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PROJECT

Structured Product Analysis - Cirdan Phoenix Autocallable Worst of Certificates

By Tomás Teixeira

SUPERVISION

Professor Gil Nogueira, Ph.D.

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AI Disclaimer

I disclose that AI tools were employed during the development of this thesis:

- AI-based research tools were used to assist in literature review and data collection.
- AI-powered software was utilized for data analysis and visualization.
- Generative AI tools were consulted for brainstorming and outlining purposes. However, all final writing, synthesis, and critical analysis are my own work. Instances where AI contributions were significant are clearly cited and acknowledged.

I have ensured that the use of AI tools did not compromise the originality and integrity of my work. All sources of information have been appropriately cited in accordance with academic standards. The ethical use of AI in research and writing has been a guiding principle throughout the preparation of this thesis.

I understand the importance of maintaining academic integrity and take full responsibility for the content and originality of this work.

Tomás Teixeira, 20th of June 2024

Abstract

This report was elaborated with the goal to analyse the" Cirdan Phoenix Autocallable Worst of Certificates" product – a structured product created and operated by the firm Cirdan Capital Management.

The report can be deconstructed in four chapters: Product Overview (Composition, Advantages and disadvantages and risks), Valuation Methods, Product Valuation and Risk Level, and Delta Hedging.

The analysis was conducted using the fundamentals learned from my master's in finance.

At the beginning of the report, we can observe a more qualitative analysis, where I explain the structure of the product, elaborate some analysis on the advantages and disadvantages, and the risks that come with it. I also decompose the product into simpler derivatives (options), with the goal to better understand the logic behind the product behaviour and structure.

The second part of the report focuses more on a quantitative analysis, where I try to obtain a realistic theoretical price for the product. For that, I apply three models for valuation – Black-Scholes Merton Model, Binomial-Tree Model, and the Monte Carlo Simulation. The value obtained represents the payouts that an investor may receive not only at maturity, but also during the expected lifetime. Lastly, the Delta Hedging strategy simulated revealed to be a good option for the investor as it as shown an effective performance, even though it occurred during the crash related to the Covid-19 Pandemic.

Keywords: Monte Carlo Simulation, Structured Products, Exotic Options, Delta Hedging, Hedging Strategy, Black-Scholes Model, Binomial Tree Model

Jel Codes: G11, G12, G17, G23

Resumo

Este relatório foi elaborado com o objetivo the analisar o produto “Cirdan Phoenix Autocallable Worst of Certificates” – um produto estruturado criado e comercializado pela empresa Cirdan Capital Management.

O relatório pode ser decomposto em quatro capítulos principais: Visão geral do produto (composição, vantagens, desvantagens e riscos), métodos de avaliação, avaliação do produto e Delta Hedging.

A análise foi conduzida com base nos recursos aprendidos durante o meu mestrado em finanças.

No início do relatório podemos observar uma análise mais qualitativa, onde é explicado a estrutura do produto, elaborada alguma análise em torno das vantagens, desvantagens e riscos do produto. É também feita uma decomposição em derivados mais simples (opções), com o objetivo de compreender melhor a lógica e comportamento da estrutura do produto.

A segunda parte do relatório é mais direcionada para uma análise quantitativa, onde é procurado obter um preço teórico realista para o produto estruturado. Para tal, são aplicados três models de avaliação: Modelo Black-Scholes Merton, Modelo das árvores binomiais e simulação de Monte Carlo. Os valores obtidos representam os pagamentos que um investidor poderá receber não só na maturidade do produto, mas também ao longo do ciclo de vida do ativo expectável. Por último, a simulação da estratégia de Delta Hedging revelou ser uma boa opção para o investir, visto ter obtido uma performance eficaz, apesar de ocorrer no mesmo horizonte temporal do crash do mercado que ocorreu devido à pandemia causada pelo Covid-19.

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1. Introduction

Structured products are defined as financial instruments whose performance is directly correlated with an underlying asset, product, or index. Usually, this underlying is some type of equity traded in the market, which makes the price inconstant.

They can have various types of complexity, as most of them are fully customizable by the distributor, which allows to obtain products tailored to the profile of the investors according to their assets, risk appetite, their market preferences, and their knowledge. These are usually considered riskier than the underlying itself.

Investors can tailor the asset (or group of assets) in infinite ways, allowing the client to be exposed precisely to the variables he wishes. The returns, logically, will depend on this exposure, delivering higher returns for more riskier positions.

Because of this level of customization, these types of products are usually traded over-the-counter (OTC) by financial institutions, which makes them an illiquid and more expensive investment option. Nevertheless, the risk and the price are rewarded by the profitability of the product.

The complexity of the products makes them unfitted for unexperienced investors and are not recommended for most type of recreational investors., as the risks of the products are not simple and visible as the ones for a more common investment opportunity. Also, investors must resort to complex mathematical models to obtain a complete understanding of all the variables and dependencies, either to estimate risks, returns or the price of the product.

This project was made with the goal to study and deeply understand a structured product – Cirdan Phoenix Autocallable Worst of Certificates – by understanding its price, the variables that are intrinsic to it, how does it react to manipulations of the macro perspective of the market.

The analysis will be conducted in five segments - Description of the product, Valuation methods, Data analysis, Product valuation and Delta hedging strategy – with the final goal to obtain the theoretical price of the product and to develop a strategy that would counter the fluctuations in the value, mitigating the risk of the product with its own underlyings.

2. Literature Review

This project reflects the insights studied by John C. Hull in the book "Options, Futures and Other Derivatives".

One of the constants throughout the book is the risk-reward theory. In fact, Hull explains multiple pricing models, where there is the constant of incorporating the variable risk in the computations, making it a key element in the output of the model, also known as the expected value. The bigger the value of the risk that is incorporated in the model, the bigger the expected payout, and therefore the value of the product that is being priced.

In the book, Hull emphasizes the importance of using an adequate pricing model, in order to guarantee fair trading and correct function of the capital markets. Hull also emphasizes the strengths and weaknesses of the main pricing models: the Black-Scholes Model, the Binomial Model, and the Monte Carlo Simulation. He defines the Black-Scholes as a mathematical model for pricing European-style options, the binomial model as a discrete-time model for option pricing and the Monte Carlo Simulation as a versatile method for pricing complex structured products and derivatives. Applying the correct model will allow to obtain a better estimation of the theoretical price and therefore make informed decisions.

One of the key takeaways from the book is the importance of applying a strategy of hedging to reduce the risk of the investments. Hedging allows to mitigate risk and to reduce exposure to the capital market. In structured products, as there is higher risk and the derivatives can be more sensitive to variations in the market, performing a correct hedge can help on increasing stability on the variations of the price. Hedging may also be viewed as a way of regulatory compliance, as for some financial institutions may be obliged to hedge their positions to hedge certain exposures, ensuring they have enough capital reserves to face their obligations, promoting a safer and more stable financial system.

For that purpose, the Delta Hedging strategy is studied across the book, in the context of managing the risk associated with investing in derivatives. Hull defines it as a strategy that is used to neutralize the risk of an options position, by entering an opposite position through the underlying of the derivative. Through the words of John C. Hull, "Delta hedging aims to keep the value of the financial institution's position as close to unchanged as possible". This is true as the Delta measures the sensitivity of the product to changes in the price of its underlyings. By creating a Portfolio that replicates (inversely) the Delta of the product, the risk is neutralized, and the fluctuations are covered by the hedging portfolio.

This is made through a Delta-Neutral portfolio, which consists of aggregating two portfolios, one composed by options or derivatives (that may create a structured product) and another composed by the underlyings of the said derivatives. The second one is built in a way that the delta of the portfolio is the reverse value of the first one, obtaining a combined delta of zero for the Delta-Neutral portfolio. With this strategy, the fluctuations of the value of the options are offset by the increase/decrease of value of the underlyings portfolio value.

"Options, Futures and Other Derivatives" is a reference in the finance literature, and as so provides valuable insights for most of the theories that are used in the real world. The studies presented by Hull strengthen the purpose of this project, about the relevance of Pricing Models and Delta Hedge as a hedging strategy.

3. Product Overview

The Cirdan Phoenix Autocallable Worst of Certificates is a complex financial product distributed by the company Cirdan Capital Management and regulated by the U.K. Financial Conduct Authority (information as of 10 February 2020). The firm categorizes herself as an "Investment Boutique" with specialization in issuing structured products, providing various types of asset securitization and most recently provide investment technology solutions. The firm was founded in 2014 in London by Antonio de Negri, and since then it has pushed to become an innovative fintech firm, providing high quality services in the financial spectrum, separating themselves from the market.

The product is composed by an Underlying – the Reference Underlying - that will define the Payout structure. The Reference underlying is dependent on four assets.

The first part to understand is the Underlyings. They are assets traded in the capital market, which makes their price volatile. That value is what is going to dictate how the structured product works; therefore, they are directly linked to one another. For this product, there are four underlyings, and they are stocks that trade in the Italian Stock Exchange Market. The stocks are from Intesa Sanpaolo SpA (IT0000072618), UniCredit SpA (IT0005239360), Eni SpA (IT0003132476) and Fiat Chrysler Automobiles NV (NL0010877643). Together they will from the Reference Underlying, which is a theoretical underlying, composed by the worst performing stock in each moment in time (Worst of feature). This means that all four of the assets may be in this theoretical Underlying. To measure the performance, and to allow to compare stocks with different values, the price of each in the issuance date is registered and every value registered from that day forward will be divided by that said value. Therefore, the value obtained is a percentage of the value of the stock compared to its original value. The one with the lowest percentage in each time step is the one who will represent the reference underlying in that moment, and the percentage of that stock is also the percentage of the reference underlying for that month. Altogether, the four stocks form the Reference Underlying composed entirely by percentages rather than stock values.

The product has a complex payout structure, that is directly dependent on the performance of the Reference Underlying, and therefore dependent on the performance of the four underlyings. The structure can be divided into two parts.

The first one is the payout at the maturity date (Image 1). This is the amount that the investor is entitled when the product reaches maturity. This can be either a fixed value of the total Denomination, which is 1,000 euros, or that amount multiplied by the performance of the Reference Underlying on that date. The criteria that define which one happens is called the Capital Barrier and is stablished at 60%.

The second part of the payout are the Coupons, which are paid periodically during the lifetime of the product. Each Coupon has a value of 0.5% of the Denomination, which means that 5 euros can be paid at the 18^{th}, 19^{th} or 20^{th} each month while the product is still active. Similar to the previous payout, there is also a barrier that determines if there is the right to receive a Coupon in a certain month, and that is called the Coupon Barrier, which is stablished at the same level as the Capital one.

This type of payout features memory, an uncommon characteristic that increases the complexity of the product. A memory feature means that a certain type of pay-out works retroactively. Therefore, if the investor is entitled to the coupon of a month, then he is also entitled to all the previous unpaid coupons from previous months, up until the last month where a Coupon was paid.

To complete the structure of the product, there is a last feature called the Automatic Early Redemption (AER), which is, as the name suggests a redemption of the product before its maturity date, defined at 25/05/2025. This redemption is activated if the reference underlying goes above a third barrier, the AER Barrier, and it delivers the total amount of the Denomination on the month of the activation, closing the position. There is a grace period of 6 months where the AER is not possible.

The currency where the product is transacted is the Euro and there is a minimum investment of 1,000 euros on the product.

There is a Capital Protection of 10% over the nominal amount in case that the reference underlying registers a performance below 10%. Besides that, there is no type of capital protection over the investment.

The product is classified as a 6 out of 7 in a risk scale, which is the second highest value. It represents the risk of having the payouts directly linked with the capital market in a firm that is also relying on the same capital markets to be able to pay its own responsibilities. There are other multiple risks that will be investigated in detail in the next sections of the report. Altogether, it creates a product directed to risk seekers and that is highly exposed to different risks, whether they are systematic or unsystematic.

