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PROJECT

PRIIP ANALYSIS: 9.20% P.A. BARRIER REVERSE CONVERTIBLE IN EUR ON LUFTHANSA (2024-2026)

LEA RIBEIRO VAZ

JUNE - 2025

Lisbon School of Economics & Management
Universidade de Lisboa

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SUPERVISION:
PROFESSOR JOÃO LUÍS CORREIA DUQUE

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Glossary

BRC – Barrier Reverse Convertible

DIP – Down-and-In Put

GARCH – Generalized Autoregressive Conditional Heteroskedasticity

GBM – Geometric Brownian Motion

IFC – Issuer Funding Cost

SRI – Summary Risk Indicator

YTM – Yield-to-Maturity

ZCB – Zero-Coupon Bond

ABSTRACT

This study presents an in-depth quantitative analysis of a “9.20% P.A. Barrier Reverse Convertible in EUR on Lufthansa”, focusing on valuation, performance under varying market conditions, and investor suitability.

The analysis begins with a brief decomposition of the product, outlining its key features, advantages, disadvantages, and embedded risks.

To assess performance, a dual-model approach is employed: the Binomial Tree model captures early redemption logic with increasing accuracy as time-step granularity improves, while the Monte Carlo Simulation, enhanced with a GARCH (1,1) volatility model, incorporates time-varying volatility and better reflects market behavior.

A rolling historical issuance analysis over a two-year period evaluates how changing market conditions affect expected value, capital loss risk, and coupon probability. The findings show that favorable issuance timing enhances expected returns and increases the likelihood of early redemption, while high volatility increases downside risk.

Additionally, the comparison between simulation-based valuations and actual market prices suggests possible mispricings that could present investment opportunities.

Overall, the research highlights the importance of dynamically volatility modeling and reinforces the role of advanced quantitative methods in understanding and pricing complex structured products.

KEYWORDS: Structured Products; Barrier Reverse Convertible; Monte Carlo Simulation; Binomial Tree Model; Early Redemption; GARCH Modeling

JEL CODES: C53; C63; G11; G12; G13; G17

Resumo

Este estudo apresenta uma análise quantitativa aprofundada de um "9.20% P.A. Barrier Reverse Convertible in EUR on Lufthansa", com foco na avaliação do produto, no seu desempenho sob diferentes condições de mercado e na adequação ao perfil do investidor.

A análise começa com uma breve decomposição do produto, destacando as suas principais características, vantagens, desvantagens e riscos embutidos.

Para avaliar o desempenho, é utilizada uma abordagem de modelação dupla: o Modelo de Árvore Binomial capta a lógica de resgate antecipado com maior precisão à medida que a granularidade temporal aumenta, enquanto a Simulação de Monte Carlo, aprimorada com um modelo de volatilidade GARCH (1,1), incorpora volatilidade variável no tempo, refletindo de forma mais realista o comportamento do mercado.

Uma análise histórica de emissões ao longo de dois anos avalia como as mudanças nas condições de mercado afetam o valor esperado, o risco de perda de capital e a probabilidade de recebimento dos cupões. Os resultados mostram que um momento favorável de emissão melhora os retornos esperados e aumenta a probabilidade de reembolso antecipado, enquanto períodos de elevada volatilidade elevam o risco de perdas.

Adicionalmente, a comparação entre os valores obtidos por simulação e os preços reais de mercado sugere possíveis desvalorizações, que podem representar oportunidades de investimento.

No geral, o estudo destaca a importância da modelação dinâmica da volatilidade e reforça o papel dos métodos quantitativos avançados na compreensão e avaliação de produtos estruturados complexos.

Palavras-chave: Produtos Estruturados; Reverse Convertible com Barreira; Simulação de Monte Carlo; Modelo de Árvore Binomial; Resgate Antecipado; Modelagem com GARCH

Códigos JEL: C53; C63; G11; G12; G13; G17

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Disclaimer

I affirm that this master's project was developed in strict accordance with the academic integrity policies and guidelines established by ISEG, Universidade de Lisboa. All research, analysis and writing presented in this project are my own, unless otherwise cited.

In the interest of transparency, I disclose that artificial intelligence (AI) tools were employed during the development of this work as follows:

- Generative AI tools were used solely to assist with English language refinement, grammar correction, and syntactic clarity in portions of the written text.
- AI-powered software was used to assist in data analysis and visualization.

However, all final analysis, interpretation of results, and written content are entirely my own. The use of AI tools was limited to technical support in data processing and linguistic refinement, and in no way compromised the originality, critical thinking, or academic integrity of this project.

All sources of information have been appropriately cited in accordance with academic standards. I understand the importance of upholding academic integrity and take full responsibility for the content, quality, and authenticity of this work.

Lea Ribeiro Vaz

June 2025

1. INTRODUCTION

In today's evolving financial landscape, structured products have gained significant traction among both retail and institutional investors. Their growing popularity stems from their ability to offer tailored risk-return profiles, enhanced yield opportunities, and access to strategies that would otherwise be complex or costly to implement individually. By combining traditional securities – such as stock, bonds, and derivatives – structured products provide investors with customizable exposure to market performance, often with built-in features such as capital protection, barriers, or autocall mechanisms. This flexibility makes them attractive in a low interest rate or uncertain market environment, where traditional investments may not meet investors' risk or return objectives.

Structured products such as Barrier Reverse Convertibles (BRC) have grown increasingly popular due to their potential for enhanced yields and embedded optionality. Typically, BRCs offer a fixed coupon and exposure to a specific underlying asset – in this case, Lufthansa shares – while incorporating path-dependent features such as barrier levels and early redemption (autocall) options.

One such product is the "9.20% P.A. Barrier Reverse Convertible in EUR on Lufthansa", a structured note issued by Leonteq Securities AG. This product offers a fixed annual coupon of 9.20% in Euros, regardless of Lufthansa's stock performance. While it presents an appealing income stream, the product also incorporates downside risk should the underlying asset breach a predetermined barrier level.

This Master Final Project aims to conduct a comprehensive analysis of the above-mentioned structured product, by evaluating its key risk and return characteristics. The study will begin with a detailed breakdown of the product, including the product overview and decomposition, its advantages and disadvantages and a brief analysis of the risk profile.

Furthermore, the core of the study will focus on fundamental calculations, including product valuation, some key probabilities associated with the product's performance, and a sensitivity analysis through the delta. The product valuation will be conducted using both the Binomial Tree model and the Monte Carlo Simulation model. Additionally, this project will also develop an analysis of the product by incorporating a varying volatility assumption through a Monte Carlo Simulation.

To further enrich the study, this research will simulate the impact of issuing the product at alternative historical dates (e.g., one month, two months, etc., prior to the actual issuance). This retrospective evaluation will highlight how different market conditions could have influenced the product's valuation, performance, and investor appeal.

By exploring the structure, pricing, and performance drivers of this BRC, the project will deliver deeper insights into its risk-reward trade-offs and its sensitivity to market dynamics. Ultimately, this study will support more informed decision-making for investors and contribute to the broader understanding of structured product behavior under varying financial conditions.

2. LITERATURE REVIEW

This literature review explores both foundational and contemporary contributions to the valuation of structured products, with particular emphasis on BRC that include early redemption features. Key topics include the developments in derivatives pricing, binomial lattice models, Monte Carlo Simulation, and volatility modeling using Generalized Autoregressive Conditional Heteroskedasticity (GARCH) (1,1) processes.

A primary source for this project is Hull (2022), a widely recognized reference in the field. Hull (2022) offers a comprehensive and rigorous treatment of derivative instruments, establishing the foundational terminology and pricing methodologies applied throughout this project. Notably, he presents the Binomial Tree model as a discrete-time framework ideal for pricing options where the underlying asset's price follows a stochastic process with binary outcomes (up or down). This model's flexibility allows it to incorporate features such as dividends, barriers, and path dependency.

Hull (2022) also introduces Monte Carlo methods as a powerful tool for valuing complex and path-dependent derivatives. These methods approximate expected payoffs by simulating numerous price paths based on Geometric Brownian Motion (GBM) assumptions.

Further insights into these techniques are provided in Glasserman (2004), which offers an in-depth discussion of GBM and its application in simulation-based pricing and explores improvements in simulation accuracy, particularly for products with early exercise or knock-in features.

In addition, Boyle et al. (1997) presents a detailed study on Monte Carlo methods tailored to security pricing. Their work addresses key challenges in simulating payoffs for derivatives with complex features, such as early redemption and path dependency.

In modelling volatility, Hull (2022) introduces the GARCH (1,1) process as an alternative to the constant volatility assumption. The influential work of Bollerslev (1986) formalizes GARCH as a dynamic process for conditional variance, allowing volatility to respond to past shocks in a mean-reverting structure. This framework enables more realistic modeling of asset price behavior and is particularly relevant when assessing the risk and pricing of derivatives over longer horizons.

3. PRODUCT DECOMPOSITION

While this product is a complex one, it can be decomposed into simpler elements of fixed income and derivatives, allowing for a better understanding and offering high periodic coupon payments while embedding equity risk exposure. It can be broken down into four simpler components: a Zero-Coupon Bond (ZCB), a short Down-and-In Put Barrier Option (DIP), a single Bermudan Callable Option, and lastly, six Fixed-Coupon payments.

The first component, the Zero-Coupon Bond, represents the principal repayment at maturity, assuming the product remains "alive" (i.e., it is not called early, and the barrier is not breached). The ZCB is fundamental to structured products because it effectively ensures that part of the investor's capital is preserved under favorable conditions. In this case, as I will mention in the section about the payoffs, the investor will receive the denomination plus the respective coupon.

The second and third components of the product – the short DIP and the Bermudan Callable Option – are equity derivatives. Specifically, the DIP is a barrier option, a type of exotic option whose payoff depends on whether the underlying asset breaches a predefined barrier level during the life of the option. The "Down" refers to the barrier being set below the strike price, and "In" indicates that the option only becomes active if the barrier is breached (path-dependent).

If the underlying asset price remains above the barrier throughout the entire maturity period, the barrier is not triggered and the option expires worthless, resulting in full redemption in cash for the investor. However, if the underlying asset price falls below the barrier at any time, the option "knocks in", transforming the exotic option into a vanilla put option that becomes active until maturity. In this scenario, the investor may receive a predetermined number of shares of the underlying asset (or their cash equivalent) instead of the notional in cash.

Additionally, the product includes a Bermudan Callable Option, held by the issuer, which grants them the right – but not the obligation – to redeem the product early at predefined observation dates, provided that certain market conditions are met. If the issuer exercises this option, the investor receives their initial capital back and the investment is terminated prior to maturity.

This feature offers the issuer significant flexibility, as they can strategically decide when to call the product based on market conditions and according to predetermined dates and criteria. Consequently, the issuer will typically choose to exercise the call when it is financially advantageous, effectively limiting the investor's upside potential. That is, the product is more likely to be called when the market evolves favorably, making further coupon payments or risk exposure less attractive for the issuer.

From a valuation standpoint, both the short DIP and the Bermudan Callable Options increase the complexity of the pricing model, as it introduces optionality and path dependency.

The final component is the coupon stream. The BRC offers periodic fixed coupons, which are independent of the underlying asset's performance. These payments function as a fixed-income component, offering investors steady cash flows throughout the investment period, unless the product has been early redeemed. The high coupon rate (9.20% P.A.) compensates investors for the embedded risks, including the potential loss in case the barrier is breached.

