Lisbon School of Economics & Management
Universidade de Lisboa

# MASTER

## APPLIED ECONOMETRICS AND FORECASTING

# MASTER´S FINAL WORK

## DISSERTATION

# HOUSE PRICE DYNAMICS IN THE LISBON METROPOLITAN AREA


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## ABSTRACT

The Housing market plays a key role in the economy, influencing both financial stability and household wealth. Therefore, studying the dynamics of housing prices is important, particularly in identifying periods of price exuberance and understanding the mechanisms of contagion. This makes the detection of these patterns an important issue, as they provide insights of the shortcomings in the market. This dissertation examines the existence of housing bubbles and contagion across 18 municipalities of the Lisbon Metropolitan Area. Using quarterly real house price data from January 2007 to October 2021, we applied the real-time monitoring procedure developed by Phillips et al. (2015) to identify explosive behavior in house prices. Following this, we employed the time-varying Granger causality framework proposed by Shi et al. (2018) to analyze the exuberance contagion between municipalities. Our findings indicate that all municipalities experienced episodes of exuberance, which persisted until the end of the sample period, except for Lisbon. Additionally, we find evidence of exuberance contagion from Lisbon to the surrounding municipalities, with the exceptions of Oeiras and Sesimbra.



KEYWORDS: Housing prices; Exuberance; Contagion; Recursive right-tailed unit root tests; Time-varying Granger Causality; District and municipality data.

JEL CODES: C12; C32; G01; G12; R30; R31.

# Resumo

O mercado da habitação desempenha um papel fundamental na economia, influenciando tanto a estabilidade financeira como a riqueza das famílias. Portanto, estudar as dinâmicas dos preços das habitações é importante, particularmente a identificação de períodos de exuberância de preços e a compreensão dos mecanismos de contágio. Isso faz com que a detecção desses padrões seja uma questão importante, pois podem fornecer-nos informações sobre as deficiências do mercado. Esta dissertação examina a existência de bolhas imobiliárias e o contágio entre os 18 municípios da aréa metropolitana de Lisboa. Utilizando dados trimestrais dos preços reais das habitações de Janeiro de 2007 a Outubro de 2021, aplicámos o procedimento de monitorização em tempo real desenvolvido por Phillips et al. (2015) para identificar comportamentos explosivos nos preços das habitações. Posteriormente, utilizámos a causalidade de Granger variável no tempo proposta por Shi et al. (2018), para analisar o contágio da exuberância entre os municípios. Os nossos resultados indicam que todos os municípios apresentam evidência de episódios de exuberância, que persistiram até ao final do período da amostra, com exceção de Lisboa. Adicionalmente, encontramos evidências de contágio da exuberância de Lisboa para os municípios circundantes, com exceção de Oeiras e Sesimbra.

Palavras-chave: Preços das habitações; Exuberância; Contágio; Testes de raiz unitária recursivo de cauda à direita; Causalidade de Granger variável no tempo; Dados distritais e municipais.

Códigos JEL: C12; C32; G01; G12; R30; R31.

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# GLOSSARY

ADF – Augmented Dickey-Fuller

AIC – Akaike Information Criterion

BIC – Bayesian Information Criterion

BSADF – Backward Supremum Augmented Dickey-Fuller

GDP – Gross Domestic Product

GFC – Global Financial Crisis

GSADF – Generalized Supremum Augmented Dickey-Fuller

HPI – House Price Index

INE – Statistics Portugal (Instituto Nacional de Estatística)

LMA – Lisbon Metropolitan Area

MFW – Master's Final Work

OECD – Organisation for Economic Co-operation and Development

SADF – Supremum Augmented Dickey-Fuller

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# 1. INTRODUCTION

According to Kim and Park (2005), the housing market operates based on the principles of supply and demand, making housing services one of the largest household expenditures. Variations in housing prices are significant for both individuals and governments due to their far-reaching effects on socio-economic conditions and the overall economy. For example, anticipated capital gains from housing investments can drive up demand, resulting in high price volatility, particularly since housing supply cannot swiftly adjust in the short term. This volatility is particularly impactful as the housing market influences the economy through the wealth effect, where fluctuations in housing prices can alter consumer spending, savings, and investment strategies (Hossain and Latif, 2009).

The disconnection between house prices and their fundamental values, which consequently resulted in an inefficiency of resource allocation and skewed investment decisions, was one of the possible factors that contributed to the 2008-2009 global recession (Pavlidis et al., 2016). The global financial crisis (GFC) triggered an unprecedented decline in house prices globally, with the most severe impacts felt in countries that had previously experienced a real estate bubble. In the aftermath, house prices worldwide have generally seen a persistent growth, though the extent of this growth has varied significantly by country (Lourenço and Rodrigues, 2017; and Cevik and Naik, 2024).

In Portugal, property values have risen consistently in recent years, following a period of relatively modest growth during the late 1990s and early 2000s, leading up to the 2007 financial crisis (Rodrigues et al., 2022). The financial crisis negatively impacted housing prices in Portugal, particularly in the Lisbon area (Januário and Cruz, 2023). According to Statistics Portugal (INE, 2024), the house price index (HPI) in 2023 maintained its growth trajectory but slowed down, with the annual rate of change decreasing from 12.7% in 2022 to 8.2% in 2023 (a 4.4 percentage-point reduction). Meanwhile, in the first quarter of 2024, the HPI increased by 7.0% year-on-year, a 0.8 percentage-point decline compared to the previous quarter and the lowest price increase since the first quarter of 2021. During these three months, existing dwellings' prices rose more rapidly than those of new dwellings - 7.6% and 5.5%, respectively (INE, 2024).

The concept of bubbles in economics and finance, particularly in the housing market, is critical to the understanding of the dynamics of price volatility. According to Stiglitz (1990), “If the reason that the price is high today is only because investors believe that the selling price will be high tomorrow—when “fundamental” factors do not seem to justify such a price—then a bubble exists.” This definition underscores the potential for speculative behavior to drive prices beyond what is supported by underlying economic fundamentals, and as a result, an explosive behavior in prices is triggered and temporarily dominates its time series dynamics.

Asset price bubbles generally follow three phases. First, a period of large credit expansion accompanies a sustained increase in asset prices, such as real estate and stocks, inflating the bubble. Second, the bubble bursts, and prices collapse, sometimes over a short period and sometimes more gradually. Finally, the third phase is marked by defaults from agents who borrowed to buy inflated assets, potentially triggering a banking crisis (Allen and Gale, 2000).

In the housing context, explosive price behavior can lead to periods of exuberance - boom phases characterized by rapid price increases, which often culminate in market corrections or busts. These dynamics are nonlinear, as they tend to burst, and are characterized by explosive growth during the boom phase (Martínez-García and Grossman, 2020). Case and Shiller (2003) state that, “During a housing price bubble, homebuyers think that a home that they would normally consider too expensive for them is now an acceptable purchase because they will be compensated by significant further price increases” (p. 299). They also explain that, “If expectations of rapid and steady future price increases are important motivating factors for buyers, then home prices are inherently unstable. Prices cannot go up rapidly forever, and when people perceive that prices have stopped going up, this support for their acceptance of high home prices could break down. Prices could then fall as a result of diminished demand: the bubble bursts” (p. 300). This makes bubble detection crucial for policymakers, as this rupture would have serious consequences for the economy (Pan, 2019).

Following the sovereign debt crisis, house prices in Portugal experienced a significant increase, with various media outlets raising concerns about a potential housing bubble in the country. This highlights the importance of studying the Portuguese real estate market to examine possible exuberant behavior, particularly since country-specific empirical literature is limited, with most studies focused on the US. Moreover, detailed information on the Portuguese market is even scarcer (Rodrigues et al., 2022). The Portuguese real estate market has exhibited signs of exuberance, particulary in major metropolitan areas like Lisbon and Porto. A study conducted by Rodrigues et al. (2022, pp. 29-52) examined price exuberance and contagion at local level, utilizing data from 18 districts and 278 municipalities across Portugal. The study found strong evidence of exuberant behavior in Lisbon and Porto, and explored the potential contagion effects, where price bubbles in these cities influenced neighboring housing markets. The findings indicated that, albeit to a lesser degree, Lisbon and Porto had a contagious effect on nearby housing markets, suggesting a broader regional impact of housing market dynamics within the country.

This dissertation has two main objectives. Firstly, we study the existence of price exuberance in house prices within the municipalities of the Lisbon metropolitan area. Secondly, we examine the existence of contagion from Lisbon to neighboring municipalities. We analyze quarterly data from 2007:Q1 to 2021:Q4, encompassing 18 municipalities in the Lisbon metropolitan area. We focus on local-level data because treating Portugal as a single housing market may not provide sufficient insights, as real estate markets are significantly influenced by local variables.

To achieve the first objective, we employ the date-stamping strategy proposed by Phillips et al. (2015) to detect mildly explosive behavior and establish a chronology of episodes of exuberance in our data sample. This approach utilizes the Augmented Dickey-Fuller (ADF) test within a recursive evolving algorithm. The recursive evolving algorithm allows for the real-time identification of bubbles and crises, even in the presence of multiple structural breaks during the sample period. By applying a flexible window widths, the ADF test statistic is calculated recursively using a backward expanding sample sequence. When the sample period contains multiple bubbles, Phillips et al. (2015) demonstrates that this approach outperforms the forward expanding and rolling window algorithms.

Regarding the second objective, the novelty of this dissertation lies in the application of the time-varying Granger causality test to detect bubble contagion. We employ the test developed by Shi et al. (2018), which is also based on a recursive evolving window procedure. This method is specifically adapted to identify Granger causality and to datestamp the origination and termination of changes in causal relationships. Like Phillips et al. (2015), this procedure involves intensive recursive calculations of the Wald test for Granger causality, utilizing a backward expanding sample sequence, where the last observation in each sample is the current observation of interest. Shi et al. (2018) have shown that, similar to the date-stamping strategy, this algorithm is superior to forward expanding and rolling window methods.

Our findings provide evidence of several episodes of exuberance across all municipalities of the Lisbon metropolitan area, though with varying durations. Exuberance in Oeiras exhibited the longest duration in the sample period, lasting for 22 quarters. Furthermore, the exuberant behavior in most municipalities was still ongoing at the end of the sample period, except for Lisbon. Additionally, we identified causal relationships originating from Lisbon and spreading to most other municipalities, suggesting the presence of bubble contagion. However, no Granger causality was found from Lisbon to Oeiras and Sesimbra, indicating a lack of contagion for these municipalities.

The remainder of this dissertation is organized as follows: Section 2 covers the literature review on both housing bubbles and bubble contagion. Section 3 describes the methodologies used in this study, while Section 4 presents the data. Section 5 provides the empirical findings and some discussion. Finally, Section 6 presents the conclusions.

## 2. LITERATURE REVIEW

In this section, we provide a concise overview of the literature focusing on studies that employed similar methodologies for detecting and date-stamping housing bubbles, as well as research on bubble contagion.

### *2.1. Empirical Studies on Housing Bubbles*

There is a substantial body of research addressing economic bubbles, with various methods available for detecting them. Among the most commonly utilized detection techniques are those based on the present value model and the rational bubble assumption. In this framework, the asset price is considered as the sum of all discounted future incomes, excluding any bubble component. Rational bubbles occur when investors are willing to pay a higher price for an asset than its fundamental value, expecting that the asset price will greatly surpass its fundamental worth in the future. When rational bubbles are present, the asset price consists of two components: the fundamental value and the bubble.

