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DISSERTATION

GREEN INVESTMENT WITH CAPACITY CHOICE UNDER SUBSIDY RETRACTION RISK

ANA MARGARIDA CONCEIÇÃO CENTENO

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SUPERVISION:

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Glossary

CfD Contract for Difference.

FIP Feed-in Premium.

FIT Feed-in Tariff.

GBM Geometric Brownian Motion.

HJB Hamilton-Jacobi-Bellman.

JEL Journal of Economic Literature.

NPV Net Present Value.

ODE Ordinary Differential Equation.

RE Renewable Energy.

ABSTRACT

Investments in renewable energy (RE) projects have received considerable support from governments to accelerate the energy transition. However, government support also creates uncertainty for investors, as it can be changed or withdrawn at any time. This dissertation studies the investment decision of a firm planning to develop a RE project, determining the optimal investment timing and the optimal capacity to be installed. To address this problem, we develop a real options model with a feed-in tariff (FIT) subsidy, in which the subsidy withdrawal time is modeled as an exponentially distributed random variable.

The investment problem is solved analytically and the results are illustrated through a numerical example and a comparative statics analysis. The results show that subsidies do not necessarily accelerate investment. Instead, a critical level of subsidy is needed to accelerate investment. Subsidies below this level delay investment compared to the unsubsidized case. Although high levels of subsidy reduce the optimal installed capacity, low levels of subsidy increase capacity. The impact of subsidy withdrawal risk also depends on the level of financial support. For sufficiently high subsidy levels, an increase in the probability of withdrawal raises the optimal investment threshold, while installed capacity has a non-monotonic behavior. Both remain below the unsubsidized case. For lower subsidy levels, both the optimal investment threshold and installed capacity have non-uniform behaviors as the risk of subsidy withdrawal increases.

This study provides important contributions for investors and policy makers, demonstrating that the effectiveness of support policies depends not only on the level of subsidy, but also on its credibility and predictability.

KEYWORDS: Renewable energy; Real options; Feed-in tariffs; Policy uncertainty; Investment timing; Capacity installed.

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Green Investment with Capacity Choice under Subsidy Retraction Risk

By Ana Centeno

1 Introduction

The increase in greenhouse gas emissions and the limited availability of non-renewable natural resources have led governments to implement policies and establish ambitious targets to create a more sustainable energy system. The European Commission (2025) has set the target of increasing the share of renewable energy (RE) to reduce net greenhouse gas emissions by at least 55% by 2030. To further support this transition, the Renewable Energy Directive was revised in 2023, raising the European Union's binding target for the share of renewable energy in the energy mix to at least 42.5% by 2030 (European Commission (2025)). To achieve these objectives, governments have introduced a variety of policy instruments aimed at encouraging investment in RE projects, including feed-in tariffs (FITs), feed-in premiums (FIPs), green certificates and, more recently, contracts for difference (CfDs). Among these mechanisms, FITs have been one of the most widely adopted support schemes, providing RE producers with stable and predictable revenues. According to Boomsma & Linnerud (2015), the FIT is the least risky support scheme for the investor. Also, FITs encourage earlier investment, when compared with FIPs and green certificates (Boomsma et al. (2012)). Although many countries have gradually shifted from FITs to CfDs, both mechanisms are designed to provide revenue certainty and reduce investment risk. In 2024, 44 countries implemented FITs, including Hungary, Belgium and the Netherlands (REN21 (2025)).

Over time, RE support policies go through numerous changes and, in some cases, subsidies are reduced or withdrawn. Several factors may explain such changes in subsidy schemes, including technological progress. As RE technologies mature and production costs decline, projects may become sufficiently competitive to remain profitable without support. Subsidy reductions can also result from the achievement of policy targets, changes in government priorities and budget constraints. The Czech Republic provides a classic example. Following a rapid expansion of photovoltaic installations driven by generous FITs, the government progressively revised its support scheme, excluding FITs for most new RE installations in 2013. This measure was taken in response to the sharply increasing costs of the support scheme and its impact on electricity prices (Gürtler et al. (2019)). Another example is Spain, where the government significantly reduced RE subsidies from 2008 onwards in an attempt to lower the costs of support and furthermore to address the growing tariff deficit of the electricity sector (del Río & Mir-Artigues (2012)). More recently, Germany announced in 2025 its intention to phase out FITs for

RE projects, arguing that the sector had become sufficiently mature to compete under market conditions (Alkousaa (2025)). These policy changes can significantly affect the profitability of RE projects, making policy instability one of the main barriers to investment in the sector (White et al. (2013)).

In this project, we consider a firm that has the option to invest in a RE project and wants to determine the optimal time to invest and the capacity to install in order to maximize its profit. The investment cost, as well as the revenue, depends on the installed capacity. After investment, the firm receives a fixed FIT per unit of electricity produced, avoiding exposure to market price uncertainty during the subsidized period. Once the subsidy is retracted, electricity is sold at the market price.

The net present value (NPV), which is defined as the sum of all discounted inflows minus the costs, provides a now-or-never investment rule. This becomes inadequate when investments are totally or partially irreversible and there is flexibility to postpone the investment decision. By investing immediately, the firm gives up the option of waiting for new information, knowing that future market conditions can be more favorable (Dixit & Pindyck (1994)). Applying a real option approach allows us to value the flexibility associated with investment timing under uncertainty and obtain investment rules based on economic indicators (Dixit & Pindyck (1994)).

Real options analysis has been adopted for evaluating investment opportunities under uncertainty, as it explicitly accounts for managerial flexibility, which is essential in uncertain and dynamic environments (Trigeorgis (2002)). The flexibility that comes from delaying irreversible investment decisions until new additional information enables the firm to capture the upside potential of a project while limiting downside losses (Trigeorgis (2002)). As a result, waiting acquires economic value, implying that greater uncertainty increases the value of postponing investment. Thus, firms require more favorable economic conditions before investing, leading to the classical real options result that higher uncertainty delays irreversible investment decisions (Dangl (1999)). The real options theory has been used in many contexts and subsequent contributions have additionally determined the optimal size of the investment (Huberts et al. (2015)). Some authors, like Dangl (1999), show that the optimal installed capacity increases with uncertainty as well as with investment timing. Thereby, the standard real options result, stating that if uncertainty goes up the firm delays its investment, is modified to state that if uncertainty increases, a firm will invest later in a larger capacity.

Dixit & Pindyck (1994) and Hassett & Metcalf (1999) were some of the first to study regulatory uncertainty, considering the effect of subsidy size on investment timing. Subsequent contributions related to real options have examined policy uncertainty arising from the provision, revision, or retraction of RE subsidies. In this context, Boomsma et al. (2012), Adkins & Paxson (2013), Boomsma & Linnerud (2015), Ritzenhofen & Spinler (2016), and Chronopoulos et al. (2016) analyze how uncertainty regarding support schemes affects RE investment decisions. For instance, Ritzenhofen & Spinler (2016) studied how the FITs influence the investment decision and concluded that a high subsidy triggers investment while regulatory uncertainty only affects investment behavior for FIT levels near electricity spot market prices and for high probabilities of a regime switch. Adkins & Paxson (2013) state that the effect of policy uncertainty depends a lot on the subsidy scheme considered. More recently, Hagspiel et al. (2021) analyze RE investment under the risk of retraction of a FIT, which depends on time. Their model determines the optimal investment threshold for a project with fixed capacity and shows that it is optimal to invest for a certain range of FIT once the investment decision is considered. Otherwise, the firm should wait for the subsidy retraction. They also find that it is not clear if the investment incentive is higher for a time-dependent subsidy retraction risk when compared to a constant one. It depends on how long the subsidy has been active. Nagy et al. (2021) analyze RE investments under the risk of a lump-sum subsidy withdrawal within a real options framework. The subsidy can be withdrawn unexpectedly and both the optimal timing of investment and the installed capacity are determined. Their results show that a higher probability of subsidy withdrawal accelerates investment and leads to smaller project sizes.

In this thesis we develop a model in which a firm wants to invest in a RE project in a subsidized market, knowing that the support can be removed at any moment with a constant probability. We contribute to the literature with some new results. We found that subsidies do not always accelerate investment, only above a certain level, and they can in fact, delay investment. We also found that the impact of the risk of subsidy retraction is not uniform, it depends on the subsidy level. Therefore, it can generate different results, both for the optimal timing of investment and the installed capacity.

The remainder of this paper is structured as follows. Section 2 presents the investment model and its solution procedure is exhibited in Section 3. In Section 4 we present a numerical example and illustrate the comparative statics of the model. Finally, in Section 5 we summarize the main results and conclude.

2 MODEL

We introduce a model to study the optimal timing and capacity choice in order to maximize the firm's value when investing in a RE project. We consider a firm that intends to invest in an electricity production project. Since the electricity market is highly concentrated, large producers may exert some degree of market power. Following Nagy et al. (2021), it is assumed that the firm is sufficiently large to influence market prices through its production decisions and, therefore, is not considered a price taker. To achieve sustainability goals and accelerate RE investments, policy makers introduced a FIT subsidy. The FIT scheme is characterized by a fixed remuneration of F per MWh of electricity produced, granted by the government, which reduces investment uncertainty and guarantees stable revenues. Although the FIT provides a stable source of revenue, there is a possibility that the subsidy may be withdrawn due to technological progress or shifts in government policy. Let ν denote the random time of subsidy withdrawal, which is modeled by an exponential distribution with parameter λ. This means that the probability of subsidy retraction over a small time interval ds is λds.

To capture the firm's market power, we describe the electricity market price using an inverse demand function, following Nagy et al. (2021) and Huisman & Kort (2015). Under this specification, the electricity price is given by

$$P_s = X_s(1 - \eta K) \tag{1}$$

where X_s is an exogenous stochastic demand shock, K denotes the installed production capacity and $\eta > 0$ is a constant that measures the sensitivity of the electricity price to the capacity installed. This formulation reflects the idea that larger production capacity increases market supply and therefore exerts downward pressure on electricity prices. This inverse demand approach is consistent with the framework of Dixit & Pindyck (1994), who model the output price as $P = XD(K)$, where $D(K)$ is an unspecified inverse demand function. Our specification can be interpreted as a simplified version of their framework.

The exogenous shock X_s follows a Geometric Brownian Motion (GBM):

$$dX_s = \mu X_s\, dt + \sigma X_s\, dW_s, \qquad X_0 = x$$

where μ is the drift parameter, σ is the volatility parameter and $W = \{W_s : s \geq 0\}$ is a Brownian motion. The use of a GBM to model electricity price dynamics is standard in the real options literature and has been adopted, among others, by Fleten et al. (2007), Boomsma & Linnerud (2015), Ritzenhofen & Spinler (2016), Chronopoulos et al. (2016) and Hagspiel et al. (2021).

The firm's revenue depends on two possible regimes. When the subsidy is active, it receives the fixed tariff F for each unit of electricity produced. Once the subsidy is withdrawn, revenues are determined by the market price P_s. To distinguish between subsidized and unsubsidized regimes, we introduce the stochastic process θ_s, as in Hagspiel et al. (2021):

$$\theta_s = \begin{cases} 1, & s < \nu \\ 0, & s \geq \nu \end{cases}.$$

Hence, $\theta = 1$ corresponds to the regime with subsidy, whereas $\theta = 0$ corresponds to the regime without subsidy.

Let $\Pi_\theta(X, K)$ denote the firm's revenue function under regime θ, given the demand shock X and the installed production capacity K. Using the previous definitions, the firm's revenue takes the form

$$\Pi_\theta(X, K) = \begin{cases} KF, & \theta = 1 \\ KX(1 - \eta K), & \theta = 0 \end{cases}.$$

During the subsidized period, the FIT protects the firm from fluctuations in the electricity market price. Consequently, during this period, the company is exempt from the inverse demand structure that governs market price dynamics. However, as soon as the subsidy is withdrawn, the firm becomes fully exposed to market conditions and its revenues are determined by the inverse demand function. Such revenues are obtained through the installation of the production capacity K. Accordingly, we assume a sunk cost of δ per unit of installed capacity, resulting in a total investment cost of δK. The combination of investment irreversibility, market uncertainty, and the risk of subsidy withdrawal generates an option value of waiting.

The firm is willing to invest in a RE project only if it is profitable. We formalize the problem as a real options investment problem in which the firm determines both the optimal investment timing and the optimal installed capacity. The firm's investment problem is therefore given by the following optimal stopping problem:

$$V_\theta(x) = \sup_{\tau \geq 0, K \geq 0} J(x, \theta, \tau, K) \tag{2}$$

with

$$J(x, \theta, \tau, K) = \mathbb{E}_{\theta,x} \left[\int_\tau^\infty e^{-rs} \Pi_{\theta_s}(X_s, K)\, ds - e^{-r\tau} \delta K \right],$$

where $J(x, \theta, \tau, K)$ denotes the expected value of investing at time τ, where τ is a stopping time, $r > 0$ denotes the discount rate and $\mathbb{E}_{\theta,x}$ represents the expected value conditional to $\theta_0 = \theta$ and $X_0 = x$. Henceforward, we will use $V_0(x)$ to designate the value of the investment opportunity when the subsidy is not available and $V_1(x)$ to represent the value of the investment opportunity when the subsidy is initially available.

We assume that $r > \mu$, otherwise it would always be optimal to delay investment indefinitely and we would never invest. At the same time, we assume $\delta - \frac{F}{r+\lambda} > 0$ so that the expected subsidy does not compensate for the entire investment cost, implying immediate investment and eliminating the option value of waiting. Finally, we assume that $K > 0$, so that the investment decision involves a positive installed capacity.

