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Bond Value-at-Risk: a comparison of methods

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Abstract

This dissertation will compare three ways of computing the Value-at-Risk of a fixed income portfolio. The first being the Cash Flow Mapping [15], the second one being the Pull Price method [17] and the last one being the Duration Method [10]. To compare the results this study computes the VaR through the different methods for past days and then uses a backtesting tool to measure each method's consistency. This study will show that the results from the Duration Method were vastly overestimated and that the Pull Price Method got a good approval rate of the backtesting technique when applied for a portfolio. The results from the Cash Flow Mapping are subjected to a more careful analysis since the underlying risk factor used was the European Central Bank yield curve for our portfolio of corporate bonds. Even so, the method failed to meet expectations since there are numerous bonds whose rating is eligible to be used under this yield curve.

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1 INTRODUCTION

1.1 Introduction to VaR

One way to help a manager assess his portfolio's risk is the Value-At-Risk (VaR). The VaR answers the manager's problem the following way:

"We are X percent certain that we will not lose more than V dollars in the next N days." [8]

This is, in fact, one of the biggest advantages of the Value at risk as described by Pearson [12] which is its simplicity and ability to focus all the decision making into one number. But there's also criticism in all this simplicity. Taleb is an active critic of the VaR risk tool since "*it tries to estimate something that is not scientifically possible to estimate, namely the risks of rare events*"[18]. Other critics like Heihorn render the VaR useless because it neglects what happens at the extreme tails. So if, for instance, a VaR is computed at an $\alpha = 1\%$, then the VaR doesn't give us any information beyond the 1%'s quantile [6].

Despite all this, it still is a widely used tool to compute the holding's risk [9]. For this reason, VaR computations influence capital requirements for the institutions in compliance to the Basel III guidelines. In the case that the number of exceedances of the actual returns is abnormally high, compared to the VaR estimates, a penalty will be charged to the VaR estimates that can go as high as 3 times the VaR estimated [2]. Researchers found evidence that this penalty leads the way to intentionally flawed VaR estimates suggesting that companies have incentive to underestimate their VaR estimations to incur smaller penalties [14]. However studies like [16] point on the opposite direction finding evidence that banks overestimated their Value at Risk showing an high degree of conservatism. The two main causes are the institution's reputation and to minimize the chance of having to disclose information about the excessive number of exceedances. Next we will see the relationship between VaR and fixed income products.

1.2 VaR and Bonds

To reach the VaR we need to know what is the nature of the underlying asset or assets we are computing the VaR on. This is relatively simple for a stock or stock portfolio as historical data can be directly applied to the historical simulation [12] where one can use the historical prices to compute historical returns and, finally, the standard deviation of these returns. This is not as linear for a bond or a bond portfolio.

For a bond or bond Portfolio additional factors must be taken into consideration. Three aspects are to note:

- the characteristics of a bond change everyday since the maturity changes everyday
- there are cash flows in the future
- there is a pull to par effect

Bonds are more sensitive to interest rate risk factors than stocks. It is impossible to introduce interest rate risk without mentioning the yield curve. The yield curve represents, among other important things, the expectations of investors towards future economic health of the issuer. An issuer with negative future prospectives will raise the probability of default which will inflate the return asked by the investor. This also means that different maturities have different risks which is the fundamental problem of applying a bond's historical prices directly to the VaR. Despite all this, some authors still apply historical simulation to calculate VaR of bonds [19]. However there are ways of going around this. One way is the Cash Flow Mapping which stretches in a time line all the cash flows in bond portfolio either being principal payments or coupon payments and treats them like zero coupon bonds. Later, it maps these cash flows into standard maturities which have available historical yields which will be used to compute the VaR.

The Pull Price method simplifies this problem since it does not need historical yields to compute the VaR because it uses the adjusted returns based on the pulled prices.

At last, the Duration method uses the Duration of a bond to reach the VaR. Since Duration is a measure of a bond's price sensitivity to changes in the interest rates, the VaR will be the change in price correspondent to the worst, relevant quantile, change in yield. All these methods will be further developed ahead in this dissertation.

This dissertation aims to verify each method's credibility to measure VaR for bonds. To do this we will use a simple backtesting tool that will be developed next. For all the methods, the VaR was computed at an $\alpha = 5\%$ and $\alpha = 1\%$ with a time horizon of 10 days following the Bank of International Settlement's regulations. [2]

Table 1: Portfolio composition

ISSUER	ISSUE DATE	COUNTRY	RATING	MATURITY	CPN
AIB MORTGAGE BANK	26-06-2007	IE	A-	29-06-2017	4,875
AYT CEDULAS CAJAS GLOBAL	13-03-2007	ES	BBB	21-03-2017	4
AYT CEDULAS CAJAS GLOBAL	12-12-2006	ES	BBB+	20-12-2016	4
SNS BANK NV	03-03-2010	NL	AA-	10-03-2017	3,625
BANCO POPOLARE SC	24-02-2010	IT	BBB+	31-03-2017	3,625
BANCO DE SABADELL SA	09-01-2007	ES	A-	24-01-2017	4,25
BANCO BILBAO VIZCAYA ARG	05-01-2010	ES	A	18-01-2017	3,625
BANCA MONTE DEI PASCHI S	09-03-2011	IT	BBB	15-09-2016	4,875
CAIXABANK	08-02-2012	ES	A+	16-02-2017	4
BANCO SANTANDER SA	18-03-2010	ES	A	06-04-2017	3,625
INTESA SANPAOLO SPA	13-07-2012	IT	NR*	28-04-2017	3,25
BANKIA SA	24-09-2009	ES	NR	05-10-2016	3,625
BANCA POPOLARE DI MILANO	01-10-2009	IT	BBB	17-10-2016	3,5
BANK OF SCOTLAND PLC	31-05-2007	GB	AAA	08-06-2017	4,625
BANCO ESPANOL DE CREDITO	10-09-2012	ES	A	24-01-2017	4,75
BANCO SANTANDER SA	31-08-2006	ES	A	09-01-2017	4,125
CRED MUTUEL- CIC HOME LO	11-02-2010	FR	AAA	25-04-2017	3,5
DEXIA KOMMUNALBANK AG	05-01-2010	DE	NR	12-01-2017	3,375
NR = No Rating					

2 Methodology

2.1 The Portfolio

The portfolio used in this dissertation is composed by eighteen European live ¹ coupon bearing bonds with different issuers, maturities, countries and credit ratings. Since it is completely composed of European bonds, the application of the Cash Flow Mapping is done with the Euro area yield curve. The portfolio composition, and respective characteristics can be seen in Table 1.

To achieve its conclusions, this study computed the VaR of all days between the 1st of April of 2010 until the 3rd of March of 2014. The criteria for the choice of these dates were (i) wide enough to provide a good historical

¹live to 03-03-2014

background and (ii) recent data so it's easily accessible.

2.2 Backtesting

To test if the VaR results were acceptable a simple backtesting tool was used. Backtesting is a tool that assess the accuracy of the VaR estimates [10]. The backtesting tool used in this study is the Kupiec's Unconditional Coverage test [3, 5, 11, 16]. The method consists of counting the amount of samples of historical returns that actually exceeded the estimated VaR for each method. In other words, if the VaR at $\alpha = 1\%$ is X then the method is considered adequate if the amount of exceedances is around 1%. A good method will, then, offer a number of exceedances closest to 1% in this scenario. This is measured using significance test of the exceedances at 95% and 99% with a null hypothesis of

$$H_0 : \text{Correct number of exceedances}$$

and an alternative hypotheses of

$$H_1 : \text{Incorrect number of exceedances}$$

Despite its simplicity, this backtesting tool has been known to under report the VaR estimates. In other words, if the $\alpha = 1\%$ then this test has a chance to report results for an "actual" $\alpha = 2\%$ or higher [5]. Additionally [11] also concluded that the method is prone to misconclusions for limited sized samples which is not the case of the sample used in this study.

