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ECONOMIA MONETÁRIA E FINANCEIRA

TRABALHO FINAL DE MESTRADO

DISSERTAÇÃO

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Master's in Monetary and Financial Economics

Dissertation

Should Central Banks Increase the Inflation Target?

Mário André Santos de Oliveira†

Supervisor: Bernardino Adão (Bank of Portugal)

Abstract

Typically when central banks face economic slowdowns they use the interest rate channel to boost economies. However, we have seen that if the nominal interest rate is already at low levels, then their capacity to invert such economic slowdowns is little. The main objective of this dissertation is to study whether increasing the inflation target can increase the capacity of central banks to invert economic downturns. Specifically, we will study whether the real interest rate decreases more when the inflation target is higher, as a response to a negative shock in the nominal interest rate. To study this we use a general equilibrium model, where agents are heterogeneous in their amount of money holdings. Our model suggests that increasing the inflation target does not increase the real stimulus of central banks when they decrease the nominal interest rate by one percentage-point. In fact, the real interest rate declines more, the lower the target. This occurs because the degree of price stickiness is lower for higher levels of inflation.

JEL codes: E3, E4, E5

Keywords: inflation target, zero lower bound, cash-in-advance, nominal interest rate shock

† I would like to thank Bernardino Adão for his comments and suggestions. I would also like to thank all of my friends and family for their support during these years. This work is dedicated to my parents and grandparents. All possible errors are my responsibility.

Instituto Superior de Economia e Gestão - Universidade de Lisboa

Mestrado em Economia Monetária e Financeira

Dissertação

Devem os Bancos Centrais Aumentar o Target da Inflação?

Mário André Santos de Oliveira

Orientação: Bernardino Adão (Banco de Portugal)

Abstract

Tipicamente os Bancos Centrais usam as taxas de juro para inverter os efeitos das crises económicas. No entanto, temos observado que se as taxas de juro nominais já estiverem muito próximo de zero, então a capacidade que estes têm de usar este mecanismo para estimular a atividade económica é reduzido. O principal objetivo desta dissertação é estudar se aumentando o nível médio de inflação, aumenta a capacidade do bancos centrais em inverter crises económicas. Especificamente, iremos estudar se a taxa de juro real diminui mais para valores médios mais elevados da taxa de inflação, quando um choque exógeno na taxa de juro nominal ocorre. Para tal, iremos utilizar um modelo de equilíbrio geral, onde os agentes são heterogéneos na quantidade de moeda que detêm. O nosso modelo sugere que aumentar o target da inflação não aumenta o estímulo provocado pela taxa de juro real, quando um choque de 1 ponto-percentual ocorre sobre a taxa de juro nominal. De facto, o que se verifica é que a taxa de juro real diminuí mais quando menor for o nível médio de inflação. Isto ocorre porque o grau de *price stickiness* é menor para níveis mais elevados do *target* da inflação

Contents

1 Introduction

Recently the environment of low nominal interest rates and low inflation has been part of many central banks' concerns, because under this environment little room is left for monetary policy to take part in to recover economic downturns.

To recovery from the last financial crisis the major central banks, such as the Fed, the ECB and the Bank of England, decreased the short-term nominal interest rates from 5-6% in 2008 to almost 0% in 2014 (see figure 1). Despite these significant drops in the nominal interest rate, the situation demanded further steps to have a stronger stimulus, since economies were not reacting to these - hight unemployment rates and poor economic performance persisted. Hofmann & Bogdanova (2012) showed that the Taylor rule recommended a negative nominal interest for the advanced economies around minus 3%[2], in order to achieve a desired recovery. However, monetary policy measures are limited by the non-negativity constraint of the nominal interest rate, which prevents it to be set below zero. This is commonly known as the zero lower bound problem (ZLB) of the interest rate channel.

