomes active after a barrier below the strike price is breached. After that, it behaves like a vanilla put option.

The third component is a Bermudan call option with four exercise dates. This option gives the right, but not the obligation, to the issuer of calling the product by paying the denomination and a last coupon. This makes the issuer buy the obligations against itself, effectively annulling the product. The investor is technically also short the Bermudan call option.

The fourth component is a stream of six coupons. The fact that the investor is lending funds and is short both on a put option and a call option must be remunerated, and the high-yield coupons play exactly that role.

4.3 Pricing Methods

The Lufthansa BRC is valued using a Monte Carlo simulation. This simulation reproduces the evolution of the Lufthansa share price and attributes a payoff to each simulated path, according to the product's contractual conditions. The model generates 100.000 share price paths, from the initial fixing date until the final fixing date, with a fixed random seed that ensures that the results are reproducible.

The valuation is done in two different ways. First, the product is valued under a pure risk-neutral framework, where the asset price is simulated using the risk-free rate and all the payoffs are discounted using also the risk-free rate, resulting in the clean, default free, risk-neutral theoretical price. Second, another valuation is done by keeping the price evolution using the risk-free rate but changing the discounting rate to the IFC. This originates an alternative valuation that, in the perspective of the issuer, adjusts the theoretical price to their cost of funding, and in the perspective of the investor, adjusts the theoretical price to the credit risk of

Leonteq. I assume that the funding spread implied in the IFC is a good proxy to the credit-risk spread of Leonteq.

Between two consecutive business days, the Lufthansa share price is simulated according to a Geometric Brownian Motion (GBM):

$$S_{t+\Delta t} = S_t \exp\left[\left(r - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}Z_t\right], \quad (6)$$

where S_t is the Lufthansa share price at time t, r is the risk-free rate, σ is the implied volatility, Δt is the time interval and Z_t is a standard normally distributed random variable.

The model uses 6,34€ for the initial Lufthansa share price, 2,39% for the risk-free rate and 28,81% for the implied volatility. Time intervals are computed using ACT/365 fixed day-count convention and the simulation is implemented over a Monday-to-Friday grid. A discrete cash dividend of 0,30€ is incorporated on the 7th of May 2025, the ex-dividend date. That amount is subtracted from every simulated price-path that is still active at that specific date. It is introduced as a discrete downward adjustment and there is no change in the risk neutral drift.

Path dependent features are captured by monitoring the share price throughout the product's lifetime. Each business day the model checks the current simulated price and updates the relevant state variables of that specific path. This way it doesn't need to keep record of a full daily price history for each one of them.

Through this mechanism the model can monitor the path dependent events like barrier breaching and early redemption and store that information to, in the end of the simulation, compute the cashflows assigned to each path correctly and according to the product's rules.

While the barrier is monitored by checking every day if the simulated price is at or below the barrier, the early redemption process is handled differently. While it is also determined based on a pre-determined auto call price threshold, which is monitored at the redemption observation dates, the model is design to try to approximate the outcome of a real-life decision process that would involve the comparison between expected continuation value and the cost of early redemption.

Before running the final simulation, an alternative model with the same characteristics is run with a different seed, to generate a different sample of simulations. Using a pre-determined range of share prices for the auto call threshold, it chooses the one that minimizes the theoretical value of the product. This way it determines which threshold maximizes the

issuer's benefit and the auto call rule using this price level is accepted as a proxy for an optimal call decision process.

This threshold-selection procedure constitutes a methodological contribution of the project. It can be interpreted as a parsimonious approximation of the optimal-stopping framework of Longstaff and Schwartz (2001): whereas their method uses regression on simulated paths to estimate continuation values and derive a flexible, state-dependent exercise policy, this approach does not use regression and instead identifies the optimal rule within a restricted family of universal price thresholds. Therefore, the selected price level is not necessarily globally optimal, but only optimal among the rules considered.

One limitation to both methods is that actual issuers may not exercise optimally, as their decisions can also reflect hedging, liquidity, funding, operational and commercial considerations that are not captured by the model.

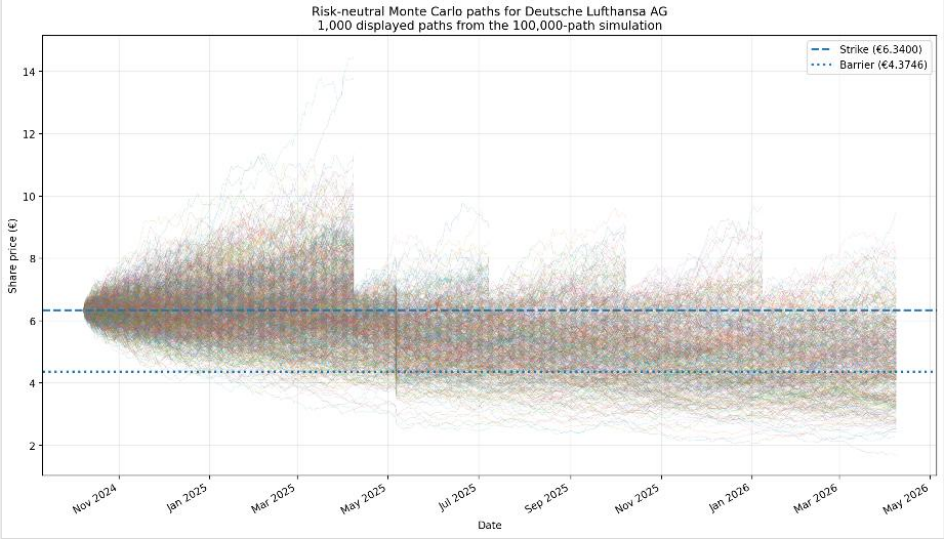

Figure 3: Monte Carlo Simulations

After all the paths are simulated, all the coupons and redemption amounts are discounted at the risk-free rate. For a simulated path i, the risk-free present value is

$$PV_i^{RF} = \sum_{j=1}^{m_i} C_j \, e^{-r\tau_j} + R_i e^{-r\tau_{R_i}}, \qquad (7)$$

where C_j represents a coupon received on path i, τ_j is the time between the valuation date and the corresponding coupon payment date, R_i is the redemption amount generated by the path, and τ_{Ri} is the time until the relevant early-redemption or maturity date.

Finally, the theoretical value of the product is estimated as the average present value of all simulated paths:

$$V_0^{RF} = \frac{1}{N}\sum_{i=1}^{N} P\,V_i^{RF}, \tag{8}$$

where N is the total number of simulations.

The adjusted theoretical value was obtained, as previously described, using the same model with a discount-rate equal to the Leonteq's estimated IFC. The determined optimal call rule threshold was the same for both valuations and is equal to 6,8742€ (108% of K). The valuations results are expressed in the following table:

Table 3: Lufthansa BRC Theoretical Values

Model	Theoretical Value	Implied Margin
Default free	967,41 €	3,26%
Credit risk / Funding Cost Adjusted	959,89 €	4,01%

Looking at the valuation results, the product appears to have been issued above its theoretical value under both valuation approaches. However, the implied margins should not be interpreted as direct profit, as they may incorporate distribution, manufacturing and hedging costs.

The difference between the two theoretical values was expected and, although relatively small, remains economically relevant. The estimated implied margins of 3.26% and 4.01% are, however, considerably lower than those documented by Henderson and Pearson (2011), who find that the offering prices of 64 retail structured equity products were, on average, almost 8% above their estimated fair values. This suggests a more moderate degree of overpricing in the present case, although the comparison should be interpreted cautiously due to differences in product structure, maturity, market conditions and valuation assumptions.

4.4 Scenario Analysis

When we are valuing a structured product and are trying to find its theoretical price, we must work under a risk neutral framework. However, if we want to do a scenario analysis where we estimate payoff probabilities and expected returns, we must transition from using risk neutral measures to using real-world measures. It is possible to use the model and the inputs I used in the previous chapter for this end, but the results are not intuitive, and do not represent real probabilities, but instead the risk adjusted probabilities that are consistent with the price of the product.