2.3 Cash Flow Mapping

In an effort to create a homogeneous market practice, J.P. Morgan released a document called RiskMetrics with guidelines on how to compute risks of a set of financial instruments like equities, FOREX, commodities, fixed income products and their derivatives. Among the instruments to compute fixed income risk there is the Cash Flow Mapping method(CFM). As said before, the CFM stretches on a time line all of the portfolio's cash flows either being coupon payments or principal payments and considers them to be separate zero coupon bonds. Later they split these zero coupon bonds into standard maturities which are maturities that have historical yields available. JP Morgan's Risk Metrics sets the following conditions for these split zero coupon bonds [15]

- Market value is preserved. The total market value of the two split zero coupon bonds' cash flows must be equal to the market value of the original cash flow.
- Market risk is preserved. The market risk of the portfolio of the split zero coupon bonds must also equal the market risk of the original cash flows.
- Sign is preserved. The split zero coupon bonds have the same sign as the original cash flows.

The standard maturities were defined according to the available historical rates at the European Central Bank's database which has more detailed information. Those maturities are 1, 5, 10, 20 and 30 years for the long term and 3, 6 and 9 months for the short term maturities. Mark Henrard argues that one could indeed add more standard maturities to the mapping procedure. However some specific maturities might not be liquid enough to extract data necessary for the method. Additionally, by adding more standard maturities into the mapping can lead to problems when creating a covariance matrix of all standard maturities since there's a risk that the matrix is singular or, in other words, some combinations of positions are riskless [7]. Henrard also argues that adding too many maturities it becomes increasingly harder to extract results as the covariance-matrix grows with the *square* of the standard maturities.

The main struggle of the method is data. This method is relatively easy to use if we are analysing government issues that have broad historical data we can use to compute the yield volatility. Corporate bonds on the other hand require some extra effort to extract yield volatilities since individual corporations issue less debt than governments. Few literature exists where the method is applied and tested for corporate debt due to the fact that historical rates for the defined standard maturities are either scarce or non-existent. A few exceptions that mention this method are [4] which applies the CFM using the no longer available RiskMetrics data set and [20] which applies the CFM to riskless derivative products. At the time of the issue of the RiskMetrics document, JP Morgan additionally released a data set containing volatilities and correlations between instruments in 33 countries [15]. Later it was named DataMetrics and, unlike in the past, it is no longer disclosed information to the public but rather limited to users of a prepaid software. For this reason, the yield curve considered for this method was the Euro area yield curve from the European Central bank.

The CFM has the following assumptions [15]:

- there's a linear path between standard maturities
- there's a linear path between standard deviation of standard maturities
- assumes normality of returns

There are four methods to allocate each individual cash flow to the corresponding bucket[1]

- Present Value Invariant and Duration Invariant Maps
- Dollar Present Value Invariant Cash Flow Maps
- Volatility Invariant Maps

The Dollar Present Value Invariant Cash Flow Map allocation method uses rates to compound 1\$ into the next standard maturities. This allocation method differs if the rates are continuously compounded (r) or discretely compounded(R). The Volatility Invariant Map allocation method requires that combination of the volatilities of the adjacent vertices of the cash flow σ_1 and σ_2 must be equal to the volatility of the original cash flow σ .

In this study, the allocation of cash flows will be done with the first method where, for a cash flow at time t (cf_t), where $t_1 < t < t_2$, the combination of positions in the adjacent buckets t_1 and t_2 will have a Present Value and Modified Duration equal to that of the original cash flow cf_t following the guidelines of JP Morgan. For the Present Value to remain invariant the following formula is applied

$$x_1 + x_2 = PV_{cf_t} \tag{1}$$

Where x_1 and x_2 are the weights of the positions in the standard maturities t_1 and t_2 respectively in present value terms. Because this condition alone is ambiguous since there are infinite ways it can be satisfied we need to add the Duration constraint

$$x_1T_1 + x_2T_2 = (x_1 + x_2)T \tag{2}$$

where T_1 and T_2 are the Durations of the standard maturities t_1 and t_2 respectively. To successfully map cf_t we need to compute a system of equations with (1) and (2). For computing the present value of cf_t , the rates were extracted from the Euro area yield curve. Next comes the computation of the VaR once cf_t has been mapped. This can be done in 4 steps [8]

- Extract volatilities of the yields of the reference maturities t_1, t_2
- compute the correlation matrix of the returns of the reference maturities' prices
- compute the variance of the theoretical portfolio composed of positions in t_1 and t_2 using the multi-asset portfolio variance formula

$$\sigma_p^2 = \sum_{i=1}^n \alpha_i^2 \sigma_i^2 + 2 \sum_{i=1}^n \sum_{j<i}^n \rho_{ij} \alpha_i \alpha_j \sigma_i \sigma_j \quad (3)$$

- compute the VaR with confidence interval X , where $0 < X < 1$, and time horizon N with the following formula

$$VaR = \sigma_p \times z_X \times \sqrt{N} \quad (4)$$

London Stock Exchange Group proposes a multiple between 2 and 4 of the riskless VaR for corporate bond VaR depending on the credit risk of the issuer [13]. Unfortunately the exact multiplier for each credit rating isn't specified. Because we are using the yield curve of the Euro area, this implies that we are calculating the CFM VaR of a bond if it were without risk.

2.4 Pull Price Method

CFM has a lot of assumptions that ruin its objectivity. Not only that but also the lack of historical yields in some cases pose some problems for the model. This method simplifies the VaR computation by using the historical prices directly. It has the following assumptions:

- yield to maturity is enough to measure return as it takes into account credit, interest rate, and liquidity risk among other.
- assumes flat forward rates for the computation of the *pulled prices*

The method can be described as the following: If the N-day return is defined by

$$HR(n, N) = \frac{p(n)}{p(n-1)} \quad (5)$$

and considering that P is the principal amount, $r(n)$ the yield at time n and T is the maturity. The method argues that, if the implied yield $r(n)$ formula is

$$\left(\frac{P}{p(n)}\right)^{\frac{1}{T-n}} - 1 \quad (6)$$

and the pulled price at time m ($n_{VaR} \leq m < T$) of a bond bought at time n ($1 \leq n < n_{VaR}$) is $v(m, n)$ then the formula for this pulled price for an asset purchased at time n at time m is

$$v(n_{VaR} + N, n) = \frac{P}{(1 + r(n))^{T-n_{VaR}+N}} \quad (7)$$

but because this implies that the rate $r(n)$ should have a maturity m makes no sense in the context of a manager wanting to compute the VaR today. Some further adjustments are in order. If n_{VaR} represents the most recent day with historical data and a time horizon N is defined for VaR purposes, the paper adjusts the historical prices the following way:

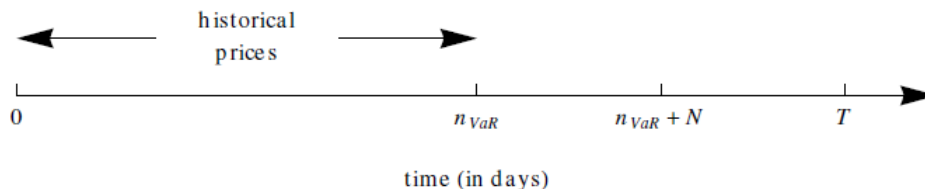
- for each historical price $p(n)$, in (5), we compute the corresponding pulled price $v(n_{VaR} + N, n)$, at time $n_{VaR} + N$, matching the VaR relevant maturity $T - (n_{VaR} + N)$;
- for each historical price $p(n - N)$, in (5), we compute the corresponding pulled price $v(n_{VaR}, n - N)$, at time n_{VaR} , matching the VaR relevant maturity $T - n_{VaR}$;
- adjust each historical return, of the HR equation, for the VaR relevant maturities by replacing $p(n)$ by $v(n_{VaR} + N, n)$ and $p(n - N)$ by $v(n_{VaR}, n - N)$.