4. Product Overview

The product that is going to be analyzed is a “9.20% P.A. Barrier Reverse Convertible in EUR on Lufthansa”, issued by Leonteq Securities AG, Guernsey Branch, under Swiss Law. It is designed for retail investors seeking an enhanced yield through coupon payments while taking on downside risk tied to the performance of Deutsche Lufthansa AG shares. Potential investors should carefully assess their risk tolerance, as this product is not suitable for those seeking capital protection or who are unwilling to accept substantial investment risks.

This product is issued as Swiss Uncertificated Security, meaning it exists in electronic form without a physical certificate. Ownership and legal rights are established through book entries in securities accounts, ensuring efficient settlement, ease of trading, and strong investor protections. This format offers both operational flexibility and legal certainty.

The primary “goal” of the product is to provide investors with a specific entitlement according to predefined conditions while offering a relatively attractive coupon yield. It also serves as an alternative to direct investment in the underlying assets, providing investors with an opportunity to gain returns without the direct risks associated with holding shares. While it offers the chance to benefit from the underlying asset’s performance, it does so with an element of protection provided by the fixed coupon.

Below I will present the specifications of the product being priced on this project.

4.1. Issuer Overview

Leonteq Securities AG is a Swiss-based financial services firm specializing in the design, structuring, issuance, and distribution of structured products and insurance solutions. Leonteq operates across Switzerland, Europe, and Asia, under the supervision of the Swiss Financial Market Supervisory Authority (FINMA) (Leonteq, 2025b).

Although Leonteq holds a BBB credit rating, it continues to maintain transparency. Recent reports show a strong Common Equity Tier 1 (CET1) ratio, high liquidity coverage, and a conservative balance sheet, all of which reinforce its fee-based, low-risk business model.

Given the path-dependent and credit-sensitive nature of many structured products, the issuer's financial health is a key factor in pricing and risk evaluation. In this regard, Leonteq's conservative capital structure, regulatory oversight, and operational transparency contribute meaningfully to investor confidence.

4.2. The Underlying Asset

The underlying asset of this BRC is Deutsche Lufthansa AG shares. Lufthansa is one of the world's largest airline groups and, as a publicly traded company, Lufthansa's share price is influenced by a variety of factors. It is organized into four primary business segments: Passenger Airlines, Logistics, Maintenance, Repair and Overhaul (MRO), and Catering (Leonteq, 2025c).

Given the cyclical nature of the aviation industry, Lufthansa's stock tends to exhibit higher volatility. This makes it a suitable underlying for structured products like BRCs, where higher volatility can contribute to more attractive coupon rates.

4.3. The Product's Structure

As said above, BRCs are structured products that not only involve elements of both derivative components and fixed income securities but also offer a high yield in exchange for the additional risk and potential loss.

This next section outlines the main features of the BRC and its timeline from issuance to redemption. It highlights the product's key components, including its barrier mechanics, fixed coupon payments, and early redemption structure – each of which plays a critical role in determining the investment's risk-return profile (Leonteq, 2025a).

The product follows a well-defined timeline from issuance to redemption:

TABLE 1: Product timeline

Event	**Date**	**Description**
Subscription Period	30/09/2024 – 08/10/2024	Period during which investors can subscribe to the product

Issue Date	10/10/2024	Day the product is officially issued, and investors become holders of the security
Initial Fixing Date	08/10/2024	Day the price of the underlying asset is recorded, determining the strike level, barrier level, and conversion ratio
First Exchange Trading Date	10/10/2024	Day the product starts trading in the secondary market
Final Fixing Date	08/04/2026	Day the final price of Lufthansa stock is recorded to determine the redemption payout
Last Trading Day	08/04/2026	Last day that the product can be traded in the market before final settlement
Redemption Date	10/04/2026	Day when investors receive their final payoff, either in cash or shares, based on performance

The first main feature of BRCs is the presence of a barrier level, set at 69% of the initial fixing level, continuously observed from October 8, 2024, to April 08, 2026. If the barrier is breached, the investor will receive a round number of the underlying asset (or its cash equivalent), through a conversion ratio, which can result in a potential loss. Further conditions regarding this outcome will be explained in the payoff section.

Another key feature of this product is that it offers a 9.20% annual coupon rate on denomination, paid regardless of the performance of the underlying asset, if the product has not been early redeemed. Coupons are paid in six installments of 23€ (totaling 138€ per year), as seen on the table below. This allows for a certain level of income stability for the investor.

TABLE 2: Coupon payment timeline

Coupon Payment Date	**Coupon Amount**
15/01/2025	23.00€
15/04/2025	23.00€
15/07/2025	23.00€

15/10/2025	23.00€
15/01/2026	23.00€
10/04/2026	23.00€

The coupon payment dates refer to the dates on which the coupon is paid to the investor (in case the product has not been early redeemed).

The third key feature is the early redemption (autocall) option, giving the issuer the right, but not the obligation, to redeem the product early if, at any observation date, the underlying trades at or above the initial fixing level. This resembles a Bermudan-style option, allowing the issuer to call the product at predetermined observation dates. When this happens, the contract terminates and no further payments are made, effectively capping the investor's upside in the event of strong Lufthansa stock performance. For both the Binomial Tree model and the Monte Carlo Simulation, early redemption will be exercised whenever the stock price is at or above the strike price, for simplicity.

Regarding the early redemption dates, there are two distinct occasions. One concerning the observation, where the issuer decides if they will call the product or not, and the other concerning the actual payment, where, if decided by the issuer, the product is early redeemed, and the payment to the investor is made, as seen on the table below.

TABLE 3: Early redemption timeline

Early Redemption Observation Date	**Early Redemption Payment Date**
08/04/2025	15/04/2025
08/07/2025	15/07/2025
08/10/2025	15/10/2025
08/01/2026	15/01/2026

4.3. The payoff

Because of the various scenarios possible at maturity, this product does not have a linear payoff. First, there are two main outcomes, one where the product is early redeemed by the issuer and the other where there is not early redemption and so, the product reaches maturity.

When early redemption occurs, the investor will receive 100% of the denomination plus the coupon amount of the respective coupon payment date. When this happens, the contract ends. If the early redemption does not occur on any of the respective dates, the product reaches maturity where it has three possible endings. The first is when the maturity is reached without a barrier event occurring, in which case the investor receives the denomination and the respective coupon. If, at any point in time the barrier event occurred, the investor has two possible payoffs. The denomination plus the respective coupon, if the final fixing level is at or above the strike level and a round number (conversion ratio) of the underlying per product plus the respective coupon, if the final fixing level is below the strike level.

TABLE 4: Payoff scenarios

Scenarios	Payoffs
Early Redemption	Denomination + Coupon (1000€ + 23€)
Maturity without Barrier Event	Denomination + Coupon (1000€ + 23€)
Maturity with Barrier Event and St >= K	Denomination + Coupon (1000€ + 23€)
Maturity with Barrier Event and St < K	Conversion Ratio × St + Coupon

This means that if the stock price is significantly lower at maturity, the investor could suffer a large loss, as the delivered shares may be worth less than the denomination. This payoff ensures three main things. One is guaranteed fixed coupons, since the investor will receive the coupons regardless of the underlying asset's movements. The second is that, if the underlying asset's price never breaches the 69% barrier level, full principal repayment is ensured. And the last one is the opportunity to earn a positive return in case the underlying asset performs well, and the product is called for early redemption.

4.4. The parameters

To accurately value the product, several financial parameters are essential.

One of them is the risk-free rate. It represents the theoretical return on zero-risk investment. Typically derived from government bond yields, the 2-year German Bund Yield was chosen (as of October 08, 2024) due to the product's 1.5-year maturity, reflecting market conditions at issuance and better aligning with the product's time horizon than the 10-year yield.

Another key parameter is volatility, which measures the price fluctuations of the underlying asset over time. In this context, higher volatility increases the probability of barrier breaches, making the product riskier. For this analysis, implied volatility was used instead of historical volatility, as it serves as a forward-looking metric that reflects market expectations of future price movements, making it more suitable for structured products valuation.

The implied volatility selected was the one observed on the initial fixing date, associated with the nearest available maturity (June 19, 2026), given the product's 1.5-year maturity. Additionally, the chosen volatility corresponded to the product's strike price, ensuring consistency with the specific characteristics of the product being analyzed.

Since Lufthansa is a dividend-paying company, dividends must be factored into the analysis since they reduce the price of the underlying asset and, consequently, the likelihood of barrier breaches or early redemption. And so, by factoring in the expected dividends, we can more accurately reflect on the real-world conditions under which the product's payoffs will be realized.

During the observation period, there is only one ex-dividend date on May 07, 2025. Because the dividend date is known, I will assume the dividend will be paid discretely, and so, I won't implement it using the dividend yield. Instead, I will adjust the underlying asset price on the ex-dividend date. This process will be explained in more detail in the section regarding the valuation methods description.

Another significant input in pricing structured products is the Issuer Funding Cost (IFC), which reflects the internal cost of capital for the issuer and directly influences pricing and product appeal. In this case, since the IFC wasn't directly observable in the market, it was estimated using interpolation between the Yield-to-Maturity (YTM) of two Leonteq bonds, as seen on the table below.

TABLE 5: Selected bonds issued by Leonteq Securities AG

Maturity	Issued	Structure	YTM
12 January 2026	09 August 2024	Callable	3.238%
13 July 2026	04 October 2024	Callable	3.057%

These bonds were selected for their callable feature and proximity to the product's timeline. The January bond aligns with the BRC's maturity, while the July bond was issued during the subscription period. The interpolation used the formula:

$$YTM = Y_{Jan} + \left(\frac{Days\ between\ 10/04/2026\ and\ 12/01/2026}{Days\ between\ 13/07/2026\ and\ 12/01/2026}\right) \times (Y_{Jul} - Y_{Jan}) \quad (1)$$

Besides these key parameters, this product also has some costs. However, for simplicity, I chose not to include them in the analysis. More relevant parameters for the pricing of the product are listed below with the corresponding observations:

TABLE 6: Key parameters

Parameters	Value	Observations
Risk-Free Rate	2.235%	2-year German Bund Yield on Initial Fixing Date
Volatility	28.81%	Derived from Bloomberg
Dividend	0.30€	Derived from Bloomberg
IFC	3.148%	Using interpolation
Currency	EUR (€)	As presented on KID
Denomination	1000€	As presented on KID
Issue Price (S0)	6.34€	As of the Initial Fixing Date
Strike Price (K)	6.34€	100% of Issue Price
Barrier Level (B)	4.37€	69% of Issue Price
Conversion Ratio	157.73	Denomination/S0
Maturity (T)	1.5	Years

5. MAIN RISKS OF THE PRODUCT

The Summary Risk Indicator (SRI) provides a visual representation of the investment risk associated with the BRC, classifying it on a scale from 1 to 7. And, like all financial instruments, it is important to analyze the risks associated with the product to make a decision about the investment. For this product, the SRI is rated as 6. This rating reflects a significant potential for losses due to fluctuations in market performance, especially concerning the underlying, Lufthansa shares.

Overall, the SRI serves as a guide to assess the likelihood of experiencing financial losses relative to other investment products and it is calculated based on different metrics related to the product, such as historical volatility or its performance under different market conditions.

This product is not capital protected, meaning investors could lose part or all their investment.

5.1. Issuer's risks

When issuing a structured product such as a BRC, financial institutions are exposed to various risks that can impact their financial stability and profitability. These risks can stem from different sources, as stated below.