Early attempts at detecting rational bubbles include the variance bound test proposed by Shiller (1981) and LeRoy and Porter (1981). This test posits the existence of a rational bubble if the variance of observed prices exceeds the bound imposed by the variance of the fundamental value. However, this test faced criticism regarding its effectiveness in bubble detection. Another method is the two-step test developed by West (1987), which involves testing two sets of estimates of the impact of the fundamental value on the asset price in the underlying equilibrium model. The first set of estimates remains valid with or without a bubble, whereas the second set is only valid in the absence of a bubble. Any disparity between these estimates may indicate the presence of a bubble. However, this test requires a well-established underlying equilibrium model; otherwise, rejection of the no-bubble hypothesis might be due to model misspecification rather than the existence of a bubble (see, for example, Gürkaynak, 2008, for further details on proposed methods). In turn, Diba and Grossman (1984, 1988) examined the presence of rational bubbles by employing stationarity and cointegration tests on asset prices and observable fundamentals. However, Evans (1991) demonstrated that these tests lack power to detect explosive rational bubbles when the sample data includes periodically collapsing bubbles.

To address this limitation, Phillips, Wu and Yu [PWY] (2011) proposed a new econometric approach based on forward recursive regression tests. This approach offers a means of testing for explosive behavior and date-stamping the onset and collapse of periods of exuberance. This test was found to have greater discriminatory power in distinguishing periodically collapsing bubbles than standard unit root and cointegration tests. Nonetheless, this method may be inconsistent when the sample period includes multiple episodes of exuberance and collapse. In response to this challenge, Phillips, Shi and Yu [PSY] (2015) introduced a more robust technique utilizing the same recursive right-tailed ADF tests, but with greater flexibility in the windows. This flexibility allows for adjustments to both the starting and ending points of the recursion, along with a recursive backward regression technique for date-stamping bubble origination and termination dates.

These recursive and rolling tests exhibit greater efficacy compared to other procedures for identifying structural breaks, such as CUSUM and Chow tests. They can be applied across different data frequencies and serve as effective tools for real-time bubble detection (Homm and Breitung, 2012; Yiu et al., 2013; and Phillips et al., 2015). Several studies have employed this methodology to detect asset price bubbles in diverse countries and markets, including stocks, bonds, commodities, and real estate. Notably, Greenaway-McGrevy and Phillips (2016) identified evidence of a housing bubble in New Zealand spanning from 1993 to 2014. Bago et al. (2021b) employed the test procedure developed by Phillips et al. (2015) to detect explosive behavior in house prices within six European countries, using quarterly housing price-to-rent ratios from 1970 to 2020. Their findings revealed that all selected countries experienced at least one bubble episode during the study period.

Rodrigues and Lourenço (2015) investigated potential exuberant periods in real house prices in Europe and the US, using quarterly data from 1970:Q1 to 2014:Q4. They employed a quantile regression approach to identify potential misalignments of house prices. Their analysis indicated periods of over and under-evaluation in most countries studied, suggesting that real house prices may be influenced by factors beyond the fundamental values, potentially indicating exuberant behavior. To complement their analysis, they utilized the method proposed by Phillips et al. (2015) to detect and date periods of exuberance. While their tests confirmed the mispricing conclusions drawn using the quantile regression approach, their overall results suggested that the aggregate house price index in Europe did not exhibit evidence of exuberant behavior in recent years, particularly since 2010.

Gomez-Gonzalez et al. (2018) utilized the PSY bubble detection test and discovered at least one instance of housing price exuberance in each OECD country. Multiple bubbles were identified during their sample period for all countries except Canada and Greece, where only one bubble was found. The early 2000s saw the development of most housing bubbles, predating the subprime crisis. While both positive and negative bubbles exist, the former type is more prevalent in their study and tends to last longer than negative bubbles.

Engsted et al. (2016) conducted a similar study across OECD countries, employing an econometric analysis of housing market bubbles. They utilized quarterly OECD data from 1970 to 2013 for 18 countries. Initially, they applied the PSY test on the price-to-rent ratio to identify and date periods of explosive behavior. Subsequently, they investigated the cointegrating relationship between prices and rents, employing the co-explosive VAR framework developed by Engsted and Nielsen (2012) to test for bubbles. Their findings supported the bubble hypothesis, revealing evidence of explosiveness in many housing markets.

Sobieraj and Metelski (2021) examined episodes of exuberance in housing prices across major Polish cities by analyzing two different sets of data: real house prices and price-to-income ratios, using quarterly data from 2006:Q3 to 2021:Q1. They utilized the test procedures provided by Phillips et al. (2011) and Phillips et al. (2015). Their results showed empirical evidence for explosive behavior in the housing markets of most Polish cities studied (13 out of 17) when considering real house prices. However, when applying the same research method to the price-to-income ratio, the tests yielded statistically insignificant results in the vast majority of cases. Sobieraj and Metelski (2021) concluded that the increase in average incomes tends to attenuate explosive dynamics and alter the context in which the issue of housing bubbles is perceived.

Petris et al. (2020) identified evidence of housing bubbles in the London housing market over the past 20 years. They initially employed the test procedures provided by Phillips et al. (2011) and Phillips et al. (2015) at an aggregate level, revealing explosive behavior in house prices both in London and across the UK. Subsequently, they extended their analysis to a regional level, examining evidence of house price exuberance in the 32 London boroughs and the City of London. However, they found evidence of house price exuberance in only 5 boroughs and identified bubbles in just 3 London boroughs out of the total 33 (including the City of London). A similar approach to detecting and dating periods of explosive dynamics was undertaken by Pavlidis et al. (2016). They found strong evidence of exuberance in real house prices and price-to-fundamentals ratios. Notably, the periods of explosiveness in the price-to-fundamentals ratios were slightly shorter. Additionally, the authors extended their analysis using a panel version of the PSY test, providing robust evidence in favor of global exuberance in their sample. Pavlidis et al. (2016), utilizing a probit model, demonstrated that long-term interest rates, private credit growth, disposable income growth, unemployment, and GDP growth are significant factors influencing the likelihood of exuberance in housing markets.

Using the PSY procedure with various data frequencies, including monthly and quarterly data for price-to-rent ratios, Shi et al. (2016) presented evidence of explosive bubbles in house prices across several Australian cities. These bubbles exhibited differing durations, and the beginnings of many of them aligned with changes in federal government policies regarding capital gains tax. Among all capital cities, Perth experienced the longest and most sustainable house price bubble.

### *2.2. Recent Advances in Bubble Detection*

As the field of bubble detection continues to evolve, recent studies have introduced innovative methodologies aimed at enhancing accuracy and robustness across diverse market conditions.

Harvey et al. (2023) contribute to this advancement by introducing a novel heteroskedasticity-robust test adapted for the detection of asset price bubbles amidst varying levels of volatility. Their study presents two variants of the test—one with an intercept and one without—both demonstrating competitive performance. To further strengthen robustness, they propose a union of rejections approach, combining both variants to achieve superior detection capabilities across a spectrum of bubble scenarios. This represents a significant advance in bubble identification, overcoming previous heteroskedasticity-robust tests.

Similarly, Whitehouse et al. (2024) propose two new procedures, $\mathrm{A}_{\mathrm{MAX}}^{\mathrm{AR}}(\mathrm{k})$ and $\mathrm{A}_{\mathrm{MAX}}^{\mathrm{TR}}(\mathrm{k})$, aimed at early detection of asset price bubbles. Building upon established methodologies, these procedures exhibit lower false positive rates and higher true positive rates compared to previous methods. Theoretical foundations ensure that these procedures do not increase the risk of false detections, and empirical validations across OECD housing markets highlight their efficacy in identifying bubbles much earlier than existing methods. Extensive Monte Carlo simulations further confirm the robustness and reliability of these procedures, suggesting their potential to provide more reliable early warnings and enable timely policy interventions.

In addition, Harvey et al. (2024) demonstrate the robustness and power of sign-based variants of the Phillips, Shi, and Yu (2015, PSY) test—sPSY and s̄PSY—for detecting explosive autoregressive regimes in financial time series with deterministic level shifts. Unlike the original PSY test, these sign-based tests maintain validity and reliability, thereby reducing the likelihood of false detections of bubbles.

These recent strides in bubble detection methodologies underscore the ongoing commitment to improving the accuracy, timeliness, and reliability of identifying asset price bubbles across various financial markets.

### *2.3. Empirical Studies on Bubble Contagion*

The contagion phenomenon refers to the migration of a bubble from one market to another (Gomez-Gonzalez et al., 2018; and Hu and Oxley, 2018). Spillover behaviors add complexity to the housing bubble issue by spreading the effects of an overheated market from one city to others. This transmission leads to inflated housing prices becoming a widespread and significant problem[1] (Tsai and Chiang, 2019).

Nneji et al. (2015) provided evidence that speculative bubbles can spill over from one region to another between regional housing markets in the United States (US). Speculative bubbles were found in five of the nine census divisions and these spillovers occurred across both contiguous and noncontiguous regions. DeFusco et al. (2013) discovered that contagion played an important role during the last housing boom (2007-2009) in the US, but no evidence of contagion was found during the bust. Furthermore, the contagion impacts were primarily observed from the closest neighbors, with no spillovers associated with more distant neighbors.

Research conducted by Berg (2002) and Oikarinen (2004) supports the idea that price appreciation typically initiates in urban cores before extending to peripheral markets with strong economic connections to the urban center. Berg (2002) observed that changes in house prices in the Stockholm region of Sweden had a contagious effect on prices in other regions, akin to the influence of the London region on house prices in England. Oikarinen (2004) investigated the diffusion of house price movements in regional markets across

[1] In housing market research, spillovers are also known as the ripple effect, contagion effect, or house price diffusion. In our work, we use these terms interchangeably.

Finland from 1987 to 2004, utilizing vector autoregressive and vector error-correction models. The study demonstrated that house price changes originating in the Helsinki Metropolitan Area, the main economic center in Finland, spilled over into peripheral regions. Additionally, appreciation in house prices in regional centers exerted influence on price changes in surrounding provinces. In summary, the findings suggest a pattern where price movements diffuse initially from the economic center to regional centers and subsequently to peripheral regions.

Teye et al. (2017), using Granger-causality tests, found that house prices in Amsterdam ripple out to all regions in the Netherlands, except Zeeland. Balcilar et al. (2013), employing Bayesian and non-linear unit root tests, provided evidence of a ripple effect on house prices in five major metropolitan areas of South Africa. Their factor analysis identified Cape Town and Durban as the primary drivers of house price shocks in the country.

Riddel (2011) developed a theoretical model that allows speculative price appreciation spread from one market (urban core) to another (peripheral) market. Using an error correction model and Granger-causality tests, the author found evidence that income and price contagion originating in Los Angeles (urban core) contributed to the rapid appreciation in home prices of Las Vegas (peripheral market). Tsai (2015) examined that house prices in London do not affect the housing markets of other regions in the UK, and in terms of spillover informativeness, the Southeast region proved to have a greater impact on the overall UK house prices than the reverse. Furthermore, during the 2008 global financial crisis, the declining house prices in the Northern region were as significant as those in the overall market but failed to rebound like the overall market did after 2009.

Greenaway-McGrevy and Phillips (2016) developed a non-parametric model with time-varying coefficients to examine contagion effects from the Auckland property market to other metropolitan centers in New Zealand. They found that the Auckland city market is the main source of bubble contagion within its metropolitan area and other regions in the country. Using the contagion coefficient proposed by Greenaway-McGrevy and Phillips (2016), Gomez-Gonzalez et al. (2018) found evidence of contagion of the US housing bubble to several countries, mostly European, but found no evidence of bubble contagion from the UK.

