UNIVERSIDADE DE LISBOA

INSTITUTO SUPERIOR DE ECONOMIA E GESTÃO

MASTER'S FINAL WORK

INTERNSHIP REPORT

ALLOCATION OF SCR BY LINES OF BUSINESS AND RORAC OPTIMIZATION

BY DANIIL PANCHENKO

MSC ACTUARIAL SCIENCE

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ABSTRACT

Nowadays topics that are related with the new insurance supervisory regime, Solvency II, have been becoming increasingly important. This is due to the fact that insurance companies must follow this regime from January 1, 2016. This project focuses on the study of risk-based capital, SCR, which is calculated using the standard formula proposed by EIOPA. However, the formula calculates the SCR of the insurance company as a whole. Which creates a problem for purposes like identification of risk concentration, perception of sensitivity of the risk and the optimization of the portfolio. Therefore, it is necessary to have an idea of risk-based capital that is necessary to allocate to each of the lines of business (sub-portfolios). Which is a very challenging task since there is some partial correlation between the risks (from where the diversification effect appears) in different levels of the formula, this effect needs to be incorporated in the allocated capital in such a way that the sum of the allocated capital would be the company's global SCR.

The main goal of this project is to allocate the SCR between sub-portfolios (lines of business), using a method developed by Dirk Tasche which is based on Euler's formula, and show how this allocation could be used in the optimization of the portfolio in such a way that the maximization of the RORAC of the company is reached.

For the academic purposes this study should contribute to the better understanding of the standard formula and the SCR, show some properties that SCR follows, how it is possible to do a fair allocation of SCR between lines of business and show a practical example of this method applied to a non-life insurance company.

For business purposes this investigation will show a practical step-by-step demonstration of the application of the model. In my opinion this project should support the analysis of decisions that are made by the management of the company.

By applying this model to a real data of a non-life insurance, we obtained a very interesting result: some LoBs that at first sight seem to be profitable, show high volatility, and we conclude that they do not fulfill the risk appetite of the company.

Keywords

Solvency II; SCR; Standard Formula; RORAC; capital allocation; Euler's allocation principle; diversification; risk appetite; optimization; profitability.

RESUMO

Atualmente, temas ligados ao novo regime Solvência II têm vindo a assumir uma importância crescente, muito devido ao facto de se exigir às seguradoras que, a partir do dia 1 de janeiro de 2016, sigam este novo regime de solvência.

Esta investigação incide sobre o estudo do capital em risco, SCR, que é calculado através da Fórmula Padrão proposta pela EIOPA. Esta fórmula calcula o SCR da seguradora como um todo, mas se se pretender fazer uma análise da concentração do risco, uma análise da sensibilidade ao risco ou da otimização do portfolio, é necessário alocar o SCR por cada uma das linhas de negócio (*sub-portfolios*) presentes na seguradora. Tal tarefa pode não se revelar fácil pois existe uma correlação parcial dos riscos (do qual resulta o efeito da diversificação), em diferentes níveis da fórmula, que tem que ser incorporada na alocação feita de modo a que a soma do capital alocado seja o SCR global.

O objetivo do trabalho é alocar o SCR por linhas de negócio através de um método, desenvolvido por Dirk Tasche que se baseia na fórmula de Euler, e mostrar como esta alocação poderá ser usada na otimização do portfolio da seguradora de modo a que a maximização do RORAC seja atingida.

A nível académico este estudo irá contribuir para uma melhor compreensão da Fórmula Padrão e do SCR, mostrar algumas propriedades do SCR, mostrar como é possível a sua alocação por linhas de negócio e a aplicar todo este modelo a um caso prático.

A nível empresarial, esta investigação irá mostrar um modelo de alocação do SCR aplicado à Formula Padrão juntamente com um exemplo da sua aplicação. Penso que este trabalho será interessante para atuários, gestores de risco ou mesmo administradores, que poderão aplicá-lo nas suas decisões de gestão da empresa.

Ao aplicar o modelo a uma seguradora não vida, foram obtidos resultados bastante interessantes pois linhas de negócio que a primeira vista parecem lucrativas, mostraram-se bastante voláteis, o que faz com que o retorno não é compensado pelo risco, ou seja é ultrapassado o limite da volatilidade proposto pela empresa (apetite ao risco da empresa).

ACKNOWLEDGEMENTS

I would like to thank the insurance company's actuarial team who proposed me this topic for my Master's final work and guided me during the internship period. I also wanted to recognize the indispensable support that was given to me by my professor from ISEG Walther Neuhaus and the author of the paper from which I based my investigation Dr. Ivan Granito.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

INTRODUCTION

After the introduction of Solvency II it is crucial that all financial institutions measure the risk in their portfolio in terms of economical capital. However, measuring it for the total portfolio does not give any information to risk managers for purposes like identification of risk concentration, risk sensitivity or portfolio optimization. That is the reason why it is important to decompose the total portfolio into sub-portfolios.

There is a lot of research being done about different methodologies for capital allocation. In this project, I will present the Euler's allocation principle, developed by Dirk Tasche and described in the paper *"Capital allocation and risk appetite under Solvency II framework"* (by Ivan Granito and Paolo de Angelis), and its application to a small Portuguese non-life insurance company. After a proper allocation is done, we will see that Euler's compatibility with the Return on Risk-Adjusted Capital (RORAC) will allow us to evaluate which Lines of Business (LoBs) create value to the company. Also, by using the same approach, I will show that it is possible to optimize the company's portfolio.

This final project was proposed by the non-life Portuguese insurance company that offered me a four-month internship. During this time, I had the opportunity not only to develop this investigation, but also to work with experienced actuaries and analyze the real problems that insurance companies are facing today. This experience will surely strengthen my knowledge and will allow me to face the next steps in my professional career.

The internship started with the presentation of techniques and procedures that actuaries use to calculate their provisions. I was shown methods that used bootstrap and chain ladder modeling which gave me the opportunity to implement my knowledge of Loss Reserving.

My first task was the calculation of the SCR of the company by applying standard formula, where I had the chance to read carefully the delegated act offered by EIOPA and construct the Standard formula in excel using the data of the company. This information allowed me to go further, explore SCR and understand how it is possible to allocate SCR by LoB, since Standard Formula allows only the calculation of SCR of the whole company.

When proceeding with the risk capital allocation I perceived that I would face a problem related to the allocation of the diversification effect by LoB. I noticed that it is not possible to analyze the capital requirements of each LoB by themselves, since it is also important to incorporate the diversification effects, which will lower this capital.

In this paper, the Euler's allocation method is proposed, since it ensures RORAC compatibility and allows practical application. This approach, applied to Standard Formula, was presented by Dr. Granito and Prof. Angelis.

1 A BRIEF INTRODUCTION TO SOLVENCY II

1.1 Risk

In this section we introduce the common elements and definitions of this topic.

Definition 1 (Risk). *It is a situation where the probability distribution of a variable is known, but the actual value of the variable is not.*

~ Is it good or bad?

Insurance companies offer products to cover many different risks. The decision whether to cover a risk or not must be taken after a proper analysis of the risk (for example look at the frequency and severity of the risk) and the market price, in order to see if it will be profitable. These decisions, made by the risk managers, will determine whether risks are good or not.

1.2 Risk Management

Analyzing the risk and whether it will be profitable to the company or not, and measuring it, is a very complex process. Therefore, it is necessary to perform a proper risk management and it consists in the following steps:

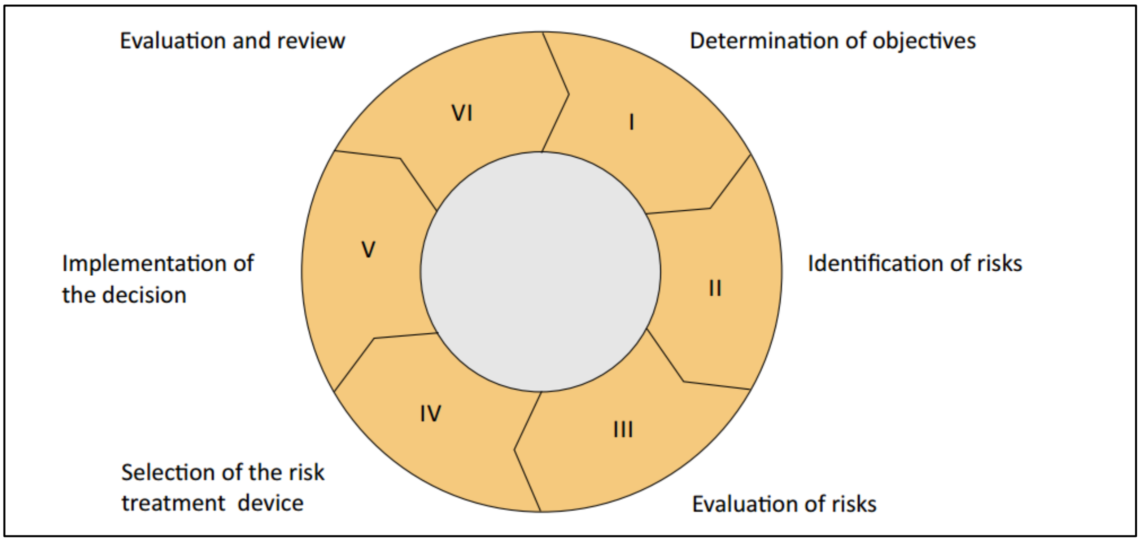

Figure 1: The risk management cycle steps according to Vaughan (2008).

A good risk management enhances the chance of the company to reach its goals ensuring that it does not go bankrupt. This is done by preventing the acceptance of "*bad risks*"[1] that have high probability of generating financial losses to the company. Poor risk management can lead to severe consequences not only to the company or to all individuals related to it, but also to economic instability through a domino effect. The 2008 crisis is an excellent example.

After the financial disaster, EU implemented new regulations in the insurance and banking industry, therefore a more intense preparation to this new regime, Solvency II, had started.

The Solvency II is a "*Directive in European Union law that codifies and harmonizes the EU insurance regulation. Primarily, this concerns the amount of capital that EU insurance companies must hold to reduce the risk of insolvency*.". The main goal of this regime is the protection of policyholders and beneficiaries, implying that insurance companies must guarantee their solvency.

[1] Risks that are unprofitable to cover, since the returns do not compensate.

