LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Master

Monetary and Financial Economics

Master's Final Work

Dissertation

Is Bitcoin a Good Investment Asset?

João Miguel Guerreiro Apolónia

December - 2021

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Supervision:

Professor Margarida Abreu

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Abstract

This dissertation aims to analyze the consequences of adding Bitcoin to an investment portfolio. The main methodology used is the Mean-Variance model combined with the Monte Carlo Simulation. Results show that Bitcoin can improve the Sharpe Ratio of an already diversified portfolio, however the inclusion of Bitcoin has to be done in proportions averaging 3.83% of the portfolio's weight. This dissertation also found that Bitcoin does not seem to behave as a safe haven/hedge asset during the Covid-19 pandemic.

Keywords: Portfolio theory, Bitcoin, safe haven asset, diversification

JEL Codes: G11; G12; G10

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1. Introduction

A cryptocurrency, as defined by the Merriam-Webster dictionary, is “any form of currency that only exists digitally, that usually has no central issuing or regulating authority but instead uses a decentralized system to record transactions and manage the issuance of new units, and that relies on cryptography to prevent counterfeiting and fraudulent transactions”.

When Nakamoto (2008) first introduced Bitcoin, the goal was to create a virtual, alternative currency that would be completely independent from any financial institution. In 2009 Bitcoin was launched, a cryptocurrency based on a peer-to-peer network that relies on a public ledger of every transaction, called the Blockchain[1]. Considered by many a breakthrough technology, the Blockchain allows every user to check the balance and the transactions of any Bitcoin wallet, and thus providing full transparency. It is not completely anonymous, but rather pseudoanonymous, in the sense that a person is linked to its Bitcoin address (the pseudonym) but not to its name or home address (Moreno & Shivangee, 2011)

Bitcoins are created by “miners”, people that use their CPU power to validate transactions, thus generating blocks, that are added on the blockchain. The supply of Bitcoins is capped at 21 million and its rate is predetermined: in 2140, the last Bitcoin will be mined. Brito & Castillo (2013), Böhme et al (2015) or Barber et al. (2012) provide a more detailed insight on the technical aspects of Bitcoin and the functioning of Blockchain.

Thousands of cryptocurrencies have ever since been created, with the total market capitalization surpassing the trillion-dollar mark on Jan 2021 and reaching two trillion dollars just three months after (Figure 1). Bitcoin has always been the most relevant cryptocurrency[2], as it detains the biggest share of the cryptocurrency market, of around 45% in May 2021 (Figure 2), and so it will be the one this work will focus on.

[1] Referred on Nakamoto (2008) as “chain”.
[2] See Appendix A for Bitcoin’s total market capitalization.

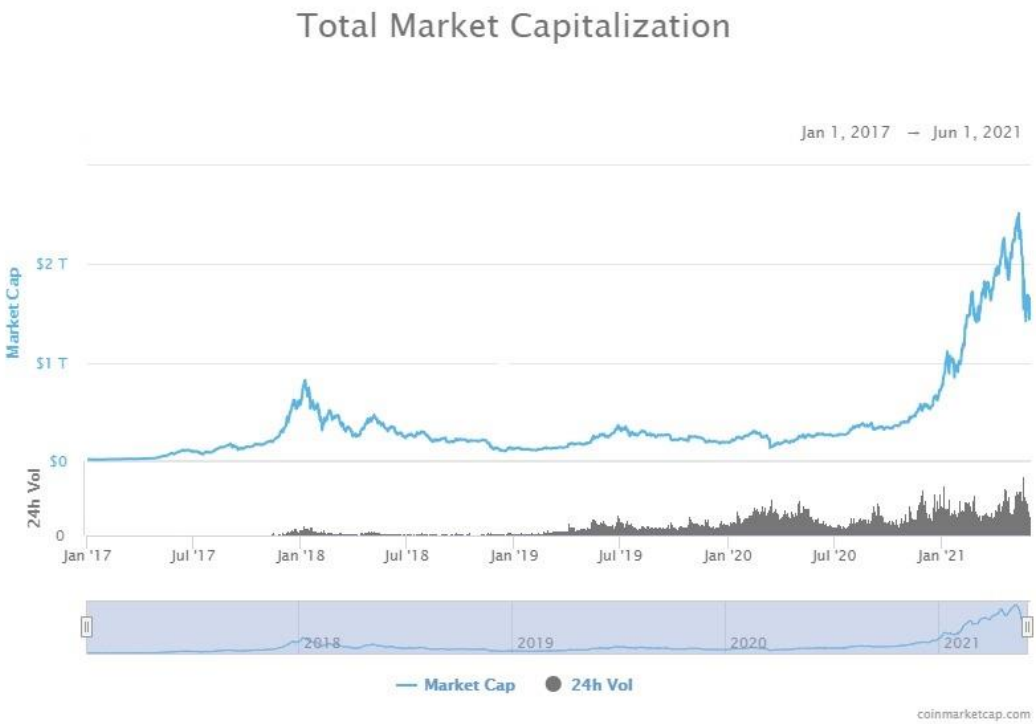

Figure 1: Total Market Capitalization of Cryptocurrencies.
Source: coinmarketcap.com

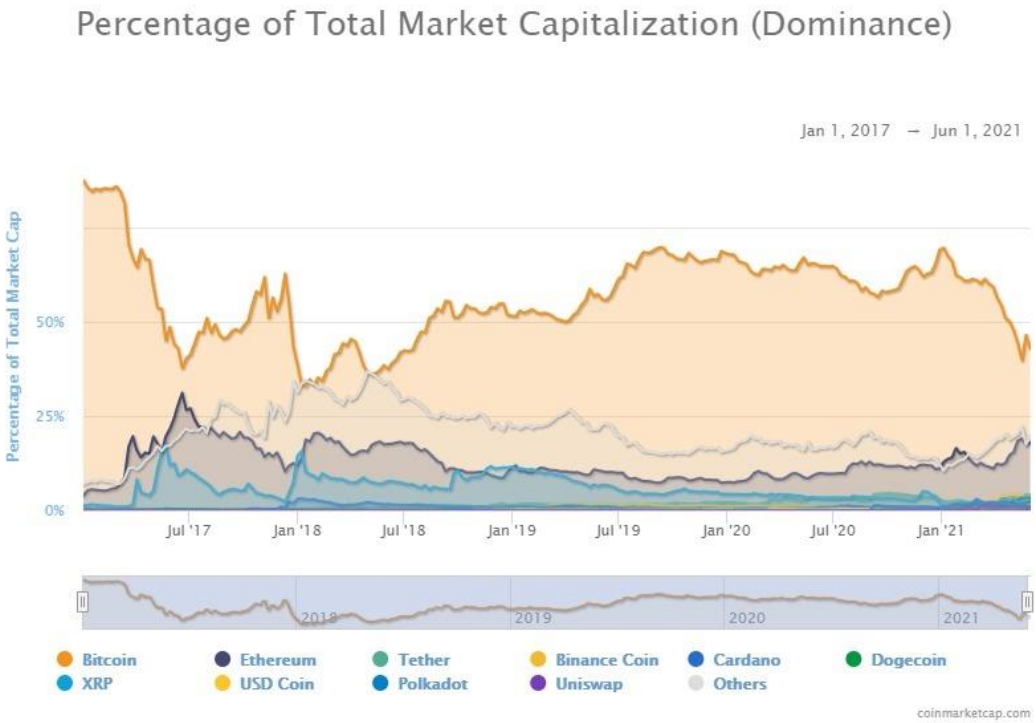

Figure 2: Percentage of Total Market Capitalization.
Source: coinmarketcap.com

Literature has shown that Bitcoin does not seem to function as a currency. Instead, it is generally seen as a speculative asset, that people seek in order to grab profits off of its increases in price. Contrasting with the literature, El Salvador has become in September of 2021 the first nation to consider Bitcoin as legal tender, making mandatory for economic agents to accept Bitcoin as a mean of payment (Hawkins, 2021). This is raising some controversy regarding the logistics associated, for instance, on how would a small business accommodate Bitcoin transactions. At the same time, the number of people voicing their concerns about the environmental issues associated with Bitcoin mining rises (Badea & Mungiu-Pupazan, 2021), making the future of Bitcoin rather uncertain.

Similarly to Klabbers (2017), this work aims to see Bitcoin not as a currency in itself, but as a financial instrument that may or may not provide a more efficient diversification of an investment portfolio or increase its overall performance. Both the data and methodology used are similar, however this work analyses a larger timeframe, not considering the early stages of Bitcoin but rather capturing it after the first big explosion in terms of price and popularity, which happened in 2013. On the brink of 2020, the Covid-19 pandemic struck, leaving the financial markets with increased uncertainty and consequent volatility. This may have led a wave of new investors to join the more traditional markets, as well as the cryptocurrencies market (Priem, 2020). This work also tests the potential of Bitcoin as an investment asset under this new, pandemic context. It is structured as follows: Section 2 is the Literature Review, Section 3 describes the Data and Methodology, Section 4 presents the results, Section 5 presents the robustness check tests, Section 6 zooms into the Covid-19 pandemic period and Section 7 concludes.

2. Literature Review

2.1. Bitcoin: Currency or investment?

A currency, generally speaking, can be described as anything that is used to buy a good or service or to pay a debt (Abreu et al, 2012). It must fulfill three functions:

- Medium of exchange: In the context of a monetary area, a currency should be, in general, accepted by everyone, everywhere. As a medium of exchange, a currency decreases the time spent in the process of exchanging goods and services and overall transaction costs, thus improving economic efficiency.

- Unit of account: A currency is the good through which the prices of all the other goods are calculated and allows us to compare any two goods by their relative price.

- Store of Value: A store of value is an asset that holds purchasing power over time, with no intrinsic depreciation.

Recent literature has since posed the question of whether Bitcoin can fulfill these requirements and be considered a currency. The answer seems to be clear: it cannot.

In 2012, the European Central Bank (ECB) defined virtual currencies as "a type of unregulated, digital money, which is issued and usually controlled by its developers, and used and accepted among the members of a specific virtual community". The stance of the ECB changed in 2015, where virtual currencies were no longer considered money, but rather "a digital representation of value, not issued by a central bank, credit institution or e-money institution, which, in some circumstances, can be used as an alternative to money."

Baur and Dimpfl (2017) showed that Bitcoin volatility can be up to 20 times larger than major currencies, thus not able to function as a currency as it fails to fulfill the store of value function of money.

Dyhrberg (2016a) and Baur et al (2017) place Bitcoin somewhere between a fiat currency and a commodity without intrinsic value. The second work also shows that Bitcoin is being used as a speculative investment: By categorising Bitcoin users into six different types, they found out that currency users[3], in 2013,

[3] Users that have made more than one transaction; have both sent and received transactions, and with sending transaction sizes below $200.

represented only 2.25% of the total Bitcoin users, and passive[4] and active[5] investors represent together more than 34% of all users.

In a similar fashion, Glaser (2014) tried to find what do cryptocurrency users seek when changing from their domestic currencies to digital currencies. Results suggest that new, uninformed users approaching Bitcoin, are not interested in its original purpose of functioning as a alternative currency, but seek for an alternative investment vehicle.

Yermack (2013) and Kancs & Ciaian (2015) breakdown the features of Bitcoin and conclude that Bitcoin does not behave as a currency since it does not fulfill the three functions of money. Yermack (2013) adds that its behaviour resembles "a speculative investment similar to the Internet stocks of the late 1990s".

Kancs & Ciaian (2015) also found that the attractiveness for investment and the market forces are the drivers of Bitcoin's price, and as long has it happens, Bitcoin cannot compete with other currencies. Macroeconomic and financial factors were found to not influence Bitcoin's price. This is one of the reasons why it could be interesting to include Bitcoin on a global market portfolio.

2.2. Bitcoin applied to an investment portfolio

A few authors have tested Bitcoin in financial portfolios, searching for hedging capabilities or other possible instruments that financial analysts could have at their disposal. One of these instruments is, for example, the fact that there are no days where trading is closed, which provides liquidity advantages (Dyhrberg, 2016b).

Klabbers (2017) includes Bitcoin in a global market portfolio and combining a mean-variance analysis with a Monte Carlo simulation, results show that Bitcoin is an effective diversifier with an allocation between 0% and 5%. However, it does not show any hedge or safe haven characteristics for a global market portfolio.

[4] Users that do not send Bitcoin, and only receive Bitcoin in transactions greater than $100; or have received one Bitcoin transaction greater than $100.
[5] Users that have made more than two transactions and only send Bitcoin in transactions greater than $2000.

On the same line, but using a dynamic conditional correlation model, Molnár et al (2017) found that the cryptocurrency acts as a poor haven in most cases, serving only for diversification purposes, but actually showed strong haven abilities on the Chinese and Asia Pacific stock markets.

Moore and Stephen (2016) examine the possibility of Bitcoin being part of the international reserves portfolio of a Central Bank, namely the Central Bank of Barbados. Barbados holds a peg against the US dollar, and thus its Central Bank needs a good amount and diversity of international reserves in order to provide some financial stability and prevent speculative attacks. In this sense, the authors use two approaches. The first is a series of four counterfactual simulations, which illustrates what would happen to the international reserves portfolio (in relation to the actual portfolio), had the Central Bank of Barbados invested a 0.01%, 0.1%, 1% or 5% of the reserves in Bitcoin, starting from November 2010, up to April 2015. Results showed that the balance would more than double with just a 0.1% investment and would be 100 times greater with 5% of reserves in Bitcoin, without increasing volatility significantly.

The second method is a Monte Carlo simulation, which provides a forecast of four different horizons (1, 2, 5 and 10 years) for the international reserves assuming there is an incorporation of Bitcoin of 0.01% of reserves by the Central Bank of Barbados. For the 1-year forecast, in no instance portfolio losses were greater or equal to the initial investment. For the 10-year horizon, there were 47 instances of portfolio losses exceeding initial investment, contrasted with 629,953 instances of portfolio gains.

