Lisbon School of Economics & Management
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Master's Final Work

Project

PRIIP Analysis: Express Certificate Linked to MSCI Emerging Markets

Diogo Miguel Martinho Pereira

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Supervisor:

Professor Raquel Medeiros Gaspar

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Abstract

This study aims to analyze and value the structured product Express Certificate Linked to MSCI Emerging Markets issued by Deutsche Bank. The certificate is an autocallable product with a maximum duration of five years, featuring the possibility of an early termination on any of the first four anniversaries if the conditions for an autocall event are verified.

The analysis of the product starts with the description of its principal characteristics, identification of the main risks, advantages and disadvantages for investors, and presentation of a possible decomposition for the certificate. Subsequently, its valuation is carried out and the probability of the product ending prematurely on each anniversary or reaching maturity under the different possible scenarios is studied. Additionally, an autocall feature sensitivity analysis is performed and the certificate's Greeks are calculated and analyzed.

To conduct the certificate's valuation, the Monte Carlo Simulation with Geometric Brownian motion is applied, and two different scenarios are considered for the volatility parameter: one based on the historical volatility and the other based on the implied volatility.

The autocall feature sensitivity analysis allows for studying the impact that modifications on three distinct characteristics of the product have on its value, as well as on the probabilities of the occurrence of an early termination or the different maturity scenarios. The three characteristics to which modifications are applied are: the number of the autocall dates, timing of these dates and the size of the barriers that trigger an early termination.

The Greek analysis focuses on understanding the behavior of Delta, Gamma, Vega and Rho over the maximum lifetime of the product and for different underlying asset prices.

The main conclusions drawn from the study of this product across its various dimensions lead us to recognize the complexity involved in the analysis of products with these characteristics.

Keywords: Structured Product; Autocallable Product; Express Certificate; Monte Carlo Simulation; Sensitivity Analysis; Greeks.
Jel Codes: G12; G17.

Resumo

Este estudo tem como objetivo analisar e avaliar o produto estruturado Express Certificate Linked to MSCI Emerging Markets emitido pelo Deutsche Bank. O certificado é um produto autocallable com uma duração máxima de cinco anos, e que apresenta a possibilidade de término antecipado em qualquer um dos primeiros quatro aniversários se as condições para este evento forem verificadas.

A análise do produto inicia com a descrição das suas principais características, identificação dos principais riscos, vantagens e desvantagens para os investidores e apresentação de uma possível decomposição para o produto. Posteriormente, efetua-se a sua avaliação e realiza-se o estudo das probabilidades do produto ter um término prematuro em cada aniversário ou de atingir a maturidade nos diferentes cenários possíveis. Adicionalmente, é realizada uma análise sensitiva da funcionalidade de autocall e são calculados e analisados os Greeks do certificado.

Para a realização da avaliação do produto, foi aplicada a simulação de Monte Carlo com Geometric Brownian motion, e foram considerados dois cenários diferentes para o parâmetro da volatilidade: um baseado na volatilidade histórica e outro na volatilidade implícita.

A análise de sensibilidade da funcionalidade de autocall permite estudar o impacto que as modificações em três características distintas do produto têm no seu valor, bem como nas probabilidades de ocorrência de um término antecipado ou dos diferentes cenários de maturidade. As três características modificadas são: o número de datas de autocall, o seu *timing* e a dimensão das barreiras que desencadeiam um término prematuro.

A análise dos Greeks incide na compreensão do comportamento do Delta, Gamma, Vega e Rho ao longo do tempo máximo de vida do produto e para diferentes níveis de preços do ativo subjacente.

As principais conclusões retiradas do estudo deste produto nas suas diversas dimensões levam-nos a reconhecer a complexidade envolvida na análise de produtos com estas características.

Palavras-chave: Produto Estruturado; Produto Autocallable; Certificado Express; Simulação de Monte Carlo; Análise de Sensibilidade; Greeks.
Códigos Jel: G12; G17.

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1. Introduction

Structured products are sophisticated instruments that combine more than one financial product into a single structure. These often include bonds, derivatives, equities, or other investment vehicles. By combining these components, structured products create complex structures designed to meet investor's risk profiles and investment goals. Their performance is linked to the performance of a single underlying or a basket of assets.

One of the most popular type of structured products is autocallable notes. These instruments are yield enhancement products characterized by having a feature that allows for an automatic termination of the product at specific autocall dates if the underlying is at or above a defined strike level. If the instruments are autocalled, investors receive the principal amount plus a premium. If they are not autocalled and reach the maturity, offer a downside protection to the investors initial investment as long as the underlying asset is not trading below the defined downside barrier level. In case of trading below that barrier, the investors lose at least part of the initial capital. These products are suitable for investors looking for high potential returns and that are willing to risk part or the entire initial investment.

The Express Certificate linked to MSCI Emerging Markets (Price Index), here under analysis, is an autocallable structured product issued by Deutsche Bank. This certificate has a maximum lifespan of five years and can be early terminated annually during the first four years. The product offers a growing premium linked to the number of years that it survives, which is only paid in the case of an autocall event or if the product survives until maturity with the underlying above a specific level. Moreover, if the maximum lifespan is reached with the underlying price below the safety threshold that guarantees the notional amount, the investors suffer losses equal to the dynamics of the underlying price.

At the time this study is conducted, the certificate is still alive, having survived to all the previous autocall dates due to the decline in the underlying asset price experienced since the issuance of the product.

The main goal of this work is to perform an extended analysis and valuation of this certificate. In this paper, we focus on explaining the key characteristics, identifying the main risks, advantages and disadvantages for investors holding this product, developing

a possible decomposition for the product, valuing the certificate and analyzing the product probabilities of being early terminated or reaching maturity under different payoff scenarios. In addition, we study the effects of changes in the autocall feature by performing an autocall sensitivity analysis, and we calculate and analyze the Greek letters of this product.

The remaining of the text is organized as follows. Section 2 addresses key contributes for the analysis of this product, Section 3 focuses on explaining the characteristics of the certificate, including the decomposition proposed, Section 4 describes the data and methodology used in the valuation, autocall sensitivity analysis and Greek analysis, Section 5 presents the results of this study and Section 6 explains the main conclusions and suggestion for complementary analysis in further studies.

2. Literature Review

In the last few years, the market for structured products has been growing with an increasing variety of characteristics and structures. According to Deng et al. (2014), this growth led to the introduction and development of different valuation approaches, as well as the optimization and improvement of existing methods. The authors review the four primary approaches for the valuation of these products and show a common valuation example using all approaches: the simulation approach; the numerical integration approach; the decomposition approach; and the PDE approach. They demonstrate that while the numerical integration and decomposition approaches are limited to value specific products, the simulation and PDE approaches can value a wider variety of products.

When it comes to the valuation of derivative securities, there is no consensus on whether historical volatility or implied volatility is the better predictor of future volatility. Canina and Figlewski (1993) in their study using options on the S&P 100 index concluded that historical volatility better predicted future volatility compared to implied volatility. Conversely, Christensen and Prabhala (1998), also based on options on the S&P 100 index, found that implied volatility outperforms historical volatility in predicting future realized volatility. These are two examples that reflect the controversy of this matter.

Deng et al. (2011) describe in their article the call feature of autocallable products, showing in an example the cost of adding this feature to these certificates. Throughout their work, they divide autocallables in two main categories: discrete autocallables, which can only be autocalled on specific dates, and continuous autocallables, which can be autocalled at any time. Furthermore, they explain a valuation model using a PDE approach and an alternative probability approach intended for the valuation of discrete autocallables. The authors conclude that discrete autocallables are less likely to be called than continuous. Additionally, the investment in products containing an autocall feature are worth less than their non-callable version.

Deng et al. (2014) affirm that call features are among the most difficult to value, with discretionary call features requiring a simulation approach. The authors emphasize the Greeks' importance for hedging risks in structured product portfolios. Concretely, they state that to calculate them using the simulation approach, an infinitesimal change should be applied to the specific parameter, and measure the change in the product's value.

3. Product Analysis

3.1 Product Description

The product under study is the Express Certificate linked to MSCI Emerging Markets (Price Index) issued by Deutsche Bank AG on May 18, 2021. As expressed in the Key Information Document (KID)[1], the certificate's currency is U.S. Dollar and the product notional amount is USD 100. The maturity date is on May 20, 2026, indicating a maximum lifespan of five years.

The Express Certificate is linked to a single underlying asset, which is the MSCI Emerging Markets (Price Index).

This certificate is designed to provide a return in the form of a cash payment once the product is terminated. The timing and the amount paid is dependent on the performance of the underlying index throughout the product's duration.

The product can terminate either on any anniversary (early termination) or alternatively upon reaching maturity. Upon termination, no further payments are made thereafter.

An early termination following an autocall occurs if, on any autocall observation date the underlying index price is at or above the autocall barrier level. If this condition is verified, the investor of the product receives the respective autocall payment on the correspondent autocall payment date. Otherwise, the certificate continues until the next autocall observation date, and so forth. The autocall barrier levels correspond to the initial reference level of the underlying (1327.54 index points) on the first two anniversaries and decrease by 5% on the third and fourth anniversary as shown in Table 1.

Autocall Observation Dates	Autocall Barrier Levels	Autocall Payment Dates	Autocall Payments
17 May 2022	1327.54 (100%)	20 May 2022	USD 105.95
17 May 2023	1327.54 (100%)	22 May 2023	USD 111.90
15 May 2024	1261.16 (95%)	20 May 2024	USD 117.85
15 May 2025	1194.79 (90%)	20 May 2025	USD 123.80

Table 1 - Important Dates, Autocall Barrier Levels and Autocall Payments

On any such early termination, the payoff of the product consists of the product notional amount (USD 100) plus an additional USD 5.95 for each year the product survives an autocall observation date. The early termination payoff structure is shown in Figure 1.

[1] The KID of the certificate is available for consultation in the appendix.

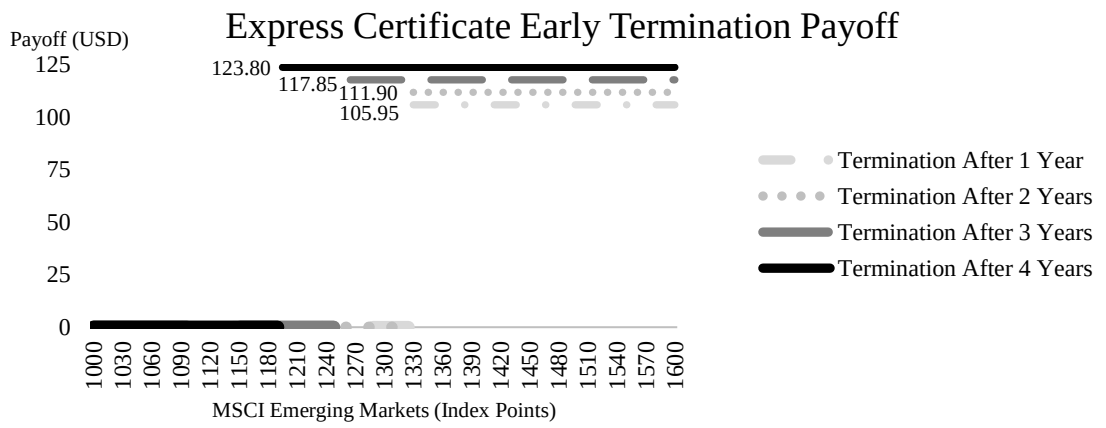


Figure 1 - Express Certificate Early Termination Payoff

If the certificate is not autocalled and survives until maturity, the payoff of the product can be expressed in Equation (1);

$$Payoff_{Maturity} \begin{cases} USD\ 100 \times \frac{Underlying_T}{1327.54}, if\ \ Underling_T < 862.901\ (pts) \\ USD\ 100,\ if\ \ 862.901 \leq Underlying_T < 1128.409\ (pts) \\ USD\ 129.75, \qquad if\ \ Underlying_T \geq 1128.409\ (pts) \end{cases} \quad (1)$$

At maturity, investors can receive the maximum possible payoff, consisting of USD 129.75, if the underlying is at or above 1128.409 index points (85% of the initial reference level). Alternatively, if the underlying is at or above 862.901 index points (65% of the initial reference level) and below 1128.409 index points, they receive the product notional amount, USD 100. On the other hand, if the underlying price falls below 862.901 index points, investors may lose the total or part of the initial money invested. In this scenario, the payoff is directly linked to the underlying's performance and investors do not benefit from capital protection. The maximum payoff in this worst scenario is close to USD 65. The certificate's payoff in the event of maturity being reached is shown in Figure 2.

