LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

MASTER IN FINANCE

MASTER´S FINAL WORK

DISSERTATION

PERFORMANCE OF ROBO-ADVISORS VERSUS MEAN-VARIANCE THEORY

JULIANA GONZALEZ FIGUEIREDO

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SUPERVISION:
RAQUEL MEDEIROS GASPAR

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GLOSSARY

AuM – Assets under Management

DB – Deutsche Bank

EF – Efficient Frontier

EFAMA – European Fund and Asset Management Association

ETF – Exchange-traded Funds

ESAS – European Supervisory Authorities

FCA – Financial Conduct Authority

FINRA – Financial Industry Regulatory Authority

FTSE – Financial Times Stock Exchange

MPT – Modern Portfolio Theory

MSCI – Morgan Stanley Capital International

MTA – Maximum Timeframe Available

MVT – Mean-Variance Theory

RAFI – Research Affiliates Fundamental Index

RF – Risk-free rate

SEC – Securities and Exchange Commission

S&P – Standard & Poor´s

SPDR – Standard & Poor´s Depositary Receipts

SR – Sharpe Ratio

SSNA – Short Selling Not Allowed

TIPS – Treasury Inflation-Protected Securities

TLH – Tax Loss Harvesting

VaR – Value at Risk

ABSTRACT

Robo advisors represent a fast-growing trend within the investment advisory industry and have the huge potential to be an alternative for retail investors. However, being such a recent technology, this is still a very unexplored area with its methods and efficiency very little studied. This study comprises a theoretical and an empirical approach on the robo-advisors investment methodology. In the theoretical part, we conduct a literature review presenting the major studies conducted about the robo-advisors and their current market status, breaking down some details about the processes and methods used by the major companies in the field through an analysis of the details elucidated in the companies´ reports. The empirical study is then conducted comparing the composition of Mean-Variance Theory efficient portfolios with real allocations proposed by robo-advisors. This is accomplished through the analysis of actual portfolio allocations provided in 2017 by four US robo-advising companies for different investor risk profiles. Besides MVT portfolios, Homogeneous and Kataoka portfolios are also used for comparison and all the analysis are conducted for in-sample and out-of-sample period.

KEYWORDS: Robo-advisors, Exchanged-traded-funds, Portfolio Management, Mean-Variance Theory, Efficient frontier, Return at Risk.

JEL CODES: D810, G110, O330

RESUMO

Os "*robo-advisors"* representam uma tendência de rápido crescimento dentro da indústria de consultoria de investimentos e têm um enorme potencial para ser uma alternativa para investidores de varejo. Porém, por se tratar de uma tecnologia tão recente, esta ainda é uma área pouco explorada e com métodos e eficiência pouco estudados. Este estudo compreende abordagens teórica e empírica sobre a metodologia de investimento dos robôs. Na parte teórica, realizamos uma revisão bibliográfica apresentando os principais estudos realizados sobre os robôs e o seu estado atual no mercado, detalhando alguns dos processos e métodos utilizados na gestão de carteiras pelas principais empresas do ramo através da análise dos detalhes elucidados nos relatórios das empresas. O estudo empírico é então realizado comparando-se a composição de carteiras eficientes da Teoria da Média-Variância com alocações reais propostas por "robo-advisors". Para isto são analisadas alocações de portfólio reais fornecidas em 2017 por quatro empresas dos EUA para diferentes perfis de risco de investidor. Além das carteiras de média-variância, carteiras homogêneas e carteiras Kataoka também são usadas para comparação e todas as análises são conduzidas para o período dentro e fora da amostra.

PALAVRAS-CHAVE: *Robo-advisors*, Fundos de investimento abertos negociados em bolsa, Gestão de Portfolio, Teoria da Variância-média, Fronteira Eficiente, Retorno em risco.

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1. INTRODUCTION

The management of a portfolio can be very defiant and a slippery terrain for people that are not inserted into the Finance area and do not work on daily basis with investment decisions. In that context, financial advice comes as a solution for investors not confident in their ability to outline well-structured portfolios. Even for professionals in that field, financial advice can be very providential, as can be difficult to detach the personal emotions from the decision-making process. The Robo-advisors (also referred here as "robos") come to the advisory sector as an alternative to the traditional human advice with the aim of digitally revolutionizing the asset management industry.

As an automated platform, all the steps are conducted online, and the robo-advisors promise to accurately define the investor´s risk profile and provide personalized well-suited and efficient portfolios. All of that while keeping the costs very low through passive investments and not requiring substantial initial capital as many traditional companies do. Therefore, it is very clear why robo-advisor can be very attractive as an investment option. The fact that they conduct all the communication with the investors digitally, also gives them an important competitive advantage during world crises like such as the one we are experiencing in 2020. However, even with the fast growth experienced by the robo-advisors, the potential on the market is very expressive since the majority of the advisory market is still controlled by traditional human advisory companies.

Unfortunately, robo-advisors do not disclose detailed information on their methodology for the portfolio allocation and management, and the real data from the portfolios´ performance is not public. That, coupled with the fact that this technology is recent in the market (first fully automatized portfolios only launched in 2010), culminates in very limited studies about these softwares. This research tries to fill that gap, examining the robo-advisors methodology and their real-world applicability. For these analysis, we had access to real robo portfolio allocations proposed on March 2017 by four of the main US robo-advisors - WealthFront, Schwab, SigFig and Tolerisk - for three different investor risk profiles – conservative, moderate and aggressive. With these real allocations, it is possible to evaluate the performance of the portfolios and compare the allocation proposed by the company with the allocation resulting from the most common methods applied on portfolio management.

In the second chapter we provide a contextualization on the robo-advisors, first providing an overview of their situation on the market and then discussing their investment methodology, from the initial profile assessment to the portfolio allocation and management. Much is not known about the details of the methods and inputs used by the companies but in this chapter we aim to shed some light on their rationale based on reports and SEC brochures provided by some of the players in this market.

The third chapter conducts a literature review of the robos presenting an evolution of the studies on this matter over the last years. It is also pointed out the important topics that we believe the past researches did not cover and that we are aiming to approach here.

The following chapters are dedicated to the analysis of the real robo-advisors allocations mentioned. Chapter 4 informs about the criteria for the Data used and the past performance of the assets that are present in these portfolios. Chapter 5 covers in detail the Methodology used in this study, where we compare the composition of MVT efficient portfolios with these robo portfolios for the in-sample (until March 2017) period and out-of-sample period (from March 2017 to December 2019). Chapter 6 presents the results obtained by the analysis.

In the final chapter, we provide some final considerations about the robo-advisors and are draw some conclusions from the results of the simulations. Some suggestions for future research are also approached since there is a lot to deepen about the knowledge of the use of this technology.

2. ROBO-ADVISORS OVERVIEW

The methodology applied by robo-advisor companies is not fully available for the investors but they disclosure some information about the guidelines and models used in their process. The information is accessible at whitepapers on the companies´ websites and the brochures provided for the US Securities and Exchange Commission (SEC).On this Chapter we present some of the information that can be taken from these companies´ reports.

The chapter is divided in four sections where are presented the methodology steps that are followed by the robo-advisors. On section 2.1 is provided an overview of the current situation of the robo advisors on the market covering its growth throughout the time as well then main characteristics of these digital advice softwares. Section 2.2 presents the process of investor assessment, that consist in the identification of the investor´s profile, understanding its goals and investment horizon. The following section, 2.3, covers the asset class selection and investment vehicles used and section 2.4 addresses the methodology behind the portfolio allocation and management.

2.1. Contextualization

After the economic crisis in 2008 the first robo-advisors were launched as a cheaper way to take part in more sophisticated investments and, since then, increased in popularity representing a very promising sector. Fisch, et al. (2018) provides a wide panorama of the evolution of these robos through the time highlighting the path of regulation in this area and the changes in this competitive market.

Robo-advisors use innovative technologies to provide digital financial advice on asset management to the investors through online algorithmic-based programs. After the online identification of the client profile, they automatically generate a portfolio allocation for the client based on these algorithms, manage and optimize clients' assets without any human interaction between investors and the advisory company.

These automated platforms have been attracting many individual investors and experiencing an expressive growth of the assets under management (AuM) on the past years. The estimative is that the robos were managing USD 200 billion in assets worldwide in 2017 (Eule, 2018) and near to USD 631 billion at the end of 2019 (Backend Benchmarking, 2020).

Notably, although robo-advisors sharp growth, their AuM still represent a relatively small fraction of the USD 80 trillion of global AuM estimated at the end of 2016 (Kelly, 2017). The potential for growth, on the other hand, is huge if we consider that there is a big parcel of investors that are difficult to be reached out by traditional advisors because of the high capital requirements and high fees charged by the companies. The real numbers of AuM worldwide show that the past predictions conducted by several banks and Research companies optimistically estimated it though, with forecasts for 2020 of USD 2.2 trillion (Regan, 2015) to USD 8.1 trillion (Statista, 2017).

Also notable, is the difference between assets of robo-advisors in US and Europe, that still have a very incipient market for that type of software (around 5-6% of the AuM under management of robos in the US according to Kaya (2017)). In the second quarter of 2019, the estimated AuM for robo-advisor in Europe were about EUR 14 billion (Hesseler, et al., 2019), close to size of the third largest robo advisor in the world, Betterment. The authors also point out that, in Europe, the UK is the largest market with AuM of EUR 5.5 billion in 2019, followed by Germany with about EUR 3.9 billion.

According to FCA (Financial Conduct Authority), the supervisory authority for traditional financial advisors, in 2016, around half of the financial advisors in the UK turned away clients due to the small size of their investments (FCA, 2017). Therefore, adding this to the current incipient market share of just a few billions under management of robo-advisors in a market with more than 20 trillion AuM, the potential of robo-advisors in this area is very clear. The European Fund and Asset Management Association (EFAMA) estimated that AuM on Europe at the end of 2016 were close to EUR 22.8 trillion (EFAMA, 2017). There is also a possible additional momentum for robo-advice in the EU, because American traditional asset managers and banks that manage large funds have begun offering robo-advice services in Europe as well.

There are many advantages of these automated portfolios, being the major ones, the lower advisory fees (even lower in US than in the EU) and the necessity of lower initial capital investment if compared with the traditional financial advisors. Unlike many traditional financial advisors, there are usually no or very low minimum volume requirements to open a robo-advisor account. Some firms even offer free services for investment of lower amounts (normally up to USD 10,000). Table 1 presents a summary of the advisory fees and initial capital requirement of the four robo-advisors studied here - WealthFront, Schwab, SigFig and Tolerisk – and traditional advisors as a way of comparison between both solutions. The data for the traditional ones relies on the annual report published by Price Metrix (2020), company specialized in providing to wealth management firms detailed information about industry best practices through collected data from the market. Although the average fee charged by traditional advisors have been decreasing according to Price Metrix, as it can be seen in Table 1, it is still more than four times what robo-advisors charge.

TABLE 1 – FEE AND INITIAL CAPITAL REQUIREMENT COMPARISON

Advisory	Advisory Fee	Initial Capital required
WealthFront	0.25% a.a.	USD 500
Schwab	zero	zero
SigFig	zero (investment < USD 10,000)	USD 2,000
	0.25% a.a.(investment > USD 10,000)	
Tolerisk	USD 99/month	Not applicable*
Fee-based traditional advisors	1.05% a.a.**	zero to USD 250,000

** Initial capital required not mentioned because company focus on financial professionals, not individual investors.*

*** Number for fee-based data based on PriceMetrix (2020).*

Automation and passive investment strategies used by the robo-advisors also minimize conflicts of interest. The fact that robo advisors receive from the investor the standard remuneration, irrespective of the type of product they allocate, is thought to lower the possibility that they will favour one product over another (Lam, 2016). Nonetheless, other conflicts can happen because some robos have affiliates that issues ETFs with high expense ratios and may receive compensation flows by using these ETFs in their portfolios, opening space for allocation on ETFs or cash deposits that are not necessarily the most suitable for the investor and with not desirable tax-return trade-off.

2.2. Investor profile assessment

For a personalized portfolio that better suits the investors, initially the robo-advisors do an assessment of their profiles determining objectives, investment horizons and personal financial profile. That is done in different ways by the companies however it mainly consists of online questionnaires that allow to gather information on the level of financial education and subjective willingness to take risk of the investor.

In this initial stage of profiling it is established what is the role of the investment, whether it is growth, income or just maintaining their wealth against adversities. That will impact the asset classes that will be considered for the investor since most robo-advisors have their assets classes grouped according to their role in the portfolio. Schwab for example have five different classes: growth, growth and income, income, inflation, defensive assets. The client´s goals will also impact on the type of accounts for taxes purposes, whether taxable or deferred. The most critical part of this assessment is the definition of the investor´s risk profile. In order to recommend a portfolio suitable to the emotional behaviour of each investor toward risk, the degree of loss aversion and risk acceptance by the investor is also analyzed in this initial phase.

Schwab in its whitepaper for example is very clear stating that allocates the cash portion of the portfolio according to an investor's risk profile, with the most risk-averse or short-term portfolios holding the highest levels of cash and the least risk-averse or longer-term ones holding the lowest levels of cash.

In their questionnaires, the companies assign objective and subjective risk scores to each individual and they weight the scores to determine the appropriate level of risk the investor should take. The questions with the focus on the definition of the risk profile are normally related to behavioural tendencies, such as the action the investor may take after experiencing significant investment loss. Lam (2016) presents questionnaires of Schwab and WealthFront and points out some particularities that can be noticed. For reference, Schwab does not ask clients about their total wealth value or annual income but, to define the investment horizon and goals, asks direct questions about when investor intend to withdraw the investment and when intends to retire. WealthFront, on other hand, provides great importance to current portfolio value and the excess income (annual after-tax income to expense ratio in retirement) considering the higher excess incomes correspond to higher capacity for risk.

However, there is a lot of discussion whether these questionnaires applied by the companies can efficiently access the risk profile of the client. The major criticism is regarding the fact that the questions tend to be very few and very simple. Kaya (2017) calls attention for the fact that being multiple-choice questions, they are eliciting only basic information about investors without providing a trustworthy overview of a user's financial situation. In the same study, the author also points out that the questionnaires can be considered too narrow, leading to different interpretations of the question, what may induce biases in the individual´s answers. Some of these shortcomings could be minimized with more meticulous questionnaires however that also could represent a problem where the individual loses concentration and patience during the answers. This issue about the possible shortcomings and biases related to the risk profile assessment by the robo-advisors will not be developed further in this study though since it is not our focus.

It is important to mention that one of the robo-advisors analyzed, Tolerisk, differently from the other companies presented in this study, names itself as a Risk Tolerance Assessment Tool, a software for financial advisors to better serve their clients. Therefore, the company states very clearly in their website that it is designed for financial professionals, not individual investors. Since the company sells for traditional advisors its ability to provide better profile and risk assessments than traditional assessments, it does not reveal the methods used in that evaluation.