The model I used in the last chapter can be adapted by switching some inputs. In first place, we must switch the drift from the risk-free rate to the expected return of Lufthansa. To estimate it I used the Capital Asset Pricing Model (CAPM) where

$$E(R_i) = R_f + \beta_i \, x \, MRP, \tag{9}$$

and $E(R_i)$ is the expected return of an asset, β_i is the sensitivity of the asset to market movements, *MRP* is the Market Risk Premium and R_f is the risk-free rate.

I derived the beta of Lufthansa shares against the EURO STOXX Index from Bloomberg's BETA function. I used weekly returns and a 2Y window to avoid potential noise from the COVID-19 shock. For the MRP I used 5,75% as the midpoint of Kroll's recommendation for Eurozone equity risk premium. According to my assumptions, the expected return of Lufthansa is 9,20%.

For the volatility I switched the implied with the historical. I retrieved it form Bloomberg using a 2Y window of daily returns for the same reason as the beta. Its value is 27,22%, which is not much higher than the implied volatility.

To make cashflows comparable and separate investment risk from reinvestment risk, for the following analyses, all early cashflows are assumed to be reinvested at the risk-free rate until maturity.

Table 4: Event Probabilities

Event	Probability
Early redemption	58,14%
Early redemption on the 1st observation date	39,29%
Early redemption on the 2nd observation date	7,52%
Early redemption on the 3rd observation date	6,44%
Early redemptionon the 4th observation date	4,97%
Reaching maturity	41,86%
Barrier not being hit	77,63%
Barrier being hit	22,37%
Receiving the denomination	79,04%
Receiving stock	20,96%
Nominal money gain	81,36%
Nominal money loss	18,64%
Positive Return	81,42%
Negative Return	18,58%

By looking at the key probabilities on Table 4 we can observe that, according to our assumptions about the underlying, the product will probably be early redeemed. The barrier is

more probable to not being breached and the investor will probably receive the denomination and not shares. The investor will also most likely experience a nominal money gain.

The probability of receiving stock is higher than the probability of money loss reflects the existence of a breakeven point (=5,4651€, X on Figure 1) where, because of the positive effect the coupons have on the total payoff, a delivery of stock represents a 0€ nominal gain/loss and above it, a positive one. In turn, the fact that the scenario of having positive returns is slightly more probable than the nominal money gain is linked with the fact that there is another lower breakeven point (=5,4521€) where the positive effect of the coupons reinvested at the risk-free rate turns a nominal money loss into a 0% expected return. Above it all money loss scenarios represent a positive expected return.

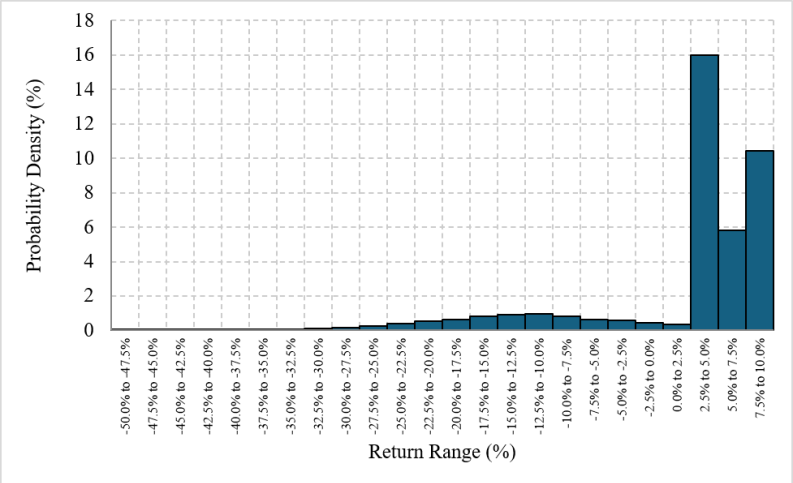

Figure 4: Return Distribution

Table 5: Scenario Return and Probabilities

Scenario Description		ETR	EAR	Prob
Early Redemption	1st Redemption Date	7,11%	4,67%	39,20%
	2nd Redemption Date	8,84%	5,79%	7,52%
	3rd Redemption Date	10,56%	6,90%	6,44%
	4th Redemption Date	12,26%	7,99%	4,97%
Reaching Maturity	Barrier **NOT** Breached **OR** Barrier **IS** Breached **BUT** $S_T \geq K$	14,00%	9,10%	20,90%
	Barrier **IS** Breached **BUT** S_T = 100% - 90% of K	9,00%	5,90%	1,46%
	Barrier **IS** Breached **BUT** S_T = 90% - 80% of K	-1,00%	-0,66%	2,93%
	Barrier **IS** Breached **BUT** S_T = 80% - 70% of K	-11,00%	-7,45%	5,11%
	Barrier **IS** Breached **BUT** S_T = 70% - 60% of K	-21,00%	-14,50%	6,24%
	Barrier **IS** Breached **BUT** S_T = 60% - 50% of K	-31,00%	-21,86%	3,64%
	Barrier **IS** Breached **BUT** S_T = 50% - 40% of K	-41,00%	-29,58%	1,32%
	Barrier **IS** Breached **BUT** S_T = 40% - 30% of K	-51,00%	-37,76%	0,26%
	Barrier **IS** Breached **BUT** S_T = 30% - 20% of K	-61,00%	-46,52%	0,02%
	Barrier **IS** Breached **BUT** S_T = 20% - 10% of K	-71,00%	-56,08%	0,00%
	Barrier **IS** Breached **BUT** S_T = 10% - 0% of K	-81,00%	-66,85%	0,00%

Table 5 shows the decomposition of the BRC's payoff profile. Each one of the payoffs is assigned its Expected Total Return (ETR) over the 1,5 years lifespan of the BRC, and the

Equivalent Annualized Return (EAR) and the respective probability. It's important to note that the scenarios involving the delivery of shares represent an interval of payoff events, therefore the ETR and EAR that are in the table represent the return of a middle point of that interval as an approximation to the weighted average of the scenarios inside it.

The most probable scenario is the redemption on the first date, which represents the smallest of the positive Expected Returns (ER), 4,67%, and includes the receiving of only two of the coupons. The second most probable scenario is also the one with highest ER of the entire distribution, 9,10%, and includes the receiving of all six coupons.

The worst scenario in terms of ER is included in the last interval of payoff events and involves the delivery of shares that are worth 0% of their market value on the initial fixing date. It represents an ER of -72,94%. Although in real life it is a scenario in the realms of possibility, the Monte Carlo simulation generated 0 scenarios that fit in that description. Finally, it is worth to highlight the interval of payoff events that involve the delivery of shares between 80% and 90% of the strike. This range contains both the nominal gain and positive ER breakeven price levels, 86,20% and 86,00% of strike, respectively.

Table 6: Best vs Worse Scenarios

Scenario	Nominal Payoff	Terminal Wealth	ETR	EAR
Best	1 138,00 €	1 140,04 €	14,00%	9,10%
Worst	138,00 €	140,04 €	-86,00%	-72,94%

If we adjust the ER of the various payoff scenarios to their probabilities and sum them, we get the BRC's ER, according to the assumptions we make regarding the underlying behaviour.

Table 7: Lufthansa BRC Expected Return

Expected Total Return	Expected Annualized Return	Expected Excess Return
4,09%	2,70%	0,31%

The BRC's expected annualized return of 2,70% exceeds the risk-free rate by only 0,31%, which appears modest relative to the downside risk borne by the investor. This result is directly related to Henderson and Pearson (2011), who find that the overpricing of structured products substantially reduces their ERs, in some specifications to below the risk-free rate. Although the Lufthansa BRC still provides a small positive Expected Excess Return (EER),

the result is consistent with the implied margins of 3,26% and 4,01% identified in the valuation analysis, which reduce the compensation ultimately received by the investor.

5 Real life Analysis

As of the delivery and presentation of this project, the product in study has already reached its planned maturity, so to complete our analysis of the various scenarios possible, we should look at what effectively happened.

Figure 5: Lufthansa Price During Lufthansa BRC Lifespan

Figure 5 shows Lufthansa's share price history during the entire maximum lifespan of the BRC. The barrier and strike levels are represented as well as a vertical line that represents the third observation date. We can see that the barrier was not breached at any point and that the price rose well above the strike price and the auto call level of the model. The minimum price was 5,56€ on the 13th of January 2025 and the maximum was 9,59€ on the 9th of February 2026.