Further literature found hedging capabilities against the stock market: Bouoiyour and Selmi (2017) analyze the behavior of Bitcoin in relation to the U.S. stock price, in the period after Trump's presidential win, which brought some uncertainty in the markets. Using data spanning from the 8th of November, 2016 (the day of the U.S. election), to the 15th of February 2017, the results show that Bitcoin serves as a weak safe haven for the U.S. stock market in the short-run, meaning that, in times of turbulence, it is uncorrelated with other assets.

Brière et al (2015) show that including a small proportion of Bitcoin on a well-diversified portfolio may drastically improve the risk-return trade-off. Since Bitcoin

has a low correlation with traditional financial assets and other alternative investments, it offers high diversification benefits. However, the period analyzed ranges from 2010-2013, which reflects an early-stage behavior of Bitcoin that is prone to change in the medium or long-run.

2.3. Portfolio Theory

Risk, in the context of financial investments, concerns the uncertainty regarding the future returns of securities. One can differentiate between two main types of risk: Unique, or idiosyncratic risk and market risk. Unique risk is the uncertainty related only to a specific security, while the market risk affects a whole system or activity sector (Abreu et al., 2012). It is inherent to macroeconomic elements and cannot be reduced. In this sense, strategies to mitigate risk in financial investments can only be targeted to the unique risk. One of those strategies and the one that is inside the scope of this work is portfolio diversification.

Markowitz (1952) first introduced portfolio theory and hypothesized that investors are risk averse, and thus they do not maximize their returns, but rather seek for an efficient trade-off between risk and return (measured by variance or, equivalently, standard deviation, and expected value, respectively). This can be pictured by assuming a N-securities portfolio, with return equal to the weighted average of the returns of the assets:

(1) $$R_p = \sum_{i=1}^{N} w_i R_i$$

Where w_i represents the weight of each security i within the portfolio ($\sum w_i = 1$) and R_i their return. As in Markowitz (1952), we will assume a no short selling constraint, meaning that no asset can have negative weight ($w_i > 0$, for all i). Risk, measured by variance, is given by:

(2) $$V_p = \sigma_p^2 = \sum_{i=1}^{N} (w_i^2 \, \sigma_i^2) + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} w_i w_j \sigma_{ij}$$

Being σ_{ij} the covariance between two different securities i and j.

What is concluded by the above formula is that, to compute the risk associated with a portfolio, one should not only take into account the individual risk associated to all the assets present in the portfolio, but also the risk of combining those assets together (*i.e.*, the covariance). Furthermore, covariance can be breakdown into the below:

(3) $$\sigma_{ij} = \sigma_i \sigma_j \rho_{ij}$$

ρ_{ij} represents the correlation between two different assets, and can assume a value between [-1,1]. When $\rho_{ij} = -1$, the assets are perfectly negatively correlated: Their returns move in opposite directions. When $\rho_{ij} = 1$, there is a perfectly positive correlation between the assets and so they move in line with each other. If it assumes a value of 0, they are not correlated.

Given the correlation value of a given asset in relation to another asset or set of assets, we are able to categorize it into different classes which, in turn, will allow an assessment of its benefits within an investment portfolio.

These classes were defined by Baur & McDermott, (2010) and Baur & Lucey, (2011), and are as follows:

- Hedge: A strong (weak) hedge is defined as an asset that is negatively correlated (uncorrelated) with another asset or portfolio on average.

- Safe haven: A strong (weak) safe haven is defined as an asset that is negatively correlated (uncorrelated) with another asset or portfolio in certain periods only, e.g., in times of falling stock markets.

- Diversifier: A diversifier is defined as an asset that is positively (but not perfectly correlated) with another asset or portfolio on average.

Given its decentralization from traditional financial institutions, cryptocurrencies, and specifically Bitcoin, could, potentially, be safe haven assets, that investors seek for in more turbulent times. As mentioned in section 2.2, some works have already leaned on this matter and found contradictory results. Thus, this definition

of safe haven must be relativized, because it is not consensual among the literature and the safe haven property has been found to change over time and is subject to the asset class, markets studied or even the characteristics of the market turmoil (Ji, Zhang, & Zhao, 2020). In addition, Smales (2019) add that meeting the criteria of correlation should not be enough to consider Bitcoin as a safe haven, as other attributes come into play, such as the volatility, transaction costs, or liquidity.

2.3.1. The efficient Frontier

Having now these concepts of return and risk in mind, the investment opportunity set, corresponding to all attainable portfolios for the investor, and, in turn, the efficient frontier, can be drawn. Figure 3 depicts the investment opportunity set for the three aforementioned values of ρ.

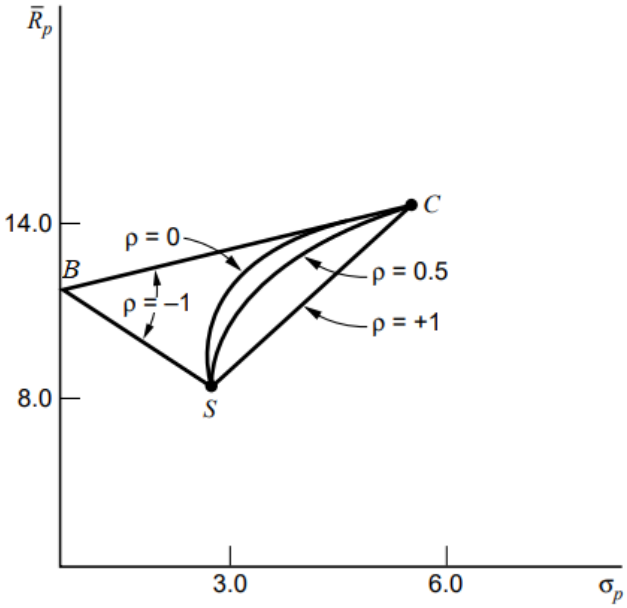

Figure 3: Investment opportunity set (N=2 case).

Source: Elton et al., (2013) [Adapted]

The space within the BSC triangle corresponds to all possible, feasible portfolios. For any other level of correlation rather than 1 or -1, the set of investment opportunities will be delimited by a hyperbole, that will be within BSC.

Also, the closer ρ is from -1, the higher the return-risk ratio, proving geometrically that lower correlation (implied by higher diversification) results in higher payoffs.

The efficient frontier is then the set of investment opportunities that, for a given level of return, has the lowest standard deviation or, for a given level of standard deviation, offers the highest return (Abreu et al., 2012). This set of portfolios can be seen on Figure 4.

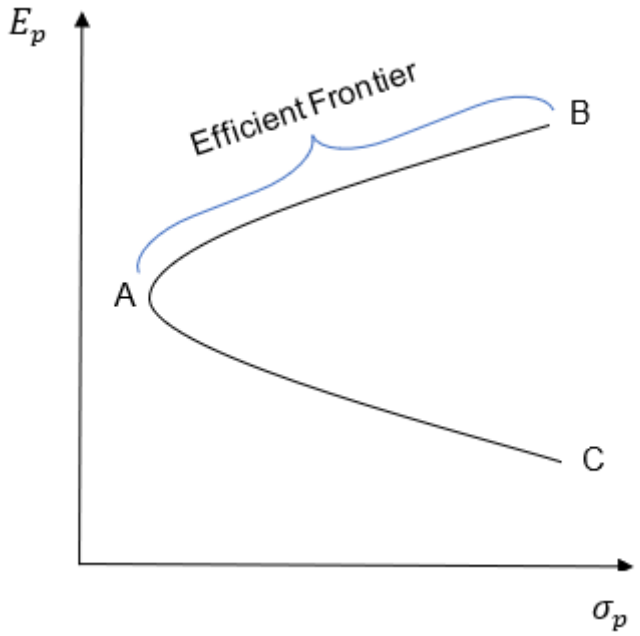

Figure 4: Efficient frontier (N-assets)

Point A represents the portfolio with the lowest possible variance, and it is called the minimum variance portfolio, while Point B represents the portfolio with the highest possible return. We can infer from the figure that point A and point B are not dominated by any other portfolio. No portfolios offer the same return with less risk, or the same risk and higher return. Also, all portfolios standing in AB are preferred to AC, as for a given level of risk, they offer a higher return. The efficient frontier is then defined by all portfolios standing between the minimum variance portfolio and the maximum return portfolio (Elton et al., 2013). According to the Markovitz model, the investor will then be able to choose an optimal portfolio, given their level of risk aversion.

2.4. The context of Covid-19

The Covid-19 pandemic and subsequent financial market crisis brought a significant increase on the volatility of markets throughout the world (Ambros et al, 2020). This ought to be a real, empirical opportunity to test Bitcoin's hedging and safe haven capabilities against traditional assets, proposed on such works as Dyhrberg, (2016b), Moore & Stephen, (2016) or Baur et al (2015).

Figure 5 shows Bitcoin's returns, in USD, from the 13th of January, 2020, the day of the first recorded case outside of China, until the 31st of May, 2021, averaging a daily return of 0.31%. The worst day over that period, corresponding to a return of -49.66%, followed the moment the World Health Organization (WHO) announced Covid-19 as a pandemic (March 11, 2020)[6]. The worst months overall were March 2020 and May 2021, where daily returns averaged -1% and -1.31%, respectively.

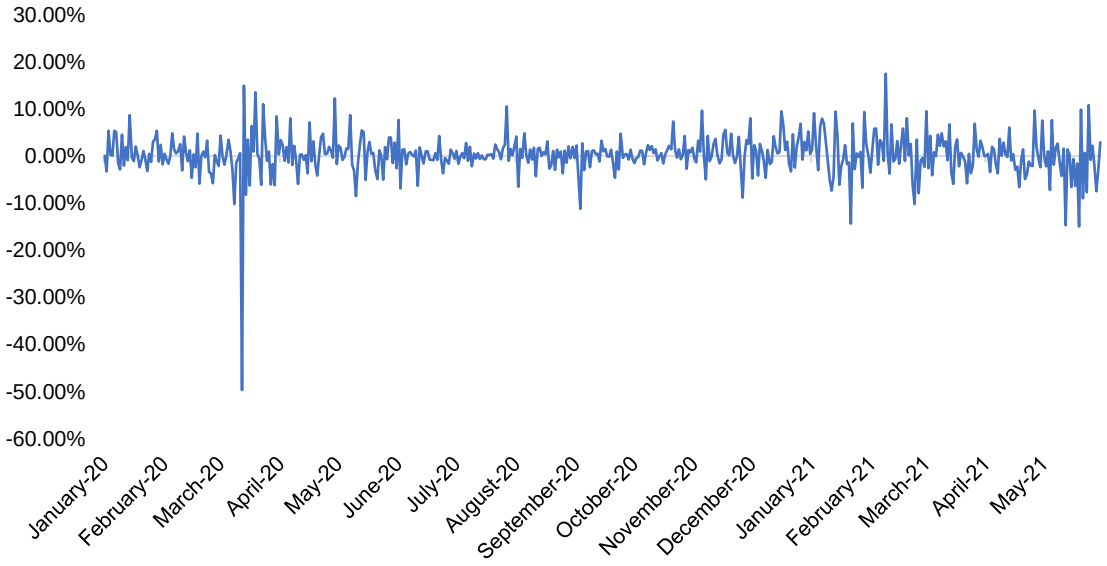

Figure 5: Bitcoin's daily returns (USD).
Data retrieved from Quandl

[6] https://www.who.int/director-general/speeches/detail/who-director-general-s-opening-remarks-at-the-media-briefing-on-covid-19---11-march-2020 [Accessed 7/6/21]

Literature regarding the performance of Bitcoin during the pandemic has been consistent and point out that Bitcoin does not seem to serve as a safe haven:

The negative returns and high volatility on the first four months of 2020 can be explained by the fear sentiment around Covid-19 (Chen, Liu, & Zhao, 2020). This indicates that Bitcoin's movement resembles those of other financial assets, rather than safe haven assets, in periods of market distress. Conlon & McGee (2020) find that Bitcoin has not shown safe haven properties during the Covid-19 pandemic for the S&P 500. Adding to that, it increased the risk (measured by the Conditional Value-at-Risk) of an investment portfolio containing both securities.

Corbet et al (2020) add that, during the pandemic, Bitcoin, and cryptocurrencies in general, besides not acting as hedges or save havens, are also contributing to extend the financial contagion effects. However, their status as new financial instruments makes their true, final capabilities as diversifiers still unclear. Section 6 will provide a more detailed analysis on Bitcoin during the Covid-19 period, using the methodology presented below.

3. Data and methodology

The data consists in weekly returns, from the 7th of January, 2013, to the 31st of May, 2021, for a total of 438 observations for each time series. The period chosen is meant to capture the first big boom on Bitcoin's price, after Cyprus' bailout, as well as a post-crisis period, following the international financial crisis of 2008, and the European sovereign debt crisis, between 2010 and 2013. We will also zoom in on the period spanning from 2020 to May 2021, to study Bitcoin's behavior during the Covid-19 pandemic. Since it consists of a global financial crisis period, it will be a test to Bitcoin's capabilities, especially as a safe haven. Bitcoin data will be the average USD market price across all major bitcoin exchange platforms and will be retrieved from Quandl, while the remaining data will be retrieved from Eikon-Datastream.

Regarding the optimal number of securities in the portfolio, there is an extensive literature on the matter and an intense ongoing discussion: While holding fewer assets may expose the investor to more idiosyncratic risk, having a large number of securities in a portfolio raises transaction costs issues (Alexeev & Tapon, 2012).