The expected value of investing introduced in Eq. (2) depends on the actual regime, meaning that the function $J(x, \theta, \tau, K)$ takes the following forms:

$$J(x, 0, \tau, K) = \mathbb{E}_{0,x}\left[\int_{\tau}^{\infty} e^{-rs} K X_s (1 - \eta K)\, ds - e^{-r\tau}\delta K\right]; \tag{3}$$

$$J(x, 1, \tau, K) = \mathbb{E}_{1,x}\left[\int_{\tau}^{\infty} e^{-rs}(KF\mathbf{1}_{\{s\leq\nu\}} + K X_s (1 - \eta K)\mathbf{1}_{\{s>\nu\}})\, ds - e^{-r\tau}\delta K\right]. \tag{4}$$

In the next section, we derive the solution of the investment problem for both regimes, $\theta = 0$ and $\theta = 1$.

3 Model solution

In this section we present the solutions for the investment problem defined in Eq. (2). A similar investment problem was studied by Hagspiel et al. (2021). However, our framework differs in important aspects. First, we incorporate the capacity decision and jointly determine the optimal investment timing and installed capacity. Second, while Hagspiel et al. (2021) model subsidy withdrawal through a time-dependent intensity, resulting in a non-homogeneous variable, we assume that subsidy withdrawal occurs at an exponentially distributed random time with constant intensity.

We first consider the absence of subsidies. This setting has already been studied in the real options literature, for instance by Dixit & Pindyck (1994), Huisman & Kort (2015) and Nagy et al. (2021), among others. Since this is a standard case, we only present the main results, while the detailed computations are in Appendix A. We then move to the case where a subsidy is available when the firm analyzes the investment decision.

3.1 Model without subsidy

In the absence of subsidies, the firm's revenue is determined exclusively by the market price. Using the strong Markov property of the process X together with the law of iterated expectations, the investment problem (Eqs. (2) and (3)) can be rewritten as

$$V_0(x) = \sup_{\tau, K} \mathbb{E}_{0,x}\left[e^{-r\tau} g_0(X_\tau, K)\right]$$

with

$$
\begin{aligned}
g_0(X_\tau, K) &= \mathbb{E}_{0,X_\tau}\left[\int_0^\infty e^{-rs} K X_s (1-\eta K) ds\right] - \delta K \\
&= \frac{X_\tau}{r-\mu} K(1-\eta K) - \delta K.
\end{aligned}
$$

where the function $g_0(X_\tau, K)$ represents the project's net present value after investment. Maximizing $g_0(x, K)$ with respect to capacity results in the optimal investment size:

$$
K_0(x) = \frac{1}{2\eta}\left(1 - \frac{\delta(r-\mu)}{x}\right)^+. \tag{5}
$$

Substituting $K_0(x)$ into the project value gives the optimized payoff

$$
g_0^*(x) = g_0(x, K_0(x)) = \begin{cases} \dfrac{x}{4\eta(r-\mu)}\left(1 - \dfrac{\delta(r-\mu)}{x}\right)^2, & x > \delta(r-\mu) \\ 0, & x \leq \delta(r-\mu) \end{cases}.
$$

The investment problem can therefore be written as

$$
V_0(x) = \sup_\tau \mathbb{E}_{0,x}\left[e^{-r\tau} g_0^*(X_\tau)\right],
$$

and its solution is presented in the following proposition.

Proposition 1. *Considering the investment problem defined in Eq.*(2) *in the absence of subsidy, the value function is given by*

$$
V_0(x) = \begin{cases} A_0 x^{\beta_{01}}, & x < X_0^* \\ \dfrac{x}{4\eta(r-\mu)}\left(1 - \dfrac{\delta(r-\mu)}{x}\right)^2, & x \geq X_0^* \end{cases}
$$

where

$$
X_0^* = \frac{\beta_{01}+1}{\beta_{01}-1}\delta(r-\mu), \tag{6}
$$

$$
A_0 = \frac{{X_0^*}^{1-\beta_{01}}}{4\eta(r-\mu)}\left(1 - \frac{\delta(r-\mu)}{X_0^*}\right)^2, \tag{7}
$$

with $\beta_{01} > 1$ *being the positive root of* $\frac{1}{2}\sigma^2\beta^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta - r = 0$. *Additionally, the optimal capacity at the investment threshold is*

$$
K_0^* = \frac{1}{\eta(\beta_{01}+1)}. \tag{8}
$$

Therefore, when the subsidy is not provided, it is optimal for the firm to wait until X reaches the threshold X_0^* and invest at that moment with an installed capacity of K_0^*. Note that $X_0^* > \delta(r-\mu)$ so that the payoff is positive.

We next consider the investment problem when a subsidy is initially available.

3.2 Model with subsidy

In this section, we present the solution of the model for the investment problem when the subsidy is initially available. The main difference relative to the unsubsidized case is that the firm faces two distinct regimes. While the subsidy remains active, revenues are determined by the FIT scheme. Once the subsidy is withdrawn, revenues are determined by the market price.

In the next proposition, we characterize the firm's payoff assuming that the subsidy is initially available and that the withdrawal time ν is exponentially distributed with parameter λ, for a given stopping time τ and installed capacity K.

Proposition 2. *The firm's expected value defined in Eq.* (4) *can be simplified as*

$$J(x,1,\tau,K) = \mathbb{E}_{1,x}\left[e^{-r\tau} g_1(X_\tau, K)\right],$$

where

$$g_1(X_\tau, K) = K\left(\frac{F}{r+\lambda} - \delta\right) + K(1-\eta K)\frac{\lambda}{\lambda+r-\mu}\frac{X_\tau}{r-\mu}$$

denotes the project payoff after investment.

The proof of this proposition is presented in Appendix A and relies on the law of iterated expectations and the strong Markov property. The payoff $g_1(X_\tau, K)$ obtained consists of two components: the first term corresponding to the expected revenues generated under the subsidy regime and the second term to the expected post-withdrawal market revenues.

If investment occurs after subsidy withdrawal, i.e. $\tau \geq \nu$, the problem reduces to the standard case without subsidy (Eq. (3)). From now on, we focus on the case in which investment occurs before subsidy withdrawal, i.e. $\tau < \nu$.

Maximizing $g_1(x, K)$ with respect to capacity yields the optimal installed capacity

$$K_1(x) = \begin{cases} \dfrac{1}{2\eta}\left[1 - \dfrac{\left(\delta - \frac{F}{r+\lambda}\right)(\lambda + r - \mu)(r-\mu)}{\lambda x}\right], & x > \tilde{X} \\ \\ 0, & x \le \tilde{X} \end{cases}.$$

The threshold $\tilde{X}$ represents the minimum value of the variable x for which it may be optimal to invest. When x is smaller than $\tilde{X}$, the optimal capacity is zero, and then investment is never optimal. Since

$$\tilde{X} = \frac{\left(\delta - \frac{F}{r+\lambda}\right)(\lambda + r - \mu)(r-\mu)}{\lambda},$$

this threshold is decreasing with the subsidy level F. Hence, a higher subsidy reduces the minimum market condition required for the firm to install positive capacity. Recall that $\delta - \frac{F}{r+\lambda} > 0$, as assumed in Section 2.

Let X_1^* be the investment threshold for the subsidized case. An illustration of the different configurations of the investment strategy is represented in Figure 1. Let $\underline{F}$ denote the subsidy level for which $\tilde{X} = X_0^*$. Since $X_1^* > \tilde{X}$, the FIT level $F = \underline{F}$ produces the condition $\tilde{X} = X_0^* < X_1^*$. Solving this equation, we obtain

$$\underline{F} = \delta(r+\lambda)\left[1 - \frac{\lambda}{\lambda + r - \mu}\frac{\beta_{01}+1}{\beta_{01}-1}\right].$$

Therefore, if $F < \underline{F}$, then $X_0^* < \tilde{X}$, meaning that the investment in the subsidized market will always occur later than in the unsubsidized market. In contrast, if $F > \underline{F}$, then $\tilde{X} < X_0^*$, and consequently, investment may be optimal after or before investment in the unsubsidized market. Note that $\underline{F}$ can be positive or negative given the parameter values. If $\underline{F} < 0$, every level of subsidy satisfies $F > \underline{F}$, implying automatically that $\tilde{X} < X_0^*$.

Let $\tilde{F}$ be the subsidy value that equals the subsidized with the unsubsidized thresholds, i.e. $X_1^* = X_0^*$. If $F < \tilde{F}$, the subsidy is not sufficiently attractive to induce earlier investment and therefore $X_1^* > X_0^*$. On the contrary, for $F > \tilde{F}$, the subsidy provides a sufficiently strong incentive for investment, implying $X_1^* < X_0^*$. Hence, $\tilde{F}$ can be interpreted as the minimum subsidy level required to accelerate investment relative to the case without subsidy.

Moreover, when $\tilde{X} = 0 = X_1^* < X_0^*$,

$$\overline{F} = \delta(r+\lambda).$$

The value $\overline{F}$ corresponds to the subsidy level for which the subsidy payment exactly offsets the investment cost and the investment is immediate. For $F > \overline{F}$, the capacity threshold disappears and the investment is immediate.

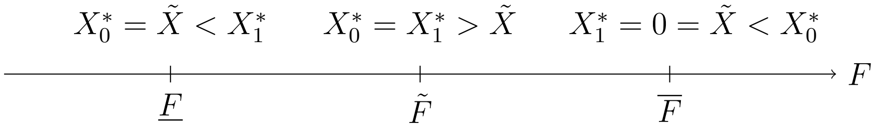


FIGURE 1: Relative position of $\tilde{X}$, X_0^* and X_1^* as functions of the subsidy level F.

Proposition 3. *$\tilde{F}$ is the largest solution of the following equation:*

$$\beta_{11}\left(\frac{F-\delta r}{2\eta(r+\lambda)}\right)X_0^* + (\beta_{11}+1)\frac{1}{4\eta}\left(\frac{F}{r+\lambda}-\delta\right)^2\frac{(\lambda+r-\mu)(r-\mu)}{\lambda}$$
$$- A_0(\beta_{11}-\beta_{01})X_0^{*\beta_{01}+1} - \frac{\lambda(1-\beta_{11})X_0^{*2}}{4\eta(r-\mu)(r+\lambda-\mu)} - \frac{\beta_{11}\lambda\delta X_0^*}{2\eta(r+\lambda)} = 0.$$

The equation defining $\tilde{F}$ is quadratic in F and therefore admits a closed-form solution, which, together with the proof of existence and uniqueness, is presented in Appendix A.

The comparison between the capacity threshold $\tilde{X}$ and the investment threshold obtained in the unsubsidized case, X_0^*, is very useful because it helps to identify the location of the investment threshold when the subsidy is available, X_1^*. Therefore, the subsidy level plays a key role in determining both the timing of investment and the firm's optimal capacity choice. Higher levels of subsidies promote earlier investment and increase the attractiveness of capacity expansion.

Substituting $K_1(x)$ into $g_1(x,K)$, we obtain the optimized payoff $g_1^*(x)$:

$$g_1^*(x) = g_1(x, K_1(x)) = \begin{cases} \frac{1}{2\eta}\left(\frac{F}{r+\lambda}-\delta\right) + \frac{1}{4\eta}\frac{\left(\frac{F}{r+\lambda}-\delta\right)^2(\lambda+r-\mu)(r-\mu)}{\lambda x} \\ + \frac{1}{4\eta}\frac{\lambda x}{(\lambda+r-\mu)(r-\mu)}, & x > \tilde{X} \\ 0, & x \le \tilde{X} \end{cases}$$

and the investment problem can therefore be written as

$$V_1(x) = \sup_{\tau} \mathbb{E}_{1,x}\left[e^{-r\tau} g_1^*(X_\tau)\right].$$

As in the unsubsidized case, the payoff is zero whenever the variable x remains below a capacity threshold. In the subsidized case this threshold is $\tilde{X}$, whereas in the unsubsidized case it is $\delta(r-\mu)$.

To characterize the investment option value, we distinguish between two regions: the investment region and the continuation region. For the investment region, $x \geq X_1^*$, the project value coincides with the payoff $g_1^*(x)$. In the continuation region, $x < X_1^*$, the firm waits and does not invest. While waiting, the subsidy may be withdrawn according to the exponential distribution. Therefore, the value of the investment opportunity is affected by the possibility of transitioning to the unsubsidized regime. Given the Markovian nature of (X, θ), the value function in the continuation region is obtained by solving the following Hamilton-Jacobi-Bellman (HJB) equation

$$rv_1 - \mu x v_1' - \frac{1}{2}\sigma^2 x^2 v_1'' - \lambda(v_0 - v_1) = 0$$

$$\Leftrightarrow (r+\lambda)v_1 - \mu x v_1' - \frac{1}{2}\sigma^2 x^2 v_1'' = \lambda v_0.$$

This equation highlights the key difference relative to the case without subsidy: the value of the subsidized investment opportunity is linked to the value of the unsubsidized opportunity through the withdrawal intensity λ. The v_0 obtained for the case without subsidy is presented in Proposition 1.

The relative position of the subsidized and unsubsidized investment thresholds plays a central role in the analysis, so we distinguish between two possible scenarios of the investment thresholds. In the first case, the subsidy accelerates investment, then we assume that $X_1^* < X_0^*$. In the second case, the subsidy is not sufficient to invest before the unsubsidized case, so we assume that $X_0^* < X_1^*$.