1.2.1 Risk management in Non-Life insurance

Risk management of the insurance company must fulfill specific requirements written in the Solvency II regime, which follow a risk-based approach. In non-life insurance, claim severities are unknown, so it is very important to do a proper risk management and evaluate the risks involved as well as their concentration in the portfolio. One possible way of lowering the implicit risk is to share it with the reinsurance companies.

However, by spreading the risk, the insurance company loses a share of the business since the expected profitability is also shared. Therefore, it´s important that the risk manager elaborates a proper contract with the reinsurer in a way that allows the reduction of the risk (by splitting it with the reinsurer) and at the same time earn profit from it.

1.2.2 Lines of Business (LoBs) in Non-Life insurance

Insurance companies sell innumerous products that cover all sorts of risks. These can lead to profit or loss, consequently, it is up to the company study these risks and decide whether to cover them or not. For the organizational purposes, EIOPA formed twelve groups, each one of them is composed by homogeneous risks[2]. These groups are called lines of businesses (LoBs).

Since insurance companies have the obligations with the policyholders that buy their products, LoBs segment the liability side of the insurance company's balance sheet, these LoBs are[3]:

- **Non-life groups**: (1) motor vehicle liability; (2) other motor; (3) marine aviation and transport (MAT); (4) Fire; (5) Third party liability; (6) Credit; (7) Legal expenses; (8) Assistance; (9) Miscellaneous.
- **Health groups**: (10) Medical Expenses; (11) Income Protection; (12) Workers' Compensation.

Each one of them will produce positive or negative results to the company, therefore, to analyze the profitability of the whole company, we should evaluate and build strategies

[2] Products that have similar characteristics.
[3] Note that the definition of each LoB as well as the type of products that can be related to each LoB is given in EIOPA delegated act.

to each LoB separately. Note that by treating them disjointedly we need to have in mind that there is a relation between sales of different products in different LoBs. Therefore, sometimes it is not possible nor desirable to sell LoBs separately, knowing this, we conclude that strategies of portfolio optimization should take into account this relation.

2 SOLVENCY II

Solvency II is a new regulatory plan for the European insurance sector, that was implemented on January 1, 2016. It considers more effective risk management approached and ensures that, theoretically speaking, ruin of the company occurs no more often than once in every 200 years (probability of default in one-year period is 0.5%). Therefore, the company needs to ensure to have necessary amounts of risk-based capital, the SCR, to guarantee that the probability of ruin will not exceed 0.5%. From this we can see that the capital that the company is required to hold on the risks it is facing, specifically, the riskier the insurance's business the more precautions it needs to take, consequently more capital is required. From investor's perspective, the capital is a scarce resource so to attract more investments insurance companies want to demonstrate their profitability, by showing their sufficiently high return per unit of capital invested and low volatility.

2.1 Why Solvency II was Implemented

We have seen recently a huge Financial Crisis starting from 2007/2008, that was the consequence of the burst of a "financial bubble" at international level. Through a "snowball effect" it contaminated the entire banking system, which led to liquidity problems and forced banks to sell assets. With a huge supply in the market the asset's prices fell drastically and since we are living in an Era of Globalization most of developed countries were affected. In order to prevent these types of crisis, the EU decided to be more demanding from the banking and insurance business[4].

2.2 Goals of Solvency II

1) **Policyholder protection**: this is the main goal of Solvency II, that ensures the policyholder's protection, so that the consumers would have confidence on insurance

[4] Note that in this paper we will only study the insurance company case.

products. This will eventually increase the demand of the insurance's products which in turn favors the grow of the insurance market.

2) **Better supervision**: supervisors have very important roles on monitoring the insurance's risk profile, risk management and administration strategies.
3) **EU Integration**: Insurers in all EU countries should obey similar rules.

2.3 Three Pillars of Solvency II

To achieve these goals Solvency II proposed following three Pillars:

i) **Pillar I** (quantitative requirements): EU expects from the insurances calculations of technical provisions, capital requirements (SCR and MCR), investments and calculation of own funds. These outputs will form the major items of the balance sheet of the insurance company;

ii) **Pillar II** (qualitative requirements): effective risk management, Own Risk Solvency Assessment (ORSA) and supervisory review process;

iii) **Pillar III** (market discipline and transparency): detailed public disclosure, improvement of market discipline by facilitating comparisons and regulatory reporting requirements.

2.3.1 Quantitative Requirements

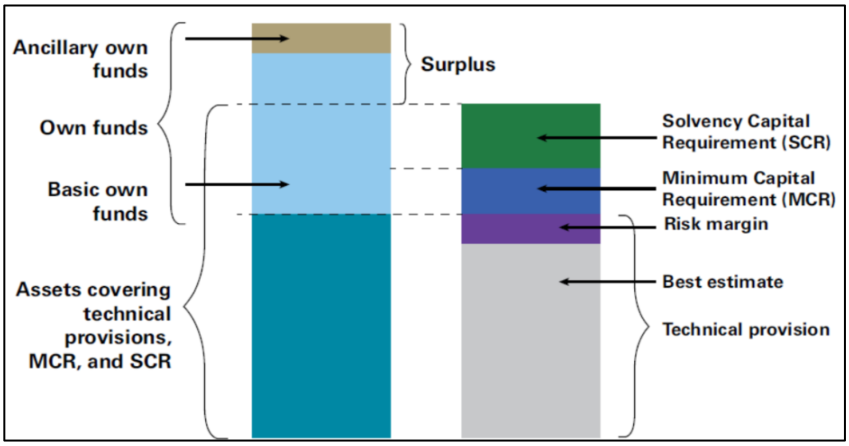

Figure 2: Solvency II Balance Sheet displaying assets (left) and liabilities (right).

Source: The Underwriting assumptions in the standard formula for the Solvency Capital Requirement calculation (EIOPA)

The main goal for the valuation of assets and liabilities “set out in Article 75 of Directive 2009/138/EC” is to have an economic and market-consistent approach.

1. Assets should be valued at the amount for which they could be transferred to knowledgeable willing parties.
2. Liabilities should be valued at the amount for which they could be settled between knowledgeable willing parties.

2.3.2 *Technical provisions*

Solvency II requires to set up Technical Provisions (TP), which correspond to the current amount that the undertakings would have to pay if they would transfer their (re)insurance obligations today to another undertaking. TP are calculated as market value and the formula is:

$$TP = BE + RM \tag{1}$$

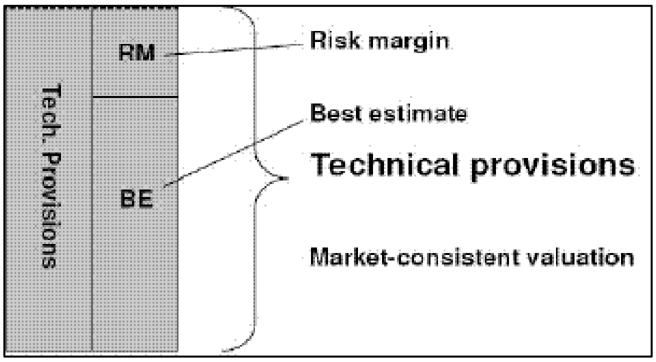

Figure 3: Technical Provision of liability side of Solvency II Balance Sheet.

Source: The Underwriting assumptions in the standard formula for the Solvency Capital Requirement calculation (EIOPA)

2.3.2.1 Best Estimate (BE)

Best Estimate (BE) is the probability weighted average of future gross cash-flows taking into account the time value of the money. In other words, through the use of actuarial approaches, it is necessary to calculate future cash-flows and after discount them at an interest rate, given by EIOPA. The projection horizon used in the calculation of BE should cover the full lifetime of all in-flows and out-flows required to settle the obligations related to the existing contracts on the date of the valuation.

2.3.2.2 *Risk Margin (RM)*

Risk Margin is the amount over BE that an independent third party (reference undertaking) would ask in order to take over the liabilities. This amount ensures that the value estimated for technical provisions is sufficient for other (re)insurer to take the obligations of the first one. It is calculated through Cost of Capital methodology, that is, by determining the cost of providing an amount of eligible own funds equal to SCR, which is necessary to support the obligations during their lifetime and it is calculated as following:

I. Calculate BE technical provision in each point (future year) during all lifetime;
II. Estimate the appropriate corresponding SCR at each future year;
III. Multiply by cost-of-capital factor;
IV. Apply the discounting factor to the sum.

$$COCM = CoC * \sum_{t \geq 0} \frac{SCR_{RU}\ (t)}{(1+r_{t+1})^{t+1}}, \tag{2}$$

where, $COCM$ is the risk margin for the whole business, CoC is the cost-of-capital rate (set at 6%), $SCR_{RU}(t)$ is the SCR as calculated for the reference undertaking at the t-th year and r_t is the risk-free rate for maturity t.

2.3.3 Minimum Capital Requirement (MCR)

The MCR is the minimum level of capital that is necessary in order for (re)insurance undertakings to be allowed to continue their operations. If the amount of eligible own funds falls below that level the policyholders and beneficiaries are exposed to an unacceptable level of risk. For this reason, the supervisors must analyze these problematic (re)insurers more carefully. If those undertakings are unable to re-establish the amount of eligible basic own funds at the MCR level within a short period of time the supervisors should take withdraw the authorization of (re)insurance business. Calculation of MCR should be simple and easy to understand such that the audit could easily verify them.

2.3.4 Solvency Capital Requirement (SCR)

The SCR is the amount of capital that ensures that the probability of default in one-year period should be no more than 0.5% and it is calculated by the usage of one of the following procedures:

i) Internal Model: is created by the (re)insurance company, using their own parameters and methodologies in order to calculate the SCR. This model should be approved by the supervisors and it should explain more precisely the situation of the company where it is being used than the Standard Formula. However, this procedure is very complex and expensive, so not every company can afford it;
ii) Using Standard Formula with company's own parameters, instead of those given by EIOPA, which result from approximation by all EU insurance companies;
iii) Using Standard Formula as it is written by EIOPA;

In this paper, I will use the third method, that is the Standard Formula with the parameters given in delegated act by EIOPA but the same procedures could be applied to those companies that use the second approach.