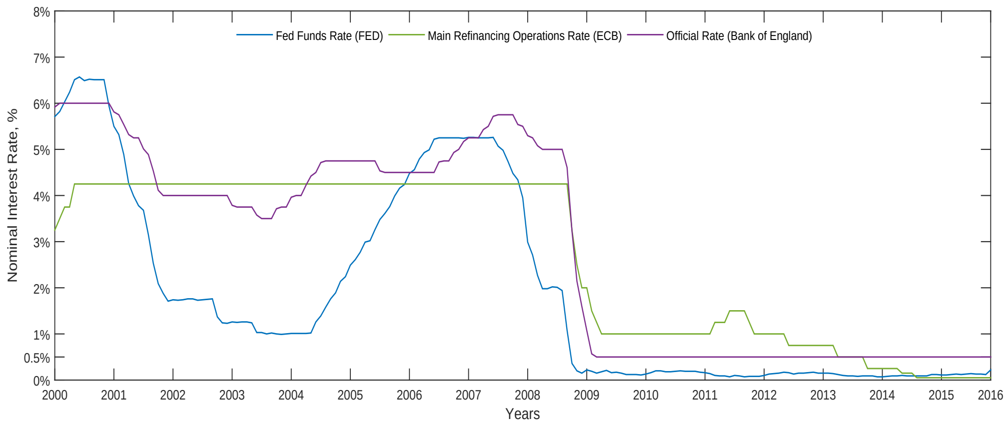

Figure 1: The problem of the zero lower bound implicit by the low levels of the interest rates of the US Fed, ECB and Bank of England.

In normal times the inflation would be expected to be near the target, which is

[2]B. Bernanke did some calculations using the Taylor rule as well but for the US case. In his article, the author shows that for the US case the recommendation is −5%. Link: http://www.brookings.edu/blogs/ben-bernanke/posts/2015/04/28-taylor-rule-monetary-policy.

broadly set at 2%. With inflation around this level, policy makers would have the capacity to change interest rates when necessary without the imminent threat of the ZLB become binding. However, lately among the developed economies inflation has been at low levels, which increased the probability of reaching the ZLB and reduced the capacity of central banks to counteract economic slowdowns.

Empirical data for the US from 1974 to 2014 shows that the correlation between inflation and nominal interest rates is high (the correlation coefficient is 0.74). Figure 2 illustrates this relationship. We can see that, typically, changes in the inflation tend to be followed by similar changes in the nominal interest rate. This relation is broken only when the ZLB becomes binding, as we can see in the period from 2009 onwards.

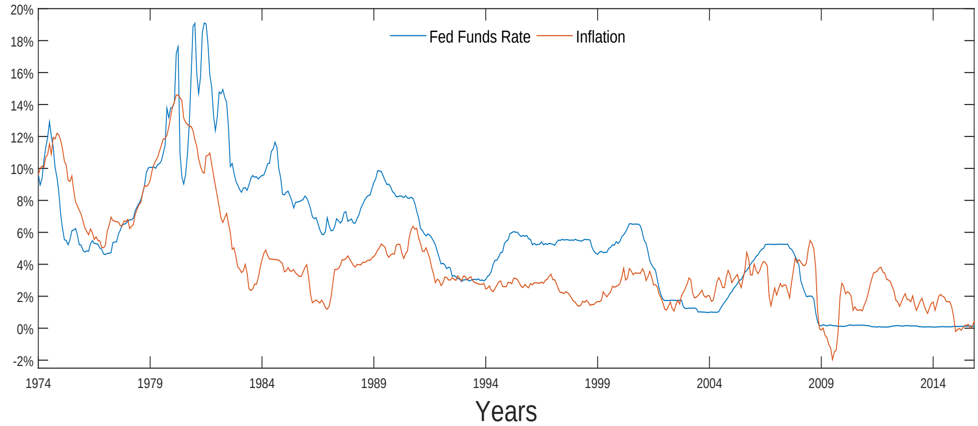

Figure 2: Correlation between the nominal interest rate and inflation rate in the US from 1974 to 2014. To calculate the inflation we used the data on the consumer price index for all consumers, and all goods.

Thus, admitting that higher average levels of inflation would be followed by higher average levels of nominal interest rates, increasing the inflation target could decrease the probability of reaching the ZLB and increase the room for monetary policy to counteract in those economic slowdowns. Some economists, such as Blanchard et al (2010), Fuhrer & Madigan (1997) and Reifschneider & Williams (2000), use this argument to support the idea of increasing the inflation target.

Following these ideas, the purpose of this dissertation is to study whether increasing the inflation target allows central banks to further reduce the real interest rate when they decrease the nominal interest rate.

In addition to this action, central banks can use the money supply channel to boost the economy, which is a good alternative when facing the zero bound. After unsuccessfully lowering the interest rates, they started a massive program of buying assets to increase the general liquidity of the economy, known as the quantitative easing (QE)[3]. Nonetheless, in this study we will discuss monetary policy in terms of changes in interest rates rather than in changes in money supply, because it has become more usual for central banks to use this channel rather than the money supply channel (see Alvarez et al (2009)).