Proposition 4. *Assume that currently there is a FIT scheme in place and that $\tilde{F} < F < \overline{F}$. Then, the value function is given by*

$$V_1(x) = \begin{cases} A_{11}x^{\beta_{11}} + A_0 x^{\beta_{01}}, & x < X_1^* \\ g_1^*(x), & x \geq X_1^* \end{cases},$$

where A_0 is given in Eq. (7)*, $\beta_{01} > 1$ is the positive root of $\frac{1}{2}\sigma^2\beta^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta - r = 0$,*

$$A_{11} = \frac{g_1^*(X_1^*) - A_0 X_1^{*\beta_{01}}}{X_1^{*\beta_{11}}}$$

and

$$\beta_{11} = \frac{\sigma^2 - 2\mu + \sqrt{4\mu^2 - 4\mu\sigma^2 + \sigma^4 + 8(r+\lambda)\sigma^2}}{2\sigma^2} > 1. \tag{9}$$

The threshold X_1^* verifies $X_1^* < X_0^*$ and is determined as the unique solution of $h(x) = 0$, where

$$h(x) = (\beta_{11} - 1)\frac{\lambda}{(\lambda + r - \mu)(r - \mu)}x^2 + 2\beta_{11}\left(\frac{F}{r+\lambda} - \delta\right)x$$
$$+(\beta_{11} + 1)\frac{\left(\frac{F}{r+\lambda} - \delta\right)^2(\lambda + r - \mu)(r - \mu)}{\lambda} + 4\eta A_0(\beta_{01} - \beta_{11})x^{\beta_{01}+1}.$$

In this case, the subsidy is sufficiently attractive to induce investment before the threshold of the unsubsidized model is reached. As a result, the firm exercises the investment option earlier and invests under less favorable market conditions than in the case without subsidy. The value function was obtained using the habitual value matching and smooth pasting conditions and its proof is presented in Appendix A.

Proposition 5. *Assume that currently there is a FIT scheme in place and that $\underline{F} < F < \tilde{F}$. Then, the value function is given by*

$$V_1(x) = \begin{cases} A_{11}x^{\beta_{11}} + A_0 x^{\beta_{01}}, & x < X_0^* \\ A_{12}x^{\beta_{11}} + B_{12}x^{\beta_{12}} + \dfrac{\lambda}{4\eta(r-\mu)(r+\lambda-\mu)}x & \\ -\dfrac{\lambda\delta}{2\eta(r+\lambda)} + \dfrac{\lambda\delta^2(r-\mu)}{4\eta(r+\lambda+\mu-\sigma^2)}\cdot\dfrac{1}{x}, & X_0^* \le x < X_1^* \\ g_1^*(x), & x \ge X_1^* \end{cases},$$

with

$$A_{11} = A_{12} + B_{12}X_0^{*\beta_{12}-\beta_{11}}$$
$$+ \left[\frac{\lambda}{4\eta(r-\mu)(r+\lambda-\mu)}X_0^* - \frac{\lambda\delta}{2\eta(r+\lambda)} + \frac{\lambda\delta^2(r-\mu)}{4\eta(r+\lambda+\mu-\sigma^2)}\frac{1}{X_0^*} - A_0X_0^{*\beta_{01}}\right]X_0^{*-\beta_{11}},$$

$$B_{12} = \frac{1}{(\beta_{11}-\beta_{12})X_0^{*\beta_{12}-1}}\left[\frac{\lambda}{4\eta(r-\mu)(r+\lambda-\mu)} - \frac{\lambda\delta^2(r-\mu)}{4\eta(r+\lambda+\mu-\sigma^2)}\frac{1}{X_0^{*2}}\right.$$
$$- \frac{\beta_{11}}{X_0^*}\left(\frac{\lambda}{4\eta(r-\mu)(r+\lambda-\mu)}X_0^* - \frac{\lambda\delta}{2\eta(r+\lambda)} + \frac{\lambda\delta^2(r-\mu)}{4\eta(r+\lambda+\mu-\sigma^2)}\frac{1}{X_0^*}\right)$$
$$\left. + A_0(\beta_{11}-\beta_{01})X_0^{*\beta_{01}-1}\right],$$

$$
\begin{aligned}
A_{12} &= g_1^*(X_1^*)X_1^{*^{-\beta_{11}}} - B_{12}X_1^{*^{\beta_{12}-\beta_{11}}} \\
&+ \left[- \frac{\lambda}{4\eta(r-\mu)(r+\lambda-\mu)} X_1^* + \frac{\lambda\delta}{2\eta(r+\lambda)} - \frac{\lambda\delta^2(r-\mu)}{4\eta(r+\lambda+\mu-\sigma^2)} \frac{1}{X_1^*} \right] X_1^{*^{-\beta_{11}}}
\end{aligned}
$$

and

$$\beta_{12} = \frac{\sigma^2 - 2\mu - \sqrt{4\mu^2 - 4\mu\sigma^2 + \sigma^4 + 8(r+\lambda)\sigma^2}}{2\sigma^2} < 0.$$

The threshold X_1^* verifies $X_0^* < X_1^*$ and is determined as the unique solution of $f(x) = 0$, where

$$f(x) = \beta_{11}Mx + (\beta_{11}+1)N + (\beta_{12}-\beta_{11})B_{12}x^{\beta_{12}+1},$$

with

$$M = \frac{F - \delta r}{2\eta(r+\lambda)}$$

and

$$N = \frac{1}{4\eta} \left[\frac{\left(\frac{F}{r+\lambda} - \delta\right)^2 (\lambda + r - \mu)(r-\mu)}{\lambda} - \frac{\lambda\delta^2(r-\mu)}{r+\lambda+\mu-\sigma^2} \right].$$

In contrast with Proposition 4, when $X_0^* < X_1^*$, the subsidy is not sufficiently attractive to induce earlier investment. Consequently, the firm delays investment beyond the threshold obtained in the unsubsidized model. This result leads us to the conclusion that the existence of a subsidy does not necessarily accelerate investment. When the subsidy level is low, the revenues generated during the subsidized period are insufficient to induce earlier investment. As a consequence, investment is postponed despite the existence of the subsidy. The constants A_{11}, A_{12} and B_{12}, together with the threshold X_1^*, are determined by value matching and smooth pasting at X_0^* and X_1^*, also in Appendix A.

The resulting equations presented in Proposition 4 and Proposition 5 are both nonlinear, so closed-form solutions for the subsidized thresholds are not available. The analysis of the function $h(x)$ is provided in Appendix A. The optimal installed capacity at the investment threshold is given by $K_1^* = K_1(X_1^*)$.

In the next section we introduce a numerical example of the model and present the comparative statics to examine how the optimal thresholds and capacities evolve with model parameters.

4 Results and Discussion

In this section, we present a numerical example based on the parameterization of Nagy et al. (2021). The corresponding parameter values are presented in Table I.

Notation	Parameter	Value
μ	Electricity price trend	2%
σ	Electricity price volatility	5%
r	Risk-free interest rate	6%
δ	Investment cost per unit of capacity	350 € /MWh
η	Slope of the linear demand curve	0.01
λ	Subsidy retraction intensity	1

Table I: Parameter values used in the numerical example.

Investment threshold	Value
X_0^*	30.371503
X_1^*	20.642417
Capacity installed	**Value**
K_0^*	26.952079
K_1^*	20.379365

Table II: Optimal thresholds and capacities for a subsidy of $F = 59.4$.

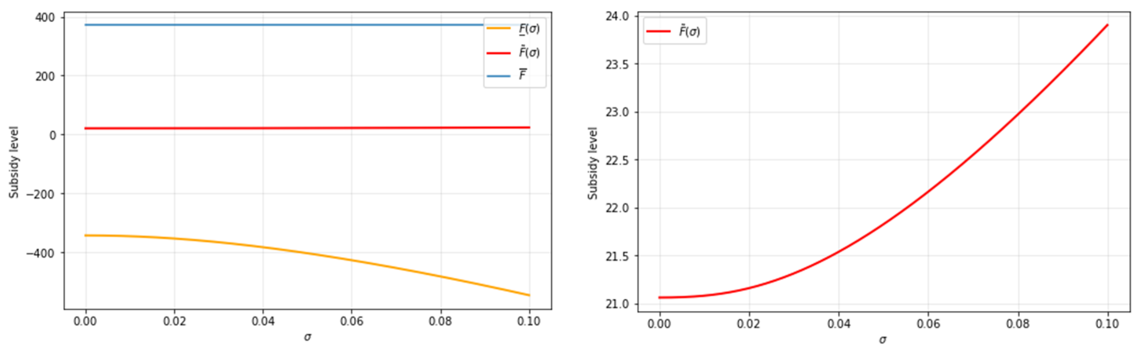
Table II reports the optimal investment thresholds and installed capacities obtained for the parameter values presented in Table I and for a FIT subsidy of $F = 59.4$, following Hagspiel et al. (2021). In the absence of the subsidy, the firm invests only when the stochastic demand variable reaches the threshold $X_0^* = 30.37$. When the subsidy is available, the investment threshold decreases to $X_1^* = 20.64$. Hence, investment occurs earlier under the subsidy regime, since, $X_1^* < X_0^*$. This result indicates that a sufficiently high level of subsidy accelerates investment. Regarding the optimal capacity, under the chosen parameters, the results show a lower capacity in the subsidized case, $K_1^* = 20.38$, than in the unsubsidized case, $K_0^* = 26.95$. Therefore, although the subsidy accelerates investment, it leads the firm to invest in a smaller project. This result is also consistent with the evidence reported by Dangl (1999), according to which greater uncertainty delays investment and is associated with larger project sizes. Additionally, the numerical analysis yields a capacity threshold of $\tilde{X} = 12.23$, which represents the minimum level of electricity demand for which it may be optimal to invest. For the parameter values considered, we verify that $\tilde{X} < X_1^* < X_0^*$. This means that even though market conditions are sufficiently favorable, the subsidy is high enough to accelerate investment. In addition, the critical subsidy levels that characterize the different investment regimes are reported in Table III.

Critical level	Value
$\underline{F}$	-402.889
$\tilde{F}$	21.825
$\overline{F}$	371.000

TABLE III: Critical level for F obtained for the baseline parameterization.

As reported in Table III, $\underline{F} = -402.889$. Since $\underline{F} < 0$, all subsidy levels considered in the numerical analysis satisfy $F > \underline{F}$. The level $\tilde{F} = 21.825$ represents the tariff for which the investment thresholds of the subsidized and unsubsidized cases coincide. Subsidies below $\tilde{F}$ are not sufficiently high to compensate the risk of retraction. In contrast, any level of subsidy higher than $\tilde{F}$, induces earlier investment. Therefore, under the defined parameters, policy makers should provide a minimum subsidy of 21.825 to accelerate investment relative to the unsubsidized case. Finally, a tariff of $\overline{F} = 371$ fully compensates the investment cost, meaning that investment becomes optimal independently of market conditions.

Figure 2 illustrates the effect of volatility on the three critical subsidy levels. In panel (a), $\overline{F}$ remains constant, since it does not depend on σ. Meanwhile, $\underline{F}$ decreases with volatility, which means that the condition $F > \underline{F}$ holds. Panel (b) shows that $\tilde{F}$ increases with volatility, revealing that stronger policy support is required to compensate for the larger effect of uncertainty and induce investment at the same threshold as in the unsubsidized case.

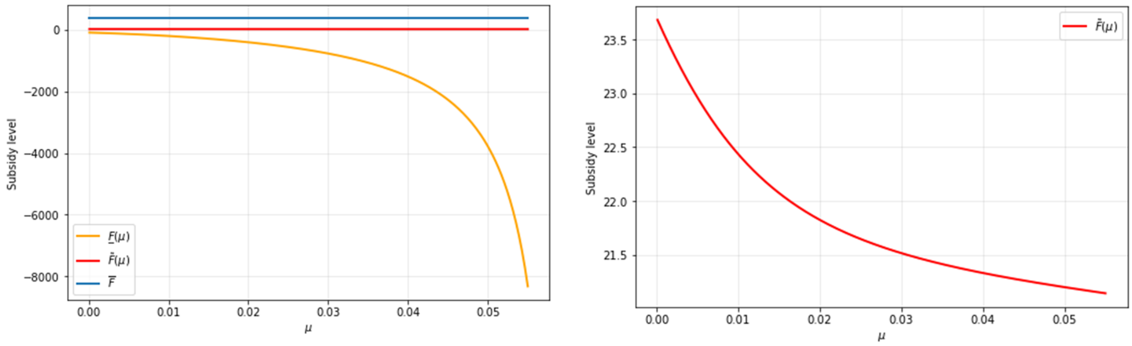


(a) (b)

FIGURE 2: Critical levels for F as a function of volatility (panel (a)) with a zoom on $\tilde{F}$ (panel (b)).