The purchase of the product has an additional cost, which is the consulting fee and is charged by the bank in question. This value is, in the worst-case scenario, 3.93% of the initial investment.

The bank defines the product as suitable for retail investors who are experienced in structured products and in the market of complex securities, who have capacity to bear a total capital loss in the worst case, and that are seeking for a timeline of investment of medium term. The investment is also suited for buyers who are looking for a type of product with a decreasing tendency on the value of the underlying.

The product is classified as a Senior, which ranks it above normal products. It is also a Bearer Product, meaning that it is issued by a company and sold directly to an investor, without being registered.

Lastly, the bank recommends the buyer to hold the product for five years, which classifies as a medium-term investment, but there can be an early termination at wish, where the product will be sold at the market price if there is demand for the product.

3.1 Product Decomposition

The Cirdan Phoenix Autocallable Worst of Certificates is a complex product. To analyse deeply his structure and behaviour, a decomposition must be made, dividing the product into sections that are simpler to understand.

Starting by the end, one of the main sections is the at maturity payout section. This part displays the payout structure that an investor is entitled to if the product stays alive until the predefined maturity. At this date, the performance of the reference underlying is compared with the Capital Barrier that were defined at the issuance, and if it is performing at or above 60% (price is at least 60% of the original value), the investor receives the total denomination, that is 1,000€ in this case. If it is underperforming that barrier, then the investor will only receive the same denomination, but multiplied by the performance of the reference underlying. In a scenario where the performance is 50%, the investor would only receive 500€ (50%*1,000€).

The other type of payout is during the lifetime of the product, up until it is terminated, which can happen before maturity. Every month the reference underlying performance is compared with the thresholds, in this case the Coupon Barriers, and if it as at or above, the investor is entitled to a coupon in the value of 0.5% of the investment. The Coupon Barriers are at the same level as the Capital Barrier.

The investor may receive more than one coupon in a single Coupon Date, as the product features memory for the coupon's payout. In practical terms, this means that if an investor is entitled to a coupon, than he will not only receive that one from the month in question, but all the previous unpaid coupons up until the last month where a coupon was paid (and, if it is the first coupon to be paid ever, than all the coupons from the issuance of the product). Analysing this feature, we can conclude that there is only one possibility for the investor to not receive a coupon, which is in the months between the last coupon paid and the Maturity Date – the closer the last coupon paid is to the maturity, the more coupons the investor will receive.

All the payout scenarios are hypothetical, as there is a second feature in the product, which is the Automatic Early Redemption (AER), that creates the possibility for the position to be terminated earlier. As the name suggests, if the reference underlying performance goes above a certain level – the AER Barrier (which is decreasing along the years) – the product is terminated earlier, and the investor is entitled to the total amount of the Denomination. There is, however, a grace period in the first six months where the AER is not available. This feature protects both the buyer against unwanted future fluctuations in the underlying's price, and the issuer against an increased loss, by cutting the amount of possible (and, at this level of performance, probable) coupons to be paid.

Each feature reacts differently with the movements of the market and therefore have different roles on the product. The memory feature poses as a more aggressive feature, as it increases the price with the outcome of increasing the probability of receiving more coupons. Statistically, if the coupons didn't had memory, the overall payout average would decrease. However, this does not mean that the investor will end up with profit, as the biggest stake of the payout is due when the product is terminated. Hypothetically, an investor could receive all the coupons and still register a loss because of the capital received in the denomination. The combination of both the coupons and the payout at maturity is what will define the overall result.

The AER feature, despite not increasing the amount of possible payout, it increases the probability of the investor to receive the total denomination. By not having to wait until the maturity date to receive the denomination, the investor eliminates a lot of movement in the underlyings, and cashes out as soon as possible, meaning it is a more conservative feature. It also protects the issuer, as it eliminates a scenario where the investor would continue to receive coupons and receive the same Denomination at maturity. Therefore, this feature benefits both parties of the contract.

3.2 Advantages and disadvantages

Like any type of investment, the investor must make a choice. For that, he has to compare the advantages and disadvantages of the investments available.

In this chapter, the advantages and disadvantages of this unique product will be analyzed, followed by the risks that come along with buying and holding the position.

The product can be looked from different perspectives. Each one of them will have a different interpretation on what may be an advantage and a disadvantage of the product. For this analysis, the focus will be on the investor's perspective, and the advantages and disadvantages will be mostly analytical to avoid taking parts in the investment decision.

The product presents two strong and unique characteristics – the memory feature and the early redemption feature. The memory creates the possibility of receiving 60 coupons along the lifetime of the product. This means a possible 30% profit if all the coupons are redeemed. Considering the Coupon Barrier, this scenario can be considered probable, being only offset by the possibility of early redemption. The memory feature increases the average amount of coupons paid by a significant amount. This effect on the profit side as a cost, which is reflected in the theoretical price.

The early redemption can be considered a defensive feature, as it creates a safety net for the investor, that, although limited by the barrier, it allows for an early termination of the product, securing the money of the coupons (since the coupon barrier is always lower than the AER barrier) and of the Denomination.

Moving on to the disadvantages that the buyer may face when buying or holding the product, we can start by analysing the complexity of the asset. It is composed by multiple unorthodox features like the memory, the early redemption, and the worst of basket of underlyings, that complexify the interpretation of the product. There are multiple risks that come with this type of features and that unexperienced investors may not recognize. We must acknowledge that are linkages and details in the product that are very difficult to valuate and therefore, as recommended by the bank, only experienced investors should consider this investment.

The fact of being a structured product creates a problem common to this type of investments, which is liquidity. In the capital market, currency market, commodities, among others, investors can terminate their position and receive a market price close to the fair price, which allows to switch positions significantly fast and at a small cost. In the structured products market however, this is much more complicated, as the demand for this type of investments is much narrower, and they are usually traded over-the-counter. Therefore, if an investor wishes to sell his asset, he will pay higher fees that will cut the profit or increase the loss, if there is even a market for the product. If not, the investor will be stuck with the position.

Still in the profitability area, we can observe that the denomination of the product is capped and, consequently, the total profit of the product. When having total exposure to the capital markets, investors may have their total capital at risk, but they also have unlimited potential for returns. This happens as there is no limit to the price of a stock, which means that, although the value won't increase continuously, in theory, there is no limitation to how high a stock can reach. With this asset, only the first part stands, as the investor may lose close to the total amount invested, but is limited on the potential gains, as both types of return on the investment have a fixed amount (or a fixed maximum amount in the case of the payout at maturity). For a product considered risky by the bank, this limitation is a downside and can be considered inadequate.

Another disadvantage of the product is that is not as diversified in terms of underlyings as it may seem. If we look deeper into the basket, the four underlyings are quoted in the same stock exchange, operate in similar countries, and are exposed to the same market risks. Therefore, the only diversification that happens is in the sector of activity of each underlying. This creates some correlation between the assets, and, in a stressed scenario, it can gravitate the reference underlying into lower values. Nevertheless, as the valuation of the reference underlying is through a "worst of" method, meaning that correlation is relevant if it leverages the asset to underperform. If not, then the correlation factor is irrelevant, as only the lowest performing one will count.

In this case, the worst of feature can also be seen as an advantage, as it may gravitate the lowest performing into higher values – if one asset is performing well and correlation is high, then it is unlikely that the other assets aren't performing similarly.

Moving on to the third main disadvantage of the product – the level of the Barriers of the product – Coupon, Capital, and AER. The gap between the defined values of the three can be considered relatively short. This not only implies that if the investor is in a favourable position, the product may terminate earlier, and if not, it may continue to lose the money of the investor, but also implies a triple penalisation for the investor in case of poor performance. In fact, if the reference underlying is below the sixty percent level and stays there, the investor will not redeem any coupon, the product will not terminate earlier and, at maturity, it will not receive the total denomination, after holding the position for five years. This means that the protection in the investor's perspective is quite limited.

Lastly, the product doesn't pay dividends despite being linked to stocks that do pay them. Considering that in the ex-dividend date the shares price tends to drop, the performance of the reference underlying may suffer with it, and consequently affect the coupons redemption and/or the nominal pay-out. In the investor's perspective, this creates a risk that is being forcedly incurred, but not compensated by the market.

3.3 Risks

There are a few risks that must be considered for the valuation, as they have an important role in the price and the client must be aware of them.

There are 10 main risks identified that are associated with this product. Most of them are macro economical risks, as they are related to the market itself and therefore hard to avoid, even when opting by other types of investment.

Starting by the market risk, this one can be found in almost any type of investment that is not categorized as risk-free. The capital market is a volatile environment and to this day there still isn't a strategy reliable enough to predict the market reaction to certain scenarios. Therefore, there is a risk of loss for the Cirdan product, as it is directly exposed to the Italian capital market, inheriting the risks associated with it. This will be reflected both in the price of the product and the payout that the investor may receive from it.

Another type of risk common to exotic options and structured products is the complexity risk. The complexity of the product creates a challenge when it comes to find the fair value. An investor that is not used to these types of products will overlook certain relationships inside the product composition, dependencies that are invisible to the common man, and overall details that will not enter the equation due to a lack of knowledge. This means that there is always a risk that the investor may be overlooking some details and deciding without knowing all the information.

The fact that the product includes an early termination feature creates a scenario of unpredictability along the timeline of the product. This creates doubt and it challenges the investor to estimate what the investment horizon of the product may be, as he may be facing from a six month to a five-year investment horizon. It also makes it harder to develop an investment strategy, as you will always have to consider that the product may terminate in each month.

There is also interest rate risk, as the product was issued at a time where the interest rates were facing a low season. If the interest rates change, there may be fluctuations in both the price of this product and its substitutes. Since shares and interest rates are negatively correlated, and as the interest rates are below average, the investor may face an increase in the interest rates, and therefore the stock's price may start to decrease, which will automatically increase the worth of assets directly related with the said interest rates and may become a more attractive alternative.

In these types of instruments investors must always account for credit risk. Since the product is issued by a private firm, and therefore non-government backed, the risk of total loss of investment due to default is higher. According to the technical sheet of the product, the bank indicates that, if severely negatively affected by the capital markets, there is a risk of default of the company, and the buyer of the product is totally exposed.

The structured product market has less transaction volume than the regular capital market, and therefore it is considered less liquid. The illiquid markets face higher transaction fees due to the increased complexity of the trade, and there is often a bigger gap between the bid and ask price. This presents a risk for the investor if he wishes to sell his position on the product, as he may be unable to do it, or may be forced to accept a price lower than what he would expect or consider to be fair.

As referred previously, there is the possibility of total loss of capital, and that there isn't guaranteed or backed amount in the investment, which therefore means that there is capital risk, corresponding to the price paid by the investor.

Other factor that may affect the price and the payout of the product is inflation. As inflation is what generates the real value of investments, this puts at risk the real value of the payout of the product. As fluctuations may defer from what is accounted for in the computations, errors could be made, and the results may defer from what is expected. This could create a profit or a loss for the investor, as the product is not linked to any type of inflation-linked Note and therefore the performance of the expected inflation versus the observed inflation will affect the fair value of the product.

Another feature that presents a risk to the investor is the Worst of method. This implies that the performance of the Reference Underlying, and therefore the payout of the product, is dependent on what could be an outlier of the market. The product is at risk of performing below the market averages due to an outlier, which removes the diversification advantage. However, the opposite doesn't happen, as if there is an outlier that is outperforming the market, it would not affect the product as there would still be a worse performing stock that would limit the performance of the reference underlying.

Lastly, there is correlation risk. As referred in the disadvantages, there is correlation between the stocks, mainly due to being traded in the same national capital market. Although the computations that will be presented in the following sections have accounted for this correlation when generating the performance of each stock through the Cholesky decomposition, it is always a risk for the investor, as it may leverage the values. It also reduces the diversification effect of the basket of underlyings.