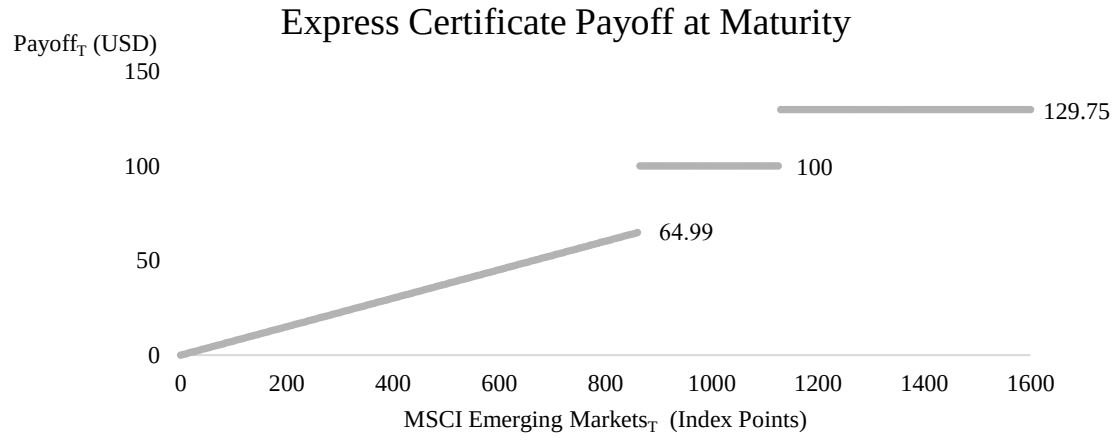


Figure 2 - Express Certificate Payoff at Maturity

3.2 Underlying Asset

The MSCI Emerging Markets Index[2] (Price Index) tracks the performance of equity markets in emerging economies. This index captures large-cap and mid-cap companies across 24 emerging markets countries in the Europe, Middle East and Africa (EMEA), Asia-Pacific (APAC) and Americas regions. As of 2023, the APAC region leaded the influence in the index, with a weight of approximately 78.1%, followed by EMEA with 13.2% and Americas with 8.7%. The top five countries with the highest weight in the index are China (25.13%), India (17.70%), Taiwan (17.63%), South Korea (12.82%) and Brazil (5.23%).

When considering the composition of the MSCI Emerging Markets Index portfolio, it comprises 1376 constituents. The main sectors are the Information Technology sector representing 23.73%, followed closely by the Financials at 22.35% and Consumer Discretionary at 12.41%.

Since the issuance of the certificate in May 2021, the underlying index experienced a downward trend, reaching its lower values in the fourth quarter of 2022, where it dropped below the 850 index points. Between June 2021 and October 2022, the index declined by more than 39%, reflecting the impacts of the Covid-19 pandemic, geopolitical tensions and rising inflation rates, coupled with tightening monetary policies. Following this period of poor performance, there has been a gradual recovery, although the index remains significantly below the higher values observed in 2021. The MSCI Emerging Markets Index performance between the issuance date and January 2024 is shown in Figure 3.

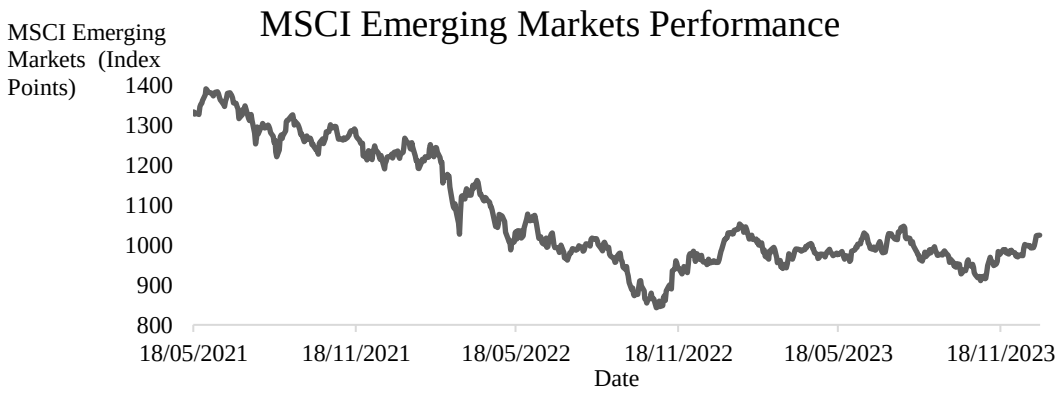


Figure 3 - MSCI Emerging Markets Performance

[2] More information concerning the underlying index can be found at: https://www.msci.com/documents/1296102/38312924/MSCI+Emerging+Markets+Indexes.pdf

3.3 Issuer

The manufacturer of the certificate is Deutsche Bank AG. Established in Berlin in 1870, the German multinational bank has grown into one of the largest banks globally, maintaining a strong position in Europe, the Americas and Asia-Pacific region.

Deutsche Bank offers financial services to companies, institutional investors, small and medium-sized business, private individuals, and governments. The bank provides a broad range of products and services across investment banking, commercial and retail banking, as well as in asset and wealth management.

Currently, Deutsche Bank is listed on both the Frankfurt Stock Exchange and the New York Stock Exchange under the ticker symbols, DBK and DB, respectively.

3.4 Principal Risks for Investors

Investors of the Express Certificate are exposed to some risks that must be considered. The main risks identified include:

Market Risk: The certificate is dependent on the performance of its underlying, and consequently, investors can make a profit or a loss based on the fluctuations of the index value. It is important for investors to be aware of the volatility of the index.

Creditworthiness of the Issuer Risk: Investors of the product are exposed to the issuer creditworthiness. In the event of Deutsche Bank failing on its financial obligations, investors could potentially lose their total investment.

Liquidity Risk: Due to the product's complexity, the investors may face some challenges to exit their positions in the secondary market. In the case of market illiquidity, the product holders may have to adjust the price of the product, which could lead to losses. The possibility of not being able to sell the structured product should be considered as well.

Interest Rate Risk: A change in interest rates can signal changes in inflation, growth expectations and overall market conditions. All of these may influence the performance of the companies within the underlying index and consequently affect the overall performance of the product.

Inflation Risk: Inflation erodes the purchasing power of money and consequently this may result in the decline in real terms of capital reimbursed.

Currency Risk: If the currency of the product differs from that of the investors' original currency, they are exposed to currency risk. For the specific case of the certificate, investors based in Europe must consider the impact of currency fluctuations when investing in the product since it is USD-denominated and is tied to an also USD-denominated index.

3.5 Principal Advantages and Disadvantages for Investors

Investors may enjoy some advantages in investing in this product, including:

Partial Capital Protection: Investors of the Express Certificate may receive predefined fixed cash payments greater than the product notional amount if an autocall event occurs.

Moreover, in the event of the product reaching maturity and the underlying being at a level above 862.901, investors are guaranteed with a cash payment equal or greater to the product's notional amount. However, in the worst-case scenario where the underlying falls below the same level, investors may incur in losses without protection, risking a total loss of their investment.

While there is a risk of partial or total loss in the latter scenario, the possibility of the others occurring, provides investors a degree of protection. Contrary to a direct investment in an ETF that tracks the underlying index, this product offers some level of protection for the amount invested, in the case of a drop in the underlying asset's price.

Potential for Enhanced Returns: Investors may achieve both in the event of an autocall or the maturity a higher return compared to investing directly in an ETF. The predefined payments can offset the returns of the ETF investment.

On the other hand, the certificate also presents disadvantages, particularly when compared to an alternative direct investment in an ETF replicating the underlying index.

Limited Returns: The autocall feature of the certificate may limit investors' returns as an early termination may result in a smaller payment compared to the highest payoff scenario at maturity.

Moreover, the predetermined payments upon both early termination or maturity, limit the potential gains for investors in comparison to a direct investment in an ETF, which may offer unlimited gains.

Complexity: The product might be difficult to understand for investors with lower financial literacy because of the existence of less usual or more complex features. This complexity may result in investors making investment decisions without full awareness.

Lack of Dividends: Unlike an ETF, the product does not provide dividends from the underlying index.

3.6 Key Information

Packaged Retail Investment and Insurance Products (PRIIPs) are often complex and difficult to understand for potential investors. Due to its characteristics, the information disclosed by the seller institutions must be clear and easy to understand.

To accomplish this, the European Union has implemented the PRIIPs Regulation, which obliges the entities that produce or sell this products to provide investors with a Key Information Document (KID) for each PRIIP that is produced. This document discloses essential information in a standard format, making it easier for investors to comprehend the features, risks, costs and eventual gains and losses associated with the products.

Among the mandatory information are the Intended Retail Investor, Summary Risk Indicator and the Performance Scenarios Analysis of the product.

As outlined in the KID provided by Deutsche Bank for the Express Certificate, this product targets private clients pursuing the objective of general capital formation and possessing a short-term investment horizon. Furthermore, the product is designed for investors with sufficient knowledge or experience that do not attach importance to capital protection and are willing to bear losses up to the entire invested amount.

Concerning the summary risk indicator, it indicates the likelihood of the product losing money due to market fluctuations or eventual inability of Deutsche Bank to fulfill payment obligations. The indicator assumes that the product is held by investors until the maturity. As disclosed, the certificate has a classification of 5 out of 7, indicating a mid-high risk.

The KID also provides a set of performance scenarios, including a stress, unfavorable, moderate and favorable scenarios. These are determined through simulations using the reference asset's past performance during a period of up to 5 years. According to the Key Information Document, the certificate generates a positive return solely in the moderate and favorable tested scenarios. However, under both stress and unfavorable simulated

scenarios, the investors experience a negative return, highlighting the risk of not receiving the amount invested in the certificate.

3.7 Product Decomposition

The complexity and payoff structure features of structured products, often allow for various approaches to their decomposition.

Adams & Smith (2019) affirms that generally, structured financial instruments have structures that combine a bond with at least one derivative.

As outlined in the Product Description Section, the Express Certificate Linked to MSCI Emerging Markets pays a predefined fixed amount if, on an anniversary, the autocall condition is met, resulting in the termination of the product on that date. According to Bellefroid (2022), this payoff structure is identical to a stream of digitals with an autocall feature.

According to Alm et al. (2013), an autocallable has a similar mechanism as a callable bond, in the sense that it can be terminated prior to its maturity on the preselected dates. The difference between the two is that autocallable products are automatically called if the defined conditions are met on one of the autocall observation dates, whereas callable bonds have to be called by the issuer.

As stated by Bellefroid (2022), it is impossible for product issuers to offer high coupons with a simple payoff structure. Therefore, to receive superior coupon payments, investors sell a down-and-in put option (DIP) with the same maturity as the certificate.

Considering the contributes outlined and the payoff structure of the certificate being analyzed, the chosen decomposition, in the investors perspective, is a long position on a callable bond, together with a short position on a down-and-in put option.

The callable bond, which partially replicates the structured product's payoffs, is an accrual bond with a face value of USD 100 and a maturity of 5 years. These bonds are issued at face value and do not make any coupon payments during its lifespan. Instead, they accrue interest, which is paid along with the principal at maturity. The interest is added to the principal and interest calculations are made based on the growing principal.

To replicate the payoff structure of the product with USD 5.95 payment increments for each year it survives an autocall date, the bond features a step down coupon rate at the

levels of 5.95% in the first year, then decreasing to 5.62% in the second year, 5.32% in the third year, 5.05% in the fourth year and finally 4.81% in the fifth year.

Due to its callable feature, this bond can be terminated annually within the first 4 years. During this period of time, investors can receive the bond's face value plus the accrued interest correspondent to the period between the issuance date and the specific autocall date. For instance, if called in the first anniversary, investors receive USD 105.95. If called in the second anniversary, they receive USD 111.90. This pattern continues, with product holders receiving USD 117.85 if called in the third anniversary, or USD 123.80 if called in the fourth.

The bond's embedded call feature allows for an automatic termination of the product on the specific dates if the underlying index is at or above the corresponding autocall barrier level, replicating a stream of digitals with an autocall feature.

Given the specific characteristics of the product, if the bond reaches maturity, there is a provision stating that the accumulated interest might or might not be paid depending on the underlying's level on that date. Consequently, the bond can either pay only the face value (USD 100) or the face value along with the total accrued interest (USD 129.75).The payoff of the accrual callable bond at maturity, if not autocalled, is illustrated in Figure 4.

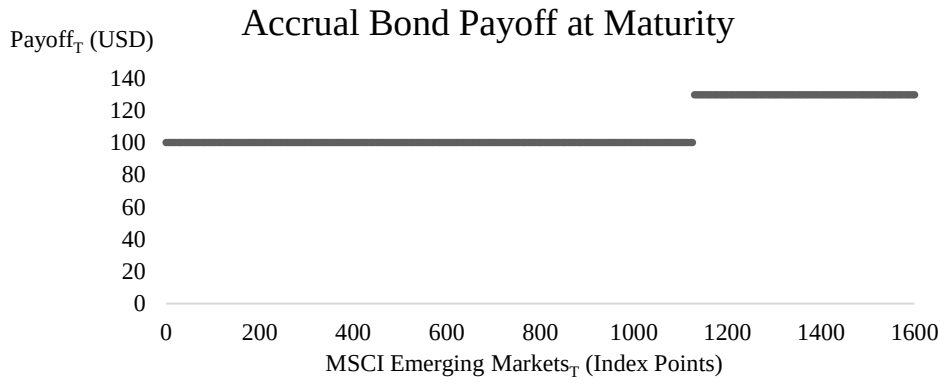


Figure 4 - Accrual Bond Payoff at Maturity

Moreover, if the product reaches maturity, in the worst payoff scenario the investor's payment is directly linked with the underlying index price. This payment scenario can be replicated by a short position on a 5-year down-and-in put option (DIP) together with the bond's payoff of USD 100 at maturity. As stated by Wilmott (1998), a down-and-in put option, is a type of barrier option that is knocked in (or activated) when the underlying falls to the barrier level, which is set below the initial value of the underlying. The payoff

received by the investor is the same as a vanilla put option when the underlying is below the barrier. The DIP that is part of the product decomposition, is an at-the-money European barrier option, given that it can only be exercised at maturity and the strike price is the same as the underlying index initial reference level. The DIP only comes into existence if the underlying is below the barrier level (65% of the underlying initial reference level) at maturity and if the Express Certificate did not terminate early in an autocall event. The payoff of the short position on the down-and-in put option is shown in Figure 5.