Evans & Archer (1968) concluded that the optimal number of securities in a portfolio should be around 10. Further beyond that, the amount of additional securities required to decrease the standard deviation significantly is substantial and not economically justifiable. Fielitz (1974), Tang (2004) and Malkiel (1999) complement Evans and Archer's work.

On the other hand, Statman (1987) shows that only from 30 stocks onwards is a portfolio well-diversified and add that diversification should be increased while the marginal benefits are greater than the marginal costs.

Despite this theoretical discussion regarding portfolio size, it does not suffice to lead an investor to an optimal portfolio. The correlation between the assets composing a portfolio, or lack thereof, is key to an efficient diversification.[7]

And so, focus should also be kept on the low correlation of assets along with its number. In that sense, the base investment portfolio (Portfolio A) constructed for this study will contain a broad variety of markets and consider traditional assets as well as more alternative investments (Table I): the stock indexes of seven of the biggest markets and three bond indexes representing the American, Asian and European continents were chosen, along with a global commodities index and a real estate index. Its size is also close to the reality of the average, individual investor. A more detailed description of the data can be found in Appendix B. We will then be adding Bitcoin to Portfolio A and further analyze this new portfolio (Bitcoin Portfolio) in the next section.

[7] See Jacob, (1974)

Table I
Base portfolio (Portfolio A)

Stocks	Bonds	Commodities	Real estate
S&P	US index	ICE BofA Index	MSCI World Index
FTSE	Asia index		
DAX	Euro index		
Nikkei			
SSE			
MSCI			
Euronext			

Similarly to Dyhrberg (2016a) and Klabbers (2017), alongside the base portfolio and the Bitcoin portfolio, a third, already well diversified, portfolio will also be constructed, with the goal of checking whether adding bitcoin to such a portfolio will bring any diversification advantages, comparing to an already hedged portfolio. For this third portfolio (Table II), the gold bullion and the USD/EUR exchange rate will be added, as they are already well-established safe haven/hedges (Baur & McDermott, 2010). We will be calling it Portfolio B.

Table II
Portfolio B

Stocks	Bonds	Commodities	Real estate	Exchange Rates
S&P	US index	ICE BofA Index	MSCI World Index	USD/EUR
FTSE	Asia index	Gold		
DAX	Euro index			
Nikkei				
SSE				
MSCI				
Euronext				

In a similar fashion to Klabbers (2017), this work will use a Mean-Variance Analysis combined with a Monte Carlo simulation.

The Mean-Variance Analysis is the model developed by Markowitz (as explained in Section 2.3) and makes use of the average returns, variance, and correlation values of each security in analysis. From this data the covariance matrix will be computed. Efficient frontiers will then be drawn by optimizing the securities weights on the portfolio that minimize the risk (measured by variance), subject to a given level of return. This analysis is considered a powerful method to assess diversification effects and is able to incorporate investor constraints, such as short sales. Jorion (1992), however, identifies a shortcoming on this analysis, which is the fact that "it does not recognize the uncertainty inherent in the input parameters, their estimation risk". Jorion highlights the importance of estimation risk in order to account for these imprecisions, and defines a series of steps to tackle this. The way these steps will be performed in practice is through the Monte Carlo simulation and are the following: For this study, we will be assuming the historical data of return and risk as true values and defining N as the number of assets and T as the number of weeks; Then, from a normal distribution with the historical values as parameters, a random sample will be pulled for each N return. This process is repeated T times. From these simulated returns, a new mean and covariance matrix will be estimated, and from then, a simulated optimal portfolio can be achieved. 100 iterations will be drawn.

Klabbers (2017) and Moore & Stephen (2016) also make use of the Monte Carlo simulation in their portfolio analysis as a method of estimation. To measure the performance of the portfolios, the Sharpe ratio, introduced by Sharpe (1994) is used. The Sharpe ratio is the most commonly used return-risk ratio and is given by $\frac{R_p - R_f}{\sigma_p}$, where R_p is the portfolio's expected return, R_f the return of the risk-free asset, and σ_p the standard deviation of the portfolio. This work will assume R_f as zero.

An additional method of assessing the performance of an investment portfolio is the Conditional Value at Risk, or CVaR, which derives from the Value at Risk (VaR) method. The VaR is used to measure the downside risk and is defined by the loss that will not be exceeded over a given time horizon at a given confidence level. It can be written as:

(4) $$\zeta_\alpha(w) = \min\{\zeta | \Psi(w, \zeta) \geq \alpha\},$$

For a confidence level α, where $\Psi(w,\zeta)$ is the cumulative distribution function of a loss $z = f(w,y)$, with:

(5) $$\Psi(w,\zeta) = P\{y|f(w,y) \leq \zeta\},$$

Where ζ is a specific loss, and y the risk associated with the loss.

The CVaR is a more accurate measure of risk since it gives an expected value of the loss, instead of a range of potential losses as in VaR:

(6) $$CVaR_{\alpha}(w) = \frac{1}{1-\alpha} \int_{f(w,y)\geq\zeta_{\alpha}(w)} f(w,y)p(y)dy$$

The definitions of VaR and CVar, as well as the above notation, were taken from Eisl et al (2015). The CVaR analysis will be used as a complement for the main results provided by the Markowitz model and the Monte Carlo simulations.

The next chapter presents the results. The different efficient frontiers and covariance matrixes computed in that chapter were derived from the average of the logarithmic weekly returns $r = \ln\left(\frac{P_t}{P_{t-1}}\right)$ which was then annualized as below:

(7) $$r_{annualized} = (1+r)^{52} - 1$$

4. Results

A summary of the weekly returns, weekly standard deviation, and respective annualized values of all the securities is presented on Table III.

All assets except the Asian markets bonds and the Commodity Index present annual positive returns from 2013 to 2021, on average. Bitcoin far exceeds all other assets in terms of return and risk, with 151.78% and 93.44%, respectively. It provides a return 11.4 times greater than S&P, the second-best option, and is 5.67 times as risky.

Table III
Descriptive Statistics (2013-2021)

Security	Avg. Weekly Return	Avg. Weekly Std. Dev.	Annualized Return	Annualized Std. Dev.
S&P	0.24%	2.28%	13.34%	16.47%
FTSE	0.03%	2.29%	1.76%	16.48%
DAX	0.16%	2.82%	8.53%	20.30%
Nikkei	0.23%	2.85%	12.61%	20.53%
SSE	0.11%	3.25%	5.61%	23.43%
MSCI	0.18%	2.21%	9.67%	15.92%
Euronext	0.13%	2.59%	7.24%	18.65%
US Government Bond index	0.00%	0.55%	0.08%	3.95%
Asia Governments Bond index	-0.01%	0.71%	-0.27%	5.14%
Euro Governments Bond index	0.02%	0.54%	0.90%	3.89%
Commodity Index	-0.03%	2.77%	-1.51%	19.96%
Gold Bullion	0.03%	2.13%	1.73%	15.34%
Real Estate Index	0.07%	2.44%	3.77%	17.58%
USD/EUR	0.02%	1.10%	0.84%	7.94%
BTC/USD	1.79%	12.96%	151.78%	93.44%

Table IV presents the correlation values between all assets in analysis, and the respective significance levels. Bitcoin does not show any negative correlation with the remaining assets; however, the values are small and very close to zero, being the highest correlation with the Nikkei (0.18). On a first analysis, it should indicate that Bitcoin presents some diversifying capabilities, or, to some extent, hedging capabilities, according to the definitions presented in Chapter 2.3. Looking at the two more traditional hedges, the gold and the US Dollar, they show rather different values: While the US Dollar has negative correlation with a decent number of securities, gold seems to underperform both the Dollar and Bitcoin, as it is quite highly correlated with all bonds and only showing significantly negative correlation with Nikkei and the US Dollar. Yet, the standout is the US Government Bond Index, which, apart from the other two bond indices and gold, shows fairly negative values or is uncorrelated with all securities. .

Table IV
Correlation Matrix 2013-2021

	S&P	*FTSE*	*DAX*	*Nikkei*	*SSE*	*MSCI*	*N100*	*US bonds*	*Asian bonds*	*Euro bonds*	*Commodity Index*	*Gold*	*Real Estate*	*USD/EUR*
FTSE	0.79***	1.00												
DAX	0.75***	0.82***	1.00											
Nikkei	0.67***	0.61***	0.65***	1.00										
SSE	0.35***	0.32***	0.32***	0.35***	1.00									
MSCI	0.98***	0.84***	0.81***	0.72***	0.37***	1.00								
N100	0.80***	0.89***	0.94***	0.68***	0.34***	0.86***	1.00							
US Bonds	-0.32***	-0.28***	-0.31***	-0.33***	-0.14***	-0.29***	-0.31***	1.00						
Asian Bonds	0.34***	0.34***	0.29***	0.18***	0.15***	0.40***	0.32***	0.50***	1.00					
Euro Bonds	0.11**	0.15***	0.17***	0.05	-0.01	0.13***	0.21***	0.54***	0.51***	1.00				
Commodity	0.45***	0.45***	0.38***	0.31***	0.23***	0.48***	0.42***	-0.36***	0.12**	-0.10**	1.00			
Gold	0.00	-0.02	-0.09*	-0.18***	0.02	0.05	-0.07	0.42***	0.34***	0.28***	0.09*	1.00		
Real Estate	0.75***	0.65***	0.60***	0.54***	0.26***	0.79***	0.64***	0.06	0.61***	0.34***	0.28***	0.21***	1.00	
USDEUR	-0.03	0.11**	0.20***	0.10**	0.03	-0.11**	0.19***	-0.20***	-0.23***	0.12**	-0.07	-0.38***	-0.17***	1.00
BTCUSD	0.12***	0.09*	0.10**	0.18***	0.09*	0.12**	0.10**	-0.01	0.01	0.12**	0.10**	0.15***	0.09*	-0.02

Note: The correlation values are marked according to their significance level (* = 10%; ** = 5%; *** = 1%)

Figure 6 presents the efficient frontiers of the three portfolios constructed for the whole time series.[8] The hedged portfolio, portfolio B, very slightly outperforms the Base portfolio, while the Bitcoin portfolio's efficient frontier is much steeper and provides a better risk-return ratio than the other two portfolios from around the 4% level of return and beyond. To compare the performance of the three portfolios throughout the years, efficient frontiers were constructed for each of the years in analysis (See appendix C).

The Bitcoin portfolio had a better performance in most of the years analyzed. Highlight to 2013 and 2016, where the Bitcoin Portfolio far outperformed the other two portfolios. In 2015, 2018 and 2021 Portfolio B performed better. Only in 2014 it had an equal performance to Portfolio A and B. Portfolio B is always equal or better than the base Portfolio, which showcases the advantages of Gold and the US Dollar of being included in an investment portfolio. In this context, it makes sense to only compare Portfolio B to the Bitcoin Portfolio on the estimation risk analysis and, as such, we will be discarding the Base Portfolio for that section of the results.

[8] The leftmost point on the efficient frontiers that will be shown throughout this work may not always correspond to the minimum variance portfolio. In some cases, the minimum variance portfolio corresponds to a negative return, which is not economically viable.

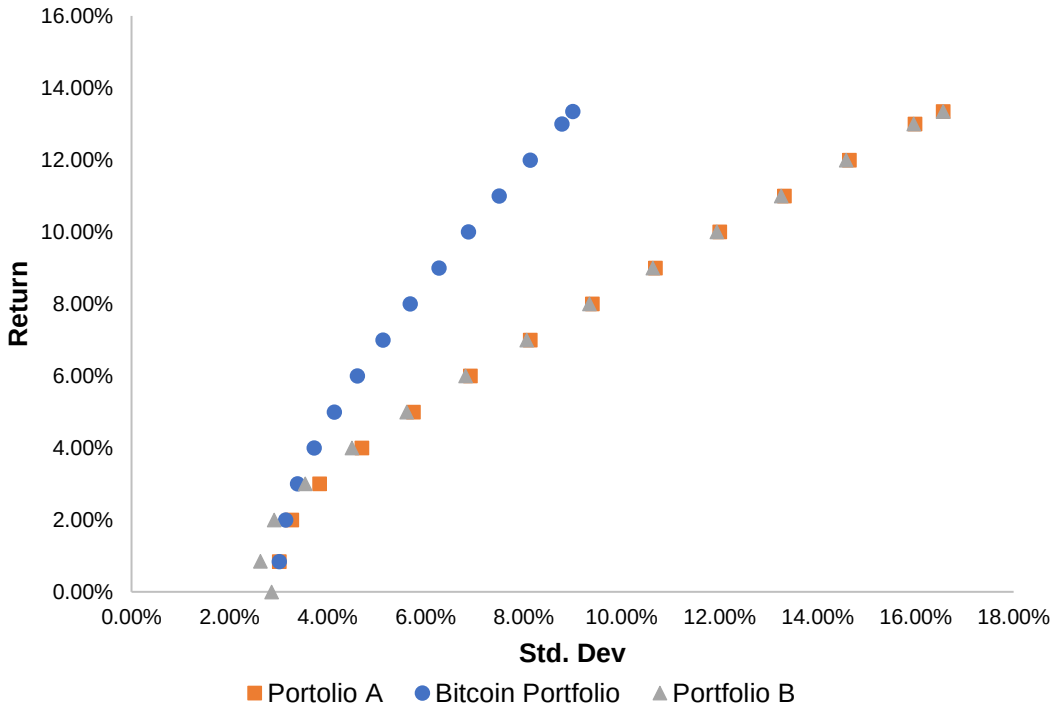

Figure 6: Efficient frontiers 2013-2021

Figure 7 illustrates the simulated portfolios resulting from the estimation risk, measured through the Monte Carlo Simulations. From the 100 iterations, the 5% worst and 5% best portfolios, according to the Sharpe Ratio, were removed, to improve the significance of the results, and thus leaving 90 simulated portfolios from the Bitcoin portfolio and Portfolio B.