2.3. *Asset class selection and Investment Vehicles*

With the main goal of minimizing costs and ensuring high liquidity for investors, the focus of robo-advisors is on passive investment, excluding strategies such as actively managed domestic or foreign equity mutual funds. Kaya (2017) mention that roughly 60% of robo-advisors base their investment approach exclusively on index-based Exchange Traded Funds (ETFs) that are an excellent choice of investment vehicle because they allow to minimize the idiosyncratic risks through diversification of assets without incurring in unnecessary costs that would be required to invest in such a vast pool of assets. The four robo-advisors studied here (WealthFront, Schwab, SigFig and Tolerisk) use only ETFs, except for Schwab that also invest on cash. The company informs in its paper (Schwab Intelligent Portfolios' Guide to Asset Classes Whitepaper) that aims a diversification choosing at least twenty ETFs but also a cash allocation. Schwab is very transparent regarding their available pool of assets, listing in its Whitepaper all the ETFs that could be potentially part of Schwab´s portfolio.

The researches on the performance of active versus passive portfolio management shows that in general, there is little to no advantage on the returns provided by fund actively managed in comparison with the ones passively managed (Bogle, 2009; Malkiel, 2012). According to Kaya (2017), since 2012, in general, actively managed funds have slightly underperformed compared with ETFs and on average only 40% of actively managed funds were able to beat ETF returns between 2014 and 2016. So, the higher fees charged in active management, customarily used by the traditional advisory sector, do not attract the retail investor.

In the way the portfolio management of robo-advisors is conducted, the investors have much less influence on the portfolio decisions since they can only impact on them through updates in their profiles and questionnaires in the robos´ website, not being able to choose for example which type of asset and what ETF´s area they desire to invest in. Still, picking stocks or trying to "beat the market" is not only time consuming, but it also seems to produce poor results on average (See Malkiel (2012)). This is in line with the standards of diversification and the findings of Markowitz (1952); Sharpe (1964); Brinson, Hood & Beebower (1986); Brinson, Singer & Beebower (1991) and Ibbotson & Kaplan (2000). The studies conducted showed that the overwhelming contributor to performance of portfolios was the choice of asset class, with the individual securities chosen impacting very little on the results. Following that mindset, the first step for the definition of robo-advisors portfolio is to identify diversified asset classes that will guide their portfolios. The classes vary between the companies, but the broad categories are domestic Stocks, foreign Stocks for developed and emerging markets, domestic Bonds, foreign Bonds, inflation-protected securities, Real State, Natural Resources and Commodities.

After decided the asset classes, robos follow a top-down approach, reducing the pool of available assets through analysis of the options that allow the best minimization of the management fees and offer ample market liquidity. Normally the companies disqualify assets with high management fees, that are leveraged, poorly diversified, with short history on the market, with insufficient market liquidity or with consistent poor performance, The robo-advisors attention is also on the minimization of tracking errors, avoiding index funds with high volatility from its benchmark. Normally, funds with the desirable low tracking error have higher management fee so this trade-off is analyzed.

Another concern of them is regarding the lending of the underlying securities by the ETF issuers. The robo-advisors try to prioritize ETFs issuers that minimize the lending of their securities (WealthFront Investment Methodology Whitepaper), a manoeuvre to enable short sale, or at least share the lending revenues, what also allows to lower the management fees.

There is though some criticism about the restrictiveness of assets available for the portfolio´s composition after the exclusion of actively managed products and common restrictions mentioned. According to Kaya (2017), the final set of ETFs available for robo-advisory purposes usually comes down to approximately 3 to 6% of all investable ETFs. This fact may raise questions on investors about the company´s ability to adjust the portfolio to the appropriate client´s profile and its objectives.

2.4. Portfolio allocation and management

Once well-defined the asset class for the investment, robo-advisors proceed for the determination of the optimal mix of the chosen asset classes. Unfortunately, as mentioned before, little is known on how they do the allocations, nor on how they estimate returns, variances and correlations but some information can be taken from their reports and papers.

On WealthFront Investment Methodology White Paper, the company reveals that, for estimation of returns, it uses Black-Litterman model (more details of this model can be seen on Walters (2014)) to blend the expected returns obtained from the Capital Asset Pricing Model (Sharpe, 1964) with returns obtained from WealthFront own multi-factor model for long-term expectations. However, are not provided many details on this multi-factor model. For the optimization of the chosen assets is disclosed that is applied the

MVT (Markowitz, 1952) and that for the covariance matrix the company relies on historical data.

Schwab portfolios are also determined according to MVT but, to take into account the emotional behaviour of investors, a full-scale optimization is also carried out to incorporate investor´s preference of loss aversion (Schwab Intelligent Portfolios Asset Allocation White Paper). In this optimization, it is considered a return threshold of zero considering that this is the point where the pain of losses exceeds the joy of a similar sized gain. Then, adjustments are made considering the risk levels (providing less weight to riskier assets classes) and qualitative considerations (not detailed by the company) to ensure that the investment strategies meet investors' preference and intuition.

There is a specific concern with Schwab cash allocation regarding a questionable conflict of interest, as cash investments from Schwab robos are deposited at Schwab Bank, which profits from the spread between the interest rate it pays on deposits and the amount it earns on the investment of such deposits. As remarked by Lam (2016), Schwab essentially admits to offsetting part of the costs of the robos´ program by allocating more of client assets to cash than it would under different investment programs. The company presents this information at the filing with the SEC (Schwab, 2015) where the company also acknowledges that a cash allocation can hurt investment performance. Therefore, is highly questionable if Schwab’s relatively large cash allocation and high ETF expense ratios are linked to compensation flows to Schwab affiliates.

Sig Fig Investment Methodology Report (2016) points out that the company analysis the asset classes according to their performance in different market and economic conditions with focus on class returns, volatility and correlation among the classes, using for that, observations over the last twenty years, weighted towards more recent history. Regarding the method used, Sig Fig informs that mean-variance optimization is applied but does not provide more details about their estimative approach.

All four robo-advisors studied here inform that they continuously monitor and periodically rebalance portfolios to ensure they remain optimally diversified. These changes are conducted to re-adapt the portfolio for price-level fluctuations and macroeconomic changes, allowing to keep the portfolio´s risk level aligned with the goals, risk tolerance and risk capacity of the target asset allocation. It is not informed clearly though with what frequency that re-optimization and rebalancing are performed. The representative assets of each class are also re-analyzed from time to time as an attempt to minimize the taxes likely to be generated by each asset class and to create allocations that are specifically customized for different taxable profile of investor.

3. LITERATURE REVIEW

Digital investment platforms cover a wide range of different technologies and services and the use of the term automated process on the Financial Digital Industry can bring some confusion when we are discussing Robo-advisors literature. Deloitte (2016) outlines the different generations that can be considered for the Robo-advisors since the first automatizations of questionaries and proposals (first and second generation) to more robust processes that include quantitative methods and algorithms to construct and rebalance portfolios (third and fourth generation). The literature review for the last two generations is the one covered in this chapter, since they are the ones that really refer to fully automated portfolio management.

The first robo-advisor, Betterment, only launched its robo service in 2010. All the available information about this industry and its practices is still scarce and recent. This is also evident from the published studies on robo-advisors. Up to 2016 there was not much published about them. On this year, the first studies are conducted, mainly consisting of online publications (i.e. BlackRock, 2016; Kramer, 2016 and Weisser, 2016). Those first analysis are focused on the overview of this new type of software, touching, in a broad way, the benefits and possible concerns. Stands out at that time the study of Lam (2016), that provides a deeper insight into the robo-advisor model, approaching not only the characteristics of these softwares but also the methodology behind it and of Klass & Perelman (2016), that explores the application of the fiduciary standards to digital advisers, not very approached until that point.

During the end of 2015 and 2016, the regulatory bodies worldwide start to pay closer attention to robos, issuing papers about the regulatory requirements that these digital advisers may be submitted. In Europe, the joint committee of the European Supervisory Authorities (ESAs) publishes a preliminary high-level assessment that aims to discuss which, if any, regulatory and/or supervisory actions may be needed to mitigate risks involved on the robo-advisors (ESAs, 2015). In the US, the Financial Industry Regulatory Authority (FINRA) point out what they consider the best effective practices with respect to technology management, portfolio development and conflicts of interest mitigation (FINRA, 2016).

From 2018, more in-depth studies began to be published focusing on the core portfolio optimization and asset allocation methods applied by the robo-advisors. Beketov, et al. (2018) conducts an interesting work, analyzing robo-advisors worldwide and showing that Modern Portfolio Theory (MPT) remains the main framework used in these softwares. For these analysis, they investigated the occurrences of the methods, their combinations and the respective assets under management (AuM). Strub, et al. (2019) follows a similar line of study approaching the use of the MVT by robo-advisors. The authors point out possible drawbacks of the use of this methodology, proposing an enhanced mean-variance framework for robo applications which is based on the equivalence between the mean-variance objective and quadratic utility functions.

Despite the abundance of information about the robo-advisors currently available online, little is known about the performance of these robos from the investors´ point of view. When mentioned the good performance of these softwares normally is considered only the AuM of the major companies and the growth in the number of clients (i.e. Eule, 2018 and Backend Benchmarking, 2020) but not much about the individual performance of the portfolios suggested by these companies. Backend Benchmarking (2020) mentions the performance of the average portfolio of the robos but do not provide further details.

It is understandable that the information about the real portfolio allocations proposed and the performance of such portfolios are not fully disclosure by the companies but we believe that these analysis are extremely important in order to understand if robo-advisory services are worth the investment for the client. Stein (2016) touched briefly that point when he tested four of the leading robo-advisors at the time and published the exact portfolio allocations proposed by these companies for the same investor profile. It is analyzed the risk profile assessment conducted by the companies and the final ETF allocation, which differs a lot, not only in term of assets, but also asset classes. Similarly, Gil, et al. (2017) look at the exact compositions proposed by robo-advisors to three different investor profiles. Our study relies on the portfolio composition from this study. Unfortunately, no further research is conducted on the implementation of these portfolios.

Since, on the literature, no concrete analysis has been made for the performance of real robo portfolios and no comparison has been made between the allocation of these portfolios and possible allocations from current methods applied on portfolio management, our study aims to fill this gap. The following Chapters 4 to 7 will present our analysis on the matter.

4. DATA

For the four robo-advisors studied here - WealthFront, Schwab, SigFig and Tolerisk – we collected daily close prices for all the ETFs embedded in the real robo portfolios proposed by the companies on 31st March 2017. The prices collected are adjusted for dividends and splits. The data is retrieved from Yahoo Finance and extracted for the entire timeframe available in the market for these ETFs. In terms of sample size used for estimates of MVT inputs, we consider both maximal data for each ETF and a fixed 5-year period.

Most assets can be classified as Equities or Bonds ETFs, with exception of one ETF classified as Commodity ETF (DGL) and some allocation into cash deposits. Each robo provided three different portfolio allocations depending on the investor´s risk profile being identified as conservative, moderate or aggressive. Therefore, we have twelve different robo allocations.

For all the portfolios, the investment horizon proposed was of five years, starting at 31st March 2017. Table 2 lists all assets considered and provides their profile, with description, benchmark, asset class and area of exposure. Information about the allocations is provided in Table 3, showing the allocations for these twelve robo portfolios, and in Figure 1 the allocation per asset class. As per details provided on Chapter 2, the robo-advisors do not disclose all the details on how they estimate their inputs. However, as many of them claim to rely mostly on historical data, we determine on our analysis the mean-variance inputs based upon historical data. Figure 2 presents in a mean-variance space the evolution of these assets (ETFs and cash allocation) through the time for the whole period when they were available on the market, exhibiting the annual returns and volatility. Table 4 presents the same results analytically.

WealthFront portfolios included an equity ETF linked with natural resource´s companies in United States (VNRSQ), mostly connected to oil and natural gas production in this country. However, since in the pre-investment period the annual returns estimated for this asset were highly negative and the volatility unusually high, we chose to consider instead another similar ETF (IEG) that allowed far more reasonable input estimates. With this artifice of replacement, it is still possible to obtain the market trends, as this secondary fund follows the same trend, without compromising our analysis with unstable data.

TABLE 2 - ASSETS PROFILE

Asset	Description	Benchmark	Asset Class	Area of exposure
IGOV	iShares International Treasury Bond ETF	Bonds issued by governments outside the U.S.	Bond	Global ex-US
SPMB	SPDR Portfolio Mortgage Backed Bond ETF	US agency mortgage pass-through debt	Bond	US
STIP	iShares 0-5 Year TIPS Bond ETF	Short-dated TIPS	Bond	US
TFI	SPDR Barclays Capital Municipal Bond ETF	Municipal Bonds	Bond	US
VCIT	Vanguard Intermediate-Term Corporate Bond ETF	US investment grade corporate Bonds	Bond	Global
VGIT	Vanguard Intermediate-Term Treasury ETF	Intermediate-term US Treasuries	Bond	US
VWOB	Vanguard Emerging Markets Government Bond ETF	International emerging market government Bonds	Bond	Global
Cash	Cash Deposits	National rate on jumbo deposits	Cash	US
DGL	Invesco DB Gold Fund	Future contracts on gold and other precious metals	Commodity	Global
IEG *	iShares North American Natural Resources ETF	S&P North American natural resources sector	Equity	US
IEMG	iShares Core MSCI Emerging Markets ETF	MSCI emerging market stocks	Equity	Asia Pacific ex-Japan
PDN	Invesco FTSE RAFI Developed Markets ex-U.S. Small-Mid ETF	International developed markets small-mid stocks	Equity	Global ex-US
PRF	Invesco FTSE RAFI US 1000 ETF	1000 US large company stocks	Equity	US
PRFZ	Invesco FTSE RAFI US 1500 Small-Mid ETF	1500 US small-mid stocks	Equity	US
PXF	Invesco FTSE RAFI Developed Markets ex-U.S. ETF	International large company developed markets stocks	Equity	Global ex-US
PXH	Invesco FTSE RAFI Emerging Markets ETF	International emerging market stocks	Equity	Global
SPY	SPDR S&P 500 ETF	S&P 500 - large and mid-cap US stocks	Equity	US
VB	Vanguard Small Cap ETF	US small company stocks	Equity	US
VEA	Vanguard FTSE Developed Markets ETF	International large company developed markets stocks	Equity	Western Europe and Asia Pacific
VOO	Vanguard S&P 500 ETF	S&P 500 - large and mid-cap US stocks	Equity	US
VSS	Vanguard FTSE All-World ex-US Small-Cap ETF	International developed markets small stocks	Equity	Global ex-US
VYM	Vanguard High Dividend Yield ETF	Dividend paying large-cap US stocks	Equity	US
VNQ	Vanguard Real Estate Index Fund	US real estate stocks	Real State	US
VNQI	Vanguard Global ex-U.S. Real Estate Index Fund ETF	International real estate stocks on developed markets	Real State	Global ex-US