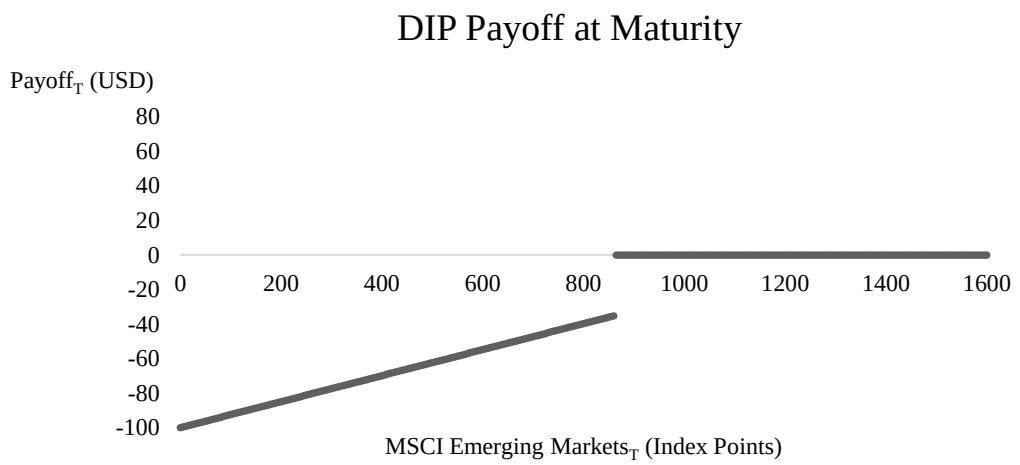


Figure 5 - DIP Payoff at Maturity

In summary, Table 2 discloses the Express Certificate decomposition.

Products	Position	Maturity	Strike	Barrier
Accrual Callable Bond	Long	5 Years (maximum)		
Down-and-In Put Option	Short	5 Years	100%	65%

Table 2 - Decomposition of the Express Certificate

4. Data & Methodology

4.1 Data

The analysis of the certificate is based on two main datasets: the historical prices of the MSCI Emerging Markets Index and the market prices of the Express Certificate, listed on the Luxembourg Stock Exchange.

Regarding the underlying index, historical closing prices of the five-year period prior to the product's issuance are used in the valuation of the certificate. This dataset covers the time span between May 17, 2016 and May 17, 2021.

Concerning the Express Certificate, we collected historical market closing prices since the issuance on May 18, 2021 until the January 1, 2024. The data is used for a comparison of the market's price of the product with its theoretical value during this interval of time.

Beyond the datasets, the detailed explanation of the other key inputs used for valuation purposes are explained in detail in what follows.

4.2 Product Analysis & Valuation

The present subsection, explains the methods followed for the analysis and valuation of the product, which is divided in: volatility employed, product valuation and certificate expected life and scenario probabilities.

4.2.1 Volatility

To perform the valuation of the Express Certificate, the volatility of the underlying index is a crucial input that is subject to uncertainty. Both historical volatility or implied volatility are used for this purpose.

Historical volatility measures the dispersion of returns of the underlying asset over a specific past period, reflecting historical price fluctuations. In contrast, implied volatility represents a marked derived estimate of future volatility inferred from the prices of derivatives. Implied volatility, unlike historical volatility which looks backwards, can be seen as the market's view of volatility over the life of the derivative instrument.

To enhance the depth of our study, we use both historical and implied volatility of the underlying index, creating thereby two different scenarios for this parameter.

To obtain the historical volatility, we compute daily log-returns, u_i, based on the daily closing prices of the index, S_i, during a five-year period prior to the issuance of the product. This period is chosen to match the maximum duration of the Express Certificate. According to Hull (2018), an hypothesis for the chosen time interval of past performance of the underlying is the number of days to which the volatility is to be applied. The calculous are made using Equation (2):

$$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right) \; for \; i = 1, 2, 3, \dots, n \tag{2}$$

Subsequently, the daily standard deviation (s) is computed and used to determine the annualized volatility (σ). Equations (3) and (4) are used for these steps:

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \overline{u})^2} \tag{3}$$

$$\sigma \; (annualized) = s\sqrt{252} \tag{4}$$

The obtained index historical volatility is 15.834%

Concerning the implied volatility, we retrieved from a Bloomberg terminal, the implied volatility of an option with a maturity as close as possible to that of the Express Certificate. The volatility obtained is interpolated using Bloomberg's BVOL engine, based on an at-the-money European Vanilla Call Option as of May 18, 2021, with a maturity at the end of 2026. The volatility obtained was derived from the Black-Scholes model, standing at 22.240%.

Since the issuance of the product on May 18, 2021 until the January 1, 2024, the underlying index exhibited an annualized volatility of 16.194%. This figure suggests a closer level to the historical volatility rather than the implied. As a result, the former is considered as a baseline scenario, while the latter is considered as an alternative scenario.

4.2.2 Product Valuation

Products with complex features are in most cases difficult to value using closed form solutions. The Monte Carlo Simulation is therefore selected as the preferred method for valuing the Express Certificate due to its flexibility in considering potential early termination and the complex payoff structure of the product.

For the valuation of the certificate, the Monte Carlo Simulation is applied for both products that decompose the certificate, the accrual callable bond and the DIP option.

Given that these products perfectly replicate the certificate's payoff structure, this process could be applied directly to the certificate itself.

The rationale behind the use of this method for the valuation is simulating a defined number of random paths for the underlying asset's price and obtain the associated payoff of the products that decompose the certificate for each path simulated. These payoffs are then averaged and discounted in order to obtain the product's value.

In practice, we simulate 10,000 paths[3] for the underlying asset's price, following Equation (5). Each path simulated accounted for daily time-steps, totaling 1248 steps. For this simulation, we use the Geometric Brownian motion stochastic process, which is a commonly adopted approach, as stated by Hull (2018), and is determined by:

$$S_i(t + \Delta t) = S_i(t)e^{\left(\mu - \frac{\sigma^2}{2}\right)\delta t + \sigma\varepsilon_i\sqrt{\delta t}} \quad (5)$$

where $S_i(t)$ is the value of the underlying asset at a specific time; μ is the annualized risk-free rate adjusted for the dividend yield (drift-term); σ is the implied or historical volatility; δt is the length of the time steps and ε_i is a random number from a normal distribution with mean 0 and standard deviation 1.

Concerning the drift-term, μ, and using risk-neutral valuation, we consider the risk-free rate and the dividend yield. The risk-free rate used is the US fitted yield on a 5-year zero coupon bond on the issue date, May 18, 2021. The correspondent rate is of 0.827%. The use of this adjusted rate is to control for the problem of mismatched duration that could occur if a conventional government bond with the same maturity as the product was used. The dividend yield[4] in the year of 2021 is of 2.430%. As a result, the drift-term is calculated as the risk-free rate minus the dividend yield, assuming the value of -1.603%.

The volatility, σ, considered for the baseline scenario is 15.834%, which corresponds to the historical volatility of the underlying index. As for the alternative scenario, it is applied the implied volatility of 22.240%, as described in Section 4.2.1.

The next step consists of computing the payoffs of the accrual callable bond and DIP option in each path simulated. Regarding the accrual callable bond, the conditions for an early termination are evaluated in the autocall observation dates. Starting from the earliest

[3] The Monte Carlo simulation was performed on Python.
[4] The information was extracted from a Bloomberg Terminal.

date, if the condition is verified the bond assumes the correspondent payoff. If not, the next autocall observation date condition is checked, and so forth until the maturity. At maturity, the bond can assume one of the two possible payoffs, depending on the underlying level at that date. Concerning the DIP option, since it only comes into existence if the product was not previously autocalled and the underlying assumes a price below 862.901 index points, for each path simulated, this conditions are verified. If met, the payoff of the DIP option is directly linked to the performance of the underlying.

Finally, we calculate the expected payoff of each product by averaging the payoffs obtained throughout the 10,000 simulations and compute the present value by discounting the payoffs from the moment they occur to moment 0. During this step we take into consideration the different timing of bond's payoffs for discounting purposes. The equations behind the computations are the following:

$$E[c_T] = \sum_{i=1}^{N} \frac{1}{N} c_{T,i} \tag{6}$$

$$c_0 = E[c_T] e^{-rT} \tag{7}$$

where $E[c_T]$ is the expected payoff of the products; N is the number of path simulations; $c_{T,i}$ is the payoff of the option or bond in the i^{th} path simulated; r is the funding rate and T is the time to termination of the products.

The funding rate employed, r, is the average funding cost, obtained by dividing the total interest expense by the average interest bearing liabilities of the issuer, as defined by Aymanns et al. (2016). The data used to perform the computations is sourced from Deutsche Bank's balance sheet for the year of 2020.

Considering the mismatch between the currency of the funding rate (EUR) and the currency of the risk-free rate (USD), we opted to compute the spread between the funding rate based in euros and the 5-year euro area yield on May 18, 2021 (-0.494%), and subsequently sum it to the risk-free rate utilized. The obtained value for the funding rate is therefore 2.237%.

In the end, the certificate's value corresponds to the value of the accrual callable bond subtracted by the value of the DIP option, given that the second involves a short position.

4.2.3 Certificate Expected Life And Scenario Probabilities

The Express Certificate, with its automatic call feature on the autocall observation dates, has the chance of not reaching its maximum duration of 5 years. Instead, it could have a shorter lifespan of 1, 2, 3 or 4 years.

The extent of time that the product is alive directly influences the payoff distributed to investors, thereby impacting the certificate's value. Considering the significance of the certificate's duration, the probability of each life duration scenario occurring is estimated for analysis. These also correspond to the probabilities associated with the payoff received by investors.

In addition, we assess the probability of each maturity payoff scenario occurring, with particular attention to the worst payoff scenario where the capital invested is at risk.

For the probabilities calculations, we consider the number of paths among the 10,000 simulated in the Monte Carlo Simulation, that satisfy the conditions for an autocall event at each anniversary of the product. Regarding the probabilities of each maturity payoff scenario occurring, we consider the number of paths, out of the 10,000 simulated, that result in the investors receiving the corresponding payoff of each scenario.

Based on the probabilities of the certificate surviving a specific number of autocall observation dates, the expected lifespan of the product is calculated. This reflects the probability of the product terminating in each anniversary, given the extent of time that elapsed since the issuance up to that date.

4.3 Product's Market Value vs Theoretical Value

With the aim of comparing the market value of the product to its theoretical value, an analysis of the two figures is conducted every two months from the issuance date of the certificate until January 1, 2024.

In this analysis, we utilize the Monte Carlo Simulation method with the parameters outlined in Section 4.2.2. The product's value is computed using actual underlying asset prices on the valuation dates for the price simulations. Both volatility scenarios are considered in this analysis.

4.4 Autocall Feature Design Sensitivity Analysis

The characteristics of the design of the autocall feature, such as the number and timing of autocall observation dates and the size of the autocall barrier levels have an effect on the outcome of the certificate, both in terms of product's duration as well as the payoff received by investors.

To gain a deeper understanding of the Express Certificate's behavior, the autocall feature design is explored for these characteristics. A sensitivity analysis is conducted using various scenarios to understand their impact on the product's value, probability of being autocalled or reaching the maximum duration and the payoff earned by investors.

Regarding the number of autocall observation dates, it is tested the effect that a lower number of these callable dates have on the product. Apart from the original certificate, four different modifications are simulated. As shown in Table 3, these modifications include: the certificate composed by a single autocall observation date on the first anniversary; two autocall observation dates in the first and second anniversaries; three autocall observation dates in the first three anniversaries; and zero autocall observation dates.

Modifications	Obervation Date 1	Observation Date 2	Observation Date 3	Observation Date 4
Original Product	Yes	Yes	Yes	Yes
Modification 1	Yes	No	No	No
Modification 2	Yes	Yes	No	No
Modification 3	Yes	Yes	Yes	No
Modification 4	No	No	No	No

Table 3 - Product Modifications - Number of Autocall Observation Dates

To assess the impact of the timing of the autocall observation dates, four different modifications with a single observation date are simulated. As shown in Table 4, in each modification, the observation date is applied in the first, second, third and fourth anniversary, respectively.

Modifications	Obervation Date 1	Observation Date 2	Observation Date 3	Observation Date 4
Modification 1	Yes	No	No	No
Modification 2	No	Yes	No	No
Modification 3	No	No	Yes	No
Modification 4	No	No	No	Yes

Table 4 - Product Modifications - Timing of Autocall Observation Dates

As previously disclosed in Section 3.1, the Express Certificate has decreasing autocall barrier levels after the second observation date, i.e. 100% of the underlying initial

reference level on the first and second observation dates, 95% on the third and 90% on the forth.

Regarding this feature, six different modifications are simulated. The effect of constant barriers throughout the four autocall observation dates is assessed, with the levels set at 80%, 90%, 100% up to 120% of the initial reference level of the underlying asset. Moreover, an average of the autocall barrier levels of the original product is computed, and the certificate is valued with constant barriers at that level as well. The tested modifications can be seen in Table 5.

Modifications	Barrier Level 1	Barrier Level 2	Barrier Level 3	Barrier Level 4
Modification 1	80%	80%	80%	80%
Modification 2	90%	90%	90%	90%
Modification 3	96.25%	96.25%	96.25%	96.25%
Modification 4	100%	100%	100%	100%
Modification 5	110%	110%	110%	110%
Modification 6	120%	120%	120%	120%

Table 5 - Product Modifications - Autocall Barrier Levels

For this analysis, the Monte Carlo Simulation is applied with the same parameters and number of simulations as disclosed in Section 4.2.2. The only differences are the modifications applied to the number of dates when the bond can be autocalled, timing of callability and the level of the barriers that trigger an early termination.

Both volatility baseline and alternative scenarios are considered for the autocall feature design sensitivity analysis.

4.5 Greek Analysis

The Greeks are a set of risk measures that help investors assessing the sensitivity of the option prices to multiple factors, such as movements in the underlying asset's price, volatility of the underlying or interest rates. For this analysis, we calculate and analyze four different Greeks of the Express Certificate: Delta, Gamma, Vega and Rho.