As in Klabbers (2017), it should be noted that most simulated portfolios lie above their respective efficient frontiers. That can be explained by the properties of the Markowitz model, which tends to be very sensitive with assets with high returns. Also, the portfolios do not necessarily have to be suboptimal, since we are drawing a random sample from the distribution.

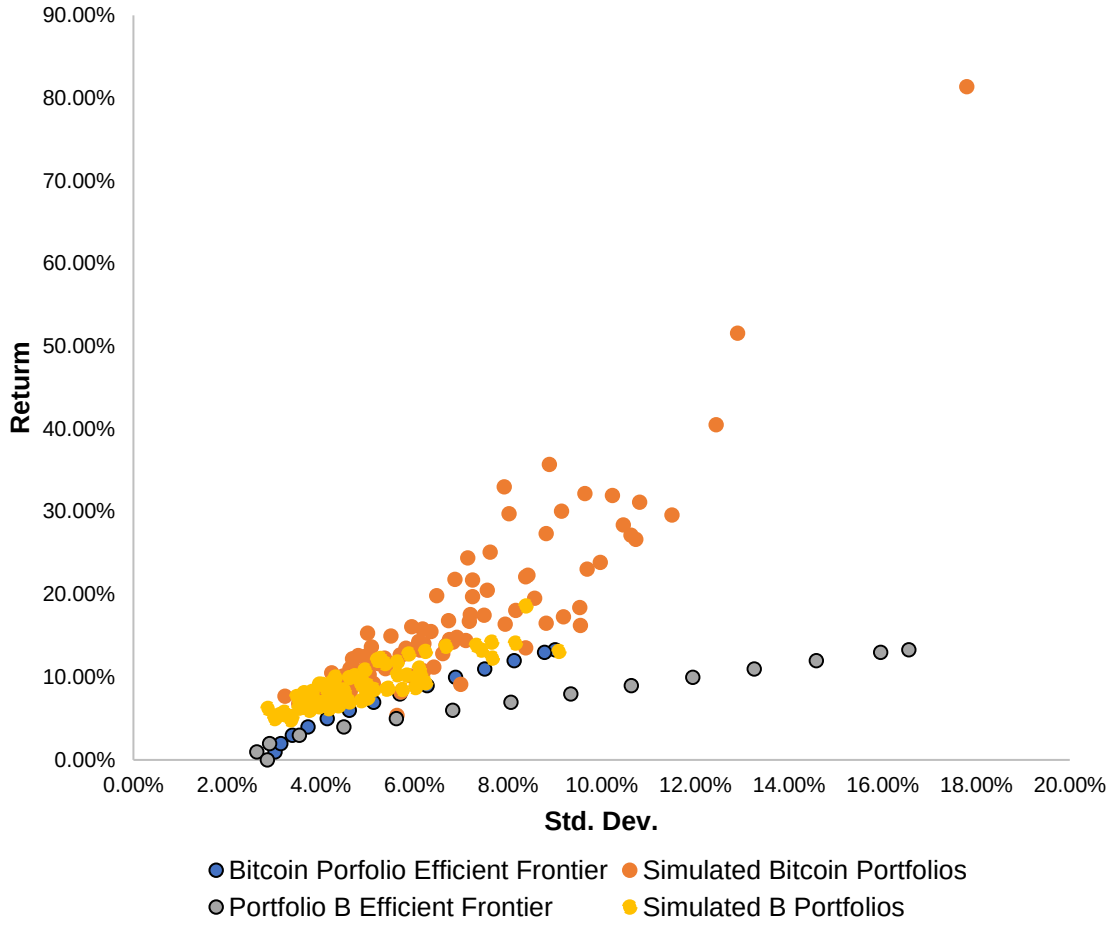

Figure 7: Simulated Portfolios

The simulated Bitcoin Portfolios show a lot more dispersion that the Simulated B portfolios, however their overall performance is better. The average Sharpe ratio is 4.85 while the B portfolios average Sharpe Ratio is 1.85. An average weight of 3.83% of the portfolio is invested on Bitcoin, and for most cases, the share of Bitcoin on a portfolio is relatively small (See Figure 8).

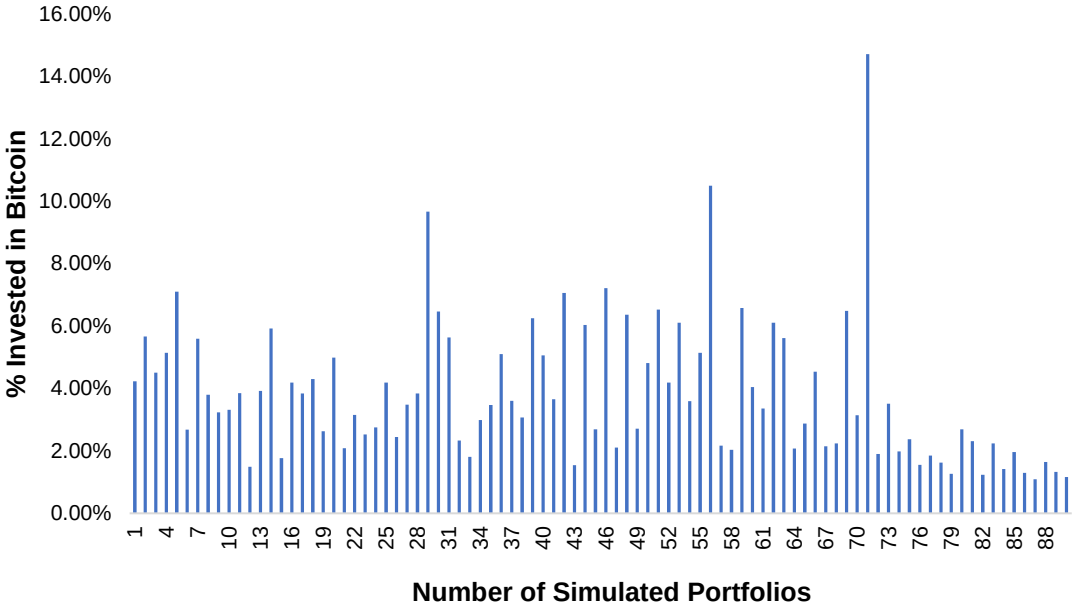

Figure 8: Share of Bitcoin in the simulated portfolios

Table V shows the results by performing the optimization of the portfolios using the CVaR method, at the 1% and 5% level. As before, on this optimization short sales were restricted ($w_i > 0$) and the total portfolio weight was capped at 100% ($\sum w_i = 1$). The objective of this analysis is to complement the Markowitz model, in the sense that this methodology requires returns to be normally distributed, which is not the case for the assets in analysis[9]. The CVaR method can tackle that and help build a more consistent assessment of the results.

At the 5% level, including Bitcoin in the portfolio increases the CVaR from 0.90% to 1.03%, in comparison with Portfolio B, meaning that in the worst 5% of returns, the average loss will be 1.03%. The Bitcoin portfolio yields, however, a better Sharpe Ratio. The optimal weight of Bitcoin is 0.20%. At the 1% level, which considers only the more extreme values, the optimal portfolio does not include Bitcoin at all, and Portfolio B provides a higher Sharpe ratio and a lower CVaR, which may retract more risk-averse investors to include Bitcoin in their portfolios. These results are consistent with Eisl et al (2015). Although increasing the CVaR, Bitcoin can be included in an investment portfolio and improve its Sharpe ratio, even if in very small proportions.

[9] See Eisl et al (2015)

Table V
Portfolio Optimization using CVaR

	Weekly return	***Annualized Return***	***Weekly-CVaR (5%)***	***BTC weight***	***Sharpe Ratio***
Bitcoin Portfolio	0.02%	0.89%	1.03%	0.20%	0.29
Portfolio B	0.01%	0.54%	0.90%	-	0.20
	Weekly return	***Annualized Return***	***Weekly-CVaR (1%)***	***BTC weight***	***Sharpe Ratio***
Bitcoin Portfolio	0.00%	0.08%	1.83%	0.00%	0.03
Portfolio B	0.01%	0.44%	1.24%	-	0.15

5. Robustness tests

In order to test the consistency of the previous results, two different, additional simulations will be run, each with a slight change on the Mean-Variance model. The remaining analysis and steps taken will be the same as the previous analysis. The first will be done by removing the no short sales constraint ($w_i > 0$ constraint is removed from the model), meaning that an investor would now be able to allocate a negative weight to their securities, a tool used when an investor expects the price of a security to drop. Many cryptocurrency exchange platforms already offer the possibility of trading Bitcoin futures, so this short selling can be achieved in practical terms also for Bitcoin.

The second analysis will consider a weight constraint: No securities can have more than 25% of the portfolio's total weight ($w_i > 0.25$ constraint added). This follows the premise of Conover et al (2009), that an investor is unlikely to allocate more than 25% of their portfolio into a single asset.

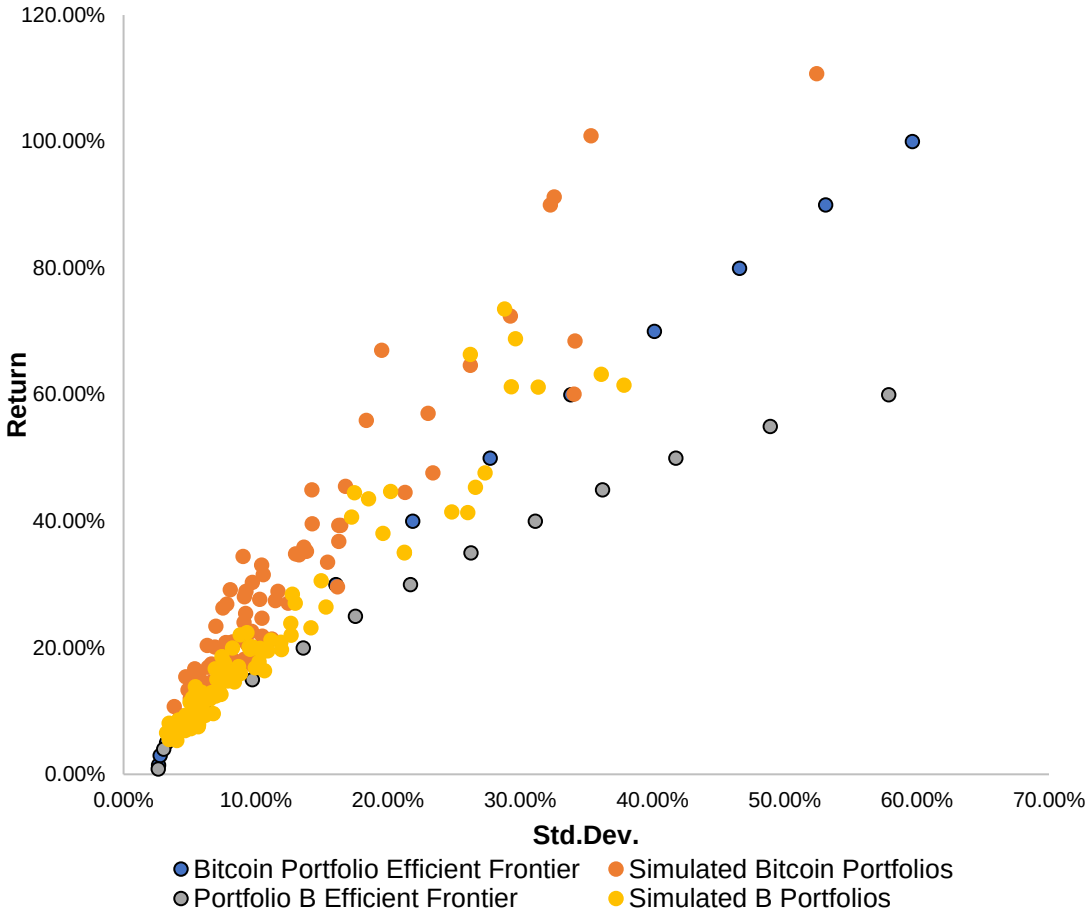

Figure 9: Simulated portfolios with no short sales constraint

Figure 9 depicts the efficient frontiers of the Bitcoin Portfolio and Portfolio B with the removal of the short sales constraint, along with the simulated portfolios computed with Monte Carlo Simulations. As in the previous section, 100 iterations were performed, and the 5% worst and 5% best portfolios were removed. The results are in line with the main analysis:

Bitcoin portfolio's efficient frontier starts yielding better results than Portfolio B's from the 4% return onwards. Regarding the simulated Bitcoin portfolios, they are more disperse than Portfolio B's, although providing a better Sharpe Ratio: Bitcoin's portfolio has a Sharpe Ratio of 2.49 while Portfolio B has 1.95. The average weight of Bitcoin on the portfolios is 6.20%, and in only one iteration Bitcoin was shorted. (See appendix D.1). Comparing with the original analysis, we can

see that here, the Bitcoin portfolios provide a better return rate in average, however, it is the greater risk associated that worsens the Sharpe Ratio.