Figure 3 illustrates the effect of the drift on the three critical levels of F. Similarly to the volatility case (panel (a) of Figure 2), $\overline{F}$ remains constant since it does not depend on

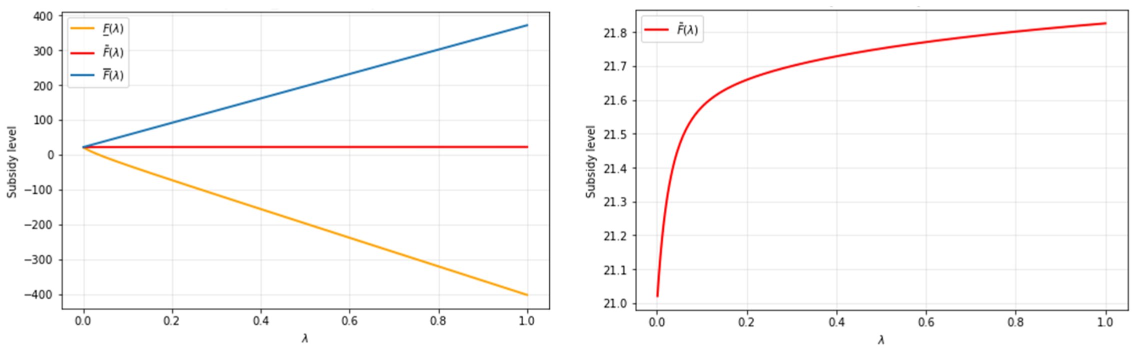
μ and $\underline{F}$ is decreasing with μ. Unlike panel (b) of Figure 2, $\tilde{F}$ decreases with μ. In this case, a lower subsidy is required for the subsidized investment threshold to coincide with the unsubsidized threshold. Since stronger expected market growth already encourages investment, less policy support is required to achieve the same investment timing.



(a) (b)

FIGURE 3: Critical levels for F as a function of the drift μ (panel (a)) with a zoom on $\tilde{F}$ (panel (b)).

Figure 4 illustrates the effect of subsidy withdrawal on the three critical subsidy levels. First, unlike the volatility and drift effects, $\overline{F}$ increases with λ. This means that the greater the risk of the subsidy being withdrawn, the higher its value will have to be to fully compensate for the investment costs. As in the previous cases, $\underline{F}$ decreases with λ. In panel (b), we see that $\tilde{F}$ increases with λ. This result indicates that a higher subsidy is required to offset the greater risk of subsidy withdrawal and induce the same investment timing as in the unsubsidized case.



(a) (b)

FIGURE 4: Critical levels for F as a function of λ (panel (a)) with a zoom on $\tilde{F}$ (panel (b)).

Taken together, Figures 2–4 show that the subsidy level $\tilde{F}$ adjusts according to investment attractiveness. $\tilde{F}$ increases with factors that discourage investment, σ and λ, and decreases with factors that improve expected profitability, μ.

In what follows, we will study the impact of the parameters on the investment decision for both the subsidized and unsubsidized cases. When analytical results are possible to obtain, we will present them. The next proposition characterizes the behavior of the investment threshold and the optimal capacity to install in the absence of a subsidy.

Proposition 6. *The optimal investment threshold is affected by the volatility parameter σ, the drift μ and the cost δ in the following way:*

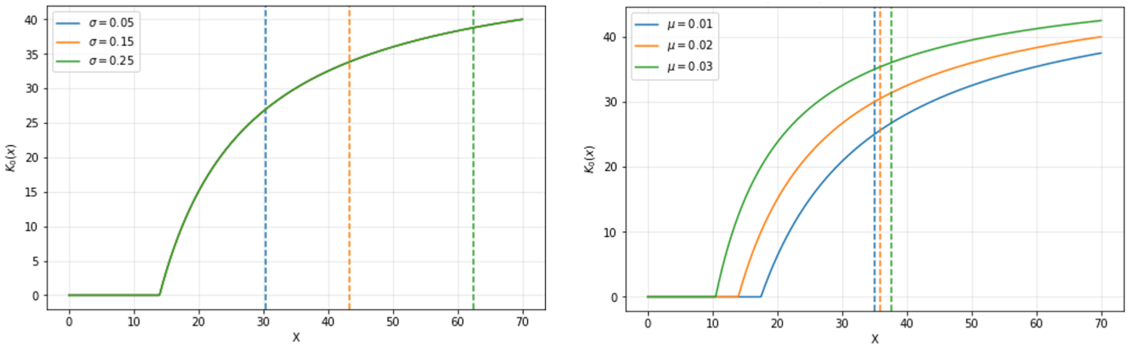
$$\frac{\partial X_0^*}{\partial \sigma} > 0, \qquad \frac{\partial X_0^*}{\partial \mu} \begin{cases} < 0, & \textit{if } \mu < \dfrac{\sigma^2}{2}, \\ = 0, & \textit{if } \mu = \dfrac{\sigma^2}{2},, \\ > 0, & \textit{if } \mu > \dfrac{\sigma^2}{2}. \end{cases} \qquad \frac{\partial X_0^*}{\partial \delta} > 0.$$

The optimal investment size is affected by the volatility parameter σ, the drift μ and the sensitivity constant η in the following way:

$$\frac{\partial K_0^*}{\partial \sigma} > 0, \qquad \frac{\partial K_0^*}{\partial \mu} > 0, \qquad \frac{\partial K_0^*}{\partial \eta} < 0.$$

Proposition 6 shows that a higher volatility increases uncertainty regarding future market conditions, raising the value of waiting and delaying investment. Therefore, X_0^* increases as seen in panel (a) of Figure 5. Consequently, investment occurs with more favorable expected market conditions and the firm installs a larger K_0^*. Regarding the drift parameter, a higher value of μ can either decrease or increase investment timing X_0^*. The non-monotonic behavior of the investment threshold is explained by two contrary effects. On the one hand, a higher drift increases the optimal installed capacity, as shown in Proposition 6. On the other hand, a higher drift encourages the firm to anticipate investment given the favorable conditions, which in some cases can mitigate the increase in the in the installed capacity. As a consequence, for relatively low values of μ, the firm anticipates investment as this will bring additional revenue to the firm. However, once the drift becomes sufficiently high, the firm expects the demand shock to reach high values rapidly. Therefore, it becomes optimal to postpone investment to boost the increase in the installed capacity. A higher installation cost δ reduces the project's profitability and therefore requires more favorable market conditions before investment becomes optimal, leading to a later investment. Finally, the higher the η, the lower the K_0^*. A larger value of η implies that additional capacity has a stronger depressing effect on the market price

(Eq. (1)). As a result, the profitability of expanding capacity decreases, leading the firm to choose a smaller optimal project size. From Figure 5 and Eq. (5) we conclude that the optimal capacity function does not change with σ, only with μ. This means that the behavior of the optimal capacity with volatility is entirely driven by the variation of the threshold.

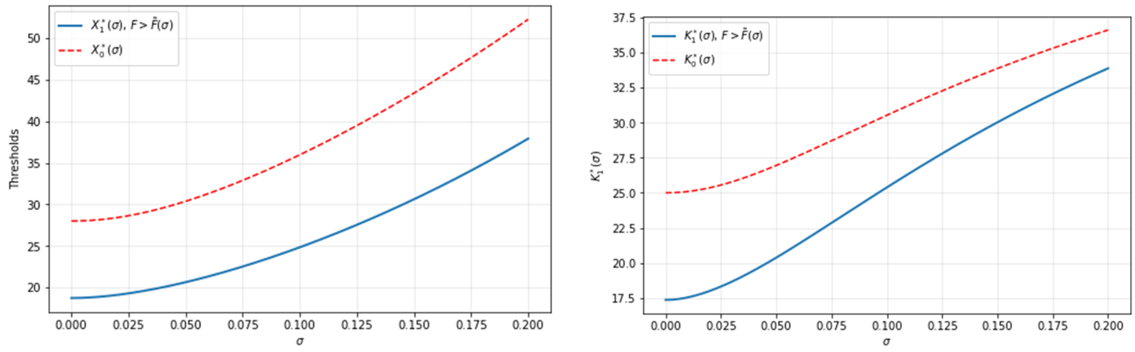


(a) (b)

FIGURE 5: Optimal capacity and investment thresholds for different parameter values of σ (panel (a)) and μ (panel (b)).

We now analyze the subsidized case. We will show how the investment threshold X_1^* evolves with the main parameters. Although the monotonicity of X_0^* was already studied, we also represent it in the next figures to allow the comparison between subsidized and unsubsidized cases.

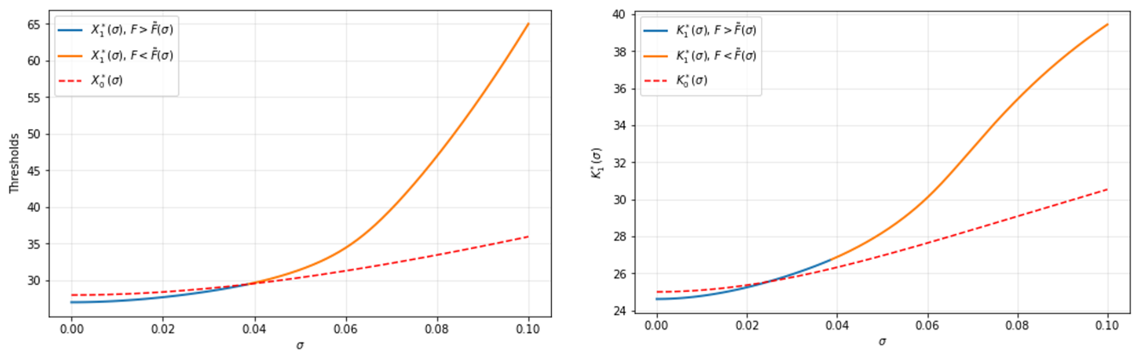
Figure 6 shows how the investment thresholds (panel (a)) and the optimal capacity (panel (b)) vary with σ. Both the thresholds X_0^* and X_1^* increase with volatility, as seen in panel (a), indicating that greater uncertainty raises the value of waiting and therefore delays investment. Nevertheless, the subsidized threshold remains below the unsubsidized threshold, showing that the subsidy anticipates the investment. Panel (b) shows that optimal capacity increases with volatility. This result comes from the fact that higher volatility delays investment in order to obtain better market conditions. Consequently, the firm chooses to install a larger capacity. Since the subsidy lowers the investment threshold and induces earlier investment, firms invest under less favorable market conditions and therefore choose smaller capacities than in the unsubsidized case. Overall, higher volatility delays investment but leads to larger project sizes once investment takes place.



(a) (b)

FIGURE 6: Investment thresholds (panel (a)) and optimal capacities (panel (b)) for different values of the volatility parameter σ with $F = 59.4$.

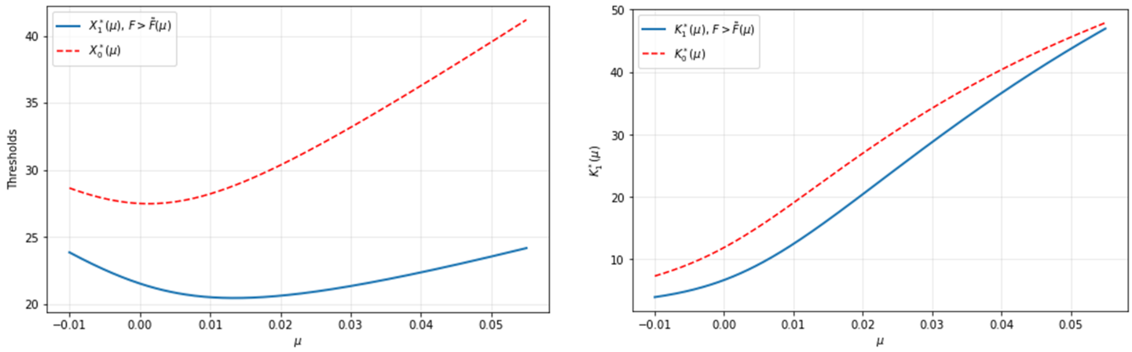
Figure 7 presents the effect of volatility for a lower subsidy level, $F = 21.5$. The main difference when compared to the case with $F = 59.4$ is that, beyond a certain level of volatility, the subsidized threshold and capacity surpass the unsubsidized results. As volatility increases, the subsidy is no longer sufficiently attractive to compensate for the benefits associated with waiting for additional information, unlike the previous case, and the firm postpones investment beyond the unsubsidized threshold. The same reasoning applies to the optimal capacity scenario.



(a) (b)

FIGURE 7: Investment thresholds (panel (a)) and optimal capacities (panel (b)) for different values of the volatility parameter σ with $F = 21.5$.

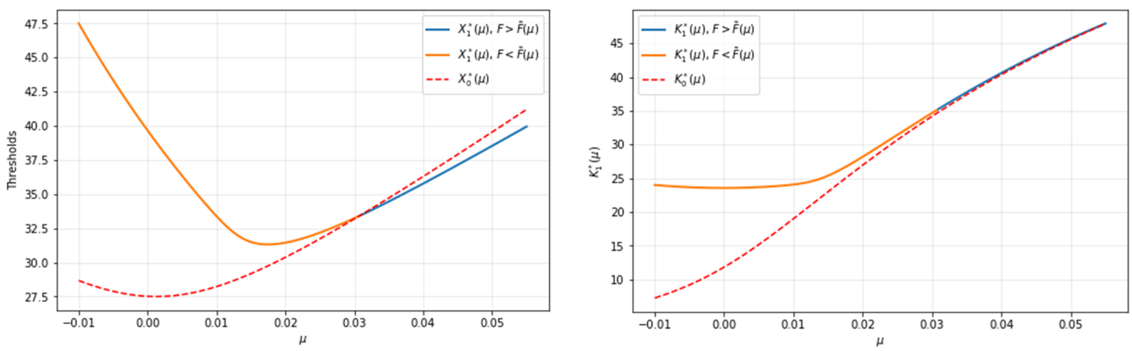
Figure 8 shows how the investment thresholds and optimal capacities vary with μ. Panel (a) shows that both X_0^* and X_1^* exhibit a non-monotonic relationship with μ. As seen in Proposition 6, X_0^* decreases until $\mu > \frac{\sigma^2}{2}$ and then increases. The same pattern happens with X_1^*. Also, X_1^* stays below X_0^* throughout the parameter range. Panel (b) shows that, similar to Proposition 6, K_1^* increases with μ, indicating that higher expected demand growth leads the firm to install larger capacities and therefore increase the profitability of additional installed capacity.