2.3.5 Definition of Non-Life insurance risks

When actuaries calculate predictions, it is always necessary to remember that no model is perfect, so there are always deviations from the predictions that were made. Even if they use the best model possible there are still some unpredictable anomalies that can occur.

There are enormous variety of risks that an insurance company is facing that could put it in insolvent position, since we cannot incorporate all risks in the model, EIOPA decided to select those that are the most important and use them in the calculation of the SCR as the figure below shows:

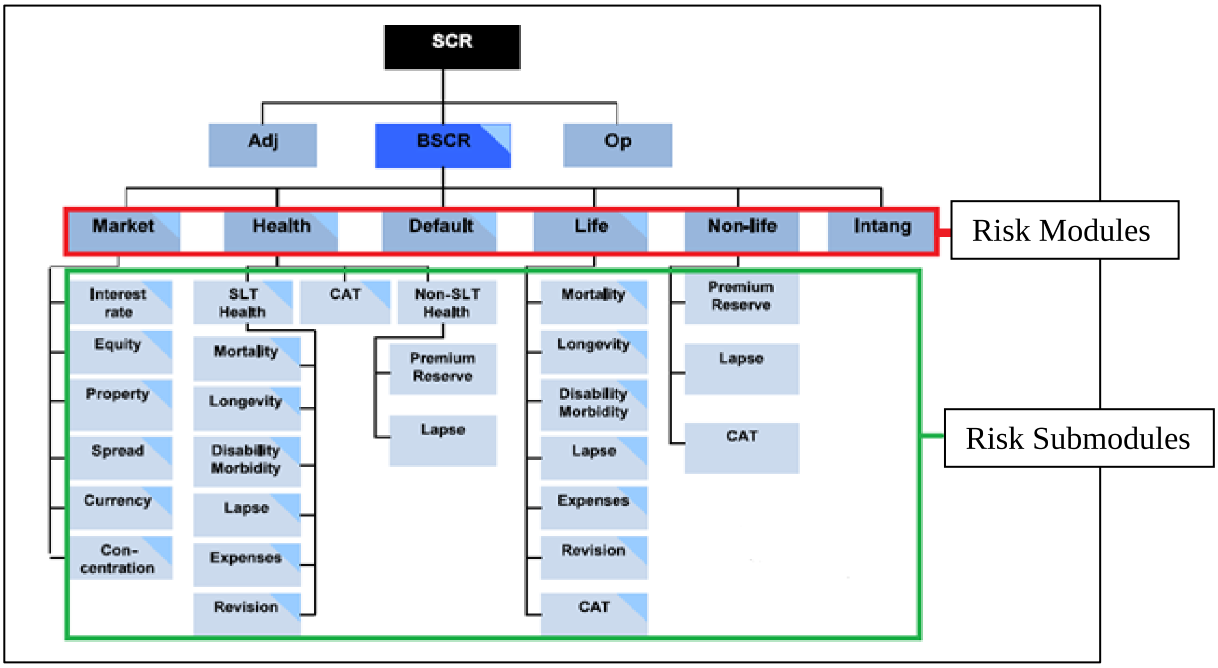

Figure 4: Hierarchy of Risks.

Source: EIOPA Delegated Act

The figure above shows the combination of risks that are involved in the calculation of SCR. When observing Figure 4, by doing the general-specific analysis, we notice that we can divide risks in different levels: **BSCR** (that results from the combination of risk modules), **risk modules** (that results from the combination of risk submodules), **risk submodules** (that result from the combination of lines of business (LoBs)) and **LoBs**. Since the analysis is done to a non-life insurance company the only risk modules that required capital by Standard Formula are: Non-life, Health, Market and Default. Giving a brief explanation of each[5]:

~ For Non-Life:

- *Premium Risk*: the risk that the premiums will not be sufficient to cover the future liabilities and the expenses that have resulted from claims;
- *Reserve Risk*: the risk that the liabilities that come from past claims will turn out to be higher than expected;
- *CAT*: the risk of the catastrophe, which means if single or series of correlated events will cause huge deviation in actual claims from the total expected claims;

[5] Assuming that there is no intangible risk.

- *Lapse Risk*: the risk that the insurance company have higher than expected premature contract termination;

~ For Health:

- *Health Similar to Life* (SLT) divided into:
  - *Longevity Risk*: the risk that person live longer than expected, this will put more weight on the pension provision thus higher costs;
  - *Disability Morbidity*: the risk that more people will have higher disability pension than expected;
  - *Expense Risk*: the risk of possible increase in expenses;
  - *Revision Risk*: the risk of unexpected revision of the claims, which can lead to higher liabilities (this is applied to the annuities).
  - *Mortality Risk*: "is the risk of loss, or of adverse change in the value of re(insurance) liabilities, resulting from changes in level, trend, or volatility of mortality rates."
  - *Lapse Risk*: "is the risk of loss, or of adverse change in the value of re(insurance) liabilities, resulting from changes in the level or volatility of the rates of policy lapses, terminations, renewals and surrenders."

- *CAT*: the risk of the catastrophe, that is, if single or series of correlated events will cause huge deviation in actual claims from the total expected claims (mass accident, concentration scenario and pandemic scenario).

- *Health Non-Similar-to-Life* (Non-SLT) divided into:
  - *Premium Risk*: the risk that the premiums will not be sufficient to cover the future liabilities and the expenses that have resulted from claims;
  - *Reserve Risk*: the risk that the liabilities that come from past claims will turn out to be higher than expected;
  - *Lapse Risk*: the risk that the insurance company have higher than expected premature contract termination;

~ **For Market** (definitions given by EIOPA)**:**

- *Interest Rate Risk*: "the sensitivity of the values of assets, liabilities and financial instruments to changes in the term structure of interest rates, or in the volatility of interest rates";
- *Equity Risk*: "the sensitivity of the values of assets, liabilities and financial instruments to changes in the level or in the volatility of market prices of equities";
- *Property Risk*: "the sensitivity of the values of assets, liabilities and financial instruments to changes in the level or in the volatility of market prices of real estate";
- *Spread Risk*: "the sensitivity of the values of assets, liabilities and financial instruments to changes in the level or volatility of credit spreads over the risk-free interest rate term structure";
- *Currency Risk*: "the sensitivity of the values of assets, liabilities and financial instruments to changes in the level or in the volatility of currency exchange rates".

~ **For Default:**

- This module reflects possible losses due to unexpected default of the counterparties and debtors of undertakings over the forthcoming twelve months.

If we want to calculate SCR by LoB, as you can see, it is not straightforward since the risks presented above are correlated with each other in different levels, and because of that, a diversification effect is produced each time we go from one level to the next.

2.4 Solvency II Standard Formula (SF)

The insurance company, that is being analyzed, uses the SF (with parameters given by EIOPA) to calculate its SCR. As it was seen previously, this was one of three methods to calculate SCR and it is not perfect. SF aims to capture the risks that most undertakings are exposed to. However, it might not cover all risks that a specific undertaking is exposed to, also the parameters that are used in standard formula are an average at EU level and do not reflect the reality of a specific insurance.

For this reasons the standard formula might not reflect the true risk profile for a specific insurance and, consequently, the level of own funds it needs. Yet, creating internal models can be very expensive and not all the insurances can afford it, this is why SF is being used by a lot of insurance companies around the EU.

2.5 Capital Allocation

In order to analyze the business strategy of the company it is necessary to evaluate the profitability and the risk that each LoB produce. As it was previously shown, the SF calculates SCR of the company as a whole. Thus, to do a proper analysis it is important to allocate this risk-based capital to each LoB, such that the sum of allocated SCRs gives us the total SCR of the company. In other words, the allocation must be done in such a way that the diversification effect would be incorporated in the allocated capital.

2.6 Return on Risk-Adjusted Capital (RORAC)

This is very popular measure that is used in the financial analysis every time it is necessary to evaluate risky investments. It is based on the ratio of earnings divided by the risk-based capital, from which it is possible to determine the percentage of return that a particular investment obtained weighted by the capital that was invested in order to get this return.

2.7 Optimization Strategy

After a proper allocation is done, it is possible to analyze the RORAC not only of a present situation, but also compare it with other RORAC obtained by different strategies, that the management board of the company propose, in such a way that the one that maximizes the company's RORAC is chosen.

3 MATHEMATICAL FRAMEWORK

It was seen that the risks that are involved in the calculation of SCR are not perfectly correlated with each other, in different levels of SF, from where the diversification effect appears. To allocate SCR by LoBs a proper mathematical approach should be applied. There are several approaches that can be used to allocate risk capital, the method that is being used in this thesis is based on Euler's principle.

3.1 General Basis

Let's consider an insurance company that has a portfolio that is composed by n-homogeneous sub-portfolios, each of those sub-portfolios can bring profit or loss to the global result of the company. Define a set of random variables X_i (i =1, …, n) , where X_i represents the risk of the $i - th$ sub-portfolio. It is clear that the portfolio-wide risk that the company is facing is:

$$X = \sum_{i=1}^{n} X_i \quad (3)$$

To do a proper analysis of the risk of an insurance company, it is necessary to apply a risk measure that calculates capital that is necessary to be kept in the company, so that the risk would be acceptable.

3.2 Risk Measure

Let π be the risk measure that quantifies the level of risk, then π(X) is the real number that represents the capital that is necessary to cover risk X. As we saw previously, $SCR(X)$ is a measure of the risk that calculates the risk capital that is required by the regulators for the amount of the risk X.

$$SCR(X) = \pi(X) \quad (4)$$

It is clear that the riskier the (re)insurance strategy (higher X) the more capital is required by the authority (higher the π(X)), but this relation is not linear because of the correlation between risks. In order to proceed to the allocation problem a desirable risk measure must satisfy the following properties:

Definition 2 (Coherent Risk Measure). *A risk measure π is considered coherent if it satisfies the following properties:*

i) ***Subadditivity:*** *For all bounded random variables X and Y we have:*

$$\pi(X+Y) \leq \pi(X) + \pi(Y) \quad (5)$$

ii) ***Monotonicity:*** *For all bounded random variables, such that $X \leq Y$ we have:*

$$\pi(X) \leq \pi(Y) \quad (6)$$

iii) ***Positive Homogeneity:*** *Consider $\lambda \geq 0$ and bounded random variable X we have:*

$$\pi(X\lambda) = \lambda\pi(X) \tag{7}$$

iv) ***Translation invariance:*** *for a fixed return $\alpha \in \mathbb{R}$, bounded random variable X and riskless investment whose price today is 1 and price at some point in the future is B*

$$\pi(X + \alpha B) = \pi(X) - \alpha \tag{8}$$

Remark 1. From above, property: (i) shows that risk-based capital of holding two risky sub-portfolios at same time is smaller or equal than when we are holding them separately, this happens due the imperfect correlation between X and Y; (ii) shows that if the loss of sub-portfolio X is, in all scenarios, less or equal than the loss Y, then X is less risky than Y , thus needs less capital; (iii) explains that the risk of a portfolio is proportional to its size, and (iv) tells us that if we add some riskless investment to the portfolio it will reduce the risk of the company by the return of that riskless investment.