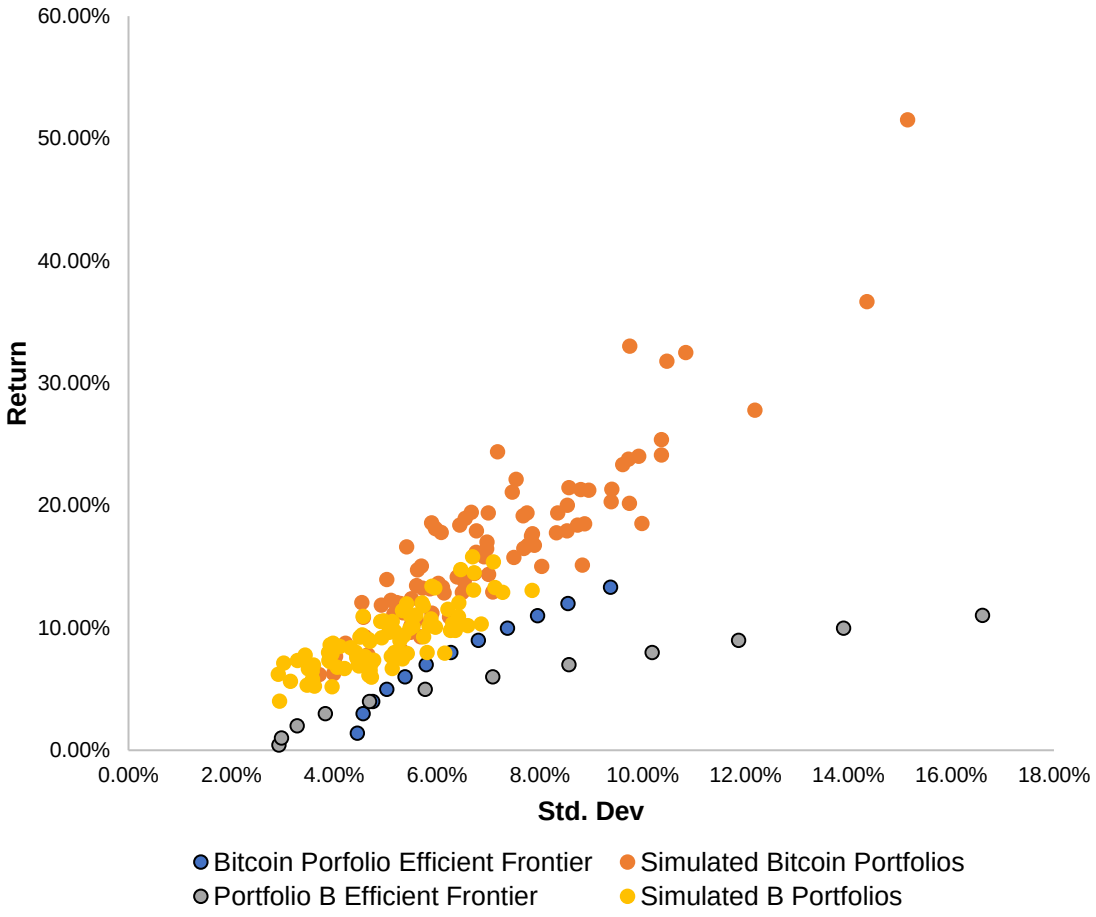

Figure 10: Simulated Portfolios with weight constraint

Similarly to previous results, with the addition of the weight constraint of 25% (Figure 10), Bitcoin Portfolio's efficient frontier is also steeper than the B portfolio's efficient frontier, and is above the latter as from the 4% level of return. Bitcoin's simulated portfolios are also more disperse than the simulated B portfolios and yield a better Sharpe Ratio (2.32 compared to 1.82).

Bitcoin's allocation is also relatively small, with an average weight of 3.70% of the portfolio (See Appendix D.2).

6. Zooming in the Covid-19 Pandemic Period

As briefly mentioned of Section 3 of this work, the Covid-19 pandemic brought a period of sanitary, social, and economic distress, which translated into increased uncertainty and volatility of the financial markets (Zhang, Hu, & Ji, 2020). So far, there hasn't been an event of such magnitude during Bitcoin's lifespan. In the context of this work, this can be seen as a real test for Bitcoin's capabilities on periods of greater volatility, namely as a safe haven, as pointed out by Kristoufek (2020). The author sees the claim that Bitcoin is a safe-haven as unsubstantiated, although with the reservation that the Covid-19 pandemic was only the first big assessment to its capacities.

But before going deeper into further analysis on Bitcoin during the pandemic, we should also try to look at the financial landscape, that faced some changes over the last year and a half, since the Covid-19 global outbreak:

The lockdowns issued throughout the world, which gave people more spare time, as well as the increase in unemployment, and the low interest rate environment, may have pushed a new wave of individuals to start investing (Priem, 2020). These new investors are mainly young males, with less financial literacy than regular investors, who invest in smaller amounts than the latter (Frenay & Bonnet, 2020). For example, 15% of the U.S. stock market investors started investing in 2020 (Schwab, 2020).

Regarding the cryptocurrencies market, it has been heavily referred across the media and by online influencers (BritainThinks, 2021). Additionally, Li, Chen, & Dong, (2021) show that Bitcoin-related events heavily influence Bitcoin's price. Recently, the most notable case is Tesla's CEO, Elon Musk, who announced in February of 2021 that Tesla bought 1.5 billion USD worth of Bitcoin and that they will be accepting it as payment, only to revert that decision months later for envi-

ronmental reasons[10]. These announcements were followed by significant increases and decreases in Bitcoin's price. China's crackdown on crypto mining also caused prices to go down[11].

Park & Chai (2020) and Naeem et al (2021) have shown that the cryptocurrency market is an inefficient market and that the Covid-19 pandemic has worsen this inefficiency: Prices are heavily influenced by privileged information. Also, the former add that many traders are sentiment-driven, rather than making information-based decisions when investing in the cryptocurrency market. This may be derived by the amount of individual, less sophisticated investors, that compose this market (Panos & Karkkainen, 2019).

For this section, we will be using data from two distinct periods: A pre-pandemic period, comprising the whole year of 2019 and the first week of 2020, for a total of 53 observations, and the pandemic period, from the 13th of January, 2020, the day of the first recorded case of Covid-19 outside China, until May 2021, consisting in 73 observations. The same analysis as in section 4 will be made, in terms of the descriptive statistics, correlation matrix, creation of the efficient frontiers, and estimation risk, but this time zooming only on the pre-pandemic, and pandemic period, to observe more thoroughly what impact does a period of high uncertainty can have on the performance of Bitcoin, and subsequently, what impact does Bitcoin have on an investment portfolio, in such periods, and if safe haven abilities were demonstrated.

Table VI summarizes the weekly returns, weekly standard deviation, and respective annualized values of all the securities during this period:

[10] https://www.coindesk.com/business/2021/05/12/elon-musk-says-tesla-is-suspending-bitcoin-payments-over-environmental-concerns/ [Accessed: 5/9/21]
[11] https://www.coindesk.com/markets/2021/05/21/bitcoin-falls-as-china-calls-for-crackdown-on-crypto-mining-trading/ [Accessed: 5/9/2021]

Table VI
Descriptive Statistics January 2019 - May 2021

Security	Pre-Pandemic Period		Pandemic Period	
	Avg. Weekly Return	Avg. Weekly Std. Dev.	Avg. Weekly Return	Avg. Weekly Std. Dev.
S&P	0.49%	1.74%	0.35%	3.77%
FTSE	0.22%	1.79%	-0.10%	3.54%
DAX	0.41%	1.79%	0.22%	4.23%
Nikkei	0.28%	2.01%	0.30%	3.69%
SSE	0.40%	2.40%	0.22%	2.87%
MSCI	0.43%	1.58%	0.31%	3.73%
Euronext	0.43%	1.71%	0.11%	3.94%
US Govt. Bond index	0.09%	0.59%	0.01%	0.78%
Asia Govt. Bond index	0.16%	0.39%	-0.01%	1.12%
Euro Govt. Bond index	0.09%	0.46%	-0.03%	0.69%
Commodity Index	0.33%	2.35%	0.23%	3.85%
Gold Bullion	0.38%	1.64%	0.27%	2.64%
Real Estate Index	0.33%	1.24%	0.07%	4.62%
USD/EUR	0.05%	0.66%	-0.12%	1.18%
BTC/USD	1.26%	8.46%	2.12%	12.19%

During the Covid-19 outbreak, all the securities in analysis had their returns decreased compared to the period before the pandemic, except for Bitcoin and the Nikkei. The FTSE, the Asian and Euro bond indexes, and the USD/EUR exchange rate even saw negative average returns. Also, the standard deviation has risen for all assets. Bitcoin had its return increased from 1.26% to 2.12% and its standard deviation from 8.46% to 12.19%. Comparing with the security that comes second in terms of return, S&P, Bitcoin is 3.23 riskier but provides a return 6.05 times greater than the American index.

Table VII
Correlation Matrix on the Pre-pandemic period

	S&P	*FTSE*	*DAX*	*Nikkei*	*SSE*	*MSCI*	*N100*	*US bonds*	*Asian bonds*	*Euro bonds*	*Commodity Index*	*Gold*	*Real Estate*	*USD/EUR*
FTSE	0.78***													
DAX	0.79***	0.67***												
Nikkei	0.77***	0.55***	0.79***											
SSE	0.38***	0.34**	0.48***	0.33**										
MSCI	0.99***	0.77***	0.83***	0.82***	0.43***									
N100	0.87***	0.82***	0.86***	0.72***	0.47***	0.88***								
US Bonds	-0.56***	-0.46***	-0.66***	-0.73***	-0.28**	-0.59***	-0.55***							
Asian Bonds	0.02	-0.06	-0.21	-0.30**	0.01	0.01	-0.04	0.65***						
Euro Bonds	-0.15	-0.06	-0.18	-0.36***	-0.06	-0.18	-0.09	0.62***	0.64***					
Commodity	0.61***	0.51***	0.56***	0.57***	0.52***	0.65***	0.52***	-0.57***	-0.16	-0.24				
Gold	-0.20	-0.25*	-0.39***	-0.36***	0.01	-0.17	-0.32**	0.54***	0.52***	0.43***	-0.07			
Real Estate	0.38***	0.10	0.15	0.11	-0.01	0.36***	0.22	0.08	0.32**	0.11	0.11	0.17		
USDEUR	0.04	0.26*	0.08	-0.13	-0.10	-0.06	0.16	-0.08	-0.16	0.15	-0.08	-0.53***	-0.11	
BTCUSD	-0.26*	-0.24*	-0.15	-0.18	0.24*	-0.22	-0.31**	0.14	0.01	-0.01	0.05	0.38***	-0.19	-0.28**

Note: The correlation values are marked according to their significance level (* = 10%; ** = 5%; *** = 1%)

Table VIII
Correlation Matrix January 2020 - May 2021

	S&P	*FTSE*	*DAX*	*Nikkei*	*SSE*	*MSCI*	*N100*	*US bonds*	*Asian bonds*	*Euro bonds*	*Commodity Index*	*Gold*	*Real Estate*	*USD/EUR*
FTSE	0.88***													
DAX	0.86***	0.90***												
Nikkei	0.82***	0.83***	0.81***											
SSE	0.44***	0.37***	0.38***	0.44***										
MSCI	0.99***	0.90***	0.89***	0.87***	0.46***									
N100	0.87***	0.94***	0.96***	0.86***	0.41***	0.90***								
US Bonds	-0.31***	-0.36***	-0.31	-0.25**	-0.10	-0.27**	-0.32***							
Asian Bonds	0.51***	0.42***	0.43***	0.44***	0.36***	0.56***	0.40***	0.36***						
Euro Bonds	0.25**	0.25**	0.28**	0.36***	0.01	0.30***	0.33***	0.50***	0.45***					
Commodity	0.52***	0.58***	0.58***	0.48***	0.36***	0.53***	0.59***	-0.52***	0.20*	0.03				
Gold	0.23*	0.23*	0.20*	0.25**	0.18	0.28**	0.23**	0.38***	0.39***	0.50***	0.02			
Real Estate	0.87***	0.79***	0.76***	0.77***	0.41***	0.90***	0.77***	-0.09	0.70***	0.38***	0.39***	0.38***		
USDEUR	-0.33***	-0.21*	-0.12	-0.23*	-0.23*	-0.37***	-0.12	-0.30***	-0.57***	-0.20*	-0.02	-0.36***	-0.49***	
BTCUSD	0.29**	0.36***	0.34***	0.40***	0.21*	0.33***	0.39***	-0.01	0.17	0.35***	0.34***	0.33***	0.27**	-0.07

Note: The correlation values are marked according to their significance level (* = 10%; ** = 5%; *** = 1%)

Table VII and VIII show the correlation between all the securities in analysis, during the pre-pandemic and pandemic period, respectively. As expected, the correlation values rose, in general, especially between the more traditional assets. This follows the results of Zhang et al (2020). The US Dollar showed good safe haven capabilities, being negatively correlated with every asset. On the opposite direction, Bitcoin and the gold bullion increased their correlation values significantly. Bitcoin only maintained or decreased its correlation level with the US bonds and the SSE index. This may disprove the hypothesis that Bitcoin could act as a safe haven, as in times of market volatility like the Covid period, the cryptocurrency tends to accompany other, more traditional, securities.

The estimation risk performed with the Monte Carlo Simulation, and the efficient frontiers constructed, are shown on figure 11*,* for the period preceding the covid-19 outbreak, and on figure 12, for the pandemic time frame. Again, 100 iterations were performed and the 5% worst and 5% best portfolios removed, ranked with the respective Sharpe Ratio. Regarding the efficient frontiers during the Covid-19 pandemic, the results remain in line with the main results. Bitcoin Portfolio's efficient frontier is steeper and from the 4% return onwards is always to the left of Portfolio B. Portfolio A was discarded, as it always performs equal or worse that Portfolio B on both periods (See Appendices C.10 and C.11). Bitcoin simulated portfolios show way more dispersion in this period compared to the pre pandemic period and the Sharpe ratio dropped by half, from 9.26 to 4.63. They also show more dispersion relative to the other portfolio estimated, Portfolio B. The average allocation of Bitcoin is 6.39%, a significant increase in the relative weight on the portfolio, which had 1.01% allocated to Bitcoin before the Covid-19 outbreak.