- Operational risk: This encompasses potential losses arising from pricing errors, settlement failures, or disruptions in the issuance and management of the product.
- Credit risk: It is the risk of the issuer defaulting on its financial obligations, leading to a deterioration in the issuer's creditworthiness, affecting investor's confidence.
- Non-compliance and reputational risk: Issuers must adhere to regulatory requirements when structuring and selling financial products. Any failure to comply with these regulations can lead to legal penalties and reputational damage, which in turn can affect investor's trust and the issuer's market position.
- Liquidity risk: A liquidity crisis could hinder the issuer's ability to meet coupon payments and redemption commitments, leading to default.
- Legal risk: Given the complexity of structured products, there may be some legal disputes, especially if the investor claims they were misinformed about the risks involved. Issuers must ensure that all product documentation is accurate and comply with any legal and regulatory requirements.

5.2. Investor's risks

On the other hand, there are also some risks for the investor, such as:

- Barrier risk: This risk arises when the underlying asset breaches the barrier level, potentially resulting in the investor receiving the asset itself rather than the entire cash payout, resulting in a loss if the underlying asset's value has declined.
- Credit risk: There is also the risk of the issuer not being able to meet its financial obligations (coupon payments and principal repayment), as this depends on the issuer's financial health.
- Market risk: Because the BRC depends on the price changes of the underlying asset, a significant price drop can lead to substantial losses, especially given the limited downside protection and the high risk of the product.
- Volatility risk: Higher price fluctuations increase the likelihood of the barrier being breached, raising the chances of principal loss.
- Liquidity risk: BRC's are generally less liquid than traditional assets and so, investors may struggle to sell their holdings at fair value due to a lack of buyers, potentially leading to a forced sale at a discount.
- Early redemption risk: By terminating the investment early, the issuer limits upside potential preventing investors from benefiting from future coupon payments or favorable market conditions.
- Currency risk: For those investing in a foreign currency, exchange rate fluctuations may affect the value of returns and result in gains or losses that are unrelated to the BRC's real performance.
- Counterparty risk: It is important for the investors to know beforehand the issuer's financial health, as there is always the risk of them not fulfilling their financial obligations, resulting in missed payments and increased investor losses.
- Interest rate risk: When interest rates rise, the market value of fixed-income securities typically declines, potentially reducing the attractiveness of the BRC. However, given the relatively short maturity of this product, its exposure to interest rate fluctuations is limited

It is important to note that, for both the issuer and the investor, there are more risks involved, but these are the ones I consider most important.

6. ADVANTAGES AND DISADVANTAGES

6.1. For the Issuer

This structured product offers several advantages for the issuer. One key advantage is that, if the underlying asset's price is below the strike at maturity following a barrier breach, the issuer can return the underlying stock instead of cash, reducing their financial obligation. The embedded short DIP option transfers downside risk to the investor, further limiting the issuer's exposure. Additionally, the issuer benefits from leveraging the capital raised during the product's term, potentially investing it elsewhere at a lower funding cost. Due to its high coupon payments, the BRC appeals to investors looking for enhanced yields, expanding the issuer's market reach.

The issuer embeds a short put option within the product, meaning they effectively sell a down-and-in put option to investors. If the barrier is breached, investors bear the risk of receiving shares rather than cash, limiting the issuer's exposure. If the issuer holds underlying shares, this setup provides a natural hedge.

Market volatility also plays a role in making this product attractive to issuers. When volatility is high, they can collect a higher option premium. Furthermore, the issuer retains control over when the product ends, effectively capping the investor's upside potential and reducing market exposure by ending the investment early when conditions are favorable. The high coupon payments serve as an incentive for investors, though they come with stock performance risk.

However, this product also brings some disadvantages for the issuer. First, because the high coupon payments, while attractive to investors, make the product more expensive compared to regular bonds. Second, because, if the barrier is breached and the underlying asset's price falls significantly, the issuer is required to deliver shares at a loss. Additionally, the higher coupon payments compared to traditional bonds also introduce unpredictability, as the product could end early. Furthermore, the issuer has a coupon payment obligation, meaning they must consistently pay out returns, which can be a financial burden. Lastly, the issuer can face reputation risk, especially if investors experience significant losses due to the structure of the product.

6.2. For the Investor

Likewise, this structured product also offers a range of advantages and disadvantages for the investors.

One of its key benefits is the high fixed coupon payments, which provide an attractive yield compared to traditional fixed-income products, especially in low-interest-rate environments. Additionally, the product offers conditional capital protection, ensuring that if the barrier is not breached, the investor is guaranteed to receive their full principal amount at maturity. And even in cases where the barrier is breached, the investor can still recover their full investment if the underlying asset rebounds to at least the strike level by the final fixing date. Furthermore, this product allows investors to benefit from stock price stability. If the underlying asset remains stable or experiences low fluctuations, the investor continues to collect high coupon payments while securing their principal, making it a potentially attractive alternative to direct equity investment.

However, it also brings some disadvantages. If the barrier is breached and the underlying asset is below the strike price at maturity, investors may receive depreciated shares instead of cash, leading to potential capital loss. Moreover, the investor's upside potential is capped, as returns are limited to the fixed coupon payments – unlike direct equity investments, which offer unlimited gains.

Another major drawback is the risk of early redemption. Since the issuer has the discretion to call the product early, the investor may lose the opportunity to continue receiving high coupon payments. As a result, the investor may not fully realize the expected yield if the product is redeemed prematurely.

The complexity of this structured product is another concern. Its multi-faceted nature, including coupon payments, barriers and auto call features, can make it difficult for retail investors to fully grasp the associated risks. Finally, there is also issuer credit risk to consider. The investor's returns and principal are dependent on the issuer's financial stability, meaning that in the event of an issuer default, the investor could face significant losses.

7. VALUATION METHODS DESCRIPTION

For product evaluation, two widely used models in financial engineering were employed: the Binomial Tree model and the Monte Carlo Simulation model. Given the embedded derivative components, accurately pricing these products is essential for both issuers and investors. These models will provide different approaches to estimating expected values, the respective probabilities, and sensitivity measures such as delta.

While both models handle path-dependent features – crucial for barrier and autocallable structures – they offer different perspectives. The Binomial Tree model, a discrete-time method, provides transparency in tracking specific price paths. In contrast, the Monte Carlo Simulation captures a continuous range of potential price evolutions, offering greater flexibility, especially when modeling more complex payoffs.

Below, I will go through each of these models in more detail.

7.1. Binomial Tree

The first model used was the Binomial Tree model, a discrete-time pricing model (Hull, 2022) that represents an asset's price evolution over multiple time-steps (δt). The model assumes that in each time-step, the underlying asset price can either move up or down, with a certain probability, until the maturity of the product, creating a tree-like structure of possible future prices. The up and down movements are defined as:

$$u = e^{\sigma\sqrt{\delta t}} \tag{2}$$

$$d = \frac{1}{u} = e^{-\sigma\sqrt{\delta t}} \tag{3}$$

The tree starts at S0 and at each step the price will either move up (S_u) or down (S_d) based on the up and down factors, making the asset price evolve as:

$$S_t \begin{cases} S_{t-1} \times u = S_{t-1} \times e^{\sigma\sqrt{\delta t}}, & with\ probability\ p \\ S_{t-1} \times d = S_{t-1} \times e^{-\sigma\sqrt{\delta t}}, & with\ probability\ 1 - p \end{cases} \tag{4}$$

Where the risk-neutral probability (i.e., the probability under the risk-neutral measure) is given by:

$$p = \frac{e^{r\delta t} - d}{u - d} \tag{5}$$

Where σ is the implied-market volatility (annualized), δt is the time-step, computed as T/Number of Steps (where T is the total time to maturity) and r is the risk-free rate.

This formulation ensures that the model is arbitrage-free under the risk-neutral framework, a central concept emphasized by Hull (2022).

For dividends, a discrete approach was adopted. Specifically, on the day of the ex-dividend date, the dividend amount was directly subtracted from the underlying asset price. This adjustment reflects the immediate drop in the asset's price after the dividend payout, maintaining the consistency of the price path within the Binomial Tree structure.

After making this tree, a second one is generated containing the value of the expected payoff at each node. This is done using backward induction (Hull, 2022), starting from the terminal nodes where the final payoffs are known based on the payoff structure and working backward through the tree using the formula:

$$f_t = [pf_u + (1-p)f_d]e^{-r\delta t} + Coupon\ (if\ applicable) \tag{6}$$

Where f_t is the expected payoff at a certain node, f_u is the expected payoff on the up node and f_d is the expected payoff on the down node.

This backward valuation method enables the model to compute the present value of the structured product by recursively discounting expected future payoffs. For this model, a risk-neutral valuation is assumed, meaning that future payoffs are estimated under the risk-neutral measure and discounted using the risk-free rate, a methodology central to modern derivative pricing as outline by Hull (2022).

This structure makes the model particularly useful for evaluating path-dependent products, such as autocallables or barrier options, where tracking the history of price movements is essential.

7.2. Monte Carlo Simulation

The second model used was the Monte Carlo Simulation, implemented in Python. This method is a flexible and powerful numerical approach for derivative pricing, especially for path-dependent instruments. It relies on the generation of a large number of simulated price paths for the underlying asset price, using stochastic modeling techniques. As described by Glasserman (2004) and Hull (2022), Monte Carlo methods assume that the asset price evolves according to a GBM, where each simulated path reflects potential future asset prices.

The asset price at each step is modeled as:

$$S_t = S_{t-1} e^{\left(r - \frac{\sigma^2}{2}\right)\delta t + \sigma\sqrt{\delta t} Z} \tag{7}$$

Where Z ~ N (0,1) is a standard normally distributed random variable.

In this study, 10000 independent paths were simulated. Each path was then analyzed to determine whether a barrier breach or early redemption event occurred. If there was early redemption, the simulation recorded the corresponding payoff and coupon. Otherwise, the payoff was computed at maturity, depending on the final underlying price relative to the strike level and barrier.

Regarding the dividends, they were applied in the same way as the Binomial Tree model. Through a discrete dividend approach where, on the ex-dividend date, the dividend amount was subtracted from the simulated underlying asset price.

Each payoff was then discounted to present value using:

$$Discount\ Factor = e^{(-IFC \times Time\ Remaining)} \tag{8}$$

Where the time remaining depends on whether there was early redemption, or the product reached maturity. If there is early redemption the difference between the observation dates and the actual payment dates will be considered.

Coupon payments were also incorporated at the appropriate dates across the simulation paths. These too were discounted back to time $t = 0$ using the same formula. Finally, the expected value of the product was computed by averaging all discounted payoffs across the simulated paths, a standard practice emphasized by Boyle et al. (1997).

This method's strength lies in its statistical convergence to the true expected value as the number of simulations increases, as detailed in Glasserman (2004).

8. PRODUCT VALUATION

Given the complex nature of BRCs – particularly their path-dependency and embedded options – choosing appropriate models is crucial.

In this section, I will present the key results obtained from the two pricing models used. These models allow for a detailed assessment of the product under realistic market assumptions and help capture the effect of volatility, interest rates, and barrier events.

The analysis includes the expected value of the product, probability estimates (such as the likelihood of early redemption), and the product's delta, which measures sensitivity to changes in the underlying asset price.

