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DISSERTATION

PREDICTIVE MODELS OF THE PROBABILITY OF DEFAULT: AN EMPIRICAL APPLICATION

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Abstract

This study intends to conduct a survey of Probability of Default models to listed companies. The methodologies of Merton (1974) model, Accounting model and Hybrid were addressed. We tested a sample of 172 American companies in the sectors of Consumer Products, Distribution, Manufacturing and Telecommunications in which 82 entered into default. For each methodology, the predictive ability was tested with Type I and II errors. The results suggests that the Hybrid model, i.e. a combination of market models and accounting analysis, have a better performance in the classification of credit default than each model individually.

Keywords: Merton; Credit Risk; Accounting Model; Hybrid Model; Black-Scholes; Probability of Default

Resumo

Este estudo tem como objetivo realizar uma pesquisa dos modelos de previsão do incumprimento a empresas listadas em bolsa. Foram abordadas as metodologias do modelo de Merton (1974), modelo Contabilístico e Híbrido. Testou-se uma amostra de 172 empresas presentes no mercado Americano dos setores do Consumo, Distribuição, Produção e Telecomunicações nas quais 82 entram em incumprimento. Para cada metodologia, a capacidade preditiva foi testada através dos erros Tipo I e II. Os resultados sugerem que o modelo Híbrido, i.e., a combinação de modelos de mercado e análise contabilística, confere maior poder de precisão na classificação de incumprimento, ao invés de cada modelo individualmente.

Palavras-Chave: Merton; Risco de Crédito; Modelo Contabilístico; Modelo Híbrido; Black-Scholes; Probabilidade de Incumprimento

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1. Introduction

Evaluating the performance of companies and their ability to pay off their debt is a fundamental activity for commercial banks. An essential ingredient of this activity consists of measuring the credit risk of loans due to the uncertainty of the counterparty to fulfill its obligations. The creditworthiness of a company affects its cost of capital and credit spreads. In a credit transaction it is necessary to estimate the components that affect the credit risk of a borrower: the probability of default; the exposure at default; and the recovery rate. This study addresses the problem of evaluating the probability of default.

A popular technique for forecasting defaults are the accounting models (see, e.g. Beaver, 1966; Altman, 1968; Ohlson 1980). These models evaluate the creditworthiness of borrowers through accounting and financial ratios, and usually provide satisfactory results. A different approach, that has revolutionized the way that the probability of default is measured, was proposed in the seminal paper of Merton (1974). This is a structural model, using the concept of a call option where the strike price is the debt of the company, and the price of the underlying asset is the company's assets. A shortcoming of these models is that they can only be implemented for listed companies. Nevertheless, structural models have been the subject of various developments over the years (see, e.g. Moody's KMV model).

In a recent study, Tudela and Young (2003) suggest combining a structural model with an accounting model to obtain better accuracy in forecasting defaults. This "hybrid" model showed a very good performance. The objective of this study is to replicate this finding using an original data set. This data set was constructed using information from Moody's Ultimate Recovery Database and Datastream. It is shown that a hybrid model resulting from the combination of a structural model and an accounting model does indeed give better accuracies with respect to the individual models.

The remainder of this study is organized as follows. Chapter 2 presents a literature review. Chapter 3 describes the framework of the models used in this study: the structural and the accounting models. Chapter 4 describes the data used in the empirical work. The forecasting accuracies, such as type I[1] and II[2] errors, are reported and discussed in Chapters 5. Finally, Chapter 6 provides some concluding remarks.

[1] Type I error: error of rejecting the null hypothesis when it is true. In other words, it means classifying a company as 'defaulted' when it is 'non-defaulted'.

[2] Type II error: error of not rejecting the null hypothesis when it is false, i.e., classifying a company of 'non-defaulted' when it is 'defaulted'.

2. Literature Review

Several articles explore models of credit analysis to companies, but not all reach the same conclusion. Therefore, this section aims to review some types of credit analysis models such as structural, the reduced form, accounting based and hybrid models.

In 1968 and most recently in 2000, Altman presents a model of accounting analysis to evaluate the probability of bankruptcy of companies. This is a traditional approach where the author uses five financial ratios to evaluate companies' health.

On the other hand, in recent years several authors have attempted to assess the default risk of companies using models based on the theory of options, the Black Scholes (1973) model. This is the case of Merton (1974) which is a structural one. Although several authors refer that this type of model presents quite significant predictions, others consider the opposite stating that traditional (i.e. Altman Z-score and Ohlson's O-score) models can provide better information of the state of enterprises. Apart from structural models, reduced form models have been developed. These models introduce explicit assumptions regarding the dynamics of the variables of default which are modeled independently from the structural characteristics of the enterprise.

2.1 Merton Model (1974)

According to the literature, there are several advantages of using structural models. Sundaram (2001) states that the fact that being a model based on economic information adds value because it uses information from market prices and the incorporation of equity prices makes it a prospective model (as opposed to models based on accounting information). He claims that the model appears to have some predictive power for default and ratings transition.

On the other hand, the model has several assumptions that are mostly infringed in practice. From the structure of the capital, where if it is too complex the application of the model can become extremely challenging, as Sundaram (2001) and Elizalde (2005) mentioned.

Elizalde (2005) presents in its paper a guide to the literature on Merton (1974) suggesting possible extensions in the model for future development. The author states that one of the problems of the Merton's model is the time restriction until maturity of the debt default. For instance, in the original model it is not possible that failure happens before maturity.

Leland (2002) examines the differences in default probabilities generated by two types of structural models. In the first group, called the 'exogenous default threshold' approach, Merton (1974) and Black and Scholes (1973) models are used. According to the author, the model works for long-term horizons, albeit for short maturities the predictive default frequency is low. In addition, endogenous models predict a rise in the probabilities of failure with the costs of default and a decrease with the maturity of the obligation, unlike the default probabilities derived from exogenous models which do not vary under these parameters.

Huang and Huang (2002) propose a calibration approach based on historical data on defaults. They show that one can obtain consistent estimates of credit spreads through various economic considerations through a structural framework for the assessment of credit risk.

Vassalou and Xing (2004), as a way to realize the effect that the default risk has on stock returns, have conducted a study to assess default probabilities for individual companies using the Merton (1974) model. They concluded that both the size and the book-to-market have a strong impact on default risk.

On the other hand, Jarrow (2011) argues that structural models are not useful neither for pricing nor risk management due to inconsistency with the balance of the credit market. As a result, he states that structural models are not useful to infer default probabilities.

Hamilton, Sun and Ding (2011) compare a point-in-time (PIT) and a through-the-cycle (TTC) structural models. A PIT uses all available and pertinent information in a given date to estimate a firm's expected probability of default over some time horizon. This model is very reactive to all news affecting the firm but also highly volatile. On the other hand, a TTC reflects a firm's long-run, a credit risk trend. They argue that a complete credit risk management system requires both PIT and TTC credit risk measures.

2.2 KMV-Merton Model

Crosbie and Bohn (2003) state that the most effective way to measure the default derives from models using both market prices and financial statements. In their study, they explain how to calculate the expected default frequency (EDF) with the KMV model. This consists on estimating the market value and volatility of companies assets, calculate the distance to default and turn it into expected default frequency using it for an empirical distribution of defaults.