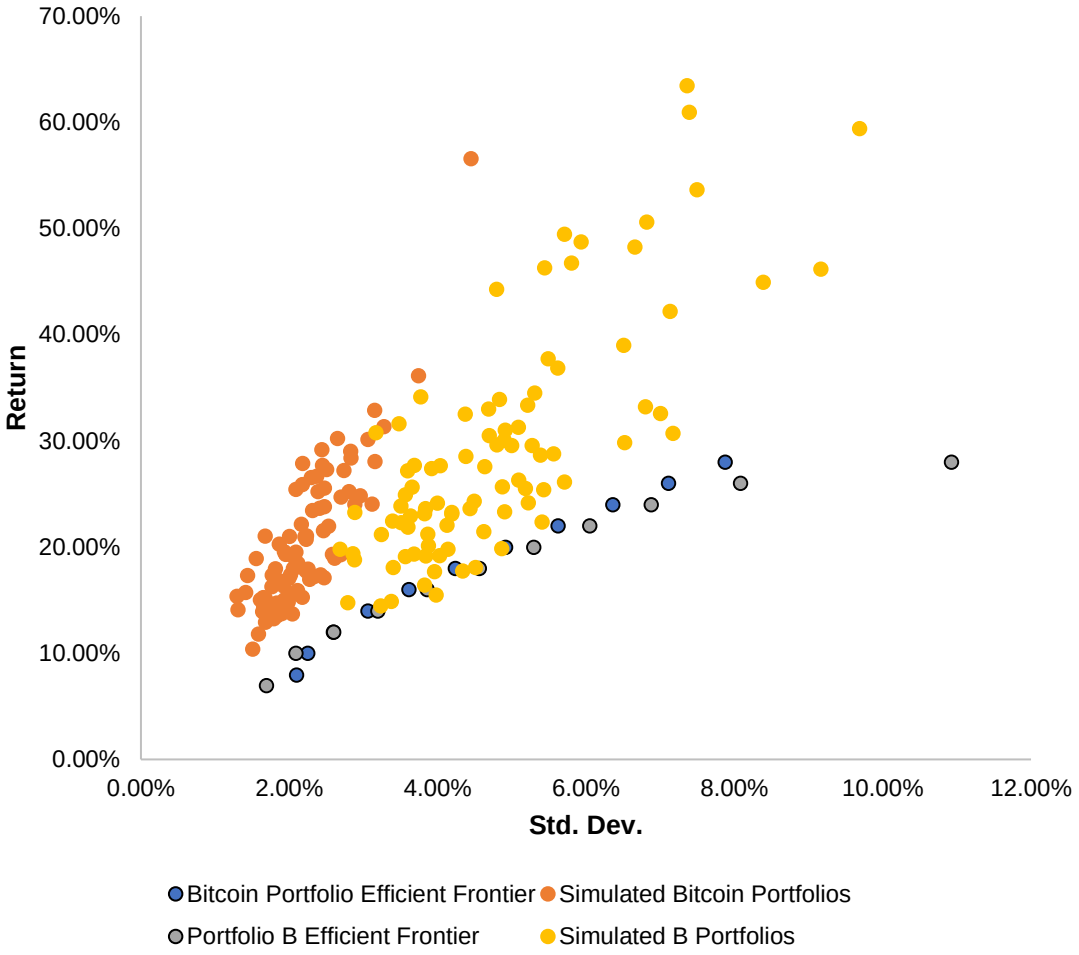

Figure 11: Simulated Portfolios on the Pre-pandemic period

Portfolio B saw its Sharpe Ratio decrease 40%, from 6.09 to 3.65, but still staying below the Bitcoin Portfolio despite the lesser decrease. Regarding the traditional hedges, they had opposite movements in terms of their average weights: the weight of the USD decreased from 6.63% to 3.78%, while the gold saw an increase to 12.24% comparing to the average of 8.02% pre-pandemic.

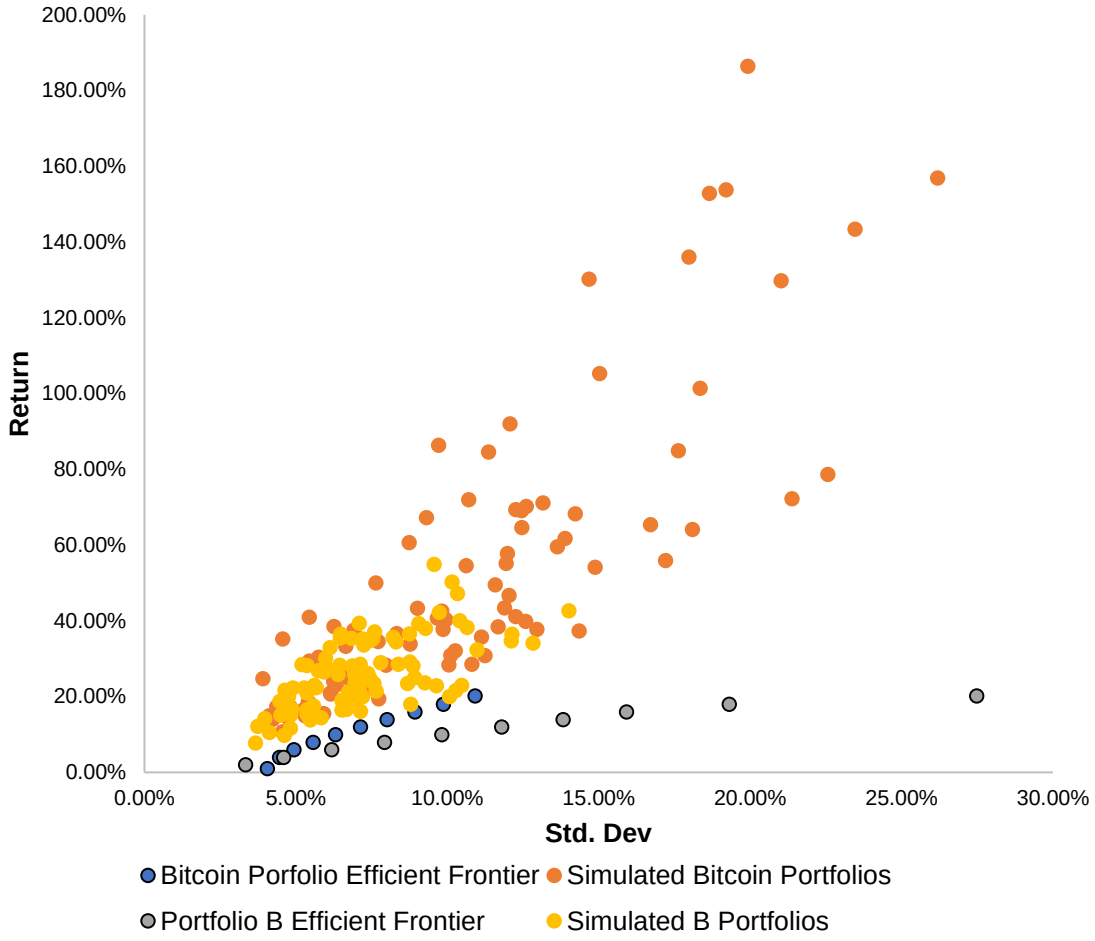

Figure 12: Simulated Portfolios January 2020 – May 2021

7. Conclusion

The literature has proven that Bitcoin has not been used as a currency, as originally intended by Nakamoto (2008), but rather as an investment asset. In this sense, this work sees Bitcoin as a potential tool that could be capable of improving an investment portfolio, in terms of its Sharpe ratio, or as a diversifier that could be uncorrelated with the more traditional assets.

The results of this work, using the Mean-Variance, or Markowitz model, showed that, when applied on an investment portfolio, Bitcoin improves the efficient frontier of that portfolio, compared to an already hedged portfolio, from the 4% level of return onwards. Moreover, the Monte Carlo Simulations reveal that the Sharpe Ratio rises from 1.85 to 4.85. However, this improvement can only be achieved if Bitcoin is allocated in small percentages (average of 3.83%), which is in line with

what other similar works have reached (Eisl et al., 2015; Klabbers, 2017). These results are robust when the short-sales constraint is removed and when a ceiling of 25% weight for each individual asset is included. Performing the portfolio optimization with the CVaR also yields similar results and complements the analysis: Bitcoin improves the Sharpe ratio of the portfolio if included in very small proportions, but at the same time it increases the CVaR of the portfolio.

Bitcoin, in general, does not show a good level of uncorrelation (expected from a hedge of safe haven asset) with other traditional assets, in times of crisis, and particularly, during the covid-19 pandemic, where the correlation levels rose compared to a pre-pandemic period. This, allied with its high volatility, does not work in favor of considering Bitcoin as a safe haven.

During the pandemic, its Sharpe Ratio decreased by half, while the hedge portfolio suffered a smaller decreased, but still with a ratio below the Bitcoin portfolio's. There is definitely a place for Bitcoin in regard to its inclusion on an investment portfolio, however the mixed signals from the different economic agents, namely its acceptance from companies and countries on one side, and the concerns about its volatility and environmental issues on the other side, leads to believe that the future of Bitcoin is still uncertain, either as a currency, or as an investment asset.

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Appendices

Appendix A: Bitcoin's Market Capitalization

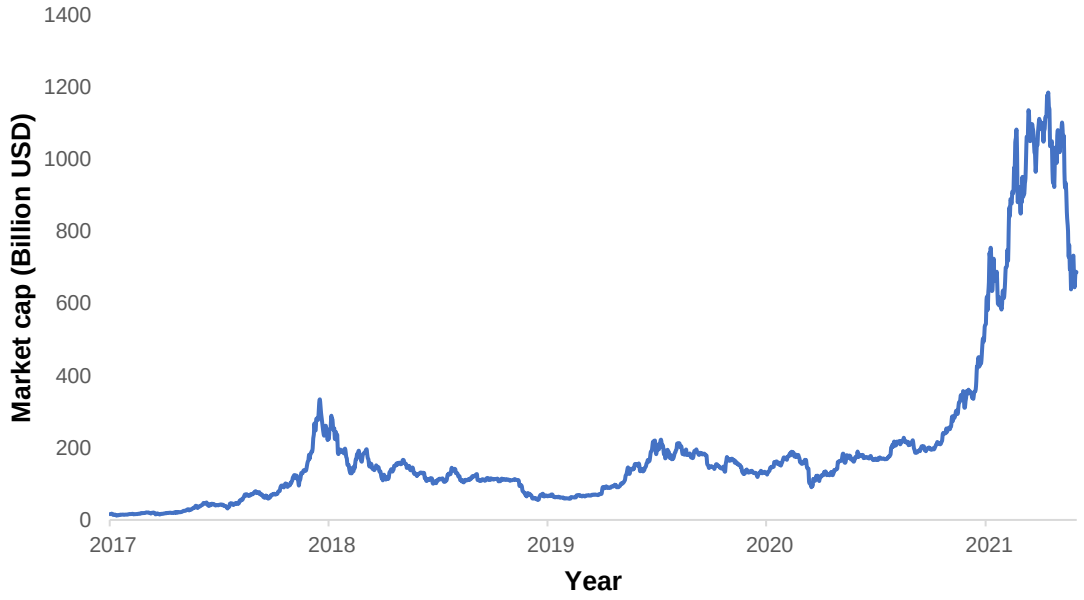

Figure A.1: Bitcoin's Market Capitalization (Billion USD)

Data retrieved from Quandl

Appendix B: Data retrieved

Table B.1
Data retrieved from Datastream

Security	Datastream Name	Asset Class	Datastream Code
S&P	S&P 500 COMPOSITE	Equity	S&PCOMP
FTSE	FTSE100	Equity	FTSE100
DAX	DAX 30 PERFORMANCE	Equity	DAXINDX
SSE	SHANGHAI SE A SHARE	Equity	JAPDOWA
Nikkei 225	NIKKEI 225 STOCK AVERAGE	Equity	CHSASHR
MSCI	MSCI WORLD U$	Equity	MSWRLD$

Euronext	EURONEXT 100	Equity	EUNX100
US bonds	ICE BofA US Treasury Index	Fixed Income	MLTRSML
Asian Bonds	ICE BofA Asian Dollar Govern-ment Index	Fixed Income	MLAGTSL
Euro bonds	ICE BofA Euro Government Index	Fixed Income	MLDGVCL
Commodity Index	MLCX Spot Index	Commodity	MLCXSPT
Gold Bullion	Gold Bullion LBM $/t oz	Commodity	GOLDBLN
Real Estate Index	MSCI WORLD REAL ESTATE $	Real Estate	M2DWR2$
USD/EUR	EURO TO US $ (RFV)	Exchange Rate	USEURO.