The product ended up being early redeemed on the third early redemption date on the 15th of October 2025, being the price of Lufthansa on the respective observation day 7,04€. The investors in the Lufthansa BRC that held it until early redemption received 1.092€ as nominal payoff and, if we assume reinvestment at the risk-free rate until maturity, a return of 6,90% on their investment.

The fact that the price was 7,40€ on the second early redemption day, well above the 6,8742€ auto call threshold of the model, and it was not early redeemed, shows the limitations of the model in approximating the true early redemption decision logic.

While an EAR for Lufthansa of 9,20% was assumed, in real life, the observed EAR was 20,82% (ETR of 32,81%). For the volatility, while the 2Y historical volatility used was 27,22%, the true observed volatility during that period was 36,72%. The choice regarding the dividend used in the model was already made according to what was observed, so it coincides in both the amount and the ex-dividend date.

The disparities between the assumptions made and the observed reality highlight the importance of doing a sensitivity analysis to complement our results.

6 Sensitivity Analysis

6.1 Underlying Asset

This chapter aims to study the impact that different assumptions regarding the underlying can have on the valuation of the product, the estimation of expected returns for the investor and the estimation probabilities of barrier or early redemption events. The sensitivity analysis done regarding theoretical value and expected returns offer the same interpretable results, so only the sensitivity of the expected returns and the probabilities are computed to restrict analysis to real-world measures.

Expected Return of the Underlying

The Lufthansa BRC's payoffs don't depend entirely on the underlying's performance, only partially. Furthermore, because of their path dependent nature, we cannot trace conclusions about the BRC's returns just by looking at the price of the underlying on maturity. Nonetheless, by tracking the sensitivity of the Lufthansa BRC's expected return to that of the underlying, we can clearly comprehend the nature of their relationship.

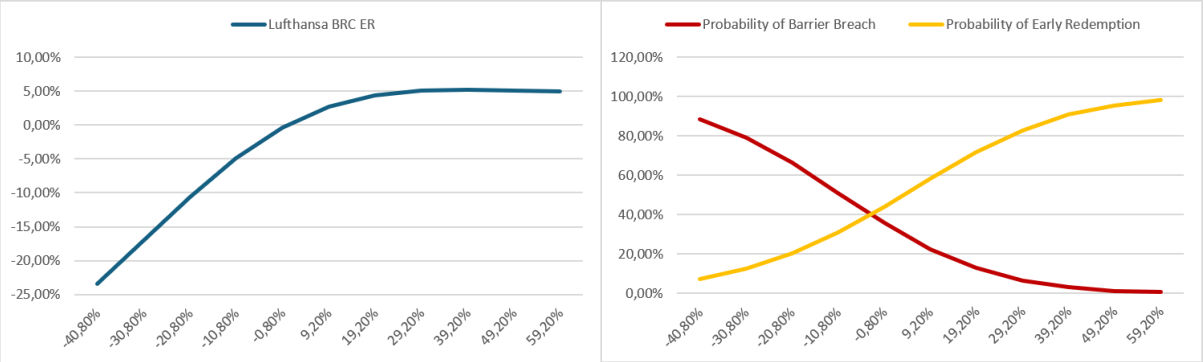

Figure 6: Sensitivity to Underlying Expected Return

Figure 6 shows the ER of the Lufthansa BRC plotted against a wide range of expectations for the performance of the underlying. The function resembles the payoff diagram of a put option, the dominant component of a BRC, and makes it clear that its investor is exposed to downside risk while it has a caped gain possibility at around 5% ER.

The probability of the barrier being breached decreases with the increase of the underlying's expected return because the paths on average are moving away from the barrier level faster. The early redemption probability increases with the underlying expected return for the opposite reason. For both probabilities, the sensitivity to the expected return of the underlying decreases near the extremes reflecting the more marginal effect when the prices are already very close or far from either the barrier or the model implied auto call threshold.

Volatility

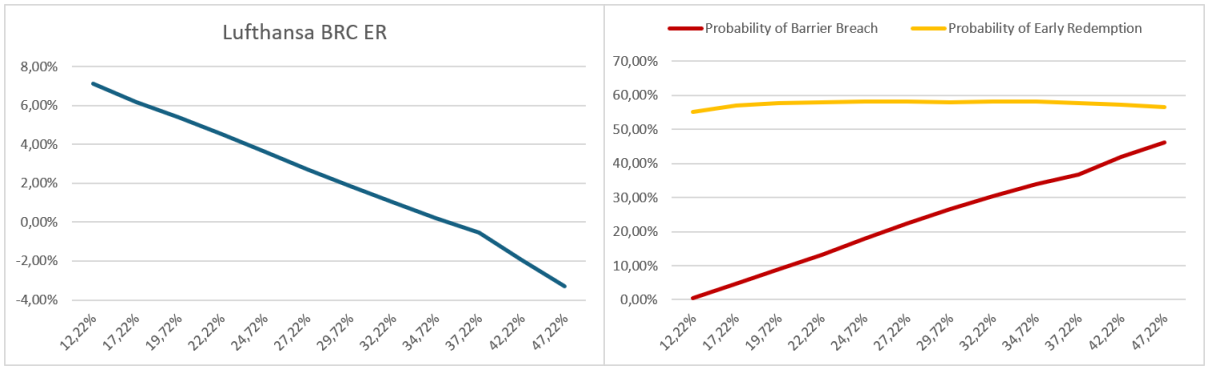

Figure 7: Sensitivity to Volatility

Another important assumption that our model relies on is the volatility. The type of derivative product we are focusing this analysis on, callable BRCs, intuitively seem to be very sensitive to volatility. Logic seems to point that if there is a big dispersion of possibilities for the movement of the price of the underlying, this will result in a more probable barrier breaching or, in a model which simplifies the call option of the issuer as an optimal auto call threshold, a higher probability of early redemption.

When we look at Figure 7, we realize that, although our assumption seems to materialize regarding the probability of the barrier being breached, it doesn't hold true vis a vis early redemption. While at first they may seem two similar events, they are of a very different nature. While the barrier is monitored throughout the life of the product and its breaching is irreversible, early redemption requires the price to be above a specific level at a specified date, with only four tries until maturity. Volatility increases the probability of touching extremes, but

it doesn't necessarily increase the probability of a number being above a certain level at a specific date.

In the end, the negative effect that volatility has regarding activating the barrier create a clear negative sensitivity of the Lufthansa BRC's ER to the underlying's volatility that can be seen in the Figure 7.

Dividends

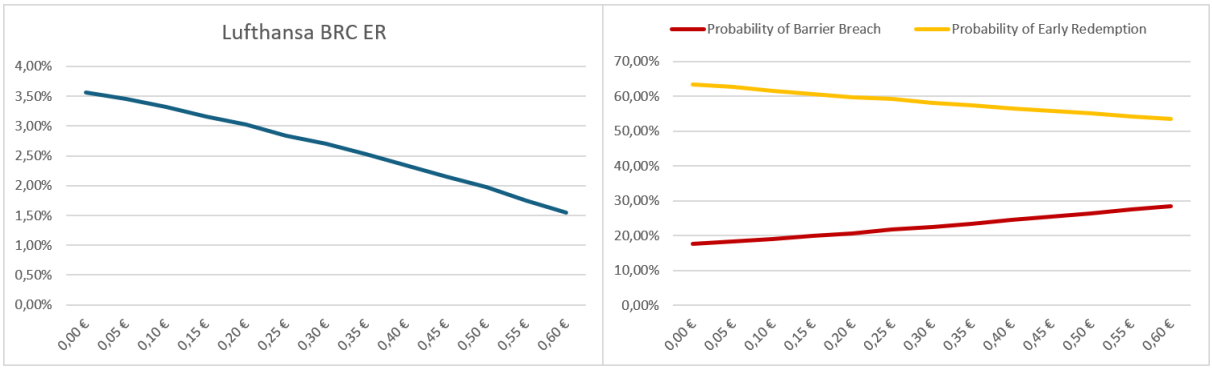

Figure 8: Sensitivity to the Dividend

Another underlying assumption that our Lufthansa BRC model uses is the dividend that is hardcoded in the simulated paths of the Monte Carlo simulation.