4. Valuation Methods

To obtain the fair value of the product and therefore discuss the quality of the investment opportunity, a model should be created and tailored to reproduce accurately all the characteristics that make this product unique, and therefore obtain the theoretical price. For that, there are 3 models available, the Black-Scholes Merton Model, the Binomial Tree Model and the Monte Carlo Simulation Model.

For comparison purposes, a simulation of the theoretical price was reproduced using each model. However, and considering the unique features of the product, the most accurate and the one that should be considered as correct is the result obtained from the Monte Carlo Simulation. This is mainly due to two of the features of the product, the Memory feature incorporated in the coupons, and the Automatic Early Redemption.

For the Black-Scholes method, it is not possible to include these features in the pricing, as it is a closed-form model and can't be adapted to non-default derivatives.

The Binomial Model allows to program it to include the Automatic Early Redemption, but it does not allow to include the memory feature as each coupon is treated as a separate option and it cannot activate options that have already expired.

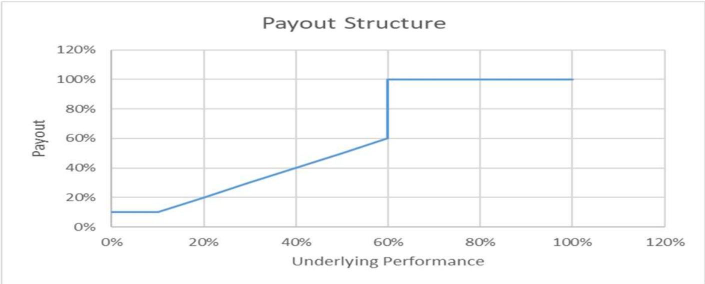
4.1 Black-Scholes Merton Model

The Black-Scholes Merton Model is the simplest of the three, as it has a predefined structure, where it returns the theoretical price after including the necessary inputs. It is composed by closed-form formulas, therefore there isn't room for adaptation, which makes it unfitted for more complex derivatives. In fact, the model only allows to compute European Calls or Puts, therefore the only way to obtain the price of a structured product is by decomposing it in simpler derivatives.

For this case, the Cirdan Certificates were decomposed in multiple derivatives, to apply the model for each one of them and combine the individual value of each to obtain the theoretical value (Table 1).

Starting by the end, the payout at maturity is composed by one long binary call option with strike at 60% of the value of the reference underlying at moment 0, and a payout of 40% of the total Denomination (Table 2), creating the straight line in the payout structure graph (figure 1). The binary (or digital) call is combined with a long call spread strategy (investor goes long in a call option with a lower strike price and goes short in another call with a higher strike price), that will generate the payout if the reference underlying is performing below the threshold. This will create a slump around the capital barrier and a declined line from that reference (figure 1), while also pushing up the straight line.

Figure 1 - Payout Structure



Source: Own elaboration based on the product technical sheet

This part of the payout is not fixed, but proportional to the performance of the underlying. Which means that the underlying price at moment 0 needs to be the same value as the Denomination (1,000€) to obtain the desired payout (in digital options, we can incorporate an independent value for the payout). The combination of calls is composed by a long call with a strike price of 10% of the initial price (Table 3), and a short call with 60% of the initial price (Table 4). This creates an interval of profit with a floor and a roof, as it will never be below 0, and will never exceed 500€ of payout when the underlying is at 60% performance (investor can buy at 100€ in the call and sell in the market at 600€). If the underlying value keeps rising, the profit from the long call will be offset by the money lost in the short call, as the buyer will exercise it, caping the profit at 500€ - 50% of Denomination.

However, it needs to be combined with a long, guaranteed capital instrument that would push the combination of the two up to the point where it would create the 10% capital protection, creating the desired structure. This ZCB will however push the binary call option up as well, and therefore needs to be considered, so that the combined payout with the ZCB is at the 100% level. In the best-case scenario, the investor will receive 100€ from the ZCB, 500€ from the combination of the calls, and 400€ from the digital option, which adds up to the required 1,000€.

For the rest of the lifetime of the product, there are two types of payouts to be considered: the coupons and denomination in case there is an early redemption of the product. Those were generated by creating one long digital call option for each coupon date and one for each AER date. The ones that would replicate the coupon payout, had a fixed strike price of 60% and a fixed payout of 0.5% of the denomination (Table 5). The ones replicating the AER had different strike prices, as the AER barrier decreases over time, and a fixed payout of 100% of the denomination (Table 6). Overall, it was created 60 digitals replicating the payout of the coupons and 54 digitals replicating the payout of the early redemption, in case of occurrence.

There are six variables needed to operate the formula: volatility, price of the underlying asset, the strike price of the option, the time until the expiration of the option and the risk-free rate and dividend yield. There are also a few assumptions that must be considered. The main one is that it assumes that the prices follow a random walk, which means a lognormal distribution with constant drift and volatility of its values. From that, there are 6 assumptions that are implied in the price obtained from the model:

- Market movements are random and therefore cannot be predicted.
- No transaction costs are applied,
- The risk-free rate and the volatility are considered constant.
- The returns of the underlying follow a normal distribution.
- The option can only be exercised at maturity (European option)

To compute the theoretical price applying this model, firstly I had to obtain the necessary inputs for the calculations. The strike price and the time until the expiration were obtained from the product technical sheet. The price of the underlying was considered 100, as the product is evaluated in a performance base, therefore it looks into a percentage, and the price at moment zero is always 100% (for the call options, this could not be applied because of the payout amount, as explained previously). Lastly, the risk-free rate represents a product with maturity of 1-month, that will be rolled over of each time step. This is due to the possibility of an early redemption, and in that case, the entire product needs to be terminated and the position closed. For a product with guaranteed capital with longer duration, there was the risk that the derivatives had already closed the position, but this product would still be active.

The volatility was obtained by calculating the observed standard deviation from a Monte Carlo simulation that generated 10,000 simulations of the evolution of the price for each of the four underlyings, considering their specific characteristics (historical volatility, dividend yield, etc.) and combined the lowest performing one in each observation in the reference underlying. The dividend yield used was the average of the four dividend yields that were used in the referred simulation.

I then combined them to apply the Back-Scholes equation (equation nº3) to obtain the d2 value. From that, I used the Excel formula to obtain the normal distribution of that value to compute the digital option value. From the Black-Scholes method, the digital option price is obtained through the following formula (equation nº1):

$$V(S,t) = e^{-r(T-t)}N(d_2) \quad (1)$$

This method was repeated for all the binary call options, making the necessary changes. For the call options at maturity, the Black-Scholes formula used was slightly different, as the model needs to be adapted for the options. For the call options, the original formula was applied, so I had to compute d1 and d2 from the same formula (equation nº3 and 4), and then applying the same method as before to get the normal distribution of both. They are then combined in the original formula of the model (equation nº2) to obtain the theoretical put price of the option.

$$C = N(d_1)S_t e^{-qt} - N(d_2)Ke^{-rt} \quad (2)$$

$$d_1 = \frac{ln\frac{St}{K}+\left(r-q+\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \quad (3)$$

$$d_2 = d_1 - \sigma\sqrt{t} \quad (4)$$

The combined value of all the derivatives used and the guaranteed capital product considered in the calculations is 20,918.82€ (Table 7).

However, as referred, this value I not accurate, as the Black-Scholes Merton Model is a closed formula model and doesn't have the capacity to consider out-of-the box characteristics that are included in exotic options. The model cannot consider the memory in the coupon's payout, and therefore only pays one coupon at the time. It also cannot consider the early termination, and so if an AER occurs in one of the binary options, the other would still be active and so there is the possibility of multiple AER. Therefore, the best proxy to consider would be to only account for the coupon's payout and the payout at maturity, which combine in a smaller value of 789.38€ (Table 7).

The value obtained can be compared to the Monte Carlo theoretical price of the product without AER and without memory, as it also ignores the same details and therefore should represent the same variation of the product.

4.2 Binomial Tree Model

The second method is the Binomial Tree Model. This one allows for more personalization in accordance with the specifications of the product. The model follows an iterative path that allows for specification of nodes or paths, or even barriers that may affect the price of the product. It consists of a tree where each node has two paths to follow, to move up or to move down, and that process is repeated for each timestep. Each node represents the price of an underlying in a certain point in time. Altogether, it will generate a path representing the evolution of the underlying from the moment zero up until the maturity of product, generating a value for each timestep defined.

From that point, a second tree is generated, that will calculate the value of the option in each node, according to the price associated with it from the first tree. In the last timestep, it calculates the price based on if the option was exercised or not and the profit obtained. From that, the time-value and the intrinsic value is accounted for, and the value of the option is obtained by discounting this profit for the previous nodes.

If an early redemption occurs, then in that node, the value will be of the denomination paid and not the discounted profit at the redemption. After computing everything until the issuance date, there will only be one value left in the tree, which represents the theoretical price of that option.

This model presents a few advantages when comparing to the Black-Scholes model. It allows for visualization of the evolution of both the option and the underlying. It also allows to incorporate early termination of the option (American options) and it is mathematically less complex.

Nevertheless, there are a few disadvantages when applying this specific model. The first one is that it is hard to incorporate the memory feature, since that would imply that an option wouldn't have a defined strike price, and it would change accordingly to the payment of previous coupons. This would over complexify the model, and therefore is not optimal for the price estimation. Nevertheless, it allows to incorporate the early redemption feature, although it implies the computation of 56 binomial trees, one for each possible early redemption.

Similarly to the Black-Scholes Method, the Binomial Tree also requires inputs, which are the price of the reference underlying at the issuance date, the risk-free rate, the time horizon between timesteps and the dividend yield. The process to obtain these inputs was the same as the one applied in the Black-Scholes Model.

To obtain the theoretical price of the Cirdan Certificates, the first step was to compute the variations that each iteration must have in case the underlying rises or falls in price. The factor that determinates the variation in case of a rise is the u factor and is calculated through the following formula (equation nº5). Following the same logic, the factor that determines the variation in case of a downward move is the d factor and is the opposite fraction of the u factor, obtained by the following formula (equation nº6).

$$u = e^{\sigma\sqrt{\Delta t}} \tag{5}$$

$$d = \frac{1}{u} \tag{6}$$

For this model, similarly to the Black-Scholes one, instead of an absolute value, a percentage was considered, since the Cirdan Product focuses the payout on the performance rather than the price of the underlyings itself. Therefore, the 100% value assumed at the moment zero was multiplied by the u factor and the d factor to obtain a binomial tree that expands as the time horizon increases. This tree represents the evolution of the reference underlying along the timesteps. Therefore, to obtain the payout related to the performance of the underlying, a second tree was created, that identifies in which time steps is there a payout and then discounts them to the present value. This discount was made using the normal inputs of the timestep (delta t), the risk-free rate, and a new variable, the p value. This new variable represents the binomial (two-way possibility) probability of a certain scenario to happen. Let's imagine that we have two possible prices in a moment t+1, one is the payout of the option that comes from the upward movement of the underlying in moment t, and the other comes from the downward movement of the same underlying at the same moment in time. At moment t, the expected value of the option is the combined expected value of those two scenarios. The probability of the up scenario is given by the binomial probability (p-factor), and is given by the equation below (equation nº7), and the down scenario is the remaining probability, meaning 1-p. This process is repeated several times, until all the possible scenarios simulated combine into a single expected value. As we can observe, the p-value is a combination of both the u and d factor calculated previously.

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d} \tag{7}$$

Considering that the product is composed by a possible of 60 coupons (Table 8), 54 early redemptions (Table 9), and then a three-way payout at the maturity (Table 10) (10% of denomination for a performance at or above 10%, 100% of denomination for a performance at or above 60%, and the equivalent amount of denomination according to the performance for the remain values), this process was repeated 115 times. All the trees also had to consider the early redemption of the product, and therefore, if, in a certain iteration, the AER was activated, all the subsequent trees would have a value of 0.

