

**MASTER
FINANCE**

MASTER'S FINAL WORK
DISSERTATION

PORTFOLIO DIVERSIFICATION USING BITCOIN

CARLOS MANUEL GERALDES ESTEVES

NOVEMBER - 2020

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**SUPERVISION:
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To my parents

GLOSSARY

ATM – Automatic Teller Machine.

BTC – Bitcoin.

BTC/EUR – Bitcoin/Euro conversion rate.

CBOE – Chicago Board Options Exchange

CCMP – Nasdaq Composite Index.

CME – Chicago Mercantile Exchange.

EUR – Euro.

FTSE – Financial Times and the London Stock Exchange.

HSI – Hang Seng Index.

OEX – S&P 100 Index.

P2P – Peer-to-Peer.

S&P – Standard and Poor's.

SPX – S&P 500 Index.

SX5E – Euro Stoxx 50.

UKX – FTSE 100.

USD – United States Dollar.

VIX – CBOE's Volatility Index.

XAU – Philadelphia Gold and Silver Index.

ABSTRACT

Usually, we associate Bitcoin with the dark side of the finance world - Bitcoin as a mean for online blackmail or scam, the black market or even for Ponzi schemes, where Bitcoin and other digital currencies are used as mean of payment, instead of physical currency.

But, there are also investors who are using Bitcoin as an investment asset, whether for buy and hold strategies or trading (Lee, Guo, & Wang, 2018).

The downside of this investment asset it is the volatility. Decentralization is the main aspect here, as there are no financial institutions between transactions, these are performed “Peer-to-Peer”, or from user to user (Nakamoto, 2008). This lack of oversight brings speculation, and speculative agents are a major player here, with big influence on the trading prices (Baur & Dimpfl, 2017).

Although risky and legally in a grey zone, it can be used in an investment portfolio as a diversification agent, an odd one but perhaps feasible.

The aim of this thesis is linked to the above sentence: to see whether bitcoin play a significant effect in portfolio deficiency and whether volatility is a key variable in the decision-making process of portfolio allocation. The analysis goes from 2012’s first semester until 2019’s second semester, providing 16 semesters in analysis.

We found different outcomes based on our analysis. Some outcomes agree (in a way) with authors (Eisl, Gasser, & Weinmayer, 2015), and other outcomes completely rule out the possibility of Bitcoins being part of a efficient and diversified portfolio.

KEYWORDS: Bitcoin; Digital Currencies; Peer-to-Peer; Volatility; Portfolio; Diversification.

JEL CODES: C63; G10; G11; G15; G19; G21; G23.

RESUMO

Normalmente, as Bitcoins são associadas a um lado mais controverso e ilegal – Bitcoin como meio de chantagem a pessoas ou empresas. Esquemas de pirâmide (Ponzi) ou ainda meio de pagamento no mercado negro, geralmente na “dark-web”.

Mas, existem investidores que estão a utilizar Bitcoin como um ativo nos seus investimentos, seja numa estratégia mais passiva seja mais ativamente, com compra e venda consoante as flutuações cambiais (Lee et al., 2018).

O aspeto negativo deste ativo financeiro é a sua volatilidade. A sua principal característica é a descentralização, ou seja, não existem instituições financeiras entre transações, ou intervenção de reguladores. Estas transações são efetuadas Ponto-a-Ponto (Peer-to-Peer, ou P2P), ou de utilizador para utilizador (Nakamoto, 2008). Esta falta de fiscalização, ou regulamentação, promove a especulação. Daí que os agentes especulativos têm um papel de maior importância nestes mercados, influenciando os preços (Baur & Dimpfl, 2017).

Apesar de, atualmente, Bitcoins e outras cripto moedas se encontrarem numa zona cinzenta, ou vazio legal, e serem um ativo de elevado risco, existe a possibilidade de estas pertencerem a portfolios de investimento, como agente de diversificação. Um agente diferente e recente, mas algo possível.

O objetivo desta dissertação está diretamente ligado à frase acima descrita: observar, e analisar, durante um espaço de tempo, se portfolios de investimento ótimos (ou eficientes), tiveram uma melhor performance, com e sem Bitcoin. Este espaço temporal inicia-se no primeiro semestre de 2012 e termina no segundo semestre de 2019. A análise é semestral, contemplando 16 semestres analisados, e irá ser estudado o fator volatilidade: se é realmente um fator decisivo quando do peso do investimento para cada ativo.

Esta dissertação tem, portanto, como objetivo analisar se a Bitcoin pode ser um agente diversificador num portfolio eficiente e bem diversificado.

Descobrimos diferentes resultados nesta análise. Uns confirmam (de certa forma) o que é enunciado por Eisl (Eisl et al., 2015) e outros resultados põem completamente de parte a introdução de Bitcoins fazerem parte de um portfólio eficiente e diversificado.

Palavras-Chave: Bitcoin; Moedas Digitais; Ponto-a-Ponto; Volatilidade; Portfolio; Diversificação.

Classificação JEL: C63; G10; G11; G15; G19; G21; G23.

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Veni, Vidi, Vici – Julius Caesar

1. INTRODUCTION

Risk is a part of our life. In the finance world, risk can be seen in two opposite perspectives: the chance of making profits or the chance of losing money.

Most investors are risk averse. Therefore, they seek investment opportunities that provide lesser risk, combining risky assets with less riskier assets, called “safe havens”. Safe havens behave as a refuge during times of crisis, or market shocks, where other assets such as stocks can get a big loss in value. The most known safe haven is gold. Gold is usually used to hedge against United States Dollar. So, during dark times for stock investments, gold’s nominal value tends to rise covering for the other assets’ loss (Baur & McDermott, 2010).

In Finance, safe havens are expected to have negative correlations with other assets. And negative correlations can bring diversification benefits for an investor’s portfolio (Hood & Malik, 2013). In a common language, diversification is translated as “Don’t put all of your eggs in one basket”. Markowitz (1952) clarifies about this subject, stating that “our assets should be from different industries or backgrounds”.

In this work, a set of different asset categories is used as a proxy of a globally diversified portfolio, combining indices from different geographies and areas including commodities, gold (as safe haven), a proxy for a European risk-free asset (the German 10-year Treasury Bond) as well as VIX Index (the Volatility Index, a less traditional way of investment diversification).

Bitcoin is the “new” asset that is added to this portfolio. There are many other cryptocurrencies, but Bitcoin is the oldest and most traded one. It appeared in 2008, when someone self-entitled “Satoshi Nakamoto” published an online paper and a domain called “bitcoin.org” appeared.

This paper was presenting a revolutionary way of transactions: they were P2P – Peer to Peer – and there was no need for a double-check from a central or regulatory entity (Nakamoto, 2008). Performed transactions are in cryptocurrency, i.e., virtual currency with no physical form, like bills or coins. In theory, these currencies can be used as money, as they have four qualities for exchange: portability, durability, divisibility, and

security. There are in some places around the world, including Portugal, with Bitcoin ATM machines, providing the liquidity quality (Harwick, 2016).

