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MASTER IN
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MASTERS FINAL WORK
INTERNSHIP REPORT

METHODS OF CAPITAL ALLOCATION IN A SOLVENCY II ENVIRONMENT

RAQUEL SEQUEIRA CORREIA

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Abstract

Under Solvency II regulation the SCR is mainly calculated using a standard formula which considers the risks that an insurer faces. Due to this aggregation of risks, a diversification benefit is achieved and the global SCR is smaller than the sum of the capital requirements of each risk. To take these diversification benefits into account the total capital should be allocated back to the lower levels of risk by applying a proper method of capital allocation. This report is the result of a curricular internship that took place at EY. One of the goals was to find the most appropriate method to perform a capital allocation of the SCR of an insurance company. Five methods of allocation were studied, Proportional, Variance-Covariance, Merton and Perold, Shapley and Euler. The methods were compared theoretically by analyzing their respective properties, and based on several studies in the literature it is concluded that the Euler method is the most appropriate to apply. This report contributes to a better understanding of capital allocation methods and allows to demonstrate how to allocate the SCR. It also contributes to show how to construct the SES for the purpose of the calculation of the adjustment of LAC DT. Since this task was one of the difficulties enumerated in the Fifth Quantitative Impact Study (QIS 5), this work can serve as a literary base, being useful to overcome these difficulties.

Keywords: Solvency II; SCR; capital allocation; Proportional method; Variance-Covariance method; Merton and Perold method; Shapley method; Euler method; Single equivalent scenario.

Resumo

De acordo com a regulamentação de Solvência II, o SCR é geralmente calculado usando uma fórmula padrão que considera os riscos que uma seguradora enfrenta. Devido à agregação dos diferentes riscos, são originados benefícios de diversificação e um valor de SCR total menor que a soma dos requisitos de capital de cada risco. Para ter em conta estes benefícios de diversificação, o capital total deve ser alocado de volta aos níveis mais baixos de risco, aplicando um método apropriado de alocação de capital. Este relatório é resultado de um estágio curricular que decorreu na EY. Um dos objetivos foi encontrar o método mais apropriado para realizar a alocação do SCR de uma empresa de seguros. Foram estudados cinco métodos de alocação, Proporcional, Variância-Covariância, Merton e Perold, Shapley e Euler. Os métodos são comparados teoricamente, analisando as suas respetivas propriedades e, com base em vários estudos presentes na literatura, conclui-se que o método de Euler é o mais apropriado. Este trabalho contribui para uma melhor compreensão dos métodos de alocação de capital e permite demonstrar como alocar o SCR. Contribui também para mostrar como construir o SES para fins do cálculo do ajustamento LAC DT. Visto que esta tarefa foi uma das dificuldades referidas no QIS 5, este trabalho pode servir como base literária, sendo útil para superar essas dificuldades.

Palavras-chave: Solvência II; SCR; alocação de capital; método Proporcional; método de Variância-Covariância; método de Merton e Perold; método de Shapley; método Euler; Cenário único equivalente.

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Acronyms and Abbreviations

BSCR - Basic Solvency Capital Requirement

CAT - Catastrophe

DT - Deferred taxes

DTA - Deferred tax assets

DTL - Deferred tax liabilities

EIOPA - European Insurance and Occupational Pensions Authority

Eul. - Euler allocation method

LAC - Loss Absorbing Capacity

LAC DT - Loss Absorbing Capacity of Deferred Taxes

LAC TP - Loss Absorbing Capacity of Technical Provisions

LoB - Line of Business

M & P - Merton and Perold allocation method

MCR - Minimum Capital Requirement

P & R - Premium and Reserve

Prop. - Proportional allocation method

QIS5- Fifth Quantitative Impact Study

SII - Solvency II

SCR - Solvency Capital Requirement

SES - Single Equivalent Scenario

Shap. - Shapley allocation method

TP - Technical Provisions

TVaR - Tail Value at Risk

VaR - Value at Risk

VarCov. - Variance Covariance allocation method

1. Introduction

This work is the result of a curricular internship at EY - Ernst & Young, S.A. in the Actuarial services which started on 13th February and ended on 30th June of 2017. During the internship I was assigned with tasks related with Solvency II and Pension Funds allowing me to apply many concepts that I have learned from my masters. Since the theoretical background of the tasks was so different I chose to write and do some research on one main topic, Capital Allocation, which I found interesting and that was not so familiar to me in the beginning of the internship. Additionally I also got the opportunity to get more insight about concepts such as the Single Equivalent Scenario (SES) approach and the Loss Absorbing Capacity of Deferred Taxes (LAC DT)[1], which connect easily to the concept of capital allocation. With the new regulation standards implied by Solvency II many rules have been established with the aim to provide a safe and stable environment for all financial institutions including insurance companies. Any risk faced by these institutions should be quantified, managed and reported, which allows an increase of the stability of the financial system. It is mandatory to determine the amount of capital, Solvency Capital Requirement (SCR), that an institution needs to hold in order to remain solvent. There are two possibilities to compute the SCR, using a standard model or using an internal model if the same is approved and shown to be more efficient and suitable for the risk profile of a certain insurance company. It is also possible to determine the risk capital as a combination of both models. In this paper the standard model is considered entirely. After computing the SCR an

[1] For further details see chapter 7.

important step to take would be its allocation back to each risk module, sub module or even to each line of business in such a way that the sum of each individual risk add up to the total risk. This allows to get some knowledge about the benefits from diversification effects resulting from the aggregation of all risks.

Allocation of capital has many different applications in a financial institution, such as, the division of capital reserve among business units, support on strategic decision making regarding new lines of business, for pricing, assessment of performance of each portfolio and of managers, settlement of risk limits and also portfolio optimization. For an insurance company the advantages are also many and similar to the ones stated before and capital allocation methodologies can also be used to find the SES. To determine the best way to do this, five different allocation methodologies were studied in order to determine which method could be more appropriate. In chapter 2 a bibliographic review is provided given that some conclusions and assumptions in this work were based on previous research articles. Chapters 3 and 4 give a brief overview about risk measures and Solvency II regime, chapters 5 and 6 require the most attention, since the risk capital allocation problem is defined, as well as the different methodologies, its properties and a theoretical decision about which method to use. Subsequently, the practical results of the final chosen method are showed and for a particular case all the methodologies were applied in order to know how different the results were. Also, some insight about the SES, LAC DT and possible applications of capital allocation are given in chapter 7. At last, it is possible to find final conclusions and possibilities for further research in chapter 8.

2. Bibliographic review

This chapter provides a summary of the available literature on capital allocation methods. Several authors have contributed to this area by explaining allocation methods in detail, or bringing new perspectives and possible applications of it. Merton and Perold (1993) provides an allocation method that is based on option pricing theory. The work of Tasche (1999, 2004, 2007,2008) provides a wide range of information regarding the Euler method and proves that this method is the only one suitable for measurement of performance. Overbeck (2000) introduced the Variance-Covariance method. Denault (2001) presents the coherence of an allocation method and explains the Shapley, Aumann-Shapley and Euler methods. Urban *et al.* (2003) compares and analyzes different methods of capital allocation providing some equivalences between them. Buch and Dorfleitnet (2008) has as a main topic the coherence of risk measures and allocation methods. More authors continued to study this topic, for instance, Furman and Zitikis (2008), Corrigan *et al.* (2009), Balog (2011), Dhaene *et al.* (2012), Gulicka *et al.* (2012) and Karabey (2012). Regarding the applications of an allocation method, Cummins (2000), Panjer (2002), Gründil and Schmeiser (2005), Buch *et al.* (2011) and Asimit *et al.* (2016) provide different ideas and perspectives on the matter. Some conclusions presented in this report were based on Balog *et al.* (2017) which also presents various allocation methods and focus on the properties of coherence that each one satisfies. EIOPA regulations and guideline papers also provided a strong background and knowledge for the elaboration of this internship report.