Table B.2
Data retrieved from Quandl

Security	Quandl Name	Asset Class	Quandl Code
BTC/USD	Bitcoin Market Price USD	Alternative Investment	BCHAIN

Appendix C: Yearly Efficient frontiers

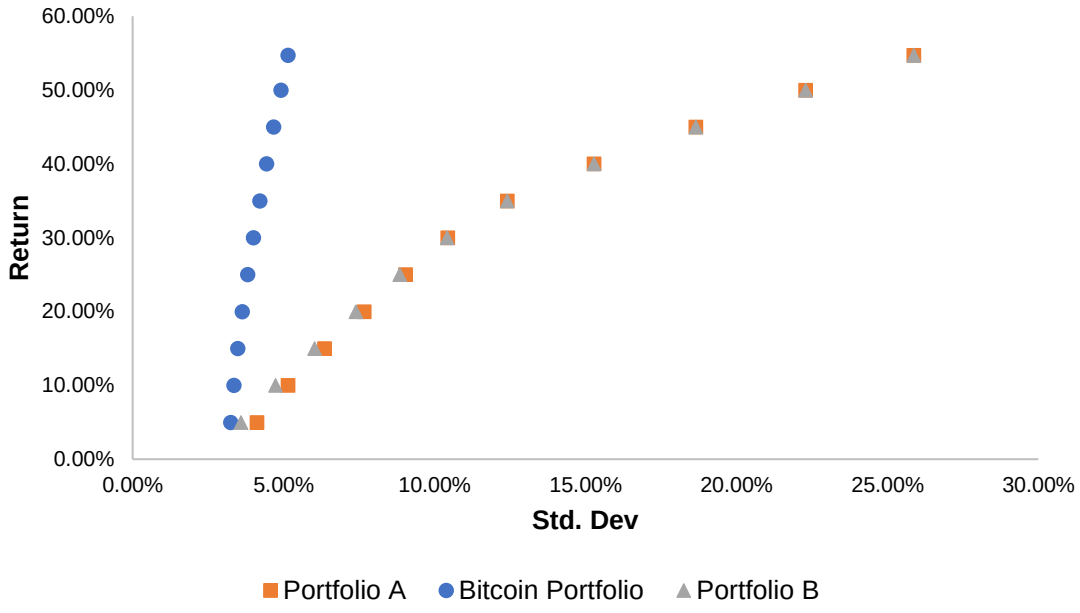

Figure C.1: Efficient Frontiers 2013

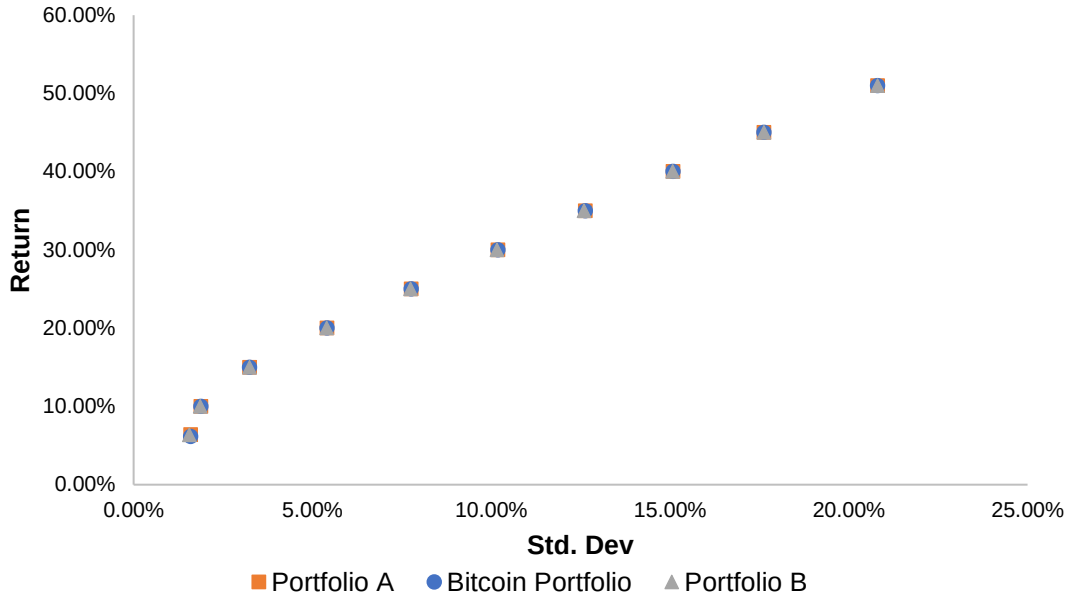

Figure C.2: Efficient Frontiers 2014

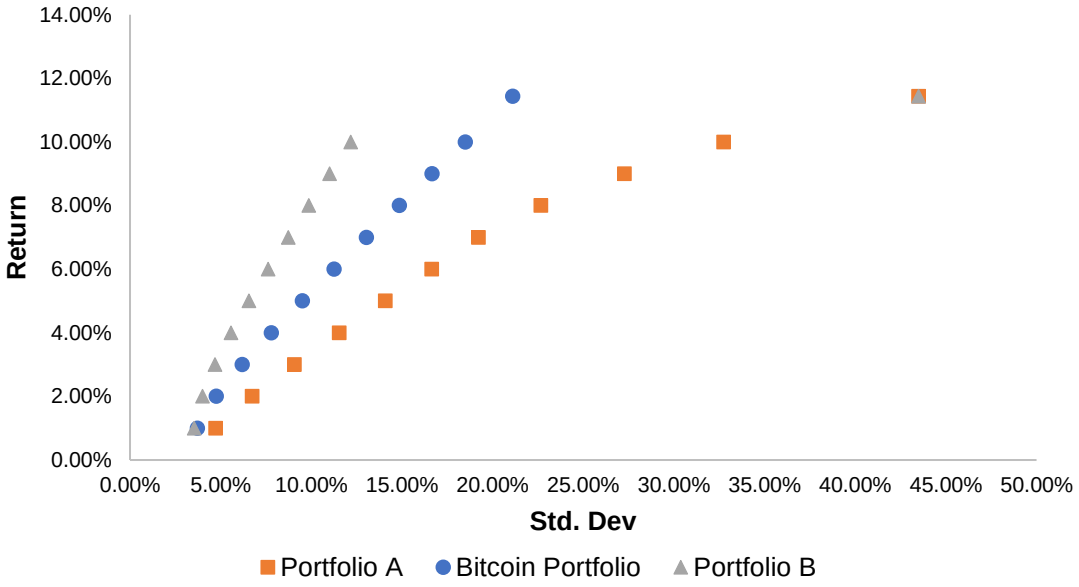

Figure C.3: Efficient Frontiers 2015

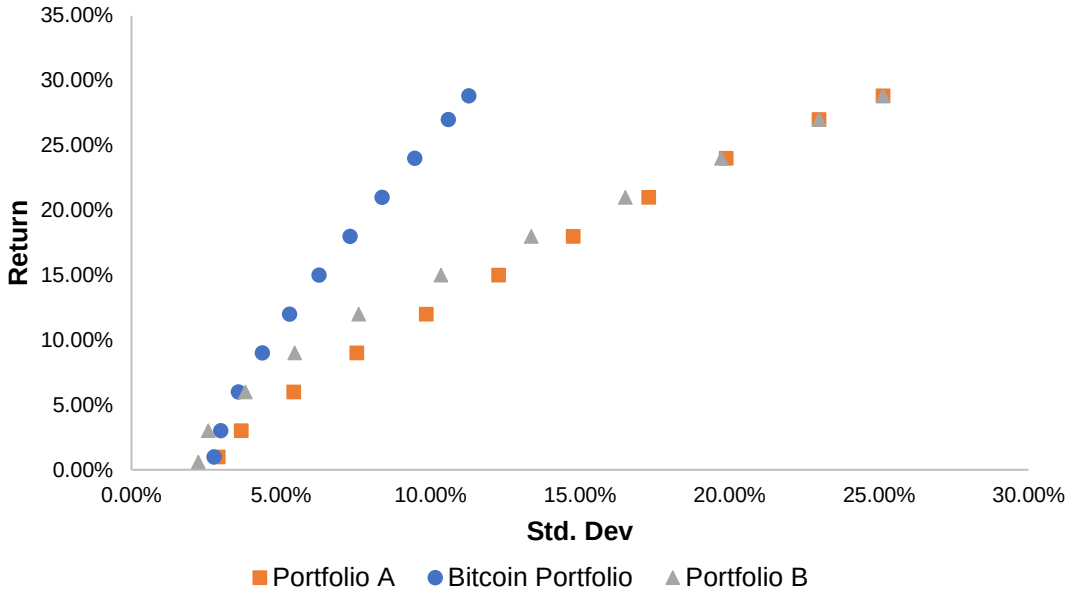

Figure C.4: Efficient Frontiers 2016

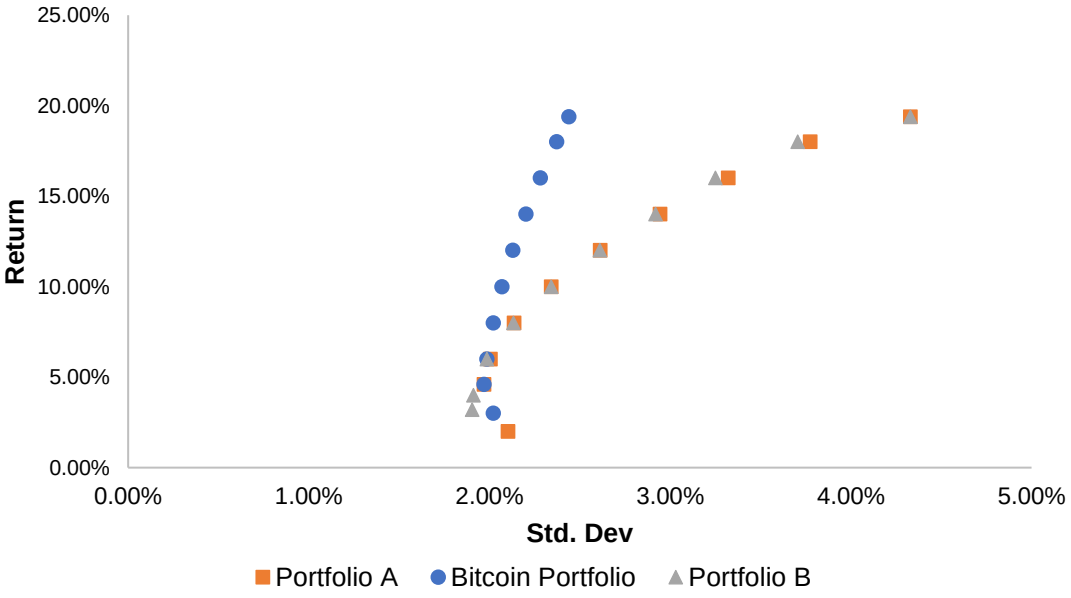

Figure C.5: Efficient Frontiers 2017

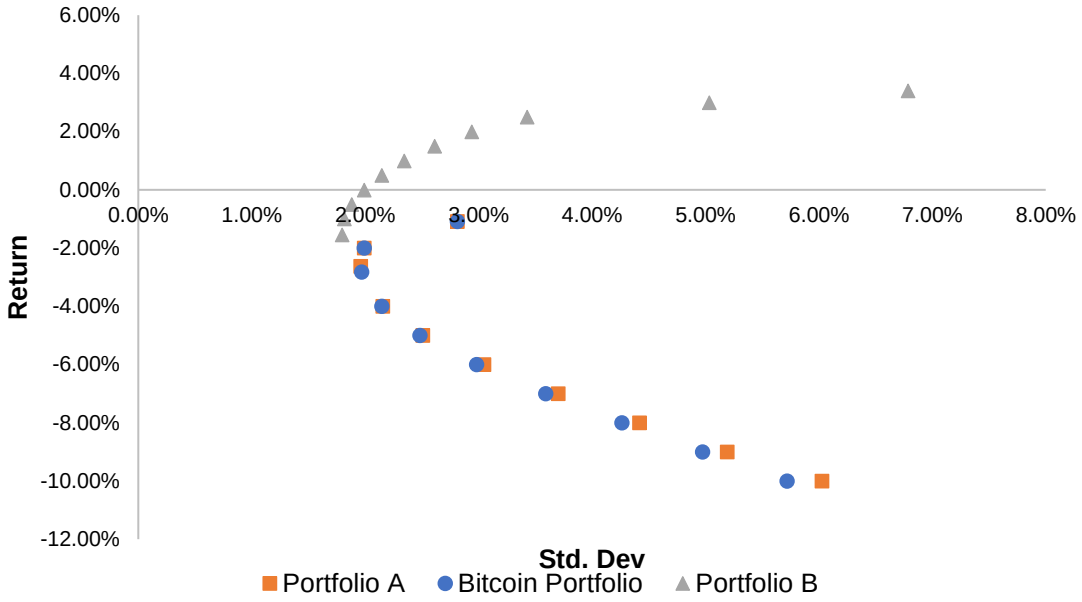

Figure C.6: Efficient Frontiers 2018

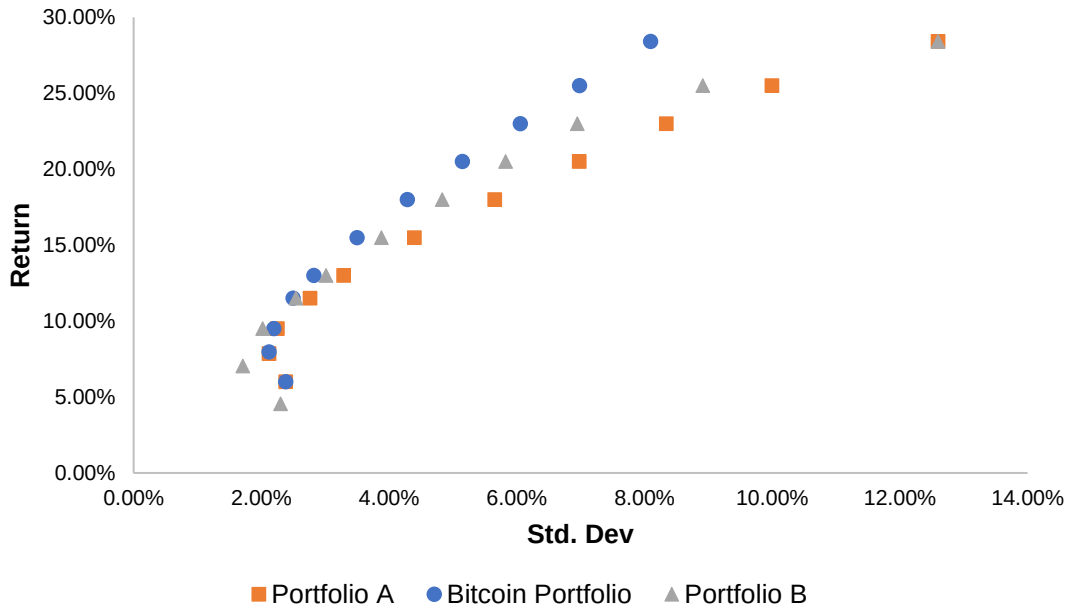