Authors (Eisl et al., 2015) have been studying the correlations and relationships between cryptocurrencies and other investment assets. Mainly the low correlations, or even negative, as mentioned above.

In 2015, studies led to a range between 1.65% and 7.69% Bitcoin weight in an efficient investment portfolio (Eisl et al., 2015).

In this work, optimal portfolios with and without Bitcoin are going to be tested, and the goal is to find the efficient Bitcoin weight in a globally optimal portfolio. Every semester will be analyzed and compared without Bitcoin and after introducing Bitcoin as an investment asset. The results will then be discussed in the final chapters.

This empirical approach uses data retrieved from Bloomberg and “Investing.com” platforms, followed by the mathematical treatment, dealing with correlations and covariance matrices, and ending with the portfolios’ construction. The portfolios’ inputs are detailed further on.

The work is structured as follows: the first section has a literature review. This review addresses the main aspects of Bitcoin, how it was created, the technology behind this digital revolution; then there are empirical studies using Bitcoin – EUR exchange rate, intrinsic volatility, and correlation with other investment assets. Finally, the literature review provides information about risk and investors, as well as portfolio diversification using cryptocurrencies. The second section has all the gathered historical data as explained, and the used methodology to provide the empirical results. The third section provides the discussion of the results obtained in the previous section. The fourth section includes the conclusions and presents the author’s limitations as well as suggestions for upcoming research.

Digital currencies are here to stay, and mankind is witnessing a digital revolution. There are still flaws, and regulatory entities are not existent (dedicated to cryptocurrency) and the other financial entities disapprove this type of currency. So, this dissertation can act as an incentive for upcoming studies about not only about Bitcoin, but other cryptocurrencies currently being traded in the market.

2. LITERATURE REVIEW

Bitcoin, blockchain, digital currencies.

These are some new words that appeared on recent years. Blockchain is the actual technology behind cryptocurrencies. Cryptocurrencies are a fusion between cryptography and currencies. These digital currencies are protected with highly advanced codes generated whenever there's a transaction (Gurusamy, 2018). All transactions are recorded in ledgers, in this case virtual and decentralized, because transactions are peer-to-peer, this means there are no central authorities to intermediate. As we can see in the figure below, everything is managed between users.

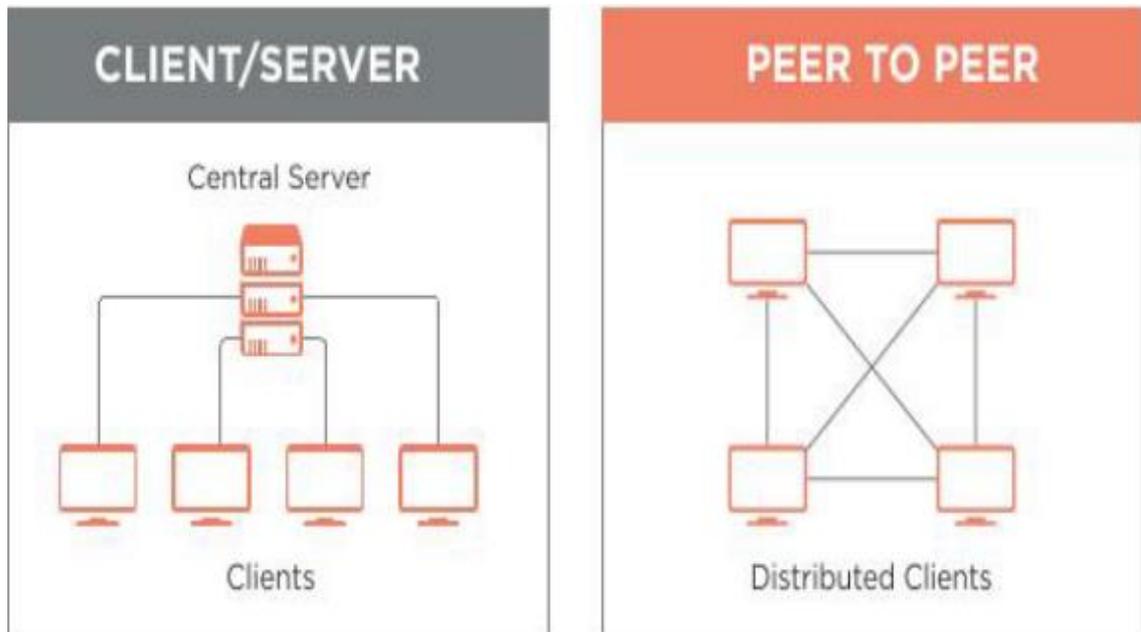


Figure 1 – Architecture of Bitcoin. Source: Gurusamy (2018)

This architecture provides user advantages like tax-free transactions, mobility, and anonymity.

Recent studies and papers show little correlation between cryptocurrencies and other assets, even indices. With this belief, Alexander Eisl 2015 tested if the inclusion of Bitcoin on an investment portfolio could have impact on efficiency, i.e., the Sharpe Ratio. The market data can be easily found on the internet, but many authors use “coinmarketcap.com” to get their databases.

The risky nature of cryptocurrencies is an opposite to haven, like gold or platinum, or bonds. These assets are a way to ensure some returns in bad times.

2.1. The origins of Bitcoin

Blockchain technology is the basis behind Bitcoin, it is a virtual ledger without the need of a central authority to maintain it (usually banks). Every time a new transaction occurs, the ledger is updated and advanced cryptography ensures the safety aspects (Gurusamy, 2018). Without a central authority means we have decentralization because transactions use the Peer-to-Peer protocol. This means that money movements are directly established between users. This technology provides some advantages, like user anonymity, tax free transactions, low transaction fees and mobility. Gurusamy (2018) only addresses the theoretical basis of Bitcoin, not the related problems associated with it namely technical and legal challenges.

To perform a transaction, each owner validates his/her ownership using his/her private key. This will generate an encrypted code to record the transaction and sends the amount to the new owner. When we have several blocks, they are stored in a chain-like disposition, getting the name “blockchain”. The security and payment verification are made by independent users, who use highly computational power to create the blocks. These users are also known as “miners” and are paid in Bitcoins. This lead in recent years for a new and specific demand for certain computer parts, like graphic cards and cooling systems. (Li & Wang, 2017)

Most miners are in China because electricity in some places is very cheap, due to over-productive electricity plants in dams and because this mining process requires a lot of electricity consumption. As they also need to cool down the computers in the mining process, this is one strong reason why there are also many miners in northern latitudes.

Bitcoin was born in 2008 from someone self-entitled “Satoshi Nakamoto” and quickly gained notoriety for mean of payment inside the dark web. These safe transactions were helping armed groups and criminals getting away with their activities. This dark past (and present) is a major turndown when a regular or honest person is thinking about an investment with cryptocurrencies.