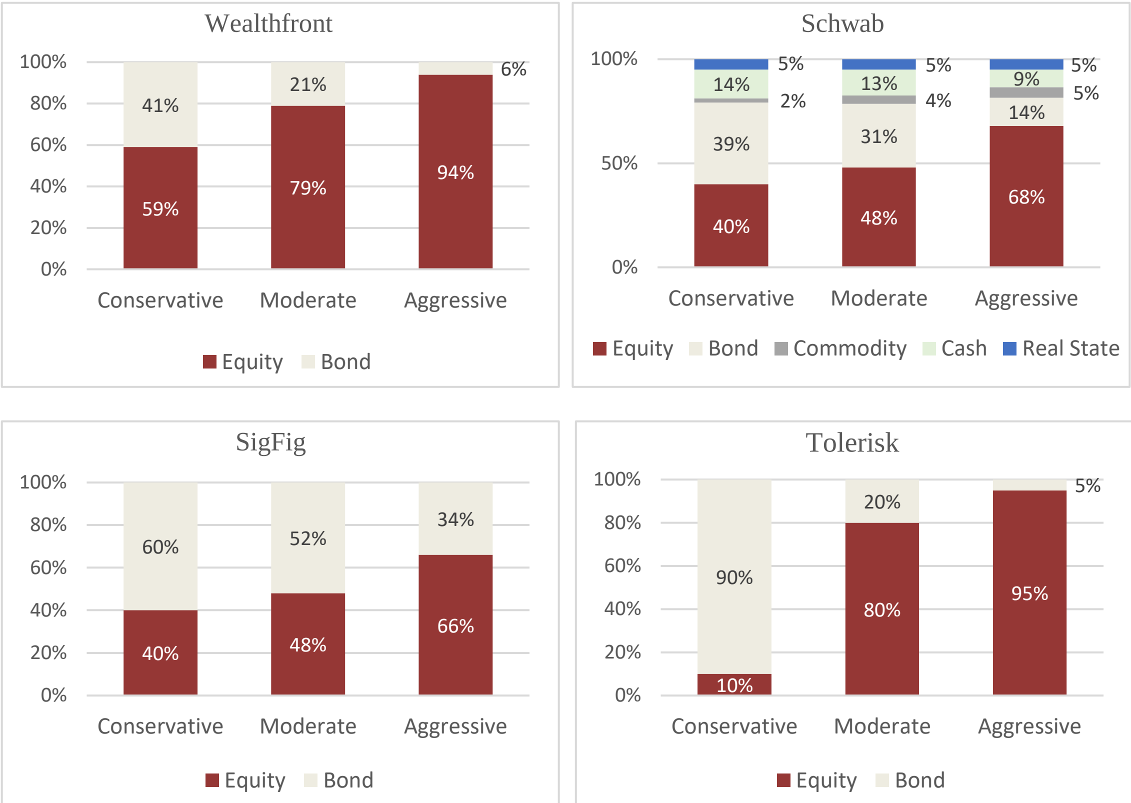
** Replaced VNRSQ – Vanguard Natural Resources LLC*

TABLE 3 – ROBO PORTFOLIOS ALLOCATION

Asset	Asset Class	WealthFront			Schwab			SigFig			Tolerisk		
		C	M	A	C	M	A	C	M	A	C	M	A
IGOV	Bond	-	-	-	5%	5%	3%	-	-	-	-	-	-
SPMB	Bond	-	-	-	11%	9%	3%	-	-	-	-	-	-
STIP	Bond	6%	0%	0%	5%	1%	0%	4%	12%	24%	-	-	-
TFI	Bond	35%	21%	6%	-	-	-	14%	10%	7%	-	-	-
VCIT	Bond	-	-	-	6%	6%	1%	22%	0%	0%	90%	20%	5%
VGIT	Bond	-	-	-	8%	6%	0%	0%	0%	0%	-	-	-
VWOB	Bond	-	-	-	4%	4%	7%	20%	30%	3%	-	-	-
Cash	Cash	-	-	-	14%	13%	9%	-	-	-	-	-	-
DGL	Commodity	-	-	-	2%	4%	5%	-	-	-	-	-	-
IEG	Equity	6%	5%	5%	-	-	-	-	-	-	-	-	-
IEMG	Equity	6%	15%	19%	2%	5%	3%	13%	35%	41%	-	-	-
PDN	Equity	-	-	-	2%	3%	4%	-	-	-	-	-	-
PRF	Equity	-	-	-	7%	8%	11%	-	-	-	-	-	-
PRFZ	Equity	-	-	-	4%	5%	8%	-	-	-	-	-	-
PXF	Equity	-	-	-	5%	5%	8%	-	-	-	-	-	-
PXH	Equity	-	-	-	2%	0%	5%	-	-	-	-	-	-
SPY	Equity	27%	35%	35%	-	-	-	5%	13%	25%	10%	80%	95%
VB	Equity	-	-	-	2%	3%	4%	-	-	-	-	-	-
VEA	Equity	12%	18%	25%	3%	4%	5%	22%	0%	0%	-	-	-
VOO	Equity	-	-	-	4%	5%	9%	-	-	-	-	-	-
VSS	Equity	-	-	-	1%	2%	3%	-	-	-	-	-	-
VYM	Equity	8%	6%	10%	8%	8%	8%	-	-	-	-	-	-
VNQ	Real State	-	-	-	3%	3%	3%	-	-	-	-	-	-
VNQI	Real State	-	-	-	2%	2%	2%	-	-	-	-	-	-

** C – Conservative, M – Moderate, A-Aggressive*

FIGURE 1 – ALLOCATION OF ROBO PORTFOLIOS PER ASSET CLASS



** Portfolio allocations provided on 31st March 2017 by the Robo-advisors.*

FIGURE 2 – HISTORICAL IN-SAMPLE EFFICIENCY OF ALL ROBO ASSETS ON A MEAN-VARIANCE SPACE

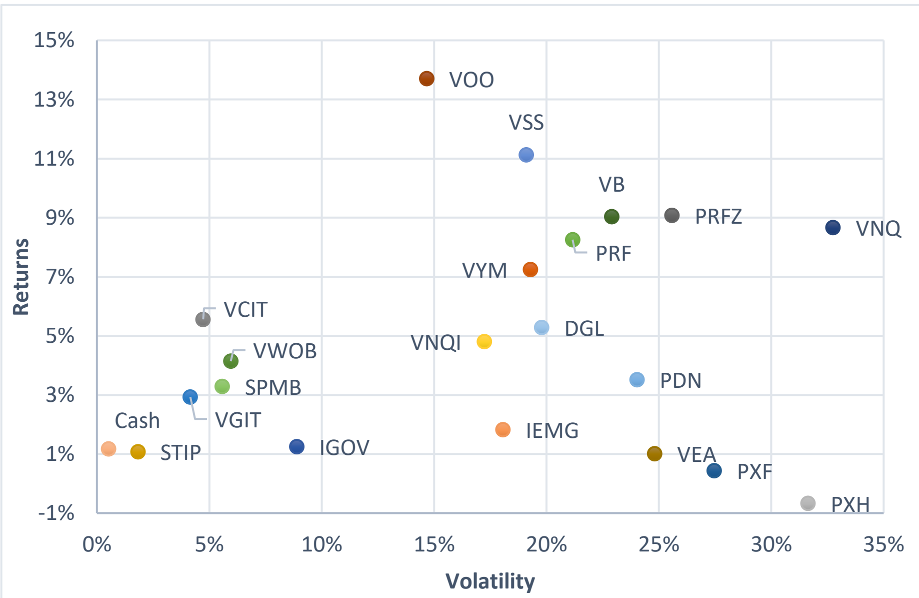


TABLE 4 – HISTORICAL IN-SAMPLE EFFICIENCY OF ALL ROBO ASSETS

Asset	Asset Class	Average Return	Volatility	SR
IGOV	Bond	1.25%	8.89%	-0.08
SPMB	Bond	3.29%	5.58%	0.24
STIP	Bond	1.08%	1.82%	-0.47
TFI	Bond	3.98%	6.73%	0.30
VCIT	Bond	5.56%	4.73%	0.77
VGIT	Bond	2.93%	4.16%	0.24
VWOB	Bond	4.14%	5.95%	0.37
Cash	Cash	1.18%	0.52%	-1.44
DGL	Commodity	5.29%	19.78%	0.17
IEG	Equity	6.78%	27.79%	0.17
IEMG	Equity	1.83%	18.07%	-0.01
PDN	Equity	3.52%	24.03%	0.07
PRF	Equity	8.26%	21.16%	0.30
PRFZ	Equity	9.08%	25.58%	0.28
PXF	Equity	0.44%	27.46%	-0.05
PXH	Equity	-0.67%	31.64%	-0.08
SPY	Equity	8.84%	18.60%	0.37
VB	Equity	9.03%	22.91%	0.31
VEA	Equity	1.01%	24.81%	-0.04
VOO	Equity	13.70%	14.67%	0.80
VSS	Equity	11.12%	19.11%	0.48
VYM	Equity	7.24%	19.28%	0.28
VNQ	Real State	8.66%	32.75%	0.21
VNQI	Real State	4.80%	17.24%	0.17

We consider a risk-free rate (RF) of 1.93% as it was the U.S. 5-year Treasury on the day that the portfolios were defined, 31st March 2017. A 5-year rate was chosen to match exactly with the investment horizon proposed.

For the cash deposits present on Schwab portfolio, we extracted data from the Federal Reserve Bank of Saint Louis for national rate on jumbo deposits (greater or equal to USD 100,000). The collected national rates consider an investment horizon of sixty months (5-year period) with annual rates provided weekly. The rates are afterward converted into daily data to match the ETFs daily returns.

5. METHODOLOGY

Plainly, there is no absolute consensus in the literature or market about the best and more efficient methodology to be applied in portfolio management. Several approaches can be used on the determination of a portfolio allocation, varying in terms of assumptions and/or inputs required.

The Mean-Variance Theory (MVT) developed by Markowitz (1952, 1959) has been the basis of modern portfolio theory being also widely used by advisory companies as a leading method in their recommendations. Despite its popularity, the theory has also some drawbacks. The more important of them is related to estimation errors or uncertainty in the parameters. In particular, MVT relies on the expected returns that, in practice, are very difficult to estimate. This may lead to theoretical optimal portfolios. For further details on this matter see DeMiguel, et al. (2009); Michaud (1989); Litterman (2003); Black & Litterman (1992) or the more pedagogically oriented work of Cardoso and Gaspar (2018).

In this study, we use Mean-Variance Theory (MVT) to determine in-sample efficient frontiers (EFs), as well as *Minimum-Variance* (MV), *Tangent* (T) and *Kataoka* (K) portfolios without short selling. Naïve *Homogeneous* portfolios (H) are also analyzed. The idea is to compare the composition of these portfolios with the real allocations of the twelve robo portfolios proposed on 31st March 2017. Our analysis is based upon the same set of basic assets used by robo-advisors and it relies on the historical inputs presented on Chapter 4. We then compare the perform of out-of-samples estimates of the twelve robo portfolios with the fifteen MVT (MV, T and K) portfolios and five H portfolios, from March 2017 until December 2019.

5.1. Estimation of Mean-Variance inputs

Using the historical approach, the daily prices before 31st March 2017 are used to estimate the mean-variance inputs i.e. the vector of expected returns and the variance covariance matrix. Daily prices are utilized in this study aiming to better reflect the market trends during the period analyzed and avoid that return events occurring within the long sampling are obscured, which could confound the estimates.

This estimative is conducted for the subset of ETFs used by each robo (presented on Table 3). We consider also a fifth set, which englobes the twenty-four ETFs, named as the Blended set. The goal is to use that set, more diversified, to compare against the results of the original subsets of the four robo-advisors.

For robustness check we use two different historical timeframes for estimative: a maximum timeframe (with all historical data available for each ETF until the 31st March 2017 in the market) and a 5-year timeframe (from 31st March 2012 until 31st March 2017). The aim with this check is to understand the effect that the timeframe assumption can have on the estimates. At Table 4 presented previously on Chapter 4 were presented the inputs and Sharpe Ratios of the assets (ETFs and cash allocation) and at this Chapter, on Table 5, are presented the historical performance of the robo portfolios based on these inputs. The results show that, for the 5-year timeframe, the historical returns and SRs are higher than the ones for the maximum timeframe available (MTA) while the volatilities are lower. The only exceptions are the *Conservative* portfolios of SigFig and Tolerisk, for which returns are slightly lower over the 5-year period. These differences found are important to show the impact that the timeframe used can impact the inputs used on the portfolio management.

TABLE 5 – HISTORICAL INPUTS FOR THE ROBO PORTFOLIOS

Portfolio		Average Return		Volatility		Sharpe Ratio	
		Max	5y	Max	5y	Max	5y
WealthFront	C	5.06%	6.09%	14.76%	9.98%	0.21	0.42
	M	5.16%	6.97%	17.65%	12.48%	0.18	0.40
	A	5.00%	7.55%	19.87%	14.20%	0.15	0.40
Schwab	C	4.35%	4.87%	12.68%	8.58%	0.19	0.34
	M	4.88%	5.38%	14.18%	9.78%	0.21	0.35
	A	5.58%	6.67%	18.46%	12.48%	0.20	0.38
SigFig	C	3.88%	3.62%	7.60%	5.70%	0.26	0.30
	M	4.19%	5.80%	13.05%	9.73%	0.17	0.40
	A	4.64%	7.31%	18.64%	13.88%	0.15	0.39
Tolerisk	C	5.89%	4.85%	6.11%	5.13%	0.65	0.57
	M	8.18%	10.71%	15.83%	11.03%	0.39	0.80
	A	8.67%	11.97%	17.91%	12.29%	0.38	0.82

** C – Conservative, M – Moderate, A-Aggressive*

At the end of 2011 and beginning of 2012, the markets were greatly affected by the solvency crisis, experiencing large drops in the asset prices. Some assets covered by these portfolios suffered with highly negative annual returns, especially the ones connected to commodities and equities of emerging markets. That fact impacted too much the returns in this 5-year period and for that reason we opted to proceed the study only with the MTA estimates, that are the only ones for which the analysis are presented in the main text.

With the data presented at Table 5 for the MTA were estimated the correlations between assets and, afterwards, the variance-covariance matrices for the five set of assets (four robo-advisors plus the Blended set). The variance-covariance matrix for the whole set of assets involved in this study is presented in the Appendix at Figure A. 1.

5.2. MVT portfolios and efficient frontiers

To analyze the portfolios´ performance, we consider a scenario where is possible to deposit money at the risk-free rate, but there is no credit to invest in risky assets and short selling is also not allowed (SSNA), considering that leverage is not possible with the robo-advisors. Das, et al. (2010) points out to the fact that portfolios with the constraint of short selling not allowed can lead to minor reductions in efficiency relative on the optimal portfolios, but that this loss of efficiency is small relative to the damage that investors can suffer due to this increased risk.