Hu and Oxley (2018) utilized the contagion cofficient to show evidence that, in Japan, the bubble in the stock market (core market) migrated to the real estate market during the 1980s to 1990s. Bago et al. (2021a) observed house price transmission between the Japanese real estate market, the US, the Eurozone, and the UK over several periods, noting that the intensity of the contagion has decreased after the year 2000. Deng et al. (2017) showed that stock market bubbles in China migrated to the real estate market between 2005 and 2010. Shih, Li and Qin (2014), after identifying house price bubbles in most provinces of China, combined three possible contagious regions. The authors observed that spillover effetcs were mainly from Beijing and Shangai, the two biggest cities in China, to surrounding provinces.

Gomez-Gonzalez and Sanin-Restrepo (2018) tested for house price bubble migration within Canadian provinces by applying the approach of Phillips and Yu (2011) and the contagion coefficient. Their results revealed that the bubble originated in British Columbia and migrated to other provinces, such as Saskatchewan, Alberta, Newfoundland and Labrador, and Ontario. Furthermore, these migrations became more intense when house prices exhibited their maximum peak.

Rherrad et al. (2021) found evidence of price bubble transmission between the new housing and the resale housing markets within and between four Canadian census metropolitan areas (Vancouver, Toronto, Hamilton, and Victoria). Bubble contagion was noted from Vancouver to Victoria and from Toronto to Hamilton in the resale market. Bago et al. (2021b, 2022) found evidence that bubbles in real estate markets migrated between six selected European countries during several periods and between the Scandinavian housing markets, including Denmark, Norway, Finland, and Sweden.

Chen et al. (2011) applied causality tests using the Toda-Yamamoto approach to investigate the origin of the ripple effect in Taiwan and Teng et al. (2017) used Engle-Granger cointegration and Granger-causality tests to show that house prices diffuse from the city center (Taipei City) to the suburbs (New Taipei City) via bubble contagion, with larger bubbles observed in the suburbs post-diffusion. More recently, Kim and Cho (2023) investigated the transmission of house prices between Gangnam and Gangbuk regions in Seoul, Korea, focusing on how speculation in one region affects the other. The authors found evidence of short-run price diffusion from Gangnam to Gangbuk, particularly in the bubble components, indicating unidirectional bubble contagion. Additionally, the study identified Gangnam as the likely "price-leader" between the two submarkets, implying a dominant role in price diffusion within Seoul.

## 3. METHODOLOGIES

In this section, we present the methodologies used to analyze asset price bubbles, detect explosive behavior, and examine bubble contagion. We start with a conventional framework based on the present value model, incorporating rational expectations and informational efficiency to understand fundamental housing prices and the formation of rational bubbles. Next, we introduce advanced econometric techniques, including the Supremum Augmented Dickey-Fuller (SADF), Generalized SADF (GSADF), and Backward SADF (BSADF) tests, which are designed to identify and date periods of explosive behavior in asset prices. Finally, we explore the phenomenon of bubble contagion using time-varying Granger causality tests, which help to uncover the interconnections and directional influences between housing markets in different regions. These methodologies collectively offer a robust approach to studying the dynamics and propagation of asset price bubbles.

### *3.1. Conventional Framework for Studying Asset Price Bubbles*

Most asset pricing tests begin with the present value model. Following Cuthbertson (1996), we adopt a model featuring homogeneous and risk-neutral agents, rational expectations, informational efficiency (without any informational asymmetries), and a constant real rate of return, r, on the asset, $E_t R_t = r$. In this model, the house price is determined by the Euler equation:

$$P_t = \delta\,(E_t\, P_{t+1} + E_t\, D_{t+1}) \tag{1}$$

where $P_t$ denotes the house price at time t, $\delta = 1/(1 + r)$ is the discount factor, $E_t$ represents the conditional expectations operator for information at time t, and $D_{t+1}$ signifies the income generated by owning a house between t and t+1. Solving equation (1) under rational expectations through repeated forward substitution yields:

$$P_t = P_t^f = \sum_{i=1}^{\infty} \delta^i \, E_t \, D_{t+i}. \tag{2}$$

Assuming that the transversality condition holds, i.e., that $\lim_{n\to\infty} \delta^n \, E_t D_{t+n} = 0$, a unique solution (price) given by equation (2), can be found, corresponding to the fundamental house price, $P_t^f$. The fundamental notion behind a rational bubble is that there exists another mathematical expression for $P_t$ that satisfies (1), namely:

$$P_t = \sum_{i=1}^{\infty} \delta^i \, E_t \, D_{t+i} + B_t = P_t^f + B_t, \tag{3}$$

where $B_t$ represents a rational bubble. Consequently, the actual house price $P_t$ deviates from its fundamental price $P_t^f$ by the amount of the rational bubble $B_t$. Equation (3) constitutes the baseline model for most empirical studies of asset price bubbles.

However, for (3) to satisfy (1), certain restrictions must be imposed on the dynamic behavior of $B_t$. These restrictions can be imposed by assuming that (3) is a valid solution to (1), thereby restricting the dynamics of $B_t$. Initiating equation (3) at time t+1 and taking expectations at time t yields,

$$\begin{aligned} E_t \, P_{t+1} &= E_t[\delta \, E_{t+1} D_{t+2} + \delta^2 E_{t+1} D_{t+3} + \cdots + B_{t+1}] \\ &= \delta \, E_t D_{t+2} + \delta^2 E_t D_{t+3} + \cdots + E_t B_{t+1}, \end{aligned} \tag{4}$$

where the second equality involves the use of the law of iterated expectations, i.e., $E_t(E_{t+1} D_{t+i}) = E_t D_{t+i}$. Hence, considering (1) and (4), we observe that:

$$\begin{aligned} P_t &= \delta(E_t P_{t+1} + E_t D_{t+1}) \\ &= (\delta^2 E_t D_{t+2} + \delta^3 E_t D_{t+3} + \cdots + \delta E_t B_{t+1}) + \delta E_t D_{t+1}. \end{aligned} \tag{5}$$

Consequently, given the results in (2) and (5), we have:

$$P_t = P_t^f + \delta E_t B_{t+1}. \tag{6}$$

For (3) and (6) to be a valid solution to (1), it is necessary that $\delta E_t B_{t+1} = B_t$, or equivalently, $E_t B_{t+1} = B_t / \delta = (1 + r) B_t$. Thus, apart from the discount factor $\delta$, $B_t$ must behave as a martingale, i.e., the best forecast of all future values of the bubble depends only on its current value. However, although the bubble solution satisfies the Euler equation, it violates the transversality condition (for $B_t \neq 0$), and because $B_t$ is arbitrary, the house price in (3) is not unique (see, Cuthbertson, 1996, for more details).

The rational bubble model described above can be extended to allow for strictly positive bubbles that collapse almost surely in finite time (see, among others, Blanchard, 1979; Evans, 1991; and Diba and Grossman, 1988). This type of bubble is defined as:

$$B_{t+1} = \begin{cases} (1+r)\, B_t u_{t+1} & \text{if } B_t \leq \alpha \\ \{\, \delta + \pi^{-1}(1+r)\, \theta_{t+1} \left[B_t - (1+r)^{-1}\delta\right] \}\, u_{t+1} & \text{if } B_t > \alpha \end{cases},$$

where $\delta$ and $\alpha$ are positive paremeters with $0 < \delta < (1 + r)\, \alpha$ and $\alpha > 0$, $u_{t+1}$ is an exogenous i.i.d. positive random variable with $E_t u_{t+1} = 1$, and $\theta_{t+1}$ is an exogenous i.i.d. Bernoulli process (independent of $u_{t+1}$) that takes the value 1 with probability $\pi$, and 0 with probability 1- $\pi$, whereby $0 < \pi \leq 1$.

These two models of rational bubbles are not informative about how bubbles start or end; they only provide insights into the time properties of the bubble once it is underway. In these models, the bubble is exogenous to the fundamental model of expected returns. Under the condition that $B_t \leq \alpha$, the bubble grows at mean rate 1+ r. Eventually, when $B_t > \alpha$, the bubble erupts and grows at a faster mean rate (1+r) $\pi^{-1}$, but may collapse with probability 1- $\pi$ per period. When the bubble collapses, it falls to a mean value of $\delta$, and then the process begins again. One implication of rational bubbles is that they cannot be negative.

Froot and Obstfeld (1991) proposed an alternative concept of bubbles, wherein the bubbles' existence is linked to dividend levels, termed as intrinsic bubbles. Driffill and Sola (1998) extended the approach of Froot and Obstfeld (1991) by incorporating regime-switching models. For further insights into models that vary depending on different regimes, see, for instance, Funke et al. (1994); Hall et al. (1999); van Norden and Vigfusson (1998); and Psaradakis et al. (2001).

### *3.2. Testing for Explosive Behavior*

As highlighted at the beginning of Section 2, to address the limitations of traditional unit root tests (e.g., Dickey-Fuller test) and cointegration tests in detecting bubbles, and in response to Evans's critique, Phillips et al. (2011, PWY) and Phillips et al. (2015, PSY) introduced innovative procedures for real-time testing and dating of bubble phenomena. Our study employs the PSY procedure to identify exuberant behaviors, in line with the rational bubble concept discussed in Section 3.1, which captures the martingale characteristics of asset price bubbles.

This method is designed to detect mildly explosive behavior in time series data, using the following empirical regression model:

$$\Delta y_t = a_{r_1,r_2} + \beta_{r_1,r_2} y_{t-1} + \sum_{j=1}^{k} \psi_{r_1,r_2}^{j} \Delta y_{t-j} + \epsilon_t \tag{7}$$

where $y_t$ represents the time series of interest, $\Delta$ is the first difference operator, $a_{r_1,r_2}$ is the intercept, and $\beta_{r_1,r_2}$ is the autoregressive coefficient of interest. The coefficients $\psi_{r_1,r_2}^{j}$(for *j = 1,…,k*) pertain to the lagged first differences, $\Delta y_{t-j}$, and $\epsilon_t \sim$ i.i.d $(0, {\sigma^2}_{r_1,r_2})$. The parameters $r_1$ and $r_2$ indicate fractions of the total sample size that define the start and end points of a subsample period, respectively, with k being the optimal lag order.

The null hypothesis posits the presence of a unit root in $y_t$, ($H_0$: $\beta_{r_1,r_2} = 0$) against the alternative hypothesis of mildly explosive behavior, ($H_1$: $\beta_{r_1,r_2} > 0$). The test statistic for this hypothesis is given by:

$$\mathrm{ADF}_{r_1}^{r_2} = \frac{\hat{\beta}_{r_1,r_2}}{\text{s.e. } (\hat{\beta}_{r_1,r_2})} \,. \tag{8}$$

Setting $r_1 = 0$ and $r_2 = 1$ yields, under the null hypothesis, the standard ADF test statistic, $\mathrm{ADF}_0^1$. The limit distribution of $\mathrm{ADF}_0^1$ is:

$$\frac{\frac{1}{2}\left[w(1)^2-1\right]-w(1)\int_0^1 w(s)ds}{\left[\int_0^1 w(s)^2ds-\left(\int_0^1 w(s)ds\right)^2\right]^{1/2}}, \tag{9}$$

where W (.) is a standard Wiener process. In testing for explosive dynamics, the ADF test compares the $\mathrm{ADF}_0^1$ statistic with the right-tailed critical value from its limit distribution. If the test statistic exceeds the critical value, the null hypothesis of a unit root is rejected in favor of the alternative hypothesis of explosive behavior. However, the standard ADF test has very low power in detecting episodes of explosive behavior when periodically collapsing bubbles are present in the entire sample. It might not detect a bubble even if explosive behavior occurs in one or more subsamples of the time series.

### *3.2.1. The Supremum ADF (SADF) Test*

The approach adopetd by PWY uses a supremum ADF (SADF) based on a sequence of forward recursive right-tailed ADF unit root tests, allowing for the identification of single-bubble episodes in sample data. This procedure involves repeatedly estimating equation (7) on an expanding sample, while keeping the starting point of the subsample fixed at $\mathrm{r}_1 = 0$ and extending only the ending point of the sample, $\mathrm{r}_2$, from $\mathrm{r}_0$ (the minimum window size for the initial sample) to 1 (the last available observation).