With the passage of time, Bitcoin has been the flagship for every cryptocurrency and the most studied by authors. There are several currencies in market nowadays, but this work will be focused solely on Bitcoin, as this is the oldest and the most important cryptocurrency until now.

2.2 Bitcoin and other currencies

Bitcoin has been on the media for some years now, whether for the good and the bad reasons. December 2017 was a big challenge for Bitcoin resistance for economic shocks. The price was very high at the time, peaking almost at \$20.000,00 on December 8, 2017. As we can see by the chart below its price and, subsequently, its market cap, was facing a slightly upward trend. Then, in November 2017, the price began to rise as demand increased. Japan legalized payments by cryptocurrency on April 2017 and CME (Chicago Mercantile Exchange) announced in December 2017 that they were going to launch futures' contracts on Bitcoin.

As Bitcoin had lack of regulation, speculation rose, and the traded volume was immense. After the peak, it started falling in value, as many people were selling and could not cash in the revenues. This was a major turndown in Bitcoin exchanges.

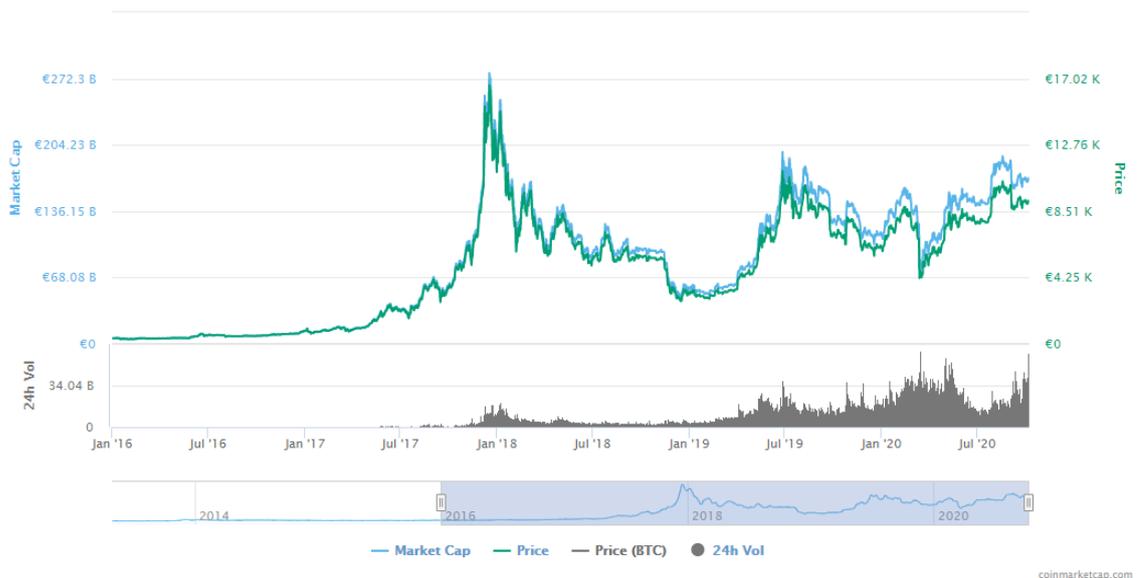


Chart 1 – Bitcoin's Price and Market Cap between Jan1, 2016 and Oct6, 2020. Source: Coinmarketcap

United States Dollar is the reference for Bitcoin trading. Of course, we can exchange with another currency like the Euro. For this dissertation, the used currency is Euro.

Until 2014, nearly all cryptocurrencies on the market were based on the same configuration as Bitcoin. This brings us evidences in favor of Bitcoin's strength and substitution effect. Also, these other cryptocurrencies depreciated against the USD, while Bitcoin appreciated against USD (Li & Wang, 2017).

2.3. Bitcoin and Portfolios

Bitcoin's versatility provides good diversification in good and also in bad times. An analysis to the public ledger shows about one third of Bitcoins are held only, meaning they are part of investment portfolios, or single investment assets. There is a minority of users using Bitcoin as mean of exchange. For now, the number of transactions has been small, very small comparing to other investment assets. So, cryptocurrencies are not seen as a risk or a threat to financial stability. Although small, these transactions are not regulated by a central authority and can affect major fiat currencies in the future.

Chinese authors Xin Li and Chong Wang compared time periods to study the dynamics of the Bitcoin exchange rate. The periods in study were Mt. Gox post-Mt. Gox.

Mt. Gox was, at the time, the largest Bitcoin exchange place. Based in Shibuya, Japan, the company reported 850.000 Bitcoins missing, with a value of \$450 million at the time. Evidence shows mismanagement and careless, as Bitcoins were being stolen since 2011 from a cyber-attack.

The future challenges are directly linked to central bank acceptance of these currencies and to a creation of a regulatory entity to lower the volatility. (Baur & Dimpfl, 2017).

In well-diversified portfolios bitcoin has a place; around 1.65% and 7.69% in the portfolio weight can be used with Bitcoin.(Eisl et al., 2015) This implies higher risk-return ratios, as Bitcoins are very volatile. As documented, both private and institutional investors can use them as another way to diversify their portfolios, as the returns outweigh the additional risk factor taken by the investor. (Eisl et al., 2015)

Authors (Halaburda & Gandal, 2014) found a positive compensation for volatility risk using this two-factor decomposition. Portfolio returns, or expected returns, depend on market volatility and the portfolio's components volatility.

These expected returns depend positively on long-run volatility but negatively on short-run volatility. Using regressions and controlling for the market return, (Adrian & Rosenberg, 2008) found that growth stocks “have positive exposure to short-run volatility, while the exposure of value stocks is negative”. This means investors want a premium for holding assets vulnerable to a rise in volatility.

Using regressions and controlling for the market return, authors discovered that growth stocks “have positive exposure to short-run volatility, while the exposure of value stocks is negative”. This means investors want a premium for holding assets vulnerable to a rise in volatility (Adrian & Rosenberg, 2008). These same authors stated that rising volatility led investor to hedge their positions to secure their returns or to minimize them, getting a protection against. High volatility means small investment transactions, as investors prefer taking long positions.

2.4. VIX

VIX (Volatility Index), is the more popular name for the Chicago Board Options Exchange's CBOE Volatility Index. Also known as the “fear Index”, this Index is measured by volatility instead of price, based on S&P500's prices.

VIX was introduced in 1993 by Whaley, and, originally, was based on S&P100 (with ticker OEX), because this Index's options were the most traded in the United States, accounting for 75% of the total volume (Whaley, 2008).

On the following years, S&P500 (ticker SPX) changed positions with OEX, having most of the traded options in the market. This is the reason why VIX is based on SPX prices instead of OEX.

Fear can be translated, in financial terms, as volatility. Volatility has a major role when building investment portfolios, as investors are do their allocations accordingly with their risk acceptance.