Wang et al. (2009) study the measurement of credit risk of listed companies using different types of credit risk models such as KMV model. The results show that the KMV model can reflect the risk of default.

Bohn, Arora and Korablev (2005a) validate the performance of the model MKMV EDF in its ability to differentiate good firms from bad firms in North America's market. They compare the KMV model with other popular alternatives such as rating agencies, Z-scores and the simplest version of the Merton model. Tests indicate that the EDF model MKMV had a good and consistent performance across different time periods and different sub-samples based on the company's size and credit quality. According to the authors, the MKMV model is superior compared to other alternatives until the date of the study (2005).

Bharath and Shumway (2008) examine the accuracy and the contribution of the Distance to Default model (DD) and conclude that this model is useful in default predictions but they state that it is not statistically sufficient. They also report that the structural models provide a useful guidance to build predictive models of default.

2.3 Reduced-form Models

The main characteristic of a reduced form model is given by two assumptions: an exogenously given process for the firm's default time; a given process exogenously for recovery. On the word of Jarrow (2011) *"these are asymmetric information assumptions"*.

Jarrow and Turnbull (1995), as a way to change the assumptions of symmetric information of structural models, introduce the reduced form models. Other authors, such as Lando (1998), Jarrow et al. (1997) Duffie and Singleton (1999), have extended these type of models using market prices of companies, i.e. bonds or Credit Default Swaps (CDS), to extract both the probability of default and credit risk dependencies. They assume that the market is the only source of useful information to structure the credit risk companies.

According to Elizalde (2005), the reduced form models require the link between credit risk and the information about the financial situation of companies incorporated in the structure of its assets and liabilities. *"A reduced form model, despite not compromising the theoretical question of complete information, suffers from other weaknesses including the lack of a clear economic rationale to define the nature of the infringement procedure."* (Arora, Bohn, & Zhu, 2005)

One advantage of the structural models versus the reduced form is the ability to link the company's value to its probability of default, where reduced-form cannot. Notwithstanding, Arora, Bohn and Zhu (2005) argues that reduced form models are flexible in functional form, which could be a strength and a weakness. However, *"this flexibility in their functional form can result in a model with strong engagement of the sample properties but poor predictive abilities outside the sample."* (Arora, Bohn, & Zhu, 2005)

Uhrig-Homburg (2002) reports that the reduced form models appear to be ideal for evaluating complex credit derivatives, they easily adapt to different situations and there are several variations that are flexible enough to be calibrated to arbitrary data market.

As stated by Teixeira (2011) reduced form models do not condition default on the firm's value. Hence, the assumptions about the dynamics of the default variables are modeled independently from the characteristics of the company, asset volatility and leverage.

Comparing reduced form models with the structural models, the latter takes into account symmetric information. According to Jarrow (2011), both the management and the market have the same information about the process of asset value. He states that there is no adverse selection or moral hazard in structural models. In contrast, and shown by

Jarrow and Protter (2004), reduced models are consistent with asymmetric information in credit markets.

2.4 Accounting Models

We can define the accounting analysis models as retrospective, because they indicate the health of a company at a certain time. According to Beaver (1966) the analysis of financial ratios can be useful to predict bankruptcy up to 5 years before it happens.

Altman’s (1968) accounting analysis model is one of the most popular. He uses five financial ratios to measure the probability of a company enters into bankruptcy, through the Multiple Discriminant Analysis (MDA). According to Ohlson (1980), the MDA presents some problems in predicting business failures: the need for equality of variance-covariance matrices of the predictors of the two groups of companies; intuitive interpretation of the Z-score and the arbitrary nature of the process of matching the sample, since *"failed and non-failed companies are categorized according to criteria such as size and industry, and these tend to be of any arbitrary shape."* (Ohlson, 1980) Other methodologies have been addressed in the analysis of financial ratios, such as Probit and Logit.

According to Ferrando and Blanco (1998) one of the advantages of the Logit model is the possibility of categorizing the independent variables, admitting not only the economic and financial ratios or metric variables, and allowing the use of non-financial or qualitative information.

Lo (1986) states that, for purposes of parameter estimation, the Logit has been shown to be more robust than Discriminant Analysis. Nevertheless, under certain distributional assumptions both procedures yield consistent estimates. Furthermore, the Logit model has simpler representation and mathematical treatment compared to the Probit one.

2.5 Hybrid Models

Sobehart and Stein (2000) use financial statements and market information to build a hybrid model to predict the failure of public companies based on methods of nonlinear regression. According to the authors, *"market information provides an overview, and has already proved to be useful in predicting defaults. It also reflects changes in the investors' preferences not related to the solvency of the company."* (Sobehart & Stein, 2000) On the other hand, they argue that the financial statements contain specific information that demonstrates the financial state of the company.

Tudela and Young (2003) developed a hybrid approach to evaluate the default risk of listed companies through a version of Merton's (1974) model with the use of accounting data. The results show that the combination of the accounting data to the default probabilities derived from Merton model produces better results compared to the implementation of each model individually. In the Merton model developed by the authors, a barrier option is used allowing the default to occur at any time rather than specifically at the end of the maturity of debt.

Benos and Papanastasopoulos (2007) combine fundamental analysis with structural analysis in a hybrid model for measuring credit risk. These two authors develop the initial Merton model assuming a more complex structure of capital (adjusted for dividend payments) introducing randomness to the time of default, allowing a fractional recovery at the time of default. Thus, using financial ratios and the distance to the default of the structural model, a hybrid model is estimated through a Probit regression model. Its main conclusion is that the accounting data and financial ratios contain information that provides additional value to the Merton model.

Another type of approach is used by the risk assessment company Kamakura Corporation (2011), with the structural model to evaluate private companies (Merton model), the Jarrow Merton Hybrid model and also the Jarrow Chava model. The latter is a statistical model hazard that includes financial information and ratios of stock market prices. Here the failure can occur at any time during the period of analysis. In Jarrow Merton, a statistical hazard model is used with the same explanatory variables as the Jarrow Chava model and incorporates the Merton structural model as an additional variable.

3. Theoretical Framework

3.1 Structural Models

This section describes the model of credit rating companies based on Black and Scholes (1973), the Merton model (1974) and Moody's KMV. In general terms, this model uses the concept of a call option where the strike price is the debt of the company and the price of the underlying asset is the company's assets, so if this is lower than the value of the debt then the company is considered to enter into default.

According to Cossin and Pirotte (2001), it is considered a structural approach since it depends on the sharing of value between the shareholders and debt holders, which is the structure of the company itself.

One of the constraints of the model is that the assets of the company are neither traded nor observable and to test the model, parameters have to be estimated implicitly. *"This creates a strong joint hypothesis in empirical testing of the models."* (Jarrow, 2011)

3.1.1 Merton Model (1974)

According to the Merton's (1974) model of credit analysis, the shareholders of a company by contracting a loan are actually transferring the control of the company to creditors. However, if the liabilities of the company are liquidated we have the ability to retrieve this control, so the value of equity can be seen as the price of an option on assets of the company, with the strike price corresponding to the value of debt.

The model thus assumes that the capital structure of the company comprises equity and a zero coupon bond with maturity T and face value of D, where the values in period *t* are entitled V_E and $z(t,T)$ respectively for $0 \leq t \leq T$. The asset value of the company V_A is the sum of equity V_E and debt D values.