After combining all the simulations, a theoretical price of 899.36€ was reached (Table 11). This value can be compared to the Monte Carlo price with early redemption and without memory, as it is not possible to consider memory between separate simulations in the binomial model.

This simulation shows that the Binomial model, although it displays a more flexible structure, being able to consider non-standard characteristics like the AER in the computations, it has some limitations when it comes to higher levels of complexity, being also a harder model to develop and adapt to the circumstances.

Nevertheless, it features one big advantage for analysis and educational purposes, as it is very visual and we can observe the evolution across the timesteps with this model, something that is not as straightforward with the other two alternatives.

4.3 Monte Carlo Simulation

The model chosen to do the computations and obtain the most accurate theoretical price possible was the Monte Carlo Simulation due to its high ability of adaptation to unique cases. It is also the most widely used model in the market.

This model consists of generating a large sample of randomly distributed observations, that follow a normal distribution and that incorporates characteristics defined by the asset that is being estimated, to generate multiple scenarios that represent the infinite possibilities of the future performance of that asset.

Again, there were some parameters that are common between the models and that also need to be included in the Monte Carlo to perform the simulations. By inputting the volatility, risk-free ratio, dividend ratio, timestep interval and the spot price of each underlying – this creates a sample of randomly generated scenarios that, through the Central Limit Theorem, generates a reliable prediction of what the prices of the shares may be in each timestep. This sample will be the basis of the computation, and all the steps after it are specific of each scenario and won't be equal to every application of the model. In this case, this process was repeated four times, creating a price prediction for each share included in the basket of underlyings. After that the prices had to be converted into percentages to obtain the performance of the shares, and then create a new sample which was based on the asset with the lowest performance. If we considered the share price, the reference underlying would consist mainly in the cheapest stock and would not reflect the intent of the product.

The next steps were computations that based on the reference underlying, for each observation and in each timestep, calculated if the holder of the product was entitled to an early redemption and, if not, if it was entitled to a coupon. After considering all this information, the values in each timestep were discounted to their real value and considered into the theoretical price. The discounted values of both the coupons and the denomination received, whether it was through an early redemption or at maturity, are all added up to obtain the total value hypothetically received for that observation. By repeating this process for the 10,000 observations, we will find ourselves with a large sample of different theoretical discounted total values. To represent the theoretical price of the product these values were averaged, since the average value of a large sample is usually considered the expected value. This means that the average discounted total value received represents the theoretical price obtained through the simulation.

4.4 Data Analysis

To perform the price estimation, there were inputs that were needed to complete the computations. In this section, those values will be analysed to better understand variables such as risk-free rate, dividend rate, volatility, and correlation between assets.

The risk-free rate was obtained from Bloomberg. To represent as accurate as possible the risk-free rate implied in the product, the asset chosen was a German Bond maturing at the same time as the structured product. This allows to represent the estimated rate in Europe for the next five years, in one of the highest rated bonds in the world. Also, the currency of the security is the Euro, the same as the product, so we don't have to consider any conversion rate in the computations, avoiding currency risk.

The dividend yield was also obtained from Bloomberg, and it comes directly from the balance sheet of each of the four firms that compose the basket of underlyings. All the yields are according to the last dividend paid and are not fixed, as each company has the power to compensate their shareholders according to their unique policies.

The volatility was also extracted from Bloomberg, and it represents the volatility implied in the stock variations of each firm. The value is computed automatically from their historical prices of the five years before the issuance date.

The discount factor is based on the financial reports of the Cirdan Capital Management. Since no other information was disclosed by the firm, the value was defined as the percentage of the interest paid by the firm on the long-term and short-term debt disclosed in the balance sheet of the company.

Lastly, the correlation between the assets was also obtained through Bloomberg. This was computed by regressing the registered daily prices of each stock and understand their movements in the market when compared to each other.

For simplification purposes, I assumed that the issuance date was equal to the strike date, and that the maturity date is equal to the redemption date, across the three models.

5. Product Valuation and Risk Level

As referred, the product valuation was based in the Monte Carlo Simulation (MCS) model due to the complexity of the product.

The basis of every MCS is to obtain a sample of random observations that follow a normal distribution. This is to incorporate the random outcome variable into the computations. In fact, the premiss of the MCS is to obtain an outcome estimation when the potential for random variables is present.

The images presented as support of the report are just examples of the total matrixes, as the tables were too large. For a detailed overview on the computations, please use the support Excel files of this project.

Following that, a 60 by 10,000 matrix was created (Table 12 to Table 15), which represents a sample of 10,000 observations for each date stipulated in the product. Those values were obtained by combining the norm.inv() and rand() excel formulas that generated random probabilities that followed a normal distribution, and that will dictate the behaviour of the price and performance of the underlyings. Since each underlying acts separately, the process was repeated until each underlying had its own, independent 60 by 10,000 matrix.

Following that, and before going through the process of converting the probabilities into actual underlying prices, there is a step that needs to be performed. Although independent, each underlying has some level of correlation with the others. In this case, since the underlyings all belong to the same capital market, their level of correlation is even more relevant and must be considered in the sample. The MCS however, doesn't account for correlation, but there is a method that allows simulations to convert independent random samples to correlated random samples, by including the adequate level of dependency in the probabilities generated. That is called the Cholesky Decomposition.

In theory, the Cholesky Decomposition or Cholesky Factorization is a decomposition of a Hermitian, positive-definitive matrix into the product of a lower triangular matrix and its conjugate transpose. In practice, and following the formula presented above, the independent random samples are converted into an image of themselves, but considering the correlation that there is between the assets in question. For this case, the samples will only account for the correlation between the four underlyings, and not external factors.

For simplification purposes, the equation was divided into steps. Firstly, and to obtain the matrix that will define the rest of the process, the L matrix was obtained (Table 16) throught the formula below (equation nº8). For confirmation purposes, the L matrix was multiplied by its own transpose version (Table 17) and the correlation matrix was obtained (Table 18). After that, the independent samples (X1, X2, X3 and X4 – Table 12 to 16) were multiplied by their correspondent value in the L matrix and the correlated samples (E1, E2, E3 and E4 – Table 19 to Table 22) were obtained. For the rest of the model, only the correlated samples will be considered.

$$A = L * L^T \tag{8}$$

The next step was to convert the correlated samples format, from probabilities into actual prices of the stocks, considering the characteristics of each one of the assets (Table 23 to Table 26). For this step, an assumption was made – that the stocks followed a Geometric Brownian Motion (stochastic process that is continuous in time, where the logarithm of the random quantity follows a Wiener Process with drift). The computations were made following the equations below (equation nº9, 10 and 11).

$$S_k = S_0 * e^{\left(\mu-\frac{1}{2}\sigma^2\right)} * e^{\sigma b_1} * \ldots * e^{\sigma b_k} \tag{9}$$

$$S_k = S_0 * e^{\left[\left(\mu-\frac{1}{2}\sigma^2\right)*k\right]} * e^{\sigma \sum_{i=1}^{k} b_i} \tag{10}$$

$$S_k = S_0 * e^{\left[\left(\mu-\frac{1}{2}\sigma^2\right)*k+\sigma \sum_{i=1}^{k} b_i\right]} \tag{11}$$

In a simplistic point of view, this means that the prices obtained combined the random correlated probabilities with the risk-free rate and the time interval between observation dates (common inputs to all the assets), the price of each stock at moment 0, the standard deviation observed for each underlying and the dividend yield.

In structured products, if the underlying is composed by only 1 stock, then the computations can be made considering the real price of the asset. However, it would create a biased estimation, since the shares with the highest absolute price would always be superior, not reflecting how did the group of shares behave. For that reason, the prices computed in the step before were then converted to relative values, dividing the computed price by their original price, therefore obtaining the performance of each underlying (Table 27 to Table 30).

As referred to previously, the reference underlying follows a worst of method, and therefore its matrix will have the same size as the four previously calculated – 60 by 10,000 – and it will be formed by the worst performance percentage of the four underlying matrixes, in each cell (Table 31). This means that there are only four alternatives for each matrix input and that each observation is not dependent on the previous one, meaning that one of the assets may represent the reference underlying in a certain timestep, and not in the following one.

This reference underlying will be the base of all computations. The analysis is made month to month in the five-year span, and it will be made in two parts: the Coupons Payout and the Principal Payout.

Starting by the Coupons Payout, the first step in the computations is to define when does an AER occurs (Table 32). For that, a matrix was created, which defines in each of the 10,000 simulations if the product hasn't been redeemed, represented by the value 0, if the product is redeemed in that timestep – represented by a 1 in the cell, and if a AER has already happened and therefore that product is terminated, it is represented by a " –" in the observations until the end of the 5 years. As explained in the previous sections, the AER is dictated by the AER barrier.

A Coupon Payout matrix is then created (Table 33). The cells where an AER hasn't occurred yet, and if the reference underlying is above the Coupon barrier, then a coupon must be paid. The value to be paid is calculated by multiplying the defined coupon value – 0.5% of the denomination – with the previous unpaid coupons, covering all the previous months where a coupon wasn't redeemed (Memory feature) (Table 34).

The next step was to obtain the principal payout, using the same matrix method (Table 35). For the cases where an early redemption occurred, a payout of the total denomination was considered and included in the principal payout matrix, in the correspondent timestep where the redemption occurred. For the observations where there wasn't an AER and the product reached its maturity, then the payout at redemption rules were applied. This means that, if the reference underling is above the Capital Barrier, the final settlement amount in the last timestep would be the total denomination (as in AER). If not, then the payout would be the denomination times the performance percentage correspondent.

At this point, there are two payout matrixes, that are dependent on the performance of the reference underlying, to define the payout of the coupons, and to define the payout of the denomination.

The last step of the computations is to combine the two matrixes in one, that would represent the amount paid in each timestep for each observation and discounting it properly accordingly to the month in question (Table 36). The value used to discount the observations into their real value was the discount factor.

The last matrix of the total discounted payouts in each observation is condensed in a single column, that sums the payouts received in each observation for both the coupons and the denomination, representing the real value received by an investor in each simulation (Table 36). Following the Monte Carlo theory, the theoretical price was then obtained by averaging the total amount obtained in each simulation -the average expected payout – which sums up to 720.43 EUR.

There are two main factors influencing this price: the memory feature and the EAR feature. The memory, by increasing in large the profit capacity of the product through increasing the average coupon payout along the lifetime of the product, is one of the factors that increase the price. As mentioned previously, this feature increases the chance of profit and therefore works as a leverage for the investor.

To understand how deep the impact of the memory is, a theoretical price following the exact same method but removing the memory feature from the calculations was computed, and the suggested theoretical price is 717.36 EUR (Table 37). To obtain the value, the only difference made was to consider a payout of only one coupon for each month where the Reference Underlying is above the Coupon Barrier.

The difference is relatively short, despite the features increasing drastically the probability of receiving a coupon, at any point during the lifetime of the product. This is due to the AER feature. The AER shortens deeply the average lifetime of the Cirdan Product, and therefore also shortens the average coupon paid. Also, the fact that the Coupon Barrier is at a significant low level, especially for the first months, where a drop in the stock prices in unlikely, means that for those months, the difference in the average amount of coupons paid is similar with and without the feature. Only in later months it could have an impact in the total amount of coupons paid. However, the probability of being redeemed before is high due to the Automatic Early Redemption.

The second feature – the AER – is the biggest buffer on the price, and therefore on the profitability of the product. In fact, most of the payout of the product comes from the denomination, which is paid at termination. That can happen only at the end, and with different values, or, with the feature, at any month (except the first five months) in the lifetime of the product and at the maximum value that the investor could receive without the feature.