According to Hull (2018), Delta (Δ) is the rate of change in value of a derivative, with respect to the underlying asset's price; Gamma (Γ) is the rate of change in the value of Delta, with respect to the underlying asset's price; Vega (ν) measures the rate of change in value of a derivative, with respect to the volatility of the underlying asset; and Rho (ρ) measures the rate of change in the derivative's value, with respect to the interest rate, in this case, the risk-free rate.

According to Wilmott (1998), one straightforward method for calculating the Greeks of an option using the Monte Carlo Simulation, is by estimating the value of the option twice. This technique is applied in this analysis of the certificate, wherein the value of the product is calculated twice, each time varying the variable associated with the respective Greek letter. Concerning the Delta, an increase of 1% in the underlying asset's price is applied. For Gamma, the change in the Delta for a 1 index point increase in the underlying is considered. For Vega, an increase of 1% in the underlying asset volatility is applied and lastly, for Rho, a 1% increase on the risk-free rate is considered.

For a better understanding of the behavior of Delta, Gamma, Vega and Rho over time, these four Greek letters are computed for an extended range of underlying asset prices and throughout the maximum duration of the Express Certificate.

Due to the computational intensity of the calculations, for this section, the product is valued using the Monte Carlo Simulation as explained in Section 4.2.2, but considering bimonthly time steps, rather than daily time steps. Additionally, only the baseline volatility scenario is considered.

5. Results

The present section presents the results of the analysis carried out in this study, and is divided as follows: index price simulations; scenario probabilities; payoff scenarios; Express Certificate expected life; product valuation; product analysis since issuance; expected life given survival; autocall sensitivity analysis; and Greek analysis.

5.1 Index Price Simulations

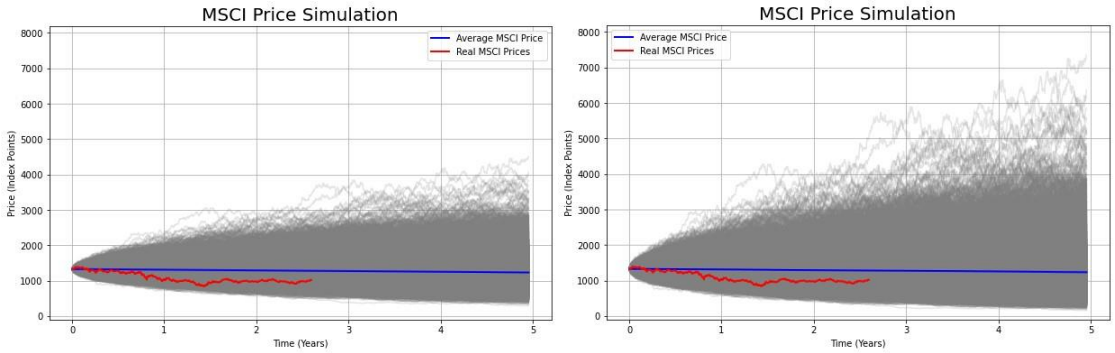


Figure 6 - MSCI Emerging Markets Index Price Simulation (Volatility of 15.83%) (Left Picture) and (Volatility of 22.24%) (Right Picture)

For the analysis and valuation of the product, we simulate the underlying price paths as described in Section 4.2.2. Figure 6 show the results obtained for the 10,000 price simulations using the baseline and alternative volatility scenarios, respectively. In addition, the real underlying path is plotted since the issuance until the beginning of 2024.

Compared with the baseline volatility scenario, the simulation with the alternative volatility scenario exhibits a greater dispersion of the underlying index prices. Under the historical volatility simulation, the MSCI index do not exceed 4,500 index points, whereas the simulation that incorporates the implied volatility achieve values surpassing 7,000 index points. Furthermore, the latter assumes lower values and reaches prices closer to 0, as the product nears its maximum lifespan. This behavior contributes to a more difficult early termination and affects negatively the payoff received by investors if the product is not autocalled.

Table 6, shows the proportion of prices in the anniversaries of the product that are at or above the initial reference level for both volatilities used in this study. As time goes by, the frequency of prices above the initial level is decreasing, reflecting the effect of the negative drift-term applied for the price simulations.

Scenarios	2022	2023	2024	2025	Maturity
σ = 15.83%	43.03%	40.35%	38.53%	36.98%	34.72%
σ = 22.24%	42.94%	40.17%	38.30%	36.78%	34.51%

Table 6 - Frequency of MSCI Prices At or Above the Initial Reference Level

5.2 Scenario Probabilities

Table 7 shows the probabilities of the outcome/payoff scenarios of the Express Certificate for the two volatilities considered. The product exhibits the highest probability of terminating in the first autocall observation date for both volatilities. Under the baseline volatility scenario this probability is of 43.03%, whereas in the alternative is slightly lower, at the level of 42.94%.

Despite the product featuring decreasing barrier levels after the second autocall observation date, the probabilities of the certificate being autocalled decrease as the time to maturity diminishes. Following the high probability of an early termination one year after issuance, with a likelihood greater than 30% is the product reaching its maturity 5 years after issuance. Under the baseline scenario, the likelihood of occurrence is of 31.90%, while slightly higher at the level of 33.59% in the alternative scenario.

In the case of the product surviving until the maturity, both scenarios of a payoff equal to USD 100 and USD 129.75 present a higher probability of occurrence in the baseline volatility scenario. However, the probability of the worst payoff scenario occurring, which is the most probable maturity outcome, is 6.5% higher in the alternative volatility scenario.

Scenarios	Early Finish (2022)	Early Finish (2023)	Early Finish (2024)	Early Finish (2025)	Maturity (USD 129.75 payoff)	Maturity (USD 100 payoff)	Maturity (below USD 100 payoff)
σ = 15.83%	43.03%	11.00%	8.40%	5.67%	4.52%	12.01%	15.37%
	68.10%					31.90%	
σ = 22.24%	42.94%	10.93%	7.52%	5.02%	3.62%	8.10%	21.87%
	66.41%					33.59%	

Table 7 - Scenario Probabilities

5.3 Payoff Scenarios

The Express Certificate has fixed predefined payments to investors in the event of an early termination or if one of the two maturity scenarios that offer capital protection occurs. However, if the product is not autocalled and reaches the maturity with the

underlying below 862.901 index points (worst payoff scenario), the payoff to investors is directly linked to the MSCI Emerging markets index performance. Therefore, we analyze the payoffs associated with this scenario under the tested volatility levels, as shown in Figure 7.

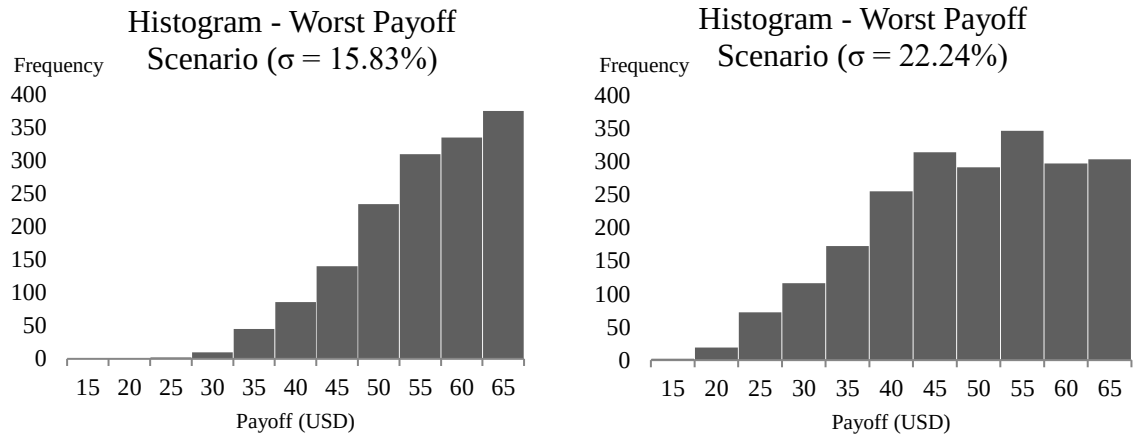


Figure 7 - Worst Payoff Scenario Distribution (Baseline Volatility Scenario) (Left Picture) and (Alternative Volatility Scenario) (Right Picture)

The histograms reveal that, under the baseline volatility scenario, in the paths that the product reaches maturity and the conditions for the occurance of the worst payoff scenario are verified, the distribution of the payoffs has an upward trend. There is a low frequency of lower payoffs and a higher frequency of payoffs closer to the maximum possible under this scenario, USD 65. On the other hand, in the alternative higher volatility scenario, the payoff ditribution reaches lower values close to USD 15, and the higher payoffs occur with a lower frequency.

5.4 Express Certificate Expected life

Table 8 presents the expected lifespan of the product under the two volatility scenarios being analyzed. In the baseline scenario, the Express Certificate has an expected life of 2.72 years, whereas in the alternative of 2.75 years. This result indicates that, at issuance, the certificate was expected to survive the first two autocall observation dates. Furthermore, the longer expected life in the second scenario, reflects the lower probability of the product terminating early at any autocall observation date and the higher chance of reaching maturity.

Scenarios	Expected Life
σ = 15.83%	2.72 Years
σ = 22.24%	2.75 Years

Table 8 - Express Certificate Expected Life

5.5 Product Valuation

Table 9 shows the values of the accrual callable bond, down-and-in put option and the Express Certificate for the two volatility scenarios considered.

Scenarios	Products		
	Accrual Callable Bond	DIP	Express Certificate
σ = 15.83%	$101.80	$6.48	$95.31
σ = 22.24%	$101.20	$10.47	$90.72

Table 9 - Express Certificate Value at Inception

The Express Certificate's value at issuance in the baseline scenario is USD 95.31 and in the alternative is USD 90.72. The product exhibits an inverse relationship with volatility which is mainly caused by the change in value of the down-and-in put option. The increase in the DIP value is explained by the linkage of the option payoff, barrier level and underlying index price. When volatility assumes a higher value, the likelihood of the index reaching maturity with a price below 862.901 index points without being autocalled earlier is higher, thus increasing the chance of the DIP being activated. Additionally, lower prices of the index at maturity, contribute positively for the DIP value. On the other hand, the accrual callable bond does not reveal a big variation in value, despite a small decrease. The bond's value is only affected by the probabilities of each scenario occurrence, given that the payoffs are fixed and do not change with the underlying movements.

5.6 Product Analysis Since Issuance

The Express Certificate has been out on the market since the issuance in 2021 without being autocalled in 2022 and 2023. To understand how the product has evolved in the market in comparision with its theoretical value, the product is valued according with the two volatility scenarios every two months during the period between the issuance and the beginning of 2024. Additionally, a performance analysis is carried for the same period.

As shown in Figure 8, the market's value of the product closely followed its theoretical value under both volatility scenarios. From the last quarter of 2021 until November 2022, the market price closely tracked the theoretical value of the Express Certificate under the alternative volatility scenario. After this period, until the beginning of 2024, it aligned more closely with the theoretical value under the baseline volatility scenario.

The Express Certificate experienced a significant market price decline of over 30% from its issuance until the last quarter of 2022. The downward trend during this period behaved similarly to the underling index asset. After the poor performance period, the certificate

has been showing signs of appreciation, reaching the price of USD 90 at the beginning of 2024.

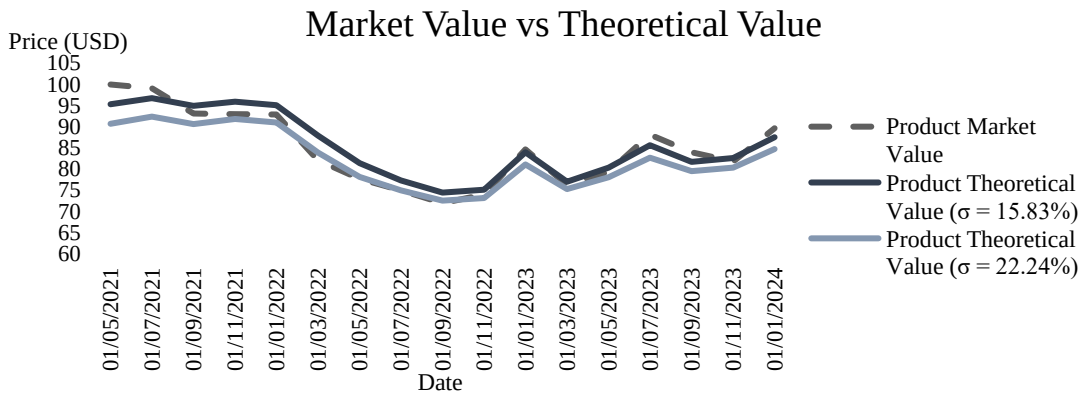


Figure 8 - Express Certificate Market Value vs Theoretical Value

The similarity in market prices and theoretical values, demonstrate that both the implied and historical volatility models reveal adequacy for the valuation of this product.

5.7 Expected Life Given Survival

Since the issuance, the Express Certificate has survived two autocall observation dates in 2022 and 2023, showing an alignement with the initial expected life of over 2 years.

The product's life duration is dependent on the underlying asset price movements, and consequently the expected life measure is very volatile. Therefore, it is recalculated one day after each autocall observation date that the certificate has already survived, and the results can be seen on Table 10.