The unregulated cryptocurrencies present higher risks to investors, but, in other hand, can be used as hedging instrument during bad times, against SPX or VIX, i.e., Bitcoin's volatility "behaves differently across time". So, when stock markets have bigger volatilities, Bitcoin can act as a hedge, if proved. On the other hand, if stock markets stabilize, Bitcoin attracts speculative investors, increasing volatility. (López-Cabarcos, Pérez-Pico, Piñeiro-Chousa, & Šević, 2020).

2. DATA AND METHODOLOGY

The purpose of this analysis is to verify whether the inclusion of Bitcoin in diversified portfolios brings financial advantage to the investors. The method will be thoroughly shown below, with the various indices and investment assets to create well-diversified portfolios.

Diversification is a tool to reduce uncertainty within the various assets that compose the portfolios. Of course, the investor is free to invest 100% (or more) in certain assets depending on his/her risk aversion and analysis. An efficient portfolio would be one with maximum expected return and minimum variance. (Markowitz, 1952).

To add robustness to our analysis, volatility and the Sharpe ratio were calculated for every portfolio. The Sharpe ratio helps investors to understand the return of their investments. This is calculated by subtracting the risk-free rate from the mean returns and dividing these results by the portfolio's volatility (standard deviation). This ratio can only be negative if the risk-free return is higher than the portfolio's expected return providing negative amount since the standard deviation cannot be negative. If this ratio is positive, it means the excess return is higher than the risk free and then the higher it gets the better the portfolio performs. This can provide an insight of low volatility or a bigger expected return. (Sharpe, 1994)

The Sharpe ratio equation is given by the following equation:

$$S = \frac{R_p - R_f}{\sigma_p}$$

Where R_p stands for the portfolio's expected return, R_f for the risk-free rate (in this research we will use the German 10-year government bond), and σ_p for the portfolio's standard deviation. This equation measures the excess return (portfolio's return above the risk-free) divided by the standard deviation

3.1. Data

To build a diversified portfolio, I choose indices from around the world, as well as a government bond and data from VIX.

The chosen indices are from the USA – SPX (S&P500 or Standard & Poor’s 500), XAU (Philadelphia Gold and Silver Index) and CCMP (Nasdaq Composite Index) -, Hong Kong – HSI (Hang Seng)-, UK – UKX (FTSE 100 Index) -, and SX5E (Euro Stoxx 50).

The data was collected on a weekly basis, from January 2012 until December 2019, from two different sources: Bloomberg for indices and VIX and the “Investing.com” platform for BTC/EUR (Bitcoin/Euro conversion) and Germany’s 10-year Government Bond.

The data was then grouped by semesters in order to proceed with the analysis in Microsoft’s Excel® software.

3.2. Methodology

The analysis was treated by semester, being the first analysis the first semester from 2012 and the last semester the second semester of 2019.

The analysis was performed using Excel.

Initial treatment consisted of collecting the weekly returns from the database and then calculating the expected returns, the variance and the covariance and correlation matrices for each semester

For each semester we estimated four different portfolios’ types:

- A. A diversified portfolio including VIX and excluding BTC;
- B. A diversified portfolio excluding both the VIX and the BTC;
- C. A diversified portfolio excluding VIX, but including BTC;
- D. A fully diversified including both VIX and BTC.

For each portfolio type I created three sub portfolios each one respecting a different objective function:

1. Maximum expected return;

2. Minimum variance;
3. Maximum Sharpe ratio.

For better understanding: for example, portfolio A1 means this is a portfolio which is diversified including VIX and excluding BTC and calculated the maximum expected return.

For a more solid analysis, four additional constraints were created for portfolios composition based on the weight of the components (constraints):

- A. Naive (equal weights);
- B. No constraints;
- C. Max 20% per asset;
- D. Max 30% per asset.

So, for instance, D1D means this is a fully diversified portfolio with maximum expected return and constraint for maximum 30% per asset investment. For minimum variance portfolios, after trial and error, I found out the two better portfolios constraints with maximum expected returns: this would be “no constraints” and “max 30% per asset”.

In addition to these portfolios, a reference portfolio was created, using the best Sharpe ratio from the first semester in analysis. This portfolio was replicated throughout the entire timeframe and analyzed. The reference portfolios were A3D and D3D, resulting in 30 more portfolios.

There are 32 portfolios per semester, for 16 semesters, for a total of 512 investment portfolios built, plus the 30 reference portfolios. The grand total is 542 portfolios.

The assets weights were calculated using the solver routine, four kinds of portfolios were created: maximum expected return, minimum variance, and maximum Sharpe (most efficient), and the reference portfolio, computed with maximum Sharpe, but using the reference weights.

4. ANALYSIS OF RESULTS

The analysis was entirely made in Excel®. Firstly, the desired analysis was for 10 years, this meant from 2010 to 2019. That would be possible if this portfolio was entirely made in US dollars, because the Bitcoin was initially traded in US dollars when it was created, and the oldest obtained data with Euro comes from 2012. So, this analysis starts in the first semester of 2012 and finishes on the second semester of 2019.

The analysis covers 16 semesters, with four different approaches regarding portfolios' asset weights:

- Higher Expected Return Portfolios;
- Minimum Variance Portfolios;
- Maximum Sharpe Ratio Portfolios;
- Reference Sharpe Ratio Portfolios.

4.1 Variance-Covariance Analysis

With the historical returns from each semester the average returns were calculated. With these returns, the variance-covariance matrices can be built. VIX and BTC are very volatile and the calculations may present some odd results. Therefore, it was better to build different matrices as seen below.

Table I - Variance-Covariance matrix from 2012 S1 (all assets excepting BTC)

	VIX	SX5E	UKX	HSI	XAU	CCMP	SPX	DE Bond
VIX	6,0638	0,5369	0,3774	0,3959	0,0798	0,1013	0,1483	-4,0512
SX5E	0,5369	0,0663	0,0429	0,0390	0,0293	0,0100	0,0133	-0,5120
UKX	0,3774	0,0429	0,0415	0,0403	0,0169	0,0092	0,0119	-0,0959
HSI	0,3959	0,0390	0,0403	0,0517	0,0045	0,0099	0,0134	0,2070
XAU	0,0798	0,0293	0,0169	0,0045	0,0393	0,0019	0,0009	-0,2701
CCMP	0,1013	0,0100	0,0092	0,0099	0,0019	0,0027	0,0032	-0,0115
SPX	0,1483	0,0133	0,0119	0,0134	0,0009	0,0032	0,0044	-0,0262
DE Bond	-4,0512	-0,5120	-0,0959	0,2070	-0,2701	-0,0115	-0,0262	42,6414

Table II - Variance-Covariance matrix from 2012 S1 (all assets excepting VIX)