Like the previous feature, a price estimation without the AER feature was obtained, to understand the deepness of the impact it has on the price of the product (Table 38). The suggested theoretical price is 588.73 EUR, which is a much smaller number than the original one. This value was obtained by removing the AER matrix mentioned previously, and adapting all the computations that were dependent on that to perform without it. The change impacted both the Coupons Payout and the Payout at Maturity. With that, the number of coupons being paid increased as the expected lifetime of the product increased as well – now the only way to terminate the position earlier is by selling in the secondary market, which, as specified by the bank, is an illiquid market that features high costs. The payout at maturity also increased, but not in function of the value being paid, but rather because of the increase in the probability of occurrence, as it became much more likely that the buyer would hold the product until the maturity, even if he was already expecting a loss, as selling would increase costs. Consequently, this increased the average lifetime of the product.

The feature can, therefore, be considered the major factor that increases the value of the product. This is due to the safeness that the AER creates around the investment, as in a five-year span, there is a significant chance that the stock prices would gradually decrease and may stay below the Capital Barrier at the termination date. In this case, the stock is less likely to have a sudden drop, and even if so, it is most likely to suffer a following correction in the prices, as it is hard to have a sustained, drastic decrease in the stock market.

On the other hand, the feature also protects the bank in case of a favorable scenario. In this case, the issuer would likely have to pay a large number of coupons (as the barrier is at a significantly low level), while also having to pay the same amount of the denomination at maturity (Final Settlement Amount), which would represent a higher loss for the firm. With that being said, the feature then works for both sides of the investment and is a feature that both parties would agree to add on.

Lastly, and still in the AER subject, the feature also decreases the average lifetime of the product, and therefore increases the average return per year, as the payout values aren't really affected by the early terminations.

If we remove both memory and early redemption from the computations, we get a theoretical price of 584.62 EUR. This substantially lower value reflects the impact of combining both features in the product and its importance for a risk-seeker investor, as he is willing to pay 23 % more to increase the potential payout, even if that translates into an increase in the risk taken.

Without the features we could argue that the product would sit at a lower place in the ranking, but that would attract other type of investor, and drift the whole purpose of the product and the market strategy of the Cirdan Capital Management.

6. Delta Hedging

By entering the position, the investor will assume a certain amount of risk. That risk, as analysed in previous sections, is directly linked to the performance of the underlyings. In fact, most of the exposure of the product derives from it.

One of the most common strategies to reduce the risk incurred is to create a strategy that will work as the opposite of the initial investment. Therefore, when the main product decreases in value, the second one would increase and offset the loss of the first investment. This is called a hedging strategy, meaning that the investor is hedging his risk, by “betting” against himself. In theory, the perfect hedge is when the second portfolio inversely replicates exactly the fluctuations of the values of the underlyings in the first one. However, that is not possible which means that there will always be a loss or a win deriving from the hedging strategy.

There is another factor that can affect the level of a hedge, which is the risk appetite of the investor, since the closer to perfection the hedging is, the less profit the investor can make. This means that certain people may prefer to hedge less or not hedge at all in order to leverage their profits, accepting the consequences that come with it.

The biggest question in this subject is how to perform that hedge. The answer would be to find the perfect relationship between a structured product and the underlyings that are linked to it. There are multiple types of relationships, which are called “The Greeks”, because each relationship is called a certain Greek letter (e.g.: Vega, Theta, Gamma, etc.). The most common and the one that we will be using for the strategy is the Delta.

The Delta represents the direct relationship between an underlying and a structured product. In other words, it represents how much the value of a certain product would change (positively or negatively) when there is an infinitesimal change in the price of the underlying asset.

In practical terms, a delta of 0.5 means that if the price of the underlying changed by 1, the product value would change by 0.5, and therefore the delta hedging portfolio would be composed by the value of the product time the delta, so that the values would coincide. However, the portfolio must be in the opposite position of the structured product. So, if the product is long on the underlying, the hedging portfolio must be shorting the stock.

In that case, if the stock would increase its value by 1, the product would increase by 0.5, and the portfolio would decrease also by 0.5, keeping a neutral position despite the fluctuation in the value of the underlying.

For the Cirdan product, to obtain the Delta Hedging Portfolio, the first step was to obtain its Delta. As the product is dependent on not 1, but 4 underlyings, this means that there are 4 Deltas to be considered that will explain the variations of the product. Therefore, the following steps were repeated equally between the Deltas.

The first step was to create a new simulation, where every step was equal to the original ones, but the price of one of the stocks was increased by 1 percent (Table 39). By doing so and repeating all the steps that would generate inputs based on the stock price, a theoretical value of the product considering a value of the stock A 1% higher than its original price was obtained. The value of the product will logically go up as the underlying also rises in value - the higher the stock the more in the money the options will be (or less out of the money).

After subtracting the new value by the original one, we can observe the variation of the theoretical price, which represents the sensitivity of the product for a 1% change in the underlying (Table 40). From that, we can extrapolate the true Delta, which is the variation of the product for a 1-unit change in the underlying, which in this case, it represents an increase of 1 euro in each of the stocks (Table 41). As expected, the more expensive a stock is, the smaller the delta is, since the unit measure has different proportions for different stock prices.

It is important to refer that, after changing the price of the stock, the new theoretical value of the product is computed considering again the Cholesky decomposition, which means that the new theoretical value already includes the correlations between assets, and therefore the correlation that is implied in the variation of the stock. In other words, if we change the price of one of the stocks by 1%, the others will also increase because of the correlation between the assets – not only the prices, but the Deltas are also correlated among underlyings.

The Deltas obtained at issuance date for a 1 unit change in the price were 173.22 for the Intesa Sanpaolo SpA stock, 12.37 for the UniCredit SpA, 25.79 for the Eni SpA and 0.0271 for the Fiat Chrysler Automobiles NV. There is a clear tendency for the Intesa Sanpaolo SpA exposure. As explained before, the discrepancies in the values are caused by a comparison on the absolute value on the change of the underlying. If we look at the changes of a 1% change, we can observe that the values have a smaller variance (4.34, 1.71, 3.32, 0.0032, respectively).

The values obtained are higher than what is common for the delta variable. Indeed, in most vanilla derivatives, the delta stays at a value between -1 and 1. However, this interval can expand when the derivative is a structured product, as the complexity allows for them to be extremely sensitive to variations in the underlying. Considering the values of all the barriers that define the payout of the product, we can say that the product is extremely in the money, and therefore is extremely sensitive to variations in any of the underlyings.

The conclusion to be taken is that the product is actually highly sensitive to a change in certain stocks, and that the diversification of the basket can be obsolete if the market affects the right equities.

6.1 Hedging Portfolio

To perform the hedging of a portfolio, or, in this case, a product that can be deconstructed in a portfolio, investors create what is called a Delta-Neutral Portfolio. This is a portfolio with a Delta that is exactly the opposite value of the main one. This can be done through derivatives or, in an indirect way, through equities. For that, as explained before, the Delta of the main portfolio will dictate the number of shares and the position on those that will be hold for the hedge, in order to replicate the fluctuations of the value of the main portfolio. This doesn't represent a Delta-Neutral position, but it does equal the values of both portfolios in absolute terms, hedging the position.

To test the accuracy of the hedge, I simulated the performance of a hedging portfolio composed by the four underlyings of the product and compared the fluctuations in the value of both portfolios in each timestep.

As the Delta represents an instantaneous rate of exchange, it is not constant among time and therefore needs to be recomputed for different timesteps to obtain a correct hedging that would accompany the fluctuations of the market. For this case, that recalculation was made monthly at the coupons Date, for the following five months. In real terms, this would mean that there was a need to readjust the hedging portfolio according to the recalculations.

For this, the real shares prices were obtained and applied in those months, as done in the issue date (Table 42). The model was kept the same, except that the timesteps were adjusted as the months were passing by. The rest of the inputs were kept constant, as was firstly made in the original simulation, to keep the same parameters, for comparisons purposes. The reference value that was considered to obtain the performance of the underlying was also kept constant, generating performances relative to the price at the issuance date.

The performance of the hedging portfolio was calculated by accounting the variations on the prices of the stock with the exchange on the positions of the shares (since Delta is not constant, to keep an efficient hedge, the portfolio must be adjusted accordingly), the Profit and Loss was computed and compared to the changes in value of the structured product (keeping in mind that the objective of this portfolio is to reduce risk and not to make a profit, the closer the overall results are to zero, the more efficient the portfolio was) (Table 43 and Table 44).

In this case, and since the strategy was made from an investor's perspective (not focusing however, in gains and losses that derive from costs that may surge from borrowing securities and interest that may be received on cash), the portfolio would be composed by the long position in the product and a short position in the shares.

The standard deviation of the portfolio returns is 8.83%, which is a fraction of the unhedged portfolio standard deviation (30.90%) (Table 45). To measure how efficient the hedging was, I compared the variance of the Delta-Neutral portfolio with the unhedged product. The result was obtained by dividing the first by the second and subtracting it to 1 and the result was 91.83% of effectiveness (Table 45).

If we look in detail into the performance of each of the portfolios, we can observe that there was a big slump in the months following the issuance date. This was due mostly due to the Covid-19 pandemic, where the global economy was drastically affected, and by consequence the capital markets as well. This made the product lose 63.9% of its value in the first month of issuance, which could've settled the path for a large loss in the original investment. However, the delta portfolio, being short on the underlyings, provided a huge win during that month that allowed to offset that loss of value. During the next months, the underlyings started to gain value again, following a correction in the market, due to the panic generated by the pandemic crisis.

Up until July of 2020, the underlyings value increased steadily, and the theoretical value of the product followed that behavior. The hedging kept reducing the spread between the product performance and the delta portfolio performance, generating a delta-neutral portfolio with a performance of -0.91% - a minimum benchmark for the months analyzed.

For the last month, the underlyings decreased in value again, but in a much more acceptable ratio, reducing the value of the product as well, and leaving the delta-neutral portfolio with a performance of 1.57%.

The conclusion obtained from the values is that the delta hedging strategy was effective in its effort of offsetting market risk and protecting against large fluctuations. Being the product so sensitive to each of its four underlyings, it was important to prove that the strategy could perform, as the risk of capital loss was at a large stake.

The ultimate test for the strategy was the massive effect that the pandemic had on the four underlyings. Nevertheless, the delta portfolio replicated the performance of the product in relatively accurate manner, being more than 90% effective in reducing fluctuations in the value of the Cirdan Product.

7. Conclusion

One of the main challenges in the structured products market is to define precisely how much a product is worth, that would allow to determine if the market price is over or under the intrinsic value of the product. Throughout this investigation I was able to explore some of the main challenges of developing a model that would calculate the true value of a structured product with a certain degree of confidence.

Along this report, the Cirdan Phoenix Autocallable Worst of Certificates were decomposed and looked at from different perspectives, identifying some of the advantages, disadvantages and risks that the product may represent for an investor. One of the main objectives of this study was to calculate a theoretical price that would represent the true value of the product, which was made from different perspectives, and by applying different models. The use of different tools to try to obtain the same result helped to realize the limitations of each of the tools, and the importance of applying the most adequate model, since the variations in the result may conduct to poor investment decisions.

Lastly, the study focused on analyzing the possibility of using a delta hedging strategy to hedge the position of an investor. For that, the model used to obtain the theoretical price had to be adapted to calculate the deltas of the product, and then to perform an evaluation of the said delta hedging. From this section, we could conclude that the Delta hedging strategy was a success, as it was able to reduce value fluctuations on the position of the investor. In fact, the prices of the underlyings fluctuated in a drastic manner, mainly due to the Covid-19 pandemic, and therefore the product value was massively affected. The delta portfolio replicated accurately the fluctuations in the price of the product and covered most of the fluctuations, resulting in a 91.83% effective strategy.