To find the *Tangent* portfolios (T), considering the risk-free rate and according to the MVT, we obtain the optimal allocations for each portfolio through SR maximization. For the *Minimum-Variance* portfolios, we minimize the volatility. This portfolio stands out because it minimizes risk without requiring estimative of the expected return, variance and covariance inputs, as opposed as the other portfolios at the EF. That fact also turns this into a very conservative portfolio and there are some concerns about value and small-size bias, as discussed in Clarke, et al. (2006).

It is a well-known fact that investors normally treat losses and gains asymmetrically, with several studies already discussing the tendency of loss aversion (i.e. Kahneman et al, 1990). Many investors then face a downside risk bias because of the great concern about the risk of getting lower return than the expected one. Based one that theory, some robo-advisors use downside risk models combined with MVT to maximize the returns respecting a downside risk constraint, what can lead to overly conservative portfolios though. That is the case, for instance, of Betterment, one of the oldest and largest robo advisors in the world, that considers a margin of safety for its portfolios, focusing on the 5^{th} percentile performance of each portfolio on the EF (Betterment, 2014).

The Kataoka criterion is one of the possible safety criteria typically used by advisors to take into account specific investor concerns on loss aversions. It consists in choosing the portfolio with the maximum possible “minimal return” (R_{Min}) under the constraint of limiting the probability that the portfolio return is lower than that R_{Min} (Pringent & Toumi, 2005). By applying this criterion, we are ensuring that the probability of loss will not exceed a given level α (defined percentile for the outcomes). For that approach, we use the concept of Return at Risk (RaR).

There are other safety criteria that could have been used in this study, such as the Roy criterion, which focuses on the level of probability, aiming to minimize the probability of losses applying the constraint of minimal return guaranteed. However, our research of the robo-advisors methods points out to more concern towards the fixation of a percentile to establish the portfolio with the highest return possible (Betterment, 2014).

With that in mind, we study here also Kataoka safety criterion, choosing portfolios that are defined with this exact constraint of the 5^{th} percentile for the worse outcome as mentioned in Betterment report. The idea is to find for each of the four robo-advisors and the Blended set, the *Kataoka* portfolio whose allocation provides the lowest expected losses for a 5% pre-determined confidence level (α=5%). Given this constraint, we minimize RaR.

Besides the MVT portfolios, it was also included in this study the *Homogeneous* portfolios (H) since it is a very common strategy used by investors, mainly individual ones, to distribute their wealth using a diversification approach, with that being confirmed through several different studies. See, for instance, Benartzi and Thaler (2001) or Huberman and Jiang (2006). It is also used as benchmark in the literature for performance analysis. This type of portfolio merely divides equally the investments using a weight distribution of 1/N, being N the number of assets present in the portfolio and its main advantage is that does not involve any input estimative.

It is debatable if this is a good strategy though, since investors opt for a portfolio that is not theoretically efficient nor matching their risk profile. There are several studies in the literature on whether the naïve diversification, in practice, presents itself as a more effective method compared to other strategies theoretically more efficient. Well know studies are the ones of DeMiguel, et al. (2009); You and Zhou (2011); Kirby and Ostdiek (2012); Allen, et al. (2014b) and Goetzmann and Ukhov (2006).

In this study, including the naïve diversification, it is possible for us to check whether this is a more efficient strategy as opposed to optimization models like the one proposed by Markowitz and its efficiency against the portfolios provided by the robo-advisors. The same methodology applied for the robo portfolios (estimation of MVT inputs and SR) is executed for their correspondent *Homogeneous* portfolios, considering the Robo´s set of assets with homogeneous allocation. Table 6 summarizes all portfolios analyzed in this study.

TABLE 6 – PORTFOLIOS ANALYZED

Portfolios analyzed	WealthFront	Schwab	SigFig	Tolerisk	Blended
Conservative	✓	✓	✓	✓	
Moderate	✓	✓	✓	✓	
Aggressive	✓	✓	✓	✓	
Homogeneous	✓	✓	✓	✓	✓
Minimal Variance	✓	✓	✓	✓	✓
Tangent	✓	✓	✓	✓	✓
Kataoka	✓	✓	✓	✓	✓

5.3. Out-of-sample performance analysis

On this part of the analysis we consider an initial fictional investment of USD 100.00 on the 31st March 2017 and analyze the evolution until the end of 2019 of the thirty-two portfolios on

Table 6. Unfortunately, it is not possible to evaluate the performance for the entire investment horizon of five years as the investment finishes only in 2022. The data for the year of 2020 is disregarded since the higher volatilities in this year as consequence of the economic crisis would disturb the results and could affect our interpretation of these portfolios. The analysis of behaviour during stress times is not the objective of this thesis.

We focus then on the performance of the investment for about half of the investment period and compare actual performance with the expected one estimated in 2017, using the in-sample inputs. This out-of-sample analysis is performed with rebalancing and without rebalancing to compare the effect of the rebalancing on the performance. In this study we chose to consider only monthly rebalancing, as recommended by Arnott and Lovell (1993) to investors with a long investment horizon. As the objective of this study is not focused on the effects of rebalancing on portfolios, are not considered different frequencies other than the monthly one.

6. RESULTS

The present chapter addresses the results obtained by applying the methodology explained in the previous chapter to the data gathered. It is divided into two main sections. First, Section 6.1 covers the results obtained for the in-sample period. This section is subdivided in two subsections. Subsection 6.1.2 introduces the proposed portfolios allocations. Subsection 6.1.1 shows the historical performance of all portfolios analyzed for the in-sample period. On the second part, Section 6.2, we cover the results obtained for the out-of-sample period (March 2017 to December 2019) with the performances of all portfolios evaluated and comparisons between their expected versus actual performance.

6.1. In-sample period

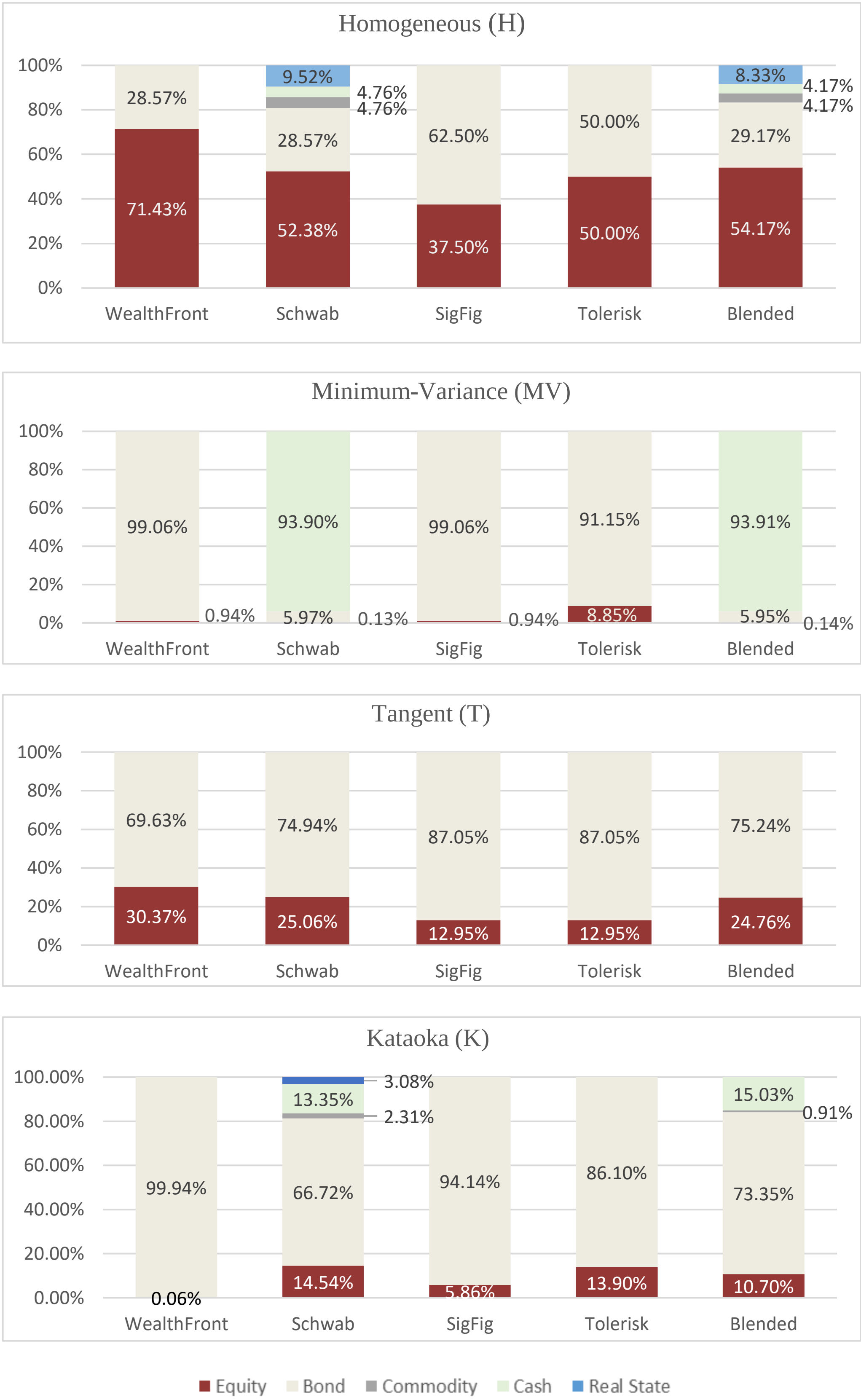
In this section, the inputs considered in the MVT analysis (expected returns and volatility), the Sharpe Ratio (SR), the RAR for each portfolio are shown. Additionally, are presented the portfolio allocations defined for the *Homogeneous* (H), *Minimum-Variance* (MV), *Tangent* (T) and *Kataoka* (K) portfolios.

6.1.1. Proposed portfolio allocations

In chapter 4 at Figure 1, were previously presented the robo-portfolios allocation per asset class based on the allocations provided by the companies with the details of allocations per asset presented in Table 3. At this section, at Figure 2, are presented the proposed portfolio allocations per asset class for all the proposed portfolios. The more detailed allocation per asset for them is presented in the Appendix at Table A. 1.

Note that the *Homogeneous* portfolios allocate the same weight to all ETFs, independent of the asset class, tending to be more concentrated in equities. It is also important to point out that the optimization of the SRs led to poorly diversified *Tangent* portfolios composed by only two assets for the robo-advisors and three for the Blended set that have twenty-four assets available. Each portfolio is composed by one Equity ETF and one or two Bond ETF with the Bonds representing more than 69% of their allocation. The Equity ETFs indicated are extremely similar as they both have the S&P 500 as benchmark. The *Kataoka* portfolios also have very high allocations in Bonds (more than 66%) and for WealthFront set of ETFs, even with 7 possible assets, the allocation led to basically one asset in the portfolio (representing more than 99%).

FIGURE 2 - PORTFOLIOS ALLOCATION PER ASSET CLASS



** Allocations defined using in-sample data (until 31ST March 2017).*

6.1.2. Historical Performance

Table 7 presents all the historical performances estimated for the in-sample period, including the results already presented for the robo-portfolios for the MTA. Figure 3 shows in a mean-variance space these historical performances presented in Table 5.

As expected, the higher the risk profile of robo portfolios, higher the expected returns and volatility. However, most of these *Aggressive* portfolios have lower SRs than the *Conservative* ones, which, although consistent across the four robos, seems to be unreasonable. As expected, in-sample *Tangent* portfolios present the highest SRs, since the MVT optimization aims exactly that, to maximize SR as a way to insure the combination of risky assets with the risk-free investment with the higher returns for the considered level of volatility.