Looking at Figure 8, we can realize that, just like with volatility, the ER of the product has an inverse relationship with the size of the dividend. The probabilities of early redemption also decrease with a bigger divided but the probabilities regarding the barrier are the opposite. This is easily explained by the fact that the dividend creates, on the ex-dividend, a jump down of 0,30€ for all simulated paths that are still alive (haven't been early redeemed). It moves every path alive away from the auto call threshold, but into the barrier.

The Lufthansa BRC ER's inverse relation with the dividend amount is explained by the way it increases the probability of barrier breaching, leading to a predominance of the worse payoff scenarios.

With the analysis of these three assumptions, the sensitivity to all the market inputs used in the GBM (the dividend usually through the dividend yield, in my model it was hardcoded) to generate the price paths are analysed.

6.2 Product's Features

This chapter analyses how distinct features of the product affect its value and return profile as well as the probabilities of barrier and early redemption events.

Barrier

The barrier is one of the most important features of the Lufthansa BRC, and it is responsible for changing the very nature of its payoff in relation to the underlying.

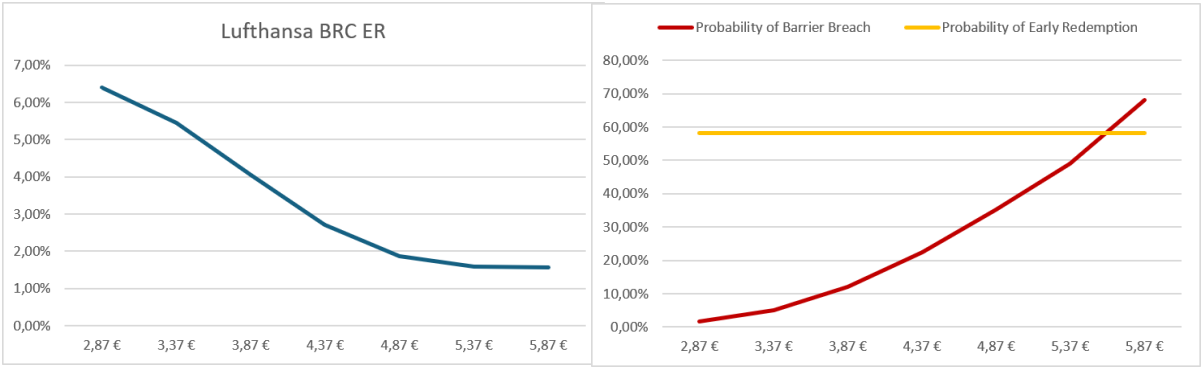

Figure 9: Sensitivity to Barrier Level

By looking at Figure 9, we can reach the intuitive conclusion that by increasing the barrier level we decrease the expected return of the product, as well as its theoretical value, and when we decrease it, it has the exact opposite effect. Looking at the variation range we included, the slope seems to be approximately linear in the middle, but the slopes get flatter as we approach the extreme ends. This happens because, in a product with a very low-level barrier, a very small number of paths will already reach there, so an even lower barrier will have a relatively smaller impact. On the other hand, when we raise the barrier very high, almost all paths that might lead to barrier breaching already do, so by raising the barrier even higher, we start to mainly influence a lot of paths that might lead to early redemption or still finish above the strike.

We can also denote that the probability of barrier breaching increases with the barrier level and the acceleration of this increase gets higher from the low-level barriers to the higher ones. There are more paths in the middle of the distribution than in the tails, so when the barrier increases it progressively gets more paths to lead to barrier breaching. Eventually, when approaching the other tail of the distribution, this probability would deaccelerate, flatten, and eventually start to decline. The probability of early redemption is independent of the barrier

according to the model I used. Because it is probably not so in real life, this highlights an important limitation of the model.

Early redemption

This product doesn't have, like many of its family, an auto call threshold that contractually obliges the issuer to redeem the product if it reaches a certain level. Instead, it gives the issuer the option to do so it if they think it is favourable to take that decision. Although it can depend on several factors, I try to simplify it by using an auto call threshold that tries to approximate the result of that decision process. The method used was already described earlier in the valuation chapter.

Taking all of that into consideration, studying the sensitivity of the product's outcomes to a change in the auto call threshold can bring insights into its behaviour regarding an either more strict or more relaxed decision process from the issuer or just a change on the issuer criteria that materializes in the product being called at higher or lower prices of the underlying.

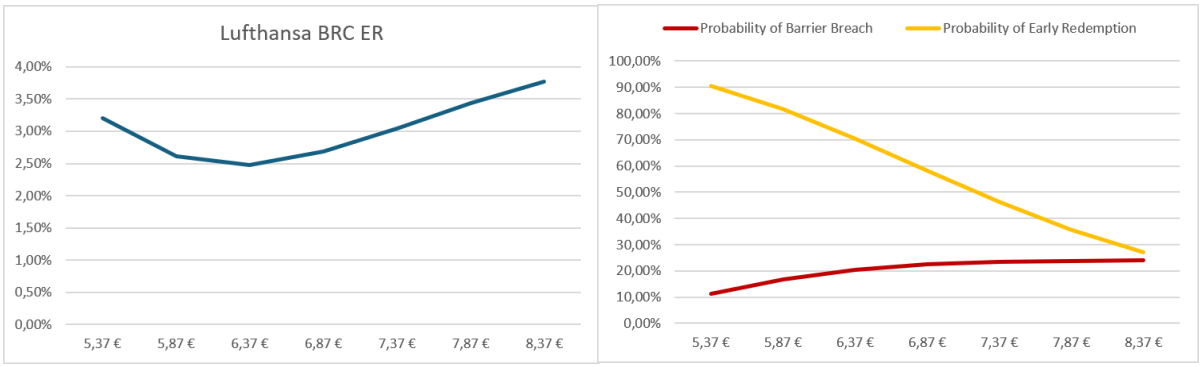

Figure 10: Sensitivity to Auto Call Threshold

Figure 10 shows that the expected return of the product is higher in both ends of the auto call threshold range. If it is very low, the product will be early redeemed very early most of the times. While this limits the gains from the coupons it also reduces the risks of barrier events and share delivery dramatically and makes sure that the investor receives the denomination very early and can invest it at the risk-free rate, according to our assumptions. When the call rule threshold is instead very high, it turns early redemption very less likely. This might expose the investor to more downside events, but it also increases the probability of receiving all the coupons by a big margin, especially when the underlying has a positive expected return. Even though the expected returns only vary by a small margin, the middle of the range ends up working as the worst of both worlds with a meaningful exposure to downside

risk with neither the capital preservation of the lower levels nor the coupon exposure of the higher ones.

The probability of early redemption expectedly rises with higher auto call thresholds and the probability of barrier breaching also increases but in a very slow way, deaccelerating as the marginal number of paths that are freed from early redemption and are now able to breach the barrier gets smaller with each rising auto call threshold.

7 Time-decay Analysis

To deepen our understanding of this product I decided to study how the passage of time affects its theoretical value. By repricing the Lufthansa BRC monthly using different constant spot prices, and assuming different states regarding the breaching of the barrier, we can create a conditional time-value profile of the product. It can show the effect of the passage of time, the coupons remaining, the probabilities of barrier breaching and early redemption and predictable future time events like a dividend.

Although we are analysing probabilities and because the dynamic change is what is relevant, the probabilities computed are risk neutral ones to keep consistency with the framework used for the theoretical value. We couldn't use the expected return like we did in the last chapter because we would have to know the market price of the Lufthansa BRC throughout its lifespan, and we don't have access to that information.

Three spot prices were used as reference for the repricing of the product: 6.34€ was chosen because it is the strike of the product and also the spot at which we start the original valuation; 7.20€ because it is above the early redemption auto call threshold of the model and 5.00€ because it is above the barrier but below the strike. Both the second and third values were chosen with the goal of still meeting their condition with the subtraction of the hardcoded 0,30€ dividend because the valuation before the ex-dividend date takes that future effect into account.

The time passage effect on the probabilities of early redemption and barrier breaching is also captured by estimating them in the same way, monthly and using the three different spots. For this goal, only the assumption of the barrier not being breached is considered. This is because the contrary assumption is irrelevant for the early redemption probability (the model's implied auto call rule is independent from the state of the barrier) and turns the probability of barrier breaching constantly 100%.