Using a general equilibrium model, where agents differ in their amount of money holdings because of the limited access to financial markets, it will be compared the effect of an unanticipated temporary decrease in the nominal interest rate on the real interest rate, in three inflation regimes. For this purpose, we will consider the case of 1%, 2% and 4% inflation. We consider the first case because the average inflation in the last two years in the US has been 1%. The second case because it is the actual target. The third is the one suggested by Blanchard et al (2010) and Fuhrer & Madigan (1997).

We use an inventory-theoretic model of money demand for two reasons. On the one hand, to compare the results of this type of models to the ones used in other papers, such as in Fuhrer & Madigan (1997) and Reifschneider & Williams (2000). In this sense, we aim to study how robust are their results. On the other hand, due to the fact that there is no significant literature using this kind of models to test their results for different inflation regimes.

This paper is organized as follows. The next section is a literature review. In section 3 we characterize the economy and the agents (households, firms and the government). In section 4 we present the conditions for the competitive equilibrium.

[3]See Reis (2009) about unconventional monetary policy.

In section 5 we calibrate the model and present the competitive equilibrium results, and compare the effects of different levels of inflation targets in the equilibrium conditions. At the end of the aforementioned section, we will study the effects of the interest rate shock under different inflation regimes, and in the cases when there is also a shock in the productivity and when there is not. In section 6 we will present the main conclusions of our study.

2 Literature Review

The model used in this paper is well-known in the literature. Baumol (1952) and Tobin (1956) both used an inventory theoretic model of money demand in which the market for consumption goods and for financial assets was segmented. The existence of transaction costs makes agents trade funds between each market infrequently. As such, they hold inventories of money to pay their consumption until the next transfer. Romer (1986) has also used this type of models to study the effects of inflation on capital stock accumulation and money demand elasticity.

In order to have real effects, changes in the nominal interest rate should not be accompanied by one-to-one opposite changes in the inflation rate, because in that case the real rate would remain unchanged. To better understand this reasoning, it is useful to use the Fischer equation to decompose the change in the real interest rate in two parts: the change in the nominal interest rate and the change in the expected inflation rate. The equation can be written as $r \approx i - \pi^e$, where r is the real interest rate, i is the nominal interest rate and π^e is the expected inflation. In other words, prices must adjust sluggishly to changes in the nominal interest rate. However, there can be various reasons for such sluggish adjustment.

Grossman & Weiss (1983) and Rotemberg (1984) have used this setup to analyse the effects of expansionary open market operations (OMO) in i and r. They concluded that OMO have real effects not because there are nominal rigidities in prices, but because people tend to spend that extra money smoothly throughout the period (sluggish expenditure). As such, prices increase only gradually. For this reason, in order for either OMO or direct changes in i to have real effects it should be the case that there are or nominal rigidities (the typical Keynesian scheme) or households have inventories of money (Baumol-Tobin scheme).

Fuhrer & Madigan (1997) and Orphanides & Wieland (1998) studied how the effectiveness of monetary policy changes as the target of inflation changes. It is unanimous among the authors that if the inflation target is lower, then the probability of the zero bound becomes binding increases. In fact, Fuhrer & Madigan (1997) claimed that central banks might not be able to invert the cycle if the short-term nominal interest rates are already close to zero due to low inflation level. The results of Orphanides & Wieland (1998) go in line with the general idea of our work, i.e., increasing inflation target increases the monetary policy effectiveness, namely by allowing a bigger fall in the interest rate, which prevents the output gap from being higher during recessions.

The model used in this work will be similar to the one used by Alvarez et al (2009) and Verheij (2012). We will construct a cash-in-advance model with exogenous market segmentation. This means that, in practice, no transfer costs will be charged to households, contrasting with Alvarez et al (2002), Alvarez & Lippi (2011) and Silva (2012), but instead we will impose a timing between each transfer (households can only transfer funds between each account every $N > 1$).

One important conclusion of Alvarez et al (2009) is that, in a setup where prices are flexible, inflation responds sluggishly to an exogenous increase in the nominal interest rate. This means that the model used is able to produce real effects in the interest rate. An important feature sustaining these results is the endogeneity of the velocity of money, V, which goes in line with the sluggish expenditure response of agents found by Grossman & Weiss (1983) and Rotemberg (1984). Making V endogenous will allow it to vary as the inflation level changes. It is expected to have a higher V when the inflation is higher than otherwise because when the inflation is higher, so it is the interest rate level. If this happens, then the money demand decreases, and so the velocity of money increases (see also Mendizábal (2006)).