This implies that the historical return, now adjusted (AHR), should be computed the following way:

$$AHR = \frac{v(n_{VaR} + N, n)}{v(n_{VaR}, n - N)} \quad (8)$$

By computing this AHR for all available n we can plot a distribution from which we can, finally, find the convenient quantile for the VaR. In figure 1 we can see the relevant maturities defined by the method for a more clear idea.

Figure 1: Relevant Maturities of the Pulled Price Method



2.5 Duration method

The Duration Method is a tool that computes the Value at Risk of a bond based on its duration. It has the following assumptions[Jorion P, 2007]:

- bond's daily returns follow a normal distribution
- bond's daily returns are independent, identically distributed

To get the VaR it starts by computing the Dollar Duration ($\$Dur$) of a bond

$$\$Dur = \left(\frac{-1}{1+y} \sum \frac{iCF_i}{(1+y)^i} \right) \quad (9)$$

where CF represents the coupon and principal cash flows and i the time discounting factor. It then extracts the annualized yield-return volatility $\sigma(dy)$ from historical data based on a minimum amount of observations. This study set these minimum observations to 250, the number of working days in a year.

It then sets the confidence interval of the VaR and deduce the corresponding $z_{statistic}$. The quantile-specific worst daily movement in yield-return, $worst(dy)$, is then

$$worst(dy) = \sigma(dy) \times \sqrt{\frac{N}{250}} \times z_{statistic} \quad (10)$$

The VaR, in money terms, of the bond will be

$$VaR_i = worst(dy) \times \$Duration \quad (11)$$

There is a drawback for this method as it requires a reasonable amount of historical data. The more observations, the better the distribution which implies that the less observations, the less representative the distribution becomes.

3 VaR and its application to Portfolios

In this study, the portfolio considered has equal weights among the different bonds. There are two methods of applying the individual VaRs to a portfolio of bonds, the Diversified and Undiversified VaR [Jorion, 2009]. Both methods deliver the VaR but with different assumptions:

- the diversified VaR accounts for diversification benefits while
- the undiversified VaR doesn't account for diversification.

To compute the Diversified VaR of a portfolio the follow equation must be computed

$$VaR = \sqrt{(xV)R(xV)} \quad (12)$$

where xV are the individual VaRs of each bond, R is the correlation matrix and $R(xV)$ is the product of each individual VaR $(xV)_i$ by the corresponding correlation vector R_i .

For the Undiversified VaR, the portfolio VaR is simply the sum of the individual VaRs. Because in this dissertation considers equal weights, the following formula applies

$$VaR = \frac{\sum VaR_i}{n} \quad (13)$$

where VaR_i are the individual VaRs and n are the number of bonds in the portfolio. In this study the method considered was the Undiversified VaR.

4 The individual results

In this Section, this dissertation will analyse the results obtained following the Methodology Section. The VaR was applied for three different methods with three different outcomes. As earlier explained, the accuracy of the VaR

was tested using a backtesting technique that involves comparing the actual historical N-day returns to the VaR estimates with a significance test where

$$H_0 : \text{"Correct number of exceedances"}$$

and an alternative hypotheses of

$$H_1 : \text{"Incorrect number of exceedances"}$$

The significance test was run at a confidence level of 95% and 99% for a 10-day VaR of $\alpha = 0.01$ and $\alpha = 0.05$.

4.1 Cash Flow Mapping

For the cash flow mapping procedure the results were, as expected, negative. For the 18 bonds in our portfolio, all H_0 were rejected due to the fact that all VaR's were underestimated and the the number of exceedances was too great. This is no surprise since the risk factor considered in this study is the European yield curve and not an appropriate yield curve. Some authors like Watanabe [20] retrieved positive results from the Cash Flow Mapping. However, the underlying risk factor in this case was the forward curve unlike in our case where the underlying risk factor are interest rates.

As said before, it is of no surprise that the results were so negative since the underlying risk factor considered was the European Central Bank yield curve but what catches our attention is that, in our portfolio, some bonds actually had a credit rating worthy of rendering this curve appropriate. Bonds like the Bank of Scotland and Credit Mutuel have a credit rating of AAA, a credit rating that is considered to be of the highest quality according to Standard & Poors. S&P differentiates this rating from the other ratings for the issuer's ability to meet their financial obligations. Even so, it appears that the method failed to meet the expectations. This data led us to believe that this method might not be the appropriate way to compute the VaR of a bond. For illustration purposes, in Figure 2 we can see how small the VaR estimates were. All these estimates were easily exceeded by the actual historical returns leading to the rejection of H_0 for all cases as we can see in Tables 2, 3, 4 and 5.

4.2 Duration Method

For the Duration method, this study's results were highly negative as we can see in Tables 10, 11, 12 and 13. For all eighteen bonds in our portfolio, the VaR estimates were overestimated for eighteen bonds. In other

Figure 2: CFM VaR for Bond 1

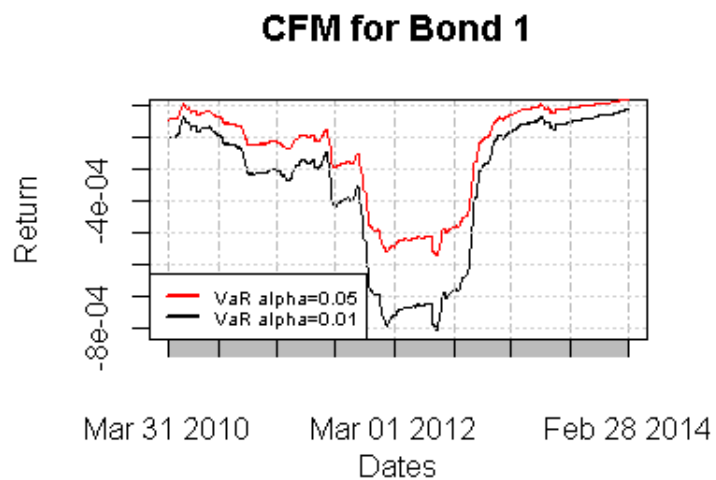
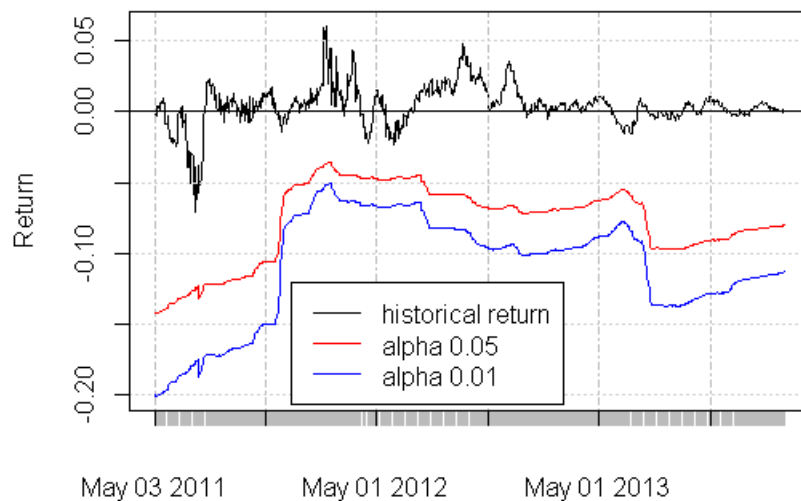