3.3 Defining the Allocation Problem

If the firm's overall risk capital ($\pi(X)$) is smaller than the sum of all sub-portfolios stand-alone risks ($\sum_{i=1}^{n} \pi(X_i)$), we have a diversification effect. This motivates the usage of an allocation principle that can separate the insurance company's overall risk capital among sub-portfolios in such a way that this effect is allocated to the sub-portfolios.

Definition 3 (Allocation Principle). *Given a risk measure π, an allocation principle is defined as a mapping Π: $A \rightarrow \mathbb{R}^n$, that maps each allocation problem into a unique allocation. Let X denote portfolio-wide risk, we have:*

$$\Pi(A) = \Pi\begin{pmatrix}\pi(X_1)\\ \vdots \\ \pi(X_n)\end{pmatrix} = \begin{pmatrix}\pi(X_1|X)\\ \vdots \\ \pi(X_n|X)\end{pmatrix} \tag{9}$$

where, $\pi(X_i|X)$ is the allocated risk capital for sub-portfolio i, such that the risk contributions $\pi(X_1|X) \ldots \pi(X_n|X)$ to portfolio-wide risk $\pi(X)$ satisfies the ***full allocation property*** *if $\pi(X) = \sum_{i=1}^{n} \pi(X_i|X)$.*

Definition 4 (Allocated Risk Capital). *This form of capital for a sub-portfolio i is the capital adjusted for a maximum probable loss that can occur and it is based on the estimation of the future earnings distribution.*

Each of the allocated risk capitals incorporates the diversification benefits that came from imperfect risk correlation. Note that the allocated risk capital does not coincide with real capital invested to fund a sub-portfolio, but it can be used to virtually express each sub-portfolio's contribution to the risk of the whole (re)insurance company and can be the point of reference to know the profitability of each sub-portfolio.

Definition 5 (Coherent Allocation)[6]. *An allocation K_i,*such that: *$i \in N$, is coherent if satisfies the following properties:*

i) ***Full allocation:*** $\sum_{i \in N} K_i = \pi(\sum_{i \in N} X_i)$

ii) ***No undercut*** $\forall M \subseteq N, \sum_{i \in M} K_i \leq \pi(\sum_{i \in M} X_i)$

iii) ***Symmetry:*** *If by joining any subset $M \subseteq N \setminus \{i, j\}$, portfolios i and j both make the same contribution to the risk capital, then $K_i = K_j$.*

iv) ***Riskless allocation*** *for a riskless deterministic portfolio L with fixed return α we have: $K_n = \pi(\alpha L) = -\alpha$.*

Remark 2. As we saw previously (i) ensures that the sum of the allocated capital by sub-portfolios would be the same as the risk capital of the whole portfolio. (ii) ensures that there is no subset M of the set portfolios which is cheaper for every single portfolio in M; (iii) guarantees that a portfolio's allocation depends only on its contribution to risk within the (re)insurance company, and (iv) says that riskless investments will lower the capital at risk of a portfolio, since the returns of that investment are guaranteed with zero risk.

4 EULER'S ALLOCATION METHOD

The Euler's allocation principle can be applied to any risk measure that is homogeneous of degree 1 and is continuously differentiable[7]. This is one of the most common allocation methods with very useful properties that allow us to study the performance of the portfolio.

[6] These properties were given in Michael Denault's work (1999)

[7] Defined in Annex D

4.1 RORAC compatibility

RORAC is a very popular measure that is being used in financial analysis that evaluates the return based on risk-based capital and it is calculated the following way:

$$E(RORAC) = \frac{E(Earnings)}{Capital\ at\ risk} \tag{10}$$

Definition 6 (Return on Risk Adjusted Capital). *Let the expected one-year income of the $i-th$-sub-portfolio be μ_i, such that $\sum_{i=1}^{n} \mu_i$ is the expected one-year income of the whole company, then the total portfolio Return on Risk Adjusted Capital is given by:*

$$E(RORAC(X)) = \frac{\sum_{i=1}^{n} \mu_i}{\pi(X)}. \tag{11}$$

If conditioned, then the i-sub-portfolio Return on Risk Adjusted Capital is:

$$E(RORAC(X_i|X)) = \frac{\mu_i}{\pi(X_i|X)} \tag{12}$$

Definition 7 (RORAC Compatibility)[8]. *Let X denote portfolio-wide profit/loss as in Definition 3, then we say that risk contributions $\pi(X_i|X)$ are $RORAC$ compatible if there are some $\epsilon_i > 0$ such that:*

$$RORAC(X_i|X) > RORAC(X) \Rightarrow \ RORAC(X + hX_i) > RORAC(X), \tag{13}$$

for all $0 < h < \epsilon_i$.

In other words, if there is $i - th$ sub-portfolio that has by its own a bigger RORAC than the RORAC of the portfolio where it is placed, than if we increase the amount invested in this sub-portfolio, the RORAC of the whole portfolio will be forced to go up.

4.2 Defining contribution of each sub-portfolio

As we saw previously, the SF calculates the risk capital of the whole company, consequently to build an optimal risk strategy it is necessary to answer the following question: *How much does each sub-portfolio i contribute to risk-based capital of the*

[8] To have a better understanding I invite the reader to look at Dirk Tasche paper (1999): *"Capital Allocation to Business Units and Sub-Portfolios the Euler Principle"* where a detailed example demonstrates.

whole company $\pi(X)$*?* From now on we denote $\pi(X_i|X)$ as the risk contribution **net of diversification effect** of i-sub-portfolio, such that $\pi(X) = \sum_{i=1}^{n} \pi(X_i|X)$.

Proposition 1. *Let* π *be a risk measure that is homogeneous of degree 1 and is continuously differentiable*[9]*. If there are risk contributions* $\pi(X_1|X) \ldots \pi(X_n|X)$ *that are RORAC compatible (see Definition 7), they can be determined as:*

$$\pi_{Euler}(X_i|X) = \pi(X_i) * \frac{\partial\pi(X)}{\partial\pi(X_i)} \tag{14}$$

where, $\pi_{Euler}(X_i|X)$ *is a uniquely allocated risk capital for the sub-portfolio* i *where* $i = 1, \ldots, n$ *and* n *is the number of lines of business of an insurance company.*

Remark 3. If π is a homogeneous of degree 1 and continuously differentiable risk measure, then using Euler allocation from equation (14), we produce Euler's contributions of each sub-portfolio. These contributions satisfy both properties stated in Definitions 3 and 7.

4.3 Euler allocation and sub-additive risk measures

From the Definition 2, risk measures that fulfill the sub-additive property are rewarded with portfolio diversification, therefore Euler's allocation principle is a very popular allocation method, since it considers the diversification effect[10] and the calculations that are involved are simple to understand.

Remark 4. Let π be a risk measure that is sub-additive, continuously differentiable and homogeneous of degree 1. After applying the allocation method given in formula (14) it is easy to obtain the following result:

$$\pi_{Euler}(X_i|X) \leq \pi(X_i) \tag{15}$$

This relation means that if we calculate Euler contributions of a risk measure, that is homogeneous and sub-additive, we conclude that the contribution to risk capital of a single sub-portfolio will never exceed the risk capital of the same sub-portfolio stand alone. This makes sense because of the benefit of the diversification effect.

[9] Defined in Annex D

[10] Remember that from Definition 2, we saw that a risk measure π is sub-additive if it follows equation (5).

5 APPLYING EULER'S METHOD

From Section 2, it was clear that the SF calculates the SCR for a company as a whole. In this calculation, many risks are involved and they are all correlated with each other at different levels as we will see.

5.1 General basis

We present some new notation that will be used in the following Sections. The SF has n risk modules, each one is represented with letter $i = 1, \ldots, n$, every i-th risk modules is composed by m_i risk submodules.

Let L_{ij} be the random variable that represents losses that can occur over the one-year period related with i-th risk modules and j-th risk submodule, and let $Y_{ij} = L_{ij} - E(L_{ij})$ be the random variable that represents the unexpected losses. The total risk that the company is facing Y can be calculated as:

$$Y = \sum_{i=1}^{n} \sum_{j=1}^{m_i} Y_{ij}, \tag{16}$$

where, $Y_i = \sum_{j}^{m_i} Y_{ij}$ such that

- $\sum_{j}^{m_i} Y_{ij}$: is the sum of risk submodules that exist in i-th risk modules;
- $\sum_{i=1}^{n} \sum_{j=1}^{m_i} Y_{ij}$: is the sum of n risk modules that exist in the whole portfolio.

As previously shown, if we want to transform the risk into risk capital, a proper risk measure should be applied. When EIOPA introduced Solvency II regime it proposed the Standard Formula (SF) which will allow us to calculate the risk-based capital (SCR). In this Section I will present the most important formulas of the SF and explain them:

$$SCR = BSCR + Adj + SCR_{OP}, \tag{17}$$

where, $BSCR$ is the Basic Solvency Capital Requirement, Adj is adjustment for the loss absorbing effect of technical provisions and deferred taxes and SCR_{OP} is the capital requirement for operational risk..