3. Risk Measures

Artzner (1999), Artzner *et al.* (1999) and Pitselis (2016) provided the main theoretical background for this chapter.

Risk can be interpreted in many ways, it can be a possible loss or its variance, a change in the future values of random variables or a set of events that can cause loss. An insurance company faces a lot of uncertainties and must be prepared to face the risks that is exposed to. Therefore, measuring risk is essential to find the capital that a company should hold is order to be able to face any unexpected losses. A risk measure assigns a real number to the random variables of a portfolio. This section gives a brief introduction to risk measures and focus on the two most known risk measures used by insurance companies.

Let $\mathcal{X}$ be the set of random variables which represent a set of events that a portfolio is exposed to and let $X \in \mathcal{X}$ be a random variable belonging to this set.

Definition 3.1: A **risk measure** ρ is a mapping from the set of variables $\mathcal{X}$ to the real line $\mathbb{R}$:

(3.1) $\rho: \mathcal{X} \longmapsto \mathbb{R}: X \in \mathcal{X} \longmapsto \rho(X).$

An appropriate risk measure should be consistent with economic and finance theory so it is important to define some properties that a good risk measure should satisfy.

Definition 3.2: A risk measure ρ is a **coherent risk measure** if it satisfies the following properties:

1. Positive homogeneity: $\rho(\lambda X) = \lambda \rho(X);\ \forall\, X \in \mathcal{X}, \lambda > 0.$

2. Translation invariance: $\rho(X+\alpha)=\rho(X)-\alpha; \forall\, X \in \mathcal{X}, \alpha \in \mathbb{R}$.
3. Subadditivity: $\rho(X+Y) \leq \rho(X)+\rho(Y); \forall\, X, Y \in \mathcal{X}$.
4. Monotonicity: If $X \leq Y$ then $\rho(Y) \leq \rho(X); \forall\, X, Y \in \mathcal{X}$.

Positive homogeneity means that scaling a portfolio implicates the same scaling for the risk, for instance, double the same portfolio also result on twice the risk. Translation invariance implies that when adding a determinist amount to the portfolio, the risk changes by the same amount. Subadditivity is related with the concept of diversification, that is, merging two or more risks/portfolios does not generate additional risk. Therefore, diversification of risks is essential in a portfolio. At last, monotonicity implies that a random variable or a portfolio with higher and better value (lower losses) originates a lower or equal risk under all the scenarios.

Besides the risk measure, an insurer must also choose the time period over which a risk is going to be measured. Under Solvency II, the risk measure used to determine the Solvency Capital Requirement (SCR) is the Value at Risk with a time horizon of one year. Another risk measure is the Tail Value at Risk which can be considered to calculate the economic capital of the company.

3.1 Value at Risk (VaR)

Known as one of the most used risk measures in the financial sector, the VaR is the maximum loss not exceeded of a given risk X over a given time horizon. For a confidence level $p, p \in (0,1)$, VaR is mathematically defined as

$$VaR_p(X)=\pi_p,\ \pi_p: P(X \leq \pi_p^-) \leq p \leq P(X \leq \pi_p) \qquad p \in (0,1). \tag{3.1.1}$$

Thus, $VaR_p(X)$ is the $p\ th$ quantile of the cumulative distribution of risk X.

The VaR is positive homogenous, translation invariant and monotone but is not subadditive in some cases which leads to it not been a coherent risk measure and the diversification effects in a portfolio of risks may be compromised. Also, the main disadvantage of using VaR is that, although it measures the maximum potential loss, it fails to measure the severity of losses that fall above the confidence level p. Despite this drawbacks, the computation of VaR is relatively simple and easy to explain leading to the most preferred risk measure of the insurers and also the elected measure by the European Commission to determine the SCR.

3.2. Tail Value at Risk (TVaR)

The Tail Value at Risk is a more robust risk measure than VaR. It can be interpreted as the mean of the expected losses above the confidence level p, given that a loss of that magnitude occurs.

Mathematically,

$$(3.2.1) \qquad TVaR_p(X) = \frac{\int_p^1 VaR_u(X)\,du}{1-p}, \qquad p \in (0,1).$$

Therefore, the TVaR provides information about the average of the tail. For normal distributions the difference between VaR and TVaR is relatively smaller when compared with other distributions with a heavier tail. The main difference is that the TVaR is a coherent measure satisfying all the properties and it gives more insight about the magnitude of the losses above the chosen confidence level. A good approach is to determine the capital requirements with both measures and analyze if there are any substantial differences. If the difference is small then it indicates that the tail of the distribution is small and the severity of losses does not reach values well beyond VaR. Thus, there is no practical reason to use TVaR since it is a more complicated measure to compute. On the other hand, if the difference is consider to be relevant then a further research on the matter should be done since it is possible that some extreme events may lead to adverse situations and different conclusions, including on capital allocation results. However and as it was mentioned before, VaR is easier to calculate, easier to explain and it was already the risk measure used as reference in the banking system.

4. Solvency II

4.1 Introduction

This chapter provides a quick and short explanation about Solvency II regulation, mentioning only the appropriate concepts for the scope of this work.

This European regulation arose as a way to ensure financial soundness of insurance undertakings providing more transparency, better management and a harmonized solvency and supervisory regime for the insurance sector.

Solvency II is divided in three components named "pillars":

- Pillar I - Quantitative Requirements: Gives orientations on the minimum capital requirement (MCR), solvency capital requirement (SCR) based on a standard approach or an internal/partial model, own funds and investments.
- Pillar II - Qualitative Requirements: Focuses on governance, risk management and internal control, and supervisory review process.
- Pillar III - Disclosure and market discipline: Comprises reporting and disclosure of information, transparency and harmonized reporting to the supervisors.

This report requires a special attention to Pillar I, specifically to what is related with the calculation of the SCR.

4.2 Solvency Capital Requirements

The SCR is the level of capital that the insurer is required to hold in order to be able to face unexpected losses. It is calculated as the Value at Risk with a confidence level of 99,5% over one year time horizon. The SCR considers all the risks that the insurer may face which under Solvency II regulation are organized as in the following scheme:

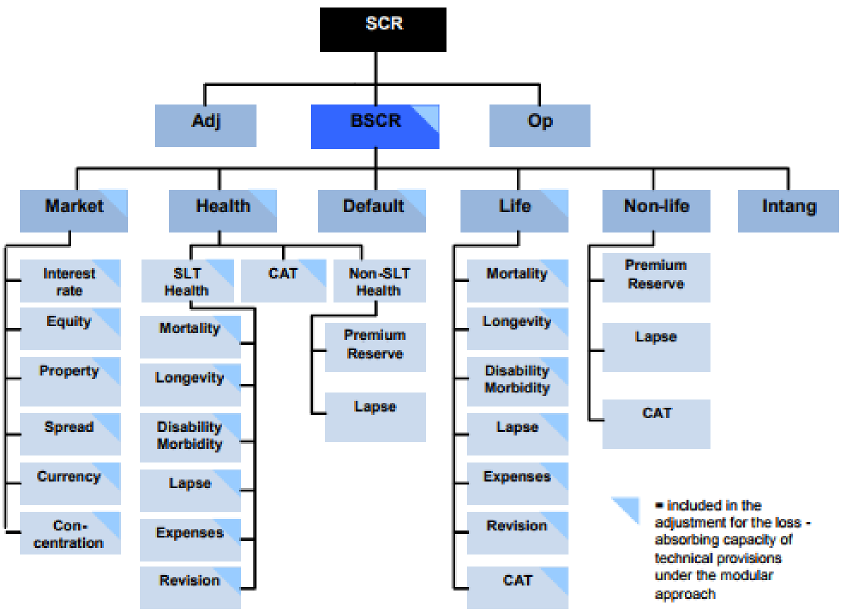

Figure 4.2.1: Risk Modules and sub modules under Solvency II regime. (*Source: The underwriting assumptions in the standard formula for the Solvency Capital Requirement calculation. EIOPA -14-322.)*

Let M= {Market, Health, Default, Life, Non-life} and S_x be the set of risks belonging to module x, $x \in M$.