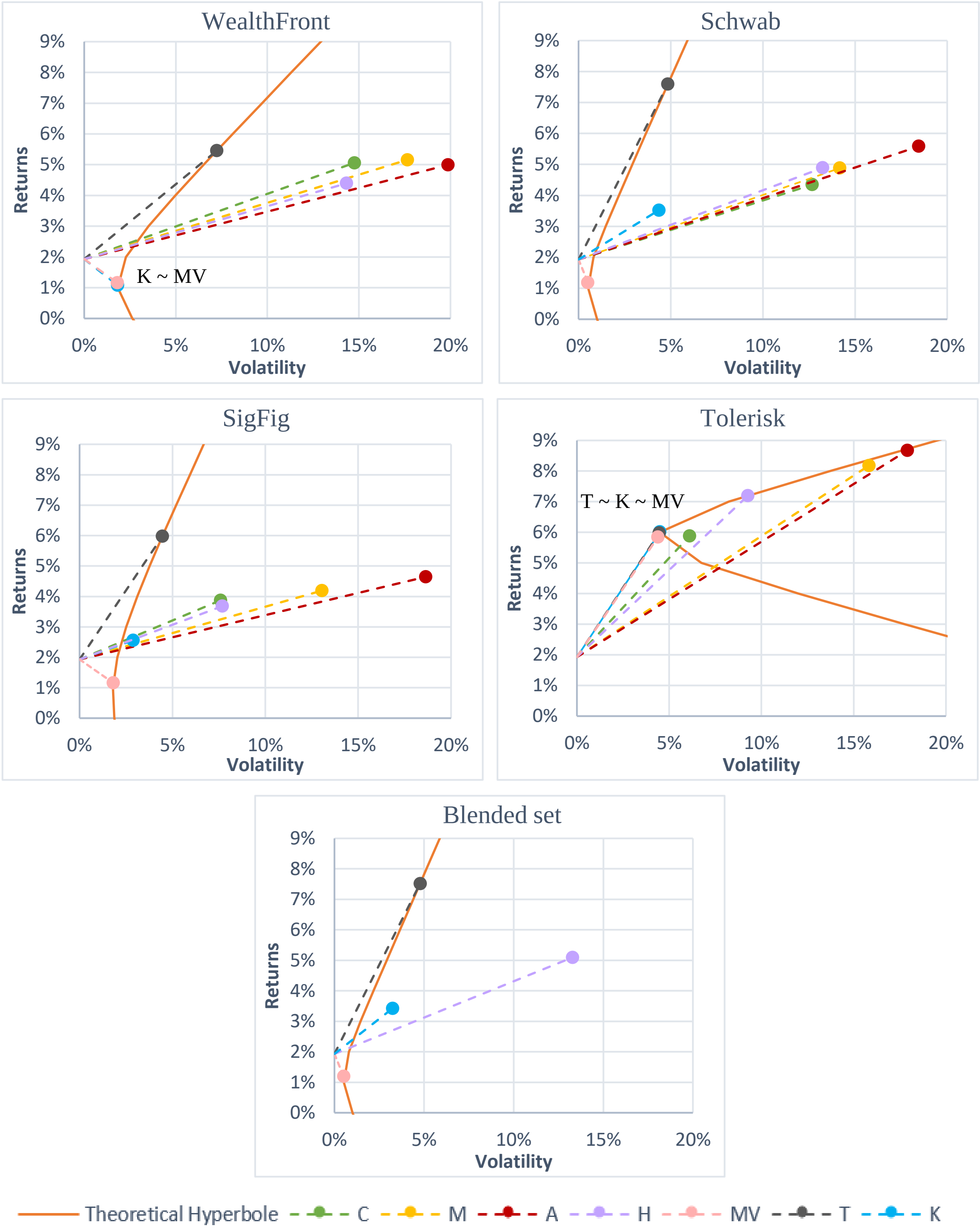
TABLE 7 – HISTORICAL IN-SAMPLE PORTFOLIOS´ PERFORMANCE FOR SSNA

Portfolio	Expected Returns (%)					Volatilities (%)				
	WealthFront	Schwab	SigFig	Tolerisk	Blended	WealthFront	Schwab	SigFig	Tolerisk	Blended
C	5.06	4.35	3.88	5.89	-	14.76	12.68	7.60	6.11	-
M	5.16	4.88	4.19	8.18	-	17.65	14.18	13.05	15.83	-
A	5.00	5.58	4.64	8.67	-	19.87	18.46	18.64	17.91	-
H	4.39	4.89	3.68	7.20	5.10	14.33	13.24	7.69	9.27	13.29
MV	1.16	1.19	1.16	5.85	1.19	1.82	0.51	1.82	4.39	0.51
T	5.45	7.60	5.98	5.98	7.52	7.24	4.85	4.46	4.46	4.78
K	1.08	3.52	2.56	6.01	3.42	1.82	4.36	2.89	4.50	3.24
Portfolio	Sharpe Ratios					RaRs (%)				
	WealthFront	Schwab	SigFig	Tolerisk	Blended	WealthFront	Schwab	SigFig	Tolerisk	Blended
C	0.21	0.19	0.26	0.65	-	-0.79	-0.60	-0.33	-0.41	-
M	0.18	0.21	0.17	0.39	-	-1.07	-0.71	-0.88	-1.25	-
A	0.15	0.20	0.15	0.38	-	-1.31	-1.05	-1.36	-1.46	-
H	0.17	0.22	0.23	0.57	0.24	-1.09	-0.87	-0.61	-0.73	-0.91
MV	-0.42	-1.45	-0.42	0.89	-1.44	-0.15	-0.01	-0.15	-0.41	-2.05
T	0.49	1.17	0.91	0.91	1.17	-0.41	-0.42	-0.39	-0.39	-0.41
K	-0.47	0.37	0.22	0.91	0.46	-0.15	-0.31	-0.18	-0.39	-0.23

** Inputs estimated using MTA.*

*** C – Conservative, M – Moderate, A-Aggressive, H – Homogeneous, MV – Minimum-Variance, T – Tangent (MVT), K – Kataoka*

FIGURE 3 - HISTORICAL IN-SAMPLE PORTFOLIOS´ PERFORMANCE FOR SSNA



** C–Conservative, M–Moderate, A-Aggressive, H–Homogeneous, MV–Minimum-Variance, T–Tangent, K–Kataoka*

*** Slopes of lines connecting RF and the various portfolios indicate the Sharpe Ratio (SR).*

**** Very similar performance for Tolerisk´s T, K and MV (difficult to distinguish in the graphic).*

***** Theoretical Hyperbole:* $\sqrt{\frac{(A*\mathbb{R}^2)-(2*B*\mathbb{R})+C}{(A*C)-B^2}}$, $A = \mathbb{1}' * \mathbb{V}^{-1} * \mathbb{1}$, $B = \mathbb{1}' * \mathbb{V}^{-1} * \mathbb{R}$, $C = \mathbb{R}' * \mathbb{V}^{-1} * \mathbb{R}$; *being* $\mathbb{V}$ *- Covariance matrix,* $\mathbb{R}$ *- Return matrix and* $\mathbb{1}$ *- Unitary matrix.*

6.2. *Out-of-sample period*

The out-of-sample performance of the twelve robo advisor portfolios, the five *Tangent* portfolios, the five naïve *Homogeneous* portfolios and the five *Kataoka* portfolios are presented at Table 8. Are shown the total portfolio value on 31st December 2019, the annual returns and volatility observed during this period and calculated the SR of each portfolio.

Figure 4 shows graphically the expected performance and the real performance at the end of 2019 on the mean-variance space. The expected ones are based on the historical data collected for the assets embedded by each portfolio and showed previously on the Table 7. On the graphics, the filled dots represent the actual performance at the end of 2019 (A) and the leaked ones, the expected performance (E).

As can be seen, for WealthFront, Schwab, SigFig and Blended the real performance overcame the expected one, with higher returns and lower volatilities for all the portfolios. For Tolerisk, the real returns are all higher, but for some portfolios the volatility is also higher than expected.

The portfolios´ performance are presented in graphics for all the thirty-two portfolios studied. The performances are shown as the cumulative portfolio value (in USD) through this entire out-of-sample period. These performances are arranged according to the robo-advisor (Figure 5) but also according to the type of portfolio considered (Figure 6).

All the graphics presented in the below figures represents the portfolios without rebalancing. The graphics for the monthly rebalancing portfolios are presented in the Appendix since the extreme similarity between them and the ones without rebalancing.

TABLE 8 – PERFORMANCE OF PORTFOLIOS ON OUT-OF-SAMPLE PERIOD

Portfolio		RaR	Performance without rebalancing				Performance with monthly rebalancing			
			$ on 31/Dec/2019	Average Return	Volatility	Sharpe Ratio	$ on 31/Dec/2019	Average Return	Volatility	Sharpe Ratio
WealthFront	C	-0.75%	120.73	7.56%	7.41%	0.76	120.89	7.62%	7.13%	0.80
	M	-1.07%	124.11	8.79%	10.01%	0.69	124.23	8.84%	9.81%	0.70
	A	-1.28%	125.46	9.29%	11.78%	0.62	125.40	9.26%	11.70%	0.63
	H	-0.99%	116.56	6.04%	9.05%	0.45	116.72	6.10%	8.90%	0.47
	MV	-0.13%	105.78	2.11%	1.28%	0.14	105.76	2.10%	1.29%	0.13
	T	-0.43%	120.06	7.31%	5.51%	0.98	120.44	7.45%	5.27%	1.05
	K	-0.13%	105.44	1.99%	1.31%	0.04	105.44	1.99%	1.31%	0.04
Schwab	C	-0.51%	115.55	5.67%	5.37%	0.70	116.04	5.85%	5.10%	0.77
	M	-0.61%	116.94	6.18%	6.36%	0.67	117.52	6.39%	6.09%	0.73
	A	-0.91%	120.06	7.32%	8.87%	0.61	120.67	7.54%	8.66%	0.65
	H	-0.74%	117.76	6.48%	7.39%	0.62	118.50	6.75%	7.15%	0.67
	MV	0.003%	103.13	1.14%	0.08%	-9.96	103.12	1.14%	0.08%	-10.02
	T	-0.40%	122.59	8.24%	4.10%	1.54	125.58	9.33%	3.84%	1.93
	K	-0.25%	111.74	4.28%	2.71%	0.87	111.94	4.36%	2.59%	0.93
SigFig	C	-0.28%	115.19	5.54%	3.09%	1.17	115.26	5.57%	2.86%	1.27
	M	-0.85%	121.90	7.99%	8.17%	0.74	121.85	7.97%	7.76%	0.78
	A	-1.29%	126.79	9.77%	11.77%	0.67	126.84	9.79%	11.64%	0.68
	H	-0.53%	117.02	6.21%	5.27%	0.81	117.37	6.34%	4.96%	0.89
	MV	-0.13%	105.78	2.11%	1.28%	0.14	105.76	2.10%	1.29%	0.13
	T	-0.31%	124.17	8.81%	4.93%	1.40	124.30	8.86%	4.58%	1.51
	K	-0.13%	109.91	3.61%	1.46%	1.15	109.80	3.58%	1.42%	1.16
Tolerisk	C	-1.24%	121.53	7.85%	11.73%	0.50	121.44	7.82%	11.73%	0.50
	M	-1.43%	135.97	13.12%	12.73%	0.88	135.83	13.07%	12.70%	0.88
	A	-1.46%	139.07	14.25%	13.15%	0.94	139.03	14.23%	13.14%	0.94
	H	-1.33%	129.78	10.86%	12.09%	0.74	129.55	10.78%	12.05%	0.73
	MV	-1.24%	121.29	7.76%	11.73%	0.50	121.21	7.74%	11.73%	0.50
	T	-1.23%	127.39	9.99%	11.92%	0.68	127.17	9.91%	11.89%	0.67
	K	-1.24%	122.33	8.14%	11.74%	0.53	122.22	8.10%	11.73%	0.53
Blended	H	-0.75%	117.40	6.35%	7.50%	0.59	118.04	6.58%	7.29%	0.64
	MV	0.003%	103.14	1.15%	0.08%	-9.98	103.13	1.14%	0.08%	-10.05
	T	-0.40%	122.37	8.16%	4.02%	1.55	125.25	9.21%	3.76%	1.93
	K	-0.18%	112.01	4.38%	2.04%	1.20	112.03	4.39%	1.95%	1.26

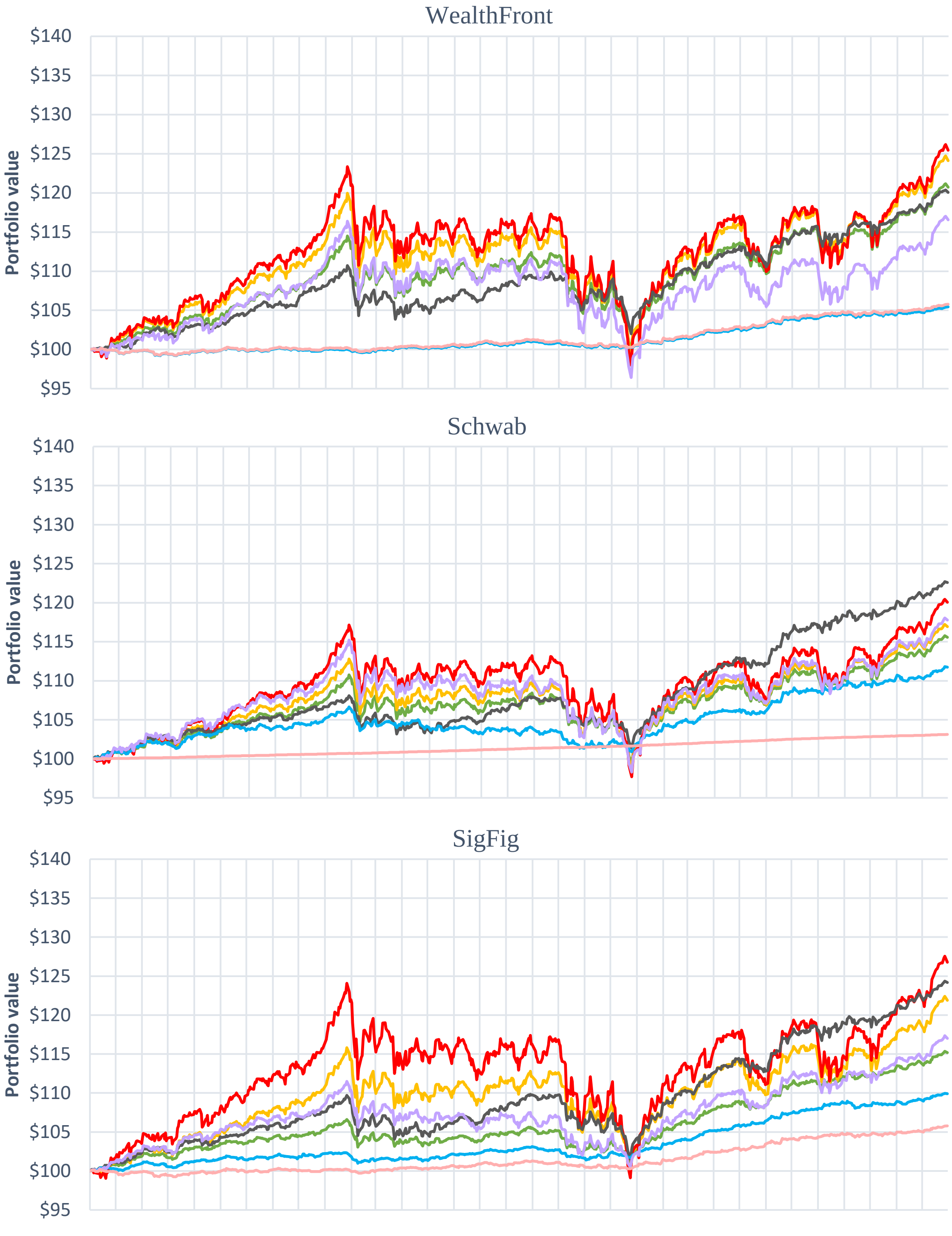
FIGURE 4 – EXPECTED X ACTUAL PERFORMANCE OF PORTFOLIOS

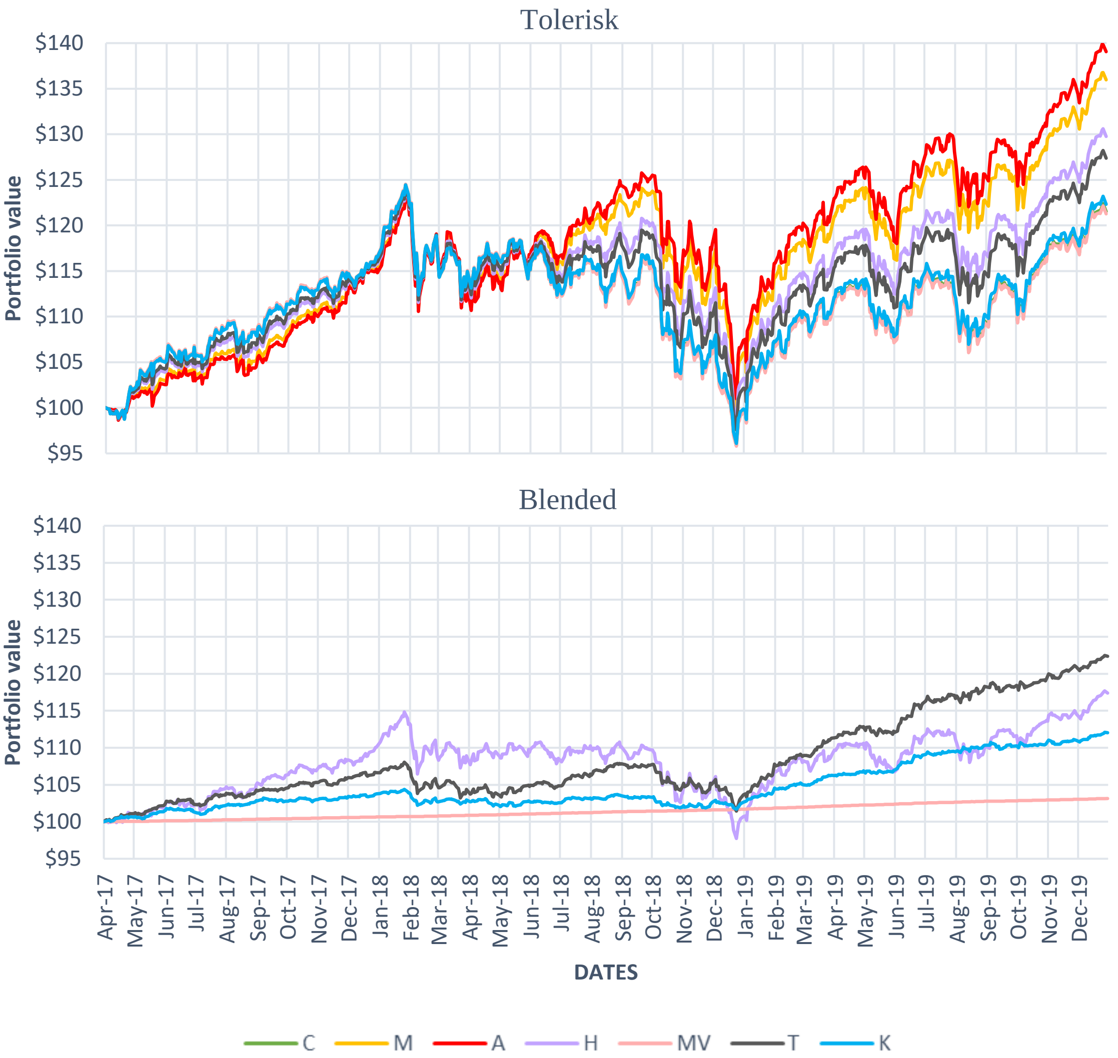
** E ( ) – Expected performance (leaked dots), A ( ) - Actual performance (filled dots)*