7.1 References

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7.2 Appendix

Table 1 – Black-Scholes Model – Payout Structure

Payout stucture	0	1	2	3	4	5	6	7	8	59	60
10% long call	647,4227										500
60% short call	212,3139										0
ZCB	103,268	103,2127	103,1574	103,1021	103,0469	102,9917	102,9365	102,8813	102,8262	100,0536	100
digital option	226,9356										400
total at maturity											1000

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 2 – Black-Scholes Model – Digital option (Payout at Maturity)

Cash-or-Nothing Call Option					
S_0	100,00	σ	23%	d1	0,6318
K	60,00	r	-0,64%	d2	0,1241
Q	400,00	T	5	N(d2)	0,5494
		n	60	c0	226,94
		Div (q)	5,71%		

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 3 – Black-Scholes Model – Call Spread (long position)

EUROPEAN EQUITY OPTIONS		
Market Premium - Call		€ (euros)
Market Premium - Put		€ (euros)
Current Equity Price	1000,00	€ (euros)
Exercise Price	600,00	€ (euros)
Time to Maturity (T-t)	1834	days (1 year=365 days)
Interest Rate	-0,64%	%
Dividend Yield	5,71%	%
Volatility	22,65%	%

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 4 – Black-Scholes Model – Call Spread (short position)

EUROPEAN EQUITY OPTIONS		
Market Premium - Call		€ (euros)
Market Premium - Put		€ (euros)
Current Equity Price	1000,00	€ (euros)
Exercise Price	100,00	€ (euros)
Time to Maturity (T-t)	1834	days (1 year=365 days)
Interest Rate	-0,64%	%
Dividend Yield	5,71%	%
Volatility	22,65%	%

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 5 –Black-Scholes Model– Digital Option (Coupons Payout)

	Binary 1	σ	23%	δτ	0,0837	c0	-	Bin
S_0	100,00	r	-0,64%	u	1,0677	d1	3,3773	
K	60,00	T	0	d	0,9366	d2	3,2308	
Q	0,50	n	5	p	44,32%	N(d2)	0,9994	
		Div (q)	5,71%	1-p	55,68%	c0	0,50	B-S

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 6 –Black-Scholes Model– Digital Option (AER Payout)

	Binary 1	σ	23%	δτ	0,0837	c0	-	Bin
S_0	100,00	r	-0,64%	u	1,0677	d1	-0,1184	
K	100,00	T	1	d	0,9366	d2	-0,2789	
Q	1.000,00	n	6	p	44,32%	N(d2)	0,3902	
		Div (q)	5,71%	1-p	55,68%	c0	391,41	B-S

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 7 –Black-Scholes Model– Theoretical Price

t	1	2	3	4	5	6	7	8	9	60
AER						391,4071	383,0366	375,3029	368,0919	
Coupons	0,500268	0,500536	0,5008	0,501012	0,501032	0,500683	0,499826	0,498393	0,496375	0,28367
At maturity										765,3125
Theroetical Price (W/O AER)	789,3781									
Theroetical Price (W/ AER)	20918,82									

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 8 –Binomial Tree Model – Coupons Payout

0	1	2	3	4	5
100,00	106,774	114,0068	121,7296	129,9756	138,7801
	93,65578	100	106,774	114,0068	121,7296
	0	87,71404	93,65578	100	106,774
	0	0	82,14927	87,71404	93,65578
	0	0	0	76,93753	82,14927
	0	0	0	0	72,05644

0,501342	0,501073	0,500805	0,500536	0,500268	0,5
0	0,501073	0,500805	0,500536	0,500268	0,5
0	0	0,500805	0,500536	0,500268	0,5
0	0	0	0,500536	0,500268	0,5
0	0	0	0	0,500268	0,5
0	0	0	0	0	0,5

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 9 – Binomial Tree Model – AER Payout

0	1	2	3	4	5	6
100,00	106,774	114,0068	121,7296	129,9756	138,7801	148,181
	93,65578	100	106,774	114,0068	121,7296	129,9756
	0	87,71404	93,65578	100	106,774	114,0068
	0	0	82,14927	87,71404	93,65578	100
	0	0	0	76,93753	82,14927	87,71404
	0	0	0	0	72,05644	76,93753
	0	0	0	0	0	67,48502

546,4593	735,4673	905,8282	1001,609	1001,073	1000,536	1000
0	395,4833	599,1525	828,715	1001,073	1000,536	1000
0	0	232,9823	415,8445	690,7212	1000,536	1000
0	0	0	87,20014	196,6428	443,4443	1000
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 10 –Binomial Tree Model – Payout at Maturity

0	1	2	3	4	5
100%	107%	114%	122%	130%	139%
	94%	100%	107%	114%	122%
	0%	88%	94%	100%	107%
	0%	0%	82%	88%	94%
	0%	0%	0%	77%	82%
	0%	0%	0%	0%	72%

92,40615	48,41958	15,5411	0	0	0
0	127,3305	74,54421	27,89684	0	0
0	0	169,2257	111,6037	50,07583	0
0	0	0	214,9299	160,4724	89,88793
0	0	0	0	258,0712	216,5031
0	0	0	0	0	290,9109

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 11 –Binomial Tree Model – Theoretical Price

t	1	2	3	4	5	6	7	8	9	60
AER						546,4593	0	55,86691	0	
Coupons	0,500268	0,500536	0,500805	0,501073	0,501342	0,50161	0,228503	0,223987	0,198216	0,008081
At maturity										92,40615
total	0,500268	0,500536	0,500805	0,501073	0,501342	546,9609	0,228503	56,0909	0,198216	92,41424
theoretical price (w/ AER)	899,3579									

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 12 – Random Sample – Underlying 1

n	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	19/10/2020
1	0,4105539	-0,2438314	-0,7537053	-0,5262565	-0,9530508	-0,4658587	-1,1290545	-0,0256895
2	-0,0823828	0,6093854	1,207325	0,2349423	-0,8665376	-2,0398928	-0,9847373	-0,604922
3	-2,2550338	-0,4281912	1,0922516	0,5697533	0,5484628	0,8424001	1,0152915	0,6535453
4	0,536837	-1,2057875	0,0992637	-0,8305947	-1,425392	-1,1060106	-0,0596061	-0,2297541
5	0,5366161	1,1777233	-0,2083974	2,059384	-0,749397	-0,0377318	-0,1233739	-1,2356085
6	0,3556005	-0,4254841	0,8542104	1,2621197	0,8548302	-0,445198	-0,2041726	-1,6665713
7	1,1246968	-0,4456687	-0,8296035	0,8794716	-0,9519924	-0,7900204	1,2807083	0,4981808
8	0,7463744	-0,2217638	0,956993	0,2059332	0,1675225	-0,1352264	-1,2200621	-0,4157769
9	0,4755784	-1,3649989	0,0772216	-0,1817727	0,4844904	-0,4856305	-0,4072734	-1,4847575
10	-0,4789018	0,8347873	0,3174768	0,2627811	0,164019	0,8876465	0,6684803	0,7224046

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 13 – Random Sample – Underlying 2

n	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	19/10/2020
1	0,8532467	-0,1622918	-0,0077412	-1,4846387	-0,1971628	0,3290178	-0,5380443	-0,9035615
2	0,8510651	-0,4889787	0,8000099	-1,1358766	0,3764035	0,5298842	-0,0387485	0,5717294
3	-0,5781452	-0,3399519	0,1807814	-0,8584063	-0,0200134	-1,4454409	-1,9857192	0,5258019
4	-0,7519139	-0,0235222	2,2123725	-0,6072062	-0,8190494	1,2399486	0,9216838	-1,4709125
5	0,136515	-0,0110729	0,7544223	-1,9936663	-0,1587896	-0,0472095	1,0290905	0,7082775
6	-0,7369791	1,5739855	-0,6325242	-0,7519144	-1,4973481	-0,1792209	-0,475198	-1,053755
7	-0,2622791	-0,1773028	0,7770929	-0,1067835	0,6513342	-0,8399193	-0,7606662	0,3728056
8	0,4443574	-0,1528295	-0,0929822	-0,9530608	0,8905287	0,466374	1,0402983	1,0387396
9	0,3521368	-0,0092352	-0,2470559	-0,193176	0,5723276	-1,4505071	-0,0818829	-2,0438101
10	-1,027295	0,1658419	-0,0795119	0,1657213	2,058281	1,2593867	0,7512198	0,5426084

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 14 – Random Sample – Underlying 3

n	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	19/10/2020
1	0,9535602	-0,4988115	-0,8469826	0,5775166	-0,657142	0,2058707	0,2977984	-0,4707724
2	0,840963	0,2210451	0,0465717	-2,5811941	1,4124461	0,4189257	-1,073093	-0,0410553
3	0,1768347	-1,0381627	0,3126568	-1,4142313	-0,8709214	0,3030633	-0,0915558	0,1466386
4	0,9244576	-0,5674152	-0,5495304	-1,0468395	0,3078151	-0,1075661	-0,3724053	0,0476223
5	-0,5074604	-0,1334762	0,1267847	0,4144215	0,6701103	-2,5211286	0,4891045	1,7641972
6	0,8404482	-1,5796668	1,3393537	0,8390044	1,8834804	0,7599063	0,0210334	-0,2690598
7	-0,7412643	0,7818884	0,4954496	1,5501944	-0,8066072	-0,7688618	0,8958409	1,4276791
8	-0,3791156	-0,6505059	-0,5409206	1,0374467	-0,6263357	1,4678776	-0,2738337	-1,9857347
9	-0,3951144	0,4114383	-1,6358998	-1,076692	0,2019307	0,2767246	-0,291199	0,187664
10	-1,2319692	-0,2858858	-2,1306656	0,2714476	-0,2215114	0,1700822	0,2836017	0,2954438

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 15 – Random Sample – Underlying 4

n	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	19/10/2020
1	-0,0560497	1,03235	1,4830356	0,3766037	-0,2900464	1,8343165	0,3257594	-0,7639083
2	0,4072331	-1,0207591	0,892696	1,0280491	0,1713603	0,932512	0,1174977	0,1509559
3	2,0188079	-1,2565806	0,7750383	0,5167206	0,5110854	-0,1465664	-1,0149604	0,0994163
4	1,4343268	1,3157435	-0,7218425	0,1928342	0,0828024	-1,1804414	0,986376	-2,1914228
5	-0,4535983	1,6266437	-0,3217098	-0,4161912	-0,1598146	0,4772319	0,8827377	2,1144372
6	-0,5049718	0,6641925	0,4700413	-1,4497602	1,1219993	-0,4048238	0,2456845	-0,9709731
7	-0,0754609	0,5217715	1,3465924	1,7240188	1,1308425	0,1716599	0,5430157	0,7410923
8	-0,9279733	-0,3249185	-2,0237071	0,0658252	0,8700826	0,8585198	0,372376	-2,1666893
9	0,1123963	-0,1797551	-0,412157	1,8037604	-1,3287655	-2,1020081	1,1759904	-0,0014313
10	-0,261409	-0,584954	1,0891327	0,4159941	0,2009373	-0,2122284	0,9479989	-0,7148269

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 16 – Cholesky Decomposition– L Matrix

L	ISP	UCG	ENI	STLAM
ISP	1	0	0	0
UCG	0,795	0,607	0	0
ENI	0,552	0,051	0,832	0
STLAM	0,428	0,049	0,211	0,878

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 17 – Cholesky Decomposition – L Matrix Transposed

LT	ISP	UCG	ENI	STLAM
ISP	1	0,795	0,552	0,428
UCG	0	0,60660943	0,05136748	0,0490266
ENI	0	0	0,8322604	0,21054183
STLAM	0	0	0	0,8775446

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 18 – Cholesky Decomposition – Correlation Matrix