- n_x is the number of risks in module x;
- SCR_x is the required capital for module x;
- $SCR_{x,i}$ is the required capital for the risk i of module x, $i \in S_x$;
- $Corr_{x,y}$ is the correlation between modules x and y, $\forall\, x, y \in M$;
- $Corr_{xi,xj}$ is the correlation between risks i and j, $\forall\, i, j \in S_x$.

The overall SCR is given by the formula

$$(4.2.1) \qquad SCR = BSCR + Adj + SCR_{OperationalRisk},$$

where Adj is the adjustment for loss absorbing capacity of technical provisions and deferred taxes ($Adj = Adj_{LAC\ TP} + Adj_{LAC\ DT}$) which takes a null or negative value and $SCR_{OperationalRisk}$ is the capital requirement for Operational risk.

The $BSCR$ is the result of the aggregation of the risk modules in M, which can be calculated as

$$(4.2.2) \qquad BSCR = \sqrt{\sum_{x,y \in M} Corr_{x,y} SCR_x SCR_y} + SCR_{Intang}.$$

In most cases, the capital requirement for each risk module x is given by

$$(4.2.3) \qquad SCR_x = \sqrt{\sum_{i,j \in S_x} Corr_{xi,xj} SCR_{x,i} SCR_{x,j}}\ , \qquad x \in M.$$

Intangible and Default risk modules are determined in a different way since they do not have any submodules. Moreover, the calculation of the capital charges of each risk sub module is different and to provide more insight on this topic the reader is advised to consult the Commission Delegated Regulation (EU) 2015/35 and EIOPA-14-322 guideline regarding the capital requirements calculations.

For this work is also important to refer the Premium and Reserve risk for both Health NSLT and Non-Life risks and explain how the capital charge is determined. All the formulas and assumptions related to this topic can be consulted in section 6.1 - Data and data treatment.

The correlation tables regarding the different risks are presented in Appendix A.

5. Risk capital allocation

5.1 Introduction to the capital allocation problem

When a portfolio is composed by different risk units and its risk capital is computed with a risk measure, diversification effects are in place. Usually, the sum of the individual risk contributions of the risk units is larger than the risk capital of the whole portfolio. Therefore, it is important to allocate the risk capital in a fair way, to each risk unit, in order to evaluate its contribution to the total diversification effects.

To provide a more general notation, consider the following definitions for a portfolio composed by different risks:

- $N = \{1,2, \dots, n\}$, represents the set of all risk units.
- X_i, $i \in N$, is a random variable representing the amount of loss due to the risk unit i.
- $Y = \sum_{i=1}^{n} X_i$, is the aggregate loss of the whole portfolio, dependent on all the individual losses X_i.
- ρ is an appropriate risk measure that quantifies the amount of losses at the level of a risk unit or portfolio and represents the capital necessary to cover that same risk.
- $\rho(X_i)$ is the risk capital required to hold for unit i.
- $\rho(Y)$ is the total risk capital required to hold for the portfolio.
- $\rho(X_i|Y)$ represents the risk capital that is allocated "back", after diversification effects, to each risk unit i.

In an insurance company, it will be assumed that N is the set of all risk modules, $\rho(Y)$ is equivalent to the $BSCR$, and $\rho(X_i)$ is the SCR of the risk module i. The notation can also be extended to the level of risk submodules and lines of business of an insurance company. The allocation must be backwards, that is, one should first compute the allocated capital of each module, then use it to compute the allocated capital to each submodule and only in the end to the lines of business if applicable.

Following Denault's (2001) approach, let us defined the allocation problem:

Definition 5.1.1. Let A be the set of risk capital allocation problems and (N, ρ) composed by a set of n portfolios and a risk measure ρ. An **allocation principle** is a function $\Pi: A \longmapsto R^n$ that maps each allocation problem (N, ρ) into a unique allocation:

$$(5.1.1) \qquad \Pi: (N, \rho) \longmapsto \begin{bmatrix} \Pi_1(N, \rho) \\ \Pi_2(N, \rho) \\ \vdots \\ \Pi_n(N, \rho) \end{bmatrix} = \begin{bmatrix} \rho(X_1|Y) \\ \rho(X_2|Y) \\ \vdots \\ \rho(X_n|Y) \end{bmatrix} \text{ and } \sum_{i=1}^{n} \rho(X_i|Y) = \rho(Y).$$

Definition 5.1.2: The **allocation ratio,** also called diversification factor, represents the portion of capital of a risk unit i that was allocated back to that same risk unit. Mathematically,

$$(5.1.2) \qquad AR_i = \frac{\rho(X_i|Y)}{\rho(X_i)}, \qquad i \in N.$$

Definition 5.1.1 leads to the need to establish which conditions make the function Π a good allocation principle and what properties should it satisfy.

5.2 Properties of risk capital allocation methods

Definition 5.2.1. An allocation Π is **a coherent allocation principle** if it satisfies the following properties:

1. Full allocation: $\sum_{i=1}^{n} \rho(X_i|Y) = \rho(Y)$.
2. No undercut: $\forall\ S \subseteq N, \sum_{i \in S} \rho(X_i|Y) \leq \rho(\sum_{i \in S} X_i)$.
3. Symmetry: Let i and j be two risks that make the same contribution to the risk capital. If they join any subset $S \subseteq N\backslash\{i,j\}$, then $\rho(X_i|Y) = \rho(X_j|Y)$.
4. Riskless allocation: If the n^{th} unit is riskless with worth 1 at time 0 and worth r at any point T, then $X_n = \alpha r$ and $\forall\ \alpha \in \mathbb{R},\ \rho(X_n|Y) = \rho(X_n) = \rho(\alpha r) = -\alpha$.

Full allocation property implies that the sum of the individual allocated capital amounts add up to the total risk, that is, the capital is fully allocated. No undercut property means that the standalone allocated capital of a risk or a subset of risks is smaller than the total risk capital of the whole risk set. Symmetry implies that identical risks should be treated in the same way. More specifically, when adding two risks to any disjoint subset which result in the same amount of capital contribution then the allocated capital should also coincide. Finally, riskless allocation means that the allocation of a determinist variable, that is, a riskless component, has no impact on the total capital being allocated to the risk units.

Also, an allocation principle is a **non-negative coherent allocation** if it satisfies all the previous properties and if $\rho(X_i|Y) \geq 0, \forall\ i \in N$.

Additionally, there are also some more properties that can be useful to compare different principles but that are not required for the allocation principle's coherence. Namely and according to Balog *et al.* (2017), the Diversification, Strong Monotonicity, Incentive Compatibility, Covariance and Decomposition Invariance properties. It is considered that the violation of a property can occur in theoretical situations and it might not be relevant in practical situations.

5.3 Proportional allocation

Proportional allocation is the easiest method to be applied where the diversification effect is proportionally distributed to all the risk units.

The "new" contribution of the risk unit i, is given by the following formula:

$$\text{(5.3.1)} \qquad \rho^{prop}(X_i|Y) = \frac{\rho(X_i)}{\sum_{j \in N} \rho(X_j)} \rho(Y), \qquad i \in N.$$

This method satisfies the full allocation property but it does not take into account the dependence structures between risks.

5.4. Variance Covariance Allocation

This principle of allocation is given by

$$\text{(5.4.1)} \qquad \rho^{VarCov}(X_i|Y) = \frac{Cov(X_i, Y)}{Var(Y)} \rho(Y), \qquad i \in N.$$

Where $Var(Y)$ is the variance of the aggregate loss Y, that is, of the whole risk portfolio, and $Cov(X_i, Y)$ is the covariance between the loss X_i and Y. Risk units facing a loss that is more correlated with the total loss are required to hold more capital than the less correlated. Thus, the method focuses on how each risk unit contributes to the variance of the portfolio. Depending on the available data, the following relations may be helpful for the application of this method.