Figure 3: Duration Method VaR



words, the realized exceedances were, eighteen out of the eighteen times, zero most of the times. For illustration purposes, at Figure 3, we can find the VaR estimates for the first bond. Take note that all the remaining bonds followed this overestimating tendency. This makes us believe that some further refinement of the method is in order since the Duration method failed to perform due to its extremely "pessimistic" forecasts. One aspect that might have contributed to these results is the fact that historical volatilities of the yield-returns are used. This can partially explain the problem since the conditions of a bond change everyday due to the pull-to-par effect. In other words, the yield returns calculated in the beginning of the time series will have different dimensions from yield returns calculated close to the maturity. This will have consequences in the volatility of these yield returns which will have an impact on the overall model's returns. A good counter-argument would be the fact that this dissertation defined an horizon of 250 observations so this effect wouldn't be felt so harshly. This is, in fact, the reason why we believe that this only explains the negative results partially.

A good approach would be to make a distribution out of the yield-returns and compute the quantile-specific best yield return registered and follow the method's steps from there. This way the model would receive the quantile-specific best shift in the yield and compute the VaR from there on. Here follows a simple example. What the original method does is to compute the yield-returns and then its corresponding standard deviation $\sigma(dy)$. What we propose is that instead of computing this standard deviation, we build a distribution of all the yield-returns and compute the quantile-specific biggest yield returns which will be used for the rest of the computations. Note that the biggest quantile-specific yield return represents the quantile-specific worst movement in price due to the inverse relationship between yields and prices. This study did not include this approach in its calculations.

4.3 Pull Price Method

For the Pull Price Method, the VaR throughout the historical data appears to have a unique behaviour when it's applied for days closer to the coupon dates. This behaviour can be observed in Figure 6. Upon closer inspection, it is believed that this behaviour happens due to VaR computations around coupon dates where $nVaR$ and $nVaR+N$ are around a coupon delivery date. The problem resides in the fact that, when "pulling" the future cash flows, at $nVaR$ the accrued coupon rate is very high since it's delivery is imminent

and on the other hand, at $nVaR+N$, the accrued coupon rate is very low since it's shortly after the delivery of the said coupon. The gap between accrued rates led to an abnormally high loss in return since the day $nVaR$ registered a high accrued and, therefore, a higher "gross pulled price" and $nVaR+N$ a low accrued and, there fore, a lower "gross pulled price" and computing the return between these two, the results tended to be negative. In Figure 4 we can see that the gross returns of the pulled prices at a $nVaR$ before a coupon payment date are consistently negative. This led to a VaR estimation abnormally low that stands out when we compute VaR's everyday as we can see in Figure 6. To test this behaviour, this study used the corresponding clean prices, the results were favourable as we can see in Figure 4 and 5. The returns shifted from a consistently negative return to a return with average closer to zero. To reach these clean prices, this study deduced, from the gross prices, the corresponding accrued coupon rates at time $nVaR$ and $nVaR+N$.

$$Clean\ Pull\ Price = v(n_{VaR} + N, n) - Accrued_{n_{VaR}+N}$$

Here follows an example. Assume $N = 10\ days$ and take the first bond for example, a 10-year coupon bond with yearly coupon payments at the 29th of June every year between 2007 and 2017. The n_{VaR} relevant maturity is the 25th of June of 2011 and n_{VaR+N} is the 5th of July of the same year. At the 6th of April of 2011 the yield-to-maturity of the bond is 9.2% and the time to maturity from n_{VaR} and n_{VaR+N} is 6.02 and 5.99 years. When pulling all the future cash flows post the n_{VaR} and n_{VaR+N} we have

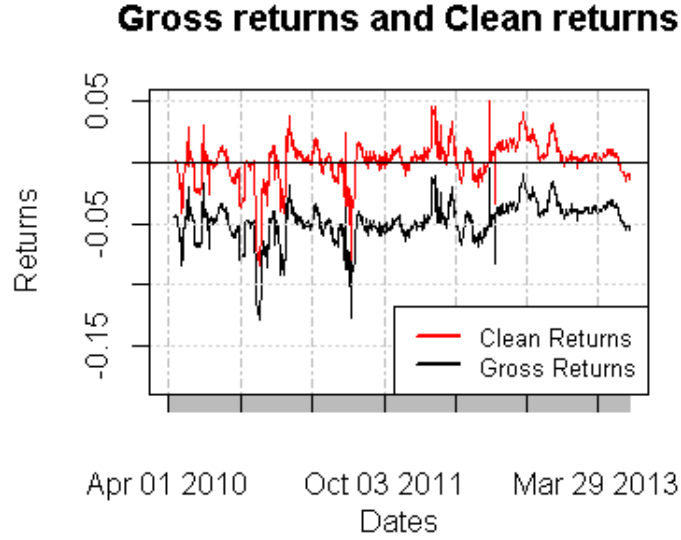
$$v(n_{VaR}, n) = \frac{4.875}{(1 + 0.092)^{0.02}} + \frac{4.875}{(1 + 0.092)^{1.02}} + \dots + \frac{4.875}{(1 + 0.092)^{5.02}} + \frac{104.875}{(1 + 0.092)^{6.02}} = 85.44$$

and

$$v(n_{VaR+N}, n) = \frac{4.875}{1 + 0.092^{0.99}} + \frac{4.875}{1 + 0.092^{1.99}} + \dots + \frac{4.875}{1 + 0.092^{4.99}} + \frac{104.875}{1 + 0.092^{5.99}} = 80.78$$

We can see immediately that there is a great loss in this random date's pulled price. A loss of roughly 5.4% in just 10 days which is very unlikely. This behaviour is repeated for every other random day prior do n_{VaR} which will tilt down the VaR for dates around the coupon payment date when finding the desired quantile. Our proposal corrects this issue by deducing the corresponding accrued rate of n_{VaR} and n_{VaR+N} from the pull price, smoothing the return between these prices.

Figure 4: 10-day Gross returns and Clean returns



$$v_*(nVaR, n) = 85.44 - accrued_{nVaR} = 85.44 - 4.83 = 80.61$$

and

$$v_*(nVaR+N, n) = 80.78 - accrued_{nVaR+N} = 80.78 - 0.093 = 80.69$$

We can now observe that the return between these two pulled prices has turned into a more acceptable return of 0.01% 10-day adjusted return of the pulled prices.

The backtesting technique's conclusions, for the Pull Price Method with Gross prices, can be seen in Tables 6, 7, 8 and 9.