Correlations	ISP	UCG	ENI	STLAM
ISP	1	0,795	0,552	0,428
UCG	0,795	1	0,47	0,37
ENI	0,552	0,47	1	0,414
STLAM	0,428	0,37	0,414	1

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 19 – Random Correlated Sample – Underlying 1

n	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	19/10/2020
1	0,4105539	-0,2438314	-0,7537053	-0,5262565	-0,9530508	-0,4658587	-1,1290545	-0,0256895
2	-0,0823828	0,6093854	1,207325	0,2349423	-0,8665376	-2,0398928	-0,9847373	-0,604922
3	-2,2550338	-0,4281912	1,0922516	0,5697533	0,5484628	0,8424001	1,0152915	0,6535453
4	0,536837	-1,2057875	0,0992637	-0,8305947	-1,425392	-1,1060106	-0,0596061	-0,2297541
5	0,5366161	1,1777233	-0,2083974	2,059384	-0,749397	-0,0377318	-0,1233739	-1,2356085
6	0,3556005	-0,4254841	0,8542104	1,2621197	0,8548302	-0,445198	-0,2041726	-1,6665713
7	1,1246968	-0,4456687	-0,8296035	0,8794716	-0,9519924	-0,7900204	1,2807083	0,4981808
8	0,7463744	-0,2217638	0,956993	0,2059332	0,1675225	-0,1352264	-1,2200621	-0,4157769
9	0,4755784	-1,3649989	0,0772216	-0,1817727	0,4844904	-0,4856305	-0,4072734	-1,4847575
10	-0,4789018	0,8347873	0,3174768	0,2627811	0,164019	0,8876465	0,6684803	0,7224046

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 20 – Random Correlated Sample – Underlying 2

n	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	19/10/2020
1	0,8439778	-0,2922937	-0,6038916	-1,3189697	-0,8772762	-0,1707724	-1,2239811	-0,5685321
2	0,4507698	0,1878423	1,4451169	-0,5022544	-0,4605675	-1,300282	-0,8063713	-0,1340966
3	-2,1434602	-0,54663	0,9780038	-0,0677635	0,4238876	-0,20711	-0,3973993	0,8385249
4	-0,0293327	-0,9728699	1,4209607	-1,0286598	-1,6300297	-0,1271139	0,5117153	-1,0749239
5	0,5094211	0,9295731	0,2919637	0,4278335	-0,6920939	-0,0586345	0,5261737	-0,552661
6	-0,1643561	0,6165346	0,2954021	0,5472668	-0,2287155	-0,4626495	-0,4505768	-1,9641419
7	0,735033	-0,4618601	-0,188143	0,6344041	-0,3617285	-1,1375692	0,5567358	0,6222011
8	0,862919	-0,26901	0,7044055	-0,4144188	0,6733835	0,1754019	-0,3388946	0,2995667
9	0,5916943	-1,0907762	-0,0884753	-0,2616917	0,7323492	-1,2659676	-0,3734533	-2,4201767
10	-1,0038938	0,7642572	0,2041614	0,309439	1,3789678	1,4696348	0,9871389	0,903463

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 21 – Random Correlated Sample – Underlying 3

n	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	19/10/2020
1	1,0640653	-0,5580725	-1,1213531	0,1138885	-1,083125	-0,0689152	-0,4030303	-0,4523995
2	0,698142	0,4952302	0,7462977	-2,0768847	0,7165291	-0,7501468	-1,4386582	-0,3387173
3	-1,1273041	-1,1178457	0,8724211	-0,906599	-0,42311	0,6429838	0,3822412	0,5098076
4	1,0270996	-1,1390402	-0,2889148	-1,360922	-0,5727066	-0,6363478	-0,2954962	-0,1627472
5	-0,1191146	0,5384475	0,0292353	1,379277	0,1358825	-2,1214885	0,3918217	0,822598
6	0,8579064	-1,4687097	1,553724	1,3563363	1,9624974	0,3774845	-0,1196077	-1,1980039
7	-0,0095649	0,395618	-0,0056808	1,7701485	-1,1633496	-1,119129	1,4134503	1,4823467
8	0,1193013	-0,6716544	0,0732971	0,9281446	-0,3830578	1,1709679	-0,8479377	-1,8287997
9	-0,0482305	-0,41153	-1,331559	-1,0063496	0,4648967	-0,11227	-0,4713744	-0,7683862
10	-1,3424425	0,23139	-1,6021058	0,3794829	0,011912	0,6962251	0,6436199	0,6725259

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 22 – Random Correlated Sample – Underlying 4

n	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	19/10/2020
1	0,369127	0,688596	0,8001392	0,1540534	-0,8104564	1,469782	-0,1610464	-0,8247745
2	0,5408881	-0,6123784	1,3491427	0,4035767	0,095331	0,0594266	-0,5461887	-0,10705
3	0,8153261	-1,5212147	1,2223048	0,3574602	0,4988957	0,2248711	-0,5727577	0,4236115
4	1,6462251	0,5179287	-0,5981982	-0,4364466	-0,5127522	-1,4711192	0,8068577	-2,0834934
5	-0,2685298	1,9028728	-0,3078285	0,5057004	-0,327685	-0,1304745	0,8752674	1,7328342
6	-0,1501204	0,1453326	1,0290637	-0,5922603	1,6736133	-0,3945902	0,1093443	-1,673675
7	0,2462241	0,4230592	0,9690357	2,2104628	0,4470202	-0,3905455	1,1759827	1,1824265
8	-0,5529241	-0,5244968	-1,4847453	0,3176046	0,7470256	1,0274269	-0,2020612	-2,4464733
9	0,2362563	-0,6557904	-0,6851729	1,268922	-0,888115	-2,065307	0,792347	-0,6974222
10	-0,7441138	-0,2080945	0,6391502	0,5427995	0,3008047	0,2912257	1,2145607	-0,2292978

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 23 – Stock Price – Underlying 1

n	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	19/10/2020
1	2,5369221	2,4834284	2,3693936	2,2866551	2,1598491	2,0907817	1,9573996	1,9373058
2	2,474679	2,528929	2,6634265	2,6709363	2,5338429	2,2657716	2,1367091	2,0539374
3	2,2180375	2,1511889	2,2524968	2,2972833	2,340448	2,420006	2,524165	2,5852466
4	2,5531182	2,3810163	2,3714597	2,2538164	2,0787572	1,9484034	1,9251089	1,8858533
5	2,5530898	2,6848642	2,6329484	2,8946385	2,7623213	2,7323061	2,690978	2,5058146
6	2,5299063	2,4539932	2,5389216	2,6813444	2,7742276	2,6883113	2,6368901	2,4026963
7	2,6298844	2,5483778	2,422079	2,5090949	2,37008	2,2571155	2,3859642	2,4246435
8	2,5802204	2,5286242	2,629721	2,6332834	2,6317516	2,5903968	2,4140448	2,342754
9	2,5452488	2,3547089	2,3426543	2,30044	2,3361206	2,2591644	2,1933872	2,0169786
10	2,4257206	2,5072152	2,5247638	2,5354371	2,5335148	2,6256156	2,6911769	2,7658811

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 24 – Stock Price – Underlying 2

n	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	19/10/2020
1	14,738813	14,336027	13,614341	12,237739	11,380091	11,172938	10,11677	9,6336239
2	14,30015	14,432106	16,042645	15,354496	14,743015	13,271333	12,408672	12,217177
3	11,715674	11,17496	11,98406	11,859413	12,187917	11,932694	11,513232	12,215202
4	13,78221	12,722525	14,116058	12,974962	11,387546	11,217827	11,606626	10,630526
5	14,364743	15,347571	15,613616	16,050978	15,139999	14,99304	15,529928	14,806312
6	13,639954	14,22683	14,477272	15,020016	14,681093	14,094174	13,543278	11,585053
7	14,615944	14,032481	13,758669	14,370371	13,903276	12,67288	13,157549	13,729646
8	14,760281	14,382617	15,10308	14,553127	15,245747	15,371728	14,898201	15,165313
9	14,455844	13,22401	13,065655	12,738513	13,405372	12,099076	11,695265	9,6597472
10	12,787814	13,490289	13,631856	13,886794	15,358216	17,104296	18,355578	19,57215

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 25 – Stock Price– Underlying 3

n	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	19/10/2020
1	13,737074	13,120813	12,065088	12,058038	11,116447	10,973834	10,591699	10,188892
2	13,402242	13,74333	14,333738	12,357898	12,862941	12,127786	10,915871	10,581546
3	11,849929	10,89904	11,464351	10,695688	10,309269	10,677501	10,866133	11,153635
4	13,702872	12,585294	12,240881	11,075572	10,568275	10,041028	9,7619081	9,5758885
5	12,6836	13,044359	12,962527	14,108966	14,121652	12,138481	12,360907	12,958431
6	13,547416	12,168953	13,401942	14,564695	16,488746	16,77466	16,502947	15,096852
7	12,777647	13,015117	12,903051	14,419339	13,221635	12,159621	13,265584	14,539534
8	12,889171	12,217013	12,176498	12,856289	12,425326	13,3356	12,490787	10,950668
9	12,744374	12,293535	11,145261	10,328282	10,569495	10,403439	9,994994	9,4121664
10	11,679252	11,765285	10,473514	10,65656	10,577344	10,994555	11,387753	11,818027

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 26 – Stock Price– Underlying 4

n	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	19/10/2020
1	11,849149	12,349327	12,973398	13,015351	12,189576	13,431882	13,175917	12,327362
2	11,995168	11,393922	12,447597	12,712033	12,69985	12,655237	12,077776	11,89332
3	12,23222	10,889975	11,789928	12,000864	12,339396	12,44197	11,851765	12,120847
4	12,978873	13,363118	12,706146	12,221628	11,691796	10,4461	10,979251	9,390341
5	11,322436	12,867683	12,491043	12,849636	12,455875	12,245174	12,933082	14,521004
6	11,418442	11,448245	12,224717	11,628635	13,001377	12,542983	12,54349	11,046343
7	11,745758	12,01196	12,77188	14,836822	15,199026	14,667378	15,827132	17,086439
8	11,095136	10,605254	9,4661213	9,6081358	10,055525	10,736283	10,50093	8,7517611
9	11,737412	11,114626	10,502856	11,408679	10,625835	9,0998649	9,5544146	9,0206387
10	10,944899	10,700368	11,112802	11,462111	11,620141	11,772307	12,73814	12,43473

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 27 – Stock Performance – Underlying 1

n	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	19/10/2020
1	1,011733649	0,9904002	0,9449227	0,9119263	0,8613556	0,8338113	0,780618	0,7726045
2	0,986910863	1,008546	1,0621841	1,065179	1,0105056	0,9035979	0,8521273	0,8191176
3	0,884561309	0,8579019	0,8983038	0,9161648	0,9333791	0,9651071	1,0066461	1,0310056
4	1,018192721	0,9495578	0,9457466	0,8988301	0,8290158	0,7770303	0,7677403	0,7520851
5	1,018181391	1,0707335	1,0500293	1,1543922	1,1016237	1,0896535	1,0731717	0,9993278
6	1,008935725	0,9786613	1,012531	1,0693298	1,1063719	1,0721082	1,0516012	0,9582039
7	1,048807325	1,0163022	0,9659338	1,000636	0,9451964	0,9001458	0,9515311	0,9669565
8	1,029001151	1,0084244	1,0487422	1,0501629	1,049552	1,0330595	0,9627297	0,9342987
9	1,015054359	0,9390664	0,9342589	0,9174237	0,9316533	0,9009629	0,8747307	0,8043783
10	0,96738608	0,9998864	1,0068849	1,0111414	1,0103748	1,0471049	1,073251	1,1030433