Scenarios	Expected Life (at 18/05/2022)	Expected Life (at 18/05/2023)
σ = 15.83%	3.58 Years	2.82 Years
σ = 22.24%	3.36 Years	2.68 Years

Table 10 - Expected Life Given the Product Survival to the First and Second Autocall Date

After the survival to the first anniversary, the certificate had an expected life of over 3.5 years in the baseline volatility scenario and a slightly lower in the alternative. Moreover, with the survival to the second autocall observation date, in the baseline volatility scenario, the product had an expected life of over 2.8 years, out of the maximum of 3 years, whereas on the high volatility scenario it was close to 2.7 years. These results suggest that the product is expected to survive all subsequent autocall dates.

Contrary to the expected life of the product at its issuance, after the survival to the first two autocall dates, the product revealed a lower expected life in the alternative volatility

scenario, reflecting the higher probability of an early termination in the subsequent autocall dates, as can be observed in Table A.1 and Table A.2.

5.8 Autocall Sensitivity Analysis

For the sensitivity analysis of the number of autocall observation dates, Figure 9 and Table A.3 demonstrate that, in the baseline volatility scenario, the incorporation of an additional autocall observation date, increase the probability of the product being autocalled. On the other hand, the probability of reaching maturity decreases across all three maturity scenarios, as the addition of each autocall observation date introduce an oportunity for an early finish. This behavior similarly occurs in the alternative volatility scenario, as shown in Figure 10 and Table A.4.

The decrease in the probability of reaching the worst payoff scenario at maturity, which does not offer capital protection, is very small for the product modifications with at least one autocall observation date. The main difference is observed when we compare the modification with zero autocall observation dates (removal of the autocall feature) with the others. This difference is greater than 4% in the baseline volatility scenario and greater than 7% in the alternative, showing that a product without the autocall feature is more likely to result in the scenario that puts the investors capital at risk.

The Express Certificate with its original characteristics reveals the lowest probability of reaching the the worst maturity payoff, in comparision with the alternative modifications.

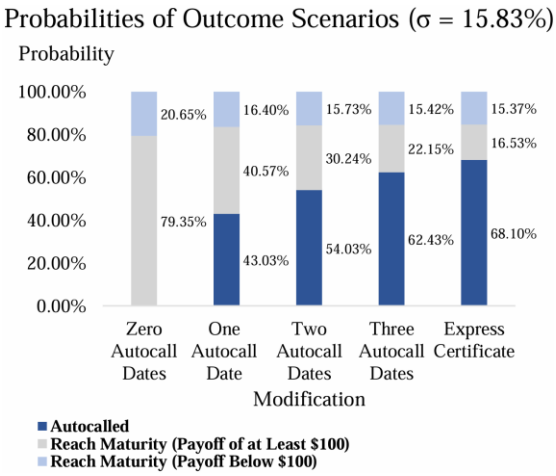


Figure 9 - Probabilities of Outcome Scenarios (σ = 15.83%) – Sensitivity Analysis of the Number of Autocall Dates

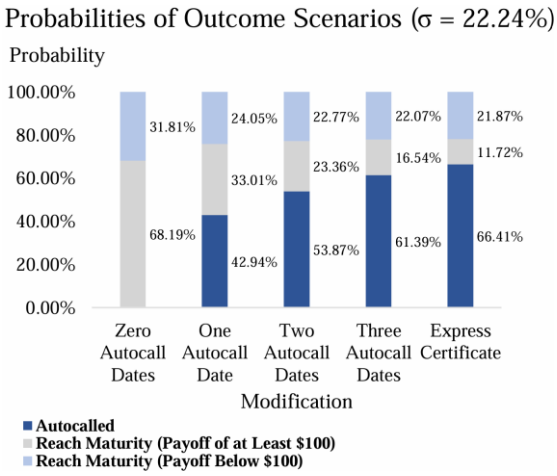


Figure 10 - Probabilities of Outcome Scenarios (σ = 22.24%) – Sensitivity Analysis of the Number of Autocall Dates

Figure 11 and Figure 12 show the product's value results for the two volatility scenarios. When considered at least one autocall observation date, the value of the product gradually increases with the inclusion of each additional early termination date. However, the value of the product modification without the autocall feature, behaves differently across the two volatility scenarios considered. In the baseline volatility scenario it falls between the values of the modifications with two and three autocall observation dates, whereas in the alternative volatility scenario, exhibits the lowest value among all modifications, significatly lower than the rest.

The higher value of the modification with zero autocall observation dates in the baseline volatility scenario is primarily explained by its greater than 50% probabilty of reaching the highest possible payoff scenario of USD 129.75, along with a close to 80% probability of the investor receiving at least the product's notional amount. In contrast, in the alternative volatility scenario, this modification has a probability of paying the product's maximum payoff of 47.28% and a probability of guaranteeing the invested capital below 70%, which negatively impact the product's value. Furthermore, as seen previously, in the higher volatility scenario, the worst maturity payoff reaches lower values compared with the baseline, worsening the product's value.

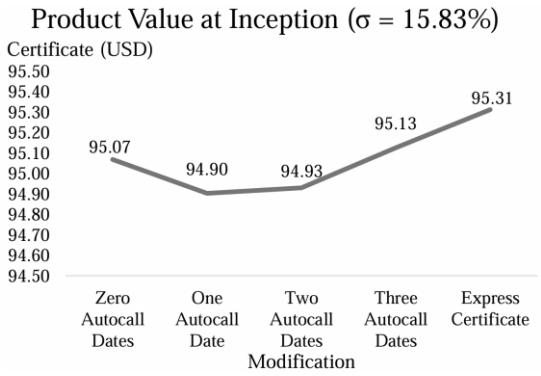


Figure 11 - Product Value at Inception (σ = 15.83%) – Sensitivity Analysis of the Number of Autocall Dates

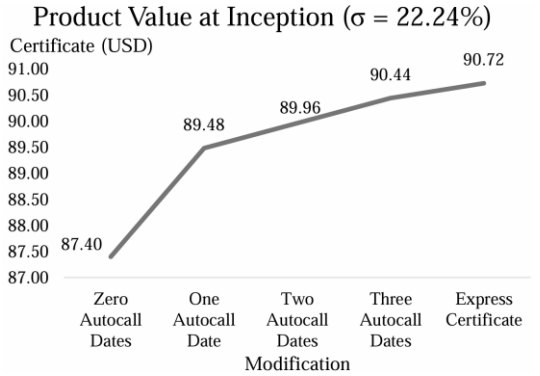


Figure 12 - Product Value at Inception (σ = 22.24%) – Sensitivity Analysis of the Number of Autocall Dates

For the sensitivity analysis of the timing of the autocall observation dates, Figure 13 and Table A.5 show the scenario probabilities of the product modifications tested under the baseline volatility scenario. The effect of the size of the barrier levels is very evident in the probabilities of these modifications being autocalled. The probability of the modification with a single autocall observation date one year after issuance terminating early is of 43.03% but decreases to 40.35% as the autocall observation date is pushed to the second year of the life of the product, as seen in modification 2. This outcome is

attributed to both modifications having the same autocall barrier level, but the second being subject to a longer duration of the underlying price movements on the simulated paths, with a negative drift-term being applied. Concerning the other two modifications with a single autocall observation date in the third and fourth year after the issuance, the probability of an early termination increases as they incorporate lower barrier levels in the third and fourth anniversaries at the levels of 95% and 90%, respectively.

As for the probabilities of the maturity scenarios, the likelihood of investors receiving USD 129.75 decreases, whereas the probability of receiving the product notional amount, USD 100, increases as the autocall observation date is pushed further in time. Moreover, the probability of the investors having their capital at risk in the worst maturity payoff scenario, rises as the autocall observation date approaches maturity. This result is a consequence of the higher paths simulated being called closer to maturity, and only the lower paths surviving until the 5 year maximum duration of the certificate.

Figure 14 and Table A.6 present the results of the alternative volatility scenario. The behavior of the product modifications tested is very similar to the baseline volatility scenario, with differences observed in the magnitude of the scenario probabilities. Across the four modifications, in comparison with the baseline volatility scenario, the probability of occurrence for all outcome scenarios is lower, except for the worst payoff scenario.

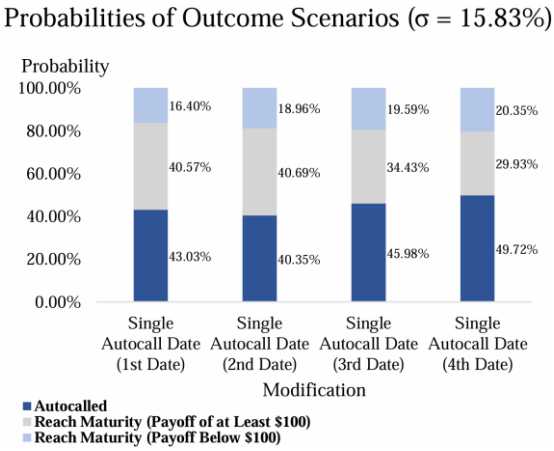


Figure 13 - Probabilities of Outcome Scenarios (σ = 15.83%) – Sensitivity Analysis of the Timing of Autocall Dates

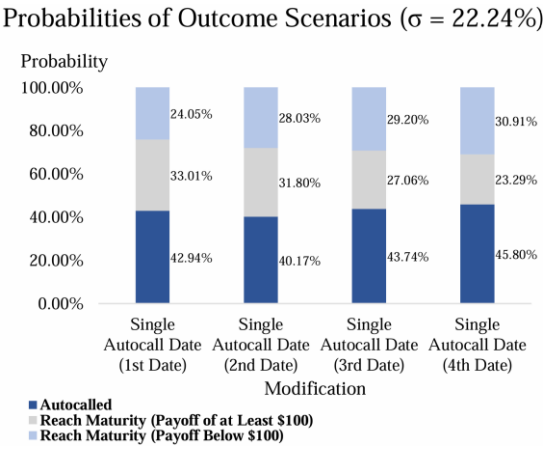


Figure 14 - Probabilities of Outcome Scenarios (σ = 22.24%) – Sensitivity Analysis of the Timing of Autocall Dates

Figure 15 and Figure 16 show the value of the product modifications in the baseline and alternative volatility scenarios, respectively. In the baseline scenario, the second, third and fourth modifications present an increase in value as the autocall observation date is pushed further in time. This behavior is related with the heightened probability of an early

termination with a superior payoff. However, the first modification, with the autocall observation date on the first year, presents a higher value than the second and third modification. This difference is due to its high probability of finish at the first autocall observation date and due to having the highest probability of reaching the maximum payoff at maturity, combined with the lowest probability of occurrence of the worst payoff scenario.

In the alternative volatility scenario, the behavior of the modifications' values is very similar, except for the fourth modification, where the only autocall observation date considered is four years after issuance. In this modification, the product's value falls between the values of the second and third modifications, reflecting the low probability of reaching the maximum maturity payoff and the significant 30.91% probability of reaching the worst payoff scenario. Additionally, in this alternative volatility scenario, the worst maturity scenario payoffs are lower compared to the baseline volatility scenario, contributing to the observed behavior variation.

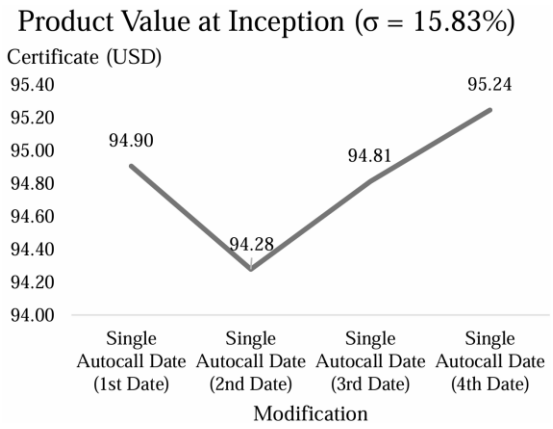


Figure 15 - Product Value at Inception (σ = 15.83%) – Sensitivity Analysis of the Timing of Autocall Dates

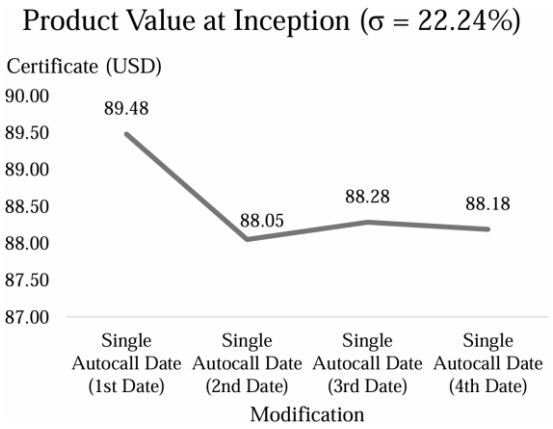


Figure 16 - Product Value at Inception (σ = 22.24%) – Sensitivity Analysis of the Timing of Autocall Dates

For the sensitivity analysis of the size of the barrier levels, Figure 17 and Table A.7, show the results of the scenario probabilities of the modifications tested with constant barrier levels in the baseline volatility scenario. Similarly, Figure 18 and Table A.8 show the results for the alternative volatility scenario. When constant barrier levels are applied, the probability of an early termination decreases as the barriers assume higher values, leading to an increase in the probability of the product reaching any of the maturity payoff scenarios. This behavior is consistent for the two volatility scenarios.

Furthermore, under both volatility scenarios, when the barrier levels are lower than 100%, the probability of the product modifications being terminated at the first autocall

observation date is notably high, assuming a probability very close to or above 50%. Conversely, product modifications with barriers greater than 100% present a higher probability of reaching maturity than being autocalled.

In contrast to the alternative volatility scenario, in the baseline scenario, if the product survives until the maturity, both the product modifications with barriers equal to or greater than 96.25% and the Express Certificate with the original characteristics, reveal a higher probability of paying the investors at least the product's notional amount than paying the worst payoff scenario. In the alternative volatility scenario, only the product modification with barriers at the level of 120% reveal this level of protection.