Therefore, equity represents a call option on assets of the company with maturity T and strike price of D. If at maturity T the value of the assets of the company V_A is insufficient to pay the face value of debt D, i.e. $V_A < D$, then the company goes into default.

Otherwise, if $V_A > D$ then the company does not enter into default and shareholders receive $V_A - D$.

The assumptions that Merton (1974) adopts are the following:

- the company can only default in T;
- the absence of transaction costs, bankruptcy costs, taxes or problems of indivisibility of assets;
- transaction in continuous time; unrestricted borrowing and lending at a constant rate r;
- no restrictions on short selling of assets;
- the company's value is invariant under changes in capital structure (Modigliani-Miller theorem);
- the company’s asset value follows a diffusion process:

$$dV_A = rV_A dt + \sigma_A V_A dW \tag{1}$$

Where σ_A is the asset volatility (relative) and W follows a Brownian motion.

The Payoffs of equity holders and bondholders at T under the assumptions of this model are respectively, $max\ \{V_A - D, 0\}$ and V_A – V_E, i.e.

$$V_A = max\ \{V_A - D, 0\} \tag{2}$$

$$z\,(T,T) = V_A - V_E \tag{3}$$

Applying Black-Scholes pricing formula, the value of equity V_E at t $(0 \leq t \leq T)$ is given by:

$$V_E(V_A, \sigma_A, T - t) = V_A N(d_1) - De^{-r(T-t)}N(d_2) \tag{4}$$

Where $N(.)$ is the cumulative normal distribution and d_1 and d_2 are given by:

$$d_1 = \frac{\ln\left(\frac{V_A}{D}\right) + \left(r + \frac{1}{2}\sigma_A^2\right)(T-t)}{\sigma_A\sqrt{T-t}} \tag{5}$$

$$d_2 = d_1 - \sigma_A\sqrt{T-t} \tag{6}$$

The probability of default in T is given by:

$$P\,[V_A < D] = N(-d_2) \tag{7}$$

Thus, the value of debt at t is equal to $z\,(t,T) = V_A - V_E$.

In order to implement Merton's model, the value of company assets V_A and its volatility σ_A must be estimated, transforming the structure of debt the company into a zero coupon bond with maturity T and face value of D.

3.1.2 KMV-Merton Model

The KMV-Merton model was developed by KMV in the late 80s, being successfully marketed by KMV in a successfully manner until it was acquired by Moody's in April 2002.

The basic methodology of this model is discussed in Crosbie and Bohn (2003), Kealhofer (2003a), Vasicek (1984) and Bharath and Shumway (2004).

This model is based on the Merton model (1974) where the major difference lies in the use of an empirical distribution of defaults of companies to transform the Distance to Default and Expected Default Frequency (EDF).

The KMV model considers the company as a perpetual entity, which issues debt on a continuous basis. Secondly, deals with different classes of liabilities, so it is able to better capture details of the capital structure. The model assumes that the company goes into bankruptcy when achieves the Default Point (DPT) and the debt plus half of long term debt to short-term. Lee (2011) indicates that there may be different DPT depending on the country of companies to study.

Like in Merton (1974), in KMV model, the equity of the company is considered as a call option based on the underlying price of the company with a strike price equal to the face value of the debt of the company with time-to-maturity T. Therefore, the value of equity is a function of the total value of the firm that can be described by the formula of Black-Scholes-Merton.

Here, two particular assumptions are taken. The first is that the value of the asset follows a Brownian motion such that:

$$dV_A = \mu V_A dt + \sigma_A V_A dZ \tag{8}$$

Where V_A is the total value of the assets of the company, μ is the expected continuously compound return V_A, σ_A is the volatility of the company and dZ is a standard Wiener process. The second assumption is that the company just issued a bond that matures in T.

According to the put-call parity, the value of the debt of the company is equal to the risk free discount bond minus the put option written on the company, similarly with the strike price of the face value of debt maturing in T. The Merton model considers the value of equity as:

$$V_E = V_A N(d_1) - e^{-rT} D N(d_2) \tag{9}$$

Where V_E is the market value of equity of the firm, D is the face value of the debt of the company, r is the instantaneous risk-free rate, $N(.)$ is the cumulative normal distribution, d_1 is given by:

$$d_1 = \frac{\ln\left(\frac{V_A}{D}\right) + \left(r + \frac{1}{2}\sigma_A^2\right)T}{\sigma_A \sqrt{T}} \tag{10}$$

And d_2 as:

$$d_2 = d_1 - \sigma_A\sqrt{T} \tag{11}$$

The KMV-Merton model uses two important functions. The first is the equation of Black-Scholes-Merton, which expresses the value of the equity of the company. The second relates the volatility of the assets of the company with the volatility of equity. Under Merton's assumptions, the value of equity is a function of the value of the company and time. From the result of a stochastic calculus known as the Ito's lemma we have:

$$\sigma_E = \frac{V_A}{V_E}\left(\frac{\partial V_E}{\partial V_A}\right)\sigma_A \tag{12}$$

In the Black-Scholes-Merton model can be shown that $\frac{\partial V_E}{\partial V_A} = N(d_1)$, so that under Merton's assumptions the volatility of the company and equity are related by:

$$\sigma_E = \frac{V_A}{V_E}N(d_1)\sigma_A \tag{13}$$

Where d_1 is defined by Equation 10. Equations 9 and 13 provide two simultaneous equations that can be solved for V_A e σ_A. In the KMV-Merton model the value of the option is seen as the total value of the equity of the company, however the underlying asset is not directly observable. Thus, while V_A needs to be deducted, V_E is easily observable in the market by multiplying the number of outstanding shares outstanding by the stock price. Similarly, the volatility of equity can be estimated and the volatility of the assets must be deducted.

The first step to implement the KMV model is to estimate σ_E, choose a forecasting horizon and measure the face value of debt. It is common to use historical data to estimate σ_E but it can also be estimated through the option implied volatility. The second step is to choose a forecasting horizon and measure the face value of debt by collecting the total book value of liabilities. Third step, collect values of risk-free rate and market value of equity of the company. The fourth step is to solve simultaneously Equations 9 and 13 numerically for values V_A and σ_A. Once obtained these values Distance to Default (DD) can be calculated by:

$$DD = \frac{\ln\left(\frac{V_A}{D}\right) + \left(\mu - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}} \tag{14}$$

The corresponding probability of failure, usually referred to as EDF is given by:

$$D = N\left(-\left(\frac{\ln\left(\frac{V_A}{D}\right) + \left(\mu - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}}\right)\right) \tag{15}$$

Equation 15 also refers to the failure rate for a given level Distance-to-default. When the Distance-to-Default descends, the company becomes more likely to default on its obligations.

According to the KMV model, default occurs when the value of assets reaches a value between total liabilities and the value of short-term debt. This point is called the default point (DPT) and is considered by KMV model as the debt plus half of long term debt to short-term.

Lee (2011) proposes a new methodology based on genetic algorithms to solve the optimal default point of the KMV model. In his empirical study they compare the GA-KMV model with the QR-KMV model and finally the KMV model. The results indicate that GA-KMV model is better suited than the other models. One of the limitations was the construction of an empirical distribution of defaults which creates limitations in predicting defaults.

3.2 Accounting Model

In this section we intend to describe the methodology used by Altman (1968), and subsequently present the one used in this work, the logit model. Generically, Altman (1968) proposes a model of credit analysis based on financial ratios using discriminant analysis (MDA).