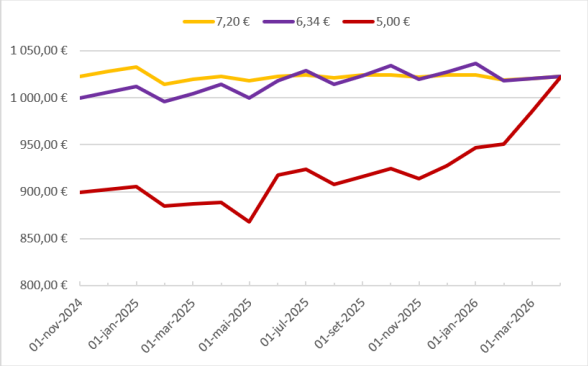

Figure 11: Theoretical Value – Assuming Barrier Not Breached

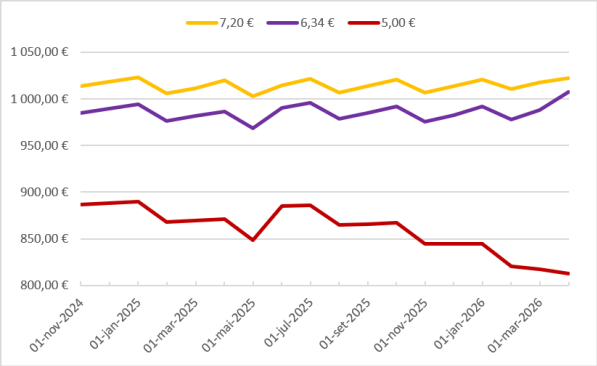

Figure 12: Theoretical Value - Assuming Barrier Is Breached

Looking at the results regarding both assumptions, we can expectedly see that, generally, the higher the spot price, the higher the valuation remains throughout the life span of the product. This is easily explainable because, the higher the spot price, the lower the probability of barrier breaching or, in case it happened already, of effective share delivery.

We can also see the down effect after each coupon payment date. After those dates, the valuation stops taking that future cashflow into account, reducing the product's value.

The difference between ex dividend and post dividend values is also very clear. The increase in the value of the product happens because prior to the dividend, the valuation is considering that future sudden reduction in price, that in turn increases the probability of barrier breaching and share delivery. After the dividend is paid and because we are repricing at a constant spot price, the value stops considering that expected negative future effect, and the valuation jumps up.

There are also relevant differences that emerge from the contrary assumption regarding the barrier state. We can observe that as the time to maturity decreases, the effect of time uncertainty fades out, and the value of the product converges to the sum of its final payoffs.

While in Figure 11 we can see all the values increasing and converging, in Figure 12, because we assume the barrier is breached, the valuation done using the spot below the strike reacts inversely to the passage of time, so the value decreases throughout the lifespan of the product and converges to the value of a lower final payoff that involves share delivery. Even between the other two spots the conversion is not as fast as in the first scenario because the valuation at the strike still involves a lot of uncertainty right until the end of the maturity, reducing its value and the speed of convergence.

In the first scenario, we can also observe that there are two points, the last two redemption dates, at which the valuation done using a spot above the auto call threshold becomes lower than the one using the spot at strike. This happens because the high probability of early redemption of the first reduces its value. After the last early redemption date, both the valuations converge.

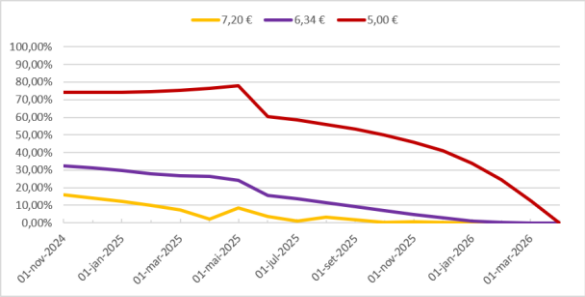

Figure 13: Probability of Barrier Breach

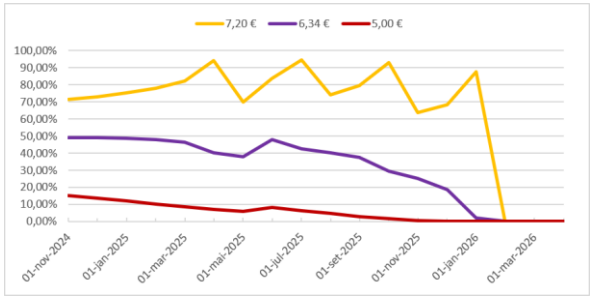

Figure 14: Probability of Early Redemption

By looking now at Figures 13 and 14 we can see the evolution of the probabilities of both barrier and early redemption events.

The probability of the barrier being breached goes down throughout the lifespan of the product, with all spots as reference, and the higher the spot the higher this probability starts. The probabilities end up merging at zero when reaching maturity.

Regarding early redemption, the probability goes down progressively regarding the spot prices on the strike and below, while for the spot above the strike, because it is above the model implied auto call rule threshold, this probability always rises to near 100% before the early redemption dates and drops afterwards. All the probabilities converge to zero after the last redemption date.

In both Figures 13 and 14, we can see the effect of the dividend. While the barrier breaching probability goes down after the ex-dividend date, because it stops considering a future drop in price that puts the spot closer to the barrier, the early redemption probability goes up because of the reverse effect.

8 Risk Analysis

The following section provides a comprehensive assessment of the Lufthansa BRC's risk profile. It focuses on measuring its market-risk exposure through the Greeks and quantifying potential losses using tail risk analysis. This section would be more complete with an examination of the Key Information Document (KID) of the product with its Summary Risk

Indicator (SRI) and Performance Scenarios. Unfortunately, because the product has already left the market, I couldn't get access to a KID from neither the issuance date nor subscription period in Leonteq's website. I tried contacting them requesting access, but I didn't get a response to date.

8.1 Greeks Analysis

The Greeks measure how the theoretical value of the Lufthansa BRC responds to marginal changes in its main market inputs. Since the product is path-dependent and callable, these sensitivities are not constant: they vary with Lufthansa's share price, the remaining maturity, the proximity of the early-redemption dates and, most importantly, whether the barrier has already been breached. The values reported at inception therefore provide an initial snapshot, while the subsequent analysis examines how each sensitivity evolves across different spot-price and barrier states.

The Greeks were estimated through a bump-and-revalue approach. Delta and gamma were calculated using upward and downward shocks equal to 1% of the relevant Lufthansa spot price. The resulting delta is expressed as the change in the product's theoretical value for a 1€ change in spot, while gamma measures the change in delta for a 1€ change in spot. Vega and rho were computed using changes of 1 percentage point in volatility and the risk-free rate, respectively, while theta was estimated using a one-day change in the valuation date.

Regarding the dynamic analysis, the evolution of each Greek was computed monthly, using the first day of each month as the valuation date. The analysis begins on the 1st of November 2024, shortly after the initial fixing date, and ends on the 1st of April 2026, immediately before the final fixing date. This monthly framework makes it possible to observe how the product's sensitivities change over time as maturity approaches and as the effects of the barrier and early-redemption features evolve.

Table 8: Greeks at Inception

Greeks	
Delta (Δ)	61,2510 €
Gamma (Γ)	-265,8518 €
Vega (V)	-5,4805 €
Rho (ρ)	-5,0316 €
Theta (Θ)	0,0869 €

At inception, with Lufthansa trading at the 6,34€ strike and the barrier still unbreached, the product presents a positive delta of 61,2510€, showing that an increase in the underlying

raises its theoretical value. The strongly negative gamma of −265,8518€ indicates that delta decreases as the share price rises, reflecting the product's limited upside participation and exposure to the embedded short-option components. Vega is also negative at −5,4805€, since higher volatility increases the probability of a future barrier breach and the subsequent delivery of Lufthansa shares below the strike. The negative rho of −5,0316€ reflects the combined effect of the risk-free rate on both the simulated share-price paths and the discounting of future cash flows. A higher rate increases the risk-neutral drift of Lufthansa's share price, but it also reduces the present value of coupons and the final payment and can increase the probability of early redemption. At inception, the latter effects dominate. Finally, the small positive theta of 0,0869€ suggests that the passage of time marginally increases the theoretical value of the product at that point.