(a) (b)

FIGURE 8: Investment thresholds (panel (a)) and optimal capacities (panel (b)) for different values of the drift parameter μ with $F = 59.4$.

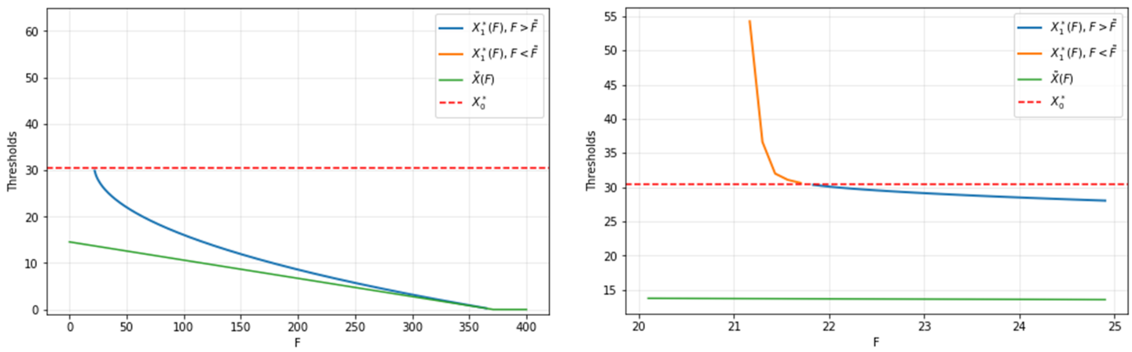
Figure 9 shows how the investment thresholds and optimal capacities vary with μ for a subsidy $F = 21.5$. As in Figure 8, panel (a) shows that the relationship between the investment thresholds and the drift parameter is non-monotonic. However, the subsidized threshold exceeds the unsubsidized threshold for sufficiently low values of μ, i.e., $X_1^* > X_0^*$. This occurs because weak expected market growth, combined with a subsidy close to $\tilde{F}$, makes policy support insufficient to compensate for the risk of subsidy retraction. Therefore, the firm invests under more favorable demand conditions and chooses a larger capacity than in the unsubsidized case.



(a) (b)

FIGURE 9: Investment thresholds (panel (a)) and optimal capacities (panel (b)) for different values of the drift parameter μ with $F = 21.5$.

We now examine how the subsidy level affects the firm's investment decision. The FIT tariff influences both the investment threshold and the optimal capacity. Figure 10 illustrates the evolution of the investment threshold as the subsidy level varies. As the subsidy increases, the subsidized investment threshold X_1^* decreases, implying that investment becomes optimal earlier, even when market conditions are less favorable. This result follows from the fact that higher subsidy payments increase the expected profitability of the project during the subsidized period. We can also see in panel (a) that, when the subsidy increases to $\overline{F}$, the investment threshold X_1^* converges to zero, as the expected subsidy received covers the investment cost. In panel (b) we can see that for low subsidy levels, $F < \tilde{F}$, the subsidized investment threshold is above the threshold obtained in the absence of subsidies, implying that investment is delayed. When $F > \tilde{F}$, the opposite occurs and the subsidy accelerates investment. Therefore, the presence of a subsidy does not necessarily accelerate investment. Only subsidies above the critical level $\tilde{F}$ induce earlier investment relative to the unsubsidized case. The effect of the subsidy on the optimal capacity is presented in Figure 11.



(a) (b)

FIGURE 10: Investment thresholds as functions of the subsidy level F.

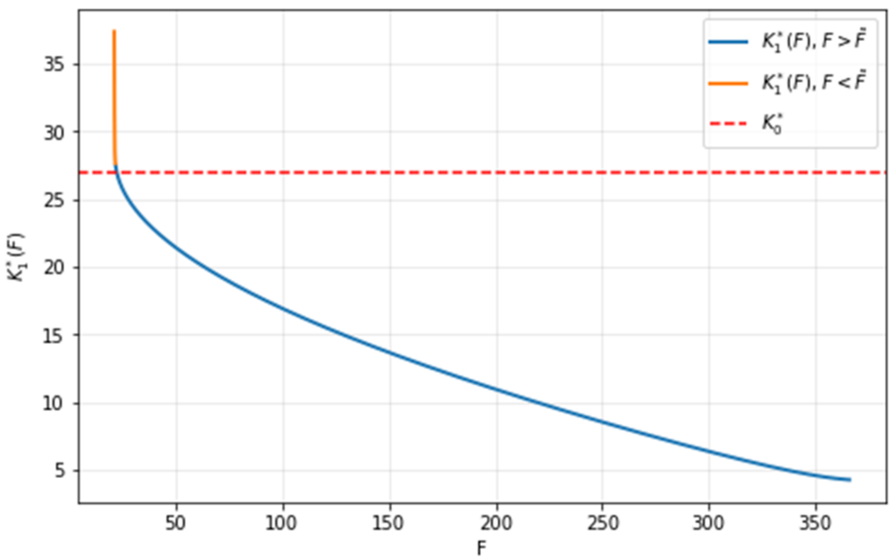
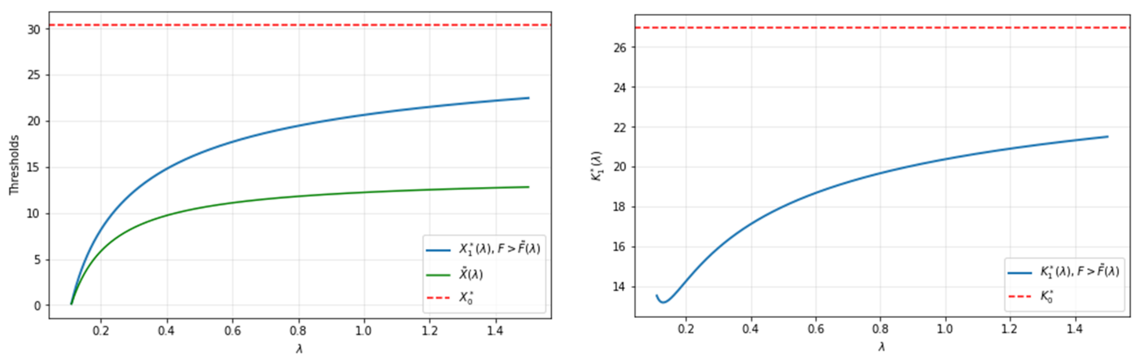


FIGURE 11: Optimal capacity as a function of the subsidy level F.

Figure 11 shows that the optimal capacity decreases with the tariff F. For low subsidy levels, the optimal capacity remains above the unsubsidized capacity. As the subsidy level increases, the optimal capacity becomes smaller than K_0^* and continues to decrease as the subsidy increases. The result that optimal capacity decreases with subsidy may seem unexpected but, as shown previously, higher subsidy levels reduce the optimal investment threshold X_1^*, inducing the firm to invest under less favorable market conditions. Although the subsidy increases expected revenues, the lower level of market demand reduces the incentive to invest in large-scale projects. On the contrary, when the subsidy is low the firm postpones investment and waits for more favorable market conditions. Since investment occurs only when the demand shock reaches a high threshold, the firm installs a larger capacity.

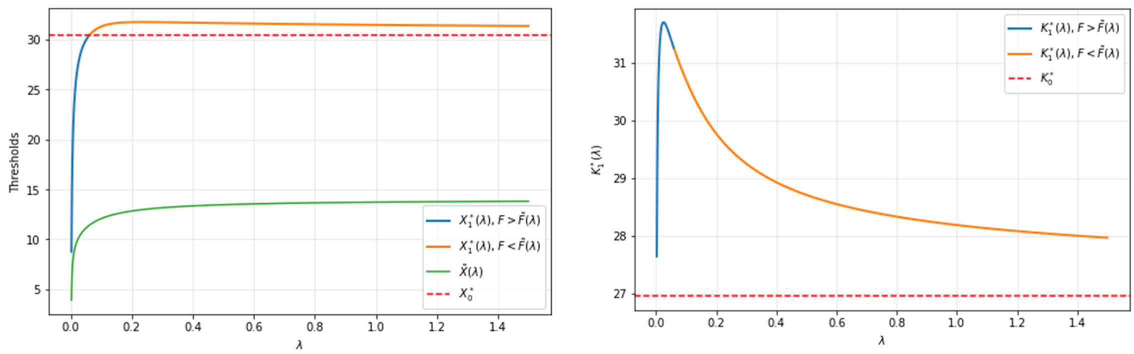
We now analyze the effect of the subsidy withdrawal intensity λ on the firm's investment decision. Figure 12 shows the behavior of the thresholds (panel (a)) and capacities (panel (b)) for different values of λ. From panel (a) we observe that both X_1^* and $\tilde{X}$ increase with λ, which means that the expected time until the subsidy retraction decreases and the firm waits for more favorable market conditions before undertaking the investment. Therefore, the investment decision is delayed. Also note that $X_1^* < X_0^*$, implying that investment still occurs earlier than in the unsubsidized case, even though that λ increases. Regarding the optimal installed capacity, panel (b) shows that K_1^* has a non-monotonic behavior. Since a higher probability of retraction raises the investment threshold, the firm invests under more favorable conditions and chooses a larger installed capacity. Nevertheless, K_1^* remains below K_0^*, indicating that the subsidy continues to affect the firm's investment strategy even when the probability of withdrawal is high.



(a) (b)

FIGURE 12: Investment thresholds (panel (a)) and optimal capacities (panel (b)) as functions of the subsidy withdrawal intensity λ, for $F = 59.4$.

Figure 13 shows the behavior of the thresholds (panel (a)) and capacity (panel (b)) for different values of λ with an $F = 21.5$. As expected, for a λ close to zero, the firm invests relatively early, as seen in panel (a). As λ increases, X_1^* rises sharply for values larger than X_0^*, meaning that the firm requires increasingly favorable market conditions before investing, even more than in the unsubsidized case. Note that for higher values of λ, X_1^* starts getting closer to X_0^*, which indicates a non-monotonic behavior. Regarding the optimal capacity in panel (b), K_1^* increases for low values of λ. This occurs because higher values of λ are associated with higher investment thresholds, leading firms to postpone investment until more favorable market conditions. As λ increases, an opposite effect dominates. As λ increases, the expected duration of the subsidy scheme becomes shorter, reducing the attractiveness of the FIT. At the same time, the slight decrease in the threshold leads to a lower installed capacity. Eventually, these effects become more important than the improvement in market conditions associated with a higher investment threshold, leading to a reduction in the optimal capacity. Nevertheless, the capacity chosen by the firm remains above K_0^*. This suggests that, although the subsidy is no longer strong enough to induce earlier investment, it still affects the scale of the project.



(a) (b)

FIGURE 13: Investment thresholds (panel (a)) and optimal capacities (panel (b)) as functions of the subsidy withdrawal intensity λ, for $F = 21.5$.

The comparative statics analysis reveals three main results. First, greater uncertainty delays investment, leading to larger project sizes, a result consistent with the real options literature (Dangl (1999)). Second, the effect of subsidies on investment timing depends on their magnitude. Although sufficiently high subsidy levels accelerate investment and reduce the optimal installed capacity, low subsidy levels may have the opposite effect, delaying investment and increasing the optimal project size. Nagy et al. (2021) conclude that both the investment timing and optimal capacity decrease with the subsidy, however they did not show that low subsidy levels may increase X_1^* and K_1^*. Third, the impact of subsidy retraction risk depends on the subsidy level. The same increase in the retraction risk can have very different implications depending on the subsidy value.

5 Conclusion

In this work, we consider a firm that has the option to invest in a RE project. Currently, there exists a FIT scheme in place that may be withdrawn with no retroactive effect. We study the investment decision of the firm, particularly how the level of subsidy affects both investment timing and capacity installed. We also study how policy uncertainty affects the investment decision, implying that after the retraction of the FIT, the firm sells electricity at the market price.

To analyze this problem, we use a real options approach to determine the optimal investment timing and installed capacity for both subsidized and unsubsidized cases. In contrast with traditional valuation methods like NPV, this approach captures the value of waiting before committing to an irreversible investment when future market conditions and government support policies are uncertain. To complement the analytical results, we analyze the solution through a numerical example using parameter values from the literature, presenting a comparative statics analysis to study how changes in the model parameters influence the optimal investment thresholds and installed capacities. This analysis, allows us to better understand the effects of market uncertainty, subsidy levels and policy uncertainty on the firm's investment decisions.

The results obtained show that the effect of subsidies on the investment decision is way more complex than expected. We find that sufficiently high levels of financial support induce earlier investment, although with smaller installed capacities. In contrast, low subsidy levels postpone investment beyond the unsubsidized threshold but increase the installed capacity. We also verified a non-uniform effect of subsidy withdrawal risk on investment and its dependence on the FIT level. For high subsidy levels, an increase in the retraction intensity delays investment and leads to a non-monotonic behavior in the installed capacity. For a high probability of withdrawal, the optimal capacity increases. Nevertheless, under high subsidy levels, both the investment threshold and the capacity remain below the unsubsidized benchmark. For low FIT levels, the investment threshold initially increases with the retraction intensity and then decreases. The optimal installed capacity remains above the unsubsidized level but with a larger size for low probability of retraction. Finally, we show that the behavior of the investment threshold with the drift parameter is non-monotonic and we confirm the standard real options result that greater market uncertainty increases the investment threshold and leads to larger optimal capacities.