In pratical terms, the ADF regression is recursively estimated by incrementing the window size $\mathrm{r}_2 \in [\, r_0, 1]$ one observation at a time, while maintaining the starting point at $\mathrm{r}_1 = 0$. Each estimation yields an ADF statistic denoted as $\mathrm{ADF}_0^{r_2}$. This method is straightforward to implement and provides a new limit theory for mildly explosive processes. The supremum of the sequence of $\mathrm{ADF}_0^{r_2}$ statistics yields the SADF, which is expressed as

$$SADF\ (r_0) = \sup_{r_2\in[r_0,1]} ADF_0^{r_2}\ . \tag{10}$$

Under the null hypothesis of a random walk, the limit distribution of the SADF statistic is given by:

$$\sup_{r\in[r_0,1]} \left\{ \frac{\frac{1}{2}r[w(r)^2 - r] - \int_0^r w(s)dsw(r)}{r^{1/2}\left\{r\int_0^r w(s)^2 ds - \left[\int_0^r w(s)ds\right]^2\right\}^{1/2}} \right\}. \tag{11}$$

The standard ADF test examines the presence of explosive dynamics over the entire sample period, whereas the SADF test focuses on specific periods within the sample. If the SADF statistic exceeds the right-tailed critical value from its limit distribution, the unit root hypothesis is rejected in favor of explosive behavior, indicating explosive dynamics in parts of the time series data.

### *3.2.2. The Generalized SADF (GSADF) Test*

The procedure aforementioned is more effective when there is a single-bubble episode in the sample data. As noted by PSY, with a long sample period, there will often be evidence of multiple asset price bubbles. Identifying multiple bubbles with periodically collapsing behavior over time is significantly more challenging than identifying a single bubble.

To address this, PSY proposed the generalized SADF (GSADF) test, which covers more subsamples than the SADF test by allowing both the ending point, $\mathrm{r}_2$, and the starting point, $\mathrm{r}_1$, to change. This additional flexibility in the estimation window results in substantial power gains compared to the SADF and performs much better in identifying explosive behavior when multiple episodes occur in the data.

The GSADF statistic is defined as:

$$GSADF\ (r_0) = \sup_{r_2\in[r_0,1],r_1\in[0,r_2-r_0]} ADF_{r_1}^{r_2}. \tag{12}$$

Under the null hypothesis, the limit distribution of the GSADF statistic is:

$$\sup_{\substack{r_1\in[0,r_2-r_0]\\ r_2\in[r_0,1]}} \left\{ \frac{\frac{1}{2}r_w[W(r_2)^2 - W(r_1)^2 - r_w] - \int_{r_1}^{r_2} W(s)ds[W(r_2) - W(r_1)]}{{r_w}^{1/2}\left\{r_w\int_{r_1}^{r_2} W(s)^2\,ds - \left[\int_{r_1}^{r_2} W(s)ds\right]^2\right\}^{1/2}} \right\}, \tag{13}$$

where the window size of each estimation is $r_w = r_2 - r_1$. Similar to other tests, rejection of the null hypothesis of a unit root indicates explosive behavior if the test statistic exceeds the right-tailed critical values from its limit distribution.

The limit distribution of the standard ADF statistic is a special case of (13) with $r_1 = 0$ and $r_2 = r_w = 1$, whereas the limit distribution of the sup ADF statistic is a further special case of (13) with $r_1 = 0$ and $r_2 = r_w \in [r_0, 1] = r$ (See, for instance, Phillips et al., 2014 and Phillips et al., 2015, for more details in the context of limit distributions).

### *3.2.3. Date-Stamping Strategy*

When the null hypothesis is rejected, indicating evidence of explosiveness, it becomes feasible to establish a chronological sequence of exuberant episodes, delineating the start and end points of booming periods. This is accomplished through the backward supremum Augmented Dickey-Fuller (BSADF) test procedure as proposed by PSY.

The BSADF test statistic, denoted as $BSADF_{r_2}(r_0)$, where $r_0$ is a function, is defined as:

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2}. \tag{14}$$

The origination date of the exuberant period is identified as the first observation for which the BSADF statistic exceeds its critical value, i.e.,

$$\hat{r}_e = inf_{r_2 \in [r_0, 1]} \{r_2 : BSADF_{r_2}(r_0) > scv_{[Tr_2]}^{\alpha}\}. \tag{15}$$

Similarly, the termination date is determined as the first observation after $\hat{r}_e$ for which the BSADF falls bellow its critical value, i.e.,

$$\hat{r}_f = inf_{r_2 \in [\hat{r}_e + \delta \log(T)/T, 1]} \{r_2 : BSADF_{r_2}(r_0) < scv_{[Tr_2]}^{\alpha}\}, \tag{16}$$

where $scv_{[r_2 T]}^{\alpha}$ is the 100(1-$\alpha$)% critical value of the sup ADF based on $\lfloor Tr_2 \rfloor$ observations at a chosen significance level $\alpha$, and $\delta$ is a frequency dependent parameter. For a bubble to be defined, its duration should exceed a minimal period represented by $\delta$log(T); see

Phillips et al. (2015) for further details. The BSADF test provides enhanced insights and improves the detection capability for bubbles within the sample, exhibiting greater efficacy in detecting multiple bubbles.

### *3.2.4. Technical Aspects*

The computation of the SADF, GSADF, and BSADF test statistics necessitates specifying the minimum window size $r_0$ and the length of the autoregressive lag k. The minimum window size must be sufficiently large to enable initial estimation but not so large that it overlooks brief episodes of exuberance, if they occur. Thus, the window size should be carefully selected to ensure that even short periods of explosiveness are detected. According to PSY, the minimum window size should be chosen based on the rule $\mathrm{r}_0 = 0.01 + \frac{1.8}{\sqrt{\mathrm{T}}}$, where T is the sample size. The length of the autoregressive lag k should be small, because a larger lag length increases the likelihood of significant size distortions compared to smaller values.

The application of the right-tailed unit root tests involves using the limit distributions of the SADF, GSADF, and BSADF test statistics which are non-standard and depend on the minimum window size. Critical values are determined through Monte Carlo simulations or Bootstrapping.

Finally, a researcher may impose a minimum duration criterion for a period to be classified as a bubble, requiring the duration to exceed a certain threshold, thereby disregarding very short periods of exuberance.

## *3.3. Testing for Bubble Contagion*

After identifying the existence of housing bubbles, the second stage of our work involves examining the presence and direction of spillover effects, i.e., bubble contagion between the municipalities of the Lisbon metropolitan area. To do this, we rely on the time-varying Granger causality test developed by Shi et al. (2018) which is based on the recursive evolving algorithm and using jointly the Fourier Flexible form approach of Gallant (1981) to allow for structural breaks and changes.

Granger's (1969) causality test is a common method for examining the relationship between two variables. According to this test, variable X is said to Granger-cause variable Y if historical values of X improve the prediction of Y's current value compared to using only the past values of Y. This suggests that past values of X contain information not found in the past values of Y. It is essential to recognize that this test does not indicate true causality but rather evaluates the presence of informational content (Berg, 2002). In the context of this study, the test can assess whether changes in real house prices in the Lisbon area can help predict price movements in another region, beyond what can be explained by the region's own historical prices.

The Granger causality test can be represented by the following equation:

$$y_t = \mu + \sum_{i=1}^{k} \alpha_i \, y_{t-i} + \sum_{i=1}^{k} \beta_i x_{t-i} + \varepsilon_t$$

In this equation, y represents the real price change in one region, while x denotes the real price change in another region. $\mu$, $\alpha_i$ and $\beta_i$ are parameters to be estimated, and $\varepsilon_t$ follows a White noise process, WN (0, $\Sigma_\varepsilon$).

Granger causality is widely used due to its reliance on the stochastic properties of variables rather than a specific structural model. It allows for a flexible approach, where variables are not rigidly defined as dependent or independent, although the results are sensitive to the period over which the estimation is conducted (Shi et al., 2018). To accommodate the time-varying nature of causal relationships in economics, Thoma (1994) introduced a method using a forward expanding window, where recursively the window expands from some minimum number of observations, while Swanson (1998) proposed a rolling window technique, where the estimation shifts forward through time. These methods facilitate testing for the joint significance of model parameters, accounting for the possibility that certain parameters may only be significant during specific periods (Shi et al., 2018). Furthermore, time-varying Granger causality tests are closely related to instability tests in econometrics, such as the Markov-switching Granger causality test, which Psaradakis et al. (2005) applied to study the link between money and output. Rossi (2005) also introduced a variety of tests aimed at detecting parameter instability.

### *3.3.1. Time-Varying Granger Causality Test under Structural Breaks*

As point out by Balcilar, Ozdemir, and Shahbaz (2019), the presence of structural breaks causes time variation in the parameters of econometric models, rendering statistical tests like Granger causality tests, which assume constant parameters, invalid. This can result in misleading inferences. To adress this question and expand our analysis, we use the Flexible Fourier form approach of Gallant (1981).

The Flexible Fourier form takes the general form:

$$fourier_t = \sum_{h=1}^{H} \gamma_{s,h} \sin\left(\frac{2\pi k_h t}{T}\right) + \sum_{h=1}^{H} \gamma_{c,h} \cos\left(\frac{2\pi k_h t}{T}\right), \tag{17}$$

where H is the number of Fourier terms, $k_h$ is the Fourier frequency for term h, and T is the number of observations in the sample. Although it is a straightforward approach, this flexible form is particularly helpful and precise to capture structural breaks in time series. It is not necessary to assume that the researcher knows the potential break dates or the number of breaks, *a priori*, making this approach empirically attractive. Futhermore, it is easy to implement and not computational intensive, even though the break date cannot be extacted from Fourier's expansion alone.

In terms of regression, Granger causality can be summarized by a Wald test of the joint significance of all lags of the variable that is considered independent and can be written generally as,

$$y_t = \mu_t + \sum_{i=1}^{k_y} \alpha_{y,i}\, y_{t-i} + \sum_{j=1}^{k_x} \beta_i x_{t-j} + \delta\, t + \sum_{h=1}^{H} \gamma_{s,h} \sin\left(\frac{2\pi k_h t}{T}\right) + \sum_{h=1}^{H} \gamma_{c,h} \cos\left(\frac{2\pi k_h t}{T}\right) + \varepsilon_t \tag{18}$$

where $1 \leq H < \frac{T}{2}$ , and $k_y$ and $k_x$ are the lag orders of the dependent and independent variables, respectively. Our analysis will focus on a model such as,

$$y_t = \mu + \alpha_y y_{t-1} + \beta x_{t-1} + \delta t + \gamma_s sin\left(\frac{2\pi kt}{T}\right) + \gamma_c \, cos\left(\frac{2\pi kt}{T}\right) + \varepsilon_t \qquad (19)$$

which contains a single Fourier frequency, i.e., H = 1 to avoid problems of over-fitting and rapid loss of degrees of freedom; see, for instance, Enders and Lee (2009). Jointly testing the null hypothesis, $H_0$: $\beta_i = 0$, for i = 1,2,…, $k_x$ against the alternative that $\exists\beta_i \neq 0$, can be done based on the Wald statistic,

$$W = \left(\hat{\beta} - \beta_0\right)'\left[Var\left(\hat{\beta}\right)\right]^{-1}(\hat{\beta} - \beta_0) \qquad (20)$$

where $\hat{\beta}$ is the vector of estimated coefficients $\beta_i$ (i = 1,2,…, $k_x$), $\beta_0$ denotes the vector of hypothesized β under $H_0$ and $Var(\hat{\beta})$ represents the covariance matrix of $\hat{\beta}$. When y experiences structural breaks, excluding the Fourier term in (19) results in inconsistent estimates of $\beta_i$, which subsequently leads to inaccurate Wald statistics and erroneous conclusions regarding the presence of Granger causality.