We assume that the BSCR is the only one that depends on the aggregation scheme allowing us to use Euler's method, Adj and SCR_{OP} depends on considerations that are made by the company.

$$BSCR = \sqrt{\sum_{iw} \rho_{iw} \cdot SCR_i \cdot SCR_w} + SCR_{intangible} \qquad (18)$$

where, ρ_{iw} is the correlation between risk modules available in delegated act, $SCR_i \cdot SCR_w$ are solvency capital requirement for risk modules and $SCR_{intangible}$ is the capital requirement for intangible asset (it is assumed that there is no intangible asset). To calculate the capital required for the i-th risk modules (SCR_i), a similar approach is applied:

$$SCR_i = \sqrt{\sum_j^{m_i} \sum_z^{m_i} \rho_{jz} \cdot SCR_j \cdot SCR_z} \qquad (19)$$

where, SCR_i is the solvency capital requirement for i-th risk modules, ρ_{jz} is the correlation between j-th and z-th risk submodule, respectively, and SCR_j, SCR_z are solvency capital requirement for risk submodule [11].

The choice of a risk measure within the overall Solvency system was not an easy task, two measures were presented Value-at-Risk (VaR) and Tail-Value-at-Risk (TVaR). After the analysis of pros and cons, it was stated that in practical work one of the most significant disadvantages using TVaR is the complexity and the scarcity of data about the tails of the distributions applicable to life or non-life insurance companies.[12] Therefore, to provide a good fit to the majority of insurance companies the SCR is calibrated using VaR of the basic own funds of an (re)insurance undertaking subject to a confidence level of 99.5% over one-year period.[13]

$$SCR_{ij} = VaR_{99.5\%}(Y_{ij})$$

5.2 Allocation Procedure

In the previous Section, we saw the definition of *coherent risk measures*, that is the necessary condition for the allocation procedure. In order for the SCR to be coherent,

[11] Note that each of the SCR for risk submodule is calculated according to SF.

[12] This could lead to an increase in modelling error and would make it difficult to calibrate any system designed to produce TVaR consistent with SF estimates, this problem can only be solved when more data is available about the company’s tail and it is only available in big insurance companies.

[13] This calibration is applied to each individual risk module and sub-module

since it is calibrated using VaR risk measure, the risk is assumed to be normally distributed.[14]

Proposition 2. *We start the allocation with BSCR, that is our final risk capital that comprises all the diversification effects according to the SF, to each of* risk modules *in such a way the condition* $BSCR = \sum_{i=1}^{n} SCR(Y_i|Y)$ *must hold.*[15] *From the Proposition D.1 (Annex D) we obtain:*

$$SCR(Y_i|Y) = SCR_i * \frac{\sum_{w=1}^{n} SCR_w * \rho_{i\,w}}{SCR_Y}, \quad (20)$$

where:

- $SCR(Y_i|Y)$ *: is the allocated risk capital to* $i - th$ *risk modules;*
- $\rho_{i\,w}$ *: is the correlation between the risk modules i and w, given by EIOPA;*
- SCR_i *: is the risk capital of* $i - th$ *risk module gross of diversification effect;*
- SCR_Y *: is the risk capital for the total company's risk* Y*, that is our BSCR.*

Proposition 3. *To realize the diversification effect that have occurred to* i*-th risk module, due to risk modules correlation, I will introduce the variable Allocation Ratio (*AR_i*):*

$$AR_i = \frac{SCR(Y_i|Y)}{SCR_i} = \frac{\sum_{w=1}^{n} SCR_w * \rho_{i\,w}}{SCR_Y} \quad (21)$$

In case of an insurance company the correlation between any different part of risks[16]*, given by EIOPA, is always less than one, which means that insurance companies are favored when they are diversifying their portfolio. This causes* $AR < 1$*, that comes from sub-additive property, as we shall see in the practical example.*

Proposition 4. *After an allocation of BSCR by macro-risks has been done, we can proceed to the allocation of our BSCR by each risk submodule, ensuring that the condition* $BSCR = \sum_{i=1}^{n} \sum_{j=1}^{m_i} SCR_{ij}$ *hold.*

$$SCR\left(Y_{ij}\middle|Y, Y_i\right) = SCR_{ij} * \frac{\sum_{z=1}^{m_i} SCR_{i\,z} * \rho_{ij,iz}}{SCR_i} * AR_i \quad (22)$$

where:

- $SCR\left(Y_{ij}\middle|Y, Y_i\right)$ *: is the allocated capital for* $j - th$ *risk submodule that is situated in* $i - th$ *risk module;*

[14] If it is not the case, then VaR does not satisfy the sub-additivity property, as it was shown by Artzner (1999)

[15] Note that each time I say allocated risk capital it is net of diversification effect.

[16] This is valid because the intangible risk is excluded from investigation.

- SCR_{ij} *: is the risk capital of* $j-th$ *risk submodule that is situated in* $i-th$ *risk module gross of diversification effect;*
- $\rho_{ij,iz}$ *: is the correlation between risk submodules* i *and* z *situated in* $i-th$ *risk module;*
- SCR_i *: is the risk capital of* $i-th$ *risk module gross of diversification effect;*
- AR_i *: is the allocation ratio for* $i-th$ *risk module.*

Remark 5: *This allocation is a general-specific process, where we start from the top level of our formula, in our case the BSCR, and we allocated it by more specific levels of risks, by risk modules (20) and by risk submodules (22), ensuring always that the sum of allocated capital would get our BSCR. The idea is to continue our allocation until we reach the lines of business, using similar methodology as in formula (22).*

6 RORAC OPTIMIZATION PROBLEM

It is clear that Solvency II regime forces insurance companies to implement risk based approaches, therefore, to build an optimal strategy, managers should, not only analyze the result of a particular LoB, but also evaluate the cost in terms of risk capital that this LoB requires and also the volatility that comes with it. After a fair allocation of the risk capital between lines of businesses, we can analyze the company's performance through the RORAC measure. It was proved that Euler's allocated contributions follow the *full allocation property* in sense of Definition 3 and are *RORAC compatible* by satisfying the condition given in Definition 7, where the second will allow us to proceed to the optimization problem.

6.1 Company's Risk Appetite

Risk appetite can be defined as *"The amount and type of risk that an organization is willing to take in order to meet their strategic objectives"*[17]. This means that similar organizations that have comparable portfolios can have very different risk appetites depending on their sector, location and objectives. In most cases Risk appetite is established by the top managers of the company. After its settlement, it should be always considered when any decision is made about the strategic plan of the company. From the study made, we can conclude that the most important strategies that non-life insurance

[17] Institute of Risk Management, UK.

companies can have are based on the optimization of the underwriting and reinsurance policies, since the quantity of risk-based capital necessary depends, mostly, on them[18]. Therefore, when dealing with the optimization of the RORAC it is better to focus on setting optimal reinsurance and underwriting policies.

6.2 Lines of business evaluation

The underwriting of insurance lines of business (LoB) is considered as a risky activity, since we cannot guarantee their returns. Consequently, if we want to compare them we cannot only analyze their returns, it is also important to look at the risk involved in those activities.

In previous Sections we saw that the allocation method applies satisfies both: Definition 3 (*Full allocation*) and Definition 7 (*RORAC compatibility*), that are essential for further investigation. With the purpose of comparing the LoB return in terms of risk capital we will use the following formulas:

$$E(RORAC_r) = \frac{E(Earnings_r)}{Allocated\ SCR_r}, \qquad \sigma(\mathrm{RORAC_r}) = \frac{\sigma(Earnings_r)}{Allocated\ SCR_r} \tag{23}$$

where:

- $E(RORAC_r)$ is expected value of the RORAC of $r-th$ LoB;
- $Allocated\ SCR_r$ is the allocated risk capital of $r-th$ LoB obtained by Euler's method;
- $E(Earnings_r) = Ps_r * (1 - \mathrm{E}(CR_r))$ with Ps_r being the estimate of the premiums to be earned by the insurance or reinsurance undertaking during the following 12 months of r-th LoB and $\mathrm{E}(CR_r)$ defined as the expected value of the combined ratio of r-th LoB.
- $\sigma(Earnings_r) = Ps_r * \sigma(CR_r)$ with $\sigma(CR_r)$ defined as the standard deviation of the combined ratio of r-th LoB.

Note that since we do not have a proper distribution for the combined ratio, it is necessary to apply a model which will allow us to calculate $\mathrm{E}(CR_r)$ and $\sigma(CR_r)$. Before going any

[18] Note that the capital-at-risk of market risk also plays a huge role, and has a high weight in SCR, but it is optimization should be performed through different methodologies that were not considered in this project.

further, I advise the reader first to understand the model of the combined ratio that is presented in Annex A.

6.3 RORAC maximization strategies

In non-life insurance, the risk that usually requires the most risk capital is the underwriting risk, therefore the purpose of the following Section is to show how it is possible to analyze different strategies, for example by changing the variables like reinsurance agreement, business volume or the premiums that the company charges, to determine the strategy that maximizes the company's RORAC.

As previously shown, each company has different risk appetite, therefore it is necessary to build the optimization problem in most general form possible, so that this procedure can be adapted to all non-life companies. The difference of the risk appetite in different companies can be seen in the proposal of different limits. Therefore, similar companies with same resources but with different risk appetites could have different strategies to optimize their portfolio.

For the evaluation of the different strategies, that were set by the managers, we suggest the following optimization problem that derived from a mean-variance model:

$$Maximize\colon\ E(RORAC)$$

Subject to:

$$a < SCR < b,$$

$$\nu_i < P_{S_i} < \varepsilon_i,$$

$$CV_i < \alpha,$$

where:

- $E(RORAC)$: is the Expected value for the RORAC of the company;
- SCR : is the Solvency Capital Required for the whole company and it should be between values a and b (limits);
- P_{S_i}: is the future premium of $i-th$ LoB and it should be between values ν_i and ε_i (limits for $i-th$ LoB);
- CV_i : is the coefficient of variation of $i-th$ LoB;
- α : could be considered as the the risk appetite of the company.

The main goal of this problem is to always maximize the global E(RORAC) of the company and considering the risk appetite of the company. We can impose limits for global SCR, business volume for each LoB (P_{S_i}) and coefficient of variation of each LoB, in such a way that the company's risk appetite will not be exceeded. From the management point of view there are not so many strategies that a company is willing to take so analyzing each of them one by one is not that time consuming.

7 APPLICATION TO A NON-LIFE INSURANCE

In this Section I will put into practice to a real non-life insurance the methodologies that were presented in last sections. This will allow us to witness the likely difficulties that can arise with the theoretical framework applied to real data and the possible solutions and simplifications that were used to address them.