Overall, the inception Greeks already reflect the key characteristics of the BRC: positive but limited exposure to the underlying, negative sensitivity to volatility, and a risk profile shaped by the interaction between the barrier and the issuer's early redemption right.

Delta

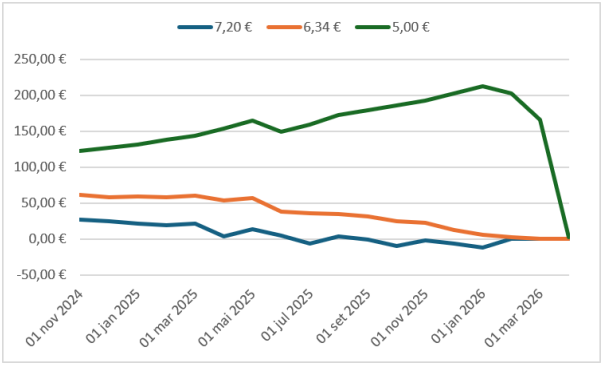

Figure 15: Delta - Assuming Barrier Not Breached

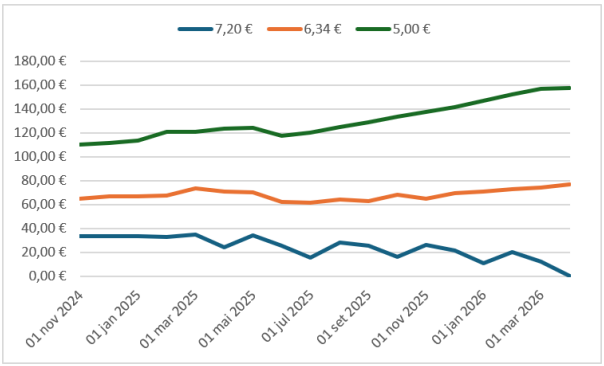

Figure 16: Delta - Assuming Barrier Is Breached

The Lufthansa BRC's delta varies substantially across spot-price and barrier states, reflecting the asymmetric payoff structure of the product. Delta is generally positive because the short barrier put creates positive exposure to Lufthansa's share price, partly offset by the negative delta of the short issuer call.

When the barrier remains unbreached, delta tends towards zero for all three spot-price scenarios as maturity approaches. Conditional on the barrier not having been reached and with very little time remaining, repayment of the 1.000€ denomination becomes increasingly likely.

Therefore, even when Lufthansa trades below the 6,34€ strike, the product gradually loses its sensitivity to small movements in the underlying.

Before maturity, however, an unbreached product can display substantial delta when Lufthansa trades closer to the 4,3746€ barrier. At a spot price of 5,00€, a change in the underlying affects both the potential value of share delivery and the probability of activating the barrier. This explains why delta can temporarily exceed the conversion ratio.

At the above-strike spot of 7,20€, delta falls before each early-redemption observation date in both barrier states. Since this price is above the model implied issuer's call threshold, a higher share price increases the probability that the product will be redeemed and that the remaining coupons will not be paid. As the observation date approaches, this call effect becomes more important and reduces the product's sensitivity to further increases in spot. When the barrier has not been breached, delta may even become slightly negative because the loss of continuation value can outweigh the benefit of a lower probability of future barrier activation. After a barrier breach, delta generally remains positive, since a higher spot still reduces the probability of finishing below the strike, but it nevertheless declines before the call dates.

The behaviour near maturity is fundamentally different once the barrier has been breached. The conditional protection provided by the barrier has permanently disappeared, meaning that delta does not converge towards zero in every spot scenario. At 7,20€, delta approaches zero because the final price is above the strike and redemption is effectively fixed at 1.000€. At 5,00€, delta converges towards the conversion ratio of 157,7287 because redemption is expected to consist of the corresponding number of Lufthansa shares.

The 6,34€ strike scenario lies between these two regions and is particularly interesting when the barrier is not breached. At this state, the maturity payoff has a turning point at the strike: immediately below it, the payoff changes with the underlying at a rate equal to the conversion ratio, while immediately above it, the payoff remains fixed at 1.000€. Consequently, delta at the strike converges towards approximately half of the conversion ratio, the average between 0 and the full conversion ratio.

Overall, an unbreached Lufthansa BRC gradually loses its equity sensitivity in every spot scenario, whereas a breached one retains material sensitivity at and below the strike. For the investor, a barrier breach therefore transforms the product from an instrument with conditional capital protection into one with substantial downside equity exposure. For the

issuer, it creates a strongly state-dependent hedge requirement, typically managed with a position in Lufthansa shares or equivalent equity derivatives.

Gamma

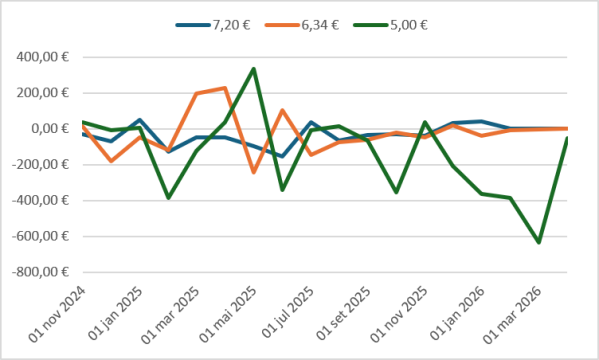

Figure 17: Gamma - Assuming Barrier Not Breached

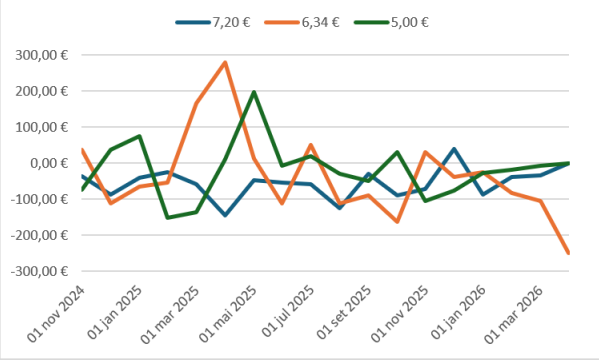

Figure 18: Gamma - Assuming Barrier Is Breached

Gamma measures how the Lufthansa BRC's delta changes when Lufthansa's share price moves. It is generally negative because the product is short both embedded options, although the barrier and call features make this curvature strongly state dependent.

Its monthly estimates vary considerably more than those of delta because gamma is a second-order sensitivity obtained from small differences between several valuations. Even when the same Monte Carlo framework is used, small spot bumps can cause some simulated paths to change payoff regime, and these discrete changes are amplified in the gamma calculation. The individual monthly values should therefore be interpreted cautiously, with greater emphasis placed on the broader patterns.

When the barrier remains unbreached, gamma converges towards zero across all spot scenarios, consistently with the convergence of delta already discussed. Before maturity, the largest movements generally appear at the 5,00€ spot, where changes in Lufthansa's price can materially affect the probability of reaching the 4,3746€ barrier.

After the barrier has been breached, gamma also converges towards zero when spot is clearly above or below the strike. At 7,20€, the product approaches a fixed payment of 1.000€, while at 5,00€ it increasingly behaves like a linear position in 157,7287 Lufthansa shares. In both cases, delta stabilises and gamma disappears.

The clearest exception occurs at the 6,34€ strike close to maturity. At this level, a small downward movement places the product in the share-delivery region, while an equivalent upward movement places it in the fixed-payment region. Delta therefore transitions sharply

from close to the conversion ratio below the strike to close to zero above it, producing a strongly negative gamma.

Overall, the most reliable conclusions are the convergence of gamma towards zero in the fixed and linear payoff regions and its strongly negative behaviour around the strike close to maturity. For the issuer, the latter implies a rapidly changing delta hedge and greater rebalancing requirements.