	SX5E	UKX	HSI	XAU	CCMP	SPX	DE Bond	BTC/ EUR
SX5E	0,0663	0,0429	0,0390	0,0293	0,0100	0,0133	-0,5120	0,6738
UKX	0,0429	0,0415	0,0403	0,0169	0,0092	0,0119	-0,0959	0,3959
HSI	0,0390	0,0403	0,0517	0,0045	0,0099	0,0134	0,2070	0,9519
XAU	0,0293	0,0169	0,0045	0,0393	0,0019	0,0009	-0,2701	0,1353
CCMP	0,0100	0,0092	0,0099	0,0019	0,0027	0,0032	-0,0115	0,1608
SPX	0,0133	0,0119	0,0134	0,0009	0,0032	0,0044	-0,0262	0,2007
DE Bond	-0,5120	-0,0959	0,2070	-0,2701	-0,0115	-0,0262	42,6414	17,5178
BTC/ EUR	0,6738	0,3959	0,9519	0,1353	0,1608	0,2007	17,5178	140,7040

Table III - Variance-Covariance matrix from 2012 S1 (all assets)

	VIX	SX5E	UKX	HSI	XAU	CCM P	SPX	DE Bond	BTC/ EUR
VIX	6,0638	0,5369	0,3774	0,3959	0,0798	0,1013	0,1483	-4,0512	6,4947
SX5E	0,5369	0,0663	0,0429	0,0390	0,0293	0,0100	0,0133	-0,5120	0,6738
UKX	0,3774	0,0429	0,0415	0,0403	0,0169	0,0092	0,0119	-0,0959	0,3959
HSI	0,3959	0,0390	0,0403	0,0517	0,0045	0,0099	0,0134	0,2070	0,9519
XAU	0,0798	0,0293	0,0169	0,0045	0,0393	0,0019	0,0009	-0,2701	0,1353
CCM P	0,1013	0,0100	0,0092	0,0099	0,0019	0,0027	0,0032	-0,0115	0,1608
SPX	0,1483	0,0133	0,0119	0,0134	0,0009	0,0032	0,0044	-0,0262	0,2007
DE Bond	-4,0512	-0,5120	0,0959	0,2070	0,2701	0,0115	0,0262	42,6414	17,5178
BTC/ EUR	6,4947	0,6738	0,3959	0,9519	0,1353	0,1608	0,2007	17,5178	140,7040

These matrices were used to calculate the portfolios. Throughout the semesters, the German bond, Bitcoin and VIX had the largest variance. This contributed to riskier portfolios with bigger volatilities. A portfolio variance is calculated using the assets weight, multiplied by their variances and then with the covariances like the below formula for two assets:

$$Portfolio\ Variance = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov_{1,2}$$

Where w is the asset's weight, σ (Sigma) is the asset's variance and $Cov_{1,2}$ is assets' covariance, the correlation coefficient between them.

4.2 Correlation Analysis

For the correlation analysis, the difference of the of the means was analyzed. Then, the mean from each upper triangular matrix without and with Bitcoin. I.e. the correlation average from all assets with the exception of Bitcoin, minus the correlation average from all assets. Below, we can see the analysis. For the positive figures in the “Difference” column, this means that, adding Bitcoin to the portfolio, the average correlation decreases, which is a good diversification factor. This indicates that Bitcoin can act as a diversifying agent for portfolio construction. Semester 1 from 2018 is the only exception to this analysis, although the difference was quite small. In this semester, the Bitcoin introduction did not act as a diversification agent.

Table IV - Correlation differences without and with Bitcoin

Date	Average	Av. w/ BTC	Difference
2012 S1	0,465211	0,410748	0,054463
2012 S2	0,043706	0,042141	0,001565
2013 S1	0,257596	0,151307	0,106289
2013 S2	0,257596	0,159708	0,097888
2014 S1	0,085517	0,044335	0,041182
2014 S2	0,428411	0,339331	0,089080
2015 S1	0,120454	0,092597	0,027857
2015 S2	0,414128	0,323012	0,091116
2016 S1	0,339082	0,258042	0,081040
2016 S2	0,210985	0,135129	0,075855
2017 S1	0,105747	0,043485	0,062263
2017 S2	0,074483	0,031398	0,043084
2018 S1	0,271661	0,275187	-0,003526
2018 S2	0,382534	0,283701	0,098833
2019 S1	0,354155	0,223984	0,130171
2019 S2	0,262169	0,199025	0,063144

4.3 Portfolio Analysis

The introduction of Bitcoins in investment portfolios may seem a good idea. There is also literature pointing on that direction. (Eisl et al., 2015) This analysis agrees with the authors in a way, but in another way, it contradicts them. We must remind Bitcoins are very recent and all the studies made regarding this usage as investment assets are scarce.

Bitcoin, as a highly volatile asset, brings returns but also risk. Therefore, the study was conducted analyzing the investment portfolios in four ways: maximizing the expected returns, minimizing variance (risk), maximizing the Sharpe Ratio and the reference portfolio's evolution throughout the timeframe.

4.3.1 Higher Expected Return Portfolios

This is how the portfolios look like:

Table V - Portfolio structure (from 2015 S2). From left to right: A1A, A1B, A1C, A1D.

Note: STD means Standard Deviation

	Weights		Weights		Weights		Weights
VIX	0,125	VIX	1	VIX	0,2	VIX	0,3
SX5E	0,125	SX5E	0	SX5E	0,2	SX5E	0,1
UKX	0,125	UKX	0	UKX	0,2	UKX	0,3
HSI	0,125	HSI	0	HSI	0,2	HSI	0,3
XAU	0,125	XAU	0	XAU	0	XAU	0
CCMP	0,125	CCMP	0	CCMP	0	CCMP	0
SPX	0,125	SPX	0	SPX	0,2	SPX	0
DE Bond	0,125	DE Bond	0	DE Bond	0	DE Bond	0
TOTAL	1	TOTAL	1	TOTAL	1	TOTAL	1
Return	4,164921	Return	16,40026	Return	5,983247	Return	7,623621
VAR	5,164283	VAR	17,92334	VAR	0,897064	VAR	1,887754
STD	2,272506	STD	4,233596	STD	0,947135	STD	1,373955
Sharpe	0,011964	Sharpe	0,035322	Sharpe	0,047903	Sharpe	0,044961

With this framework is observable a broader picture of an investor's possibilities, regarding the portfolio's structure.

A1A is a “naïve” portfolio, with equal weights for each asset. The naïve portfolio is the only one not calculated using Solver.

The remaining were calculated with Solver and had some constrains (manually inserted): All assets have positive weights (>0), and their sum is 1. Special constrains were added to **C and **D portfolios: respectively, 20% and 30% maximum weight per asset.

A1B presents the higher return combination, and throughout the semesters, VIX was the chosen asset, because provided higher returns, but also higher volatility, as it can be observed by the standard deviation, in this case the fluctuation is very big (way above 1).

A1C and A1D have specific constraints: C portfolio has a 20% ceiling for each asset and D portfolio has a 30% ceiling. These figures look more feasible, as standard deviation lowers, because VIX investment is constrained.