8.1. Binomial Tree

This model is divided into two sections. The first section focuses on a simplified binomial tree with fewer steps, design to align precisely with the product's coupon payment and early redemption dates: January 2025, April 2025, July 2025, October 2025, January 2026 and April 2026. This results in a time-step of 0.25 years (T/n). Since the ex-dividend date does not coincide with any of the time-steps, the dividend was discounted back to April and subtracted from the underlying asset price at that node. Using the formulas presented in the previous section, the following parameters were calculated: $u = 1.15, d = 0.87, p = 0.48$ and $1 - p = 0.52$.

TABLE 7: Product valuation using Binomial Tree model (6 steps)

Value at Issuing Date	948.67€
Value on January 15, 2025	970.61€
Delta	60.21

At the issuing date, the estimated expected value indicated a slight discount (94.87% of the nominal 1000€), reflecting the embedded risk of conversion into shares in adverse scenarios. An alternative scenario considering the product's valuation on January 15, 2025, the first coupon payment date, yielded a slightly higher expected value. The parameters used are provided in the attachments and $u = 1.15$, $d = 0.87$, $p = 0.48$ and $1 - p = 0.52$. The number of steps is 5 and the maturity is now adjusted to 1.25 years.

The estimated delta indicates that for a 1€ increase in the underlying asset price, the value of the structured product is expected to increase by 60.21€, all else being equal. It reflects the sensitivity of the product's price to changes in the underlying asset:

$$\Delta = \frac{f_u - f_d}{S_0 u - S_0 d} \tag{9}$$

For a more thorough analysis, I decided to also include some key probabilities related to the product's performance, to get a better understanding of how the product works. In order to get those values, I created another tree but this time with the probabilities of each node happening (e.g., in $t = 2$, with 3 nodes the probabilities are p^2, $2 \times p \times (1 - p)$ and $(1 - p)^2$) (see attachments), then I simply added each relevant probability to obtain the final probability, observing the following results:

TABLE 8: Key probabilities using Binomial Tree model (6 steps)

Probability of Early Redemption	56.56%
Probability of Losing Money	43.44%
Probability of Receiving Max Coupon	43.44%

The probability of early redemption reflects the likelihood of the BRC being called before maturity if the underlying asset's price reaches a specified level. This is usually favorable for the issuer, as it reduces exposure to long-term risk. The probability of losing money reflects the chance of the barrier being breached, leading to redemption based on the conversion ratio rather than full principal repayment. Lastly, the probability of receiving the maximum coupon corresponds to the product reaching maturity without early redemption, allowing the investor to receive all scheduled coupon payments.

The second section of this model enhances the previous framework by increasing granularity by to 18 monthly steps over the product's 1.5-year maturity, resulting in a 0.08-year time-step. This refinement allows for a more precise approximation of the underlying asset's price evolution and better captures barrier events, early redemption, and path-dependent features. For the up and down factor and the risk-neutral probability, I got the following: $u = 1.09$, $d = 0.92$, $p = 0.49$ and $1 - p = 0.51$. All other input parameters remain unchanged.

TABLE 9: Product valuation using Binomial Tree model (18 steps)

Value at Issuing Date	963.87€
Value on January 15, 2025	951.89€
Delta	66.48

This led to an expected value that showed a modest increase in value relative to the simpler model. This improvement highlights how a finer time resolution captures nuances in the payoff structure more accurately. However, considering the product's valuation on January 15, 2025, the expected value slightly decreased. The number of steps is 15, and $u = 1.09$, $d = 0.92$, $p = 0.49$, and $1 - p = 0.51$. The refined model also provided a higher delta and a new set of probabilities detailed below:

TABLE 10: Key probabilities using Binomial Tree model (18 steps)

Probability of Early Redemption	68.50%
Probability of Losing Money	29.22%
Probability of Receiving Max Coupon	31.50%

8.2. Monte Carlo Simulation

Similar to the Binomial Tree model, the Monte Carlo Simulation will be divided into two parts. The first section uses constant market-implied volatility, while the second part explores a more dynamic volatility structure using a GARCH (1,1) model. This two-tier approach allows for comparison between static and time-varying volatility assumptions when modeling the structured product's behavior.

In the first simulation, the input parameters match those used in the Binomial Tree model, with one major difference: the time-steps. Here, the asset price is simulated over 390 steps, one for each business day across the product's 1.5-year maturity. This results in a time-step of approximately 0.00399 years. After simulating all the potential price paths, I got the following values:

TABLE 11: Product valuation using Monte Carlo Simulation model

Value at Issuing Date	960.26€
Value on January 15, 2025	960.91€
Delta	34.59

The expected value at the issuing date confirms a small discount due to embedded optionality and risk, like the Binomial Tree outcomes. The product's delta is estimated using a finite difference approximation method, which involves slightly adjusting the initial asset price and recomputing the product's expected value:

$$\Delta = \frac{V(S_0 \times (1+\epsilon)) - V(S_0)}{S_0 \times \epsilon} \quad (10)$$

Where, ϵ represents a small change in S_0. As outlined by Glasserman (2004), this method is widely used to estimate option sensitivities in simulation-based models when closed-form solutions are unavailable.

To visualize how the underlying asset price may evolve over time, Figure 1 depicts 100 price paths generated during the Monte Carlo Simulation, each illustrating the market's inherent volatility and the product's path-dependent nature.

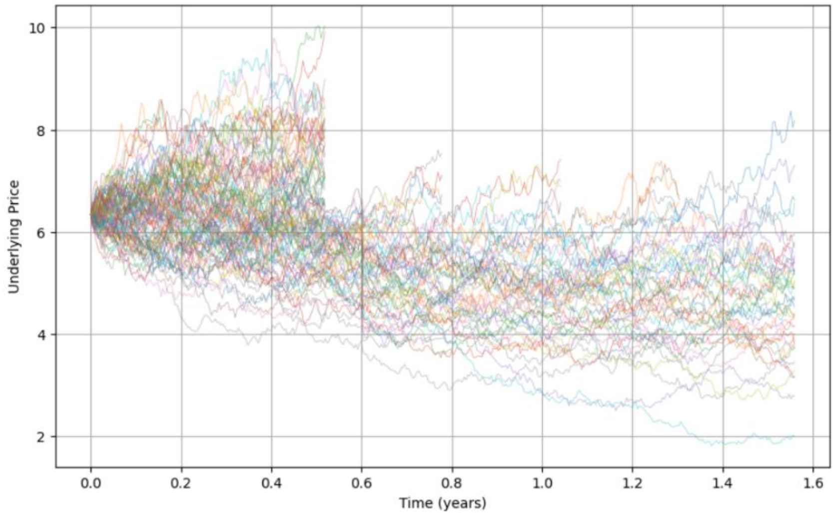

FIGURE 1: Price evolution of the underlying stock

The following histogram shows the distribution of the underlying asset's final prices across all simulated paths, summarizing the likelihood of different terminal outcomes. This distribution underpins the probabilistic estimates of early redemption, loss scenarios, and coupon payments, providing valuable insights into the product's risk profile.

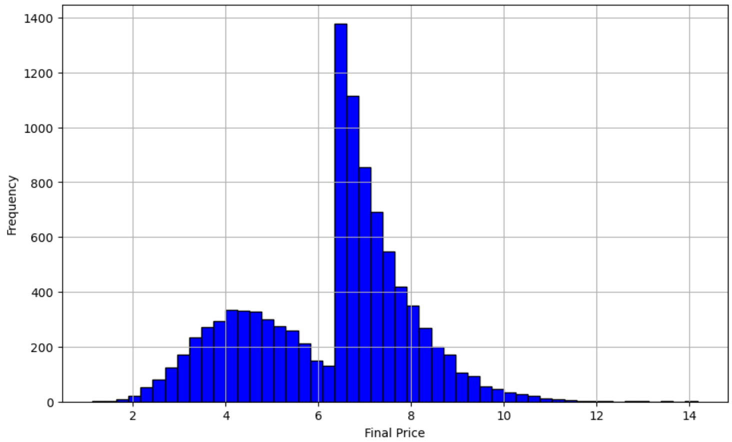

FIGURE 2: Frequency of final prices of the underlying asset

To get the value of the probabilities, I inserted three important features on my code, the early redemption count, the loss count and the reached maturity with no early redemption count, where every time one of those situations happens during the simulation, the count went up by one. At the end, I divided those numbers by the number of simulations made and got the following results:

TABLE 12: Key probabilities using Monte Carlo Simulation model

Probability of Early Redemption	61.60%
Probability of Losing Money	28.68%
Probability of Receiving Max Coupon	38.40%

As mentioned earlier, the second part of the Monte Carlo Simulation introduces time-varying volatility, a common feature observed in financial markets. Unlike the constant volatility assumption, this approach uses a GARCH (1,1) model (Bollerslev, 1986), which allows volatility to evolve dynamically over time, as discussed in Hull (2022). This approach captures the heteroskedasticity nature of financial time series, where periods of high and low volatility tend to cluster, by modeling volatility based on past returns and previous volatility.

The GARCH (1,1) model is defined as:

$$\sigma_t^2 = \omega + \alpha \times \varepsilon_{t-1}^2 + \beta \times \sigma_{t-1}^2 \tag{11}$$

With:

$$\varepsilon_{t-1}^2 = \ln\left(\frac{S_t}{S_{t-1}}\right) = \left(r - \frac{\sigma_t^2}{2}\right)\delta t + \sigma_t\sqrt{\delta t}Z \tag{12}$$

Where σ_t^2 is the conditional variance at time t, ω is the constant term, α captures the impact of past squared returns, β measures the persistence of past volatility and ε_{t-1} is the past returns.

The sum of $\alpha + \beta$ determines volatility persistence. A value close to 1 suggests that shocks to volatility decay slowly, implying long memory in the system.

To estimate the model, I used historical daily returns of Lufthansa shares, applying Maximum Likelihood Estimation (MLE). The calibrated parameters were:

$$\omega = 0.00001, \alpha = 0.049945, \beta = 0.928979$$

This results in the conditional variance formula:

$$\sigma_t^2 = 0.00001 + 0.049945 \times \varepsilon_{t-1}^2 + 0.928979 \times \sigma_{t-1}^2$$

To initialize the simulation, I calculated the long-run average volatility using:

$$\sigma_0 = \sqrt{\frac{\omega}{1 - \alpha - \beta}} = \sqrt{\frac{0.00001}{1 - 0.049945 - 0.928979}} = 0.02178$$

The simulation followed the same process as outlined in the constant-volatility model, but with volatility dynamically updated at each time-step. The simulation ran over 390 business days, with updates at each step based on new returns and volatility, accounting for coupon payments, early redemption observations, and discrete dividends. After running 10000 simulations, I got the following results:

TABLE 13: Product valuation and key probabilities using Monte Carlo Simulation model with GARCH

Value at Issuing Date	977.85€
Value on January 15, 2025	978.47€
Delta	33.26
Probability of Early Redemption	70.78%
Probability of Losing Money	22.49%
Probability of Receiving Max Coupon	29.22%

9. DISCUSSION OF RESULTS

TABLE 14: Valuation and key probabilities by method

	Binomial Tree	Binomial Tree + Steps	Monte Carlo Simulation	Monte Carlo Simulation + GARCH
Value at Issuing Date	948.67€	963.87€	960.26€	977.85€
Probability of Early Redemption	56.56%	68.50%	61.60%	70.78%
Probability of Losing Some Money	43.44%	29.22%	28.68%	22.49%
Probability of Maximum Coupon	43.44%	31.50%	38.40%	29.22%
Delta	60.21	66.48	34.59	33.26
15 January 2025	970.61€	951.89€	960.91€	978.47€

The two valuation frameworks – Binomial Tree and Monte Carlo Simulation – provide complementary insights into the pricing and risk profile of the BRC. While both models are suitable for path-dependent products, their underlying assumptions lead to notable differences in outcomes, which carry important implications for both issuers and investors.