7.1 Application of Euler method for Underwriting Risk

Consider a non-life insurance that calculates its SCR using Standard Formula with the parameters given by EIOPA and where the following results were obtained:

BSCR	27,786,074

Macro Risk	SCR_i
Market	7,573,591
Default	558,862
Health	9,756,580
Non Life	21,954,662

Table 1: Capital Requirement for i-th risk module gross of the diversification.

From Table 1 it is easy to see that the sum of the capital requirement of each risk module is different from the BSCR because of the risk correlation consequently, to allocate the risk capital we can apply the formula (20) where the correlation between risk modules is given in Annex B and the following risk module allocation is obtained:

Macro Risk	***SCR(Yi|Y)***
Market	4,263,266
Default	319,168
Health	4,139,739
Non Life	19,063,900
Total	**27,786,074**

Table 2: Allocation of risk capital between risk modules.

Table 2 shows that after incorporating the diversification effect, which lowers the risk capital, it is possible to sum the allocated risk module capital and the result will be company's BSCR. From the Formula (21) we have:

Macro Risk	AR_i
Market	0.56
Default	0.57
Health	0.42
Non Life	0.87

Table 3: First level allocation ratio.

Remark 6. It is clear that since the risk measure fulfills the *sub-additive* property the following inequality will always occur: $AR \leq 1$.

Formula (22) will allow us to continue allocate the risk capital by risk submodules, e.g. in **Non-Life** we have:

-First we calculate the capital *gross of diversification effect*:

Micro Risk	SCR_{ij}
Premium and Reserve	18,516,265
Lapse	-
CAT	8,043,084

Table 4: Capital required for j-th risk submodule gross of the diversification.

-Now, applying formula (22), where the correlation between risk submodule is given in Annex B.2:

***j-th* Micro Risk**	$SCR_{Euler}(SCR_{NL\,j} \mid BSCR, SCR_{NL})$	$AR_{NL\,j}$
Premium and Reserve	15,032,729	0,81
Lapse	-	-
CAT	4,031,170	0,50
Total	**19,063,900**	

Table 5: Allocation of risk capital between j-th mirco risk (net of the diversification).

Remark 7: The sum of NL risk submodule capital allocation given in Table 5 match with allocated non-life risk capital specified in Table 2 thus we can conclude that the **full allocation property** given in Definition 3 is fulfilled. By implementing the same methodology in different levels of Standard Formula to underwriting risk (NL and Health), we will allocate the risk capital by LoB (simplest form that SF allow)[19]:

-For Health we have:

LoB	Health SLT	Health CAT	Health Non SLT	$SCR_{Health\ LoB}$
1. Medical Expenses	-	-	4,916	4,916
2. Income Proteccion	-	629,568	1,300,614	1,930,182
3. Workers' Compensation	547,503	-	1,657,137	2,204,641
Total	**547,503**	**629,568**	**2,962,667**	**4,139,739**

Table 6: Allocated BSCR by LoB each sub-portfolios of Health risk module.

-For Non-Life we have:

LoB	NL Premium and Reserve	NL Lapse	NL CAT	$SCR_{Non-life\ LoB}$
4.Motor vehicle liability	11,572,959	-	35,052	11,608,011
5.Other motor	908,151	-	-	908,151
6.MAT	33,521	-	-	33,521
7.Fire	1,885,959	-	3,991,993	5,877,952
8.Third party liability	548,662	-	4,126	552,788
9.Credit	-	-	-	-
10.Legal expenses	17,849	-	-	17,849
11.Assistance	64,612	-	-	64,612
12.Miscellaneous	1,016	-	-	1,016
Total	**15,032,729**	**-**	**4,031,170**	**19,063,900**

Table 7: Allocated BSCR by LoB each sub-portfolio of Non-Life risk module.

The last column of above tables also shows that the **full allocation property** given in Definition 3 is fulfilled.

7.2 Possible simplifications for allocating other risk module by LoB

In Non-Life insurance company, the risk group that require the most risk capital is the Underwriting (Health and Non-Life risk modules), as we saw previously to allocate it by LoB we could use the Euler's method and we obtain the results given in Table 7 and Table 6. From Figure 4 we can see that there are also Default and Market risk modules that also needed to be allocated by LoB.

[19] Note that to understand better the allocation process, analyze the Annex C that shows the example of a complete calculation of the allocation process for NL risk module

Table 2 shows the risk-based capital net of diversification effect, but for risk modules, to allocate the risk capital that covers these risk modules by LoB we need to use some simplifications. I will present some possible simplifications that can be used.

7.2.1 Market risk allocation by LoB

For the LoB Workers' Compensation, there is an obligation to associate assets with responsibilities, in such a way that there would be sufficient assets to cover the liabilities. The (re)insurer's main goal is to guarantee that the investments made have the average duration adjusted to the liabilities, this will allow, from an economic point of view, the reduction of the interest rate risk.

Having that in mind, the approach that was used for the allocation of the SCR Market was the following:

For LoB Workers' Compensation, since there is a direct connection between LoB and the assets, it was chosen the ones that represent that LoB. For the rest, it was used the following simplification:

1st. In each LoB we sum the Best Estimate (BE) for Premiums and Reserves;

2nd. We calculate how much percentage each LoB's BE sum represent in of total of BE of the company;

3rd. We multiplied this percentage by **SCR Market Net of Diversification**, that is given in Table 2.

The following result is obtained:

LoB	SCR Market
1. Medical Expenses	4,664
2. Income Proteccion	237,209
3. Workers' Compensation	1,546,977
4. Motor vehicle liability	1,911,893
5. Other motor	158,606
6. MAT	1,596
7. Fire	332,608
8. Third party liability	68,869
9. Credit	0
10. Legal expenses	17
11. Assistance	800
12. Miscellaneous	26
Total	**4,263,266**

Table 8: Allocated Risk Capital of Market Risk by LoB *Net of Diversification*.

7.2.2 Counterparty Default risk allocation by LoB

From the delegated act, we realize that the Counterparty Default risk module is related to the risk of not fulfillment of the obligations of the different counterparties. In order to allocate the risk capital that covers this module we present the following approach:

- 1st Calculate the reinsurance recoverable for each LoBs and calculate the percentage of total of reinsurance recoverable;
- 2nd We multiplied this percentage by **SCR Default Net of Diversification**, that is given in Table 2.

The following result is obtained[20]:

LoB	SCR Default
1. Medical Expenses	50,567
2. Income Proteccion	11,089
3. Workers' Compensation	0
4. Motor vehicle liability	114,977
5. Other motor	0
6. MAT	0
7. Fire	142,535
8. Third party liability	0
9. Credit	0
10. Legal expenses	0
11. Assistance	0
12. Miscellaneous	0
Total	319,168

Table 9: Allocated Risk Capital of Default risk modules by LoB *Net of Diversification*.

7.3 Allocated BSCR by LoB

After a proper allocation of each risk modules in SF by LoB is done, it is possible to present the allocated BSCR, according to full allocation property, by simply summing the capital required for each risk module for each LoB:

[20] Note that Table 9 shows many LoBs with $SCR_{Default}$ equals to zero, this happens because only four LoBs have reinsurance capital recoverable. It is also important to mention that the reinsurance bankruptcy risk is the most noticeable, however, the counterparty default risk includes other types of risks (e.g mortage loans).

LoB	BSCR by LoB
Medical Expenses	60,148
Income Proteccion	2,178,480
Workers' Compensation	3,751,618
Motor vehicle liability	13,634,881
Other motor	1,066,757
MAT	35,117
Fire	6,353,096
3rd party liability	621,657
Credit	0
Legal expenses	17,866
Assistance	65,412
Miscellaneous	1,042
Total	**27,786,074**

Table 10: Allocated BSCR by LoB.

7.4 Allocated SCR Operational and Adjustments by LoB

SCR Operational is the risk of loss that arises from inadequate or failed internal processes, from personal and systems or from external events. This risk module is designed to address operational risks to the extent that these have not been explicitly covered in other risk modules.

The calculation of the adjustment for the loss-absorbing capacity of technical provisions and deferred taxes should ensure that there is no double counting of the risk mitigation effect provided by future discretionary benefits or deferred taxes.

Both capital requirements were calculated using the methodology given in the delegated act, but to allocate the capital by LoB, we used the same methodology as when we allocated Market risk module and the following results were obtained:

LoB	Adj by LoB	$SCR_{Operational}$
1. Medical Expenses	-9,798	4,297
2. Income Proteccion	-498,279	218,505
3. Worker's Compensation	-721,120	316,225
4. Motor vehicle liability	-4,016,100	1,761,137
5. Other motor	-333,165	146,099
6. MAT	-3,353	1,470
7. Fire	-698,672	306,381
8. Third party liability	-144,666	63,439
9. Credit	0	0
10. Legal expenses	-36	16
11. Assistance	-1,681	737
12. Miscellaneous	-54	24
Total	**-6,426,925**	**2,818,329**

Table 11: Allocated Adj and SCR Operational by LoB.

7.5 Allocated SCR

Finally, after the allocation of $BSCR,\ Adj$ and $SCR_{Operational}$ is done, we can apply the Formula (17) to calculate the SCR of each LoB, and we obtain the following outcome:

LoB	SCR by LoB
1. Medical Expenses	54,647
2. Income Proteccion	1,898,706
3. Worker's Compensation	3,346,723
4. Motor vehicle liability	11,379,917
5. Other motor	879,691
6. MAT	33,234
7. Fire	5,960,805
8. Third party liability	540,430
9. Credit	0
10. Legal expenses	17,845
11. Assistance	64,468
12. Miscellaneous	1,011
Total	**24,177,478**

Table 12: Allocated SCR by LoB.

From Table 12 we can notice that the LoBs that require the most risk capital are Worker's Compensation (3), Motor Vehicle Liability (4) and Fire (7) from where we can conclude that they incorporate the most risk therefore need more capital to cover it.

But these LoBs also represent the majority of the business volume of the company therefore the results are of greater importance.

7.6 Return per unit of Risk

In Section 6.3 we saw the structure of the optimization problem, which will allow the company to set strategic goals to maximize the usage of risk-based capital and maximize the return per unit of risk, that is the maximization of the company's E(RORAC). The starting point is to apply the method presented in Annex A which will allow us to model the combined ratio and after apply the formula (23) to each LoB.