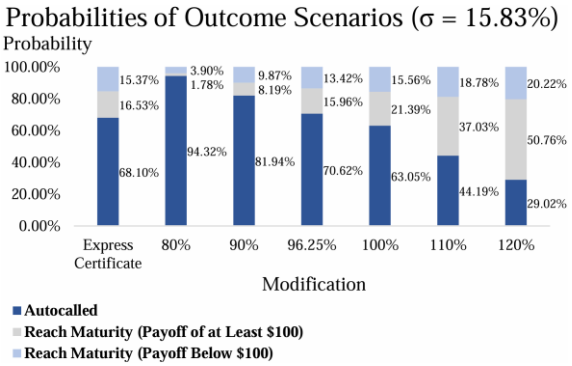


Figure 17 - Probabilities of Outcome Scenarios (σ = 15.83%) – Sensitivity Analysis of the Size of Barrier Levels

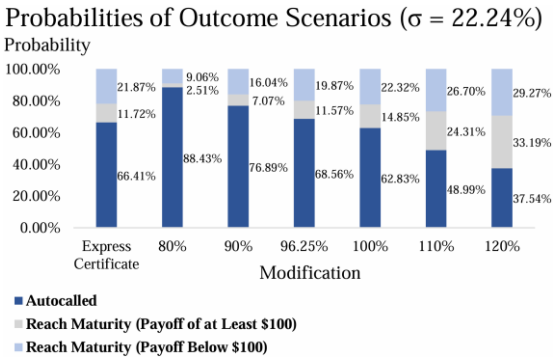


Figure 18 - Probabilities of Outcome Scenarios (σ = 22.24%) – Sensitivity Analysis of the Size of Barrier Levels

Figure 19 and Figure 20 show the value of the product across the modifications tested for both volatility scenarios. As the size of the barrier levels increase, the value of the product gradually decreases, reflecting the decreased probability of an early termination, where the invested capital is guaranteed, with an increased probability of occurrence of the scenario that does not guarantee the invested amount at maturity.

The value of the Express Certificate falls between the value of the modifications with barriers at 96.25% and 100% of the underlying initial reference level.

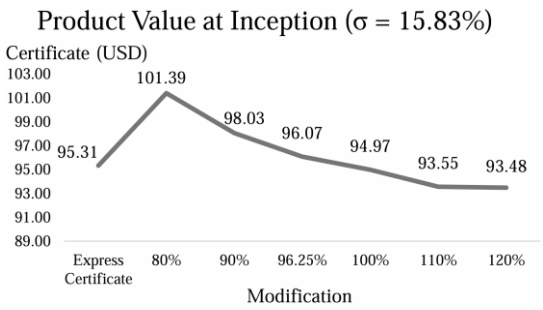


Figure 19 - Product Value at Inception (σ = 15.83%) – Sensitivity Analysis of the Size of Barrier Levels

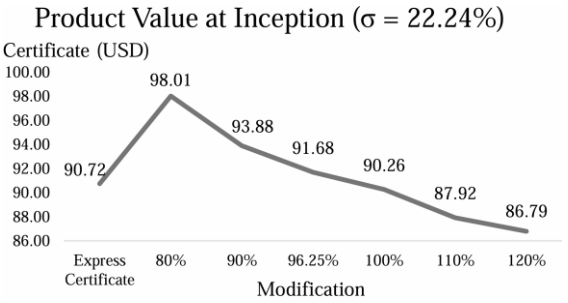


Figure 20 - Product Value at Inception (σ = 22.24%) – Sensitivity Analysis of the Size of Barrier Levels

5.9 Greek Analysis

The sensitivity of the Express Certificate's price to changes in the underlying asset's price is measured by Delta. Figure 21 represent the Delta of the Express Certificate for a range of underlying prices, and different times to maturity. As shown, the Delta of the certificate changes significantly when the underlying asset is close to the various barrier levels, at the autocall dates or at maturity.

Close to the four autocall observation dates, Delta exhibits fair jumps, representing the changes in value of the product as a small price change in the underlying, near these dates, may cause a knock-out of the product.

Similarly, close to expiry, the product's Delta experiences significant jumps near both the down-and-in put option barrier level (862.9 index points) and the underlying level (1128.4 index points) that changes the investor's payoff from USD 100 to USD 129.75, in the case of maturity being reached. At this two maturity barrier levels, a very small change in the underlying asset may cause a significant change in the investor's payoff, thereby impacting the product's value.

Throughout the described underlying levels, the product's Delta behaves similarly to that of digital options.

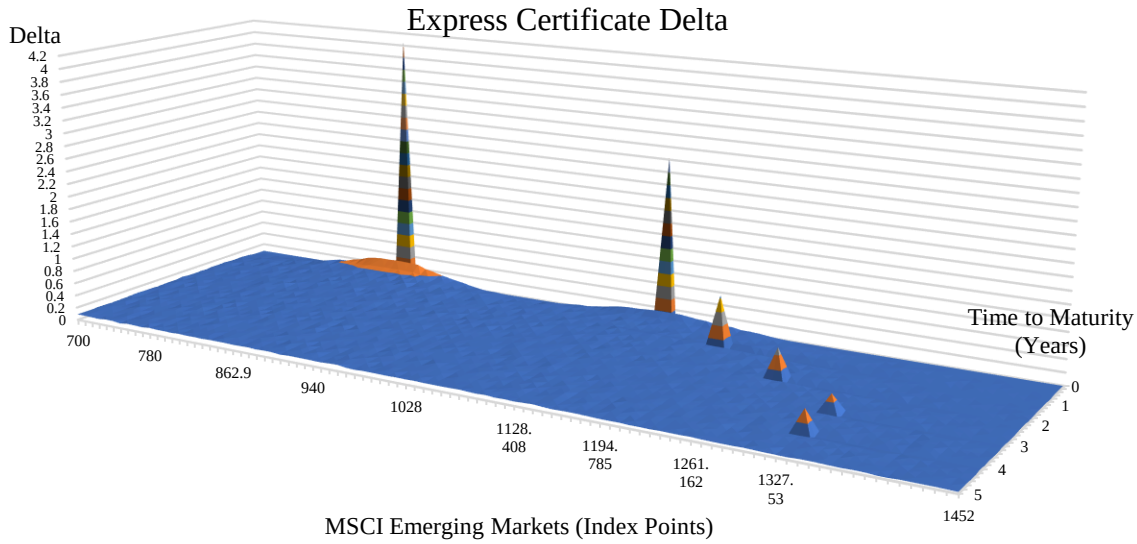


Figure 21 - Express Certificate Delta

The sensitivity of the Express Certificate's Delta to changes in the underlying asset's price is measured by Gamma. Figure 22 shows the Gamma of the Express Certificate for a range of underlying prices, and different levels of time to maturity. The Gamma of the

certificate, similarly to the Delta, experience big changes near the autocall observation dates and the maturity. The Gamma exhibits a change in sign around the autocall barrier levels, the DIP barrier level and the level that triggers the payment of the extra USD 29.75 over the product notional amount.

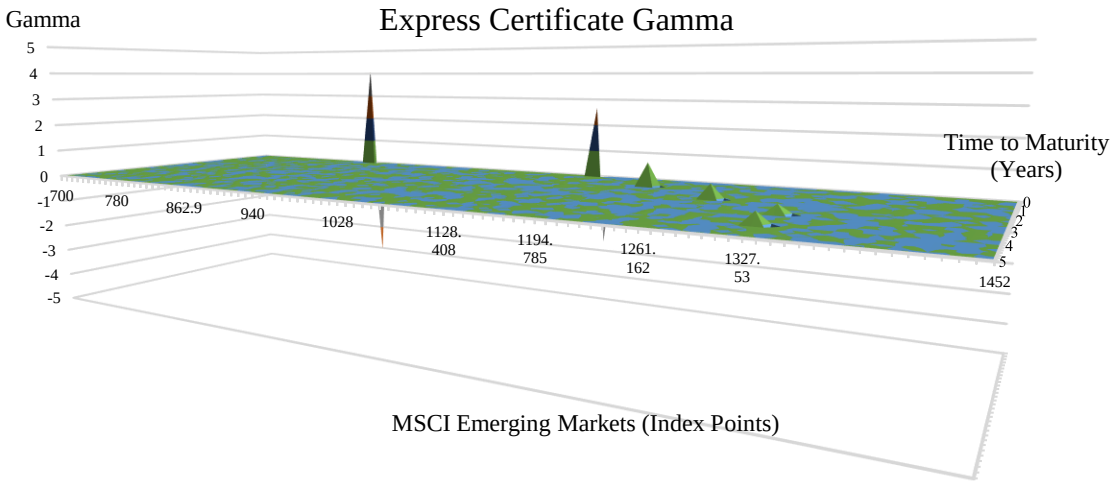


Figure 22 - Express Certificate Gamma

The sensitivity of the Express Certificate's price to changes in the underlying asset's volatility is measured by Vega. Figure 23 reveals the Vega of the certificate throughout different times to maturity since the issuance until the maximum duration of the product and for a wide range of underlying prices. As shown, the levels of the underlying asset with respect to the barrier levels, particularly the DIP barrier, drive the dynamics of Vega.

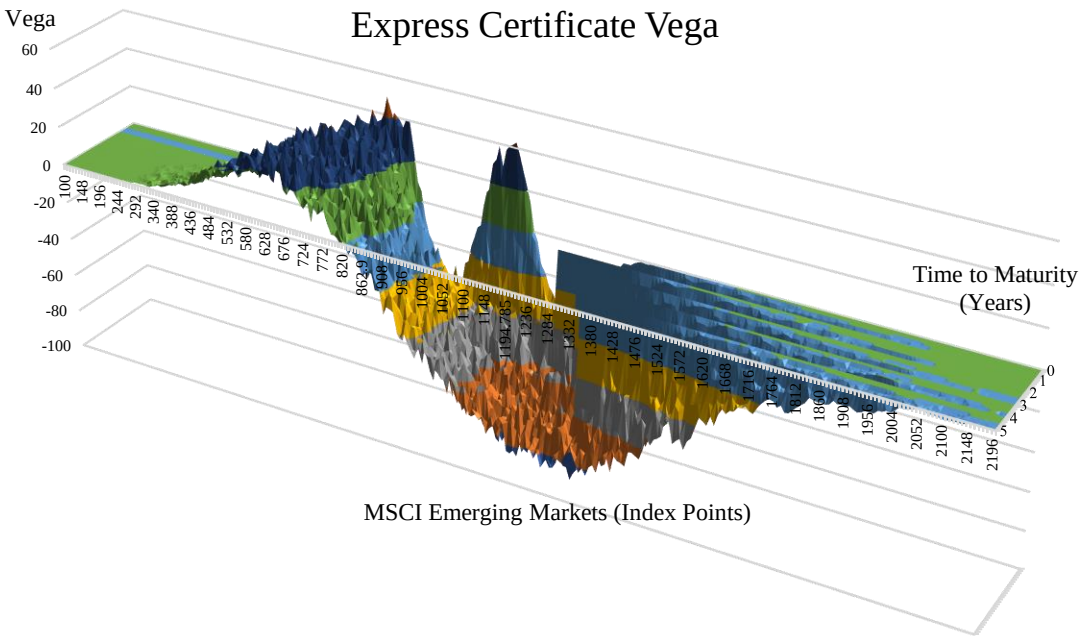


Figure 23 - Express Certificate Vega

As the underlying prices are very low or very high, Vega is close to 0, regardless of the time to maturity, revealing a low sensitivity of the product's value to a change in the underlying's volatility at this price levels. However, until a short period of time after the last autocall observation date, Vega is positive for prices below the DIP barrier level and after crossing this level, it becomes negative. As shown in Figure 23, the peak of Vega for prices below the DIP barrier moves in direction to this barrier, as the maturity becomes closer.

This behavior is mainly explained by the sensitivity to volatility of the DIP option. This options are much more sensitive to changes in volatility of the underlying than a vanilla put option with the same characteristics. Moreover, the likelihood of the product being autocalled also have an impact on this behavior.

When the underlying price is below the DIP barrier, there is some chance of the option being activated, nevertheless it is still very low as it can only occur at maturity. An increase in volatility might contribute for the activation of the DIP with a lower payoff received by investors, due to the applied negative drift-term. Nevertheless, the sensitivity of the DIP option to the volatility of the underlying is low at this levels. On the other hand, an increase in the volatility, increases the probability of the product being autocalled or paying a higher amount at maturity and therefore, the value of the product increases.

Conversely, when the underlying asset's price is above the DIP barrier, the chance of the option being activated at maturity is low. At this levels, an increase in the volatility might increase the probability of the option being activated, and therefore it becomes very sensitive to changes in volatility. Moreover, when the underlying is lower than the next autocall barrier level but higher than the DIP barrier, an increase in volatility increases the chance of the product being autocalled in the next autocall date. Conversely, as the underlying is above the next autocall barrier level and above the DIP barrier, an increase in volatility might reduce the probability of the product ending earlier in the next autocall date. In general, the high sensitivity of the product to volatility at this levels is driven by the strong sensitivity of the DIP option.

As the product survives all four autocall observation dates and approaches the maturity, Vega behaves slightly differently. Besides changing sign around the DIP barrier, it also changes around the 1128.4 index points barrier. When the underlying is to some degree lower than this threshold that triggers the payment of the maximum payoff of the product,

an increase in volatility may increase the chance of this payment occurring, increasing the product's value. Conversely, when the underling is above, a similar change in volatility may increase the chance of the underlying falling to the level that pays to the investors the notional amount of USD 100, therefore decreasing the product's value.