From these tables we can see that the performance of this method decreases when the α of the VaR is decreased from 5% to 1%. It is to note that in this study, the historical data horizon is of 250 days. Even though the method fails to achieve at the $\alpha = 1\%$, it still has a relatively low approval rate for the $\alpha = 5\%$ of 38.8% for a confidence interval of 99% and 16,7% for a confidence interval of 95%².

²Computed dividing the approved bonds over the total

Figure 5: VaR at $\alpha = 0.05$ of Gross and Clean pull price

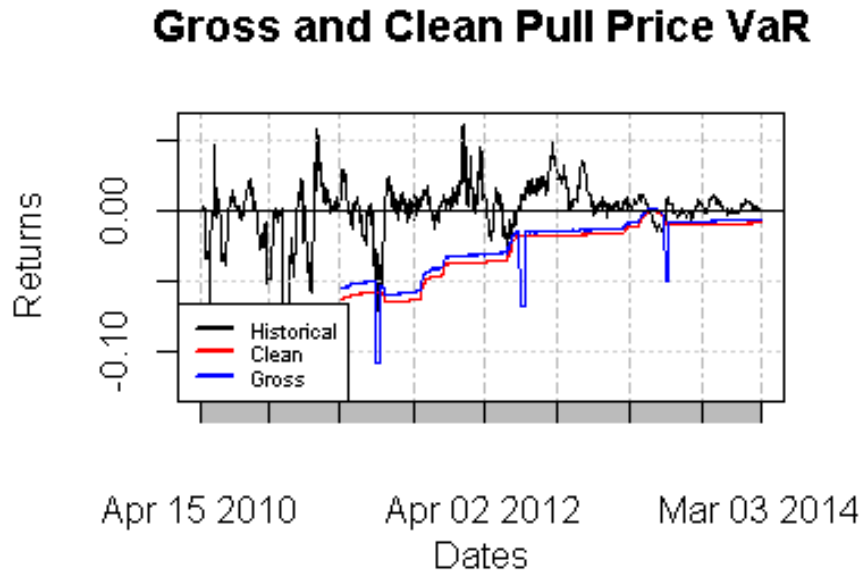
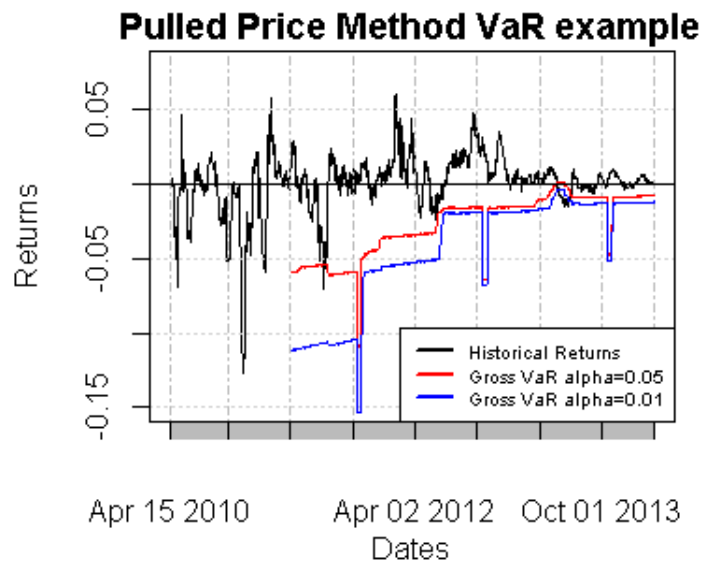
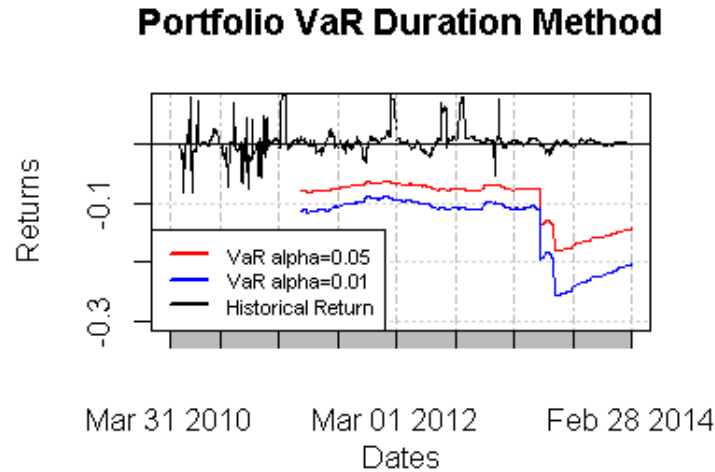


Figure 6: VaR at $\alpha=0.05$ for the AIG Mortgage Bank bond



A closer analysis to the results for $\alpha = 0.01$ indicates that the VaRs estimated with this method for this particular α is, in fact, underestimated for all the rejecting cases. In other words, the VaR expected was below the actual losses incurred in our time frame. This can have negative consequences for a company since it's expecting outcomes that are "too optimistic".

Figure 7: Portfolio Duration method VaR



5 The portfolio results

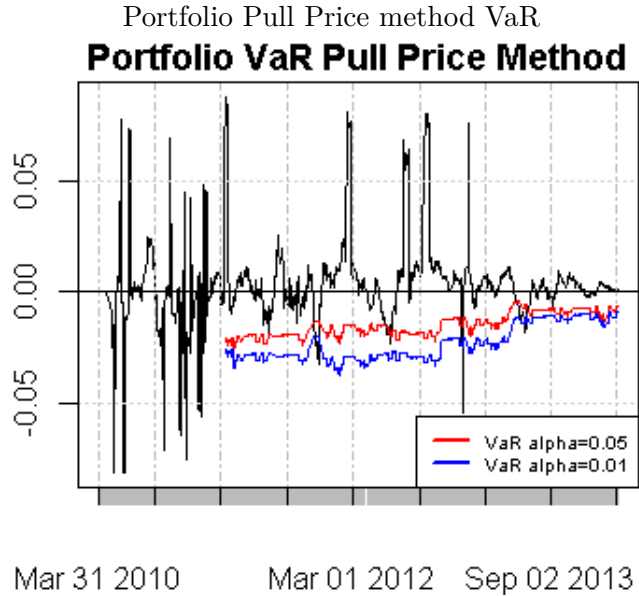
5.1 Cash Flow Mapping

Just like the individual bond VaR, the results from this method applied for a portfolio were greatly underestimated. In fact, by plotting the portfolio return and the VaR's at $\alpha = 1\%$ and $\alpha = 5\%$, the VaR's were so small (Figure 9) that they appear to be an horizontal axis. It goes without saying that the backtesting tool failed the method due to a big number of exceedances. Tables 14, 15, 16 and 17 depict the results obtained and, as we can see, the number of exceedances was too great to don't reject the H_0 .

5.2 Duration Method

Following the results from the individual bond VaR, the results of the Duration method were greatly overestimated as we can see in Figure 7. The backtesting tool failed this method due to the non-existence of exceedances of the historical returns towards the estimated VaRs. These conclusions can be easily observed after analysing Tables 22, 23, 24 and 25.