*** C–Conservative, M–Moderate, A-Aggressive, H–Homogeneous, MV–Minimum-Variance, T–Tangent, K–Kataoka*

**** For Tolerisk: very similar values for Expected performance of T and K portfolios and for Actual performance of C and MV portfolios.*

FIGURE 5 – CUMULATIVE PERFORMANCE PRESENTED BY SET OF ASSETS (ROBOS AND BLENDED SET)

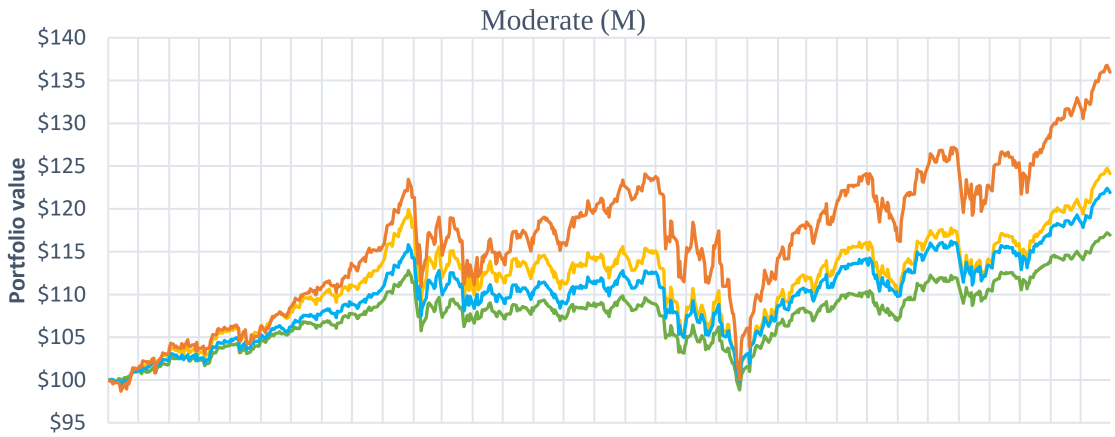


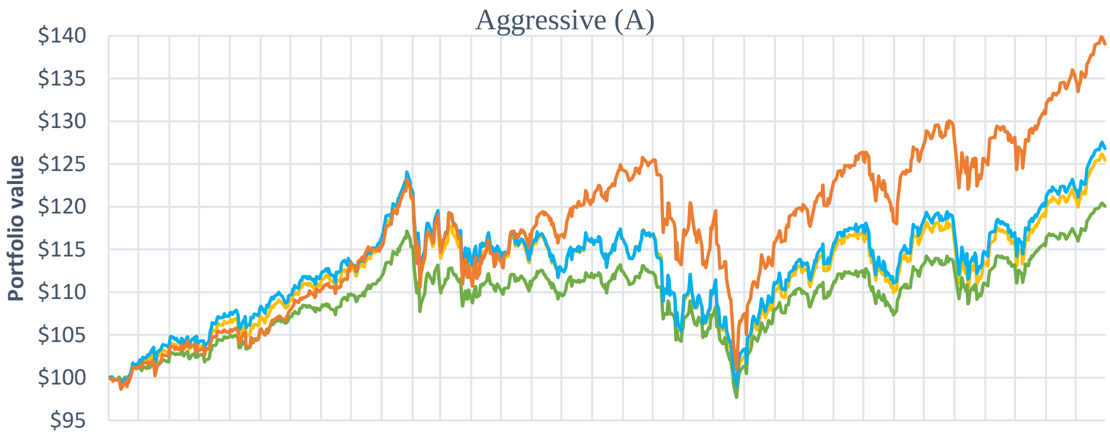


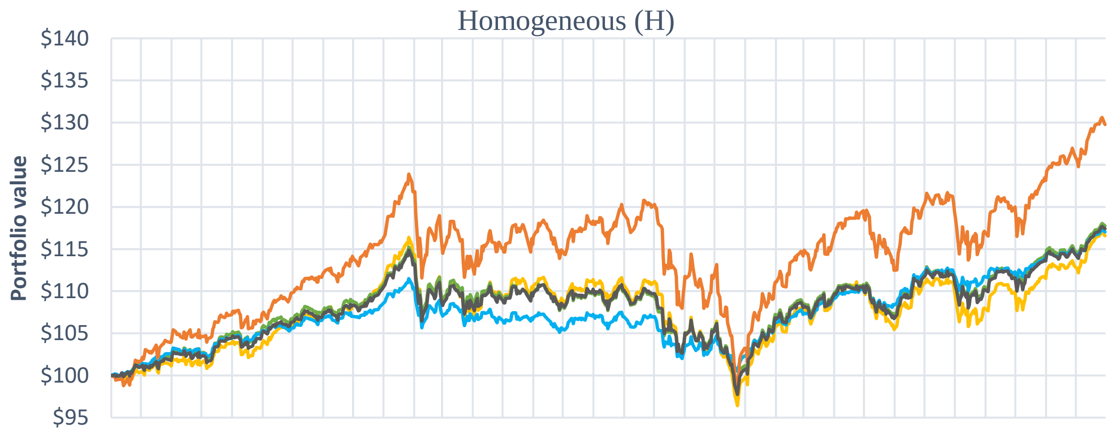
** Cumulative performance without considering rebalancing of any kind.*

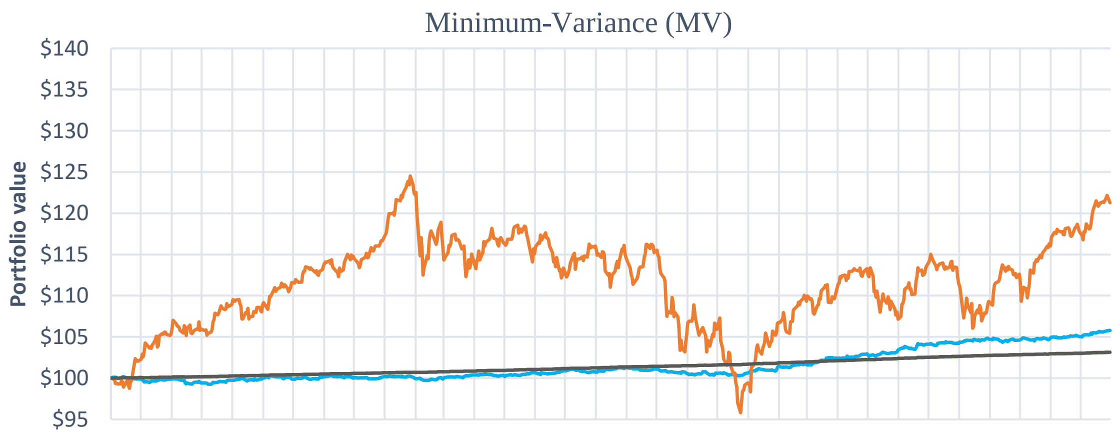
*** C–Conservative, M–Moderate, A-Aggressive, H–Homogeneous, MV–Minimum-Variance, T–Tangent, K–Kataoka*

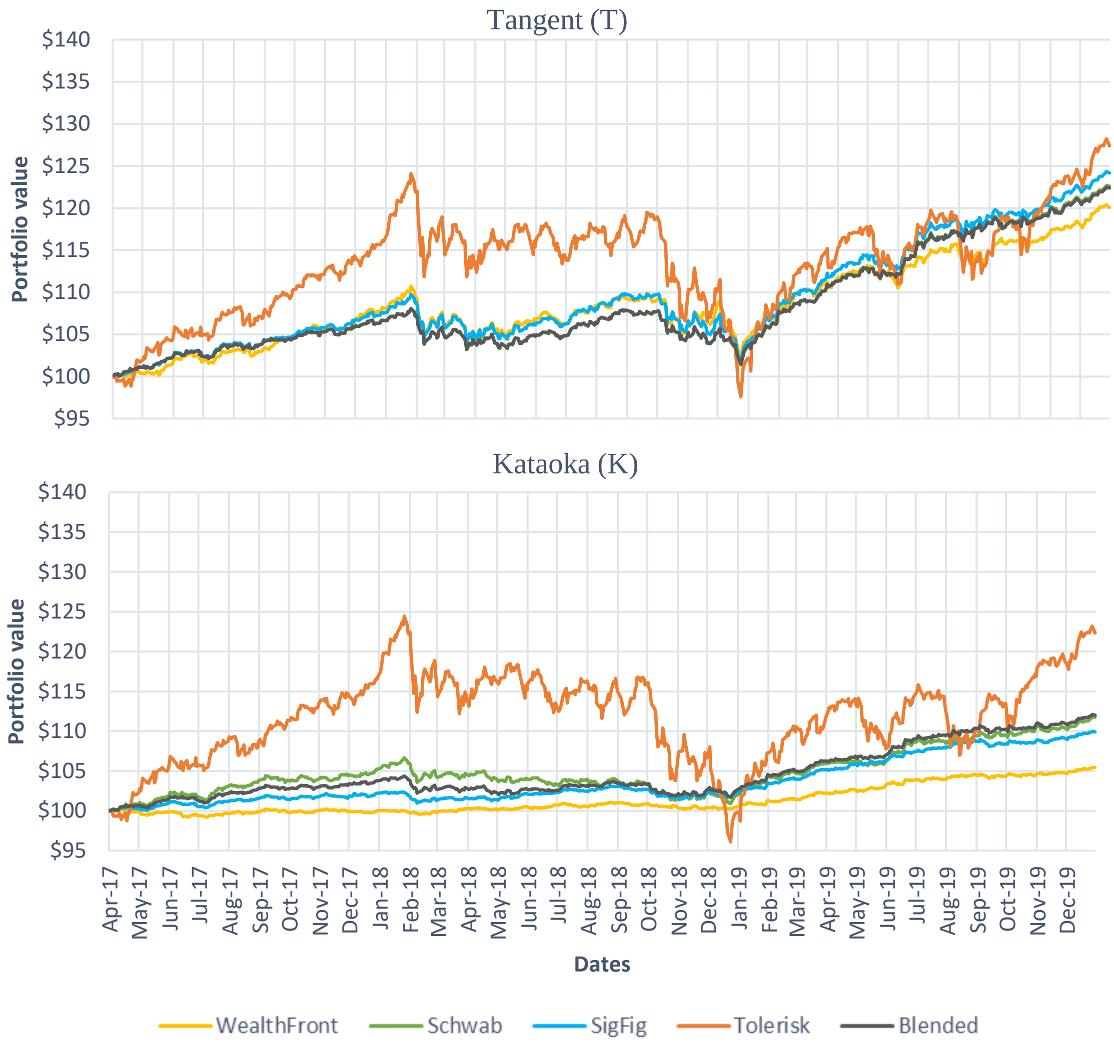
FIGURE 6 – CUMULATIVE PERFORMANCE PRESENTED BY TYPE OF PORTFOLIO


Moderate (M)
Portfolio value
$140
$135
$130
$125
$120
$115
$110
$105
$100
$95


Aggressive (A)
Portfolio value
$140
$135
$130
$125
$120
$115
$110
$105
$100
$95


Homogeneous (H)
Portfolio value
$140
$135
$130
$125
$120
$115
$110
$105
$100
$95


Minimum-Variance (MV)
Portfolio value
$140
$135
$130
$125
$120
$115
$110
$105
$100
$95



** Cumulative performance without considering rebalancing of any kind.*

Looking into the performance graphics divided by set of assets (Figure 5) is easy to notice that, for all of them, the *Aggressive* portfolios have the highest volatility and the highest cumulative performance over this entire period or, as in the case of Schwab, until the end of December, when all the portfolios faced an expressive downfall. In the case of Schwab, after the recuperation of the portfolios at the beginning of 2019, the *Tangent* portfolio is the one that better performs.

The behaviour of the *Moderate* portfolios follows close to the *Aggressive* one but with lower volatility and returns. The *Conservative* and *Homogeneous* ones have the lowest performance during this period only surpassing the *Kataoka* portfolios that, as expected, are much more stable with low volatility and suffering very low impact during times of stress in the market. The only exception is for the *Kataoka* portfolio of Tolerisk, that experiences large fluctuations and followed the same behaviour as in the *Conservative* portfolio.

The portfolio performance for the Blended set, presented at Figure 5 (e), shows the *Homogeneous* portfolio as the one with higher volatility and, at least until the downfall on December 2018, with the best performance. After that, the *Tangent* portfolio surpasses it showing very good returns.

Through the graphics presented in Figure 6, we can see that for all of them, the best performance observed is of Tolerisk. The *Homogeneous*, *Tangent* and *Kataoka* portfolios have very similar performance for all the robo-advisors, except for Tolerisk that really stands out with higher returns before the decrease in the value in December 2018. It is interesting to draw attention to the difference in the behaviour of the Tolerisk *Kataoka* portfolio in comparison with all the other ones that suffered very few fluctuations throughout the time.

For all the portfolios considered, the RaR for a 5% percentile at risk is estimated and presented on at Table 8. Only Tolerisk presented RaR higher than the ones predicted in the in-sample analysis, for some portfolios more than 3 times the estimated one. It is also very notable the higher RaR of this robo-advisor compared with the all the other ones.