3.2.1 Altman's Z-score (1968)

In 1968, Altman proposed a model of Multiple Discriminant Analysis (MDA) as a measurement of the probability of bankrupt companies.

The author selected two groups totaling sixty-six companies where thirty-three of them were bankrupt, and tries to explain bankruptcy through various financial ratios. The companies were selected according to industry and size. The group of bankrupt companies contained manufacturers who have failed between 1946 and 1965.

The author has selected a list of twenty-two financial ratios divided into five categories (liquidity, profitability, productivity, solvency and activity) based on their popularity and potential relevant for the study. After creating this five groups a set of five ratios were selected, taking into account which best explained the prediction of corporate bankruptcy using the following methodology: observation of the statistical significance of various alternative functions; evaluation of inter-correlations between relevant variables; observation of the prediction accuracy; judgment of the analyst.

The final discriminant function obtained was as follows:

$$Z = 1.2\,X_1 + 1.4\,X_2 + 3.3\,X_3 + 0.6\,X_4 + 1.0\,X_5 \tag{16}$$

Were:

X_1 Represents the liquidity ratio (Working Capital / Total Assets)

X_2 Represents the profitability ratio (Retained Earnings / Total Assets)

X_3 Represents the solvency ratio (EBIT / Total Assets)

X_4 Represents the solvency ratio (Market Value of Equity / Book Value of Total Debt)

X_5 Represents the activity ratio (Sales / Total Assets)

Z Represents Altman's Z-score

After estimating the discriminant function, the value of "Z" is quantified and companies supposed to enter bankruptcy are compared to the ones that actually failed. In the initial study in 1968, the author used a z-score cutoff of 2,675 by which if the z-score of a company to assess was below this value it was considered bankrupt and above this value the opposite.

3.2.2 Logistic Regression

“There are two primary reasons for choosing the logistic distribution. First, from a mathematical point of view, it is and extremely flexible and easily used function, and second, it lends itself to a clinically meaningful interpretation.” (Hosmer & Lemeshow, 2000)

This method estimates the probability of occurring a particular event as a function of a set of independent variables $x_1,\ldots, x_k$. Since the response variable (or dependent variable) is binary, we assigned the value one to indicate failure and zero to indicate the opposite.

In order to estimate the probability of a particular event *i* of the response variable to be "successful", $P(Y_i = 1) = \hat{p}_i$, the following logistics function is used:

$$\hat{p}_i = \frac{e^{\hat{\beta}_0+\hat{\beta}_1 x_{1i}+\cdots+\hat{\beta}_k x_{ki}}}{1 + e^{\hat{\beta}_0+\hat{\beta}_1 x_{1i}+\cdots+\hat{\beta}_k x_{ki}}} \qquad (17)$$

So that,

$$logit(\hat{p}_i) = \ln\left(\frac{\hat{p}_i}{1-\hat{p}_i}\right) = \hat{\beta}_0 + \hat{\beta}_1 x_{1i} + \cdots + \hat{\beta}_k x_{ki} \qquad (18)$$

The coefficients $\hat{\beta}_i$ are estimated by maximum likelihood method. “*In a very general sense the method of maximum likelihood yields values for the unknown parameters which maximize the probability of obtaining the observed set of data.”* (Hosmer & Lemeshow, 2000)

3.2.2.1 Testing for the Significance of the Coefficients

After fitting the logistic regression model is necessary to assess the significance of the fitted model, such as the significance of the regression coefficients. The significance of the fitted model is obtained by testing the following hypotheses:

$$H_0: \beta_1 = \beta_2 = \cdots = \beta_k = 0 \quad vs \quad H_1: \exists_i: \beta_i \neq 0 \ (i = 1, \ldots k) \tag{19}$$

In order to predict the probability of the event, from the independent variables in the model, the fitted model must be statistically significant, a condition that is satisfied when the alternative hypothesis is true.

The statistic to test the significance of the logistic regression model is called the *likelihood ratio test* and is given by:

$$G^2 = -2\ln\left(\frac{L_0}{L_C}\right) \overset{a}{\sim} X_k^2 \tag{20}$$

Where L_0 is the likelihood function for the model containing only the constant and L_C is the likelihood function for the full model. The null hypothesis H_0 is rejected if the observed G^2 p-value is less than the size of the test, α.

Another test has been suggested to identify the independent variables that significantly influence the response variable, this is the Wald test. This is obtained by comparing the maximum likelihood estimate of the slope parameter $\hat{\beta}_i$ to an estimate of its standard error, conditioned by the estimated coefficients of the other values:

$$\begin{gathered} H_o: \beta_i = 0 \mid \beta_0, \beta_1, \beta_{i-1}, \beta_{i+1}, \beta_k \ vs \ H_1: \beta_i \neq 0 \mid \\ \beta_0, \beta_1, \beta_{i-1}, \beta_{i+1}, \beta_k \ (i = 1, \ldots, k) \end{gathered} \tag{21}$$

The test statistic has the following expression:

$$T_{Wald_i} = \frac{\hat{\beta}_i}{\widehat{SE}(\hat{\beta}_i)} \overset{a}{\sim} N(0,1) \tag{22}$$

Where $\hat{\beta}_i$ is the estimator of β_i and $\widehat{SE}\left(\hat{\beta}_i\right) = \sqrt{\hat{\sigma}^2\left(\hat{\beta}_i\right)}$ is the estimator of the standard deviation of $\hat{\beta}_i$. The null hypothesis is rejected for each of the tests on β_i when the respective p-value is less than the size α of the test.

4. Description and Data Treatment

4.1 Sample

This work analyses, 172 non-financial listed companies operating in the United States of America, from the sectors of Consumer Products, Distribution, Manufacturing and Telecommunications.

Companies that met their obligations were selected through the Bloomberg website and those which defaulted were selected from Moody's URD (Ultimate Recovery Database). The last ones have default between 1989 and 2010. Ideally, it should have been chosen a sample of companies within a smaller period of time, but due to lack of available data it was considered the mentioned period. According to Moody's URD, a company is considered in default if it is in a situation of bankruptcy, distressed exchanges, and nonpayment of interest. (Moody's, 2007)

The accounting information of enterprises that have met and not met their obligations was extracted through Datastream platform. Due to missing information in Datastream, several companies from various sectors that were originally on the list of defaulted and non-defaulted were not included.

Table 1 summarizes, by sector and state of credit default, the list of companies included in the study. For further detail, Table 16 in the Appendix lists the companies participating in the study.

Table 1. Summary of Companies Included in the Study (by Sector)

Sector	Defaulted?		Total
	No	Yes	
Consumer Products	16	22	38
Distribution	28	25	53
Manufacturing	14	16	30
Telecommunications	32	19	51
Total	90	82	172

5. Implementation, Analysis and Results

This chapter describes the methodology implemented in the calculation of the probability of default (PD) for the models of Merton, Accounting Analysis and Hybrid.

For the calculation of PD a time period of 1 year was considered, i.e., calculating the probability of defaulting on its credit obligations over the next 12 months. Thus, for compliant businesses was considered accounting and market information for the year 2012, since it was the last observable data. For defaulted companies, we picked up the latest annual data available before entering into default, for example, in Eddie Bauer company for the 1 year PD assessment, which defaulted on 17-06-2009, we used accounting information relating to 2008.