It is important to remember that this is a small insurance company so due to insufficient size in some of the LoBs, we do not have a proper distribution for claims and premiums. These LoBs show to be outsiders that can be ignored. Therefore, only LoBs: 2,3,4,5,7 and 8 will be evaluated.

After applying the model, explained in Annex A, and through formula (23), we obtain the following result:

LoB	E(RORAC)	σ(RORAC)	SCR
2. Income Proteccion	32%	26%	1,870,603
3. Worker's Compensation	2%	28%	3,465,322
4. Motor vehicle liability	-9%	12%	11,235,537
5. Other motor	71%	19%	861,776
7. Fire	81%	11%	6,015,629
8. Third party Liability	113%	22%	531,254
Whole Portfolio	**42%**	**7%**	**23,980,121**

Table 13: Expected value and Standard Deviation of RORAC per LoB

Table 13 shows the E(RORAC) and σ(RORAC) of each LoB and of the whole portfolio.We realize that only LoB 4 shows negative E(RORAC)[21]. *Does this mean that this is the only LoB that does not create value to the company?* To answer which LoBs create in fact value to the company, we need to study not only the E(RORAC), but also consider the volatility (σ(RORAC)) and if these values are acceptable according to company's risk appetite plan. In order to have a better understanding, we will plot values of the table 13 into a graph:

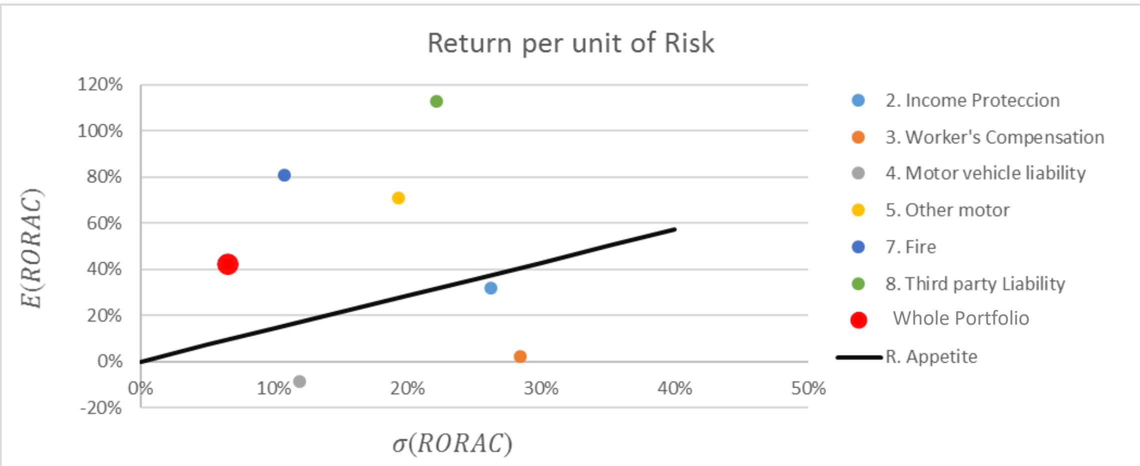

Figure 5: Return per unit of risk.

From Figure 5, it is easier to compare the return per unit of risk of different LoBs. The black line separates LoBs that create value to the company from those that do not and the

[21] This means that by investing in LoB 4 investors got 9% less than the initial capital invested in that LoB.

slope of the line comes from the risk appetite of the company that, by the assumption, was decided to be seventy percent[22].

We can see that even LoBs with positive E(RORAC) could not create value to the company because of the volatility involved with these returns, this is the case of LoB 2 and 3. From the whole portfolio, red point, we can see that globally this insurance company is fulfilling the risk appetite plan that was imposed and has a low volatility with decent return of risk-based capital. Note that the volatility of LoBs by themselves is higher than when they are together, this is justified by the fact that when we are analyzing the whole portfolio the sample size of independent policies grows.

It is important to mention that these results should be interpreted carefully, since there are other gains/costs involved that this model does not reflect, for example taxes or gains/losses of the investments, which could lower the rate of return, thus for sake of this investigation we will ignore these gains/costs. Even so, these results allow us to have an idea of overall performance of the portfolio, if it fulfills the company's risk appetite and if we change underwriting and reinsurance policies what will occur with the E(RORAC) and σ(RORAC) of each LoB and of the whole company.

7.7 Portfolio Optimization

From Figure 5, some conclusions could be taken about the rate of return and the volatility. Since each LoB has its own particularities, we need to study each one of them so that the strategies that are proposed make sense.

- *Looking at LoB 2, what could be done so that this LoB would be not only profitable but also acceptable by the Risk appetite strategy of the company?*

After a careful data analysis, we could say that it has a relatively high claim volatility, so a different reinsurance contract could be proposed. Also, the commissions paid for the intermediaries which sell this product seems to be higher than it should be, this increases the costs and consequently lowers the rate of return. Therefore, it is recommended for this

[22] This means that only LoBs that have coefficient of variation (CV) between $0 < CV < 70\%$, create value to the company. This comes from the limit created in Section 6.3 in the optimization problem. (The definition of coefficient of variation is in Annex D).

LoB to renegotiate the limits with the reinsurance and try to change the commissions given to intermediaries.

Doing this type of study, we will get closer to the optimal strategy for each LoB and consequently for the whole company. This strategy should maximize the rate of return and satisfy the Risk Appetite that was set at the beginning. For example, the limits that should be satisfied when we build our strategies should come from the type of strategy and the capital (tier 1) available of an insurance company, for example purpose we set the following limits:

$$Maximize\colon\ E(RORAC)$$

Subject to: $$20{,}000{,}000 < SCR < 30{,}000{,}000,$$

$$0.8 * \nu_i < P_{S_i} < \varepsilon_i * 1.2,$$

$$CV_i < 70\%,$$

Constant on a reinsurance program.

We observe that SCR needs to be controlled in that interval, because this insurance company cannot accept SCR higher than 30M€ since it doesn´t possess enough Tier 1[23] own funds or it just does not want to have more that 30M€ as risk-based capital. Also, the business volume should not vary too much so that realistic scenarios could be build, thus a twenty percent of future business volume variation seems to be fair.

The idea of the optimization procedure is very simple, from the study of our current situation we select realistic strategies that we would like to verify. After we recalculate all the models until we get the results as in Figure 5 but for our new possible strategy. Doing it to all the selected strategies we can compare the graphs and select the one that fulfills the company's criteria.

[23] These are the highest quality own funds and it was set in the Solvency II that for the SCR, at least 50% of the own funds will need to be tier 1.

8 CONCLUSION

It is important to understand that Solvency II is a new regime that was only implemented in the beginning of the year 2016, consequently some insurance companies lack experience of working in this regime, which make these types of investigations crucial to understand it. This work should help the risk managers and the administrators of an insurance company to: determine different strategies that could be built according to their risk appetite; uncover the risk concentration in different LoBs; study the SCR and return per unit of risk per LoB and finally for pricing determination. I also showed that risks should be measured carefully, because poor risk management in financial institutions could lead to severe consequences.

From this project we conclude that companies that use the Standard Formula can still have an idea of the amount of SCR that each LoB requires. This amount could not reflect the reality[24], but it should not differ much. Due to the allocation method and risk measure special properties we can go further with our investigation and compare the return per unit of risk of different LoBs and by implementing the risk appetite, we can realize which LoBs create more value to the company. It was also shown that it is possible to perform E(RORAC) optimization procedure using the same calculations which will allow us to build different graphs, similar of the one that I showed in Figure 5, and choose the type of strategy that best satisfies the Risk Appetite criteria that we imposed initially.

Also, it is important to mention that in real life the optimization problem is a very complicated process because after we determine which LoBs create value to the company and which do not, we cannot simply eliminate LoBs that are unprofitable. We live in a very competitive market, so when clients buy insurance products they usually try to buy a package of products, which means that different products of different LoBs (profitable and unprofitable) are sold to the same client and for them having one product without another does not make sense, so if one insurance company does not have this package he will find another one. It is easy to see it from an example, from Figure 5, we realize that LoB 5 creates value to the company and LoB 4 does not, but LoB 5 cannot exist without

[24] The calculation of the real amount of the allocation is only possible if the SCR is calculated by the usage of internal models.

LoB 4, this means if we attract more client to buy products from LoB 5, consequently the business of LoB 4 will also grow.

The main limitations that occurred during this investigation were that this is a small insurance company, with some particularities, so the data that was given showed to be insignificant for some LoBs, which made them impossible to study. Also, I understood that EIOPA proposed for SF correlation table between LoBs and for business management it is crucial to have a more specific analysis, that is by looking at the company's products. Therefore, it is necessary to have correlation tables between different products. Which will allow to do this type of investigation, but by product. This will enforce even more the importance of this work.

During my research, I noticed that there are not so many investigations about the allocation procedures applied to SCR. Since there is a decent amount of other methods that could be used to allocate capital, it would be interesting to compare results of other allocation procedures applied to SCR and compare the results.

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APPENDIX

A Combined Ratio model

Calculation of $E(RORAC_r)$ *and* $\sigma(RORAC_r)$ *found in equation (23) [see Section 6.2].*

Let us define a proper model for the combined ratio. Assume that we have some LoB with n contracts spread over t years numbered $j = 1, \dots, n$. Also, let $P_j = P_1, \dots, P_n$ be the earned premiums over t years of j-th contract and $T_j = T_1, \dots, T_n$ be the r.v. annual aggregated claim cost plus other expenses, assuming that all the contracts are independent we can formulate the Combined Ratio (CR) as:

$$CR = \frac{\sum_{j=1}^{n} T_j}{\sum_{j=1}^{n} P_j}$$

By taking the expectation and the variance of the Combined Ratio we have:

$$E(CR) = \frac{\sum_{j=1}^{n} E(T_j)}{\sum_{j=1}^{n} P_j}$$

$$Var(CR) = \frac{\sum_{j=1}^{n} Var(T_j)}{\left(\sum_{j=1}^{n} P_j\right)^2}$$

From the formulae above, it is easy to calculate the expected value and the variance of the combined ratio, assuming we know the distribution of T_j. However, this is a very difficult task, therefore we can write a following simplification:

$$T_j = I_j * X_j,$$

where,

- I_j is a dummy variable, that is equal to zero when a contract has no claim and is equal to one when a contract has one or more claims;
- X_j is annual aggregated claim cost plus other expenses of one or more claims.