The sensitivity of the Express Certificate's price to changes in the interest rate is measured by Rho. Figure 24 shows the behavior of Rho for a range of underlying asset prices and since the issuance until the maximum duration of the certificate.

Given that the risk-free rate is not used to discount the product's payoffs, the change in this variable only affects the simulation of the underlying paths. By increasing the value of the risk-free rate by 1%, the drift-term used in the simulations of the underlying paths remain negative but higher, contributing for the construction of higher priced paths.

The products Rho decreases as time to maturity decrease, and assumes its highest value when the underlying asset is close to the DIP barrier. This behavior is explained by the higher underlying price paths increasing the likelihood of the product reaching maturity above the threshold, making activation of the DIP harder and therefore increasing the product's value. Following the same rationale, if the product survives all autocall dates and gets closer to maturity, Rho is higher near the underlying levels that trigger a jump in the investor's payoff.

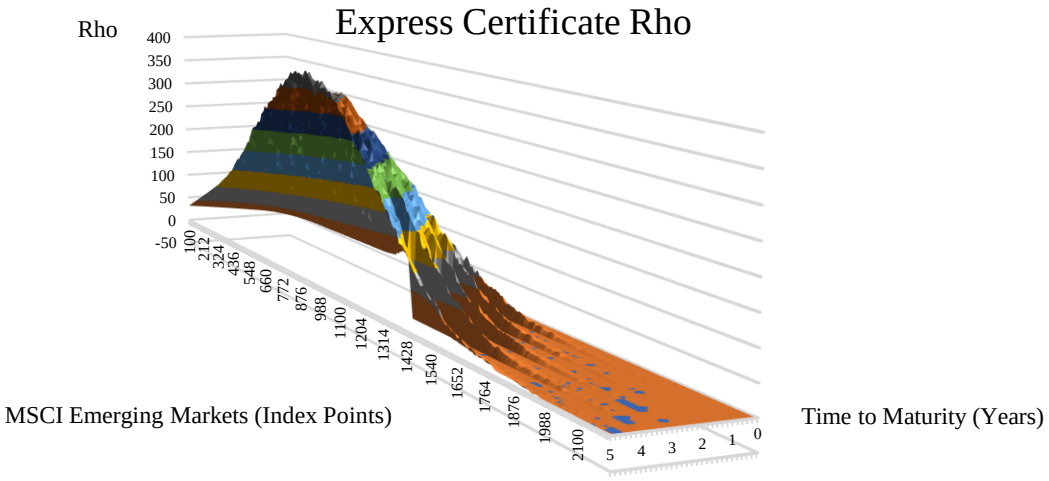


Figure 24 - Express Certificate Rho

6. Conclusions

The main goal of this project is to analyze the Express Certificate linked to MSCI Emerging Markets issued by Deutsche Bank. The analysis goes far beyond valuation as we also look into the product's autocall sensitivity and Greeks.

Investors in this product have a short position on a down-and-in put option and a long position on an accrual callable bond.

In terms of valuation, we use both historical and implied volatilities, as there is no literature consensus on which is best. Pricing and all other analysis is done using Monte Carlo Simulation.

The Express Certificate, under the baseline volatility scenario, has the value of USD 95.31, whereas when under the higher alternative volatility scenario it presents a lower value of USD 90.72. At inception, the certificate presents a considerably high probability of termination on the first anniversary.

The main conclusions from the autocall feature sensitivity analysis are: a higher number of autocall dates increase the value of the product and decreases the likelihood of the product reaching the maturity under the worst payoff scenario; Considering a single autocall date, the closer it is to maturity, the higher the risk of investors losing part or all of their investment; The incorporation of constant barrier levels reduces the chance of an early termination and decreases the product's value, as the barriers are set at higher levels.

With respect to Greeks, Delta and Gamma exhibit an extreme behavior near the autocall observation dates and the maturity, Vega is mainly driven by the down-and-in put option barrier level and Rho decreases as the time to maturity diminishes.

To complement the analysis made, it would be interesting for further studies to analyze and compare the behavior of the Greek letters of similar products with the application of different valuation models, as the standard Monte Carlo Simulation might produce unstable Greeks. Additionally, given the dynamic behavior of the Greeks, the development of a comprehensive hedging strategy would be interesting. The explosive changes around the barriers of the product might require sophisticated techniques to effectively manage risk. Despite the importance of this area, still remains a lack of literature addressing this topic, making further research essential for a better understanding on this field.

References

Adams, J. F., & Smith, D. J. (2019). *Fixed Income Analysis* (4th ed.). John Wiley & Sons, Inc.

Alm, T., Harrach, B., Harrach, D., & Keller, M. (2013). A Monte Carlo Pricing Algorithm for Autocallables that Allows for Stable Differentiation. *The Journal of Computational Finance*, 17(1), 43-70.

Aymanns, C., Caceres, C., Daniel, C., & B Schumacher, L. (2016). Bank Solvency and Funding Cost. *IMF Working Paper*. International Monetary Fund.

Bellefroid, M. d. (2022). The Derivatives Academy. Available at: https://bookdown.org/maxime_debellefroid/MyBook/

Bloomberg L.P. (n.d.): Bloomberg terminal.

Börse Frankfurt. (n.d.). Accrual Bond. Available at: https://www.boerse-frankfurt.de/en/know-how/glossary/accrual-bond

Canina, L., & Figlewski, S. (1993). The Informational Content of Implied Volatility. *The Review of Financial Studies*, 6(3), 659-681.

Christensen, B. J., & Prabhala, N. R. (1998). The Relation Between Implied and Realized Volatility. *Journal of Financial Economics*, 50(2), 125-150.

Deng, G., Husson, T., & McCann, C. (2014). Valuation of Structured Products. *The Journal of Alternative Investments*, 16(4), 71-87.

Deng, G., Mallett, J., & McCann, C. (2011). Modeling Autocallable Structured Products. *Journal of Derivatives & Hedge Funds*, 17, 326-340.

Deutsche Bank. (2020). Annual Report 2020. (2020).

Deutsche Bank. (2021). MSCI Emerging Markets Index (Preisindex) Express Certificate. Available at: https://www.xmarkets.db.com/LU/Product_Detail/XS0459851647

Deutsche Bank. (2024). Deutsche Bank - Who we are. Available at: https://www.db.com/who-we-are/

European Central Bank (2024) Yield Curve Spot Rate, 5-year maturity - Euro area. Available at: https://data.ecb.europa.eu/data/datasets/YC/YC.B.U2.EUR.4F.G_N_A.SV_C_YM.SR_5Y

European Insurance and Occupational Pensions Authority. (2024). Packaged retail and insurance-based investment products (PRIIPs). Available at: https://www.eiopa.europa.eu/browse/regulation-and-policy/packaged-retail-and-insurance-based-investment-products-priips_en

Federal Reserve Economic Data (n.d) Fitted Yield on a 5 Year Zero Coupon Bond. (n.d) Available at: https://fred.stlouisfed.org/series/THREEFY5

Hull, J. (2018). *Options, Futures And Other Derivatives*, 10th Ed. New York: Pearson.

MSCI Emerging Markets Index (USD). (2023) Available at: https://www.msci.com/documents/10199/c0db0a48-01f2-4ba9-ad01-226fd5678111

Wilmott, P. (1998). *Derivatives: The Theory and Practice of Financial Engineering*. John Wiley & Sons, Inc.

Appendix

Figure A. 1 - Key Information Document

Key Information Document

Purpose

This document provides you with key information about this investment product. It is not marketing material. The information is required by law to help you understand the nature, risks, costs, potential gains and losses of this product and to help you compare it with other products.

Product

Product name	Express Certificate linked to MSCI Emerging Markets (Price Index)
Product identifiers	ISIN: XS0459851647 | WKN: DB2DVN
PRIIP manufacturer	Deutsche Bank AG. The product issuer is Deutsche Bank AG, Frankfurt.
Website	www.db.com/contact
Telephone number	Call +49-69-910-00 for more information.
Competent authority of the PRIIP manufacturer	German Federal Financial Supervisory Authority (BaFin)
Date of production	11 October 2023

You are about to purchase a product that is not simple and may be difficult to understand.

1. What is this product?

Type

German law governed certificates

Term

The product has a fixed term and will be due on 20 May 2026, subject to an early redemption.

Objectives

(Terms that appear in **bold** in this section are described in more detail in the table(s) below.)

The product is designed to provide a return in the form of a cash payment on termination of the product. The timing and amount of this payment will depend on the performance of the **underlying**.

Early termination following an autocall: The product will terminate prior to the **maturity date** if, on any **autocall observation date**, the **reference level** is at or above the relevant **autocall barrier level**. On any such early termination, you will on the immediately following **autocall payment date** receive a cash payment equal to the applicable **autocall payment**. The relevant dates, **autocall barrier levels** and **autocall payments** are shown in the table(s) below.

Autocall observation dates	*Autocall barrier levels*	*Autocall payment dates*	*Autocall payments*
17 May 2022	1,327.54	20 May 2022	USD 105.95
17 May 2023	1,327.54	22 May 2023	USD 111.90
15 May 2024	1,261.163	20 May 2024	USD 117.85
15 May 2025	1,194.786	20 May 2025	USD 123.80

Termination on the maturity date: If the product has not terminated early, on the **maturity date** you will receive:

1. if the **final reference level** is at or above 1,128.409, a cash payment equal to USD 129.75;
2. if the **final reference level** is at or above 862.901 and below 1,128.409, a cash payment equal to USD 100; or
3. if the **final reference level** is below 862.901, a cash payment directly linked to the performance of the **underlying**. The cash payment will equal (i) the **product notional amount** multiplied by (ii) (A) the **final reference level** divided by (B) 1,327.54.

Under the product terms, certain dates specified above and below will be adjusted if the respective date is either not a business day or not a trading day (as applicable). Any adjustments may affect the return, if any, you receive.

You do not have any entitlement to a dividend from the **underlying** and you have no right to any further entitlement resulting from the **underlying** (e.g., voting rights).

Underlying	MSCI Emerging Markets (Price return index) (ISIN: CH0007292201)	**Initial reference level**	1,327.54
Underlying market	Equity	**Reference level**	The closing level of the **underlying** as per the **reference source**
Product notional amount	USD 100	**Reference source**	MSCI Inc., New York
Product currency	U.S. Dollar (USD)	**Final reference level**	The **reference level** on the **valuation date**
Underlying currency	U.S. Dollar (USD)	**Valuation date**	15 May 2026
Issue date	18 May 2021	**Maturity date / term**	20 May 2026

The issuer may terminate the product with immediate effect in the event of obvious written or mathematical errors in the terms and conditions or if certain extraordinary events provided in the terms and conditions occur. Examples of extraordinary events include (1) material changes, particularly in connection with the **underlying**, including where an index ceases to be calculated, and (2) events, in particular due to changes in certain external conditions that hinder the issuer in meeting its obligations in connection with the product or – depending on the terms and conditions of the security – otherwise affect the product and/or the issuer. In case of immediate termination, the return (if any) may be significantly lower than the purchase price, but will reflect the product's market value and, if higher, any minimum redemption (alternatively, in some cases the corresponding compounded amount may be paid out at the product's scheduled maturity). Instead of immediate termination, the issuer may also amend the terms and conditions.

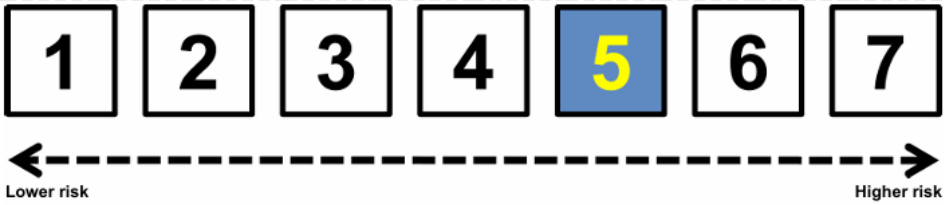
Provided that in the event of any inconsistency and/or conflict between the foregoing paragraph and any applicable law, order, rule or other legal requirement of any governmental or regulatory authority in a territory in which this product is offered, such national requirements shall prevail.

Intended retail investor

The product is intended for private clients who pursue the objective of general capital formation/asset optimization and have a short-term investment horizon. This product is a product for clients who have sufficient knowledge and / or experience to make an informed investment decision. The investor can bear losses up to the total loss of the capital invested and attaches no importance to capital protection.

2. What are the risks and what could I get in return?

Risk indicator



The risk indicator assumes you keep the product until 20 May 2026. The actual risk can vary significantly if you cash in at an early stage and you may get back less. You may not be able to sell your product easily or may have to sell at a price that significantly impacts on how much you get back.

The summary risk indicator is a guide to the level of risk of this product compared to other products. It shows how likely it is that the product will lose money because of movements in the markets or because we are not able to pay you.

We have classified this product as 5 out of 7, which is a medium-high risk class. This rates the potential losses from future performance at a medium-high level, and poor market conditions are very unlikely to impact our capacity to pay you.

To the extent the currency of the country in which you purchase this product or your account currency differs from the product currency, please be aware of currency risk. You will receive payments in a different currency so the final return you will get depends on the exchange rate between the two currencies. This risk is not considered in the indicator shown above.

Inflation erodes the purchasing value of cash over time and this may result in the decline in real terms of any capital reimbursed.

This product does not include any protection from future market performance so you could lose some or all of your investment.

If we are not able to pay you what is owed, you could lose your entire investment.

Performance scenarios

What you will get from this product depends on future market performance. Market developments in the future are uncertain and cannot be accurately predicted.

The scenarios shown are illustrations based on results from the past and on certain assumptions. Markets could develop very differently in the future.