These findings give valuable insights for both investors and policy makers. Investors should account for the expected level of financial support and for the credibility and stability of the policy framework, as both factors significantly influence investment decisions. For policy makers, the results show that simply implementing a subsidy is not enough to increase participation in green projects. Subsidies only accelerate investment above a certain value, otherwise, they may even delay it. The stability of the support scheme should also be considered, since policy uncertainty may substantially alter firms' investment behavior and reduce the intended impact of public incentives.

This model assumes a probability of subsidy retraction with constant intensity. For future research would be interesting to allow the withdrawal intensity to change over time, making the retraction risk time-dependent. Another additional extension could be to change the GBM that models the demand shock to another stochastic process.

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A Appendices

Proof of Proposition 1. Recalling Eqs. (2) and (3), the investment problem can be written as

$$V_0(x) = \sup_{\tau,K} \mathbb{E}_{0,x}\left[\int_\tau^\infty e^{-rs} K X_s (1-\eta K)\, ds - e^{-r\tau}\delta K\right].$$

We start by computing the project's NPV, denoted by $g_0(x, K)$. Since X follows a GBM with initial value $X_0 = x$, we have $\mathbb{E}_{0,x}[X_s] = xe^{\mu s}$. Thus, by Fubini's theorem,

$$\begin{aligned} g_0(x,K) &= \mathbb{E}_{0,x}\left[\int_0^\infty e^{-rs} X_s K(1-\eta K)\, ds\right] - \delta K \\ &= K(1-\eta K)\int_0^\infty e^{-rs}\mathbb{E}_{0,x}[X_s]\, ds - \delta K \\ &= \frac{x}{r-\mu} K(1-\eta K) - \delta K. \end{aligned}$$

Applying the strong Markov property together with the law of iterated expectations, we obtain

$$\begin{aligned} &\mathbb{E}_{0,x}\left[\int_\tau^\infty e^{-rs} K X_s (1-\eta K)\, ds - e^{-r\tau}\delta K\right] \\ &= \mathbb{E}_{0,x}\left[\mathbb{E}_{0,x}\left[\int_\tau^\infty e^{-rs} K X_s (1-\eta K)\, ds - e^{-r\tau}\delta K \,\middle|\, \mathcal{F}_\tau\right]\right] \\ &= \mathbb{E}_{0,x}\left[e^{-r\tau}\left(\mathbb{E}_{0,X_\tau}\left[\int_0^\infty e^{-rs} K X_s (1-\eta K)\, ds\right] - \delta K\right)\right] \\ &= \mathbb{E}_{0,x}\left[e^{-r\tau} g_0(X_\tau, K)\right]. \end{aligned}$$

Therefore, the investment problem reduces to

$$\begin{aligned} V_0(x) &= \sup_{\tau,K} \mathbb{E}_{0,x}\left[\int_\tau^\infty e^{-rs} K X_s (1-\eta K)\, ds - e^{-r\tau}\delta K\right] \\ &= \sup_{\tau,K} \mathbb{E}_{0,x}\left[e^{-r\tau} g_0(X_\tau, K)\right] \\ &= \sup_{\tau} \mathbb{E}_{0,x}\left[e^{-r\tau} \sup_K g_0(X_\tau, K)\right]. \end{aligned}$$

To obtain the optimal size of investment, we maximize $g_0(x, K)$ with respect to K. Note that $\frac{\partial^2 g_0}{\partial K^2} < 0$, so g_0 is strictly concave, which implies that the maximum is unique. Solving the first order condition $\frac{\partial g_0}{\partial K} = 0$ for K, we obtain the following result, given by Eq. (5):

$$K_0(x) = \frac{1}{2\eta}\left(1 - \frac{\delta(r-\mu)}{x}\right)^+.$$

Substituting $K_0(x)$ into the project value gives the optimized payoff

$$g_0^*(x) = g_0(x, K_0(x)) = \begin{cases} \dfrac{x}{4\eta(r-\mu)}\left(1 - \dfrac{\delta(r-\mu)}{x}\right)^2, & x > \delta(r-\mu) \\ 0, & x \leq \delta(r-\mu) \end{cases}.$$

The investment problem can therefore be rewritten as

$$V_0(x) = \sup_{\tau} \mathbb{E}_{0,x}\left[e^{-r\tau} g_0^*(X_\tau)\right].$$

To characterize the value function, we distinguish between the stopping and continuation regions. When $x \geq X_0^*$ we are in the stopping region, the firm invests immediately, and the option value coincides with $g_0^*(x)$:

$$V_0(x) = \frac{x}{4\eta(r-\mu)}\left(1 - \frac{\delta(r-\mu)}{x}\right)^2.$$

In the continuation region, i.e., $x < X_0^*$, it is optimal to delay investment. Therefore, the option value satisfies

$$\frac{1}{2}\sigma^2 x^2 v_0''(x) + \mu x v_0'(x) - r v_0(x) = 0.$$

Solving this ordinary differential equation (ODE) yields

$$V_0(x) = A_0 x^{\beta_{01}} + B_0 x^{\beta_{02}}$$

with $\beta_{01} = \frac{\sigma^2 - 2\mu + \sqrt{4\mu^2 - 4\mu\sigma^2 + \sigma^4 + 8r\sigma^2}}{2\sigma^2} > 1$ and $\beta_{02} = \frac{\sigma^2 - 2\mu - \sqrt{4\mu^2 - 4\mu\sigma^2 + \sigma^4 + 8r\sigma^2}}{2\sigma^2} < 0$. Since $V_0(0) = 0$ and $\beta_{02} < 0$, it implies that $B_0 = 0$. Hence $V_0(x) = A_0 x^{\beta_{01}}$. Combining both results we obtain:

$$V_0(x) = \begin{cases} A_0 x^{\beta_{01}}, & x < X_0^* \\ \dfrac{x}{4\eta(r-\mu)}\left(1 - \dfrac{\delta(r-\mu)}{x}\right)^2, & x \geq X_0^* \end{cases}.$$

The unknowns X_0^* (Eq. (6)) and A_0 (Eq. (7)) are determined from the value matching and smooth pasting conditions:

$$\begin{cases} A_0 {X_0^*}^{\beta_{01}} = g_0^*(X_0^*) \\ \\ \beta_{01} A_0 {X_0^*}^{\beta_{01}-1} = \frac{\partial g_0^*}{\partial X_0^*}(X_0^*) \end{cases} \Leftrightarrow \begin{cases} A_0 = \frac{{X_0^*}^{1-\beta_{01}}}{4\eta(r-\mu)}\left(1 - \frac{\delta(r-\mu)}{X_0^*}\right)^2 \\ \\ X_0^* = \frac{\beta_{01}+1}{\beta_{01}-1}\,\delta(r-\mu) \end{cases}.$$

Finally, substituing X_0^* into $K_0(x)$, we obtain the optimal capacity (Eq. (8)):

$$K_0^* = [\eta(\beta_{01}+1)]^{-1}.$$

□

Proof of Proposition 2. When the subsidy is initially available, two scenarios exist. In the first scenario, the firm invests when the subsidy is still available, $\tau \leq \nu$. In the second scenario, the subsidy is withdrawn before investing, $\tau > \nu$. Recalling Eq. (4),

$$J(x,1,\tau,K) = \mathbb{E}_{1,x}\left[\int_\tau^\infty e^{-rs}\left(KF\mathbf{1}_{\{s\leq\nu\}} + KX_s(1-\eta K)\mathbf{1}_{\{s>\nu\}}\right)ds - e^{-r\tau}\delta K\right].$$

$J(x,1,\tau,K)$ can be decomposed in the following way:

$$\begin{aligned} J(x,1,\tau,K) = \mathbb{E}_{1,x}\Bigg[&\int_\tau^\nu e^{-rs}KF\mathbf{1}_{\{\tau<\nu\}}\,ds + \int_\nu^\infty e^{-rs}KX_s(1-\eta K)\mathbf{1}_{\{\tau<\nu\}}\,ds \\ &+ \int_\tau^\infty e^{-rs}KX_s(1-\eta K)\mathbf{1}_{\{\tau\geq\nu\}}\,ds - e^{-r\tau}\delta K\Bigg]. \end{aligned}$$

For the case where $\tau \leq \nu$:

$$J(x,1,\tau,K) = \mathbb{E}_{1,x}\left[\int_\tau^\nu e^{-rs}KF\,ds + \int_\nu^\infty e^{-rs}KX_s(1-\eta K)\,ds - e^{-r\tau}\delta K\right].$$

Applying the strong Markov property together with the law of iterated expectations, we get

$$J(x,1,\tau,K) = \mathbb{E}_{1,x}\left[e^{-r\tau}\left(\underbrace{\mathbb{E}_{1,X_\tau}\left[\int_0^\nu e^{-rs}KF\,ds\right]}_{\text{(I) before withdrawal}} + \underbrace{\mathbb{E}_{1,X_\tau}\left[\int_\nu^\infty e^{-rs}KX_s(1-\eta K)\,ds\right]}_{\text{(II) after withdrawal}} - \delta K\right)\right].$$

Since $\nu \sim \text{Exp}(\lambda)$ with density $f_\nu(t) = \lambda e^{-\lambda t}, t \geq 0$, from (I) we obtain:

$$\text{(I)} \qquad \mathbb{E}_{1,X_\tau}\left[\int_0^\nu e^{-rs}KF\,ds\right] = \frac{KF}{r+\lambda}.$$

After withdrawal, the value of the project was already computed for the case without subsidy. However, for this case, $x = X_\nu$, thus

$$\text{(II)} \qquad \mathbb{E}_{1,X_\tau}\left[\int_\nu^\infty e^{-rs} K X_s (1-\eta K)\, ds\right] = \mathbb{E}_{1,X_\tau}\left[e^{-r\nu}\cdot \frac{X_\nu}{r-\mu} K(1-\eta K)\right]$$
$$= \frac{K(1-\eta K)}{r-\mu}\mathbb{E}_{1,X_\tau}\left[e^{-r\nu} X_\nu\right]$$
$$= \frac{K(1-\eta K)}{r-\mu}\mathbb{E}_{1,X_\tau}\left[e^{-r\nu}\, \mathbb{E}[X_\nu \mid \nu]\right]$$
$$= \frac{X_\tau K(1-\eta K)}{r-\mu}\mathbb{E}\left[e^{-\nu(r-\mu)}\right]$$
$$= \frac{X_\tau K(1-\eta K)}{r-\mu}\cdot \frac{\lambda}{r-\mu+\lambda}.$$

Therefore,

$$J(x,1,\tau,K) = \mathbb{E}_{1,x}\left[e^{-r\tau}\left(K\left(\frac{F}{r+\lambda}-\delta\right) + K(1-\eta K)\frac{\lambda}{\lambda+r-\mu}\cdot\frac{X_\tau}{r-\mu}\right)\right]$$
$$= \mathbb{E}_{1,x}\left[e^{-r\tau} g_1(X_\tau, K)\right].$$

For the case where $\tau > \nu$, we get Eq. (3).

□

Proof of Proposition 4. From Proposition 2, we know that when investment occurs before subsidy withdrawal,

$$J(x,1,\tau,K) = \mathbb{E}_{1,x}\left[e^{-r\tau} g_1(X_\tau, K)\right].$$

Therefore, the value function can be written as

$$V_1(x) = \sup_{\tau,K} \mathbb{E}_{1,x}\left[e^{-r\tau} g_1(X_\tau, K)\right].$$

To obtain the optimal size of investment, we maximize $g_1(x,K)$ with respect to K. Note that $\frac{\partial^2 g_1}{\partial K^2} < 0$, so g_1 is strictly concave, which implies that the maximum is unique. Solving the first order condition $\frac{\partial g_1}{\partial K} = 0$ for K, we obtain the following result:

$$K_1(x) = \begin{cases} \dfrac{1}{2\eta}\left[1 - \dfrac{\left(\delta - \frac{F}{r+\lambda}\right)(\lambda+r-\mu)(r-\mu)}{\lambda x}\right], & x > \tilde{X} \\ 0, & x \le \tilde{X} \end{cases}.$$

with $\tilde{X} = \frac{\left(\delta - \frac{F}{r+\lambda}\right)(\lambda + r - \mu)(r-\mu)}{\lambda}$. Substituting $K_1(x)$ into the project value gives the optimized payoff

$$g_1^*(x) = g_1(x, K_1(x)) = \begin{cases} \frac{1}{2\eta}\left(\frac{F}{r+\lambda} - \delta\right) + \frac{1}{4\eta}\frac{\left(\frac{F}{r+\lambda} - \delta\right)^2 (\lambda + r - \mu)(r-\mu)}{\lambda x} \\ + \frac{1}{4\eta}\frac{\lambda x}{(\lambda + r - \mu)(r-\mu)}, & x > \tilde{X} \\ 0, & x \leq \tilde{X} \end{cases}.$$