If I_j and X_j are independent, we can write expected value and variance of T_j as:

$$E(T_j) = E(I_j)E(X_j);$$

$$Var(T_j) = Var(I_jX_j) = E\left[(I_jX_j)^2\right] - \left[E(I_jX_j)\right]^2$$
$$= E^2(I_j) * E^2(X_j) - \left[E(I_j) * E(X_j)\right]^2$$
$$= \left(E^2(I_j) - E(I_j)^2 + E(I_j)^2\right) * \left(E^2(X_j) - E(X_j)^2 + E(X_j)^2\right)$$
$$- E(X_j)^2 E(I_j)^2$$
$$= Var(I_j)Var(X_j) + Var(X_j)E(I_j)^2 + Var(I_j)E(X_j)^2.$$

It is clear that $I_j \sim \text{Bernoulli}(q_j)$.

Let us assume that $q_j = P_j * \lambda$ where P_j is the premium and λ is the claim propensity per premium unit, than an estimator of λ is:

$$\lambda^* = \frac{\sum_{j=1}^{n} I_j}{\sum_{j=1}^{n} P_j}.$$

Since the majority of the contracts has one or zero claims in a one-year period, we assume that X_j are *i.i.d.* and for a *future* portfolio with premiums $P_1', \ldots, P_m'$[25] we have the following estimates for the expected value and variance of the combined ratio:

$$E(CR)^* = \frac{\sum E(T_i)^*}{\sum P_i'} = \frac{\sum P_i' \lambda^* E(X)^*}{\sum P_i'} = \lambda^* E(X)^*$$

$$Var(CR)^* = \frac{\sum Var(T_i)^*}{(\sum P_i')^2}.$$

Recall that $I_j \sim \text{Bernoulli}(q_j = P_i'\lambda)$, therefore by taking the variance and the second absolute moment of Bernoulli distribution, it is possible to simplify the equation for $Var(T_i)$:

$$Var(T_i) = Var(I_i) * Var(X_i) + Var(X_i) * E(I_i)^2 + Var(I_i) * E(X_i)^2$$
$$= q_i(1 - q_i)(\,Var(X_i) + E(X_i)^2) + q_i^2 Var(X_i)$$
$$= P_i'\lambda(Var(X_i) + E(X_i)^2) - (P_i'\lambda E(X_i))^2.$$

Thus,

[25] The estimate of the future premiums could be considered the premiums received in last year.

$$Var(CR)^* = \frac{\lambda^*(Var(X)^* + E(X)^{2*})}{\sum P_i'} - \frac{\sum P_i'^2}{(\sum P_i')^2}(\lambda^* E(X)^*)^2.$$

B Data

These parameters are an average of EU that were calculated by EIOPA.

Table B1: Correlation between risk modules.

Corr ij	SCRmkt	SCRdef	SCRlife	SCRhealth	SCRnl
SCRmkt	1	0.25	0.25	0.25	0.25
SCRdef	0.25	1	0.25	0.25	0.5
SCRlife	0.25	0.25	1	0.25	0
SCRhealth	0.25	0.25	0.25	1	0
SCRnl	0.25	0.5	0	0	1

Table B2: Correlation between Non-Life risk submodules.

Corr NL	NL P&R	NL Lapse	NL CAT
NL P&R	1	0	0.25
NL Lapse	0	1	0
NL CAT	0.25	0	1

Table B3: Correlation between Non-Life LoBs.

Corr LoB	1	2	3	4	5	6	7	8	9
4.Motor vehicle liability	1	0.5	0.5	0.25	0.5	0.25	0.5	0.25	0.5
5.Other motor	0.5	1	0.25	0.25	0.25	0.25	0.5	0.5	0.5
6.MAT	0.5	0.25	1	0.25	0.25	0.25	0.25	0.5	0.5
7.Fire	0.25	0.25	0.25	1	0.25	0.25	0.25	0.5	0.5
8.Third party liability	0.5	0.25	0.25	0.25	1	0.5	0.5	0.25	0.5
9.Credit	0.25	0.25	0.25	0.25	0.5	1	0.5	0.25	0.5
10.Legal expenses	0.5	0.5	0.25	0.25	0.5	0.5	1	0.25	0.5
11.Assistance	0.25	0.5	0.5	0.5	0.25	0.25	0.25	1	0.5
12.Miscellaneous	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	1

Table B4: Correlation between Health risk submodules.

Corr Health	Health SLT	Health NonSLT	Health CAT
Health SLT	1	0.5	0.25
Health NonSLT	0.5	1	0.25
Health CAT	0.25	0.25	1

Table B5: Correlation between Health LoBs.

Corr LoB	Medical Expenses	Income Proteccion	Workers' Compensation
Medical Expenses	1	0.5	0.5
Income Proteccion	0.5	1	0.5
Workers' Compensation	0.5	0.5	1

C Non-Life Premium and Reserve allocation for LoB 4

C1 Allocation of risk capital between risk modules

The results found in Table 2 can be deduced by applying the formula found in (17) for a Non-Life scenario, thus:

$$SCR_{Euler}(SCR_{NL}|\text{BSCR}) = \text{SCR}_{\text{NL}} * \frac{\sum_{w}^{q} SCR_{w} * \rho_{NL,w}}{BSCR} = 19{,}063{,}900€$$

where,

- $\sum_{w}^{q} SCR_{w} * \rho_{NL,w} = 21{,}954{,}662(1) + 7{,}573{,}591(0.25) + 558{,}862(0.5) + 0,$
- the correlation between NL and other risk modules are given in table B1 of annex B.

C2 Allocation Ratio

To calculate the Allocation Ratio of Table 3, we apply the following formula found in (18) to obtain:

$$AR_{NL} = \frac{19{,}063{,}900}{21{,}954{,}662} = 0.87$$

To allocate risk capital by Non-Life risk submodules, we apply (19), where "gross" is defined as the gross of diversification effect, and so:

$$SCR_{Euler}(SCR_{P\&R}|SCR_{NL}) = SCR_{P\&R-Gross} * \frac{\sum_{w}^{q} SCR_{w-Gross} * \rho_{i,w}}{SCR_{NL-Gross}} * \frac{\text{SCR}_{\text{NL-Euler}}}{\text{SCR}_{\text{NL-Gross}}}$$
$$= 15.032.729€,$$

where,

- $SCR_{P\&R-Gross} = 18.516.265;$

- $\frac{\sum_w^q SCR_{w-Gross} * \rho_{i,w}}{SCR_{NL-Gross}} = \frac{18{,}516{,}265(1)+8{,}043{,}084(0.25)}{21{,}954{,}662}$;
- $\frac{\mathrm{SCR_{NL-Euler}}}{\mathrm{SCR_{NL-Gross}}} = AR_{NL} = 0.87$;
- the correlation between NL and other risk modules is given in table B2, annex B

C3 Allocation of risk capital for Motor Vehicle Liability

To calculate the allocation of risk-based capital for LoB 4. Motor Vehicle Liability (Motor), we use:

$$SCR_{Euler}(SCR_{P\&R-Motor} | SCR_{NL-P\&R})$$
$$= SCR_{Motor-Gross} * \frac{\sum_w^q SCR_{w-Gross} * \rho_{i,w}}{SCR_{P\&R-Gross}} * \frac{SCR_{P\&R-Euler}}{SCR_{P\&R-Gross}}$$
$$= 11{,}572{,}959€$$ [26],

where,

- $SCR_{Motor-Gross} = 18{,}516{,}265$;
- $\frac{\sum_w^q SCR_{w-Gross} * \rho_{i,w}}{SCR_{P\&R-Gross}} = \frac{1{,}892{,}912 * 0.5 + \cdots + 2{,}019 * 0.5}{18{,}516{,}265}$;
- $\frac{SCR_{P\&R-Euler}}{SCR_{P\&R-Gross}} = \frac{15{,}032{,}729}{18{,}516{,}265}$;
- the correlation between NL and other risk modules is given in table B3 of annex B.

[26] Risk capital to cover Premium and reserve risk for LoB 4. Motor Vehicle Liability, as shown in Table 7.

D Definitions

Definition D.1: *A risk measure π is homogeneous of degree k if for some h > 0 it satisfies:*

$$\pi(hX) = h^k \pi(X)$$

The interest comes for risk measures that are positively homogeneous, because this is one of the properties that *coherent measures* have. When decomposing risk measures, positive homogeneity is very important property that ensures that when all the allocated sub-portfolios are multiplied by the same factor *h > 0*, the overall portfolio is also multiplied by the same factor.

Proposition D.1 (Euler's Formula): *Let π be a homogeneous risk measure of degree k. If π is partially differentiable with respect to* m_i, $i = 1, \dots, n$, *then:*

$$\pi(X) = \frac{1}{k} * (m_1 \frac{\partial\pi(X)}{\partial m_1} + \cdots + m_n \frac{\partial\pi(X)}{\partial m_n})$$

Applied to Proposition 1 we have: $k = 1$ and $m_i \frac{\partial\pi(X)}{\partial m_i} = \pi(X_i) * \frac{\partial\pi(X)}{\partial\pi(X_i)}$. Each element $m_i \frac{\partial\pi(X)}{\partial m_i}$ denote the *risk contribution of asset* i, that is the amount of risk contributed to the global risk by investing m_i in *asset i*. It is easy to see from above formula that $\frac{\partial\pi(X)}{\partial m_i}$ represents the *marginal risk*, which means the marginal impact on the total risk from change in size of portfolio *i.* Applied to SF the partial derivatives can be presented as:

$$\frac{\partial SCR_G}{\partial SCR_r} = \frac{\sum_{r=1}^{z} SCR_r * \rho_{t\,r}}{SCR_G},$$

where, SCR_G is the global risk-based capital, SCR_r is the sub-module risk-based capital and $\rho_{t\,r}$ are the correlation between sub-module *t* and other *r* sub-modules ($r = 1, \dots, z$).

Definition D.2: *Coefficient of variation (CV) is a measure of dispersion of the data, it represents the degree of volatility of the amount compared with the expected value and it is calculated by* $CV = \sigma/\mu$.

This financial measure is used to compare the degree of variation of two risky investments. It is easy to observe that the higher CV the less return we get from unit of risk of that investment.