7. Conclusion

We list some of the many benefits that robo-advisors can bring to the financial advisory area and the void that these softwares can fill, not only among retail investors who normally do not have contact with such accessible advisory services, but also between institutional investors in search of cheaper means of investment.

There are many criticisms regarding the restrictive pool of assets that each robo-advisor normally uses in their portfolio management, raising doubts about their ability to really personalize portfolios to each client, as promised. For the four companies analyzed here - WealthFront, Schwab, SigFig and Tolerisk – the portfolios´ assets suggested in 2017 were very similar through the different risk profiles – conservative, moderate and aggressive. The difference in the portfolio between risk profiles is basically the asset classes percentage. For an increase in the investor risk profile, are defined higher allocations in equity and commodities ETFs and lower allocations in bonds ETFs and cash, but the assets choose are kept basically the same.

Through the companies reports and brochures delivered to SEC, is possible to get some information about the investment methodology used in the portfolio allocation but, understandably, the details of the methods and/or assumptions are not totally disclosed.

In this study we compare compositions as well as in-sample and out-of-sample performance of twelve portfolios proposed by robos with standard MVT based portfolios – T, MV, K – and the naïve *Homogeneous* portfolios. In terms of the exposition, is possible to conclude that the robo portfolios allocations diverge a lot from the theoretical efficient portfolios that have much higher concentration in just a few assets. We also notice that higher risk profile portfolios tend to be less efficient than the *Conservative* ones.

For the out-of-sample period (from April 2017 to December 2019), we can see that all the portfolios analyzed performed better than predicted during the in-sample period (until March 2017), with higher returns and lower volatilities. Also, it is important to point out that the *Tangent* portfolios did not really stand out in relation to the robo portfolios, with returns lower than the robo portfolios during most of the almost three years of data analyzed. However, the volatility experienced during the period was lower than the on the robo portfolios (only for SigFig and Tolerisk that the volatility of the *Conservative* portfolio was a little lower than for the *Tangent*).

A discussion always present is whether the investor would be better off with a *Homogeneous* portfolio (easier to allocate and an option that the investors could do by themselves) than with structured and personalized portfolios provided by advisory companies. According to our analysis, the *Homogeneous* portfolio had a good performance for the out-of-sample period but for most of the time its returns were lower than the robos and *Tangent* ones, at least in comparison with the *Moderate* and *Aggressive* portfolios. The *Homogeneous* portfolios performance compared to the rate of return on the risk-free investment falls short. Returns at Risk are also higher than the ones of the *Conservative* and *Tangent* portfolios, what can represent an important downside risk for the investor.

We covered here the evolution of portfolios defined by the robo-advisors in March 2017 for a 5-year investment period. It would be interesting to re-analyze these results at the end of March 2022. That, combined with a deeper study into the rebalancing and adjustment strategies of robo-advisors during crisis periods, would make possible to analyze the behaviour of these portfolios during the current economic crisis caused by the Covid-19 virus in the world.

While an assessment of robo-advisors' approaches is a very interesting research, a deeper comparison between the historical performances of these portfolio management softwares and the traditional human advisory companies would be a noteworthy study. This would allow the comparison between the robos and the next best alternative available on the market, addressing the question of how much better or worse robo-advice is than human advisory.

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APPENDIX

FIGURE A. 1 – VARIANCE-COVARIANCE MATRIX

	PRF	PXF	VOO	PRFZ	VEA	VNQ	VB	PDN	IEMG	PXH	VNQI	VSS	SPMB	VGIT	VYM	VCIT	STIP	IGOV	VWOB	DGL	SPY	TFI	IEG	Cash
PRF	0.0448	0.0480	0.0306	0.0504	0.0465	0.0562	0.0457	0.0386	0.0293	0.0556	0.0302	0.0352	-0.0006	-0.0037	0.0394	-0.0014	0.0000	0.0031	0.0042	0.0013	0.0382	-0.0002	0.0463	0.0000
PXF	0.0480	0.0754	0.0352	0.0546	0.0624	0.0573	0.0493	0.0547	0.0408	0.0696	0.0414	0.0485	-0.0011	-0.0042	0.0417	-0.0014	0.0003	0.0082	0.0068	0.0063	0.0425	-0.0009	0.0575	0.0000
VOO	0.0306	0.0352	0.0215	0.0340	0.0323	0.0355	0.0314	0.0283	0.0202	0.0367	0.0211	0.0242	-0.0012	-0.0024	0.0276	-0.0007	0.0000	0.0015	0.0029	-0.0002	0.0272	-0.0011	0.0322	0.0000
PRFZ	0.0504	0.0546	0.0340	0.0654	0.0526	0.0669	0.0572	0.0441	0.0316	0.0633	0.0337	0.0404	-0.0006	-0.0042	0.0440	-0.0017	0.0001	0.0035	0.0046	0.0020	0.0430	-0.0005	0.0540	0.0000
VEA	0.0465	0.0624	0.0323	0.0526	0.0615	0.0560	0.0483	0.0506	0.0376	0.0674	0.0387	0.0451	-0.0008	-0.0038	0.0408	-0.0011	0.0003	0.0077	0.0063	0.0060	0.0419	-0.0004	0.0562	0.0000
VNQ	0.0562	0.0573	0.0355	0.0669	0.0560	0.1072	0.0605	0.0452	0.0319	0.0684	0.0393	0.0429	0.0008	-0.0024	0.0515	0.0010	0.0009	0.0059	0.0079	0.0013	0.0470	0.0000	0.0505	0.0001
VB	0.0457	0.0493	0.0314	0.0572	0.0483	0.0605	0.0525	0.0400	0.0291	0.0580	0.0309	0.0369	-0.0006	-0.0038	0.0403	-0.0014	0.0001	0.0032	0.0045	0.0018	0.0395	-0.0004	0.0498	0.0000
PDN	0.0386	0.0547	0.0283	0.0441	0.0506	0.0452	0.0400	0.0577	0.0335	0.0590	0.0351	0.0412	-0.0007	-0.0031	0.0334	-0.0007	0.0005	0.0070	0.0058	0.0071	0.0343	-0.0007	0.0479	0.0000
IEMG	0.0293	0.0408	0.0202	0.0316	0.0376	0.0319	0.0291	0.0335	0.0327	0.0546	0.0263	0.0301	-0.0007	-0.0010	0.0266	0.0006	0.0006	0.0013	0.0054	0.0030	0.0256	-0.0005	0.0330	0.0000
PXH	0.0556	0.0696	0.0367	0.0633	0.0674	0.0684	0.0580	0.0590	0.0546	0.1001	0.0458	0.0529	-0.0010	-0.0039	0.0490	-0.0004	0.0008	0.0072	0.0096	0.0076	0.0496	-0.0004	0.0699	0.0000
VNQI	0.0302	0.0414	0.0211	0.0337	0.0387	0.0393	0.0309	0.0351	0.0263	0.0458	0.0297	0.0298	-0.0007	-0.0019	0.0271	0.0003	0.0004	0.0040	0.0050	0.0036	0.0267	-0.0005	0.0341	0.0000
VSS	0.0352	0.0485	0.0242	0.0404	0.0451	0.0429	0.0369	0.0412	0.0301	0.0529	0.0298	0.0365	-0.0006	-0.0027	0.0315	-0.0005	0.0004	0.0058	0.0053	0.0074	0.0311	-0.0009	0.0435	0.0000
SPMB	-0.0006	-0.0011	-0.0012	-0.0006	-0.0008	0.0008	-0.0006	-0.0007	-0.0007	-0.0010	-0.0007	-0.0006	0.0031	0.0011	-0.0005	0.0012	0.0003	0.0006	0.0007	0.0009	-0.0007	0.0007	-0.0008	0.0000
VGIT	-0.0037	-0.0042	-0.0024	-0.0042	-0.0038	-0.0024	-0.0038	-0.0031	-0.0010	-0.0039	-0.0019	-0.0027	0.0011	0.0017	-0.0032	0.0015	0.0004	0.0010	0.0006	0.0021	-0.0033	0.0011	-0.0037	0.0000
VYM	0.0394	0.0417	0.0276	0.0440	0.0408	0.0515	0.0403	0.0334	0.0266	0.0490	0.0271	0.0315	-0.0005	-0.0032	0.0372	-0.0012	0.0000	0.0028	0.0040	0.0003	0.0341	-0.0004	0.0404	0.0000
VCIT	-0.0014	-0.0014	-0.0007	-0.0017	-0.0011	0.0010	-0.0014	-0.0007	0.0006	-0.0004	0.0003	-0.0005	0.0012	0.0015	-0.0012	0.0022	0.0005	0.0014	0.0011	0.0021	-0.0012	0.0013	-0.0012	0.0000
STIP	0.0000	0.0003	0.0000	0.0001	0.0003	0.0009	0.0001	0.0005	0.0006	0.0008	0.0004	0.0004	0.0003	0.0004	0.0000	0.0005	0.0003	0.0005	0.0004	0.0012	0.0000	0.0003	0.0008	0.0000
IGOV	0.0031	0.0082	0.0015	0.0035	0.0077	0.0059	0.0032	0.0070	0.0013	0.0072	0.0040	0.0058	0.0006	0.0010	0.0028	0.0014	0.0005	0.0079	0.0013	0.0062	0.0027	0.0011	0.0060	0.0000
VWOB	0.0042	0.0068	0.0029	0.0046	0.0063	0.0079	0.0045	0.0058	0.0054	0.0096	0.0050	0.0053	0.0007	0.0006	0.0040	0.0011	0.0004	0.0013	0.0035	0.0018	0.0037	0.0010	0.0059	0.0000
DGL	0.0013	0.0063	-0.0002	0.0020	0.0060	0.0013	0.0018	0.0071	0.0030	0.0076	0.0036	0.0074	0.0009	0.0021	0.0003	0.0021	0.0012	0.0062	0.0018	0.0391	0.0013	-0.0002	0.0152	0.0001
SPY	0.0382	0.0425	0.0272	0.0430	0.0419	0.0470	0.0395	0.0343	0.0256	0.0496	0.0267	0.0311	-0.0007	-0.0033	0.0341	-0.0012	0.0000	0.0027	0.0037	0.0013	0.0346	-0.0003	0.0426	0.0000
TFI	-0.0002	-0.0009	-0.0011	-0.0005	-0.0004	0.0000	-0.0004	-0.0007	-0.0005	-0.0004	-0.0005	-0.0009	0.0007	0.0011	-0.0004	0.0013	0.0003	0.0011	0.0010	-0.0002	-0.0003	0.0045	-0.0005	0.0000
IEG	0.0463	0.0575	0.0322	0.0540	0.0562	0.0505	0.0498	0.0479	0.0330	0.0699	0.0341	0.0435	-0.0008	-0.0037	0.0404	-0.0012	0.0008	0.0060	0.0059	0.0152	0.0426	-0.0005	0.0772	0.0000
Cash	0.00001	0.00001	0.00000	0.00002	0.00001	0.00006	0.00002	0.00002	0.00001	0.00003	-0.00001	0.00002	0.000006	0.000009	0.000009	0.00001	0.000007	0.00001	0.000003	0.000052	0.000001	0.000001	0.000003	0.0000

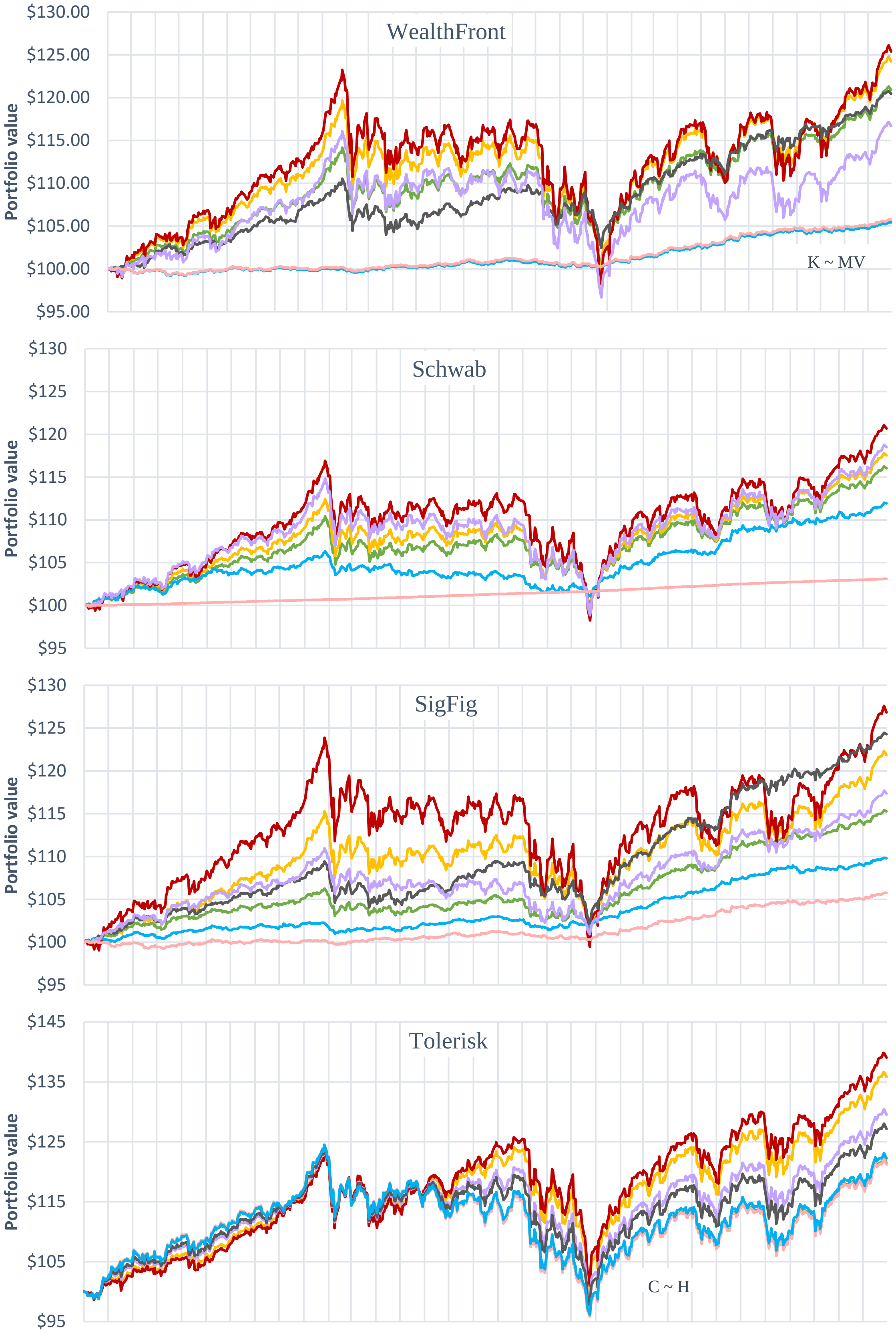
TABLE A. 1 – PORTFOLIO ALLOCATION PER ASSET (H, MV, T AND K PORTFOLIOS)