The Binomial Tree offers a more transparent and intuitive view of early redemption mechanics. It is computationally efficient and ideal for discrete decision points aligned with fixed coupon dates. However, it is sensitive to granularity: increased steps led to significant improved accuracy, with higher expected values and a clearer picture of early redemption likelihood.

Probability estimates also shifted with a higher step count. The probability of early redemption increased from 56.56% to 68.50%, suggesting that the refined model better captures early redemption scenarios. Additionally, the probability of losing money decreased from 43.44% to 29.22%, which aligns with the higher expected value. However, the probability of receiving the maximum coupon also declined from 43.44% to 31.50%, indicating a greater likelihood of early redemption rather than full-term coupon payments. The delta increased slightly from 60.21 to 66.48, suggesting an increase in sensitivity to changes in the underlying asset's price.

Conversely, the Monte Carlo Simulation, particularly the GARCH (1,1) extension, provides greater flexibility by capturing continuous price movements and time-varying volatility. This makes it better suited for evaluating more complex or volatile market environments, particularly when volatility is expected to change over time.

Regarding the Monte Carlo Simulation, the fixed-volatility one produced an expected value of 960.26€, while the expected value for the GARCH-based model is 977.85€. This higher value under GARCH reflects the model's ability to capture volatility clustering, which may affect the probabilities of barrier breaches and early redemption, leading to more favorable payoff paths.

Notably, the probability of early redemption under GARCH increased to 70.78%, the highest among all models, while the probability of losing money decreased to 22.49%, suggesting a more optimistic risk profile. The probability of receiving the maximum coupon, however, decreased to 29.22%, aligning with the prevalence of early redemption.

Delta remained more stable when using Monte Carlo Simulation, with values of 34.59 with fixed volatility and 33.26 under GARCH, supporting a smoother and potentially more accurate hedge ratio.

Both models confirm that the BRC offers enhanced yield potential through periodic coupon payments, at the cost of downside risk in the event of a barrier breach. The Monte Carlo model with GARCH volatility, reflects a more optimistic expected value and a lower capital loss probability – suggesting that the BRC's path-dependency might favor investors in volatile conditions more than simpler models predict.

10. FURTHER ANALYSIS

While static valuation models provide a snapshot of a structured product's performance under current market conditions, they may not fully capture how its risk-return profile fluctuates over time. To gain deeper insights into the sensitivity of the BRC to market dynamics, a rolling historical issuance analysis was conducted. This approach helps assess how different issuance environment – such as changes in volatility, asset price, and interest rates – affect the product's valuation, early redemption likelihood, and downside risk.

The analysis involved simulating the BRC as if it had been issued on a monthly basis over the past two years. For consistency, the initial fixing date for each scenario was set as the 8th of the month, or the preceding business day if the 8th was not a trading day. This resulted in 24 distinct simulations, each reflecting the market environment prevailing at that time.

The simulation included some key steps, namely the extraction of the underlying asset price at the issuance date, the implied volatility and the risk-free rate. With the implied volatility and the risk-free rate, I used the same logic as the original simulation, but on the new initial fixing date and with the new initial stock price. However, as I went further back in time, the availability of bonds with maturities close to that of the product became non-existent and so, due to this constraint, I assumed the same IFC derived from the original issuance date for all 24 simulations. This assumption is not ideal, but it allows for a comparative assessment across different issuance periods.

Additionally, during these 2 years, another date for the dividends was included. Given the observation period, not only was considered the initial dividend date (May 7, 2025), but also the previous one, on May 8, 2024. I did not include any other dividend date, as the previous one was in 2019, not being relevant for the period being worked.

A summary of the parameters used is provided in the attachments.

After running all the 24 simulations, the outputs – expected value, probability of early redemption, probability of capital loss, and likelihood of maximum coupon – were recorded and compared.

TABLE 15: Valuation and key probabilities by issuing month

Issuing Month	Expected Value	Prob ER	Prob LM	Prob RMC	Delta	Expected Return
Oct-24	960.26 €	61.60%	28.68%	38.40%	34.59	-3.97%
Sep-24	959.30 €	61.08%	28.81%	38.92%	52.30	-4.07%
Aug-24	948.78 €	61.01%	30.67%	38.99%	60.33	-5.12%
Jul-24	962.34 €	64.55%	26.78%	35.45%	38.79	-3.77%
Jun-24	963.45 €	64.84%	26.37%	35.16%	45.72	-3.66%
May-24	958.85 €	64.70%	27.20%	35.30%	41.96	-4.12%
Apr-24	941.68 €	58.75%	33.01%	41.25%	50.80	-5.83%
Mar-24	941.46 €	58.10%	32.64%	41.90%	51.07	-5.85%
Feb-24	948.01 €	58.96%	31.40%	41.04%	34.97	-5.20%
Jan-24	941.04 €	60.29%	32.12%	39.71%	32.11	-5.90%
Dec-23	942.15 €	60.65%	31.47%	39.35%	35.37	-5.79%
Nov-23	948.30 €	60.73%	30.09%	39.27%	39.41	-5.17%
Oct-23	959.38 €	63.17%	28.28%	36.83%	34.63	-4.06%
Sep-23	964.72 €	64.14%	26.98%	35.86%	34.18	-3.53%
Aug-23	958.72 €	63.22%	28.04%	36.78%	36.04	-4.13%
Jul-23	958.49 €	65.14%	27.54%	34.86%	30.27	-4.15%
Jun-23	948.82 €	64.07%	29.51%	35.93%	30.69	-5.12%
May-23	942.75 €	63.74%	30.49%	36.26%	27.58	-5.73%
Apr-23	938.32 €	64.06%	30.77%	35.94%	23.04	-6.17%
Mar-23	946.46 €	65.03%	29.09%	34.97%	19.77	-5.35%
Feb-23	937.67 €	64.26%	30.48%	35.74%	24.20	-6.23%
Jan-23	936.05 €	64.89%	30.51%	35.11%	26.83	-6.40%
Dec-22	928.33 €	64.05%	31.73%	35.95%	31.63	-7.17%
Nov-22	898.51 €	62.08%	35.02%	37.92%	29.49	-10.15%
Oct-22	898.15 €	61.15%	35.90%	38.85%	33.58	-10.19%

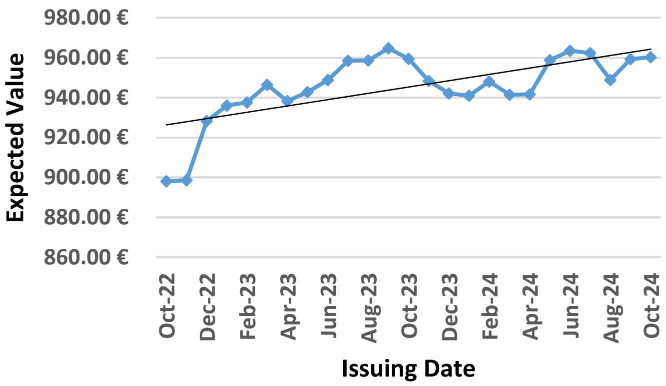

FIGURE 3: Expected value through time

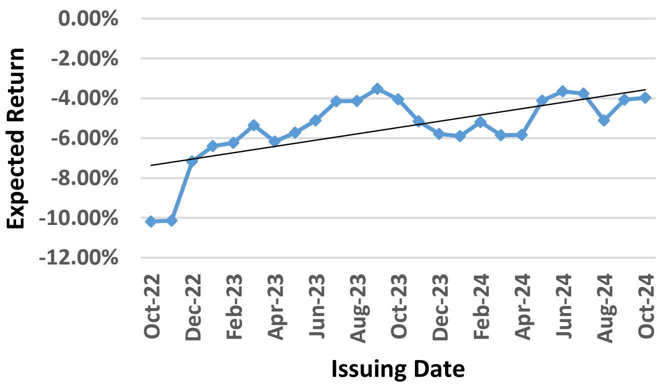

FIGURE 4: Expected return through time

The simulation results reveal several important trends regarding the sensitivity of the BRC to market conditions at issuance. The evolution of the expected value over time reveals significant variations driven by market conditions at each issuance date. Certain issuance dates exhibit higher expected values, reflecting more favorable market conditions, such as a higher initial asset price or lower implied volatility. The highest expected value was in September 2023 (964.72€), driven by a relatively high asset price and moderate volatility. Conversely, in periods of high volatility, the probability of barrier breaches increases, leading to a higher probability of loss and a corresponding decline in expected value, as it happens in October 2022, with an expected value of 898.15€, an initial asset price of 6.055€ and a volatility of 48.79%.

This relationship between volatility and expected value is further confirmed by other high-volatility months, where sharp increases in implied volatility led to reductions in the product's attractiveness. These volatility spikes amplify the downside risk by increasing the probability that the barrier will be breached, directly eroding the product's potential return profile.

The product's delta, which reflects the product's sensitivity to movements in the underlying asset, shows a strong correlation with expected value, reaching 60.33 in August 2024 and falling to just 19.77 in March 2023. This means the product becomes more reactive to market changes under favorable conditions and less responsive during riskier periods.

The probability of early redemption remains consistently above 60% in most scenarios, with the highest observed in July 2023 at 65.14%, reflecting favorable market conditions at issuance. The probability of capital loss generally increases during periods of higher volatility or lower expected values, peaking at 35.90% in October 2022.The probability of receiving the maximum coupon varies notably, with the highest values in February, March and April 2024, at 41.04%, 41.90% and 41.25% respectively, reflecting increased downside risk. In contrast, March and July 2023 show lower probabilities (34.97% and 34.86%), suggesting stable or bullish outlooks. This underscores the importance of issuance timing, as prevailing conditions significantly shape the product's appeal.

Expected returns are always negative, highlighting the cost of the embedded options and credit risk premiums within the product structure. October 2022 stands out with the worst expected return at -10.19%, while more favorable months such as September 2023 or June 2024 yield relatively modest losses of -3.53% and -3.66% respectively.

Overall, this rolling issuance analysis reinforces how sensitive structured products like BRCs are to issuance-time market conditions. Parameters such as volatility, asset price, and interest rates directly impact the product's valuation and risk. As such, careful timing can materially enhance both product attractiveness and investor outcomes.

11. CONCLUSION

This project explored the valuation, performance, and investor suitability of the "9.20% P.A. Barrier Reverse Convertible in EUR on Lufthansa", with particular emphasis on how market conditions at issuance influence the product's risk and return characteristics. Through a combination of quantitative modeling and historical simulations, this research provided a comprehensive analysis of a structured product whose complexity demands careful consideration from both issuers and investors.

To better understand the behavior of the BRC under changing market environments, a rolling historical issuance analysis was conducted. This involved simulating the product as if it had been launched at different points in time over a two-year period. By doing so, the analysis revealed how evolving factors such as the underlying asset price, implied volatility, and interest rates affected expected values, redemption probabilities, and downside risks. This dynamic approach underscored a critical insight into structured finance: context matters. The timing of issuance, shaped by prevailing market conditions, plays a defining role in determining the product's attractiveness and its performance profile.

The simulations confirmed that favorable issuance environments – characterized by high underlying asset prices and low volatility – correlated with increased expected values and a higher likelihood of early redemption, benefiting both issuers with reduced exposure and investors with reduced risk. Conversely, high volatility and uncertainty led to lower expected values and greater capital loss risk due to increased barrier breaches, reinforcing the importance of market timing and empirical analysis.