### *3.3.2. Recursive Evolving or Double Recursive Test*

Based on the new procedure proposed by Phillips et al. (2015) for detecting and date-stamping financial bubbles in real-time, Shi et al. (2018) and Shi et al. (2020) introduced a new recursive evolving test procedure that provides a mechanism for detecting and dating changes in causal relationships. Shi et al. (2018) investigated the causal relationship between the slope of the yield curve and real economic activity, while Shi et al. (2020) the causal link between money and output. The authors compared the performance of the recursive evolving algorithm to forward expanding window and rolling window algorithms. Their simulation results indicated that, in terms of change detection, the recursive evolving procedure was superior compared to both the forward expanding window and the rolling window. The forward expanding window showed the worst performance.

According to Hammoudeh et al. (2020), the time-varying Granger causality test developed by Shi et al. (2018) offers several advantages. It can accurately identify the start and end dates of causality episodes, track shifts in causal relationships, and detect real-time economic turbulence and instability between variables. Additionally, this method applies robust econometric techniques, eliminating the need for detrending or differencing the data.

The recursive evolving, or double recursive procedure, is an extension of the forward expanding window by Thoma (1994)[2] and the rolling window by Swanson (1998)[3]. A key empirical feature of the three procedures is that they can be applied in real-time, since they depend only on data available at the current moment.

In this algorithm, a minimum window size, $r_0$, is required to perform the regression. For each observation of interest, $r$, the Wald statistics are calculated using a sequence of backward expanding samples. The endpoint of the sequence is $r_2 = \{r_0,\ldots,T\}$, where $T$ is the total number of observations. However, the starting point of the estimation covers all possible values from 1 to $r_2 - r_0 + 1$.

The Wald statistic obtained for each subsample regression over $[r_1, r_2]$, with a window size of $r_w = r_2 - r_1 \geq r_0$, is denoted by $\mathcal{W}_{r_2}(r_1)$ and the sup Wald statistic is given by:

$$SW_r(r_0) = \sup_{(r,r_2)\in\Lambda_0, r_2=r} \{\mathcal{W}_{r_2}(r_1)\}, \tag{21}$$

where $\Lambda_0 = \{r_1, r_2 : 0 < r_0 + r_1 \leq r_2 \leq 1, \text{and } 0 \leq r_1 \leq 1 - r_0\}$.

The origination date in the causal relationship is identified as the first chronological observation for which the sup Wald statistic exceeds its critical value, i.e.,

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[2] In the forward expanding window method by Thoma (1994), the starting point $r_1$ is fixed at the first observation ($r_1 = 1$), and the regression window size expands from $r_0$ to $T$.

[3] In the rolling window method introduced by Swanson (1998), the window size $r_w$ is fixed, with $r_w = r_2 - r_1 = r_0$. The starting point $r_1$ moves from the first observation to $T - r_0 + 1$ (i.e., $r_1 = r_2 - r_0 + 1$), and the endpoint $r_2$ ranges from $r_w$ to $T$.

$$\hat{r}_e = \, inf_{r\in[r_{0,1}]}\{r : S\mathcal{W}_r(r_0) > scv\}. \tag{22}$$

In the same way, the termination date is established as the first chronological observation for which the sup Wald statistic falls bellow its critical value, i.e.,

$$\hat{r}_f = \, inf_{r\in[\hat{r}_{e,1}]}\{r : S\mathcal{W}_r(r_0) < scv\}, \tag{23}$$

where $scv$ is the corresponding critical value of the $S\mathcal{W}_r$ statistics. For multiple switches in the sample period, the origination and termination dates are computed similarly.

## 4. DATA

We use data for 18 municipalities (concelhos) that comprises the Lisbon metropolitan area namely: Alcochete, Almada, Amadora, Barreiro, Cascais, Lisboa, Loures, Mafra, Moita, Montijo, Odivelas, Oeiras, Palmela, Seixal, Sesimbra, Setúbal, Sintra and Vila Franca de Xira. Our dataset provided by Confidencial Imobiliário, INE and Banco de Portugal, contains quartely observations spanning from 2007:Q1 to 2021:Q4. Data on real house prices were collected from Confidencial Imobiliário, and the HPI consists in transaction prices.

Before starting our analysis it is relevant to have an overview of the situation in the housing market of the Lisbon metropolitan area (LMA). To do this, Figure 1 presents the house price indices for each municipality. It is possible to observe that during the period of the 2007-2009 global financial crisis that started in US, the house price index in most municipalities of Lisbon metropolitan area probably saw declines, reflecting the broader economic downturn. Areas like Cascais that is considered a more luxurious or of heavy investment region witnessed sharper drops, while more suburban areas like Amadora might have experienced smaller decreases.

The post-crisis period was characterised by a slow and irregular recorvery in the early years, followed by more pronounced growth from 2013 forward. Whereas Lisbon and Cascais lead the recovery, with early signs of a housing boom, a more gradual growth was experienced by suburban areas like Amadora, Loures, and Almada as affordability concerns began to push the demand for houses beyond the city center.

From 2015 period onwards, house prices across the Lisbon metropolitan area increased dramatically, with central areas like Lisbon, Cascais and Oeiras seeing particulary sharp growth. Probably the surge in tourism, foreign investment, and economic recovery may have all played crucial roles in driving this boom. Nonetheless, signs of market saturation may have began to appear by 2018-2020, specially in the most expensive areas, while more affordable suburban areas continued to experience robust growth. It is noted that as central Lisbon became more and more unaffordable, mainly for the national residents, house prices in surrounding municipalities saw also a significant growth, the reason for this may be because investors and buyers turned to these areas, leading to rising demand and property value. This dynamics around the Lisbon metropolitan area suggests a spillover effect, that not only reshaped the housing scenario of the Lisbon region, but also highligthed the interconnectedness of the market, where changes in one area could ripple out to impact the entire metropolitan area. Consequently, the housing market in suburban and peripheral areas became an essential part of the wider Lisbon market, contributing to its overall growth and resilience.

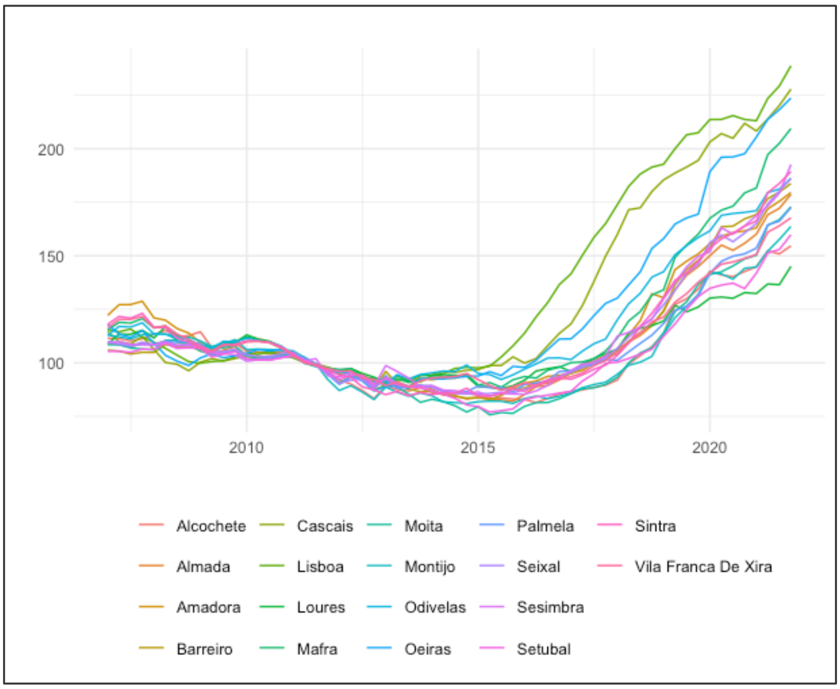


Figure 1 - House price index (HPI) for each municipality (concelho) in the LMA

## 5. EMPIRICAL RESULTS AND DISCUSSION

In this section, we begin by establishing a chronology of exuberance episodes using the PSY procedure. Following this, we apply the time-varying Granger causality test of Shi et al. (2018) to assess whether the house prices of a determined municipality help predict the aforementioned chronology of exuberance in the Lisbon metropolitan area.

### *5.1. Bubble Detection at Local (Municipalities) Level*

We begin our real-time monitoring of house price bubbles using the PSY (2015) test from the second quarter of 2010 onwards, focusing on the HPI for each of the 18 municipalities in the Lisbon metropolitan area. To compute the PSY test statistics, we used a minimum window size of 14 observations, with the lag order in the ADF regressions selected by AIC, and a maximum lag order of 2. The test was applied to each subsample. To address potential issues of unconditional heteroskedasticity and multiplicity in recursive testing, the critical values were obtained via bootstrapping[4] with 999 simulations.

Figure 2 presents the date-stamping outcomes for each municipality of the Lisbon metropolitan area. The shaded orange areas represent periods where the PSY statistic exceeds its 95% bootstrapped critical value[5]. A bubble episode is identified as beginning with the first observation where the BSADF test statistic (blue solid line) exceeds its corresponding critical value, and ending when the statistic falls below this threshold.

At first glance, evidence of bubble episodes is apparent in all municipalities, though with varying durations. Apart from some short-lived explosive periods, there seems to be significant synchronicity in bubble occurrences across municipalities. Before 2016, instances of exuberant behavior are sporadic. However, from late 2016 onwards, we observe the emergence of prolonged and widespread exuberant behavior across all municipalities.

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[4] See Phillips and Shi (2020) for more details in the new bootstrap procedure.

[5] The plots identify only bubble periods that last more than a month. The reason is because periods of just one month of high house price inflation are too short to be visually distinguishable.

For instance, the results in Figure 2 indicate the presence of only one bubble epsiode in Amadora, Mafra, Palmela, and Vila Franca de Xira, during the periods 2018:Q1 – 2021:Q4, 2017:Q4 – 2021:Q4, 2017:Q4 – 2021:Q4 and 2018:Q2 – 2021:Q4, respectively. Other municipalities experienced two bubbles, such as Alcochete (2011:Q4 - 2012:Q4 and 2018:Q2 – 2021:Q4), Barreiro (2011:Q4 – 2012:Q1 and 2017:Q2 – 2021:Q4), Cascais (2016:Q4 – 2020:Q4 and 2021:Q2 – 2021:Q4), Lisboa (2012:Q4 – 2013:Q1 and 2015:Q4 – 2018:Q3), Loures (2018:Q2 – 2020:Q4 and 2021:Q2 – 2021:Q4), Moita (2013:Q3 – 2014:Q4 and 2018:Q1 – 2021:Q4), Odivelas (2012:Q3 – 20213:Q1 and 2017:Q4 – 2021:Q4), Oeiras (2012:Q3 – 2012:Q4 and 2016:Q3 – 2021:Q4), Seixal (2011:Q4 – 2012:Q2 and 2017:Q2 – 2021:Q4), Sintra (2012:Q3 – 2013:Q1 and 2017:Q3 – 2021:Q4), and Sesimbra (2011:Q4 – 2012:Q1 and 2017:Q4 – 2021:Q4). Figure 2 also shows that three bubbles epsiodes were observed in Montijo (2011:Q3 – 2012:Q1, 2012:Q3 – 2012:Q4 and 2017:Q3 – 2021:Q4) and Setúbal (2014:Q1 – 2015:Q4, 2019:Q1 – 2020:Q2 and 2021:Q1 – 2021:Q4). Notably, Almada experienced five distinct bubble episodes for the periods 2011:Q4 – 2012:Q1, 2012:Q3 – 2012:Q4, 2014:Q2 – 2014:Q4, 2018:Q2 – 2020:Q2 and 2020:Q4 – 2021:Q4. The longest and most intense being from 2018:Q2 to 2020:Q2, peaking in 2019:Q2.