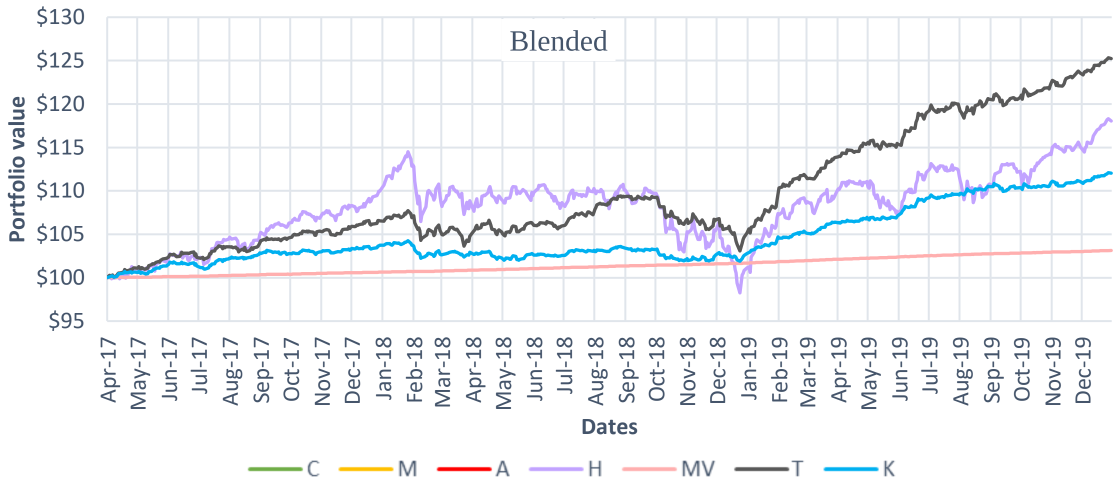
Asset	Asset Class	WealthFront (%)				Schwab (%)				Sig Fig (%)				Tolerisk (%)				Blended (%)			
		H	MV	T	K	H	MV	T	K	H	MV	T	K	H	MV	T	K	H	MV	T	K
IGOV	B	-	-	-	-	4.76	-	-	10.03	-	-	-	-	-	-	-	-	4.17	-	-	5.58
SPMB	B	-	-	-	-	4.76	0.03	-	12.27	-	-	-	-	-	-	-	-	4.17	0.01	-	12.23
STIP	B	14.29	98.82	-	99.94	4.76	5.94	-	12.09	12.50	98.82	-	56.92	-	-	-	-	4.17	5.76	-	12.56
TFI	B	14.29	0.24	69.63	-	-	-	-	-	12.50	0.24	-	31.54	-	-	-	-	4.17	0.18	3.72	12.24
VCIT	B	-	-	-	-	4.76	-	74.94	11.05	12.50	-	87.05	0.05	50.00	91.15	87.05	86.10	4.17	-	71.51	9.01
VGIT	B	-	-	-	-	4.76	-	-	11.87	12.50	-	-	5.04	-	-	-	-	4.17	-	-	11.02
VWOB	B	-	-	-	-	4.76	-	-	9.41	12.50	-	-	0.59	-	-	-	-	4.17	-	-	10.72
Cash	C	-	-	-	-	4.76	93.90	-	13.35	-	-	-	-	-	-	-	-	4.17	93.91	-	15.03
DGL	Com	-	-	-	-	4.76	-	-	2.31	-	-	-	-	-	-	-	-	4.17	-	-	0.91
IEG	E	14.29	-	-	0.02	-	-	-	-	-	-	-	-	-	-	-	-	4.17	-	-	-
IEMG	E	14.29	-	-	-	4.76	-	-	-	12.50	-	-	0.01	-	-	-	-	4.17	-	-	-
PDN	E	-	-	-	-	4.76	-	-	3.66	-	-	-	-	-	-	-	-	4.17	-	-	0.66
PRF	E	-	-	-	-	4.76	-	-	-	-	-	-	-	-	-	-	-	4.17	-	-	-
PRFZ	E	-	-	-	-	4.76	-	-	-	-	-	-	-	-	-	-	-	4.17	-	-	0.47
PXF	E	-	-	-	-	4.76	-	-	-	-	-	-	-	-	-	-	-	4.17	-	-	0.12
PXH	E	-	-	-	-	4.76	-	-	-	-	-	-	-	-	-	-	-	4.17	-	-	-
SPY	E	14.29	0.94	30.37	-	-	-	-	-	12.50	0.94	12.95	5.85	50.00	8.85	12.95	13.90	4.17	-	-	4.82
VB	E	-	-	-	-	4.76	-	-	0.01	-	-	-	-	-	-	-	-	4.17	-	-	0.13
VEA	E	14.29	-	-	0.03	4.76	-	-	0.12	12.50	-	-	-	-	-	-	-	4.17	-	-	0.09
VOO	E	-	-	-	-	4.76	0.13	25.06	-	-	-	-	-	-	-	-	-	4.17	0.14	24.76	0.02
VSS	E	-	-	-	-	4.76	-	-	4.57	-	-	-	-	-	-	-	-	4.17	-	-	0.25
VYM	E	14.29	-	-	0.01	4.76	-	-	6.18	-	-	-	-	-	-	-	-	4.17	-	-	4.13
VNQ	RS	-	-	-	-	4.76	-	-	-	-	-	-	-	-	-	-	-	4.17	-	-	-
VNQI	RS	-	-	-	-	4.76	-	-	3.08	-	-	-	-	-	-	-	-	4.17	-	-	-

** Portfolio: H – Homogeneous, MV – Minimum-Variance, T – Tangent and K – Kataoka*

*** Asset Class: B – Bonds, C – Cash, Com – Commodities, E – Equities and RS – Real State*

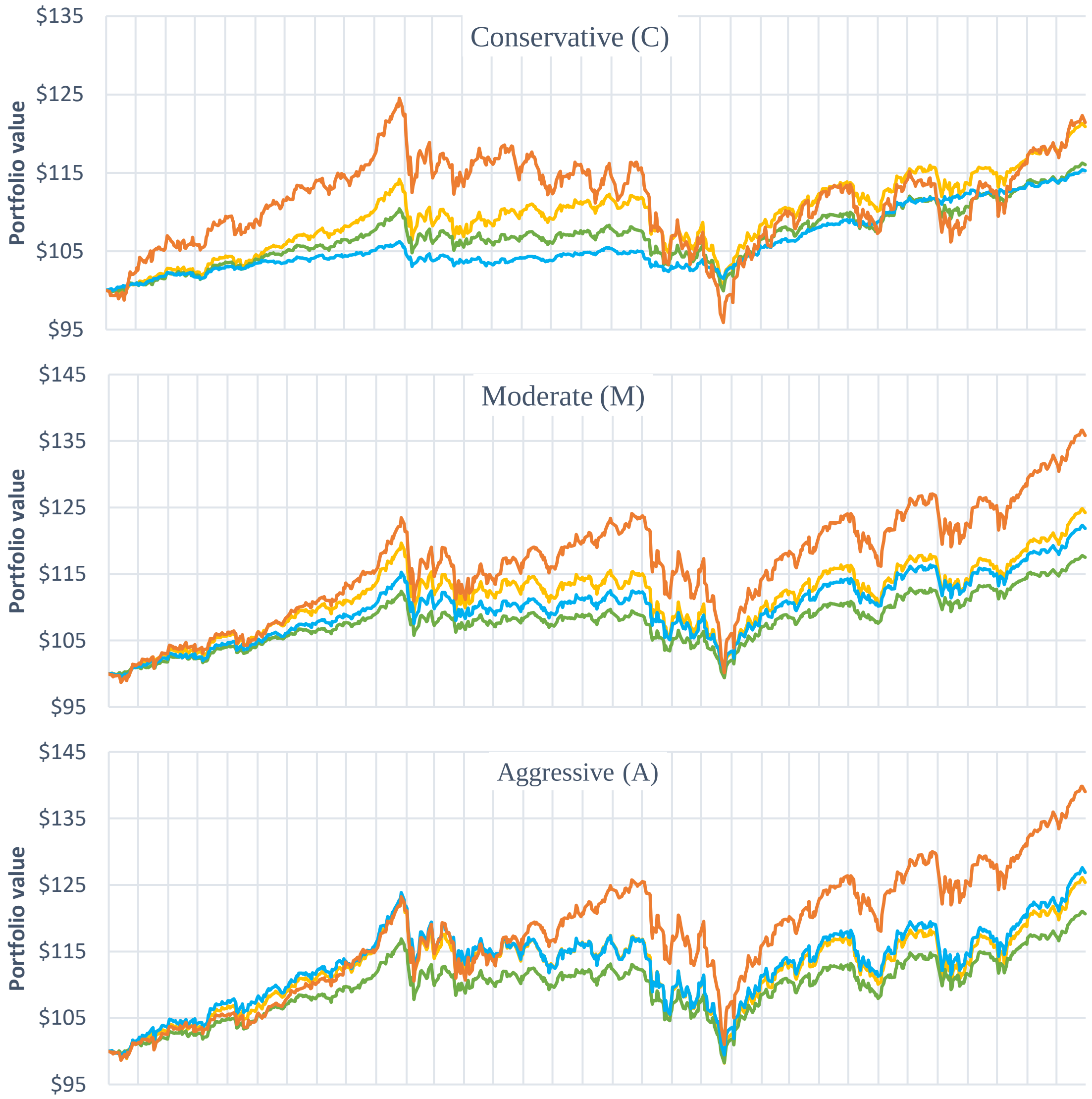
FIGURE A. 2 - CUMULATIVE PERFORMANCE PRESENTED BY SET OF ASSETS (ROBOS AND BLENDED SET) - MONTHLY REBALANCING

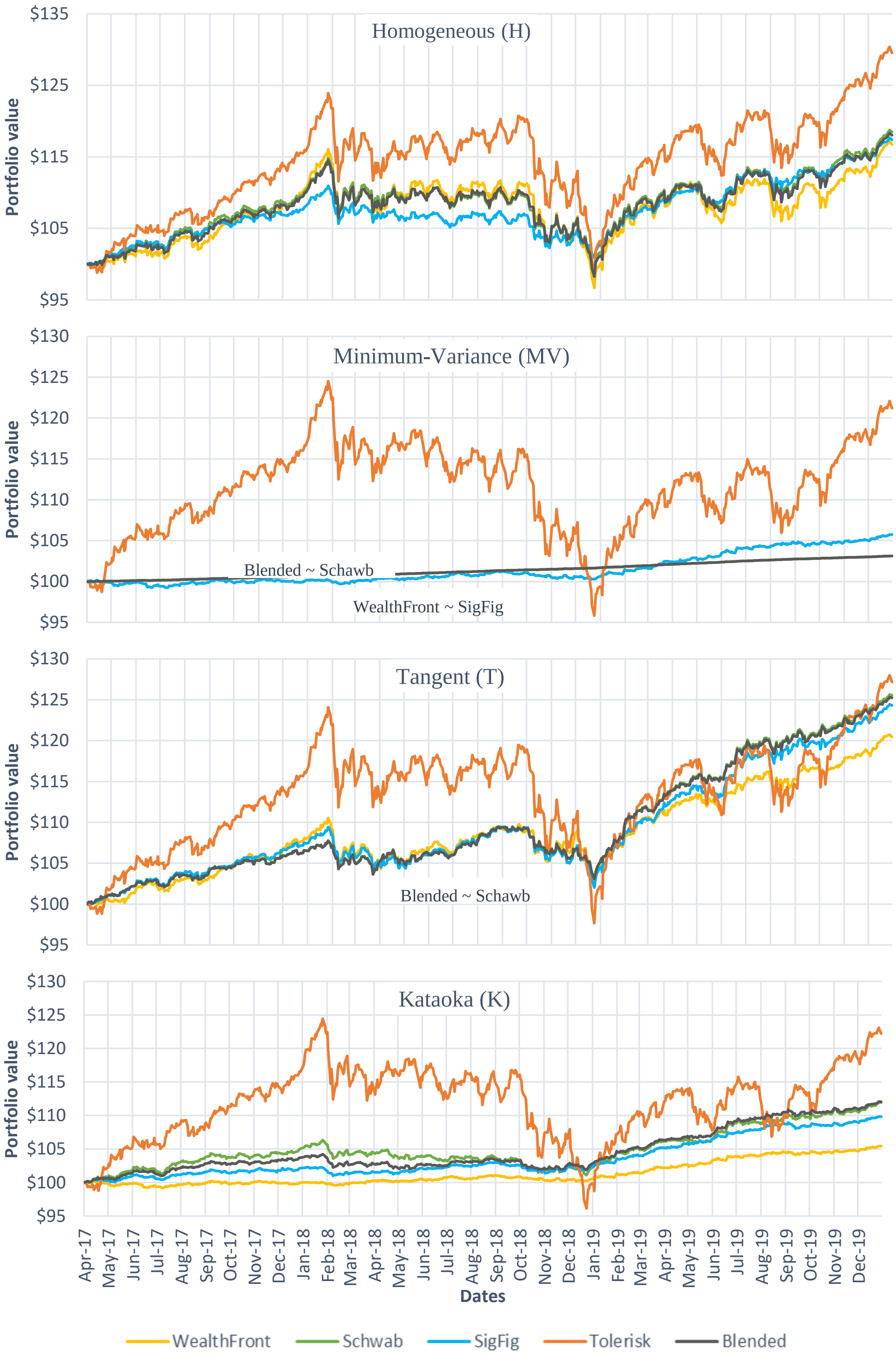




* *C–Conservative, M–Moderate, A-Aggressive, H–Homogeneous, MV–Minimum-Variance, T–Tangent, K–Kataoka*

FIGURE A. 3 - CUMULATIVE PERFORMANCE PRESENTED BY TYPE OF PORTFOLIO - MONTHLY REBALANCING





Homogeneous (H)
$135
$125
$115
$105
$95
Portfolio value
Minimum-Variance (MV)
$130
$125
$120
$115
$110
$105
$100
$95
Blended ~ Schawb
WealthFront ~ SigFig
Tangent (T)
Blended ~ Schawb
Kataoka (K)
Apr-17
May-17
Jun-17
Jul-17
Aug-17
Sep-17
Oct-17
Nov-17
Dec-17
Jan-18
Feb-18
Mar-18
Apr-18
May-18
Jun-18
Jul-18
Aug-18
Sep-18
Oct-18
Nov-18
Dec-18
Jan-19
Feb-19
Mar-19
Apr-19
May-19
Jun-19
Jul-19
Aug-19
Sep-19
Oct-19
Nov-19
Dec-19
Dates
WealthFront
Schwab
SigFig
Tolerisk
Blended