Valuation methodologies applied in this study provided complementary perspectives on the product's behavior. While the Binomial Tree model helped illustrate the structured payoff mechanics and allowed for intuitive interpretation of potential outcomes at each node, the Monte Carlo Simulation offered a more nuanced and realistic distribution of potential returns. The contrast between the two methods highlighted the trade-off between simplicity and realism. Together, they enabled a more holistic assessment of the BRC's valuation and risk profile, supporting better-informed structuring and pricing decisions.

The investor-focused component of this research emphasized that the BRC is most suitable for retail investors with a higher tolerance for risk and a solid understanding of structured financial products. While the fixed coupon provides an attractive yield and serves as a cushion during periods of mild market fluctuations, the exposure to the performance of Deutsche Lufthansa AG introduces substantial downside risks.

Moreover, the study stressed the importance of aligning investment decisions with investor profiles that go beyond conventional risk categorization. Factors such as the investor's market experience, familiarity with derivatives, and their financial objectives should be considered. Structured products like BRCs require not just risk appetite, but also a nuanced appreciation of how market dynamics interact with product features. This alignment is essential to ensure that investors can fully understand the product's benefits and risks, thereby enhancing the quality of their financial decisions. While the BRC offers attractive yields, the incorporation of hedging strategies could significantly enhance its appeal and safety.

In conclusion, this project reveals the intricate relationship between product design, market timing, valuation methodology, and investor suitability. Through rigorous simulation and analysis, this study supports the notion that informed and strategic decision-making – rooted in both quantitative modeling and market understanding – is essential for maximizing the benefits and minimizing the risks of such investments. Applying this framework to the January 15, 2025 valuation, specifically considering the probabilities of early redemption and its slight undervaluation, a buy recommendation is made. With a Monte Carlo estimated expected value of 960.91€ and 978.47€ against a market quotation of 946.10€, this undervaluation suggests an attractive entry point with potential for capital appreciation. The high probability of early redemption further enhances expected returns, especially if market volatility stabilizes. However, this recommendation is critically contingent on the investor's risk profile. As a complex, high-risk product with significant underlying exposure, it is best suited for investors with high risk tolerance, experience, and a strong understanding of structured product mechanics.

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APPENDICES

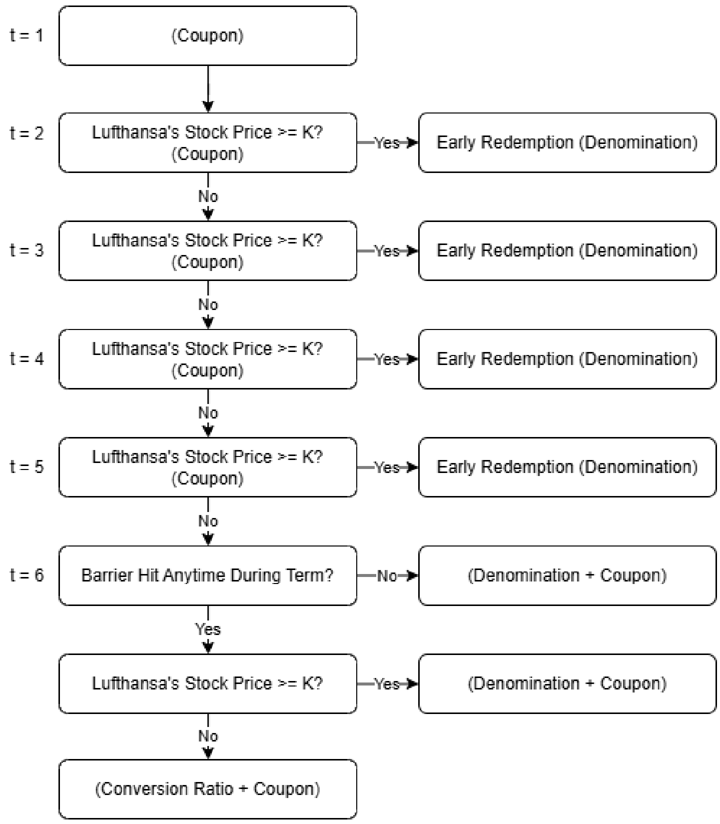

FIGURE 5: Flowchart of product's payoff

TABLE 16: Key parameters on January 15, 2025

Parameters	Value
Initial Price (S0)	5.726€
Strike Price (K)	5.726€
Barrier Level (B)	3.95€
Risk-Free Rate	2.249%
Volatility	28.80%
Conversion Ratio	174.642
Maturity (T)	1.25

TABLE 17: Key Monte Carlo Simulation outcomes

	MC Simulation	**MC Simulation GARCH**
ER 1st Date	4687	5080
ER 2nd Date	655	982
ER 3rd Date	464	599
ER 4th Date	354	417
No Barrier hit	851	506
Barrier Hit	3103	2731
Barrier Hit and St >= K	98	143
Barrier Hit and St < K	2891	2273

TABLE 18: Lufthansa's relevant dividend payments and yields

Ex-Dividend Date	**Dividend**	**Payment Date**	**Yield**
07 May 2025	0.30€	09 May 2025	4.13%
08 May 2024	0.30€	13 May 2024	4.40%

TABLE 19: Binomial Tree Model (6 steps): Paths and payoffs

Date	Oct-24	Jan-25	Apr-25	Jul-25	Oct-25	Jan-26	Apr-26
Time-Step	0	1	2	3	4	5	6
Year	0	0.25	0.5	0.75	1	1.25	1.5
	6.34 €	7.32 €	8.16 €				
	948.67 €	1,010.99 €	1,023.00 €				
		5.49 €	6.04 €	6.98 €			
		900.63 €	965.94 €	1,023.00 €			
			4.45 €	5.14 €	5.94 €	6.86 €	
			804.51 €	878.25 €	955.11 €	1,023.00 €	
				3.86 €	4.45 €	5.14 €	5.94 €
				699.45	771.07	857.17	960.00
					3.34	3.86	4.45
					595.24	654.09	725.46
						2.89	3.34
						501.84	549.62
							2.50 €
							417.80 €

TABLE 20: Binomial Tree Model (6 steps): Paths and payoffs on January 15, 2025

Date	Jan-25	Apr-25	Jul-25	Oct-25	Jan-26	Apr-26
Time-Step	0	1	2	3	4	5
Year	0	0.3	0.6	0.9	1.2	1.5
	5.73 €	6.41 €				
	960.85 €	1,023.00 €				
		4.59 €	5.38 €	6.29 €		
		870.88 €	948.87 €	1,023.00 €		
			3.92 €	4.59 €	5.38 €	6.29 €
			764.97 €	847.59 €	937.14 €	1,023.00 €
				3.35 €	3.92 €	4.59 €
				653.39 €	730.63 €	824.79 €
					2.86 €	3.35 €
					545.35 €	607.85 €
						2.44 €
						449.61 €

TABLE 21: Binomial Tree Model (6 steps): Probabilities

Date	Oct-24	Jan-25	Apr-25	Jul-25	Oct-25	Jan-26	Apr-26
Time-Step	0	1	2	3	4	5	6
Year	0	0.25	0.5	0.75	1	1.25	1.5
	6.34 €	7.32 €	8.16 €				
		48.34%	23.37%				
		5.49 €	6.04 €	6.98 €			
		51.66%	49.95%	24.15%			
			4.45 €	5.14 €	5.94 €	6.86 €	
			26.68%	38.70%	18.71%	9.04%	
				3.86 €	4.45 €	5.14 €	5.94 €
				13.78%	26.66%	22.55%	10.90%
					3.34 €	3.86 €	4.45 €
					7.12%	17.21%	19.97%
						2.89 €	3.34 €
						3.68%	10.67%
							2.50 €
Total	**1.00**						1.90%

TABLE 22: Binomial Tree Model (18 steps): Paths and payoffs

Date	Oct-24	Nov-24	Dec-24	Jan-25	Feb-25	Mar-25	Apr-25	May-25	Jun-25	Jul-25	Aug-25	Sep-25	Oct-25	Nov-25	Dec-25	Jan-26	Feb-26	Mar-26	Apr-26
Time-Step	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Year	0.00	0.08	0.17	0.25	0.33	0.42	0.50	0.58	0.67	0.75	0.83	0.92	1.00	1.08	1.17	1.25	1.33	1.42	1.50
	6.34 €	6.89 €	7.49 €	8.14 €	8.84 €	9.61 €	10.44 €												
	963.87 €	1,001.43 €	1,027.70 €	1,040.30 €	1,019.20 €	1,021.10 €	1,023.00 €												
		5.83 €	6.34 €	6.89 €	7.49 €	8.14 €	8.84 €												
		931.24 €	979.81 €	1,019.34 €	1,019.20 €	1,021.10 €	1,023.00 €												
			5.37 €	5.83 €	6.34 €	6.89 €	7.49 €												
			887.90 €	945.35 €	977.99 €	1,021.10 €	1,023.00 €												
				4.94 €	5.37 €	5.83 €	6.34 €												
				835.87 €	872.18 €	940.08 €	1,023.00 €												
					4.55 €	4.94 €	5.37 €	5.53 €	6.01 €	6.54 €									
					758.76 €	810.01 €	863.73 €	896.95 €	956.67 €	1,023.00 €									
						4.18 €	4.55 €	4.64 €	5.04 €	5.48 €	5.95 €	6.47 €	7.03 €						
						712.21 €	761.28 €	789.70 €	842.76 €	896.34 €	925.93 €	976.06 €	1,023.00 €						
							3.85 €	3.88 €	4.22 €	4.59 €	4.98 €	5.42 €	5.89 €	6.40 €	6.95 €	7.55 €			
							667.59 €	691.50 €	741.52 €	794.27 €	825.92 €	881.07 €	934.46 €	959.70 €	998.47 €	1,023.00 €			
								3.24 €	3.52 €	3.83 €	4.16 €	4.52 €	4.91 €	5.34 €	5.80 €	6.31 €	6.85 €	7.45 €	8.09 €
								601.80 €	645.89 €	693.47 €	721.51 €	775.86 €	832.91 €	868.37 €	925.90 €	978.52 €	997.28 €	1,021.10 €	1,023.00 €
									2.98 €	3.24 €	3.52 €	3.83 €	4.16 €	4.52 €	4.91 €	5.34 €	5.80 €	6.31 €	6.85 €
									561.56 €	602.47 €	623.81 €	671.85 €	723.78 €	756.62 €	816.18 €	878.64 €	918.82 €	978.02 €	1,023.00 €
										2.75 €	2.98 €	3.24 €	3.52 €	3.83 €	4.16 €	4.52 €	4.91 €	5.34 €	5.80 €
										524.24 €	538.92 €	579.85 €	624.33 €	649.61 €	702.07 €	759.07 €	797.96 €	865.22 €	938.30 €
											2.53 €	2.75 €	2.98 €	3.24 €	3.52 €	3.83 €	4.16 €	4.52 €	4.91 €
											466.81 €	501.49 €	539.17 €	557.07 €	601.50 €	649.78 €	679.19 €	736.15 €	798.04 €
												2.32 €	2.53 €	2.75 €	2.98 €	3.24 €	3.52 €	3.83 €	4.16 €
												435.14 €	467.07 €	478.71 €	516.34 €	557.24 €	578.63 €	626.86 €	679.28 €
													2.14 €	2.32 €	2.53 €	2.75 €	2.98 €	3.24 €	3.52 €
													406.01 €	412.35 €	444.24 €	478.88 €	493.47 €	534.32 €	578.71 €
														1.97 €	2.14 €	2.32 €	2.53 €	2.75 €	2.98 €
														356.17 €	383.18 €	412.52 €	421.36 €	455.96 €	493.56 €
															1.81 €	1.97 €	2.14 €	2.32 €	2.53 €
															331.48 €	356.34 €	360.31 €	389.61 €	421.45 €
																1.67 €	1.81 €	1.97 €	2.14 €
																308.76 €	308.61 €	333.43 €	360.39 €
																	1.53 €	1.67 €	1.81 €
																	264.83 €	285.85 €	308.69 €
																		1.41 €	1.53 €
																		245.56 €	264.91 €
																			1.30 €
																			227.84 €