Below, for the same period the D1* portfolios: the portfolios including all the assets:

Table VI - Portfolio structure (from 2015 S2). From left to right: D1A, D1B, D1C, D1D

	Weights		Weights		Weights		Weights
VIX	0,111111	VIX	1	VIX	0,2	VIX	0,3
SX5E	0,111111	SX5E	0	SX5E	0,2	SX5E	0,1
UKX	0,111111	UKX	0	UKX	0,2	UKX	0,3
HSI	0,111111	HSI	0	HSI	0,2	HSI	0,3
XAU	0,111111	XAU	0	XAU	0	XAU	0
CCMP	0,111111	CCMP	0	CCMP	0	CCMP	0
SPX	0,111111	SPX	0	SPX	0	SPX	0
DE Bond	0,111111	DE Bond	0	DE Bond	0	DE Bond	0
BTC	0,111111	BTC	0	BTC	0,2	BTC	0
TOTAL	1	TOTAL	1	TOTAL	1	TOTAL	1
Return	3,972323	Return	16,40026	Return	6,045799	Return	7,623621
VAR	5,997015	VAR	17,92334	VAR	3,68452	VAR	1,887754
STD	2,44888	STD	4,233596	STD	1,91951	STD	1,373955
Sharpe	0,010316	Sharpe	0,035322	Sharpe	0,023963	Sharpe	0,044961

From this chart it is possible to verify how the volatility from VIX is very big, as it is opposite of the other assets: VIX is riskier during stable times, as it varies with the market volatility.

Again, the 30% ceiling in D portfolio is the more feasible portfolio structure that allows higher expected returns, without big fluctuations (volatility is 1.37).

For investors that have a higher risk acceptance, this structure could be one to use. Note that this portfolio has 0% with Bitcoin. But, in C portfolio, Bitcoin has a 20% weight, and the variance was about 2 times higher.

For a deeper analysis, the Sharpe ratios were calculated for all portfolios. The below chart has a summary of the analysis between B and D portfolios. B and D were chosen because, throughout the semesters, the portfolios with no constraints and the ones with 30% ceilings per asset showed higher returns.

Table VII - Sharpe ratio summary for B (no constraints) and D (30% ceiling) portfolios

Date	A1B	A1D	D1B	D1D	B	
					Difference	D Difference
2012 S1	0,056279	0,076750	0,538043	0,334755	0,481764	0,258005
2012 S2	0,078919	0,076750	0,078919	0,113427	0,00E+00	0,036677
2013 S1	0,066384	0,099671	0,066384	0,009583	0,00E+00	-0,090088
2013 S2	0,081470	0,110683	0,004374	0,009027	-0,077096	-0,101656
2014 S1	0,068969	0,137829	0,068969	0,137829	0,000000	8,40E-10
2014 S2	0,053898	0,103790	0,053898	0,103790	0,000000	4,28E-10
2015 S1	0,022817	-0,000857	0,022817	-0,000857	0,000000	-5,67E-17
2015 S2	0,035322	0,044961	0,035322	0,044961	0,000000	-4,23E-16
2016 S1	0,068969	0,165884	0,071762	0,165884	0,002793	-4,16E-16
2016 S2	0,047022	0,061363	0,047022	0,061363	0,000000	-3,45E-15
2017 S1	0,016847	-0,059114	0,016847	-0,059114	0,000000	-5,22E-15
2017 S2	0,057541	0,094857	0,057541	0,009541	0,000000	-0,085316
2018 S1	0,036208	0,056642	0,036208	0,056642	0,000000	1,38E-10
2018 S2	0,030463	0,045123	0,030463	0,045123	0,000000	-6,88E-11
2019 S1	-0,000047	-0,002007	-0,000047	-0,001998	0,000000	0,000008
2019 S2	0,051913	0,071164	0,051913	0,071164	0,000000	-2,91E-10

This table shows the difference between portfolios with Bitcoin and without. So, the B difference comes from D1B minus A1B; for the D difference comes from D1D minus A1D.

As observed, the differences are very small. This happened because VIX has higher returns throughout the semesters. Thus, adding bitcoins to the portfolios did not make an influence on the overall return. This is the full picture, where all the semesters were studied, same way as table VII. In the table VIII, we can observe the VIX's higher return and higher variance. These two characteristics led to a small Sharpe ratio which we can observe from all portfolios throughout the timeframe. In this case, the lower ratio comes primarily from the high volatility (higher than 1, in this case, even greater than 2).

4.3.2 Minimum Variance Portfolios

For the minimum variance portfolios, the analysis was made for the entire framework. However, it was concluded the portfolios that provide higher returns were the B and D ones: No constrains (B) and 30% ceiling (D). In this case, B2D and D2D.

For comparison analysis, the below portfolios come from 2015 S2.

Table VIII - Portfolio Structure (2015 S2). From left to right: A2B, A2D

	Weights		Weights
VIX	0	VIX	0
SX5E	0	SX5E	0
UKX	0	UKX	0,167547
HSI	0	HSI	0
XAU	0,049677	XAU	0,232379
CCMP	0,950323	CCMP	0,3
SPX	0	SPX	0,3
DE Bond	0	DE Bond	7,38E-05
	1		1
Return	1,26464	Return	2,09714
VAR	0,002035	VAR	0,005089
STD	0,04511	STD	0,071336
Sharpe	-0,04024	Sharpe	0,091256

As VIX is highly volatile, it has no weight in these minimum variance portfolios. In this case, the best option was the D portfolio, where we also have a higher diversification, by allocating in 5 assets out of 8. The return was more than twice (109,7%) with 7% standard deviation. The small Sharpe ratio means this portfolio does not perform well, but that is one of the outcomes when we are more focused in minimum risk.

A similar situation can be observed when including all the assets, like the below chart:

Table IX - Portfolio Structure (2015 S2). From left to right: D2B, D2D

	Weights		Weights
VIX	0	VIX	0
SX5E	0	SX5E	0
UKX	0	UKX	0,142839
HSI	0	HSI	0
XAU	0,05618	XAU	0,254752
CCMP	0,943528	CCMP	0,3
SPX	0	SPX	0,3
DE Bond	0	DE Bond	0
BTC	0,000292	BTC	0,002409
TOTAL	1	TOTAL	1
Return	1,268076	Return	2,039539
VAR	0,002031	VAR	0,004714
STD	0,045061	STD	0,06866
Sharpe	-0,03952	Sharpe	0,086424

The new factor here is the Bitcoin introduction (0,2% of the total investment), although quite residual.