These findings align with the analysis in Figure 1, which suggests that exuberant behavior began around 2016. Interestingly, while Lisbon initially appeared to show the most rapid house price growth (Figure 1), Figure 2 reveals that this exuberance ended in the third quarter of 2018. In contrast, Oeiras exhibited the longest exuberance duration within the sample period, spanning 22 quarters.

Overall, our analysis provides evidence of multiple bubble episodes in the housing markets of the Lisbon metropolitan area from 2010 to 2021. On average, the major exuberance periods lasted 17 quarters. Furthermore, the results indicate that during the Covid-19 period, all municipalities, except Lisbon, experienced explosive behavior. It is worth noting that the exuberance periods in all municipalities, except for Lisbon once more, is still ongoing since the last quarter of 2021.

The high degree of synchronization observed across municipalities suggests the possible influence of a common factor, which may have facilitated the spread of house price exuberance throughout the region. In the next step of this study we will explore the hypothesis of bubble contagion as a potential common factor driving the synchronization of bubble episodes across these municipalities.

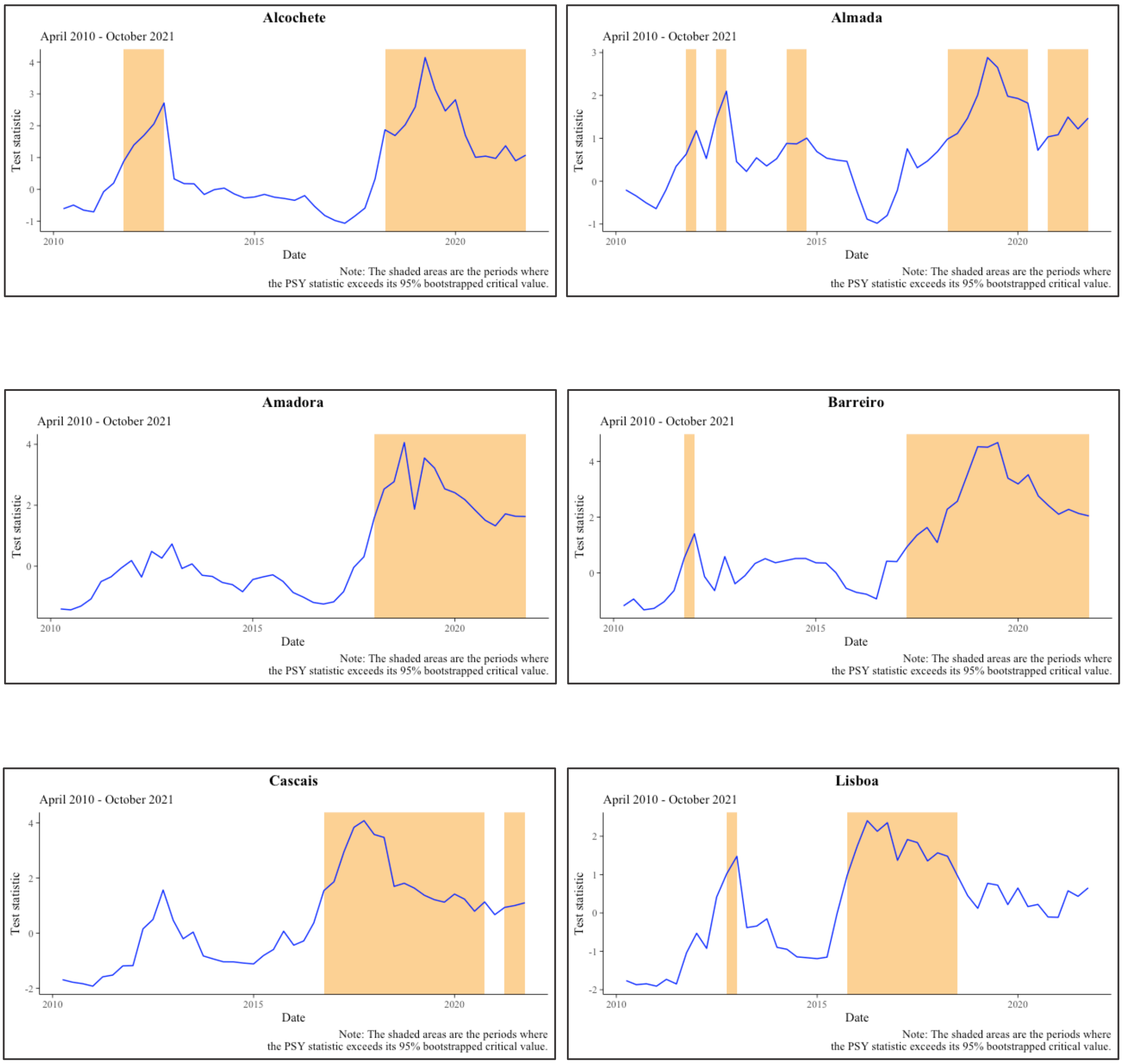


Figure 2 - Chronology of bubble episodes in the municipalities of the LMA

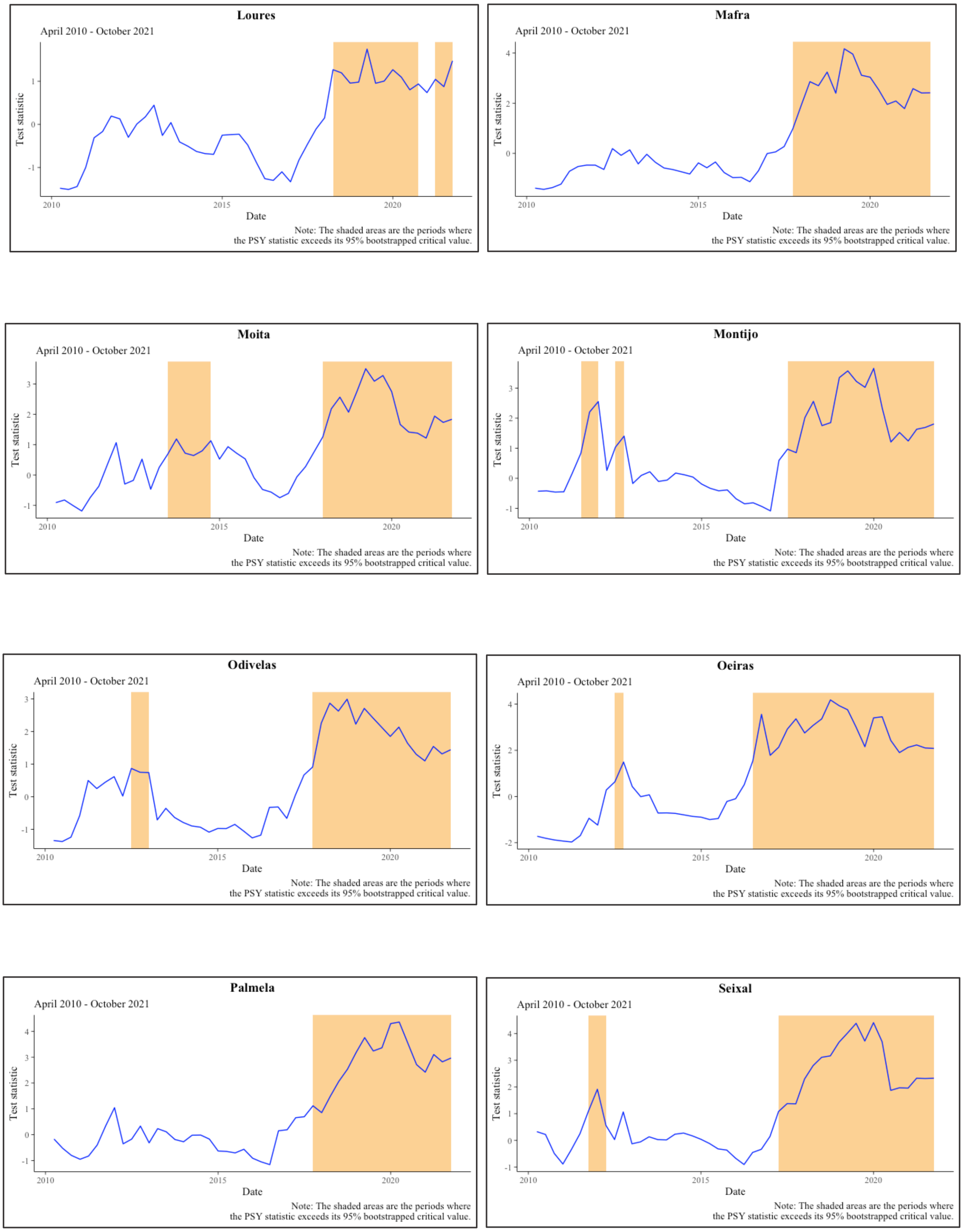


Figure 2 – Cont.

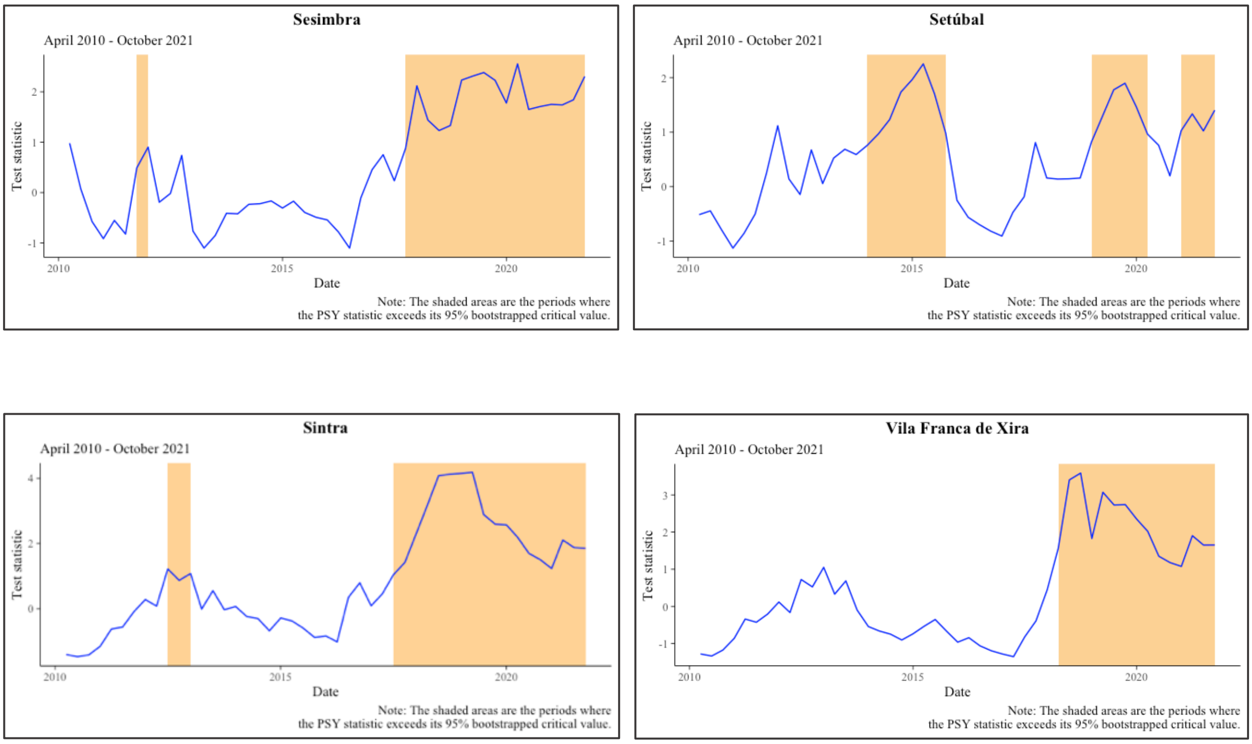


Figure 2 – Cont.