TABLE 23: Binomial Tree model (18 steps): Paths and payoffs on January 15, 2025

Date	Jan-25	Feb-25	Mar-25	Apr-25	May-25	Jun-25	Jul-25	Aug-25	Sep-25	Oct-25	Nov-25	Dec-25	Jan-26	Feb-26	Mar-26	Apr-26
Time-Step	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Year	0.00	0.08	0.17	0.25	0.33	0.42	0.50	0.58	0.67	0.75	0.83	0.92	1.00	1.08	1.17	1.25
	5.73 €	6.27 €	6.87 €	7.53 €												
	942.62 €	988.24 €	1,020.70 €	1,023.00 €												
		5.23 €	5.73 €	6.27 €												
		903.03 €	961.45 €	1,023.00 €												
			4.77 €	5.23 €	5.43 €	5.94 €	6.51 €									
			850.96 €	906.66 €	937.21 €	985.22 €	1,023.00 €									
				4.36 €	4.47 €	4.90 €	5.37 €	5.88 €	6.44 €	7.05 €						
				801.29 €	836.19 €	895.29 €	953.32 €	982.83 €	1,020.70 €	1,023.00 €						
					3.68 €	4.03 €	4.41 €	4.83 €	5.29 €	5.80 €						
					726.19 €	783.19 €	843.59 €	884.05 €	950.85 €	1,023.00 €						
						3.36 €	3.68 €	4.03 €	4.41 €	4.83 €	5.29 €	5.80 €	6.35 €			
						674.71 €	728.72 €	763.33 €	823.87 €	885.83 €	923.52 €	978.80 €	1,023.00 €			
							3.07 €	3.36 €	3.68 €	4.03 €	4.41 €	4.83 €	5.29 €	5.80 €	6.35 €	6.96 €
							625.88 €	653.57 €	708.63 €	768.07 €	808.43 €	874.57 €	940.71 €	977.98 €	1,020.70 €	1,023.00 €
								2.80 €	3.07 €	3.36 €	3.68 €	4.03 €	4.41 €	4.83 €	5.29 €	5.80 €
								556.93 €	603.64 €	654.75 €	687.58 €	748.55 €	814.98 €	863.95 €	941.31 €	1,023.00 €
									2.55 €	2.80 €	3.07 €	3.36 €	3.68 €	4.03 €	4.41 €	4.83 €
									514.58 €	557.27 €	580.98 €	632.13 €	688.14 €	726.43 €	793.57 €	867.10 €
										2.33 €	2.55 €	2.80 €	3.07 €	3.36 €	3.68 €	4.03 €
										475.89 €	491.83 €	534.48 €	581.18 €	609.28 €	665.24 €	726.54 €
											2.13 €	2.33 €	2.55 €	2.80 €	3.07 €	3.36 €
											417.53 €	453.09 €	492.04 €	511.63 €	558.29 €	609.38 €
												1.94 €	2.13 €	2.33 €	2.55 €	2.80 €
												385.26 €	417.74 €	430.25 €	469.14 €	511.74 €
													1.77 €	1.94 €	2.13 €	2.33 €
													355.81 €	362.42 €	394.84 €	430.35 €
														1.62 €	1.77 €	1.94 €
														305.88 €	332.91 €	362.52 €
															1.48 €	1.62 €
															281.30 €	305.98 €
																1.35 €
																258.86 €

TABLE 24: Binomial Tree model (18 steps): Probabilities

Date	Oct-24	Nov-24	Dec-24	Jan-25	Feb-25	Mar-25	Apr-25	May-25	Jun-25	Jul-25	Aug-25	Sep-25	Oct-25	Nov-25	Dec-25	Jan-26	Feb-26	Mar-26	Apr-26
Time-Step	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Year	0.00	0.08	0.17	0.25	0.33	0.42	0.50	0.58	0.67	0.75	0.83	0.92	1.00	1.08	1.17	1.25	1.33	1.42	1.50
	6.34 €	6.89 €	7.49 €	8.14 €	8.84 €	9.61 €	10.44 €												
		49.041%	24.051%	11.795%	5.784%	2.837%	1.391%												
		5.83 €	6.34 €	6.89 €	7.49 €	8.14 €	8.84 €												
		50.959%	49.982%	36.768%	24.042%	14.738%	8.673%												
			5.37 €	5.83 €	6.34 €	6.89 €	7.49 €												
			25.968%	38.205%	37.472%	30.628%	22.531%												
				4.94 €	5.37 €	5.83 €	6.34 €												
				13.233%	25.958%	31.826%	31.216%												
					4.55 €	4.94 €	5.37 €	5.53 €	6.01 €	6.54 €									
					6.743%	16.535%	24.327%	11.930%	5.851%	2.869%									
						4.18 €	4.55 €	4.64 €	5.04 €	5.48 €	5.95 €	6.47 €	7.03 €						
						3.436%	10.111%	17.355%	14.591%	10.137%	4.971%	2.438%	1.196%						
							3.85 €	3.88 €	4.22 €	4.59 €	4.98 €	5.42 €	5.89 €	6.40 €	6.95 €	7.55 €			
							1.751%	6.011%	11.792%	13.218%	11.648%	8.246%	5.286%	2.592%	1.271%	0.623%			
								3.24 €	3.52 €	3.83 €	4.16 €	4.52 €	4.91 €	5.34 €	5.80 €	6.31 €	6.85 €	7.45 €	8.09 €
								0.892%	3.501%	7.726%	10.525%	11.097%	9.644%	7.423%	4.962%	3.081%	1.511%	0.741%	0.363%
									2.98 €	3.24 €	3.52 €	3.83 €	4.16 €	4.52 €	4.91 €	5.34 €	5.80 €	6.31 €	6.85 €
									0.455%	2.007%	4.921%	7.777%	9.469%	9.558%	8.470%	6.682%	4.847%	3.147%	1.921%
										2.75 €	2.98 €	3.24 €	3.52 €	3.83 €	4.16 €	4.52 €	4.91 €	5.34 €	5.80 €
										0.232%	1.136%	3.065%	5.466%	7.506%	8.552%	8.510%	7.579%	6.187%	4.638%
											2.53 €	2.75 €	2.98 €	3.24 €	3.52 €	3.83 €	4.16 €	4.52 €	4.91 €
											0.118%	0.637%	1.874%	3.705%	5.642%	7.124%	7.831%	7.702%	6.930%
												2.32 €	2.53 €	2.75 €	2.98 €	3.24 €	3.52 €	3.83 €	4.16 €
												0.060%	0.354%	1.129%	2.441%	4.072%	5.628%	6.750%	7.235%
													2.14 €	2.32 €	2.53 €	2.75 €	2.98 €	3.24 €	3.52 €
													0.031%	0.195%	0.671%	1.573%	2.847%	4.264%	5.531%
														1.97 €	2.14 €	2.32 €	2.53 €	2.75 €	2.98 €
														0.016%	0.107%	0.395%	0.995%	1.939%	3.123%
															1.81 €	1.97 €	2.14 €	2.32 €	2.53 €
															0.008%	0.059%	0.230%	0.620%	1.292%
																1.67 €	1.81 €	1.97 €	2.14 €
																0.004%	0.032%	0.133%	0.381%
																	1.53 €	1.67 €	1.81 €
																	0.002%	0.017%	0.076%
																		1.41 €	1.53 €
																		0.001%	0.009%
																			1.30 €
																			0.001%
Total		1.00																	