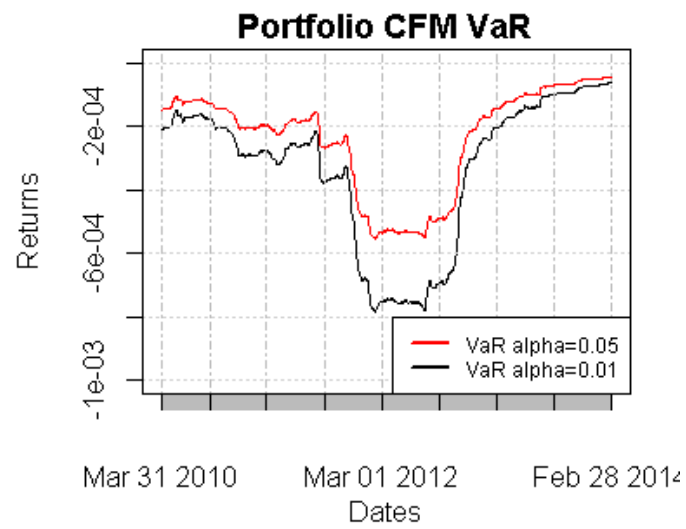
Figure 8:



5.3 Pull Price Method

For the Pull Price method, using the Undiversified portfolio VaR, the back-testing method concluded positively for an $\alpha = 5\%$ for both confidence intervals considered, 95% and 99%. On the other hand, for an $\alpha = 1\%$ the results were negative for both confidence intervals (Tables 18, 19, 20 and 21). The graphical representation of the VaR for this method can be seen in figure 8. After analysing the figure we can see that the systematic issue of the accrued rates was softened by the diversification of the portfolio. The fact that there are more bonds in the portfolio made the yearly-shocks seem less evident even though they still persist.

Figure 9: Portfolio CFM VaR

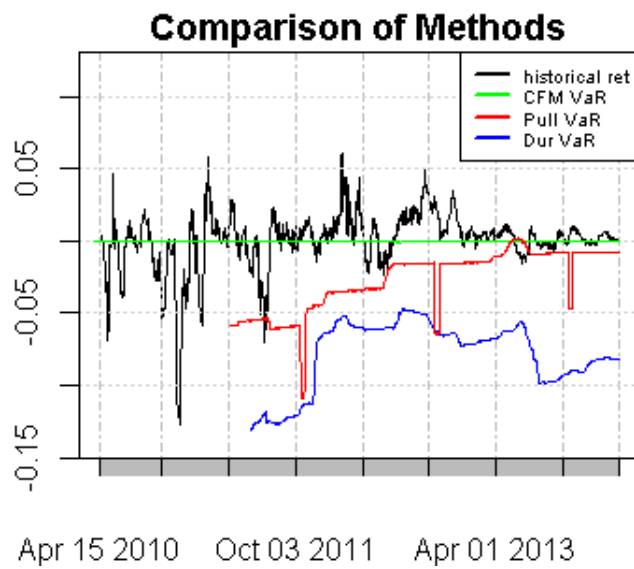


6 Conclusions

As we mentioned before, computing the VaR for bonds requires an extra effort when compared to stocks. The pull to par effect makes sure that applying an historical simulation directly, with bond prices, is ineffective since different prices have different underlying conditions. From the three methods, the one who received the most positive feedback from the backtesting tool was the Pull Price method with an approval rate (for a VaR $\alpha = 5\%$) of 38.8% for a significance of 99% and 16.7% for an significance of 95%. On the other hand, when the VaR's $\alpha = 1\%$ the results declined. This leads us to believe that the method isn't sensitive enough to the shift from $\alpha = 5\%$ to an $\alpha = 1\%$ because, for the later, the results were largely underestimated. This means that the portfolio incurred losses larger to those forecast by the method. This demands extra attention and should be revised by future literature. Additionally, the method isn't flawless when applying for coupon bonds since it doesn't account for difference in accrued rates between the relevant maturities n_{var} and n_{var+N} . This study included some minor adjustments that yielded positive results by removing the accrued rates of the same relevant maturities. This method also received good results for a portfolio of bonds when $alpha = 5\%$ and negative results when $alpha = 1\%$ following the trend from the individual bond computations.

Both the other methods, the CFM and the Duration, failed for opposite reasons. The CFM was very underestimated with a large number of exceedances making us believe that the method underestimates the VaR. Even that the yield curve considered isn't the appropriate for most of the bonds in this portfolio, some bonds fit the criteria and still followed the same trend as others. Retrieving data for the computation of this method is one of its biggest downsides when the portfolio is composed of corporate debt. The Duration method failed the backtesting test for the opposite reason of the CFM since its results were overestimated. For many of these bonds, the number of exceedances were zero which meant that, throughout the historical computations, the returns never went below the VaR estimations. In Figure 10 we can see the the difference between the three methods for an $\alpha = 5\%$ and each method appeared to have its own independent behaviour throughout time. Each method should carefully be revised in a near future.

Figure 10: Comparison of results for AIB Mortgage Bank bond



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Table 2: Backtesting - CFM $\alpha = 5\%$, *confidence interval* = 95%

Bond	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	42.00	287.00	686.63	3.84	0.00	Reject H0
2	20.00	103.00	189.78	3.84	0.00	Reject H0
3	20.00	101.00	182.33	3.84	0.00	Reject H0
4	20.00	173.00	509.53	3.84	0.00	Reject H0
5	20.00	125.00	278.35	3.84	0.00	Reject H0
6	20.00	119.00	253.02	3.84	0.00	Reject H0
7	20.00	123.00	269.81	3.84	0.00	Reject H0
8	15.00	90.00	194.26	3.84	0.00	Reject H0
9	10.00	50.00	84.15	3.84	0.00	Reject H0
10	20.00	110.00	216.66	3.84	0.00	Reject H0
11	9.00	39.00	55.56	3.84	0.00	Reject H0
12	20.00	108.00	208.85	3.84	0.00	Reject H0
13	20.00	134.00	317.88	3.84	0.00	Reject H0
14	20.00	174.00	514.86	3.84	0.00	Reject H0
15	9.00	54.00	109.55	3.84	0.00	Reject H0
16	20.00	126.00	282.65	3.84	0.00	Reject H0
17	20.00	165.00	470.50	3.84	0.00	Reject H0
18	20.00	170.00	493.66	3.84	0.00	Reject H0

Table 3: Backtesting - CFM $\alpha = 5\%$, *confidence interval* = 99%

Bond	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	42.00	287.00	686.63	6.63	0.00	Reject H0
2	20.00	103.00	189.78	6.63	0.00	Reject H0
3	20.00	101.00	182.33	6.63	0.00	Reject H0
4	20.00	173.00	509.53	6.63	0.00	Reject H0
5	20.00	125.00	278.35	6.63	0.00	Reject H0
6	20.00	119.00	253.02	6.63	0.00	Reject H0
7	20.00	123.00	269.81	6.63	0.00	Reject H0
8	15.00	90.00	194.26	6.63	0.00	Reject H0
9	10.00	50.00	84.15	6.63	0.00	Reject H0
10	20.00	110.00	216.66	6.63	0.00	Reject H0
11	9.00	39.00	55.56	6.63	0.00	Reject H0
12	20.00	108.00	208.85	6.63	0.00	Reject H0
13	20.00	134.00	317.88	6.63	0.00	Reject H0
14	20.00	174.00	514.86	6.63	0.00	Reject H0
15	9.00	54.00	109.55	6.63	0.00	Reject H0
16	20.00	126.00	282.65	6.63	0.00	Reject H0
17	20.00	165.00	470.50	6.63	0.00	Reject H0
18	20.00	170.00	493.66	6.63	0.00	Reject H0