(5.4.2) $$Cov(X_i, Y) = Cov\left(X_i, \sum_{j=1}^{n} X_j\right)$$

$$= \sum_{j \in N} Cov(X_i, X_j) = \sigma_i \sum_{j \in N} Corr(X_i, X_j)\sigma_j, \qquad i \in N.$$

Where σ_i and σ_j are the standard deviations of X_i and X_j and $Corr(X_i, X_j)$ is the correlation between losses X_i and X_j , $i, j = (1, \dots n)$.

(5.4.3)[2] $$Var(Y) = \sum_{i=1}^{n} Cov(X_i, Y), \qquad i \in N.$$

Moreover, relation (5.4.3) proves that the variance of the portfolio can be written as the sum of the individual covariances between the risks and the whole portfolio which means that the full allocation property is satisfied.

Given the previous relations and to have the individual risk contributions of each unit into account, the method can also be written as

(5.4.4) $$\rho^{VarCov^*}(X_i|Y) = \frac{\rho(X_i)Cov(X_i, Y)}{\sum_{j=1}^{n} \rho(X_j)Cov(X_j, Y)}\rho(Y), \qquad i \in N.$$

5.5. Merton-Perold Allocation

Merton – Perold methodology is an incremental allocation of capital that measures the marginal effect of a risk unit i, similar to what is done in pricing. Marginal contributions to the whole portfolio are differences between the total capital amount of the company including the risk unit i and the total capital without risk unit i. Mathematically, the allocated capital is given by

(5.5.1) $$\rho^{M\&P(1)}(X_i|Y) = \rho(Y) - \rho(Y - X_i), \qquad i \in N.$$

[2] Proof is given in appendix C.

One disadvantage of this method is that the sum of risk contributions does not add up to the total capital. A simple alteration solves this problem:

$$(5.5.2) \qquad \rho^{M\&P(2)}(X_i|Y) = \frac{\rho^{M\&P(1)}(X_i|Y)}{\sum_{j=1}^{n}\rho^{M\&P(1)}(X_j|Y)}\rho(Y) =$$

$$= \frac{\rho(Y) - \rho(Y - X_i)}{\sum_{j=1}^{n}\rho(Y) - \rho\left(Y - X_j\right)}\rho(Y), \qquad i \in N.$$

With the previous alteration the full allocation property is now satisfied.

5.6. Shapley Allocation

This methodology can be considered as a general case of the previous method since in this case the marginal effects of the risk units are studied within all the possible combinations in a portfolio composed by these risk units.

To illustrate this, consider a group of players working to find the best and fair coalition possible. The goal is to form a coalition such that all the players benefit more as a group than as a stand-alone. Game theory provides a solution for a fair and unique distribution using the Shapley value.

Let (N, c) denote a coalitional game where N is a finite set representing the number of players and c a cost function representing a real number associated to each subset $G, G \in N$.

Definition 5.6.1: A **value** is a function ψ that maps the coalitional game (N, c) into a unique allocation:

$$(5.6.1) \qquad \psi: (N, c) \mapsto \begin{bmatrix} \psi_1(N,c) \\ \psi_2(N,c) \\ \vdots \\ \psi_n(N,c) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \ \sum_{i=1}^{n} x_i = c(N).$$

Definition 5.6.2: The **core** of a coalition game (N, c) is the set of allocations $x \in \mathbb{R}^n$ for which $\sum_{i \in G} x_i \leq c(G)$ for all coalitions $G \in N$.

This ensures that players always form the largest coalition possible since the cost of each player is always minimized if they join the coalition.

The Shapley value is given by the following formula:

$$(5.6.2) \qquad x_i = \sum_{G \subseteq N} \frac{(g-1)!\,(n-g)!}{n!} \big(c(G) - c(G \backslash \{i\})\big), \qquad i \in N.$$

where g is the number of players in coalition G and n is the total number of players.

According to the previous notation this is equivalent to:

$$(5.6.3) \qquad \rho^{Shap}(X_i|Y) = \sum_{S \subseteq N} \frac{(x-1)!\,(n-x)!}{n!} \big(\rho(S) - \rho(S \backslash \{i\})\big), \qquad i \in N.$$

where x is the number of risk units in subset S and n is the total number of existent risk units.

Hence, this method takes into consideration all permutations of the risk units, computes the marginal benefit of each unit in each case and returns the allocated capital as an average of the marginal benefits.

The computation of this method can be extensive and not worthy because the higher the value of n, the higher the number of possible coalitions, leading to a number of $2^n - 1$ possible combinations to analyze.

There is also an extension of the Shapley value, the Aumann-Shapley value that allows fractional players/portfolios, also mentioned as non-atomic players in game theory. Although this is not in the scope of this work, is still important to refer since the next method was derived from this concept.

For further details on Shapley and Aumman-Shapley see Balog (2011), Denault (2001), Karabey (2012) and Kaye (2015).

5.7. Euler Allocation

The Euler allocation method is also known as the gradient allocation principle and is currently one of the most used methods. This method can be used under a differentiable and homogeneous risk measure of degree 1.

Definition 5.7.1: A risk measure ρ is **homogenous** of degree κ if for any $h > 0$,

(5.7.1) $$\rho(hY) = h^{\kappa}\rho(Y), \qquad \kappa \in \mathbb{Z}.$$

A function $f: Z \subset \mathbb{R}^n \longmapsto \mathbb{R}$ is called homogeneous of degree κ if for all $h > 0$, $z \in Z$ and $hz \in Z$,

(5.7.2) $$f(hz) = h^{\kappa} f(z), \qquad \kappa \in \mathbb{Z}.$$

Theorem 5.7.1: Let $Z \subset \mathbb{R}^n$ be an open set and $f: Z \longmapsto \mathbb{R}$ a continuous differentiable function. The function f is homogeneous of degree κ if and only if,

(5.7.3) $$\kappa f(z) = \sum_{i=1}^{n} z_i \frac{\partial f(z)}{\partial z_i}, \qquad z = (z_1, \dots, z_n) \in Z, \kappa \in \mathbb{Z}.$$

Let f be defined as $f = \rho(Y)$, if κ equals 1, ρ is a homogeneous risk measure of degree 1 and according to the theorem 5.7.1 the following equation holds:

(5.7.4) $$\rho(Y) = \sum_{i=1}^{n} \rho(X_i) \frac{\partial \rho(Y)}{\partial\, \rho(X_i)} .$$

Remember that $\sum_{i=1}^{n} \rho(X_i|Y) = \rho(Y)$ and Y also depends on the variables X_i, leading to the following formula to compute the allocated capital according to Euler method:

$$(5.7.5) \qquad \sum_{i=1}^{n} \rho^{Eul}(X_i|Y) = \sum_{i=1}^{n} \rho(X_i) \frac{\partial \rho(Y)}{\partial\, \rho(X_i)} \Leftrightarrow$$

$$\Leftrightarrow \rho^{Eul}(X_i|Y) = \rho(X_i) \frac{\partial \rho(Y)}{\partial\, \rho(X_i)}, \qquad i \in N.$$

Given these five methods the goal is to find the best one to apply in a portfolio of an insurance company and specifically to the practical example that is the object of study of this work.

5.8. Choice of method

The goal of this work was to find the best method to determine the allocated capital of each risk module, submodule and to the lines of business. The conclusions in this chapter are based on theoretical assumptions and on many researches present in the literature. Also consider the following conclusions assuming that a coherent risk measure was used.

The Proportional allocation method is the simplest method to apply but does not take into account the dependence structure between risks which is a big drawback of this method. It does not penalize portfolios that are highly correlated but also, it does not reward portfolios that improve the diversification effect. For this reason it is not considered to be a good methodology to perform the allocation of capital in an insurance company portfolio.