5.1 Merton Model Approach

The implementation of the Merton model was based on Löffler and Posch (2007) "Wiley Credit Risk Modeling using Excel and VBA", chapter 2.

Here, we set the horizon $T - t$ to one year, we take the equity value V_E from the stock market, set liabilities D equal to book liabilities, and use the one-year yield on US treasuries as the risk-free rate of return. Then we must estimate the equity volatility σ_E.

We chose to base our estimate on the historical volatility measured over the preceding 252 days. Stock prices are collected and then converted to daily log returns. Then we need to compute the standard deviation of daily log returns which are multiplied by the square root of 252, giving us the annualized equity volatility σ_E.

5.1.1 Default Probability

For the default probability formula, we need the expected change in asset values. With the asset values obtained from the accounting sheet, we can apply the standard procedure for estimating expected returns with the Capital Asset Pricing Model (CAPM). We obtained the beta of the assets regarding a market index, and then apply the CAPM formula for the return on an asset i:

$$E[R_i] - R = \beta_i(E[R_M] - R_f) \tag{23}$$

With R denoting the simple risk-free rate of return ($R = e^r - 1$). We took the S&P 500 index return as a proxy for R_M, the return on the market portfolio.

We obtained an estimate of the assets' beta by returning the asset value returns on S&P 500 returns. Like in Löffler and Posch (2007) and for simplification we assumed a standard value of 4% for the market risk premium $[R_M] - R$, then we determine μ as $\ln(1 + \beta_i(E[R_M] - R_f))$.

Now that we have estimates of the asset volatility, the asset value and the drift rate, we can determine the Default Probability.

For each company in the sample we compute d_1 using Equation 10 and d_2 using Equation 11 with initial unknown values of V_A and σ_A. The asset value V_A and asset volatility σ_A are then computed solving the two Equations 9 and 13 simultaneously, minimizing the sum of squared differences between model values and initial values of V_E and σ_E (observable values). Then we compute Distance to Default using Equation 14 and therefore the Probability of Default with Equation 15.

The logistic regression was performed where the independent variable the Probability of Default by the Merton model and dependent variable the failure which can assume the values zero or one.

Table 2. Omnibus Tests of Merton Model Coefficients

		Chi-square	df	Sig.
Step 1	Step	41.410	1	.000
	Block	41.410	1	.000
	Model	41.410	1	.000

Table 3. Merton Model Summary

Step	-2 Log likelihood	Cox & Snell R Square	Nagelkerke R Square
1	196.660[a]	.214	.285

a. Estimation terminated at iteration number 4 because parameter estimates changed by less than .001.

According to Table 4, 74.4% of events were correctly predicted. Furthermore, the model correctly predicted 82.2% of compliant events and 65.9% events of default.

Table 4. Classification Table between Defaulted and Non-defaulted Companies (with Merton model)

Observed			Predicted [a]		
			Default		Percentage Correct
			0	1	
Step 1	Default	0	74	16	82.2
		1	28	54	65.9
	Overall Percentage				74.4

a. The cut value is .500

According to Table 5, we find that the Merton variable is statistically significant (using the logit model as regressor) estimator for the probability of default where the independent variable assumes a positive value, indicating that the higher the Merton variable, the higher the probability of default.

Table 5. Variables in the Equation (with Merton model)

		B	S.E.	Wald	df	Sig.	Exp(B)
Step 1[a]	Merton	3.700	.664	31.096	1	.000	40.451
	Constant	-1.017	.227	20.049	1	.000	.362

a. Variable(s) entered on step 1: Merton.

5.2 Accounting Model

In this chapter we used financial ratios to estimate the probability of default within one year (in the Merton model is used in the same way one year). In Table 6, it is described each variable used in the economic-financial model. These ratios were selected through Beaver (1966).

Table 6. Economic-financial Ratios Used in the Estimation of the Models

Notation	Ratio
V1	Cash-flow to Sales
V2	Cash-flow to Total Assets
V3	Cash-flow to Total Debt
V4	Net Income to Sales
V5	Net Income to Total Assets
V6	Net Income to Total Debt
V7	Current Liabilities to Total Assets
V8	Long-term Liabilities to Total Assets
V9	Current plus Long-term Liabilities to Total Assets
V10	Current Assets to Total Assets
V11	Working Capital to Total Assets
V12	Cash to Current Liabilities
V13	Current Ratio (Current Assets to Current Liabilities)
V14	Current Assets to Sales
V15	Working Capital to Sales
V16	Total Assets to Sales

Source: Beaver (1966)

In order to perceive the ability to predict failure with economic and financial ratios we used the logit model. Here, we assume the dependent variable as a dummy variable that can take the value zero in case of fulfillment of obligations and one if defaults.

Table 7. Omnibus Tests of Accounting Model Coefficients

		Chi-square	df	Sig.
Step 1	Step	82.061	14	.000
	Block	82.061	14	.000
	Model	82.061	14	.000

Table 8. Accounting Model Summary

Step	-2 Log likelihood	Cox & Snell R Square	Nagelkerke R Square
1	150.741[a]	.386	.515

a. Estimation terminated at iteration number 8 because parameter estimates changed by less than .001.

Through Table 9 we find that using financial ratios as estimators of the failure we successfully predicted 82.1% of credit events. Compared to Merton, this model was slightly more robust. Unlike Merton, who successfully predicted 82.2% of compliant events and 65.9% of non-compliant, the use of financial ratios predicted 77.9% and 86.6%, respectively.

Table 9. Classification Table between Defaulted and Non-defaulted Companies (with Economic Financial Ratios)

Observed			Predicted[a]		
			Default		Percentage Correct
			0	1	
Step 1	Default	0	67	19	77.9
		1	11	71	86.6
	Overall Percentage				82.1

a. The cut value is .500

In Table 10 we find that not all accounting variables are significant for the logistic regression model. In fact, only the V3, V10, V12 and V15 variables are statistically significant for a 5% significance level. Notwithstanding, we could not say that V6 is not rejected with a p-value of 0.072, because is very close to 0.05.

On an economic basis, the variable V3 behaves as expected, i.e., a decrease of this variable represents an increase in the probability of default. The V10 and V12 variables follow the same rational and present a contrary view regarding the probability of default. On the other hand, variable V15 presents a contrary relationship with the expected. An increase in this variable would generate a decrease in the probability of default. All the other variables in the logit model are not significant.

Table 10. Variables in the Equation (with Economic Financial Ratios)

		B	S.E.	Wald	df	Sig.	Exp(B)
Step 1[a]	V1	6.360	7.925	.644	1	.422	577.972
	V2	16.304	14.526	1.260	1	.262	1.204E7
	V3	-27.999	12.389	5.108	1	.024	.000
	V4	-9.179	8.029	1.307	1	.253	.000
	V5	-10.116	14.115	.514	1	.474	.000
	V6	21.321	11.845	3.240	1	.072	1.817E9
	V7	1.792	1.130	2.516	1	.113	6.004
	V8	1.847	1.332	1.925	1	.165	6.343
	V10	-4.016	1.815	4.893	1	.027	.018
	V12	-2.169	.876	6.138	1	.013	.114
	V13	-.015	.200	.006	1	.940	.985
	V14	2.151	1.372	2.456	1	.117	8.592
	V15	2.165	.996	4.720	1	.030	8.715
	V16	-1.043	.637	2.678	1	.102	.352
	Constant	1.571	1.489	1.113	1	.291	4.810

a. Variable(s) entered on step 1: V1, V2, V3, V4, V5, V6, V7, V8, V10, V12, V13, V14, V15, V16.