The investment problem can therefore be rewritten as

$$V_1(x) = \sup_{\tau} \mathbb{E}_{1,x}\left[e^{-r\tau} g_1^*(X_\tau)\right].$$

To characterize the value function, we distinguish between the stopping and continuation regions. In the stopping region, $x > X_1^*$, the value function coincides with g_1^*:

$$V_1(x) = \frac{1}{2\eta}\left(\frac{F}{r+\lambda} - \delta\right) + \frac{1}{4\eta}\frac{\left(\frac{F}{r+\lambda} - \delta\right)^2 (\lambda + r - \mu)(r-\mu)}{\lambda x} + \frac{1}{4\eta}\frac{\lambda x}{(\lambda + r - \mu)(r-\mu)}.$$

In the continuation region, i.e., $x < X_1^*$, it is optimal to delay investment. Hence, the value function satisfies the HJB equation

$$\begin{aligned} & rv_1 - \mu x v_1' - \frac{1}{2}\sigma^2 x^2 v_1'' - \lambda(v_0 - v_1) = 0 \\ \Leftrightarrow \quad & (r+\lambda)v_1 - \mu x v_1' - \frac{1}{2}\sigma^2 x^2 v_1'' = \lambda v_0. \end{aligned}$$

Since v_0 was obtained in Proposition 1, the HJB equation can be solved separately on the intervals determined by v_0. Solving the ODE $\frac{1}{2}\sigma^2 x^2 v_1'' + \mu x v_1' - (r+\lambda)v_1 = 0$ we obtain the general solution

$$v_1(x) = A_1 x^{\beta_{11}} + B_1 x^{\beta_{12}},$$

with A_1 and B_1 being constants and $\beta_{11} = \frac{\sigma^2 - 2\mu + \sqrt{4\mu^2 - 4\mu\sigma^2 + \sigma^4 + 8(r+\lambda)\sigma^2}}{2\sigma^2} > 1$ and $\beta_{12} = \frac{\sigma^2 - 2\mu - \sqrt{4\mu^2 - 4\mu\sigma^2 + \sigma^4 + 8(r+\lambda)\sigma^2}}{2\sigma^2} < 0$. To determine a particular solution, we consider the two regions of v_0 separately. For $x < X_0^*$, we obtain

$$-\frac{1}{2}\sigma^2 x^2 v_1'' - \mu x v_1' + (r+\lambda)v_1 = \lambda A_0 x^{\beta_{01}}.$$

The right hand side of the equation has the form $v_p(x) = \alpha x^{\beta_{01}}$. Its derivatives are $v_p'(x) = \alpha\beta_{01}x^{\beta_{01}-1}$ and $v_p''(x) = \alpha\beta_{01}(\beta_{01}-1)x^{\beta_{01}-2}$. Substituting these terms into the original equation:

$$
\begin{aligned}
& -\frac{1}{2}\sigma^2 x^2 \alpha \beta_{01}(\beta_{01}-1)x^{\beta_{01}-2} - \mu x \alpha \beta_{01} x^{\beta_{01}-1} + (r+\lambda)\alpha x^{\beta_{01}} = \lambda A_0 x^{\beta_{01}} \\
\Leftrightarrow \quad & \alpha = \frac{\lambda A_0}{\lambda} = A_0.
\end{aligned}
$$

Therefore,

$$v_p(x) = A_0 x^{\beta_{01}}.$$

For the interval $x \geq X_0^*$, the HJB equation becomes

$$
\begin{aligned}
& -\frac{1}{2}\sigma^2 x^2 v_1'' - \mu x v_1' + (r+\lambda)v_1 = \lambda \cdot \frac{x}{4\eta(r-\mu)}\left(1 - \frac{\delta(r-\mu)}{x}\right)^2 \\
\Leftrightarrow & -\frac{1}{2}\sigma^2 x^2 v_1'' - \mu x v_1' + (r+\lambda)v_1 = \underbrace{\frac{\lambda}{4\eta(r-\mu)}}_{C} x - \underbrace{\frac{\lambda\delta}{2\eta}}_{D} + \underbrace{\frac{\lambda\delta^2(r-\mu)}{4\eta}}_{E} \frac{1}{x}.
\end{aligned}
$$

The right hand side of the equation has the form $v_p(x) = Cx + D + \frac{E}{x}$. Its derivatives are $v_p'(x) = C - \frac{E}{x^2}$ and $v_p''(x) = \frac{2E}{x^3}$. Substituting in the original equation:

$$-\frac{1}{2}\sigma^2 x^2 \frac{2E}{x^3} - \mu x \left(C - \frac{E}{x^2}\right) + (r+\lambda)\left(Cx + D + \frac{E}{x}\right) = \frac{\lambda}{4\eta(r-\mu)}x - \frac{\lambda\delta}{2\eta} + \frac{\lambda\delta^2(r-\mu)}{4\eta}\frac{1}{x}$$

$$\Leftrightarrow \begin{cases} C = \dfrac{\lambda}{4\eta(r-\mu)(r+\lambda-\mu)} \\ D = -\dfrac{\lambda\delta}{2\eta(r+\lambda)} \\ E = \dfrac{\lambda\delta^2(r-\mu)}{4\eta(r+\lambda+\mu-\sigma^2)} \end{cases}.$$

Therefore,

$$v_p(x) = \frac{\lambda}{4\eta(r-\mu)(r+\lambda-\mu)}x - \frac{\lambda\delta}{2\eta(r+\lambda)} + \frac{\lambda\delta^2(r-\mu)}{4\eta(r+\lambda+\mu-\sigma^2)} \cdot \frac{1}{x}.$$

Hence, the value function is given by

$$v_1(x) = \begin{cases} A_{11}x^{\beta_{11}} + B_{11}x^{\beta_{12}} + A_0 x^{\beta_{01}}, & x < X_0^* \\ A_{12}x^{\beta_{11}} + B_{12}x^{\beta_{12}} + \dfrac{\lambda}{4\eta(r-\mu)(r+\lambda-\mu)}x & \\ -\dfrac{\lambda\delta}{2\eta(r+\lambda)} + \dfrac{\lambda\delta^2(r-\mu)}{4\eta(r+\lambda+\mu-\sigma^2)} \cdot \dfrac{1}{x} & x > X_0^* \end{cases}.$$

If we analize the behavior of the fuction when $x \to 0$ for the case $x < X_0^*$, we get $x^{\beta_{12}} \to \infty$ because $\beta_{12} < 0$, which is not acceptable for the value function. Updating $v_1(x)$ we obtain

$$v_1(x) = \begin{cases} A_{11}x^{\beta_{11}} + A_0 x^{\beta_{01}}, & x < X_0^* \\ \\ A_{12}x^{\beta_{11}} + B_{12}x^{\beta_{12}} + \dfrac{\lambda}{4\eta(r-\mu)(r+\lambda-\mu)}x & \\ -\dfrac{\lambda\delta}{2\eta(r+\lambda)} + \dfrac{\lambda\delta^2(r-\mu)}{4\eta(r+\lambda+\mu-\sigma^2)} \cdot \dfrac{1}{x}, & x > X_0^* \end{cases}.$$

We now consider the two cases for the value function. The first case, where $\tilde{F} < F < \overline{F}$, and the second case, where $\underline{F} < F < \tilde{F}$. We show the first case in this proposition, while the second case is showed in Proposition 5. We assume that $\tilde{F} < F < \overline{F}$, so $X_1^* < X_0^*$. Under this assumption, the value function is given by

$$V_1(x) = \begin{cases} A_{11}x^{\beta_{11}} + A_0 x^{\beta_{01}}, & x < X_1^* \\ g_1^*(x), & x \geq X_1^* \end{cases}.$$

Applying value matching and smooth pasting to solve the system and find A_{11} and X_1^*, we obtain

$$\begin{cases} A_{11}X_1^{*\beta_{11}} + A_0 X_1^{*\beta_{01}} = g_1^*(x) \\ \\ A_{11}\beta_{11}X_1^{*\beta_{11}-1} + A_0\beta_{01}X_1^{*\beta_{01}-1} = g_1^{*\prime}(x) \end{cases}.$$

Therefore,

$$A_{11} = \frac{g_1^*(X_1^*) - A_0 X_1^{*\beta_{01}}}{X_1^{*\beta_{11}}}.$$

The threshold X_1^* is determined as a solution of $h(x) = 0$, where

$$h(x) = (\beta_{11}-1)\frac{\lambda}{(\lambda+r-\mu)(r-\mu)}x^2 + 2\beta_{11}\left(\frac{F}{r+\lambda} - \delta\right)x$$
$$+(\beta_{11}+1)\frac{\left(\frac{F}{r+\lambda}-\delta\right)^2(\lambda+r-\mu)(r-\mu)}{\lambda} + 4\eta A_0(\beta_{01}-\beta_{11})x^{\beta_{01}+1}.$$

We also find that $h(\tilde{X}) < 0$, $h'(\tilde{X}) < 0$ and $\lim_{x\to+\infty} h(x) = -\infty$, so h is either strictly decreasing or increases and then decreases.

□

Proof of Proposition 3. The subsidy level $\tilde{F}$ is defined as the value of F for which $X_1^* = X_0^*$. Since the threshold X_1^* is determined by the equation $h(x) = 0$, imposing $X_1^* = X_0^*$

gives $h(X_0^*, F) = 0$. Therefore, $\tilde{F}$ is obtained as a root of this equation and is given by:

$$\tilde{F} = -\frac{\sigma^2}{2}\beta_{11}\beta_{12}\delta\left[1-\left(\frac{2\beta_{11}\lambda}{(\beta_{11}^2-1)(1-\beta_{12})}\right)\left(\frac{\beta_{01}+1}{\beta_{01}-1}\right)\right] \pm \frac{2\lambda\delta}{\sigma^2(\beta_{11}^2-1)(1-\beta_{12})}$$
$$\sqrt{-\frac{\sigma^2}{2\lambda}\beta_{11}(\beta_{11}^2-1)(1-\beta_{12})\left(\frac{\beta_{01}+1}{\beta_{01}-1}\right)+\frac{2r(\beta_{11}^2-1)(1-\beta_{12})}{\lambda\beta_{12}}\left(\frac{\beta_{01}+1}{\beta_{01}-1}\right)}$$
$$+\sqrt{\frac{2\sigma^2}{\lambda}(\beta_{11}^2-1)(1-\beta_{12})\frac{\beta_{11}-\beta_{01}}{(\beta_{01}-1)^2}+\left(\frac{\beta_{01}+1}{\beta_{01}-1}\right)^2-\frac{2\delta(\beta_{11}^2-1)(1-\beta_{12})}{\beta_{12}}\left(\frac{\beta_{01}+1}{\beta_{01}-1}\right)}.$$

The equation admits two possible solutions, but the following analysis shows that the critical subsidy level is given by the larger solution. To prove the existence and uniqueness of the critical subsidy level, we rely on the properties of the function $h(x)$ established in the proof of Proposition 4. Consequently, if $h(X_0^*) > 0$ there exists a unique threshold $X_1^* < X_0^*$. It remains to characterize the condition $h(X_0^*) > 0$ in terms of the subsidy level F. Evaluating $h(X_0^*, F)$ as a function of F, we observe that it is quadratic in F. To study the existence and uniqueness of a positive solution when $\tilde{F} < F < \overline{F}$, we used the fact that the function $h(X_0^*, F)$ is a parabola. To identify the relevant root corresponding to the critical subsidy level $\tilde{F}$, we evaluate the signs of $h(X_0^*, \underline{F})$ and $h(X_0^*, \overline{F})$:

$$h(X_0^*, \underline{F}) = 4\delta^2(r-\mu)\frac{\beta_{01}-\beta_{11}}{(\beta_{01}-1)^2} < 0;$$

$$h(X_0^*, \overline{F}) = \frac{\delta^2(r-\mu)}{(\beta_{01}-1)^2}\left[\frac{\lambda(\beta_{11}-1)(\beta_{01}+1)^2}{r+\lambda-\mu} - 4(\beta_{11}-\beta_{01})\right].$$

We know automatically that $\frac{\delta^2(r-\mu)}{(\beta_{01}-1)^2} > 0$, so we just need to prove that

$$\begin{aligned} &\frac{\lambda(\beta_{11}-1)(\beta_{01}+1)^2}{r+\lambda-\mu} - 4(\beta_{11}-\beta_{01}) > 0 \\ \Leftrightarrow\quad &\frac{1}{4}\frac{\lambda}{r+\lambda-\mu}\frac{(\beta_{11}-1)(\beta_{01}+1)^2}{\beta_{11}-\beta_{01}} > 1. \end{aligned} \tag{10}$$

We will start by simplifying $\frac{\lambda}{r+\lambda-\mu}$. By definition, the positive roots β_{01} and β_{11} must satisfy their respective fundamental characteristic equations:

$$\frac{1}{2}\sigma^2\beta_{01}(\beta_{01}-1) + \mu\beta_{01} - r = 0;$$

$$\frac{1}{2}\sigma^2\beta_{11}(\beta_{11}-1) + \mu\beta_{11} - (r+\lambda) = 0. \tag{11}$$

To isolate λ, we set both equations equal and obtain:

$$\begin{aligned}\lambda &= \frac{1}{2}\sigma^2[\beta_{11}(\beta_{11}-1)-\beta_{01}(\beta_{01}-1)]+\mu(\beta_{11}-\beta_{01})\\ &= (\beta_{11}-\beta_{01})\left[\frac{1}{2}\sigma^2(\beta_{11}+\beta_{01}-1)+\mu\right].\end{aligned}$$