Recommended holding period:		**Until the product is called or matures** This may be different in each scenario and is indicated in the table	
Example investment:		USD 10,000	
Scenarios		***If you exit after 1 year***	***If you exit at call or maturity***
Minimum	**There is no minimum guaranteed return. You could lose some or all of your investment.**		
Stress	**What you might get back after costs**	USD 4,392	USD 4,603
(product ends after 2 years and 7 months)	Average return each year	-56.1%	-25.7%
Unfavourable	**What you might get back after costs**	USD 7,210	USD 6,177
(product ends after 2 years and 7 months)	Average return each year	-27.9%	-16.8%
Moderate	**What you might get back after costs**	USD 10,391	USD 12,282
(product ends after 2 years and 7 months)	Average return each year	3.9%	8.2%
Favourable	**What you might get back after costs**	USD 13,192	USD 15,936
(product ends after 2 years and 7 months)	Average return each year	31.9%	19.5%

The favourable, moderate, unfavourable and stress scenarios represent possible outcomes that have been calculated based on simulations using the reference asset's past performance over a period of up to 5 years. In the case of an early redemption, it has been assumed that no reinvestment has occurred. The stress scenario shows what you might get back in extreme market circumstances. This product cannot be easily cashed in. If you exit the investment earlier than the recommended holding period you may have to pay extra costs.

The figures shown include all the costs of the product itself, but may not include all the costs that you pay to your advisor or distributor. The figures do not take into account your personal tax situation, which may also affect how much you get back.

3. What happens if Deutsche Bank AG, Frankfurt is unable to pay out?

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You are exposed to the risk that the issuer might be unable to fulfil its obligations in respect of the product – e.g. in the event of insolvency (inability to pay / over-indebtedness) or an administrative order of resolution measures. In case of a crisis of the issuer such an order can also be issued by a resolution authority in the run-up of an insolvency proceeding. In doing so, the resolution authority has extensive intervention powers. Among other things, it can reduce rights of the investors to zero, terminate the product or convert it into shares of the issuer and suspend rights of the investors. With regard to the basic ranking of the issuer's obligations in the event of action by the resolution authority, please see www.bafin.de and search for the keyword "Haftungskaskade". A total loss of your capital invested is possible. The product is a debt instrument and as such is not covered by any deposit protection scheme.

4. What are the costs?

The person advising on or selling you this product may charge you other costs. If so, this person will provide you with information about these costs and how they affect your investment.

Costs over time

The tables show the amounts that are taken from your investment to cover different types of costs. These amounts depend on how much you invest, how long you hold the product and how well the product does. The amounts shown here are illustrations based on an example investment amount and different investment periods.

The duration of this product is uncertain as it may terminate at different times depending on how the market evolves. The amounts shown here consider two different scenarios (early call and maturity). In case you decide to exit before the product ends, exit costs may apply in addition to the amounts shown here.

We have assumed:

- USD 10,000 is invested
- a performance of the product that is consistent with each holding period shown.

	If the product is called at the first possible date, on 20 May 2024	*If the product reaches maturity*
Total costs	USD 0	USD 0
Annual cost impact*	0.0%	0.0% each year

*This illustrates how costs reduce your return each year over the holding period. For example it shows that if you exit at maturity your average return per year is projected to be 8.2% before costs and 8.2% after costs.

We may share part of the costs with the person selling you the product to cover the services they provide to you. They will inform you of the amount.

Composition of costs

	One-off costs upon entry or exit	If you exit after 1 year
Entry costs	0.0% of the amount you pay when entering this investment. These costs are already included in the price you pay.	USD 0
Exit costs	1.3% of your investment amount if you return this product before its settlement date. These costs are already included in the price you receive. The costs indicated assume that normal market conditions apply. If an early termination occurs, no exit costs will be incurred.	USD 123

5. How long should I hold it and can I take money out early?

Recommended holding period: 2 years and 7 months

The product aims to provide you with the return described under "1. What is this product?" above. However, this only applies if the product is held to maturity. It is therefore recommended that the product is held until 20 May 2026 (maturity).

The product does not guarantee the possibility to disinvest other than by selling the product off-exchange. Save as otherwise disclosed in exit costs (see section "4. What are the costs?" above), no fees or penalties will be charged by the issuer for any such transaction. However if you sell the product in the secondary market you will incur a bid/offer spread. By selling the product before its maturity, you may receive back less than you would have received if you had kept the product until maturity.

Exchange listing	Luxembourg Stock Exchange (Main Segment)	**Last exchange trading day**	14 May 2026
Smallest tradable unit	1 unit	**Price quotation**	Units

In volatile or unusual market conditions, or in the event of technical faults/disruptions, the purchase and/or sale of the product can be temporarily hindered and/or suspended and may not be possible at all.

6. How can I complain?

Any complaint regarding the conduct of the person advising on, or selling, the product can be submitted directly to that person.

Any complaint regarding the product or the conduct of the manufacturer of this product can be submitted in writing at the following address: Mainzer Landstrasse 11-17, 60329 Frankfurt am Main, Germany, by email to: x-markets.team@db.com or at the following website: www.xmarkets.db.com.

7. Other relevant information

Any additional documentation in relation to the product and in particular the prospectus, any supplements thereto and the final terms are published on the manufacturer's website (www.xmarkets.db.com/DocumentSearch ; after entering of the respective ISIN or WKN), all in accordance with legal requirements. In order to obtain more detailed information - and in particular details of the structure and risks associated with an investment in the product - you should read these documents. These documents are also available free of charge from Deutsche Bank AG, Mainzer Landstrasse 11-17, 60329 Frankfurt am Main, Germany, in accordance with legal requirements.

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Table A. 1 - Scenario Probabilities Given Survival to the First Autocall Date

Scenarios	Autocalled in the Second Anniversary	Autocalled in the Third Anniversary	Autocalled in the Fourth Anniversary	Maturity (USD 129.75 payoff)	Maturity (USD 100 payoff)	Maturity (below USD 100 payoff)
σ = 15.83%	3.75%	10.43%	10.27%	8.53%	26.91%	40.11%
σ = 22.24%	9.54%	12.61%	9.99%	7.46%	15.61%	44.79%

Table A. 2 - Scenario Probabilities Given Survival to the Second Autocall Date

Scenarios	Autocalled in the Third Anniversary	Autocalled in the Fourth Anniversary	Maturity (USD 129.75 payoff)	Maturity (USD 100 payoff)	Maturity (below USD 100 payoff)
σ = 15.83%	3.77%	10.55%	9.99%	31.62%	44.07%
σ = 22.24%	9.49%	12.53%	9.54%	20.32%	48.12%

Table A. 3 - Probabilities of Outcome Scenarios (σ = 15.83%) – Sensitivity Analysis of the Number of Autocall Dates

Modification	Autocalled in the First Autocall Date	Autocalled in the Second Autocall Date	Autocalled in the Third Autocall Date	Autocalled in the Fourth Autocall Date	Reach Maturity (Payoff of $129.75)	Reach Maturity (Payoff of $100)	Reach Maturity (Payoff Below $100)
Zero Autocall Dates	0.00%	0.00%	0.00%	0.00%	52.85%	26.50%	20.65%
One Autocall Date	43.03%	0.00%	0.00%	0.00%	23.24%	17.33%	16.40%
Two Autocall Dates	43.03%	11.00%	0.00%	0.00%	15.16%	15.08%	15.73%
Three Autocall Dates	43.03%	11.00%	8.40%	0.00%	8.94%	13.21%	15.42%
Express Certificate	43.03%	11.00%	8.40%	5.67%	4.52%	12.01%	15.37%

Table A. 4 - Probabilities of Outcome Scenarios (σ = 22.24%) – Sensitivity Analysis of the Number of Autocall Dates

Modification	Autocalled in the First Autocall Date	Autocalled in the Second Autocall Date	Autocalled in the Third Autocall Date	Autocalled in the Fourth Autocall Date	Reach Maturity (Payoff of $129.75)	Reach Maturity (Payoff of $100)	Reach Maturity (Payoff Below $100)
Zero Autocall Dates	0.00%	0.00%	0.00%	0.00%	47.28%	20.91%	31.81%
One Autocall Date	42.94%	0.00%	0.00%	0.00%	19.80%	13.21%	24.05%
Two Autocall Dates	42.94%	10.93%	0.00%	0.00%	12.40%	10.96%	22.77%
Three Autocall Dates	42.94%	10.93%	7.52%	0.00%	7.31%	9.23%	22.07%
Express Certificate	42.94%	10.93%	7.52%	5.02%	3.62%	8.10%	21.87%

Table A. 5 - Probabilities of Outcome Scenarios (σ = 15.83%) – Sensitivity Analysis of the Timing of Autocall Dates

Modification	Autocalled in the First Autocall Date	Autocalled in the Second Autocall Date	Autocalled in the Third Autocall Date	Autocalled in the Fourth Autocall Date	Reach Maturity (Payoff of $129.75)	Reach Maturity (Payoff of $100)	Reach Maturity (Payoff Below $100)
Single Autocall Date (1st Date)	43.03%	0.00%	0.00%	0.00%	23.24%	17.33%	16.40%
Single Autocall Date (2nd Date)	0.00%	40.35%	0.00%	0.00%	21.01%	19.68%	18.96%
Single Autocall Date (3rd Date)	0.00%	0.00%	45.98%	0.00%	14.64%	19.79%	19.59%
Single Autocall Date (4th Date)	0.00%	0.00%	0.00%	49.72%	8.76%	21.17%	20.35%

Table A. 6 - Probabilities of Outcome Scenarios (σ = 22.24%) – Sensitivity Analysis of the Timing of Autocall Dates

Modification	Autocalled in the First Autocall Date	Autocalled in the Second Autocall Date	Autocalled in the Third Autocall Date	Autocalled in the Fourth Autocall Date	Reach Maturity (Payoff of $129.75)	Reach Maturity (Payoff of $100)	Reach Maturity (Payoff Below $100)
Single Autocall Date (1st Date)	42.94%	0.00%	0.00%	0.00%	19.80%	13.21%	24.05%
Single Autocall Date (2nd Date)	0.00%	40.17%	0.00%	0.00%	17.56%	14.24%	28.03%
Single Autocall Date (3rd Date)	0.00%	0.00%	43.74%	0.00%	12.70%	14.36%	29.20%
Single Autocall Date (4th Date)	0.00%	0.00%	0.00%	45.80%	8.12%	15.17%	30.91%

Table A. 7 - Probabilities of Outcome Scenarios (σ = 15.83%) – Sensitivity Analysis of the Size of Barrier Levels

Modification	Autocalled in the First Autocall Date	Autocalled in the Second Autocall Date	Autocalled in the Third Autocall Date	Autocalled in the Fourth Autocall Date	Reach Maturity (Payoff of $129.75)	Reach Maturity (Payoff of $100)	Reach Maturity (Payoff Below $100)
Express Certificate	43.03%	11.00%	8.40%	5.67%	4.52%	12.01%	15.37%
Constant Barrier Levels at 80%	89.39%	3.07%	1.21%	0.65%	0.26%	1.52%	3.90%
Constant Barrier Levels at 90%	69.60%	7.18%	3.20%	1.96%	2.14%	6.05%	9.87%
Constant Barrier Levels at 96.25%	52.81%	10.18%	4.73%	2.90%	5.03%	10.93%	13.42%
Constant Barrier Levels at 100%	43.03%	11.00%	5.69%	3.33%	7.69%	13.70%	15.56%
Constant Barrier Levels at 110%	22.50%	11.10%	6.29%	4.30%	16.51%	20.52%	18.78%
Constant Barrier Levels at 120%	9.49%	8.80%	6.47%	4.26%	26.61%	24.15%	20.22%

Table A. 8 - Probabilities of Outcome Scenarios (σ = 22.24%) – Sensitivity Analysis of the Size of Barrier Levels

Modification	Autocalled in the First Autocall Date	Autocalled in the Second Autocall Date	Autocalled in the Third Autocall Date	Autocalled in the Fourth Autocall Date	Reach Maturity (Payoff of $129.75)	Reach Maturity (Payoff of $100)	Reach Maturity (Payoff Below $100)
Express Certificate	42.94%	10.93%	7.52%	5.02%	3.62%	8.10%	21.87%
Constant Barrier Levels at 80%	80.22%	4.58%	2.20%	1.43%	0.54%	1.97%	9.06%
Constant Barrier Levels at 90%	62.29%	8.40%	3.84%	2.36%	2.16%	4.91%	16.04%
Constant Barrier Levels at 96.25%	49.88%	10.48%	5.10%	3.10%	3.93%	7.64%	19.87%
Constant Barrier Levels at 100%	42.94%	10.93%	5.67%	3.29%	5.52%	9.33%	22.32%
Constant Barrier Levels at 110%	27.79%	11.02%	6.25%	3.93%	10.84%	13.47%	26.70%
Constant Barrier Levels at 120%	16.24%	10.29%	6.61%	4.40%	16.65%	16.54%	29.27%

Disclaimer

This masters project was developed with strict adherence to the academic integrity policies and guidelines set forth by ISEG, Universidade de Lisboa. The work presented herein is the result of my own research, analysis, and writing, unless otherwise cited. In the interest of transparency, I provide the following disclosure regarding the use of artificial intelligence (AI) tools in the creation of this project.

I disclose that AI tools were employed during the development of this project as follows:

Chat-GPT was used only for English and grammar checking on parts of the written text.

Nonetheless, I have ensured that the use of AI tools did not compromise the originality and integrity of my work. All sources of information have been appropriately cited in accordance with academic standards. I understand the importance of maintaining academic integrity and take full responsibility for the content and originality of this work.

Diogo Miguel Martinho Pereira

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