5.3 Hybrid Model

The Hybrid model includes the estimates of probability of default of the Merton model and the economic-financial ratios in a logit regression.

Table 11. Omnibus Tests of Hybrid Model Coefficients

		Chi-square	df	Sig.
Step 1	Step	99.712	15	.000
	Block	99.712	15	.000
	Model	99.712	15	.000

Table 12. Hybrid Model Summary

Step	-2 Log likelihood	Cox & Snell R Square	Nagelkerke R Square
1	133.090[a]	.448	.597

a. Estimation terminated at iteration number 8 because parameter estimates changed by less than .001.

As we can see in Table 13, and as was initially expected, the prediction of the percentage of events, 83.3%, was higher compared to each model used separately. The Hybrid model predicted correctly 86.0% of non-defaulted companies and 80.5% of defaulted companies.

Table 13. Classification Table between Defaulted and Non-defaulted Companies (with Hybrid Model)

Observed			Predicted[a]		
			Default		Percentage Correct
			0	1	
Step 1	Default	0	74	12	86.0
		1	16	66	80.5
	Overall Percentage				83.3

a. The cut value is .500

According to Table 14 we find some consistency with previous models, i.e. the Merton variable remains statistically significant with a positive contribution to an increasing risk of default.

Similarly, the V3 and V12 variables remained significant in the model adopting the same relationship to the increase or decrease of risk of default.

In contrast, the variables V10 and V15 failed to contribute to the prediction of good and bad companies. Here V6 has a p-value of .093 and V10 has one of .089, and was not rejected at a 10% significance level.

Note also that the significance of all other variables remained the same in the model, i.e., they are not statistically significant.

Table 14. Variables in the Equation (with Hybrid Model)

		B	S.E.	Wald	df	Sig.	Exp(B)
Step 1[a]	Merton	4.273	1.149	13.840	1	.000	71.731
	V1	6.964	9.052	.592	1	.442	1057.944
	V2	6.892	13.400	.265	1	.607	984.163
	V3	-19.251	9.365	4.225	1	.040	.000
	V4	-8.872	9.131	.944	1	.331	.000
	V5	-3.307	13.295	.062	1	.804	.037
	V6	15.155	9.021	2.823	1	.093	3818974.697
	V7	-.093	1.366	.005	1	.946	.912
	V8	.583	1.367	.182	1	.670	1.791
	V10	-3.455	2.031	2.893	1	.089	.032
	V12	-2.111	.928	5.173	1	.023	.121
	V13	-.019	.215	.007	1	.931	.982
	V14	2.250	1.437	2.452	1	.117	9.487
	V15	1.452	1.193	1.483	1	.223	4.273
	V16	-.914	.743	1.513	1	.219	.401
	Constant	1.465	1.591	.848	1	.357	4.328

a. Variable(s) entered on step 1: Merton, V1, V2, V3, V4, V5, V6, V7, V8, V10, V12, V13, V14, V15, V16.

In order to better compare the models, Table 15 shows a summary of the results of the three models applied in this study, the Merton model, Accounting model and Hybrid model. The latter was the one that had a better overall performance and better predictive accuracy on the non-defaulted events. Notwithstanding, the accounting model was the one with a better predictive performance on the defaulted events.

Table 15. Summary of Models' Predictive Accuracy

Observed			Predicted [a] Default 0	Predicted [a] Default 1	Percentage Correct
Merton	Default	0	74	16	82.2
		1	28	54	65.9
	Overall Percentage				74.4
Accounting	Default	0	67	19	77.9
		1	11	71	86.6
	Overall Percentage				82.1
Hybrid	Default	0	74	12	86.0
		1	16	66	80.5
	Overall Percentage				83.3

a. The cut value is .500

In Tudela and Young (2003), the Hybrid model predicted 77.09% of events of credit while the structural approach predicted 76.75% and the Accounting model predicted 42.37%. In the same way, in Benos and Papanastasopoulos (2007) the Hybrid model was the one that had a better performance in the classification of credit events, comparing with a Merton approach and an Accounting model. This study is therefore in line with this literature.

6. Conclusions, Limitations and Future Research

In this study we analyzed some forecasting techniques of credit failure to listed companies, namely the Merton model, an Accounting model using financial ratios and a Hybrid model that incorporates the two methodologies. The information used was collected in the Moody's URD database and Bloomberg. This study suggests that the Hybrid model confers greater ability to prediction of default comparing with the use of Merton's model and Accounting in a separate way. This is consistent with Tudela and Young (2003).

It should be noticed in this study the limitations concerning the database of companies analyzed. Regarding the number of companies, for future research it is suggested a more extensive companies' database to calculate the probability of default, thus increasing the robustness of the study. More, due to lack of available data it was considered the period between 1989 and 2010 for defaulted companies and 2012 for non-defaulted. Ideally, it should have been chosen a sample of companies within a smaller period of time, providing a further uniform economic climate. On the other hand, using a single companies' sector could bring more accurate results to this study.

Also it is noteworthy that the Merton model used also has several limitations in its assumptions, so it would be interesting for future research, for the same study base to apply a model that, for example, allows the default before maturity.

We can thus conclude that the Merton model, despite showing some unrealistic assumptions, gave power to the failure prediction in this study.

7. Appendix

Table 16. List of Companies Used in the Study

Company	Net Sales	Defaulted?	Sector	Default Date
Acme United Corp.	$ 84,370.00	No	Consumer Products	n.d.
Blyth Inc.	$ 1,179,514.00	No	Consumer Products	n.d.
Capstone Cos Inc.	$ 8,363.00	No	Consumer Products	n.d.
Central Garden and Pet Co.	$ 1,700,013.00	No	Consumer Products	n.d.
Clorox Co/The	$ 5,468,000.00	No	Consumer Products	n.d.
CSS Industries Inc.	$ 384,663.00	No	Consumer Products	n.d.
Helen of Troy Ltd.	$ 1,181,676.00	No	Consumer Products	n.d.
Jarden Corp.	$ 6,696,100.00	No	Consumer Products	n.d.
Kid Brands Inc.	$ 229,486.00	No	Consumer Products	n.d.
Kimberly-Clark Corp.	$ 21,063,000.00	No	Consumer Products	n.d.
Scott's Liquid Gold-Inc.	$ 16,041.00	No	Consumer Products	n.d.
Scotts Miracle-Gro Co/The	$ 2,826,100.00	No	Consumer Products	n.d.
Spectrum Brands Holdings Inc.	$ 3,252,435.00	No	Consumer Products	n.d.
Summer Infant Inc.	$ 247,227.00	No	Consumer Products	n.d.
Tupperware Brands Corp.	$ 2,583,800.00	No	Consumer Products	n.d.
WD 40 Co	$ 342,784.00	No	Consumer Products	n.d.
Eddie Bauer	$ 1,023,437.00	Yes	Consumer Products	17-06-2009
Lenox Group, Inc.	$ 452,115.00	Yes	Consumer Products	23-11-2008
Circuit City Stores, Inc.	$ 12,429,800.00	Yes	Consumer Products	10-11-2008
Hancock Fabrics, Inc	$ 376,179.00	Yes	Consumer Products	21-03-2007
Oneida Ltd.	$ 350,819.00	Yes	Consumer Products	09-03-2006
Salton Inc.	$ 1,076,735.00	Yes	Consumer Products	26-08-2005
Tropical Sportswear International Inc.	$ 386,723.00	Yes	Consumer Products	16-12-2004
Huffy Corp.	$ 437,676.00	Yes	Consumer Products	20-10-2004
Interstate Bakeries Corp.	$ 3,525,780.00	Yes	Consumer Products	22-09-2004
Dan River Inc.	$ 477,448.00	Yes	Consumer Products	31-03-2004
Fibermark Inc.	$ 397,411.00	Yes	Consumer Products	31-03-2004
Cone Mills Corp.	$ 445,600.00	Yes	Consumer Products	24-09-2003
Guilford Mills, Inc.	$ 643,519.00	Yes	Consumer Products	13-03-2002
Galey & Lord Inc.	$ 849,993.00	Yes	Consumer Products	19-02-2002
Kasper A.S.L.	$ 383,861.00	Yes	Consumer Products	05-02-2002
Vlasic Foods International Inc.	$ 901,564.00	Yes	Consumer Products	29-01-2001
Converse Inc.	$ 209,050.00	Yes	Consumer Products	21-01-2001
Pillowtex Corp.	$ 1,552,068.00	Yes	Consumer Products	14-11-2000
Fruit of the Loom, Inc.	$ 1,549,800.00	Yes	Consumer Products	29-12-1999