From Almeida et al. (2021) we know that $\mu = \frac{\sigma^2}{2}(1-\beta_{01}-\beta_{02})$. If we substitute this in the expression obtained, we get:

$$\begin{aligned}\lambda &= (\beta_{11}-\beta_{01})\left[\frac{1}{2}\sigma^2(\beta_{11}+\beta_{01}-1+1-\beta_{01}-\beta_{02})\right]\\ &= \frac{1}{2}\sigma^2(\beta_{11}-\beta_{01})(\beta_{11}-\beta_{02}).\end{aligned}$$

Looking back at equation (11), if we isolate $(r+\lambda)$ and subtract μ in both sides, we obtain:

$$\begin{aligned}r+\lambda-\mu &= \frac{1}{2}\sigma^2\beta_{11}(\beta_{11}-1)+\mu(\beta_{11}-1)\\ &= (\beta_{11}-1)\left[\frac{1}{2}\sigma^2\beta_{11}+\mu\right].\end{aligned}$$

Once again, we substitute μ, but this time by $\mu = \frac{1}{2}\sigma^2(1-\beta_{11}-\beta_{12})$. Therefore:

$$\begin{aligned}r+\lambda-\mu &= (\beta_{11}-1)\left[\frac{1}{2}\sigma^2(\beta_{11}+1-\beta_{11}-\beta_{12})\right]\\ &= \frac{1}{2}\sigma^2(\beta_{11}-1)(1-\beta_{12}).\end{aligned}$$

Returning to Eq. 10 and substituting the expressions obtained for λ and $r+\lambda-\mu$:

$$\frac{1}{4}\left[\frac{\frac{1}{2}\sigma^2(\beta_{11}-\beta_{01})(\beta_{11}-\beta_{02})}{\frac{1}{2}\sigma^2(\beta_{11}-1)(1-\beta_{12})}\right]\frac{(\beta_{11}-1)(\beta_{01}+1)^2}{\beta_{11}-\beta_{01}} = \left(\frac{\beta_{11}-\beta_{02}}{1-\beta_{12}}\right)\frac{(\beta_{01}+1)^2}{4}.$$

We also can conclude that

$$\beta_{01}+\beta_{02} = \beta_{11}+\beta_{12} \implies \beta_{11}-\beta_{02} = \beta_{01}-\beta_{12}.$$

Substituting this identity into the numerator of the first fraction:

$$\left(\frac{\beta_{01}-\beta_{12}}{1-\beta_{12}}\right)\frac{(\beta_{01}+1)^2}{4} = \left(1+\frac{\beta_{01}-1}{1-\beta_{12}}\right)\frac{(\beta_{01}+1)^2}{4} > 1.$$

Therefore, if $h(X_0^*,\underline{F})<0$ and $h(X_0^*,\overline{F})>1$, the root between them is $\tilde{F}$ and is unique.

□

Proof of Proposition 5. We now consider the case $\underline{F} < F < \tilde{F}$ for which $X_0^* < X_1^*$. Under this assumption, the value function is given by

$$V_1(x) = \begin{cases} A_{11}x^{\beta_{11}} + A_0 x^{\beta_{01}}, & x < X_0^* \\ A_{12}x^{\beta_{11}} + B_{12}x^{\beta_{12}} + \dfrac{\lambda}{4\eta(r-\mu)(r+\lambda-\mu)}x \\ -\dfrac{\lambda\delta}{2\eta(r+\lambda)} + \dfrac{\lambda\delta^2(r-\mu)}{4\eta(r+\lambda+\mu-\sigma^2)} \cdot \dfrac{1}{x}, & X_0^* \leq x < X_1^* \\ g_1^*(x), & x \geq X_1^* \end{cases}.$$

The constants A_{11} and B_{12} are obtained by imposing continuity and differentiability at X_0^*, where the first two branches meet:

$$\begin{cases} A_{11}X_0^{*\beta_{11}} + A_0 X_0^{*\beta_{01}} = A_{12}X_0^{*\beta_{11}} + B_{12}X_0^{*\beta_{12}} \\ \quad + \dfrac{\lambda}{4\eta(r-\mu)(r+\lambda-\mu)}X_0^* - \dfrac{\lambda\delta}{2\eta(r+\lambda)} + \dfrac{\lambda\delta^2(r-\mu)}{4\eta(r+\lambda+\mu-\sigma^2)}\dfrac{1}{X_0^*} \\ A_{11}\beta_{11}X_0^{*\beta_{11}-1} + A_0\beta_{01}X_0^{*\beta_{01}-1} = A_{12}\beta_{11}X_0^{*\beta_{11}-1} + B_{12}\beta_{12}X_0^{*\beta_{12}-1} \\ \quad + \dfrac{\lambda}{4\eta(r-\mu)(r+\lambda-\mu)} - \dfrac{\lambda\delta^2(r-\mu)}{4\eta(r+\lambda+\mu-\sigma^2)}\dfrac{1}{X_0^{*2}} \end{cases}$$

$$\Leftrightarrow \begin{cases} A_{11} = A_{12} + B_{12}X_0^{*\beta_{12}-\beta_{11}} \\ \quad + \left[\dfrac{\lambda}{4\eta(r-\mu)(r+\lambda-\mu)}X_0^* - \dfrac{\lambda\delta}{2\eta(r+\lambda)} + \dfrac{\lambda\delta^2(r-\mu)}{4\eta(r+\lambda+\mu-\sigma^2)}\dfrac{1}{X_0^*} - A_0X_0^{*\beta_{01}}\right]X_0^{*-\beta_{11}} \\ B_{12} = \dfrac{1}{(\beta_{11}-\beta_{12})X_0^{*\beta_{12}-1}}\left[\dfrac{\lambda}{4\eta(r-\mu)(r+\lambda-\mu)} - \dfrac{\lambda\delta^2(r-\mu)}{4\eta(r+\lambda+\mu-\sigma^2)}\dfrac{1}{X_0^{*2}}\right. \\ \quad - \dfrac{\beta_{11}}{X_0^*}\left(\dfrac{\lambda}{4\eta(r-\mu)(r+\lambda-\mu)}X_0^* - \dfrac{\lambda\delta}{2\eta(r+\lambda)} + \dfrac{\lambda\delta^2(r-\mu)}{4\eta(r+\lambda+\mu-\sigma^2)}\dfrac{1}{X_0^*}\right) \\ \quad \left. + A_0(\beta_{11}-\beta_{01})X_0^{*\beta_{01}-1}\right] \end{cases}.$$

We now impose value matching and smooth pasting at X_1^*, with the last two branches, in order to compute A_{12} and X_1^*.

$$\begin{cases} A_{12}X_1^{*\beta_{11}} + B_{12}X_1^{*\beta_{12}} + \frac{\lambda}{4\eta(r-\mu)(r+\lambda-\mu)}X_1^* - \frac{\lambda\delta}{2\eta(r+\lambda)} + \frac{\lambda\delta^2(r-\mu)}{4\eta(r+\lambda+\mu-\sigma^2)}\frac{1}{X_1^*} = g_1^*(X_1^*) \\ A_{12}\beta_{11}X_1^{*\beta_{11}-1} + B_{12}\beta_{12}X_1^{*\beta_{12}-1} + \frac{\lambda}{4\eta(r-\mu)(r+\lambda-\mu)} - \frac{\lambda\delta^2(r-\mu)}{4\eta(r+\lambda+\mu-\sigma^2)}\frac{1}{X_1^{*2}} = g_1^{*\prime}(X_1^*) \end{cases}$$

where

$$\begin{aligned} A_{12} &= g_1^*(X_1^*)X_1^{*-\beta_{11}} - B_{12}X_1^{*\beta_{12}-\beta_{11}} \\ &\quad + \left[-\frac{\lambda}{4\eta(r-\mu)(r+\lambda-\mu)}X_1^* + \frac{\lambda\delta}{2\eta(r+\lambda)} - \frac{\lambda\delta^2(r-\mu)}{4\eta(r+\lambda+\mu-\sigma^2)}\frac{1}{X_1^*} \right] X_1^{*-\beta_{11}}. \end{aligned}$$

The threshold X_1^* is determined as a solution of $f(x) = 0$, where

$$f(x) = \beta_{11}Mx + (\beta_{11}+1)N + (\beta_{12}-\beta_{11})B_{12}x^{\beta_{12}+1},$$

with

$$M = \frac{F - \delta r}{2\eta(r+\lambda)}$$

and

$$N = \frac{1}{4\eta}\left[\frac{\left(\frac{F}{r+\lambda} - \delta\right)^2 (\lambda + r - \mu)(r-\mu)}{\lambda} - \frac{\lambda\delta^2(r-\mu)}{r+\lambda+\mu-\sigma^2} \right].$$

□

Proof of Proposition 6. From Almeida et al. (2021) we know that $r = -\frac{\sigma^2}{2}\beta_{01}\beta_{02}$ and $\mu = \frac{\sigma^2}{2}(1-\beta_{01}-\beta_{02})$. Therefore, substituting these expressions in X_0^* (Eq. (6)), we obtain

$$X_0^* = \delta\frac{\sigma^2}{2}(\beta_{01}+1)(1-\beta_{02}). \tag{12}$$

Also, through Almeida et al. (2021) we know that

$$\frac{\partial\beta_{01}}{\partial\sigma} = -\frac{\sigma\beta_{01}(\beta_{01}-1)}{\sqrt{\left(\mu-\frac{1}{2}\sigma^2\right)^2 + 2\sigma^2 r}} \quad \text{and} \quad \frac{\partial\beta_{02}}{\partial\sigma} = \frac{\sigma\beta_{02}(\beta_{02}-1)}{\sqrt{\left(\mu-\frac{1}{2}\sigma^2\right)^2 + 2\sigma^2 r}}; \tag{13}$$

$$\frac{\partial\beta_{01}}{\partial\mu} = -\frac{\beta_{01}}{\sqrt{\left(\mu-\frac{1}{2}\sigma^2\right)^2 + 2\sigma^2 r}} \quad \text{and} \quad \frac{\partial\beta_{02}}{\partial\mu} = \frac{\beta_{02}}{\sqrt{\left(\mu-\frac{1}{2}\sigma^2\right)^2 + 2\sigma^2 r}}. \tag{14}$$

To compute how the unsubsidized investment threshold X_0^* varies with volatility, we use

Eq. (12) and Eq. (13):

$$\frac{\partial X_0^*}{\partial \sigma} = \delta\sigma(\beta_{01}+1)(1-\beta_{02}) + \delta\frac{\sigma^2}{2}\left[\frac{\partial \beta_{01}}{\partial \sigma}(1-\beta_{02}) - (\beta_{01}+1)\frac{\partial \beta_{02}}{\partial \sigma}\right] = \delta\sigma\frac{2\beta_{01}(1-\beta_{02})}{\beta_{01}-\beta_{02}} > 0.$$

To compute how the unsubsidized installed capacity K_0^* varies with volatility, we use Eq. (8) and Eq. (13):

$$\frac{\partial K_0^*}{\partial \sigma} = \frac{\partial K_0^*}{\partial \beta_{01}}\frac{\partial \beta_{01}}{\partial \sigma} = \left(-\frac{1}{\eta(\beta_{01}+1)^2}\right)\left(-\frac{\sigma\beta_{01}(\beta_{01}-1)}{\sqrt{\left(\mu-\frac{1}{2}\sigma^2\right)^2+2\sigma^2 r}}\right) > 0.$$

To compute how the unsubsidized investment threshold X_0^* varies with the drift, we use Eq. (12) and Eq. (14):

$$\frac{\partial X_0^*}{\partial \mu} = \delta\frac{\sigma^2}{2}\left[\frac{\partial \beta_{01}}{\partial \mu}(1-\beta_{02}) - (\beta_{01}+1)\frac{\partial \beta_{02}}{\partial \mu}\right] = \frac{\delta\left(\mu-\frac{\sigma^2}{2}\right)}{\sqrt{\left(\mu-\frac{1}{2}\sigma^2\right)^2+2\sigma^2 r}}.$$

The sign of the expression obtained depends on the sign of $\mu - \frac{\sigma^2}{2}$. So, if $\mu < \frac{\sigma^2}{2}$, then $\frac{\partial X_0^*}{\partial \mu} < 0$. If $\mu = \frac{\sigma^2}{2}$, then $\frac{\partial X_0^*}{\partial \mu} = 0$. Finally, if $\mu > \frac{\sigma^2}{2}$, then $\frac{\partial X_0^*}{\partial \mu} > 0$. To compute how the unsubsidized installed capacity K_0^* varies with the drift, we use Eq. (8) and Eq. (14):

$$\frac{\partial K_0^*}{\partial \mu} = \frac{\partial K_0^*}{\partial \beta_{01}}\frac{\partial \beta_{01}}{\partial \mu} = \left(-\frac{1}{\eta(\beta_{01}+1)^2}\right)\left(-\frac{\beta_{01}}{\sqrt{\left(\mu-\frac{1}{2}\sigma^2\right)^2+2\sigma^2 r}}\right) > 0.$$

To compute how the unsubsidized investment threshold X_0^* varies with investment cost, we use Eq. (6):

$$\frac{\partial X_0^*}{\partial \delta} = (r-\mu)\frac{\beta_{01}+1}{\beta_{01}-1} > 0.$$

Finally, to compute how the unsubsidized installed capacity K_0^* varies with the sensitivity constant η, we use Eq. (8):

$$\frac{\partial K_0^*}{\partial \eta} = -\frac{1}{\eta^2(\beta_{01}+1)} < 0.$$

□