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 28 – Stock Performance – Underlying 2

n	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	19/10/2020
1	1,0614153	1,0324087	0,9804365	0,8813005	0,8195371	0,8046189	0,728559	0,6937652
2	1,029825	1,0393278	1,1553108	1,1057537	1,0617179	0,9557347	0,8936102	0,8798197
3	0,843704	0,8047645	0,8630319	0,8540554	0,8777126	0,8593327	0,8291252	0,8796775
4	0,9925255	0,9162123	1,0165676	0,9343916	0,8200739	0,8078516	0,8358509	0,7655571
5	1,0344767	1,105255	1,1244142	1,1559108	1,0903067	1,0797234	1,1183875	1,0662763
6	0,982281	1,0245449	1,0425804	1,0816661	1,0572586	1,0149917	0,9753189	0,8342973
7	1,0525669	1,0105488	0,9908303	1,034882	1,0012441	0,9126372	0,9475406	0,9887401
8	1,0629613	1,0357639	1,087648	1,0480432	1,0979221	1,1069946	1,0728936	1,0921297
9	1,0410373	0,9523268	0,9409229	0,9173637	0,9653876	0,8713147	0,8422343	0,6956465
10	0,9209141	0,9715029	0,9816978	1,0000572	1,1060216	1,2317656	1,3218766	1,409488

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 29 – Stock Performance– Underlying 3

n	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	19/10/2020
1	1,0655502	1,0177484	0,9358585	0,9353117	0,8622748	0,8512127	0,8215714	0,7903267
2	1,0395782	1,0660355	1,111832	0,9585711	0,997746	0,9407218	0,8467166	0,8207839
3	0,9191692	0,8454111	0,8892609	0,8296376	0,799664	0,8282269	0,8428586	0,8651594
4	1,0628973	0,9762096	0,9494943	0,8591043	0,8197545	0,7788572	0,7572066	0,7427776
5	0,9838349	1,0118181	1,0054706	1,094397	1,095381	0,9415514	0,9588044	1,0051529
6	1,050839	0,9439151	1,0395549	1,1297467	1,2789905	1,3011681	1,2800921	1,1710248
7	0,99113	1,0095498	1,0008572	1,1184719	1,0255689	0,9431912	1,028978	1,1277951
8	0,9997806	0,947643	0,9445003	0,99723	0,9638013	1,0344089	0,9688789	0,8494158
9	0,988549	0,9535786	0,8645099	0,8011389	0,8198491	0,8069686	0,7752865	0,7300781
10	0,9059302	0,9126035	0,8124042	0,8266026	0,820458	0,85282	0,8833194	0,9166946

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 30 – Stock Performance– Underlying 4

n	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	19/10/2020
1	1,0187383	1,0617414	1,1153963	1,1190033	1,0480067	1,1548148	1,132808	1,0598529
2	1,0312924	0,9795999	1,0701902	1,0929253	1,0918779	1,0880422	1,0383947	1,022536
3	1,0516731	0,9362727	1,0136468	1,0317821	1,0608876	1,0697065	1,0189632	1,0420977
4	1,1158671	1,1489028	1,0924192	1,0507624	1,0052098	0,8981103	0,9439482	0,8073407
5	0,9734538	1,1063074	1,0739255	1,1047558	1,0709019	1,0527868	1,1119301	1,2484528
6	0,981708	0,9842703	1,051028	0,9997795	1,1178019	1,0783911	1,0784347	0,9497165
7	1,0098492	1,0327361	1,0980707	1,2756054	1,3067461	1,2610374	1,360748	1,4690177
8	0,9539115	0,9117936	0,8138559	0,8260657	0,8645303	0,9230589	0,9028243	0,7524384
9	1,0091317	0,9555872	0,9029899	0,9808686	0,9135631	0,7823668	0,821447	0,7755553
10	0,9409948	0,9199711	0,9554304	0,9854624	0,9990492	1,0121317	1,0951699	1,069084

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 31 – Reference Underlying

n	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	19/10/2020
1	1,0117336	0,9904002	0,9358585	0,8813005	0,8195371	0,8046189	0,728559	0,6937652
2	0,9869109	0,9795999	1,0621841	0,9585711	0,997746	0,9035979	0,8467166	0,8191176
3	0,843704	0,8047645	0,8630319	0,8296376	0,799664	0,8282269	0,8291252	0,8651594
4	0,9925255	0,9162123	0,9457466	0,8591043	0,8197545	0,7770303	0,7572066	0,7427776
5	0,9734538	1,0118181	1,0054706	1,094397	1,0709019	0,9415514	0,9588044	0,9993278
6	0,981708	0,9439151	1,012531	0,9997795	1,0572586	1,0149917	0,9753189	0,8342973
7	0,99113	1,0095498	0,9659338	1,000636	0,9451964	0,9001458	0,9475406	0,9669565
8	0,9539115	0,9117936	0,8138559	0,8260657	0,8645303	0,9230589	0,9028243	0,7524384
9	0,988549	0,9390664	0,8645099	0,8011389	0,8198491	0,7823668	0,7752865	0,6956465
10	0,9059302	0,9126035	0,8124042	0,8266026	0,820458	0,85282	0,8833194	0,9166946

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 32 –AER Distribution

t=0	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	19/10/2020
1	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0
6	0	0	0	0	0	1	-	-
7	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 33 –Coupons Distribution (example)

t=0	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	19/10/2020	18/11/2020	18/12/2020	18/01/2021	18/02/2021	18/03/2021	19/04/2021	18/05/2021
1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	5
2	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 34 –Coupons Payout (example)

Coupon Date	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	19/10/2020	18/11/2020	18/12/2020	18/01/2021	18/02/2021	18/03/2021	19/04/2021	18/05/2021
1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	5
2	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
6	1	1	1	1	1	1	-	-	-	-	-	-	-	-	-
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 35 – Denomination Payout (example)

Coupon Date	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/02/2025
1	0	0	0	0	0	0	-
2	0	0	0	0	0	0	440,27531
3	0	0	0	0	0	0	-
4	0	0	0	0	0	0	-
5	0	0	0	0	0	0	354,89659
6	0	0	0	0	0	1000	-
7	0	0	0	0	0	0	524,54683
8	0	0	0	0	0	0	564,00221
9	0	0	0	0	0	0	214,10956
10	0	0	0	0	0	0	316,38494

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 36 –Theoretical Price with memory and AER

n	1	2	3	4	5	6	7	Theoretical Price
Coupon Date	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/09/2020	720,4249444
1	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	0,9918913	
2	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	0,9918913	
3	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	0,9918913	
4	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	0,9918913	
5	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	0,9918913	
6	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	994,03869	-	
7	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	0,9918913	
8	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	0,9918913	
9	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	0,9918913	
10	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	0,9918913	

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 37 –Theoretical Price without memory

n	1	2	3	4	5	6	60	Total Payout	Theoretical Price
Coupon Date	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/02/2025	25/02/2025	717,3626054
1	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	-	986,3139884	
2	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	410,5976473	425,4554824	
3	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	-	996,9240191	
4	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	-	995,0160277	
5	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	330,9740543	350,6033583	
6	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	994,03869	-	999,021284	
7	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	489,1886665	530,9394052	
8	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	525,9844814	544,5956943	
9	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	199,6770669	214,538329	
10	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	295,0583643	317,7402515	

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 38 –Theoretical Price without AER

n	1	2	3	4	5	6	60	Total Payout	Theoretical Price
Coupon Date	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020	18/02/2025	25/02/2025	588,7344011
1	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	933,5255342	990,4998004	
2	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	410,5976473	427,4186085	
3	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	452,5436459	483,9371906	
4	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	933,5255342	990,5124273	
5	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	330,9740543	371,8235984	
6	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	933,5255342	990,2192033	
7	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	489,1886665	541,4606651	
8	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	525,9844814	580,282404	
9	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	199,6770669	214,538329	
10	0,9988376	0,9976765	0,9965168	0,9953584	0,9942013	0,9930456	295,0583643	317,7402515	

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 39 –Hedging strategy – Product Value with change in the Stock Price

Product Value w/ 1% change in stock	18/02/2020	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020
Original	720,425	260,07	297,42	309,50	372,61	408,35	381,68
+1% @ ISP	724,769	260,64	299,10	310,97	374,47	410,51	-
+1% @ UCG	722,143	260,53	298,19	310,65	373,89	409,53	-
+1% @ ENI	723,749	260,91	298,01	310,20	373,90	410,18	-
+1% @ STLAM	720,428	261,32	298,03	310,05	373,63	409,19	-

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 40 –Hedging strategy – Deltas for a 1% change in the Stock Price

Deltas for 1% change	18/02/2020	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020
ISP	4,3436	0,5639	1,6761	1,4737	1,8596	2,1637	-
UCG	1,7176	0,4618	0,7732	1,1539	1,2834	1,1763	-
ENI	3,3243	0,8347	0,5922	0,6990	1,2880	1,8343	-
STLAM	0,0032	1,2481	0,6091	0,5548	1,0205	0,8407	-

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 41 –Hedging strategy – Deltas for a 1-unit change in the Stock Price

Deltas	18/02/2020	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020
ISP	173,2239	40,0297	121,7884	101,4782	110,3092	118,1162	-
UCG	12,3694	6,5971	11,2716	17,5368	15,5756	13,3520	-
ENI	25,7854	12,4773	6,8938	8,0719	14,5702	20,8201	-
STLAM	0,0271	21,4823	8,7644	7,4978	12,8370	9,6630	-

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 42 –Hedging strategy – Stock Prices

Stock Price	18/02/2020	18/03/2020	18/04/2020	18/05/2020	18/06/2020	18/07/2020	18/08/2020
ISP	2,5075	1,41	1,38	1,45	1,69	1,83	1,81
UCG	13,886	7,00	6,86	6,58	8,24	8,81	8,16
ENI	12,892	6,69	8,59	8,66	8,84	8,81	8,02
STLAM	11,631	5,81	6,95	7,40	7,95	8,70	9,12

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 43 –Hedging strategy – Delta Portfolio Performance

	Performance				Delta Portfolio		
Short side	ISP	UCG	ENI	STLAM	Portfolio performance	Portfolio value	Portfolio return
18/02/2020						938,86	
18/03/2020	-190,36	-85,18	-159,92	-0,16	-435,61	310,85	-46,40%
18/04/2020	-1,30	-0,92	23,71	24,49	45,98	365,06	14,79%
18/05/2020	9,26	-3,16	0,48	3,94	10,53	388,14	2,88%
18/06/2020	23,71	29,11	1,45	4,12	58,39	545,16	15,04%
18/07/2020	16,11	8,88	-0,44	9,63	34,17	601,49	6,27%
18/08/2020	-2,57	-8,68	-16,45	4,06	-23,64	0,00	-3,93%

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 44 –Hedging strategy – Delta Neutral Portfolio Performance

	Performance			Delta neutral portfolio		
Long side	Options value	Options performance	Options return	performance	value	return
18/02/2020	720,4249444				-218,44	
18/03/2020	260,07	-460,352462	-63,90%	-24,74	-50,78	11,33%
18/04/2020	297,42	37,34739471	14,36%	-8,63	-67,64	16,99%
18/05/2020	309,50	12,07938579	4,06%	1,55	-78,65	-2,30%
18/06/2020	372,61	63,109259	20,39%	4,72	-172,55	-6,00%
18/07/2020	408,35	35,74081853	9,59%	1,57	-193,14	-0,91%
18/08/2020	381,68	-26,67018607	-6,53%	-3,03	381,68	1,57%

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations

Table 45 –Hedging strategy – Delta Hedging Results

	Std dev	Variance	Effectiveness
Delta Neutral Portfolio	8,83%	0,007805	91,83%
Options Portfolio	30,90%	0,095511	

Source: own elaboration based on the inputs from Bloomberg terminal and the outputs from the Excel computations