Table 16. List of Companies Used in the Study (Cont.)

Company	Net Sales	Defaulted?	Sector	Default Date
Town & Country Corp.	$ 250,578.00	Yes	Consumer Products	17-11-1997
Crystal Brands, Inc.	$ 444,302.00	Yes	Consumer Products	21-01-1994
Clabir Corp.	$ 179,359.00	Yes	Consumer Products	01-02-1989
ADDvantage Technologies Group Inc.	$ 35,216.00	No	Distribution	n.d.
Arrow Electronics Inc.	$ 20,405,128.00	No	Distribution	n.d.
Beacon Roofing Supply Inc.	$ 2,043,658.00	No	Distribution	n.d.
BlueLinx Holdings Inc.	$ 1,907,842.00	No	Distribution	n.d.
Core-Mark Holding Co Inc.	$ 8,892,400.00	No	Distribution	n.d.
Educational Development Corp.	$ 26,273.00	No	Distribution	n.d.
EnviroStar Inc.	$ 22,457.00	No	Distribution	n.d.
Fastenal Co.	$ 3,133,577.00	No	Distribution	n.d.
First Aviation Services Inc.	$ 21,579.00	No	Distribution	n.d.
Fossil Group Inc.	$ 2,857,508.00	No	Distribution	n.d.
Genuine Parts Co.	$ 13,013,868.00	No	Distribution	n.d.
Houston Wire & Cable Co.	$ 393,036.00	No	Distribution	n.d.
Huttig Building Products Inc.	$ 520,500.00	No	Distribution	n.d.
Infosonics Corp.	$ 34,294.00	No	Distribution	n.d.
Ingram Micro Inc.	$ 37,827,299.00	No	Distribution	n.d.
LKQ Corp.	$ 4,122,930.00	No	Distribution	n.d.
MWI Veterinary Supply Inc.	$ 2,075,146.00	No	Distribution	n.d.
Owens & Minor Inc.	$ 8,908,145.00	No	Distribution	n.d.
Pool Corp.	$ 1,953,974.00	No	Distribution	n.d.
Precision Aerospace Components Inc.	$ 20,240.00	No	Distribution	n.d.
ScanSource Inc.	$ 3,015,296.00	No	Distribution	n.d.
SED International Holdings Inc.	$ 577,274.00	No	Distribution	n.d.
Speed Commerce Inc.	$ 480,824.00	No	Distribution	n.d.
Titan Machinery Inc.	$ 2,198,420.00	No	Distribution	n.d.
United Stationers Inc.	$ 5,080,106.00	No	Distribution	n.d.
Vapor Corp.	$ 21,353.00	No	Distribution	n.d.
WESCO International Inc.	$ 6,579,301.00	No	Distribution	n.d.
WW Grainger Inc.	$ 8,950,045.00	No	Distribution	n.d.
FAO Inc.	$ 460,415.00	Yes	Distribution	04-12-2003
Samuels Jewelers, Inc.	$ 122,007.00	Yes	Distribution	04-08-2003
Daisytek International Inc.	$ 1,185,030.00	Yes	Distribution	03-06-2003
Pentacon Inc.	$ 259,351.00	Yes	Distribution	23-05-2002
Dairy Mart Convenience Stores	$ 723,671.00	Yes	Distribution	24-09-2001
Phar-Mor Inc.	$ 1,292,090.00	Yes	Distribution	21-09-2001

Table 16. List of Companies Used in the Study (Cont.)

Company	Net Sales	Defaulted?	Sector	Default Date
Homeland Holding Corp.	$ 600,835.00	Yes	Distribution	01-08-2001
Zany Brainy, Inc.	$ 400,479.00	Yes	Distribution	15-05-2001
US Office Products Company	$ 2,499,393.00	Yes	Distribution	05-03-2001
Waxman Industries Inc.	$ 99,116.00	Yes	Distribution	02-10-2000
Heilig-Meyers Corp	$ 2,726,358.00	Yes	Distribution	16-08-2000
Trend-Lines Inc.	$ 262,550.00	Yes	Distribution	11-08-2000
Flooring America Inc.	$ 762,808.00	Yes	Distribution	15-06-2000
Eagle Food Centers Inc.	$ 932,789.00	Yes	Distribution	29-02-2000
AgriBioTech Inc.	$ 370,453.00	Yes	Distribution	26-01-2000
Just for Feet Inc.	$ 776,162.00	Yes	Distribution	04-11-1999
Service Merchandise Company	$ 3,159,260.00	Yes	Distribution	27-03-1999
Levitz Furniture Incorporated	$ 986,622.00	Yes	Distribution	05-09-1997
Barry's Jewelers Inc.	$ 140,145.00	Yes	Distribution	12-05-1997
Fretter Inc.	$ 502,317.00	Yes	Distribution	24-09-1996
Caldor Corp.	$ 2,748,634.00	Yes	Distribution	18-09-1995
House of Fabrics, Inc.	$ 546,664.00	Yes	Distribution	02-11-1994
National Convenience Stores Inc.	$ 1,062,183.00	Yes	Distribution	09-12-1991
S.E. Nichols, Inc.	$ 214,315.00	Yes	Distribution	02-08-1990
Circle K Corp.	$ 3,493,107.00	Yes	Distribution	15-05-1990
Advanced Medical Isotope Corp.	$ 248.00	No	Manufacturing	n.d.
American Railcar Industries Inc.	$ 711,723.00	No	Manufacturing	n.d.
AptarGroup Inc.	$ 2,331,036.00	No	Manufacturing	n.d.
Cyclone Power Technologies Inc.	$ 1,134.00	No	Manufacturing	n.d.
FreightCar America Inc.	$ 677,449.00	No	Manufacturing	n.d.
Hillenbrand Inc.	$ 983,200.00	No	Manufacturing	n.d.
John Bean Technologies Corp.	$ 917,300.00	No	Manufacturing	n.d.
LGA Holdings Inc.	$ 515.00	No	Manufacturing	n.d.
Movado Group Inc.	$ 505,478.00	No	Manufacturing	n.d.
Organic Plant Health Inc.	$ 873.00	No	Manufacturing	n.d.
Ourpet's Co.	$ 20,161.00	No	Manufacturing	n.d.
Powin Corp.	$ 42,306.00	No	Manufacturing	n.d.
Spire Corp.	$ 22,110.00	No	Manufacturing	n.d.
TriMas Corp.	$ 1,272,910.00	No	Manufacturing	n.d.
Builders FirstSource, Inc.	$ 677,886.00	Yes	Manufacturing	22-01-2010
Champion Enterprises, Inc.	$ 1,033,193.00	Yes	Manufacturing	15-11-2009
Building Materials Holding Corporation	$ 1,324,679.00	Yes	Manufacturing	16-06-2009
Foamex International Inc.	$ 1,266,394.00	Yes	Manufacturing	15-09-2005