TABLE 25: Further analysis parameters

Initial Fixing Date	Final Fixing Date	First Exchange Trading Date	Last Trading Day	Redemption Date	RF Yield	Volatility	S0	K	B	Denomination	Coupon	Conversion Ratio	IFC
08-Oct-24	08-Apr-26	10-Oct-24	08-Apr-26	10-Apr-26	2.24%	28.81%	6.34 €	6.34 €	4.37 €	1,000.00 €	23.00 €	157.73 €	3.15%
06-Sep-24	08-Mar-26	10-Sep-24	08-Mar-26	10-Mar-26	2.23%	28.44%	5.79 €	5.79 €	4.00 €	1,000.00 €	23.00 €	172.71 €	3.15%
08-Aug-24	08-Feb-26	10-Aug-24	08-Feb-26	10-Feb-26	2.40%	30.53%	5.58 €	5.58 €	3.85 €	1,000.00 €	23.00 €	179.28 €	3.15%
08-Jul-24	08-Jan-26	10-Jul-24	08-Jan-26	10-Jan-26	2.91%	28.87%	6.06 €	6.06 €	4.18 €	1,000.00 €	23.00 €	164.96 €	3.15%
07-Jun-24	08-Dec-25	10-Jun-24	08-Dec-25	10-Dec-25	3.09%	28.82%	6.30 €	6.30 €	4.35 €	1,000.00 €	23.00 €	158.73 €	3.15%
08-May-24	08-Nov-25	10-May-24	08-Nov-25	10-Nov-25	2.93%	29.92%	6.50 €	6.50 €	4.48 €	1,000.00 €	23.00 €	153.94 €	3.15%
08-Apr-24	08-Oct-25	10-Apr-24	08-Oct-25	10-Oct-25	2.93%	29.80%	7.12 €	7.12 €	4.92 €	1,000.00 €	23.00 €	140.37 €	3.15%
08-Mar-24	08-Sep-25	10-Mar-24	08-Sep-25	10-Sep-25	2.74%	29.33%	6.82 €	6.82 €	4.70 €	1,000.00 €	23.00 €	146.69 €	3.15%
08-Feb-24	08-Aug-25	10-Feb-24	08-Aug-25	10-Aug-25	2.67%	29.08%	7.66 €	7.66 €	5.29 €	1,000.00 €	23.00 €	130.53 €	3.15%
08-Jan-24	08-Jul-25	10-Jan-24	08-Jul-25	10-Jul-25	2.55%	30.59%	7.93 €	7.93 €	5.47 €	1,000.00 €	23.00 €	126.17 €	3.15%
08-Dec-23	08-Jun-25	10-Dec-23	08-Jun-25	10-Jun-25	2.69%	30.66%	8.36 €	8.36 €	5.77 €	1,000.00 €	23.00 €	119.56 €	3.15%
08-Nov-23	08-May-25	10-Nov-23	08-May-25	10-May-25	3.07%	29.62%	7.67 €	7.67 €	5.29 €	1,000.00 €	23.00 €	130.36 €	3.15%
06-Oct-23	08-Apr-25	10-Oct-23	08-Apr-25	10-Apr-25	3.13%	30.10%	7.40 €	7.40 €	5.11 €	1,000.00 €	23.00 €	135.08 €	3.15%
08-Sep-23	08-Mar-25	10-Sep-23	08-Mar-25	10-Mar-25	3.07%	29.14%	8.07 €	8.07 €	5.57 €	1,000.00 €	23.00 €	123.92 €	3.15%
08-Aug-23	08-Feb-25	10-Aug-23	08-Feb-25	10-Feb-25	3.09%	30.79%	8.60 €	8.60 €	5.93 €	1,000.00 €	23.00 €	116.31 €	3.15%
07-Jul-23	08-Jan-25	10-Jul-23	08-Jan-25	10-Jan-25	3.31%	31.36%	9.02 €	9.02 €	6.22 €	1,000.00 €	23.00 €	110.90 €	3.15%
08-Jun-23	08-Dec-24	10-Jun-23	08-Dec-24	10-Dec-24	2.97%	33.39%	9.14 €	9.14 €	6.30 €	1,000.00 €	23.00 €	109.46 €	3.15%
08-May-23	08-Nov-24	10-May-23	08-Nov-24	10-Nov-24	2.66%	34.51%	9.18 €	9.18 €	6.33 €	1,000.00 €	23.00 €	108.98 €	3.15%
06-Apr-23	08-Oct-24	11-Apr-23	08-Oct-24	10-Oct-24	2.54%	36.02%	10.27 €	10.27 €	7.09 €	1,000.00 €	23.00 €	97.33 €	3.15%
08-Mar-23	08-Sep-24	10-Mar-23	08-Sep-24	10-Sep-24	3.34%	34.91%	10.93 €	10.93 €	7.54 €	1,000.00 €	23.00 €	91.46 €	3.15%
08-Feb-23	08-Aug-24	10-Feb-23	08-Aug-24	10-Aug-24	2.71%	36.68%	9.68 €	9.68 €	6.68 €	1,000.00 €	23.00 €	103.34 €	3.15%
06-Jan-23	08-Jul-24	10-Jan-23	08-Jul-24	10-Jul-24	2.59%	36.27%	8.32 €	8.32 €	5.74 €	1,000.00 €	23.00 €	120.18 €	3.15%
08-Dec-22	08-Jun-24	10-Dec-22	08-Jun-24	10-Jun-24	2.07%	37.58%	7.71 €	7.71 €	5.32 €	1,000.00 €	23.00 €	129.67 €	3.15%
08-Nov-22	08-May-24	10-Nov-22	08-May-24	10-May-24	2.21%	45.43%	7.22 €	7.22 €	4.98 €	1,000.00 €	23.00 €	138.60 €	3.15%
07-Oct-22	08-Apr-24	10-Oct-22	08-Apr-24	10-Apr-24	1.87%	48.79%	6.06 €	6.06 €	4.18 €	1,000.00 €	23.00 €	165.15 €	3.15%

```
import numpy as np
import pandas as pd
from pandas.tseries.offsets import BDay
import matplotlib.pyplot as plt

# Parameters
S0=6.34
K=S0
B=0.69*S0
r=0.02235
sigma=0.2881
n_sim=10000
denomination=1000
coupon=23
conversion_ratio=denomination/S0
dividend_value=0.30
ifc=0.03148

# Dates
start_date=pd.to_datetime("2024-10-10")
final_fixing_date=pd.to_datetime("2026-04-08")
redemption_date=pd.to_datetime("2026-04-10")
business_days_fixing=pd.bdate_range(start=start_date,end=final_fixing_date)
business_days_total=pd.bdate_range(start=start_date,end=redemption_date)
T=len(business_days_total)/252
n_steps=len(business_days_fixing)
dt=T/n_steps

print(f"Total Time to Rescue (T):{T:.4f} years")
print(f"Number of Time Steps (n_steps):{n_steps}")
print(f"Size of Each Time Step (dt):{dt:.6f} years")

def next_business_day(date):
    future_days=business_days_fixing[business_days_fixing>date]
    return date if date in business_days_fixing else future_days[0] if not future_days.empty else business_days_fixing[-1]
early_redemption_observation_dates=pd.to_datetime(["2025-04-08","2025-07-08","2025-10-08","2026-01-08"]).map(next_business_day)
early_redemption_payment_dates=pd.to_datetime(["2025-04-15","2025-07-15","2025-10-15","2026-01-15"]).map(next_business_day)
coupon_payment_dates=pd.to_datetime(["2025-01-15","2025-04-15","2025-07-15","2025-10-15","2026-01-15","2026-04-10"]).map(next_business_day)
ex_dividend_date=pd.to_datetime("2025-05-07")
```

```
# Monte Carlo Simulation
def monte_carlo_sim(S0):
    np.random.seed(42)
    all_paths=[]
    payoffs=np.zeros(n_sim)
    discounted_payoffs=np.zeros(n_sim)
    final_prices=np.zeros(n_sim)
    early_redemption_count=np.zeros(len(early_redemption_observation_dates))
    barrier_hit=np.zeros(n_sim,dtype=bool)
    early_redeemed=np.zeros(n_sim,dtype=bool)
    maturity_no_early_redemption=0
    loss_count=0
    barrier_hit_and_St_below_K=0
    barrier_hit_and_St_above_K=0
    no_barrier_hit=0
    barrier_hit_count=0

    for i in range(n_sim):
        St=np.zeros(n_steps+1)
        St[0]=S0
        discounted_coupons=0.0
        path=[S0]

        for t in range(1,n_steps+1):
            Z=np.random.normal(0,1)
            St[t]=St[t-1]*np.exp((r-0.5*sigma**2)*dt+sigma*np.sqrt(dt)*Z)
            path.append(St[t])

            current_date=business_days_fixing[t-1]
            if current_date==ex_dividend_date:
                St[t]-=dividend_value

            # Check if Barrier was Hit
            if St[t]<=B and not barrier_hit[i]:
                barrier_hit[i]=True
                barrier_hit_count+=1
```

```
            # Coupons if not Early Redeemed
            if current_date in coupon_payment_dates[:-1] and not early_redeemed[i]:
                time_to_coupon=(current_date-start_date).days/365
                discount_factor=np.exp(-ifc*time_to_coupon)
                discounted_coupons+=(coupon*discount_factor)

            # Early Redemption
            if current_date in early_redemption_observation_dates and St[t]>=S0 and not early_redeemed[i]:
                payment_date_index=early_redemption_observation_dates.get_loc(current_date)
                payment_date=early_redemption_payment_dates[payment_date_index]
                time_to_payment=(payment_date-start_date).days/365
                discount_factor=np.exp(-ifc*time_to_payment)

                payoff=denomination+coupon
                discounted_payoffs[i]=discounted_coupons+(payoff*discount_factor)
                payoffs[i]=payoff

                early_redemption_count[payment_date_index]+=1
                early_redeemed[i]=True
                final_prices[i]=St[t]
                all_paths.append(np.array(path))
                break

        if not early_redeemed[i]:
            maturity_no_early_redemption+=1
            time_to_redemption=(redemption_date-start_date).days/365
            discount_factor=np.exp(-ifc*time_to_redemption)
            if not barrier_hit[i]:
                payoff=denomination+coupon
                no_barrier_hit+=1
            elif St[-1]>=K:
                payoff=denomination+coupon
                barrier_hit_and_St_above_K+=1
            else:
                payoff=conversion_ratio*St[-1]+coupon
                barrier_hit_and_St_below_K+=1

            discounted_payoffs[i]=discounted_coupons+(payoff*discount_factor)
            payoffs[i]=payoff
            all_paths.append(np.array(path))
            final_prices[i]=St[-1]
```

FIGURE 6: Python code: Monte Carlo Simulation (Part 1)

```
        if payoffs[i]<denomination:
            loss_count+=1

    expected_value=np.mean(discounted_payoffs)
    prob_early_redemption=np.sum(early_redemption_count)/n_sim
    prob_loss=loss_count/n_sim
    prob_receive_max_coupon=maturity_no_early_redemption/n_sim
    std_dev_discounted_payoffs=np.std(discounted_payoffs)
    individual_returns=(discounted_payoffs/denomination-1)*100
    std_dev_returns=np.std(individual_returns)

    return (
        expected_value,
        final_prices,
        payoffs,
        discounted_payoffs,
        all_paths,
        early_redemption_count,
        prob_early_redemption,
        prob_loss,
        prob_receive_max_coupon,
        barrier_hit_count,
        barrier_hit_and_St_above_K,
        barrier_hit_and_St_below_K,
        no_barrier_hit,
        std_dev_discounted_payoffs,
        std_dev_returns
    )
(
    expected_value,
    final_prices,
    payoffs,
    discounted_payoffs,
    all_paths,
    early_redemption_count,
    prob_early_redemption,
    prob_loss,
    prob_receive_max_coupon,
    barrier_hit_count,
    barrier_hit_and_St_above_K,
    barrier_hit_and_St_below_K,
    no_barrier_hit,
    std_dev_discounted_payoffs,
    std_dev_returns
)=monte_carlo_sim(S0)
```

```
# Delta
delta_change=0.01
expected_value_original,_,_,_,_,_,_,_,_,_,_,_,_,_,_=monte_carlo_sim(S0)
expected_value_delta,_,_,_,_,_,_,_,_,_,_,_,_,_,_=monte_carlo_sim(S0*(1+delta_change))
delta=((expected_value_delta-expected_value_original)/(S0*delta_change))
print(f"Delta:{delta:.4f}")
```

FIGURE 7: Python code: Monte Carlo Simulation (Part 2)

```
import numpy as np
import pandas as pd
from pandas.tseries.offsets import BDay
import matplotlib.pyplot as plt
from arch import arch_model
import yfinance as yf

# Download of Historical Data from Lufthansa, using Yahoo Finance (LHA.DE)
ticker='LHA.DE'
data=yf.download(ticker,start="2021-10-08",end="2024-10-08")
data['Log Ret']=np.log(data['Close']/data['Close'].shift(1))
returns=data['Log Ret'].dropna()

# Estimating GARCH(1,1)
model=arch_model(returns,vol='Garch',p=1,q=1)
garch_fit=model.fit(disp="off")

# GARCH Parameters
omega=garch_fit.params['omega']
alpha=garch_fit.params['alpha[1]']
beta=garch_fit.params['beta[1]']

print(f"GARCH Parameters: Omega={omega:.6f}, Alpha={alpha:.6f}, Beta={beta:.6f}")
```

```
# Function to Simulate Price Paths and Volatility
def simulate_garch_vol_paths(n_paths,n_steps,S0,r,omega,alpha,beta):
    simulated_prices=[]
    simulated_vols=[]
    np.random.seed(42)

    for _ in range(n_paths):
        prices=np.zeros(n_steps+1)
        prices[0]=S0
        vols=np.zeros(n_steps+1)
        h=np.zeros(n_steps)
        h[0]=omega/(1-alpha-beta)
        vols[0]=np.sqrt(h[0])
        returns=np.zeros(n_steps)

        Z=np.random.normal(size=n_steps)

        for t in range(1,n_steps+1):
            h[t-1]=omega+alpha*(returns[t-2]**2 if t>=2 else 0)+beta*h[t-2] if t>=2 else h[0]
            sigma_t=np.sqrt(h[t-1])
            vols[t]=sigma_t
            ret=(r-0.5*sigma_t**2)*dt+sigma_t*Z[t-1]
            returns[t-1]=ret
            prices[t]=prices[t-1]*np.exp(ret)

        simulated_prices.append(prices)
        simulated_vols.append(vols)

    return simulated_prices,simulated_vols
```

FIGURE 8: Python code: GARCH