Figure C.7: Efficient Frontiers 2019

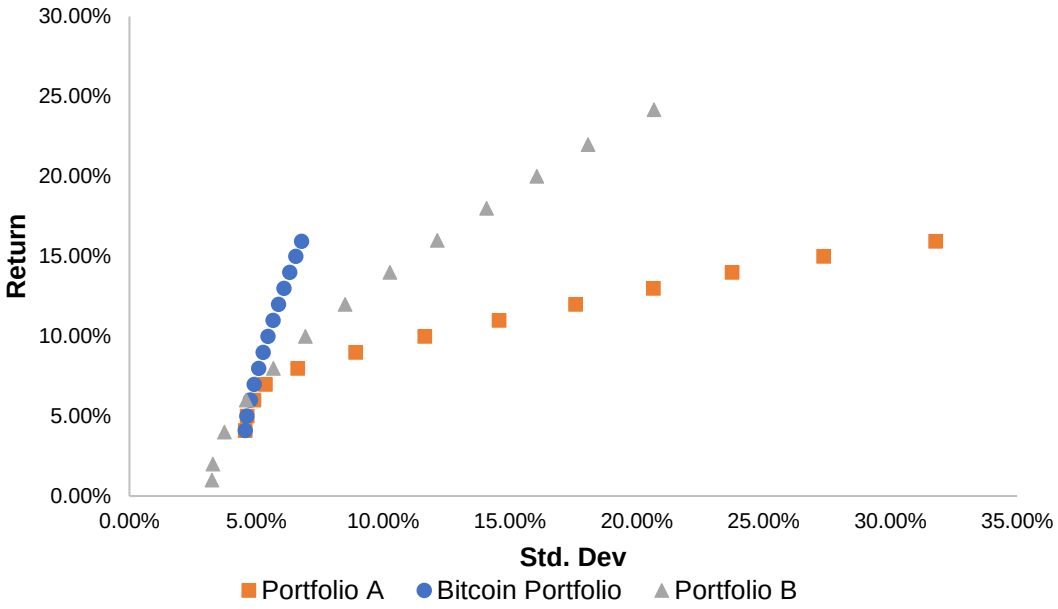

Figure C.8: Efficient Frontiers 2020

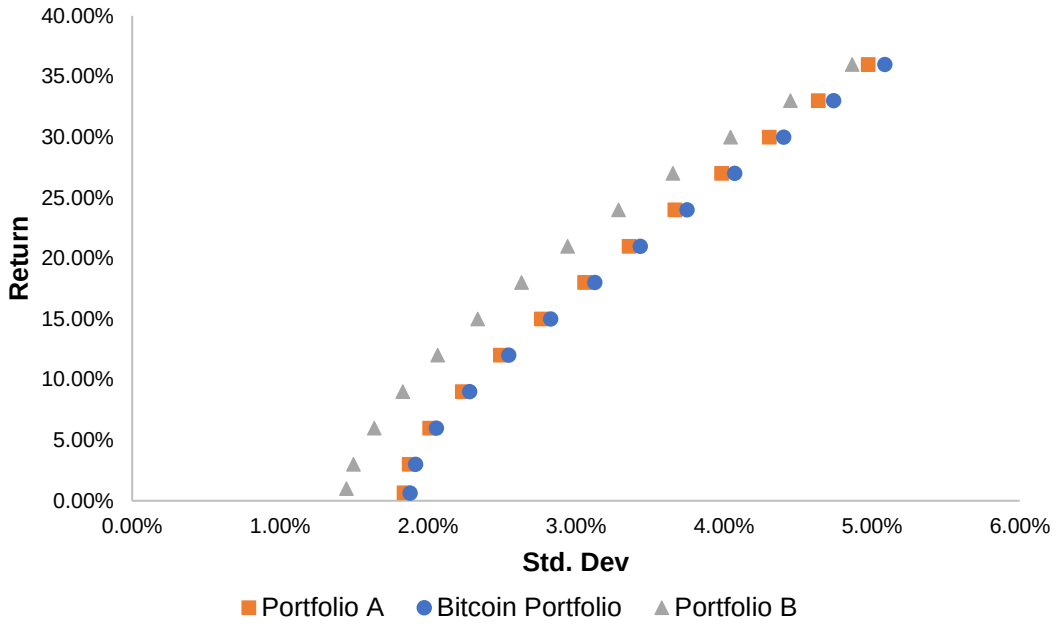

Figure C.9: Efficient Frontiers 2021 (until May)

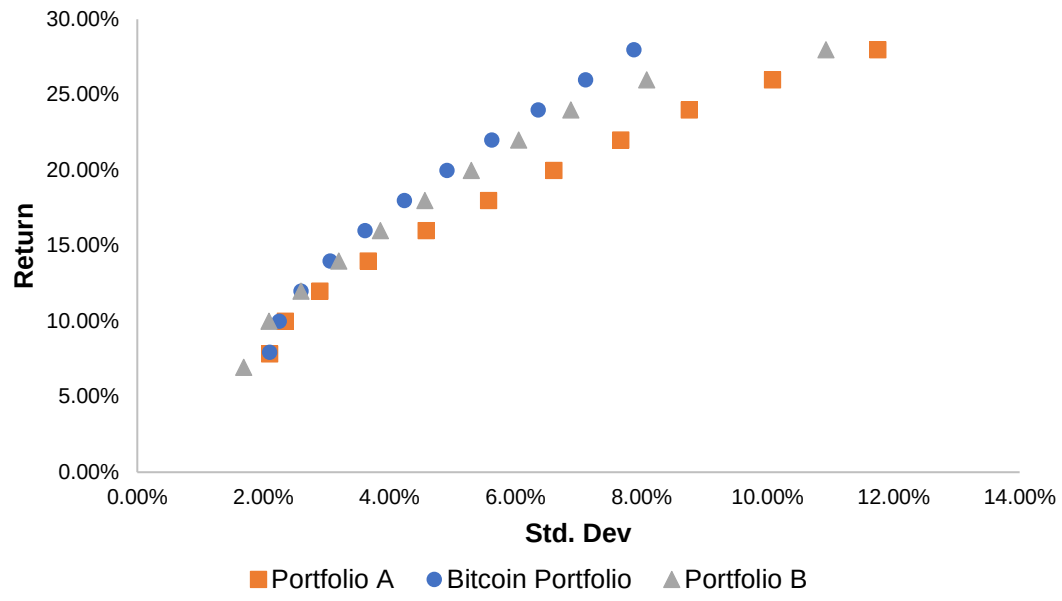

Figure C.10: Efficient Frontiers 2019 – Jan. 2020 (Pre-pandemic Period)

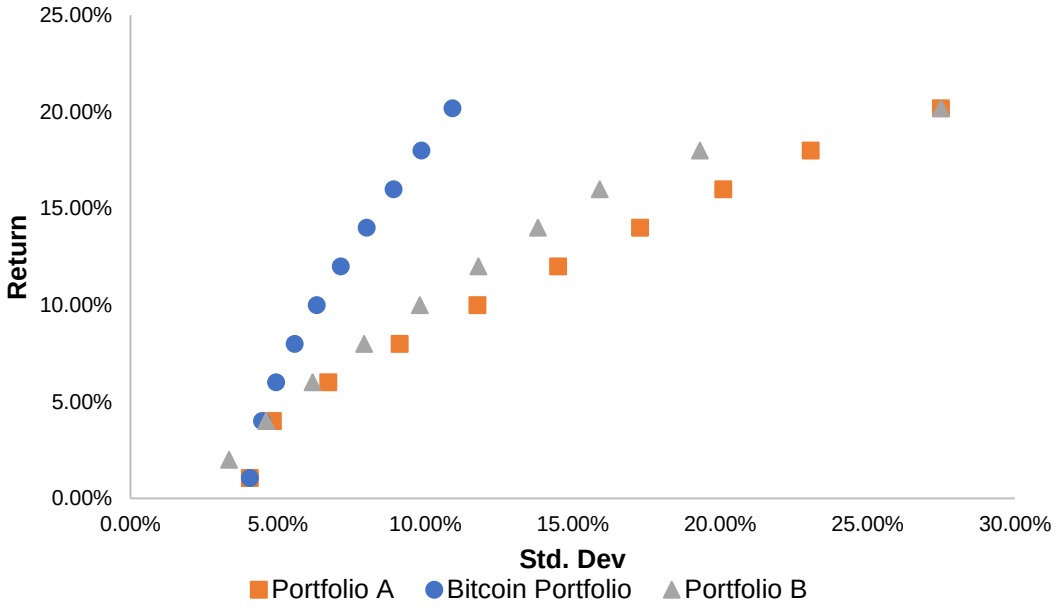

Figure C.11: Efficient Frontiers Jan. 2020 – Jan. 2021 (Pandemic Period)

Appendix D:

Bitcoin's share in Robustness tests

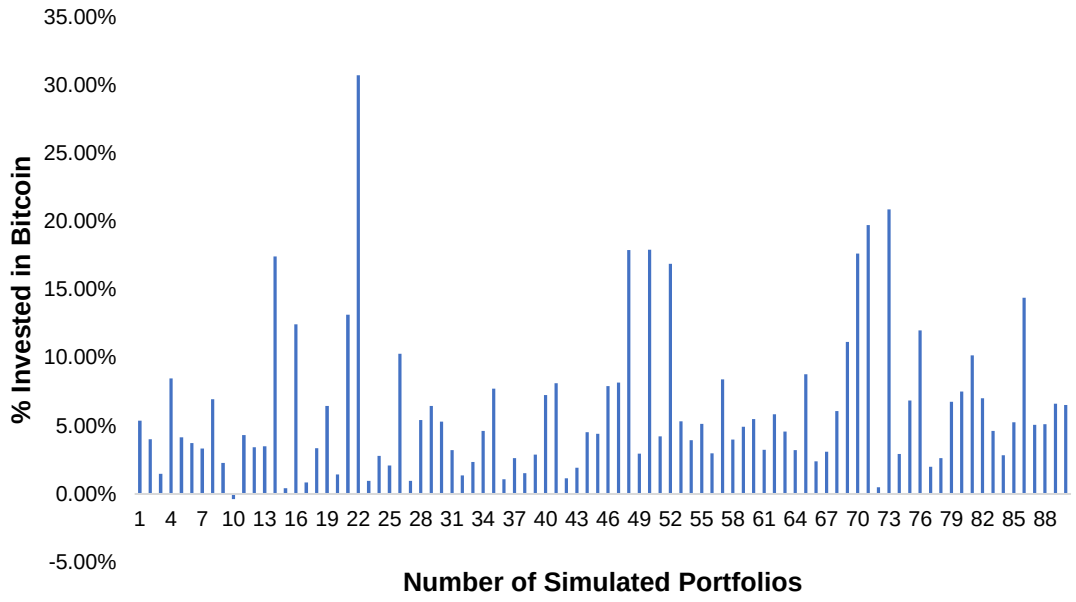

Figure D.1: Share of Bitcoin in the simulated portfolios with no short sales constraint

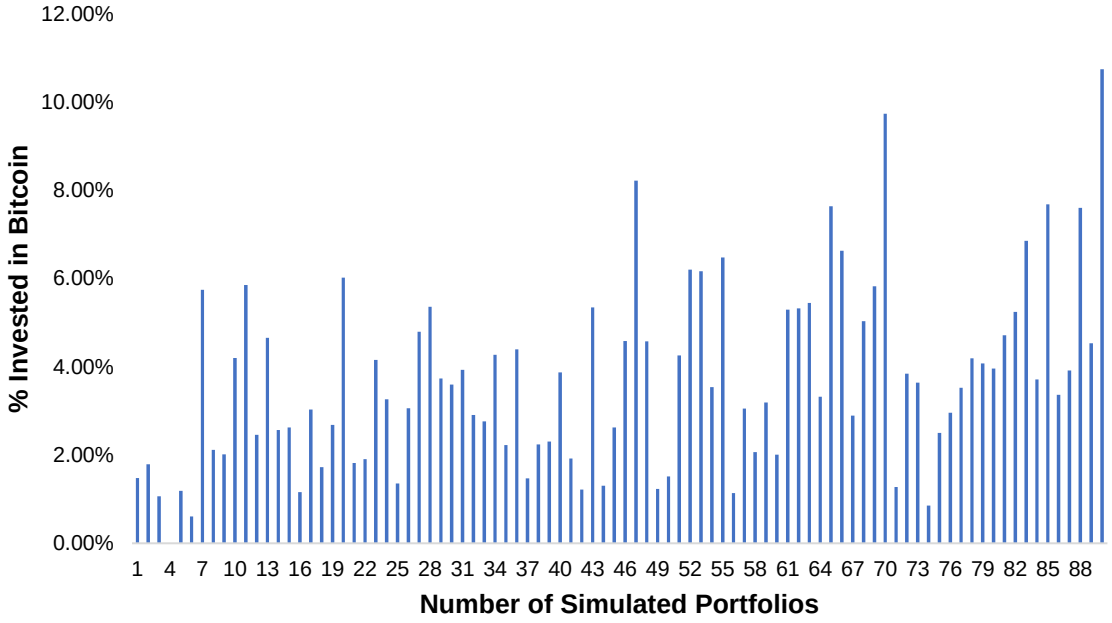

Figure D.2: Share of Bitcoin in the simulated portfolios with weight constraint