Vega

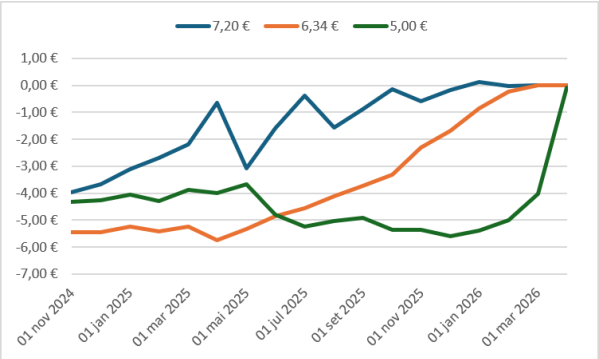

Figure 19: Vega - Assuming Barrier Not Breached

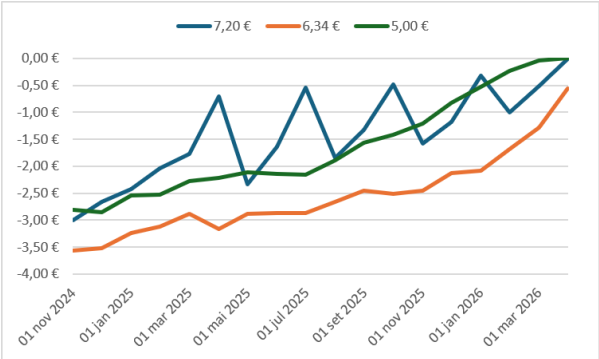

Figure 20: Vega - Assuming Barrier Is Breached

Vega measures the change in the Lufthansa BRC's theoretical value for a one-percentage-point increase in volatility. It is generally negative because higher volatility increases the value of the barrier put and issuer call held short by the investor.

When the barrier remains unbreached, vega is generally more negative because volatility affects not only the final share price but also the probability of activating share delivery. At the 5,00€ spot, vega would be expected to become progressively more negative as the barrier-breach probability becomes increasingly sensitive to volatility, before converging rapidly towards zero when insufficient time remains for a breach. The discrete dividend temporarily distorts this otherwise smoother pattern by reducing the relative importance of volatility before its payment and causing a visible adjustment once it is no longer a future event.

At 7,20€, vega repeatedly moves towards zero immediately before the early-redemption dates and becomes more negative again after them in both barrier states. Since the spot is above the call threshold, an imminent early redemption shortens the product's expected life and limits the time during which volatility can affect its value.

After a barrier breach, vega converges towards zero in all three spot scenarios. At 7,20€, the payment becomes increasingly fixed at 1.000€, while at 5,00€ the product approaches a

linear exposure to 157,7287 Lufthansa shares. At the 6,34€ strike, vega remains more negative for longer because volatility continues to affect the probability of finishing on either side of the strike. Nevertheless, it also converges towards zero as maturity approaches and the remaining uncertainty disappears.

Overall, volatility is most relevant while it can materially change the probability of a barrier breach, early redemption or delivery of shares below the strike. For the investor, higher volatility generally reduces the product's value; for the issuer, it reduces the value of the outstanding liability and creates an exposure that may be hedged through short-vega positions.

Rho

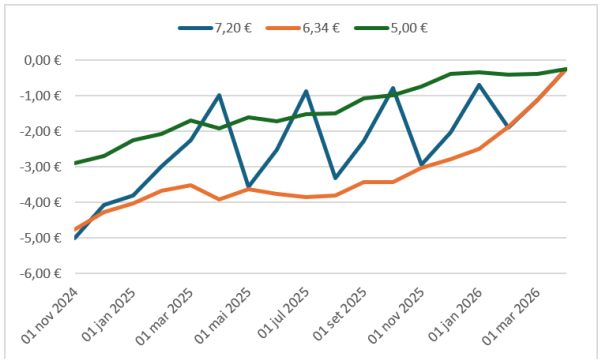

Figure 21: Rho - Assuming Barrier Not Breached

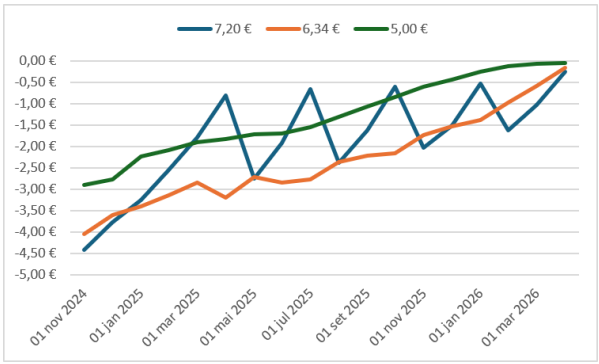

Figure 22: Rho - Assuming Barrier Is Breached

Rho measures the change in the Lufthansa BRC's theoretical value for a 1-percentage-point increase in the risk-free rate, which affects both the risk-neutral drift of Lufthansa's share price and the discounting of future cash flows. It is generally negative because higher rates reduce the present value of the bond and coupons and increase the value of the issuer call, outweighing the favourable rate effect on the short barrier put.

At 7,20€, rho exhibits the same saw-tooth pattern around the early redemption dates as vega, moving towards zero before each observation date and becoming more negative afterwards.

In the other scenarios, rho generally converges towards zero as maturity approaches. This convergence is particularly rapid at 5,00€ after a barrier breach, where the product increasingly resembles a linear position in Lufthansa shares and the effects of higher drift and discounting largely offset each other.

Overall, interest-rate risk is secondary relative to spot and volatility risk. For the investor, higher rates reduce the product's value; for the issuer, they reduce the value of the

outstanding liability, with any residual exposure manageable through bonds or interest-rate swaps.

Theta

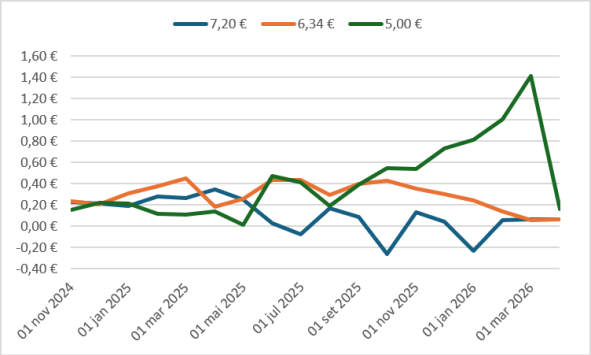

Figure 23: Theta - Assuming Barrier Not Breached

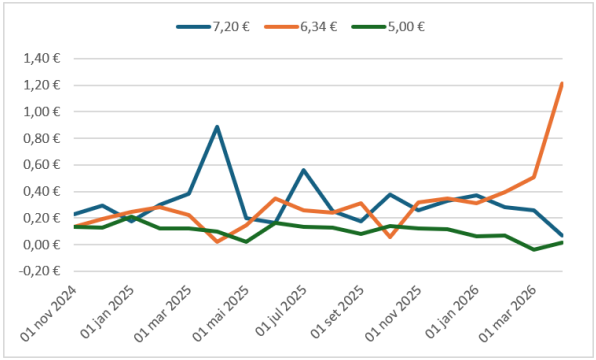

Figure 24: Theta - Assuming Barrier Is Breached

Theta measures the change in the Lufthansa BRC's theoretical value when one calendar day passes, with the spot price and other market inputs held constant. Theta is generally positive because the present value of future payments increases as they approach and because the Lufthansa BRC is short the embedded barrier put and issuer call, whose time value declines over time.

When the barrier remains unbreached, theta is positive in most scenarios. At 5,00€, it becomes particularly large at short maturities because each passing day reduces the remaining probability of reaching the barrier. Once this possibility becomes negligible, the product approaches the fixed payment of 1.000€ and theta converges towards the corresponding discounting effect. The discrete dividend temporarily reduces theta before its payment because bringing the known downward price adjustment closer offsets part of the favourable time decay.

At 7,20€, early-redemption dates produce pronounced local movements. As in the previous Greeks, the product's expected remaining life changes sharply around each observation date. Theta may become negative in the unbreached state when an increasingly likely early redemption removes valuable future coupons, whereas it is generally positive after a barrier breach when the early-redemption payment is more favourable than the riskier continuation value.

After a barrier breach, the terminal behaviour depends strongly on spot. At 5,00€, theta approaches zero as the product becomes a nearly linear exposure to 157,7287 Lufthansa shares. At 7,20€, it approaches the positive carry associated with the fixed payment of 1.000€. At the

6,34€ strike, theta becomes strongly positive close to maturity because the time value of the at-the-money short put decays particularly rapidly.