Table 4: Backtesting - CFM $\alpha = 1\%$, *confidence interval* = 95%

Bond	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	8.00	287.00	1563.59	3.84	0.00	Reject H0
2	4.00	103.00	496.58	3.84	0.00	Reject H0
3	4.00	101.00	482.52	3.84	0.00	Reject H0
4	4.00	173.00	1047.42	3.84	0.00	Reject H0
5	4.00	125.00	657.77	3.84	0.00	Reject H0
6	4.00	119.00	612.64	3.84	0.00	Reject H0
7	4.00	123.00	642.64	3.84	0.00	Reject H0
8	3.00	90.00	466.63	3.84	0.00	Reject H0
9	2.00	50.00	231.65	3.84	0.00	Reject H0
10	4.00	110.00	546.57	3.84	0.00	Reject H0
11	1.00	39.00	168.47	3.84	0.00	Reject H0
12	4.00	108.00	532.16	3.84	0.00	Reject H0
13	4.00	134.00	727.03	3.84	0.00	Reject H0
14	4.00	174.00	1056.06	3.84	0.00	Reject H0
15	1.00	54.00	271.99	3.84	0.00	Reject H0
16	4.00	126.00	665.38	3.84	0.00	Reject H0
17	4.00	165.00	982.23	3.84	0.00	Reject H0
18	4.00	170.00	1021.65	3.84	0.00	Reject H0

Table 5: Backtesting - CFM $\alpha = 1\%$, *confidence interval* = 99%

Bond	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	8.00	287.00	1563.59	6.63	0.00	Reject H0
2	4.00	103.00	496.58	6.63	0.00	Reject H0
3	4.00	101.00	482.52	6.63	0.00	Reject H0
4	4.00	173.00	1047.42	6.63	0.00	Reject H0
5	4.00	125.00	657.77	6.63	0.00	Reject H0
6	4.00	119.00	612.64	6.63	0.00	Reject H0
7	4.00	123.00	642.64	6.63	0.00	Reject H0
8	3.00	90.00	466.63	6.63	0.00	Reject H0
9	2.00	50.00	231.65	6.63	0.00	Reject H0
10	4.00	110.00	546.57	6.63	0.00	Reject H0
11	1.00	39.00	168.47	6.63	0.00	Reject H0
12	4.00	108.00	532.16	6.63	0.00	Reject H0
13	4.00	134.00	727.03	6.63	0.00	Reject H0
14	4.00	174.00	1056.06	6.63	0.00	Reject H0
15	1.00	54.00	271.99	6.63	0.00	Reject H0
16	4.00	126.00	665.38	6.63	0.00	Reject H0
17	4.00	165.00	982.23	6.63	0.00	Reject H0
18	4.00	170.00	1021.65	6.63	0.00	Reject H0

Table 6:

Backtesting - Pull Price Method $\alpha = 5\%$, *confidence interval* = 95%

Bond	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	37.00	29.00	2.39	3.84	0.12	Fail to Reject H0
2	38.00	50.00	3.29	3.84	0.07	Fail to Reject H0
3	38.00	51.00	3.86	3.84	0.05	Reject H0
4	38.00	61.00	11.78	3.84	0.00	Reject H0
5	38.00	53.00	5.13	3.84	0.02	Reject H0
6	38.00	62.00	12.78	3.84	0.00	Reject H0
7	38.00	70.00	21.99	3.84	0.00	Reject H0
8	26.00	23.00	0.45	3.84	0.50	Fail to Reject H0
9	14.00	24.00	6.03	3.84	0.01	Reject H0
10	38.00	57.00	8.15	3.84	0.00	Reject H0
11	8.00	0.00		3.84		Reject H0
12	38.00	70.00	21.99	3.84	0.00	Reject H0
13	38.00	56.00	7.34	3.84	0.01	Reject H0
14	38.00	66.00	17.12	3.84	0.00	Reject H0
15	6.00	0.00		3.84		Reject H0
16	38.00	70.00	21.99	3.84	0.00	Reject H0
17	38.00	51.00	4.18	3.84	0.04	Reject H0
18	38.00	68.00	19.49	3.84	0.00	Reject H0

Table 7:

Backtesting - Pull Price Method $\alpha = 5\%$, *confidence interval = 99%*

Bond	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	37.00	29.00	2.39	6.63	0.12	Fail to Reject H0
2	38.00	50.00	3.29	6.63	0.07	Fail to Reject H0
3	38.00	51.00	3.86	6.63	0.05	Fail to Reject H0
4	38.00	61.00	11.78	6.63	0.00	Reject H0
5	38.00	53.00	5.13	6.63	0.02	Fail to Reject H0
6	38.00	62.00	12.78	6.63	0.00	Reject H0
7	38.00	70.00	21.99	6.63	0.00	Reject H0
8	26.00	23.00	0.45	6.63	0.50	Fail to Reject H0
9	14.00	24.00	6.03	6.63	0.01	Fail to Reject H0
10	38.00	57.00	8.15	6.63	0.00	Reject H0
11	8.00	0.00		6.63		Reject H0
12	38.00	70.00	21.99	6.63	0.00	Reject H0
13	38.00	56.00	7.34	6.63	0.01	Reject H0
14	38.00	66.00	17.12	6.63	0.00	Reject H0
15	6.00	0.00		6.63		Reject H0
16	38.00	70.00	21.99	6.63	0.00	Reject H0
17	38.00	51.00	4.18	6.63	0.04	Fail to Reject H0
18	38.00	68.00	19.49	6.63	0.00	Reject H0

Table 8:

Backtesting - Pull Price Method $\alpha = 1\%$, *confidence interval* = 95%

Bond	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	7.00	21.00	16.20	3.84	0.00	Reject H0
2	7.00	22.00	17.82	3.84	0.00	Reject H0
3	7.00	21.00	15.74	3.84	0.00	Reject H0
4	7.00	23.00	20.01	3.84	0.00	Reject H0
5	7.00	41.00	71.92	3.84	0.00	Reject H0
6	7.00	35.00	52.31	3.84	0.00	Reject H0
7	7.00	31.00	40.41	3.84	0.00	Reject H0
8	5.00	8.00	1.24	3.84	0.26	Fail to Reject H0
9	2.00	15.00	26.23	3.84	0.00	Reject H0
10	7.00	27.00	29.59	3.84	0.00	Reject H0
11	1.00	0.00		3.84		Reject H0
12	7.00	30.00	37.60	3.84	0.00	Reject H0
13	7.00	36.00	55.43	3.84	0.00	Reject H0
14	7.00	30.00	37.60	3.84	0.00	Reject H0
15	1.00	0.00		3.84		Reject H0
16	7.00	30.00	37.60	3.84	0.00	Reject H0
17	7.00	26.00	27.27	3.84	0.00	Reject H0
18	7.00	34.00	49.24	3.84	0.00	Reject H0

Table 9:

Backtesting - Pull Price Method $\alpha = 1\%$, *confidence interval = 99%*

Bond	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	7.00	21.00	16.20	6.63	0.00	Reject H0
2	7.00	22.00	17.82	6.63	0.00	Reject H0
3	7.00	21.00	15.74	6.63	0.00	Reject H0
4	7.00	23.00	20.01	6.63	0.00	Reject H0
5	7.00	41.00	71.92	6.63	0.00	Reject H0
6	7.00	35.00	52.31	6.63	0.00	Reject H0
7	7.00	31.00	40.41	6.63	0.00	Reject H0
8	5.00	8.00	1.24	6.63	0.26	Fail to Reject H0
9	2.00	15.00	26.23	6.63	0.00	Reject H0
10	7.00	27.00	29.59	6.63	0.00	Reject H0
11	1.00	0.00		6.63		Reject H0
12	7.00	30.00	37.60	6.63	0.00	Reject H0
13	7.00	36.00	55.43	6.63	0.00	Reject H0
14	7.00	30.00	37.60	6.63	0.00	Reject H0
15	1.00	0.00		6.63		Reject H0
16	7.00	30.00	37.60	6.63	0.00	Reject H0
17	7.00	26.00	27.27	6.63	0.00	Reject H0
18	7.00	34.00	49.24	6.63	0.00	Reject H0

Table 10:

Backtesting - Duration Method $\alpha = 5\%$, *confidence interval = 95%*

Bond	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	35.00	0.00		3.84		Reject H0
2	36.00	11.00	25.68	3.84	0.00	Reject H0
3	36.00	0.00		3.84		Reject H0
4	36.00	0.00		3.84		Reject H0
5	36.00	0.00		3.84		Reject H0
6	36.00	0.00		3.84		Reject H0
7	36.00	0.00		3.84		Reject H0
8	24.00	0.00		3.84		Reject H0
9	12.00	0.00		3.84		Reject H0
10	36.00	0.00		3.84		Reject H0
11	6.00	0.00		3.84		Reject H0
12	36.00	0.00		3.84		Reject H0
13	36.00	0.00		3.84		Reject H0
14	36.00	0.00		3.84		Reject H0
15	4.00	0.00		3.84		Reject H0
16	36.00	0.00		3.84		Reject H0
17	36.00	0.00		3.84		Reject H0
18	36.00	0.00		3.84		Reject H0

Table 11:

Backtesting - Duration Method $\alpha = 5\%$, *confidence interval = 99%*

Bond	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	35.00	0.00		6.63		Reject H0
2	36.00	11.00	25.68	6.63	0.00	Reject H0
3	36.00	0.00		6.63		Reject H0
4	36.00	0.00		6.63		Reject H0
5	36.00	0.00		6.63		Reject H0
6	36.00	0.00		6.63		Reject H0
7	36.00	0.00		6.63		Reject H0
8	24.00	0.00		6.63		Reject H0
9	12.00	0.00		6.63		Reject H0
10	36.00	0.00		6.63		Reject H0
11	6.00	0.00		6.63		Reject H0
12	36.00	0.00		6.63		Reject H0
13	36.00	0.00		6.63		Reject H0
14	36.00	0.00		6.63		Reject H0
15	4.00	0.00		6.63		Reject H0
16	36.00	0.00		6.63		Reject H0
17	36.00	0.00		6.63		Reject H0
18	36.00	0.00		6.63		Reject H0

Table 12:

Backtesting - Duration Method $\alpha = 1\%$, *confidence interval = 95%*

Bond	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	7.00	0.00		3.84		Reject H0
2	7.00	4.00	1.82	3.84	0.18	Fail to Reject H0
3	7.00	0.00		3.84		Reject H0
4	7.00	0.00		3.84		Reject H0
5	7.00	0.00		3.84		Reject H0
6	7.00	0.00		3.84		Reject H0
7	7.00	0.00		3.84		Reject H0
8	4.00	0.00		3.84		Reject H0
9	2.00	0.00		3.84		Reject H0
10	7.00	0.00		3.84		Reject H0
11	1.00	0.00		3.84		Reject H0
12	7.00	0.00		3.84		Reject H0
13	7.00	0.00		3.84		Reject H0
14	7.00	0.00		3.84		Reject H0
15	0.00	0.00		3.84		Reject H0
16	7.00	0.00		3.84		Reject H0
17	7.00	0.00		3.84		Reject H0
18	7.00	0.00		3.84		Reject H0

Table 13:

Backtesting - Duration Method $\alpha = 1\%$, *confidence interval = 99%*

Bond	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	7.00	0.00		6.63		Reject H0
2	7.00	4.00	1.82	6.63	0.18	Fail to Reject H0
3	7.00	0.00		6.63		Reject H0
4	7.00	0.00		6.63		Reject H0
5	7.00	0.00		6.63		Reject H0
6	7.00	0.00		6.63		Reject H0
7	7.00	0.00		6.63		Reject H0
8	4.00	0.00		6.63		Reject H0
9	2.00	0.00		6.63		Reject H0
10	7.00	0.00		6.63		Reject H0
11	1.00	0.00		6.63		Reject H0
12	7.00	0.00		6.63		Reject H0
13	7.00	0.00		6.63		Reject H0
14	7.00	0.00		6.63		Reject H0
15	0.00	0.00		6.63		Reject H0
16	7.00	0.00		6.63		Reject H0
17	7.00	0.00		6.63		Reject H0
18	7.00	0.00		6.63		Reject H0

Table 14:

Portfolio Backtesting - CFM Method $\alpha = 5\%$, *confidence interval = 95%*

Portfolio	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	43.00	320.00	824.29	3.84	0.00	Reject H0

Table 15:

Portfolio Backtesting - CFM Method $\alpha = 5\%$, *confidence interval = 99%*

Portfolio	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	43.00	320.00	824.29	6.63	0.00	Reject H0

Table 16:

Portfolio Backtesting - CFM Method $\alpha = 1\%$, *confidence interval = 95%*

Portfolio	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	8.00	319.00	1800.38	3.84	0.00	Reject H0

Table 17:

Portfolio Backtesting - CFM Method $\alpha = 1\%$, *confidence interval = 99%*

Portfolio	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	8.00	319.00	1800.38	6.63	0.00	Reject H0

Table 18:

Portfolio Backtesting - Pull Price Method $\alpha = 5\%$,
confidence interval = 95%

Portfolio	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	33.00	32.00	0.05	3.84	0.82	Fail to Reject H0

Table 19:

Portfolio Backtesting - Pull Price Method $\alpha = 5\%$,
confidence interval = 99%

Portfolio	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	33.00	32.00	0.05	6.63	0.82	Fail to Reject H0

Table 20:

Portfolio Backtesting - Pull Price Method $\alpha = 1\%$,
confidence interval = 95%

Portfolio	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	6.00	15.00	7.78	3.84	0.01	Reject H0

Table 21:

Portfolio Backtesting - Pull Price Method $\alpha = 1\%$,
confidence interval = 99%

Portfolio	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	6.00	15.00	7.78	6.63	0.01	Reject H0

Table 22:

Portfolio Backtesting - Duration Method $\alpha = 5\%$,
confidence interval = 95%

Portfolio	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	31.00	0.00		3.84		Reject H0

Table 23:

Portfolio Backtesting - Duration Method $\alpha = 5\%$,
confidence interval = 99%

Portfolio	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	31.00	0.00		6.63		Reject H0

Table 24:

Portfolio Backtesting - Duration Method $\alpha = 1\%$,
confidence interval = 95%

Portfolio	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	6.00	0.00		3.84		Reject H0

Table 25:

Portfolio Backtesting - Duration Method $\alpha = 1\%$,
confidence interval = 99%

Portfolio	Expected Exceedances	Actual Exceedances	Test Statistic	Critical Value	p-Value	Decision
1	6.00	0.00		6.63		Reject H0