### *5.2. The Bubble Contagion*

Our results from the analysis above indicate for the possibility of contagion, since the bubble episodes across municipalities show a high degree of correlation.

Figure 3 illustrates the results of the time-varying Granger causality test from Lisbon to the remaining municipalities of its metropolitan area, using the recursive evolving procedure suggested by Shi et al. (2018). The dashed lines in each graph represent the critical values (red for the 0,90 quantile and blue for the 0,95 quantile). Critical values were obtained via block-bootstrapping[6] with 1000 replications. A significant causality

[6] The block-bootstrap method is employed to maintain the time dependence typical of time series data, specifically the non-overlapping block type. This method involves dividing the series into blocks of optimal size, $\sqrt[3]{T}$ (Hall et al., 1995). As a result, the number of blocks needed to reconstruct the series is $\frac{T}{\sqrt[3]{T}}$. The indexes of these blocks are then randomized, producing a new series of blocks that are shuffle versions of the original.

relationship is detected when the sup-Wald statistic sequence exceeds its corresponding critical value during a certain period.

We use a minimum window size of 29 observations (29 quarters) for all subsamples. The lag order is set to 1 for both the dependent and independent variables, selected by BIC. Additionally, our test regression includes a constant term, a linear term trend and one Fourier frequency term that is also selected by BIC.

To help us determine whether exuberance contagion exists, the direction of causal relationships needs to be categorized as unidirectional. This implies that house prices in one region cause fluctuations in another region[7]. We choose Lisbon as a core center because it represents the main city center of its metropolitan area and also the capital of Portugal. Furthermore, in Lisbon, exuberance ends earlier than in other municipalities. Using Lisbon as the core center, the remaining 17 municipalities were tested for exuberance contagion.

According to Figure 3, the test detected one episode of Granger causality running from Lisbon to Alcochete at the 0,90 quantile, during the second and third quarters of 2021. Two episodes of Granger causality running from Lisbon to Almada at the 0,90 and 0,95 quantiles are detected. The first lasts 8 quarters, starting in the first quarter of 2017 and terminating in the last quarter of 2018. The second lasts 5 quarters between the second quarter of 2019 and the second quarter of 2020.

Three main episodes of Granger causality are found from Lisbon to Amadora at the 0,90 and 0,95 quantiles. The first lasts 4 quarters between the second quarter of 2014 and the first quarter of 2015. The second lasts 11 quarters, starting in the first quarter of 2016 and terminating in the third quarter of 2018. However, the last episode only lasts one quarter. Granger causality running from Lisbon to Barreiro detected two episodes at the 0,90 and 0,95 quantiles. The first only lasts one quarter and the second lasts 6 quarters between the second quarter of 2019 and the third quarter of 2020.

Granger causality running from Lisbon to Cascais detected three episodes at the 0,90 and 0,95 quantiles. The first lasts 2 quarters between the third and fourth quarters of 2019. The second only lasts one quarter, while the third lasts 4 quarters, starting in the first quarter of 2021 and continues until the end of the sample period. This result is interesting

[7] Otherwise, if the direction of causality is categorized as bidirectional, this means that house prices between two regions affect each other simultaneously.

because it indicates that the predictive power of Lisbon over Cascais is high, given that the Granger causality between theses municipalities is still ongoing at the end of the sample period.

In Loures, Granger causality from Lisbon to this municipality detects only one episode at the beginning of the sample period at the 0,90 quantile. Similarly, the Granger causality running from Lisbon to Mafra detected just one episode at the 0,90 and 0,95 quantiles. This episode lasts 7 quarters, beginning in the first quarter of 2017 and ending in the third quarter of 2018. The same applies to Granger causality running from Lisbon to Odivelas that lasts just one quarter and only at the 0,90 quantile.

Three episodes of Granger causality from Lisbon to Moita are detected at the 0,90 and 0,95 quantiles. The first only lasts one quarter, the second lasts 4 quarters between the first and fourth quarters of 2018. The third episode lasts 5 quarters between the second quarter of 2019 and the second quarter of 2020. Two episodes of Granger causality are detected from Lisbon to Montijo at the 0,90 and 0,95 quantiles. The first lasts 6 quarters, starting in the first quarter of 2019 and terminating in the second quarter of 2020. The second episode, however, only lasts one quarter.

The graphs of Oeiras and Sesimbra of Figure 3 indicate that the test statistics of the predictive power of Lisbon for these municipalities are always below their quantiles until the end of the sample period. Consequently, we do not reject the null hypothesis of no Granger causality from the Lisbon to Oeiras and Lisbon to Sesimbra, respectively.

Granger causality running from Lisbon to Palmela detected two episodes only at 0,90 quantile. The first lasts 3 quarters between the first and third quarters of 2020. The second episode lasts 2 quarters spanning the second and third quarters of 2021. Granger Causality from Lisbon to Seixal detected at the 0,90 and 0,95 quantiles just one episode spanning the first and second quarters of 2020. Two epsiodes of Granger causality from Lisbon to Setúbal are detected at the 0,90 and 0,95 quantiles. The first starts in the third quarter of 2017 and ends in the second quarter of 2018, 4 quarters. The second lasts only two quarters, spanning the third and fourth quarters of 2020.

Granger causality from Lisbon to Sintra detected three episodes at the 0,90 and 0,95 quantiles. The first lasted 11 quarters, starting in the second quarter of 2014 and terminating in the fourth quarter of 2016. The second lasted 2 quarters, spanning the third and fourth quarters of 2017, while the third episode only lasted one quarter.

Finally, three episodes of Granger causality from Lisbon to Vila Franca de Xira are detected at the 0,90 and 0,95 quantiles. The first episode occurs between the second quarter of 2014 until the first quarter of 2015, lasting 4 quarters. The second lasts 10 quarters, starting in the fourth quarter of 2016 and terminating in the first quarter of 2019. The third episode lasts 6 quarters, spanning the fourth quarter of 2019 until the first quarter of 2021.

In general, the results of the time-varying Granger causality tests running from Lisbon to the remaining 17 municipalities provide evidence that Lisbon had a bubble contagion effect, since it is possible to see that some information contained in house prices of Lisbon, at some points in time is relevant to predict the house prices in other municipalities of its metropolitan area. However, in the case of Oeiras and Sesimbra, our results show that for these areas there is no exuberance contagion. The spillover effect seems be more pronounced for the cases from Lisbon to Almada, Amadora, Barreiro, Montijo, Sintra and Vila Franca de Xira. Conversely, the effect from Lisbon to Alcochete, Cascais, Loures, Mafra, Moita, Odivelas, Palmela, Seixal and Setúbal is weaker.

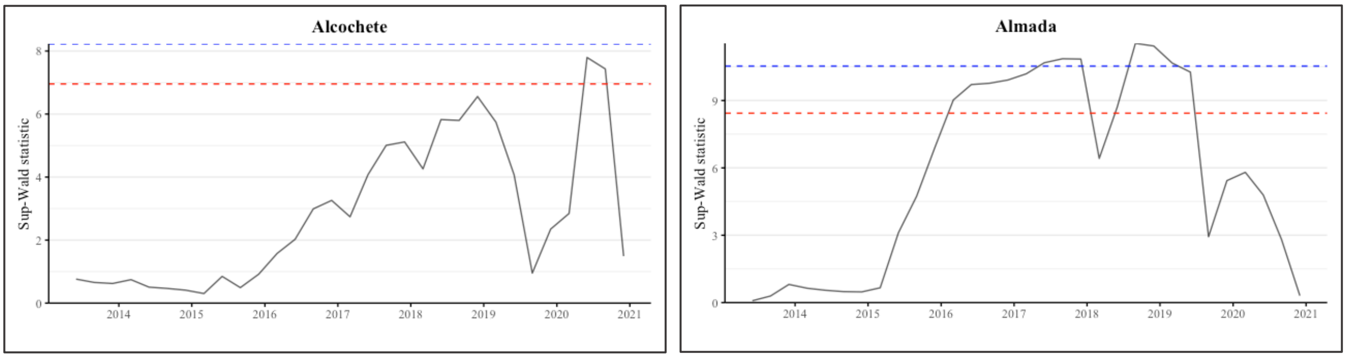


Figure 3 - Bubble contagion in the municipalities of the LMA

Note: A significant causal relationship is identified when the sup-Wald statistic exceeds its corresponding critical value within a specific period. The red dashed line represents the 0.90 quantile, the blue dashed line indicates the 0.95 quantile, and the black solid line shows the sup-Wald statistic.

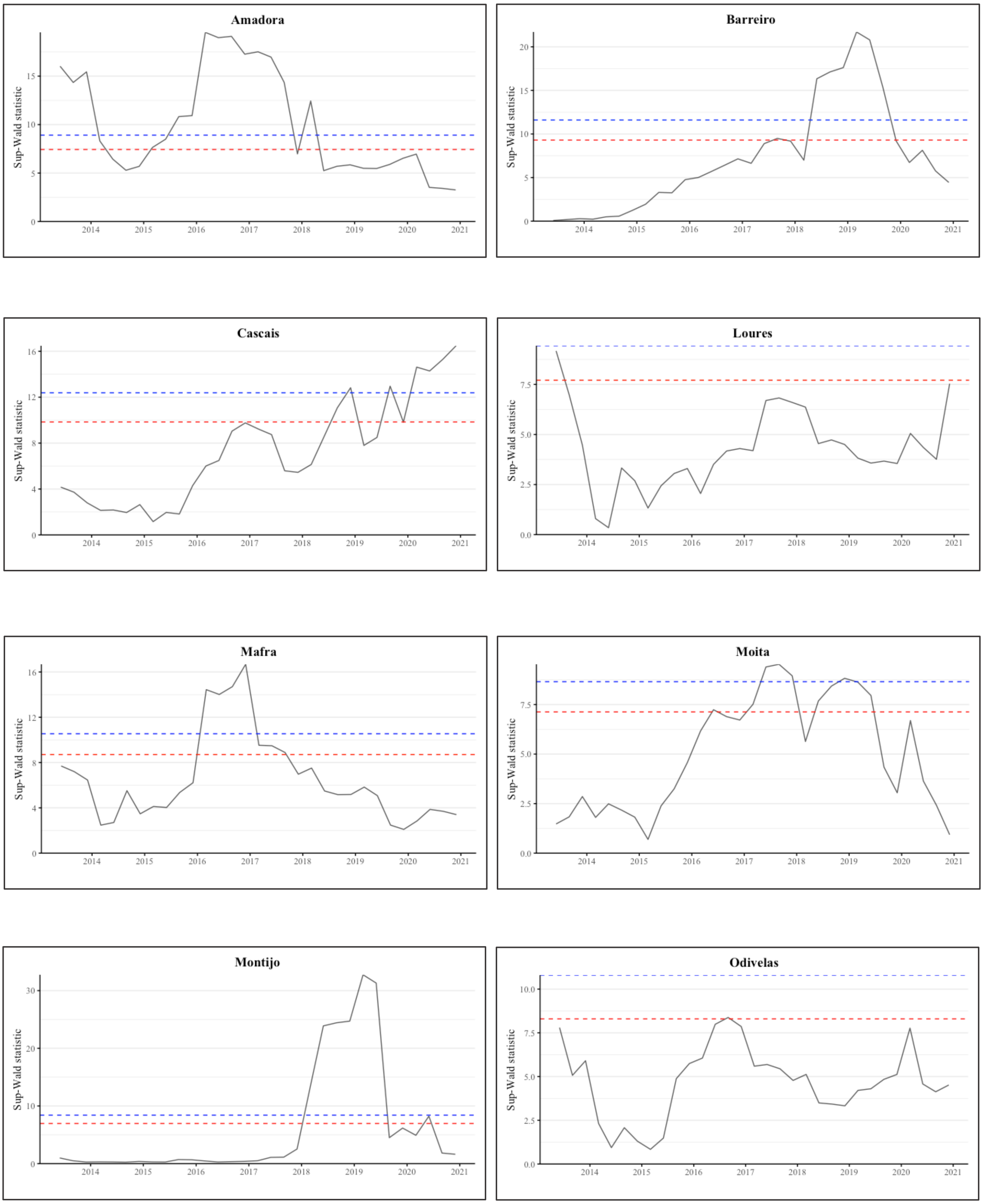


Figure 3 – Cont.