Table 16. List of Companies Used in the Study (Cont.)

Company	Net Sales	Defaulted?	Sector	Default Date
DT Industries Inc.	$ 241,066.00	Yes	Manufacturing	13-05-2004
Reunion Industries Inc.	$ 70,799.00	Yes	Manufacturing	03-12-2003
Oakwood Homes Corp.	$ 1,133,445.00	Yes	Manufacturing	15-11-2002
ACT Manufacturing Inc.	$ 1,370,597.00	Yes	Manufacturing	21-12-2001
Tokheim Corporation	$ 693,932.00	Yes	Manufacturing	28-08-2000
Trikon Technologies Inc.	$ 85,109.00	Yes	Manufacturing	14-05-1998
Cooper Companies, Inc.	$ 92,652.00	Yes	Manufacturing	06-01-1994
Emerson Radio Corp.	$ 764,152.00	Yes	Manufacturing	29-09-1993
Intermark Inc.	$ 400,845.00	Yes	Manufacturing	19-10-1992
Sudbury Inc.	$ 376,182.00	Yes	Manufacturing	10-01-1992
Eagle-Picher Industries Inc.	$ 699,347.00	Yes	Manufacturing	07-01-1991
Amdura Corp.	$ 156,715.00	Yes	Manufacturing	02-04-1990
Amdocs Ltd.	$ 190,875.00	No	Telecommunications	n.d.
Aviat Networks Inc.	$ 444,000.00	No	Telecommunications	n.d.
Consolidated Communications Holdings Inc.	$ 503,457.00	No	Telecommunications	n.d.
CPS Technologies Corp.	$ 14,052.00	No	Telecommunications	n.d.
CTI Group Holdings Inc.	$ 16,759.00	No	Telecommunications	n.d.
Digerati Technologies Inc.	$ 4,135.00	No	Telecommunications	n.d.
EarthLink Holdings Corp.	$ 1,348,977.00	No	Telecommunications	n.d.
Elephant Talk Communications Corp.	$ 29,202.00	No	Telecommunications	n.d.
FairPoint Communications Inc.	$ 973,649.00	No	Telecommunications	n.d.
Fusion Telecommunications International Inc.	$ 44,288.00	No	Telecommunications	n.d.
Global Mobiletech Inc.	$ 15,402.00	No	Telecommunications	n.d.
Glowpoint Inc.	$ 29,070.00	No	Telecommunications	n.d.
GTT Communications Inc.	$ 107,877.00	No	Telecommunications	n.d.
Hawaiian Telcom Holdco Inc.	$ 385,498.00	No	Telecommunications	n.d.
HC2 Holdings Inc.	$ 260,554.00	No	Telecommunications	n.d.
ICTC Group Inc.	$ 4,078.00	No	Telecommunications	n.d.
Inteliquent Inc.	$ 275,453.00	No	Telecommunications	n.d.
Level 3 Communications Inc.	$ 6,376,000.00	No	Telecommunications	n.d.
Lightyear Network Solutions Inc.	$ 66,441.00	No	Telecommunications	n.d.
Lumos Networks Corp.	$ 206,871.00	No	Telecommunications	n.d.
LYFE Communications Inc.	$ 532.00	No	Telecommunications	n.d.
MDU Communications International Inc.	$ 27,305.00	No	Telecommunications	n.d.
Net Talk.com Inc.	$ 5,791.00	No	Telecommunications	n.d.
NeuStar Inc.	$ 831,388.00	No	Telecommunications	n.d.
New ULM Telecom Inc.	$ 32,483.00	No	Telecommunications	n.d.

Table 16. List of Companies Used in the Study (Cont.)

Company	Net Sales	Defaulted?	Sector	Default Date
ORBCOMM Inc.	$ 64,498.00	No	Telecommunications	n.d.
Premiere Global Services Inc.	$ 504,960.00	No	Telecommunications	n.d.
RigNet Inc.	$ 161,669.00	No	Telecommunications	n.d.
Single Touch Systems Inc.	$ 6,347.00	No	Telecommunications	n.d.
Telkonet Inc.	$ 12,758.00	No	Telecommunications	n.d.
TW Telecom Inc.	$ 1,470,255.00	No	Telecommunications	n.d.
USA Mobility Inc.	$ 219,696.00	No	Telecommunications	n.d.
Suncom Wireless Holdings Inc.	$ 852,879.00	Yes	Telecommunications	31-01-2007
Choice One Communications Inc.	$ 322,891.00	Yes	Telecommunications	05-10-2004
Alamosa Holdings Inc.	$ 555,692.00	Yes	Telecommunications	11-11-2003
Allegiance Telecom Inc.	$ 770,982.00	Yes	Telecommunications	14-05-2003
NTELOS Inc.	$ 261,627.00	Yes	Telecommunications	04-03-2003
Qwest Communications International Inc.	$ 19,695,000.00	Yes	Telecommunications	26-12-2002
Focal Communications Corp.	$ 317,485.00	Yes	Telecommunications	19-12-2002
CTC Communications Group Inc.	$ 299,438.00	Yes	Telecommunications	03-10-2002
ITC Deltacom Inc.	$ 415,339.00	Yes	Telecommunications	25-06-2002
Neon Communications Inc.	$ 26,551.00	Yes	Telecommunications	25-06-2002
NTL Inc.	$ 3,699,200.00	Yes	Telecommunications	08-05-2002
Talk America Holdings	$ 495,470.00	Yes	Telecommunications	01-04-2002
Covad Communications Group Inc.	$ 158,736.00	Yes	Telecommunications	15-08-2001
Weblink Wireless Inc.	$ 289,976.00	Yes	Telecommunications	23-05-2001
Teligent Inc.	$ 152,072.00	Yes	Telecommunications	21-05-2001
Viatel Inc.	$ 749,453.00	Yes	Telecommunications	02-05-2001
GST Telecommunications Inc.	$ 321,922.00	Yes	Telecommunications	17-05-2000
Wireless One Inc.	$ 38,737.00	Yes	Telecommunications	10-02-1999
Geotek Communications, Inc.	$ 65,510.00	Yes	Telecommunications	29-06-1998

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