Table X - Sharpe ratio summary for B (no constraints) and D (30% ceiling) portfolios

Date	A2B	A2D	D2B	D2D	B Difference	D Difference
2012 S1	0,062234	0,121659	0,062234	0,121659	-1E-10	1,75E-07
2012 S2	0,119444	0,226359	0,260822	0,229488	0,141378	0,003129
2013 S1	-0,0429	0,221606	-0,04373	0,183612	-0,00083	-0,03799
2013 S2	0,277984	0,231747	0,297117	0,22438	0,019134	-0,00737
2014 S1	1,129441	1,0808	1,136606	1,127391	0,007165	0,046592
2014 S2	1,254858	0,930142	1,300368	0,934689	0,04551	0,004547
2015 S1	-3,92639	-2,70032	-3,92639	-2,7018	-9,5E-08	-0,00147
2015 S2	-0,04024	0,091256	-0,03952	0,086424	0,000719	-0,00483
2016 S1	4,273619	2,841577	4,284965	2,844916	0,011347	0,003339
2016 S2	-0,72259	-0,39853	-0,54966	-0,3991	0,172935	-0,00057
2017 S1	-3,30182	-2,72426	-5,84266	-2,64396	-2,54084	0,080296
2017 S2	-0,47963	-0,18935	-0,08192	-0,1913	0,397708	-0,00196
2018 S1	0,234184	0,350524	0,238051	0,350533	0,003867	8,71E-06
2018 S2	0,220336	0,252255	0,093534	0,158694	-0,1268	-0,09356
2019 S1	-6,79709	-4,4187	-7,30617	-4,5996	-0,50908	-0,1809
2019 S2	-0,3113	-0,05314	-0,3113	0,157752	1,88E-07	0,210888

The adding of Bitcoins to the minimum variance portfolios had little effects. As Bitcoin has a bigger variance, this was not an efficient way to analyze if this crypto currency provides diversification, while adding value for the investor.

4.3.3 Maximum Sharpe Ratio Portfolios

For this study, the Sharpe ratio provides a better insight regarding portfolio efficiency. A higher Sharpe has the optimal combination between excess return and lower volatility.

Like for minimum variance analysis, the chosen portfolios were the B and D, in this case, A3B, A3D, D3B and D3D (no constrains for B and 30% ceiling for D).

For coherency, the below charts were taken from 2015 S2.

Table XI - Portfolio Structure (2015 S2). From left to right: A3B, A3D

	Weights		Weights
VIX	0	VIX	0
SX5E	0,0806	SX5E	0,206512
UKX	0,654514	UKX	0,3
HSI	0	HSI	0
XAU	0,264886	XAU	0,193488
CCMP	0	CCMP	0
SPX	0	SPX	0,3
DE Bond	0	DE Bond	0
Total	1	Total	1
Return	3,43998	Return	2,942773
VAR	0,014736	VAR	0,010658
STD	0,121393	STD	0,103237
Sharpe	0,200998	Sharpe	0,188185

This is a more realistic solution for an optimal portfolio (without Bitcoin). The investments are well spread amongst the various assets, excepting VIX.

Table XII - Portfolio Structure (2015 S2). From left to right: D3B, D3D

	Weights		Weights
VIX	0	VIX	0
SX5E	0,069677	SX5E	0,176641
UKX	0,632214	UKX	0,3
HSI	0	HSI	0
XAU	0,295994	XAU	0,221316
CCMP	0	CCMP	0
SPX	0	SPX	0,3
DE Bond	0	DE Bond	0
BTC	0,002115	BTC	0,002043
TOTAL	1	TOTAL	1
Return	3,367526	Return	2,888611
VAR	0,013609	VAR	0,009807
STD	0,116657	STD	0,099029
Sharpe	0,202947	Sharpe	0,190712

Introducing Bitcoins, the weight combination made the variance to decrease. Thus, the Sharpe ratio increased.

The Bitcoin weight in these portfolios remains at 0,2%. Same as with the minimum variance portfolios. This may indicate that Bitcoin could prove to be a diversifying agent.

On the below chart -14- there is a full summary for the entire timeframe:

Table XIII - Sharpe ratio summary for B (no constraints) and D (30% ceiling) portfolios

Date	A3B	A3D	D3B	D3D	B Difference	D Difference
2012 S1	0,20168523	0,17652587	0,05632732	0,06192146	-0,1453579	-0,1146
2012 S2	0,07891904	0,10901026	0,07891904	0,10901026	6,6182E-11	2,87E-10
2013 S1	0,27333117	0,27086186	0,27102514	0,27430147	-0,002306	0,00344
2013 S2	0,39998668	0,15933145	0,42031292	0,29184182	0,02032624	0,13251
2014 S1	1,10693738	1,09544807	1,14561203	1,17438943	0,03867465	0,078941
2014 S2	0,72403199	0,79822035	0,71716266	0,79407246	-0,0068693	-0,00415
2015 S1	-0,7739295	-2,1086319	-2,6874719	-2,1143609	-1,9135424	-0,00573
2015 S2	0,16424545	0,14496897	0,16470201	0,14565931	0,00045656	0,00069
2016 S1	2,12727607	2,77019198	2,1272748	2,77019219	-1,267E-06	2,06E-07
2016 S2	0,03384088	0,00936206	0,03099645	0,06851062	-0,0028444	0,059149
2017 S1	-2,3230521	-0,0113543	-1,8959507	-2,6538173	0,42710144	-2,64246
2017 S2	0,16490328	0,09547903	0,1587648	0,09690321	-0,0061385	0,001424
2018 S1	0,38038067	0,37874165	0,38038072	0,37842439	5,6213E-08	-0,00032
2018 S2	0,24802335	0,25226226	0,24914901	0,31317203	0,00112566	0,06091
2019 S1	-2,5161381	-3,5100899	-2,5728137	-3,6464241	-0,0566756	-0,13633
2019 S2	0,06935886	0,10141641	0,20216638	0,18271834	0,13280752	0,081302

This chart can be the summary, or the goal, for this dissertation: the analysis of efficient portfolios with and without Bitcoins.

The analysis cannot be very conclusive, although there is a higher tendency for a Sharpe improve when the ceiling is 30% for each asset, when introducing Bitcoin. The negative ratios for 2015 S1, 2017 S1 and 2019 S1 happened because the risk-free rate was higher than the portfolio's return. Therefore, the excess return was negative.

Table XIV - Risk-free Rate (German Government Bond 10 years)

	Risk free
2012 S1	-0,263462
2012 S2	0,902308
2013 S1	0,590385
2013 S2	0,586154
2014 S1	-1,575385
2014 S2	-3,256538
2015 S1	7,835769
2015 S2	0,446154
2016 S1	-14,816923
2016 S2	1,569615
2017 S1	7,211154
2017 S2	0,808148
2018 S1	-0,816000
2018 S2	-0,563704
2019 S1	25,361923
2019 S2	0,758077

4.3.4 – The Reference Portfolio

For 2012's first semester, the portfolio's chosen structure was A3D and D3D. As a reminder, A3D and D3D are the portfolios with 30% for every asset and with most efficiency (higher Sharpe ratio); portfolio A3D does not includes Bitcoin.