The Variance-Covariance allocation method satisfies the full allocation principle but the properties of no undercut, symmetry and riskless allocation are not satisfied. No undercut may fail because the variance is not a subadditive risk measure meaning that it is possible to have a risk unit (or a set of them) with an allocated capital bigger than its initial contribution which should not happen. Also, suppose that two risk units not belonging to any existing subset have the same risk contribution to the portfolio but a different variance. If they join the subset, the allocated capital will differ because of the difference in their variances, while it should be equal to satisfy the symmetry property. For the riskless allocation property it is straight-forward that if a unit is risk free its variance is equal to zero leading to an allocated capital also equal to zero. For this reason the riskless allocation property is not satisfied. Moreover, this method gives more importance to the variance of the risk units which can be a disadvantage and really not applicable in practice. For example, most risk modules and submodules do not have any value defined in the Delegated Regulation for its standard deviation/variance which leads to the need to model the risks or have more information about the distribution function associated with each risk. In this work, all the capital requirements are computed with the standard formula and the only reference available about standard deviations of risks is for lines of business of Premium and Reserve submodules of both Non-Life and Health NSLT risk modules. Thus, it is only possible to apply this method in these particular cases.

Another method considered not the best to apply is the Shapley allocation method. Properties of full allocation, symmetry and riskless allocation are satisfied and only the no undercut property is sometimes violated. Although many researchers proved that this method, based on game theory, gives good and consistent results it is not very practical to apply to a whole portfolio. For example, for five risk modules of an insurance portfolio the number of possible combinations to analyze is $2^5 - 1$ which equals to 31 combinations and since the "order of entrance" in the portfolio matters to find the final allocated capital of each risk module then $5! = 120$ permutations have to be considered in the intermediary calculations. If the allocation is also done to the lines of business, which was the case, then for the Premium and Reserve submodule of Non-Life risk module a much more serious problem is in place. Twelve lines leads to $2^{12} - 1 = 4095$ combinations and $12!$ =479001600 permutations. Looking at this numbers and considering the chosen tool to perform the allocation of capital (Excel) the decision was to compare results for a particular case but not to apply this method for the whole portfolio of the insurance company since it is not practical and the time of computation is high.

The last two possibilities are Merton and Perold method and Euler method. The first one satisfies properties of full allocation and symmetry but does not always satisfy the no undercut and riskless allocation properties. Euler method satisfies the full allocation, no undercut and riskless allocation properties but fails to satisfy the symmetry property.

Regarding the previous methods that were studied, none satisfies all properties that define a coherent allocation principle in a theoretical point of view. Euler method seems the most appropriate to apply since it satisfies most of the properties and is not so complex to compute as the Shapley Value. Furthermore, researches done on this method refer that it is the most stable to apply even when considering different risk measures, it is the only method compatible with portfolio optimization and suitable for performance measurement.

For these reasons the method of capital allocation that was applied to the whole portfolio of the insurance company was the Euler allocation method. Introduction and results on the practical problem are present in the next chapter. To compare this method with the others studied, an extra exercise was performed in order to see if the conclusions differ significantly or not.

6. Case Study

The practical component of this work was to apply a method of capital allocation to allocate the Basic Solvency Capital Requirement (BSCR) to each risk module, submodule and lines of business of an insurance company, in such way that the allocated capital sums up to the total BSCR. First in this section is the information regarding the data available and some calculations that were required before applying any method. The second part shows some of the results relative to the application of Euler allocation method, followed by a third section were the Euler results are compared with all the other methods.

6.1. Data and data treatment

The available data to perform the capital allocation was all the capital requirements for each risk module and submodules from a composite insurance company. All the data was anonymized in order to maintain the information about the client private. Regarding the lines of business only premium and reserve volumes were provided. Part of this information can be consulted in Appendix A.

To be able to allocate the capital to each line of business and to Premium Risk and Reserve Risk separately it was necessary to compute capital charges that can be interpreted as the SCR for Premium risk and also for Reserve risk for each line of business, before applying Euler method. To solve this issue the following formulas were adapted from the ones used in the standard model for the calculation of premium and reserve risk.

(6.1.1) $$SCR_{P,k} = 3\sigma_{P,k}V_{P,k}, \qquad k \in LoBs.$$

Where the P represents the Premium component, k a Line of Business from the Premium and Reserve risk submodule and $V_{P,k}$ the volume measure of premium risk of line of business k. The amounts $\sigma_{P,k}$ and $V_{P,k}$ are available in the data.

In an equivalent way,

$$(6.1.2) \qquad SCR_{R,k} = 3\sigma_{R,k}V_{R,k}, \qquad k \in LoBs.$$

Where the R represents the Reserve component, k a Line of Business from the Premium and Reserve risk submodule and $V_{R,k}$ the volume measure of reserve risk of line of business k. The amounts $\sigma_{R,k}$ and $V_{R,k}$ are available in the data.

Also,

$$(6.1.3) \qquad SCR_{P\&R,k} = 3\sigma_k\left(V_{P,k} + V_{R,k}\right), \qquad k \in LoBs.$$

Where the $P\&R$ represents the Premium and Reserve risk submodule and k a line of business. The amount of σ_k is calculated with the orientations given by the Delegated Regulation:

$$(6.1.4) \qquad \sigma_k = \frac{\sqrt{(\sigma_{P,k}V_{P,k})^2 + 2 \times 0{,}5 \times \sigma_{P,k}\sigma_{R,k}V_{P,k}V_{R,k} + \left(\sigma_{R,k}V_{R,k}\right)^2}}{V_{P,k} + V_{R,k}},$$

$$k \in LoBs.$$

With these three formulas it is possible to have a capital charge associated with each line of business for Premium and Reserve risk and also for both components in separate. These will be used as the initial risk contributions necessary to apply Euler method. Another amount that is necessary is the capital associated with the premium component when aggregating all the lines of business and the same for the reserve risk. Using the values obtained is possible to apply the following formulas:

$$(6.1.5)\qquad SCR_P = \sqrt{\sum_{k,l \in LoBs} Corr(k,l)\ SCR_{P,k} SCR_{P,l}}.$$

$$(6.1.6)\qquad SCR_R = \sqrt{\sum_{k,l \in LoBs} Corr(k,l)\ SCR_{R,k} SCR_{R,l}}.$$

$$(6.1.7)\qquad SCR_{P\&R} = \sqrt{\sum_{k,l \in LoBs} Corr(k,l)\ SCR_{P\&R,k} SCR_{P\&R,l}}.$$

The last formula must lead to a result equal to the capital requirement for Premium and Reserve risk submodule given by the insurance company.

The results of this intermediary calculations were calculated for all the lines of business regarding the Premium and Reserve risk of Health NSLT submodule and of Non-Life module. As an example, the results related with Health NSLT submodule can be seen in the following tables.

Monetary units: Euros

$Line\ of\ business$	$V_{P,k} + V_{R,k}$	σ_k	$SCR_{P,k}$	$SCR_{R,k}$	$SCR_{P\&R,k}$
1. Medical expenses	96.669.193	4,8%	13.071.354	1.429.025	13.841.304
2. Income protection	2.556.664	8,5%	645.802	10.124	650.923
3. Workers compensation	122.161.525	7,8%	24.934.233	6.028.734	28.432.084
4. Non-proportional health	0	0,0%	0	0	0

Table 6.1.1: Capital per line of business for P&R of Health NSTL risk submodule.

Monetary units: Euros

SCR_P	SCR_R	$SCR_{P\&R}$
33.815.108	6.861.382	37.702.025

Table 6.1.2: Capital requirements for Premium, Reserve, and P&R of Health NSLT risk submodule.

Notice that the amount $\mathrm{SCR}_{\mathrm{P\&R}}$ is equal to the capital requirement given in the data for the Premium and Reserve risk of Health NSLT risk submodule. The results are consistent.

It is now possible to apply Euler allocation method to all the risks components that the insurance company faces.

6.2. Euler Allocation

The Euler allocation method was applied to all modules of risks, submodules and lines of business. Remember that the allocation is done backwards and consider the following notations and formulas that allow the application of this method.

As previously defined, let M= {Market, Health, Default, Life, Non-life} and S_x be the set of risks belonging to module $x, x \in M$. Assume always that $x, y \in M$ and $i, j \in S_x$.