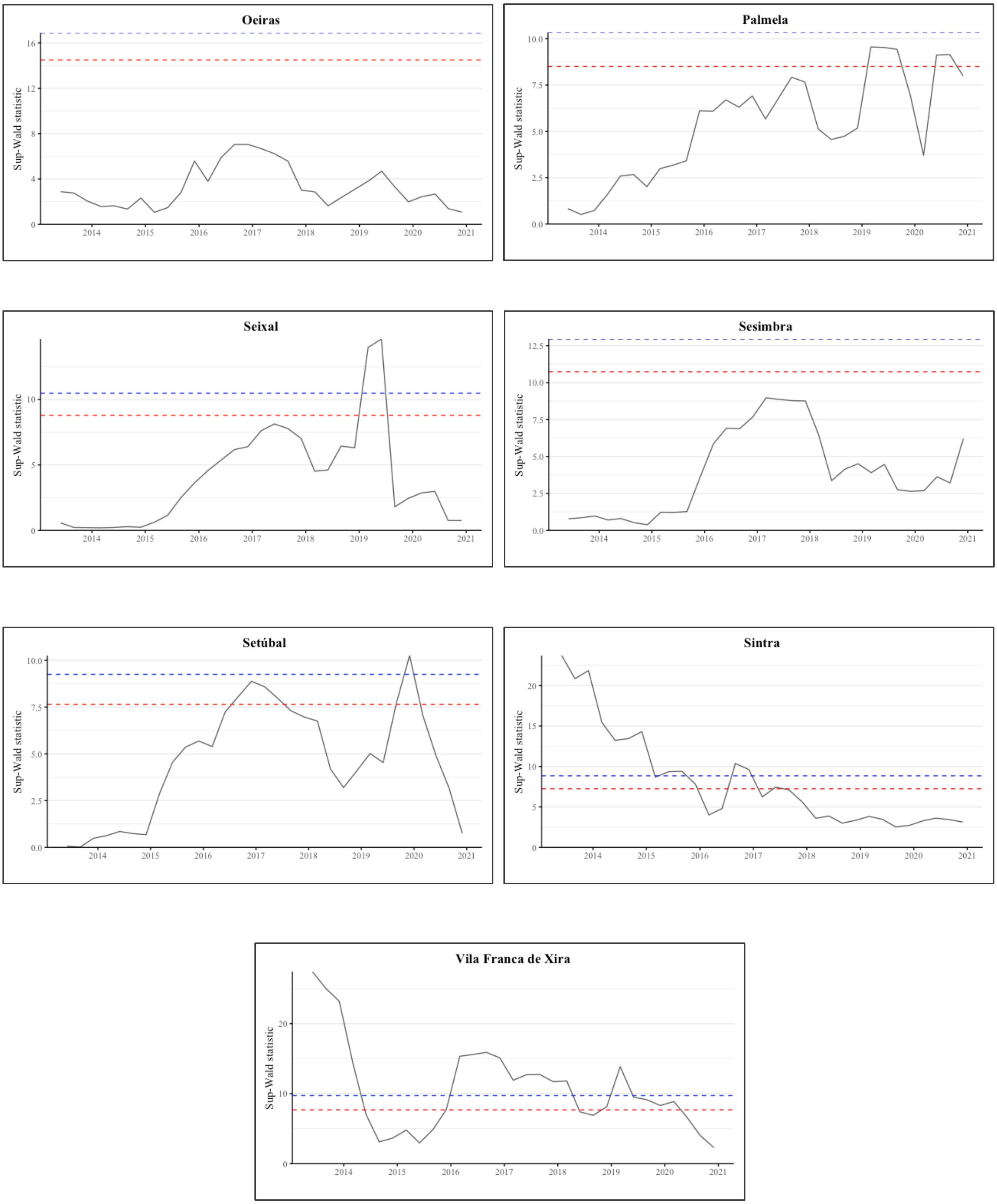


Figure 3 – Cont.

## 6. CONCLUSIONS

This study had two main objectives: first, to investigate the existence of price exuberance within the municipalities of the Lisbon Metropolitan Area (LMA); and second, to explore potential bubble contagion originating from Lisbon and spreading to neighboring municipalities. We utilized quarterly local-level data from January 2007 to October 2021, covering real house prices across 18 municipalities in the LMA. The focus on local data is crucial because it can help identify the heterogeneous nature of local housing markets, as housing markets are significantly influenced by local factors, and national-level analysis may not offer sufficient details.

We first applied the date-stamping strategy of Phillips et al. (2015) to identify and establish a chronology of exuberance episodes. Our findings reveal the existence of several episodes of exuberance across all municipalities included in our sample, with varying durations. Oeiras exhibited the longest exuberance duration, lasting 22 quarters, while the average duration across major exuberance was 17 quarters. By the end of the sample period, all municipalities displayed ongoing exuberant behavior, except for Lisbon.

Next, we employed the time-varying Granger causality test by Shi et al. (2018) to examine bubble contagion. The results indicate that Lisbon's exuberance exerted a contagion effect on the surrounding municipalities. However, no contagion was detected from Lisbon to Oeiras and Sesimbra. Interestingly, the contagion from Lisbon to Cascais persisted through the end of the sample period, indicating a strong and ongoing causal link. The contagion effect was generally more pronounced in municipalities closer to Lisbon's city center, but an intriguing finding was the weaker contagion observed in nearby municipalities like Odivelas and Loures, which might have been expected to experience stronger spillover effects given their proximity to the Lisbon region.

Overall, we found strong evidence of both price exuberance and contagion across the 18 municipalities in the Lisbon metropolitan area. However, these results should be interpreted cautiously, as explosive price dynamics can also be influenced by factors other than rational bubbles, such as explosive fundamentals or variations in discount rates (Pavlidis et al., 2016).

Although this study does not cover the possible channels for contagion, it is worth highlighting some factors that may influence the housing prices diffusion. Meen (1999) proposed four factors that could explain the ripple effect between regions, such as population movement, wealth transfer, geographic arbitrage and spatial pattern in the determinants of house prices. The first factor may help explain the contagion from Lisbon to nearby municipalities, in the sense that, as houses in the city center become increasingly inaccessible due to high prices, it is expected that households migrate to more affordable regions, thus driving price up in those areas. The geographic arbitrage noted by Meen (1999) might also explain the contagion, because when the speculative behavior begins in one area, for example Lisbon, the speculators take advantage by investing in undervalued areas expecting future price increases, then the prices in these areas increase, and consequently create a ripple effect.

The two methods employed in this study offer a key advantage: they can be applied in real-time, relying solely on past information. This feature makes them valuable tools for policymakers, enabling the development of proactive measures to prevent potential crises or mitigate their impact, thereby supporting economic and financial stability.

While this study yields important insights, it has certain limitations. First, the data used only extends to 2021, which may not fully capture more recent market developments. Second, the study does not address how housing bubbles might affect housing affordability for local residents. Third, the potential impacts of the Covid-19 pandemic were not considered, despite the likely effects on housing demand and migration patterns. Finally, other variables that might influence bubbles and contagion were not included in this analysis.

Our findings underscore the need for housing policies that specifically target the mitigation of contagion effects. Policymakers should consider localized strategies adapted to regional market conditions, as national housing policies may not suit all areas. From a risk management perspective, a better understanding of bubble contagion can help investors in managing risks more effectively.

For future research, new methods could be explored to detect early signs of bubbles, which may allow for more timely interventions. Moreover, investigating the channels and determinants of bubble contagion would help stakeholders anticipate and mitigate potential risks more effectively. Extending the analysis beyond the municipal level—perhaps to the borough level—would offer further valuable insights into housing price dynamics at better geographic scales.

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# APPENDIX

Table I - Bubble date-stamping

| Municipality | Number of bubbles | Dates of bubbles |
|---|---|---|
| Alcochete | 2 | 2011:Q4 – 2012:Q4, 2018:Q2 – 2021:Q4. |
| Almada | 5 | 2011:Q4 – 2012:Q1, 2012:Q3 – 2012:Q4, 2014:Q2 – 2014:Q4, 2018:Q2 – 2020:Q2, 2020:Q4 – 2021:Q4. |
| Amadora | 1 | 2013:Q1 – 2013:Q1, 2018:Q1 – 2021:Q4. |
| Barreiro | 2 | 2011:Q4 – 2012:Q1, 2012:Q4 – 2012:Q4, 2017:Q2 – 2021:Q4. |
| Cascais | 2 | 2012:Q4 – 2012:Q4, 2016:Q4 – 2020:Q4, 2021:Q2 – 2021:Q4. |
| Lisboa | 2 | 2012:Q4 – 2013:Q1, 2015:Q4 – 2018:Q3, 2019:Q2 – 2019:Q2. |
| Loures | 2 | 2018:Q2 – 2020:Q4, 2021:Q2 – 2021:Q4. |
| Mafra | 1 | 2017:Q4 – 2021:Q4. |
| Moita | 2 | 2012:Q1 – 2012:Q1, 2013:Q3 – 2014:Q4, 2015:Q2 – 2015:Q2, 2018:Q1 – 2021:Q4. |
| Montijo | 3 | 2011:Q3 – 2012:Q1, 2012:Q3 – 2012:Q4, 2017:Q3 – 2021:Q4. |
| Odivelas | 2 | 2012:Q1 – 2012:Q1, 2012:Q3 – 2013:Q1, 2017:Q4 – 2021:Q4. |
| Oeiras | 2 | 2012:Q3 – 2012:Q4, 2016:Q3 – 2021:Q4. |
| Palmela | 1 | 2012:Q1 – 2012:Q1, 2017:Q4 – 2021:Q4. |
| Seixal | 2 | 2010:Q2 – 2010:Q2, 2011:Q4 – 2012:Q2, 2012:Q4 – 2012:Q4, 2017:Q2 – 2021:Q4. |
| Sesimbra | 2 | 2010:Q2 – 2010:Q2, 2011:Q4 – 2012:Q1, 2012:Q4 – 2012Q4, 2017:Q4 – 2021:Q4. |
| Setúbal | 3 | 2012:Q1 – 2012:Q1, 2012:Q4 – 2012:Q4, 2013:Q3 – 2013:Q3, 2014:Q1 – 2015:Q4, 2017:Q4 – 2017:Q4, 2019:Q1 – 2020:Q2, 2021:Q1- 2021:Q4. |
| Sintra | 2 | 2012:Q3 – 2013:Q1, 2016:Q4 – 2016:Q4, 2017:Q3 – 2021:Q4. |
| Vila Franca de Xira | 1 | 2012:Q3 – 2012:Q3, 2013:Q1 – 2013:Q1, 2013:Q3 – 2013:Q3, 2018:Q2 – 2021:Q4. |