Table XV – Reference portfolios' weight structure

A3D	Weights	D3D	Weights
VIX	0	VIX	0
SX5E	0,3	SX5E	0,3
UKX	0,3	UKX	0,3
HSI	0	HSI	0
XAU	0	XAU	0
CCMP	0,095612376	CCMP	0,09561243
SPX	0,3	SPX	0,3
DE Bond	0,004387624	DE Bond	0,00438757
		BTC	0

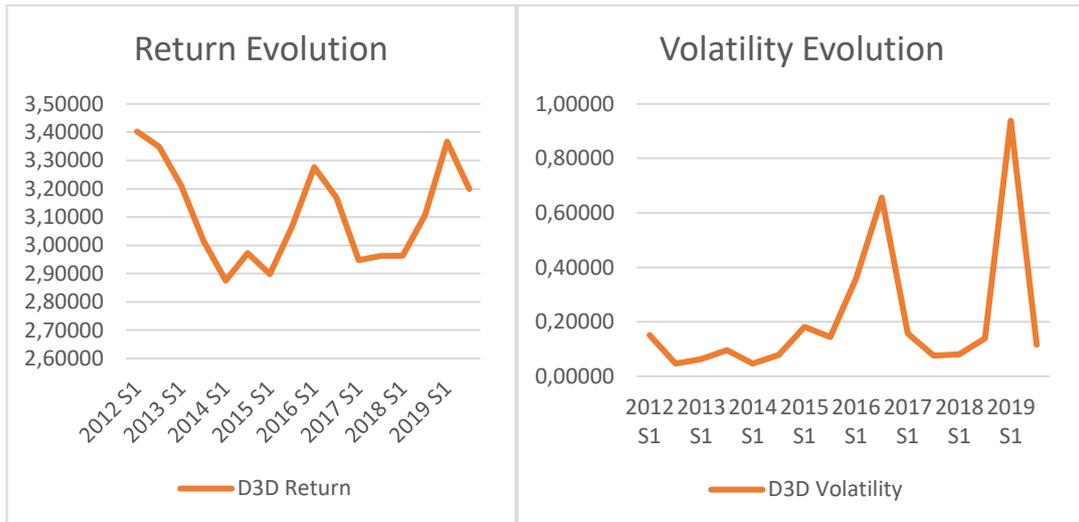


Chart 2 – Returns between 2012 S1 and 2019 S2 (on the left)

Chart 3 – Volatility between 2012 S1 and 2019 S2 (on the right)

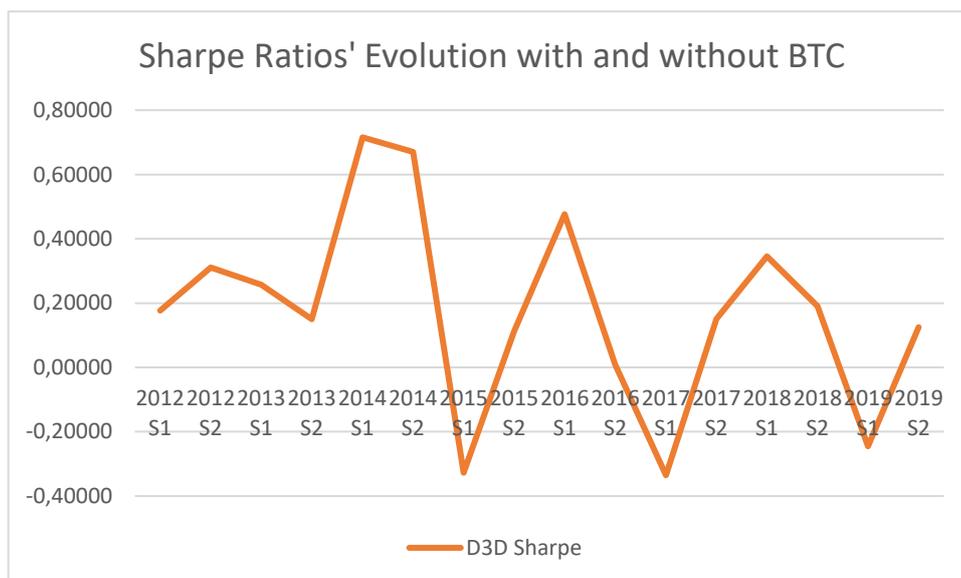


Chart 4 – Sharpe Ratios between 2012 S1 and 2019 S2

Since these portfolios do not have Bitcoin nor VIX, there are no differences between the portfolios A3D and D3D. The returns ranged between 2,875 and 3,402. The results got better after 2014, when the economic crisis was over, falling again in 2016 and rising in 2019. The return median is 3,085, which is a solid return for this investment in well-known and strong indices.

With the exception of 2016 S1, there is an inverse relationship between volatility and Sharpe ratios. Even with the social and economic turmoil, the ratios were positive, although inferior to 1, which provide an idea of a buy hold portfolio (Sharpe, 1994) (White & Haghani, 2018).

5. CONCLUSIONS, RESEARCH LIMITATIONS AND FUTURE INVESTIGATIONS

Whether we like it or not, cryptocurrencies are here to stay. Digital currency will dominate over physical currency throughout the new generations to come, and “cryptos” will become a new currency, alongside U.S. Dollar or Euro.

To include the digital currencies into society, regulations must be made for them to be accepted and more used by the people.

The aim of this dissertation was to dissect the sentence: “Portfolio Diversification Using Bitcoin”. We did not have a starting research question to answer, but this sentence can be made into a research question: “Do Bitcoins Diversify Investment Portfolios?”.

Eisl, in 2015, claimed “around 2%-8% in the portfolio weight can be used with Bitcoin”. As this analysis showed, it can be feasible. But the risk is very high as Bitcoin’s high volatility make its returns hard to predict, whether good or bad, since speculation is a major player.

Our analysis included two highly volatile assets: VIX and Bitcoin. These also presented historic high returns.

The analysis showed Bitcoin can be added to efficient and diversified portfolios. Table *XIII* summarizes the answer to the research question. This answer is not conclusive, as some semesters benefited from Bitcoin introduction while others performed worse. The weight in efficient portfolios is quite residual, around 0,2%, so its influence in the total weight is small. This past decade also had atypical years, like the crisis in the beginning and the major Bitcoin crash in December 2017.

For the future, with a longer timeframe to analyze, better and more conclusive answers can be obtained, using other investment assets or a different portfolio structure that can maximize the expected return, while having low volatilities and Bitcoin inclusion (the optimal scenario).

The major limitation to this analysis is the small timeframe where we can get the data. As previously said, if this analysis was made for U.S. Dollar portfolios, we could analyze the entire decade, but with Euros, only from 2012 onward. The other limitation is that this analysis is entirely theoretical, and do not count one big aspect: which is the investor. The

investors, although doing performance and market behavior analysis, have emotions. And these emotions sometimes influence their asset allocation.

Bitcoins are still negatively connoted, so they play a psychological influence on the society and investors.

Can Bitcoins be used as a diversifying agent? Yes.

Can Bitcoins be part of an efficient portfolio? Yes, residually.

Will cryptocurrencies be part of the future investment portfolios? Only time will tell.

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