- $\rho(X_x) = SCR_x$;
- $\rho(Y) = BSCR$;
- $\rho(X_x|Y) = SCR_{(x|BSCR)}$;
- $BSCR = \sum_{x \in M} SCR_{(x|BSCR)}$.

It is possible to deduce the amount $\frac{\partial \rho(Y)}{\partial\ \rho(X_i)}$, which in this case is given by:

(6.2.1)[3] $$\frac{\partial BSCR}{\partial SCR_x} = \frac{\sum_{x,y \in M} Corr_{x,y} SCR_y}{BSCR}.$$

Therefore, the Euler formula for a risk module is equivalent to:

(6.2.2) $$SCR_{(x|BSCR)} = SCR_x \frac{\partial BSCR}{\partial SCR_x} = SCR_x \frac{\sum_{x,y \in M} Corr_{x,y} SCR_y}{BSCR}.$$

[3] Proof is given in appendix C. Granito and Angelis (2015) also provides a similar approach and interpretation of the Euler method.

For a risk submodule,

$$SCR_{(x,i|\,SCR_{(x|BSCR)})} = SCR_{x,i}\frac{\sum_{i,j\in S_x} Corr_{xi,xj}SCR_{x,j}}{SCR_x}AR_x. \tag{6.2.3}$$

Using the same logic it is possible to continue to apply the Euler method to risk sub-submodules and to lines of business. As an example some of the results are represented in the next tables.

Monetary units: Euros

	INPUT	OUTPUT
SCR Contributions	Pre diversification	Post diversification
SCR	170.224.531	
Adj.	15.127.900	
$SCR_{Operational}$	30.655.704	30.655.704
BSCR	154.696.727	154.696.727
Market Risk	75.625.014	57.284.672
Health Risk	50.347.906	25.633.361
Health SLT	15.618.133	5.762.814
Health CAT	8.075.239	1.747.955
Health NSLT	37.704.251	18.122.592
Life Risk	19.134.942	6.846.446
Non-Life Risk	77.849.636	53.444.096
Counterparty/Default Risk	18.888.103	11.488.152
Intangible Risk	0	0

Table 6.2.1: Results of Euler capital allocation method per module of risk.

Monetary units: Euros

SCR Contributions	Pre diversification	Post diversification
Market Risk	75.625.014	57.284.672
Interest risk	7.309.779	535.200
Equity risk	2.186.194	1.368.168
Property risk	19.351.783	9.984.669
Spread risk	61.040.377	44.238.331
Currency risk	0	0
Concentration risk	10.753.693	1.158.305

Table 6.2.2: Results of Euler capital allocation method for the Market risk module.

Monetary units: Euros

SCR Contributions	Pre diversification	Post diversification
Health NSLT	37.704.251	18.122.592
Premium and Reserve risk	37.702.025	18.120.452
Premium	33.815.108	16.037.734
Reserve	6.861.382	2.082.718
Lapse risk	409.729	2.140

Table 6.2.3: Results of Euler capital allocation method for the Health NSLT risk submodule.

It is possible to see in the previous examples that the full allocation property is fulfilled and the risk components have an allocated capital lower than the initial risk contribution. This allows to measure the benefits from diversification effects resulting from the aggregation of risks.

The allocation of capital was also performed for all lines of business of Premium and Reserve risk for both Health NSLT submodule and Non-Life module. These particular cases were chosen to compare the different methods of capital allocation studied in this work. In the next section it is possible to see an example regarding to the lines of business of Premium and Reserve Risk of Health NSLT risk submodule.

6.3. Comparison of Euler method with other methods

This section relates to the comparison of Euler method with the other studied methods. Since other methods were not applied to all the risk components it was necessary to assume an allocated capital for the Premium and Reserve Risk for both Health NSLT and Non-Life risk module. Given that the Euler method is consider the best method available, the values of the allocated risk capitals for the Premium & Reserve sub-modules are equal to the ones obtain with Euler principle. This amounts are used only as a starting point to apply other methods to allocate the capital to each line of business.

As an example, the allocation to the lines of business of Premium and Reserve risk of the Health NSLT risk submodule are presented in the next table.

Monetary units: Euros

$LoBs$	$Euler$	$Proportional$	$Variance\ Covariance$	$Merton\ Perold\ (1)$	$Merton\ Perold\ (2)$	$Shapley$
$P\ \&\ R\ Health\ NSLT$	18.120.452	18.120.452	18.120.452	15.781.433	18.120.452	18.120.452
1	5.008.089	5.843.091	3.758.516	4.296.265	4.933.028	5.445.867
2	180.792	274.787	359.554	178.975	205.502	217.423
3	12.931.571	12.002.574	14.002.382	11.306.193	12.981.921	12.457.162
4	0	0	0	0	0	0

Table 6.3.1: Allocation to lines of business of P&R risk of Health NSLT risk submodule with different methods.

As it was explained before, Variance Covariance principle is proven not to be a good choice since is not possible to apply to the other submodules and modules of risk. Notice that Merton and Perold using formula (5.5.1) does not fulfill the full allocation property as it was expected. However, using formula (5.5.2) the property is now fulfilled and the results seem very similar to the Euler method.

Although is not presented in the example it is important to mention that when applying the Shapley method to the lines of business of Premium and Reserve risk of the Non-Life risk module, it was only applied to 9 lines of business due to the fact that lines 10, 11 and 12 did not have any risk contribution. The first approach was to construct an excel file able to apply this method to all lines of business and therefore prepared to receive any data, but the program was constantly shutting down and also not able to compute all the necessary permutations. Given this problem, the second tentative was to determine the allocated capital with program R, even though that was not the chosen tool on which the work had to be done, it was done to confirm the impossibility to apply Shapley method to 12 lines. Once more, no results were obtain due to lack of memory. Even with only 9 components the method was the most complicate and time-consuming to apply and therefore not recommended.

In order to have more insight about the difference between methods the Euclidian distance was applied.

Definition 6.3.1: Consider a n-dimensional space. The **Euclidean distance** $\boldsymbol{d}$ between two points $\boldsymbol{a} = (a_1, \dots, a_n)$ and $\boldsymbol{b} = (b_1, \dots, b_n)$ is given by:

$$d^{a,b} = \sqrt{\sum_{i=1}^{n} (b_i - a_i)^2}. \quad (6.3.1)$$

Euler method is used as a reference. The difference between this method and the other ones is presented in the next tables.

For the lines of business of Premium and Reserve risk of Health NSLT submodule,

Methods	*Euclidean distance*	*Difference (%)*
Proportional	1.252.637	7%
Variance Covariance	1.655.302	9%
Merton Perold (2)	93.701	1%
Shapley	646.572	4%

Table 6.3.2: Euclidean distance between Euler and other methods regarding P&R risk of Health NSLT risk submodule.

For the lines of business of Premium and Reserve risk of Non-Life module,

Methods	*Euclidean distance*	*Difference (%)*
Proportional	5.352.004	12%
Variance Covariance	3.341.560	8%
Merton Perold (2)	685.360	2%
Shapley	1.878.673	4%

Table 6.3.3: Euclidean distance between Euler and other methods regarding P&R risk of Non-Life risk module.

As it was expected Proportional allocation and Variance Covariance methods are the most distant from the Euler method and the Merton Perold method proves to be the most similar.

Within all the methods studied the preferred method is Euler allocation method and the most similar to this one is Merton and Perold method (adapted formula). However, the choice of method should take into account the risk measure, in this case the risk measure used was VaR which is not a coherent measure of risk. Theoretically, using TVaR could be more reliable in terms of results because a coherent risk measure provides a higher chance that the method of allocation is also coherent and fulfils all the required properties.

Furthermore, the choice of method should be consistent with the purpose of allocating risk capital, that is, it must be always dependent on the further uses of the allocation.