Overall, the Lufthansa BRC generally benefits from the passage of time, although callability and the barrier can produce substantial state-dependent effects. For the investor, the passing of time increases the product's value; for the issuer, it increases the value of the liability and is managed through the overall option-hedging and financing strategy rather than through a direct theta hedge.

8.2 Value-at-Risk and Conditional Expected Shortfall

A market-risk assessment was conducted using the simulated distribution of investor outcomes. Value-at-Risk (VaR) measures the minimum loss exceeded in the worst part of the distribution, while Conditional Expected Shortfall (CES) measures the average loss conditional on being within that tail. CES therefore provides additional information about the severity of losses beyond the VaR threshold.

At the 95% confidence level, the VaR is 26,49% of the 1.000€ denomination, corresponding to a monetary loss of 264,87€. This means that there is a 5% probability that the investor will lose more than 264,87€ over the lifespan of the product. The associated terminal value is 735,13€.

The 95% CES is higher, at 34,13%, or 341,30€. Therefore, when an outcome falls within the worst 5% of simulations, the average loss is approximately 341,30€, leaving an average terminal value of 658,70€. The CES exceeds the VaR because it incorporates all outcomes beyond the VaR boundary, including the most severe losses.

At the 99% confidence level, the VaR increases to 39,18%, equivalent to 391,76€. There is therefore a 1% probability that the investor will lose more than this amount. The corresponding terminal value is 608,24€. Within the worst 1% of simulations, the average loss rises to 44.09%, or 440,93€, resulting in an average terminal value of 559,07€.

The calculations are based on 100.000 simulated paths. Consequently, the 95% tail contains 5.000 observations, while the 99% tail contains 1.000 observations.

The broader outcome distribution remains favourable in most scenarios. Approximately 81,36% of the simulations generate a nominal monetary gain, while 18,64% generate a nominal loss. Similarly, 81,42% produce a positive return and 18,58% produce a negative return. Nevertheless, the VaR and CES results show that the product has meaningful tail risk. The high

coupon and the conditional capital protection produce positive outcomes in most simulations, but they do not eliminate the possibility of substantial losses when the barrier is breached and the underlying finishes significantly below its initial level.

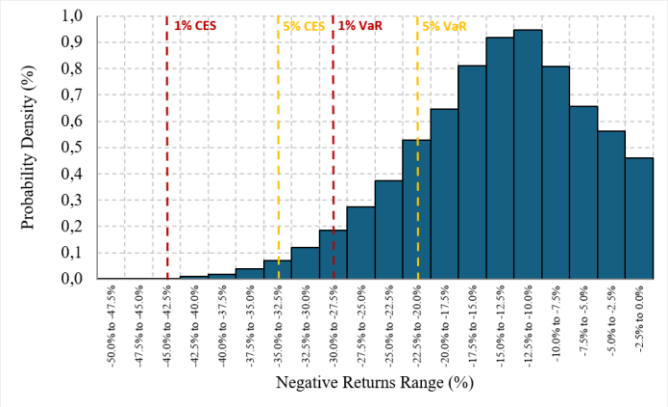

Figure 25: Negative Returns Distribution

Table 9: Value at Risk and Conditional Expected Shortfall

Confidence Level	VaR
95%	264,87 €
99%	391,76 €
	CES
95%	341,30 €
99%	440,93 €
Confidence Level	**VaR (loss %)**
95%	26,49%
99%	39,18%
	CES (loss %)
95%	34,13%
99%	44,09%

9 Cost Analysis

The costs of the product can be divided into explicit costs disclosed in the KID and implicit costs embedded in the issue price. This distinction is important because an investor will not receive a full description of most of the product's costs. Instead, they are incorporated into the price paid for the structured product.

Explicit Costs

The KID specifies certain product costs classified as entry costs and, where applicable, exit costs. These costs are generally incorporated into the prices paid or received by the investor rather than charged separately. Nevertheless, they remain economically relevant because they reduce the investor's effective return.

The adviser, distributor or financial intermediary may also charge additional fees, including advisory, brokerage, custody or distribution fees. These depend on the intermediary used and are not necessarily included in the product costs disclosed in the KID.

When the product is sold before maturity, any disclosed exit cost may be reflected in the secondary-market bid price, for example through the bid–mid spread, rather than debited as a separate fee. The sale price may also differ from the product's estimated theoretical value because of changes in market conditions, volatility, credit risk and liquidity.

Implicit Costs

The principal implicit cost is the difference between the 1.000€ issue price and the estimated theoretical value of the product.

Under the default-free risk-neutral valuation, the product has an estimated value of 967,41€ which represents a difference of 32,59€, equivalent to 3,259% of the denomination. When the cash flows are discounted using the estimated IFC, the theoretical value decreases to 959,89€, which represents a difference of 40,11€, equivalent to 4,011% of the denomination.

As discussed earlier in this paper, these differences should not be interpreted entirely as issuer profit.

Impact on Investor Return

The product's estimated annualized return of 2,70%, is computed using the 1.000€ issue price. Consequently, the effect of the embedded costs is already reflected in these results.

If the same expected cash flows could be purchased at the default-free theoretical value of 967,41€ rather than 1.000€, the estimated annualized return would increase to approximately 4,99%, a difference of 2,29 percentage points. Using the credit-adjusted value of 959,89€, the annualized return would be approximately 5,54%, a difference of 2,84 percentage points.

10 Investment Rationale and Conclusion

The Lufthansa BRC is intended for an investor seeking enhanced income and expecting Lufthansa shares to remain broadly stable or appreciate moderately. In such market conditions, the fixed coupons can provide a predictable income stream and may allow the product to outperform the underlying when the share price moves sideways. However, the product is not appropriate for an investor with a strongly bullish outlook, since gains from increases in Lufthansa's share price are capped, or for a conservative investor who requires guaranteed capital repayment.

The valuation analysis indicates that the €1.000 issue price exceeded the product's estimated economic value. The default-free and credit-adjusted valuations produced values of €967,41 and €959,89, respectively, implying embedded margins between 3,26% and 4,01%. These margins may reflect manufacturing, distribution, funding and hedging expenses rather than pure issuer profit. Nevertheless, they represent an economically relevant difference between the amount invested and the estimated value received by the investor.

Under the real-world assumptions adopted in the analysis, the expected annualized return is 2,70%, only 0,31 percentage points above the risk-free rate. This expected excess return is modest when considered against the product's asymmetric payoff. The investor's maximum gain is limited by the fixed coupons and may be further reduced by early redemption, whereas losses can become substantial if the barrier is breached and Lufthansa finishes significantly below the strike.

The simulations indicate that a positive return is considerably more likely than a negative return and that the denomination is repaid in most scenarios. However, this high probability of a moderate gain should not be interpreted as evidence of low risk. The 95% Value-at-Risk of 264,87€ and Conditional Expected Shortfall of 341,30€ demonstrate that adverse outcomes can produce economically significant losses. The investor must also accept issuer credit risk, possible secondary-market illiquidity and reinvestment risk following early redemption.

Overall, the product offers an attractive headline coupon but an unfavourable balance between expected compensation and potential loss severity. Its structure transfers substantial downside exposure to the investor while restricting participation in favourable movements of Lufthansa shares. Consequently, the Lufthansa BRC may be suitable only for a sophisticated and risk-tolerant investor with a neutral-to-moderately bullish outlook, sufficient liquidity to hold the product until redemption and a full understanding of its path-dependent payoff. At the issue price, the expected excess return does not appear sufficient to compensate for the risks assumed. The final investment assessment is therefore **Avoid** for the general investor.

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AI Disclosure

Artificial intelligence tools, including generative language models such as ChatGPT, were used for limited purposes, namely:

- improving English, grammar, clarity and readability;
- supporting brainstorming and the organisation of ideas; and
- assisting with programming syntax and code-related troubleshooting.

AI was not used to independently conduct the financial analysis, select the methodological assumptions, validate the valuation results or determine the conclusions and investment recommendation. All AI-assisted suggestions were reviewed, edited and verified by the author.

The research design, modelling, calculations, interpretation of results and final conclusions are the author's own work. The author assumes full responsibility for the accuracy, integrity and originality of the report.