7. Why allocate capital?

Capital allocation has many applications for the financial institutions. It can be used for product pricing, for strategic decisions regarding new lines of business, to decide which lines of business to expand or if a component is worth keeping or not. Allocating capital is also useful for managing the types of risk a company accepts and a helpful tool in risk budging, allowing the manager to decide which areas, for example lines of business, products or even geographical areas, to accept risk. It is also helpful to evaluate a portfolio performance or even the individual management performance. In this work the application of a capital allocation method was needed to find the SES, a concept that is clarified is the next section.

7.1 Single Equivalent Scenario and Loss-absorbing capacity of technical provisions and deferred taxes.

The Single Equivalent Scenario (SES) is one of the approaches suggested in the past by EIOPA, called "alternative approach", to calculate the loss absorbing capacity (LAC) of technical provisions (TP) and deferred taxes (DT). The SES assumes a scenario under which all the risks occur simultaneously and regarding the operational risk, it assumes that the operational loss takes a value equal to the capital charge of this same risk. To construct this scenario, the capital requirements for each risk are necessary as inputs. By default, this amounts are the gross capital requirements, which exactly correspond to the data provided for this work. After having this information the goal is to find a correspondent amount that represents the 1-in-200 scenario, which can be done by applying a capital allocation method, for instance, the outputs presented in chapter 6 may be used for other strategic decisions but also represent the SES for this particular set of data.

Regarding this approach, the following advantages were recognized by EIOPA:

- The double counting of LAC TP is avoided.
- The LAC DT can also be integrated in the scenario.
- More realist management actions.

As for the disadvantages, many undertakings are not familiar with this concept, referring that it requires more difficult calculations and therefore, this approach was not extensively tested. In fact, according to CBFA (2011), Central Bank of Ireland (2011), Dalby (2011), Danish FSA (2011), EIOPA (2011), Financial Services Authority (2011), Guiné (2011) and Hungarian FSA (2011); countries such as Belgium, Denmark, Germany, Hungary, Ireland, Norway, Portugal and the United Kingdom concluded in their QIS5 that only a few participants tried, and some unsuccessfully, to use the SES approach to calculate the LAC of TP and DT and the general conclusion was that the calculations are technically very complex. However, this disadvantage can be overcome if there is proper research and documentation on this matter.

Regarding the LAC DT, some concepts should be clarified just to provide the reader a brief idea of the following procedures after building the SES.

Deferred taxes (DT) arise from differences between an asset or a liability value, set for tax purposes, and its SII value. In the SII balance sheet, all items are valued at their economic value which recognizes unrealized gains/losses, leading to the need to also recognize the corresponding tax value. A deferred tax liability (DTL) represents a liability because it is a tax that is due during the present or has been assessed but not yet paid. A deferred tax asset (DTA) represents an asset in the balance sheet that may be used to reduce taxable income, for instance, where there are any overpaid taxes or taxes paid in advance in the balance sheet, this tax value will be returned in the future, and therefore may represent an asset.

The loss absorbing capacity of deferred taxes (LAC DT) corresponds to an adjustment that is equivalent to the change in the net tax position due to the application of a shock, arising from an instantaneous loss. Building the SES allows an allocation of this loss to each SII Balance sheet item, that is, an instantaneous loss can be allocated to the related value of assets and liabilities individually, resulting on a Balance sheet post shock. After this, is possible to compute the individual tax adjustments, item by item, which together result on the global adjustment. The DT aftershock are determined by summing the initial DT and this global adjustment. The adjustment of LAC DT is therefore, given by the difference between the value of DT in the SII balance sheet (initial DT) and the value of DT in the balance sheet post shock (under the SES). The adjustment of LAC DT is only recognized if the loss leads to a decrease in the DTL or an increase in the DTA. The decrease in DTL can be immediately recognized for the purposes of the adjustment, but that is not the case if there is an increase in the DTA, where a further test must be performed in order to ensure that enough future taxable income will be available to be used against that assets. Depending on the results of this test, it is possible that the adjustment for LAC DT has to be narrowed. For further explanations on the topic, consult CEIOPS (2009) and EIOPA (2014) (EIOPA-BoS-14/177).

8. Conclusion

The choice of subject in this report was more specific and since it was more complex, it was useful to restrict the report to one subject, allowing a greater understanding and explanation of the topic and all the calculations.

The goal was to find the best capital allocation method to be applied to the SCR. The proportional allocation is not recommended since it does not take into account correlations between risks. The variance-covariance method does not satisfy most of the coherence properties and cannot be used in all modules of risk. The Merton and Perold method presents results closer to the Euler method, however, it does not satisfy as many coherence principles as the latter. The Shapley method has the disadvantage of being difficult to calculate since it is necessary to analyze a high number of possible combinations between the risk units, resulting in a high computing time. Finally, the Euler method is the most balanced method between the ease of its application and the principles of coherence that it satisfies. It is also well defended by other authors since it is the only appropriate method for performance measurement. In short, Euler's method is the most recommended to allocate capital.

For a further research, it would be interesting to perform the allocation under the two risk measures, VaR and TVaR, to analyze whether the use of a coherent risk measure affects the results significantly.

In conclusion, this report provides a better understanding of the different allocations methods and is useful to insurance companies to understand the construction of the SES.

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Appendix

A. Data

The information provided includes all the data, which were anonymized, regarding the capital requirements for risk modules and submodules and premium and reserve volumes, from the composite insurance company that was the object of study of this work. In order to respect the privacy of the client, only the data regarding the output examples presented in chapter 6 are exposed in this section.

All the correlation matrices used and other assumptions relative to the risk components are also provided in section B.

Monetary unit: Euros

SCR Contributions	
SCR	170.224.531
Adj	-15.127.900
SCR Operational	30.655.704
BSCR	154.696.727
SCR Market	75.625.014
Interest (Scenario Up)	7.309.779
Equity	2.186.194
Property	19.351.783
Spread	61.040.377
Currency	0
Concentration	10.753.693
SCR Health	50.347.906
SLT	15.618.133
Catastrophe	8.075.239
NSLT	37.704.251
Premium and Reserve risk	37.702.025
Lapse risk	409.729
SCR Life	19.134.942
SCR Non-life	77.849.636
SCR Counterparty	18.888.103
SCR Intangibles	0

Table A.1: Capital requirements.

Monetary units: Euros

Lines of business (k)	$V_{P,k}$	$V_{R,k}$	$\sigma_{P,k}$	$\sigma_{R,k}$
Health NSLT				
1: Medical Expenses	87.142.357	9.526.836	5%	5%
2: Income Protection	2.532.558	24.106	9%	14%
3: Workers compensation	103.892.636	18.268.889	8%	11%
4: Non-proportional health	0	0	17%	20%

Table A.2: P&R volumes and the respective standard deviations for each line of business of P&R risk of Health NSLT risk submodule.

B. Correlation between risks

Correlations between risks follow the orientations given in the Delegated Regulation.

$Corr(x,y)$	$Market$	$Default$	$Life$	$Health$	$Non-Life$
$Market$	1,00	0,25	0,25	0,25	0,25
$Default$	0,25	1,00	0,25	0,25	0,50
$Life$	0,25	0,25	1,00	0,25	0
$Health$	0,25	0,25	0,25	1,00	0
$Non-Life$	0,25	0,50	0	0	1,00

Table B.1: Correlations between risk modules.

$Market$ $Corr(i,j)$ $Scenario\ Up$	$Interest$	$Equity$	$Property$	$Spread$	$Currency$	$Concen-tration$
$Interest$	1,00	0	0	0	0,25	0
$Equity$	0	1,00	0,75	0,75	0,25	0
$Property$	0	0,75	1,00	0,50	0,25	0
$Spread$	0	0,75	0,50	1,00	0,25	0
$Currency$	0,25	0,25	0,25	0,25	1,00	0
$Concentration$	0	0	0	0	0	1,00

Table B.2: Correlations between submodules of Market risk module.

$Equity$ $Corr(t_1,t_2)$	$Type\ 1$	$Type\ 2$
$Type\ 1$	1,00	0,75
$Type\ 2$	0,75	1,00

Table B.3: Correlations between equities of type 1 and equities of type 2.

$Health$ $Corr(i,j)$	$Health\ SLT$	$Health\ CAT$	$Health\ NSLT$
$Health\ SLT$	1,00	0,25	0,50
$Health\ CAT$	0,25	1,00	0,25
$Health\ NSLT$	0,50	0,25	1,00

Table B.4: Correlations between submodules of Health risk module.

$Health\ SLT$ $Corr(k,l)$	$Mortality$	$Longevity$	$Disability$ $/Morbidity$	$Lapse$	$Expenses$	$Revision$
$Mortality$	1,00	-0,25	0,25	0	0,25	0
$Longevity$	-0,25	1,00	0	0,25	0,25	0,25
$Disability$ $/Morbidity$	0,25	0	1,00	0	0,50	0
$Lapse$	0	0,25	0	1,00	0,50	0
$Expenses$	0,25	0,25	0,50	0,50	1,00	0,50
$Revision$	0	0,25	0	0	0,50	1,00

Table B.5: Correlations between submodules of Health SLT risk submodule.

$Health\ CAT$ $Corr(k,l)$	$Mass\ accident\ risk$	$Accident$ $concentration\ risk$	$Pandemic\ risk$
$Mass\ accident\ risk$	1,00	0	0
$Accident$ $concentration\ risk$	0	1,00	0
$Pandemic\ risk$	0	0	1,00

Table B.6: Correlations between submodules of Health CAT risk submodule.

$Health\ NSLT$ $Corr(k,l)$	$Premium\ \&\ Reserve$	$Lapse$	$Catastrophe$
$Premium\ \&\ Reserve$	1,00	0	0,25
$Lapse$	0	1,00	0
$Catastrophe$	0,25	0	1,00

Table B.7: Correlations between submodules of Health NSLT risk submodule.

$P\ \&\ R$ $LoBs\text{-}\ Health\ NSLT$ $Corr(m,n)$	$Medical$ $Expenses$	$Income$ $Protection$	$Workers$ $compensation$	Non $proportional$ $health$
$Medical$ $Expenses$	1,00	0,50	0,50	0,50
$Income$ $Protection$	0,50	1,00	0,50	0,50
$Workers$ $compensation$	0,50	0,50	1,00	0,50
$Non\ proportional$ $health$	0,50	0,50	0,50	1,00

Table B.8: Correlations between LoBs of P&R of Health NSLT risk submodule.

$Non - Life$ $Corr(i,j)$	$Premium\ \&\ Reserve$	$Lapse$	$Catastrophe$
$Premium\ \&\ Reserve$	1,00	0	0,25
$Lapse$	0	1,00	0
$Catastrophe$	0,25	0	1,00

Table B.9: Correlations between submodules of Non-Life risk module.

$P\ \&\ R$ $LoBs$- $Non\ Life$ $Corr(m,n)$	1	2	3	4	5	6	7	8	9	10	11	12
1: $MTPL$	1,00	0,50	0,50	0,25	0,50	0,25	0,50	0,25	0,50	0,25	0,25	0,25
2: $Motor\ Other$	0,50	1,00	0,25	0,25	0,25	0,25	0,50	0,50	0,50	0,25	0,25	0,25
3: MAT	0,50	0,25	1,00	0,25	0,25	0,25	0,25	0,50	0,50	0,25	0,50	0,25
4: $Fire\ and\ other\ damage\ to\ property$	0,25	0,25	0,25	1,00	0,25	0,25	0,25	0,50	0,50	0,25	0,50	0,50
5: $Third\ part\ liability$	0,50	0,25	0,25	0,25	1,00	0,50	0,50	0,25	0,50	0,50	0,25	0,25
6: $Credit\ and\ suretyship$	0,25	0,25	0,25	0,25	0,50	1,00	0,50	0,25	0,50	0,50	0,25	0,25
7: $Legal\ expenses$	0,50	0,50	0,25	0,25	0,50	0,50	1,00	0,25	0,50	0,50	0,25	0,25
8: $Assistance$	0,25	0,50	0,50	0,50	0,25	0,25	0,25	1,00	0,50	0,25	0,25	0,50
9: $Miscellaneous\ non\ life\ insurance$	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	1,00	0,25	0,50	0,25
10: $Non\ proportional\ casualty$	0,25	0,25	0,25	0,25	0,50	0,50	0,50	0,25	0,25	1,00	0,25	0,25
11: $Non\ proportional\ MAT$	0,25	0,25	0,50	0,50	0,25	0,25	0,25	0,25	0,50	0,25	1,00	0,25
12: $Non\ proportional\ property$	0,25	0,25	0,25	0,50	0,25	0,25	0,25	0,50	0,25	0,25	0,25	1,00

Table B.10: Correlations between LoBs of P&R of Non-Life risk module.

$Counterparty/Default$ $Corr(i,j)$	$Type\ 1\ exposure$	$Type\ 2\ exposure$
$Type\ 1\ exposure$	1,00	0,75
$Type\ 2\ exposure$	0,75	1,00

Table B.11: Correlations between submodules of Default risk module.

$Corr(P,R)$	$Premium$	$Reserve$
$Premium$	1,00	0,50
$Reserve$	0,50	1,00

Table B.12: Correlation between Premium risk and Reserve risk.

C. Mathematical proofs

Proof 1: Proof of equation (5.4.3).

$$Var(Y) = Var\left(\sum_{i=1}^{n} X_i\right) =$$

$$= E\left(\left[\sum_{i=1}^{n} X_i\right]^2\right) - \left[E\left(\sum_{i=1}^{n} X_i\right)\right]^2 =$$

$$= E\left(\sum_{i=1}^{n}\sum_{j=1}^{n} X_i X_j\right) - \left[\sum_{i=1}^{n} E(X_i)\right]^2 =$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} E(X_i X_j) - \sum_{i=1}^{n}\sum_{j=1}^{n} E(X_i)E(X_j) =$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n}\left(E(X_i X_j) - E(X_i)E(X_j)\right) =$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} Cov(X_i, X_j) =$$

$$= \sum_{i=1}^{n} Cov\left(X_i, \sum_{j=1}^{n} X_j\right) =$$

$$= \sum_{i=1}^{n} Cov(X_i, Y). \qquad \blacksquare$$

Proof 2: Proof of equation (6.2.1).

$$\frac{\partial BSCR}{\partial SCR_x} = \frac{\partial}{\partial SCR_x}\left(\sqrt{\sum_{x,y\in M} Corr_{x,y}SCR_xSCR_y} + SCR_{Intang}\right) =$$

$$= \frac{\partial}{\partial SCR_x}\left(\sum_{x,y\in M} Corr_{x,y}SCR_xSCR_y\right)^{\frac{1}{2}} =$$

$$= \frac{1}{2}\left(\sum_{x,y\in M} Corr_{x,y}SCR_xSCR_y\right)^{-\frac{1}{2}} \times \frac{\partial}{\partial SCR_x}\left(\sum_{x,y\in M} Corr_{x,y}SCR_xSCR_y\right) =$$

$$= \frac{1}{2\sqrt{\sum_{x,y\in M} Corr_{x,y}SCR_xSCR_y}}$$

$$\times \frac{\partial}{\partial SCR_x}\left(SCR_x{}^2 + SCR_y{}^2 + 2\sum_{\substack{x,y\in M \\ x\neq y}} Corr_{x,y}SCR_xSCR_y\right) =$$

$$= \frac{1}{2\left(BSCR - SCR_{Intang}\right)} \times \left(2SCR_x + 2\sum_{\substack{x,y\in M \\ x\neq y}} Corr_{x,y}SCR_y\right) =$$

$$= \frac{1}{BSCR - SCR_{Intang}{}^{(*)}} \times \sum_{x,y\in M} Corr_{x,y}SCR_y =$$

$$= \frac{\sum_{x,y\in M} Corr_{x,y}SCR_y}{BSCR}.$$

(*) Notice in the data available that $SCR_{Intang} = 0$. ■