Since our model will test different steady state inflation regimes, it must be taken into account that the number of times agents go to the bank is different in each one. As noticed by Jovanovic (1982), if inflation increases, then the number of times agents go to the bank increases, and so the holding period decreases (the time between each trip). This happens because agents have less amounts of money (due to higher interest rates). Thus, our model will have different holding periods for each different level of inflation (in part 5.1 we discuss this issue in a more detailed way).

Moreover, this work will be different from that of previous authors because while it will be applied an exogenous shock in the nominal interest rate over different levels of inflation, and then study the reaction of the real interest rate (r), inflation (π) and output (Y), they did this shock considering the same steady-state level of inflation. Besides, we will include the negative productivity shock in a second case, as in Benk (2005) and Cooley & Hansen (1989).

Unlike Alvarez et al (2009), we make the production endogenous (as in Verheij (2012)) and allow the productivity factor to vary under specific circumstances. We also contrast the representative agent models of Fuhrer & Madigan (1997) and Orphanides & Wieland (1998), since the source of frictions is the fact that not all agents are participating in the market at the same time, so they are heterogeneous and have different individual velocities of money and different amounts of money.

3 Model

3.1 Households

There will exist infinitely lived households and each one is composed by a worker, a shopper and an entrepreneur. The worker supplies units of labour to firms, the shopper purchases consumption goods and the entrepreneur is who trades bonds. As in the standard cash-in-advance models, each period is divided into two sub periods and each market opens at different sub-periods. In the first sub-period, the financial market opens and the entrepreneur trades bonds using the financial account. In the second, the goods market opens and the shopper purchases consumption goods, which can only be purchased using money held in the bank account.

Households can only transfer funds between each account every $N > 1$ periods - if $N = 1$ then the model would be reduced to the standard cash-in-advance model, in which agents could transfer funds between each account every period. As this assumption is exogenously imposed, they will have inventories of money in their bank account high enough to pay their consumption throughout the time period in which they cannot make a transfer.

Time is discrete and denoted by $t = 1, 2, \ldots$ and only a constant fraction $1/N$ of households will be active each period. For simplicity, as in Silva (2012), Alvarez et al (2009) and Verheij (2012), household s will make a transaction at $t = T_j(s)$, $j = 0, 1, 2, ...$

As only a fraction of households is active each period, they face a restriction on the number of times they can transfer funds between the each account. If they are active at time t, then they can transfer funds between the two accounts. Let us call this nominal transfer $X_t(s)$. If not, they can only purchase consumption goods using the money they hold since the last time they did a transaction. Let $p_t c_t(s)$ denote that nominal expenditure. Each worker earns $W_t h_t(s)$ from his labour, where W_t is nominal wage and $h_t(s)$ is the number hours of labour supplied by household s.

They pay lump-sum taxes as well, denoted by σ_t. Let $\gamma \in [0,1]$ be the paycheck parameter as in Alvarez et al (2009), in which γ denotes the percentage of labour earnings that is deposited in the bank account, and $1-\gamma$ in the financial account.

Each household s has to satisfy the inter-temporal budget constraint and the constraints in both accounts. First we present the constraint of both accounts and after the inter-temporal budget constraint that will be used in the household's maximization problem.

The sequence of the bank account constraint is derived as follows. If the household is not active at t, so $t \neq T_j(s)$, then the quantity of money it holds at that time is restricted to the unspent money from the previous period, $Z_{t-1}(s)$, and to the γ part of his last period earnings, $\gamma W_{t-1}h_{t-1}(s)$. On the other hand, if the household s is active at time t, so $t = T_j(s)$, then it can transfer funds from the financial account to the bank account. Therefore, the constraint on the financial account for a household s is,

$$M_t(s) = Z_{t-1}(s) + \gamma W_{t-1}h_{t-1}(s),\ t \neq T_j(s) \tag{1}$$

$$M_t(s) = Z_{t-1}(s) + \gamma W_{t-1}h_{t-1}(s) + X_t(s),\ t = T_j(s) \tag{2}$$

The cash-in-advance constraint of the bank account restricts the consumption expenditure and the amount of money carried on from the previous period to be bounded by the amount of money the households has (3). Moreover, $Z_t(s) \geq 0$, meaning that households can not borrow money.

$$p_t c_t(s) + Z_t(s) \leq M_t(s) \tag{3}$$

Now we move to the financial account constraint. Households can use the financial account to trade one-period risk-free non-contingent bonds, $B_t(s)$, with price equal to one. In the final of period t the bond pays off $(1+i_t)$, the nominal interest rate, which is deposited in the financial account in the next period. Moreover, each period the financial account will be credited by $(1-\gamma)W_{t-1}h_{t-1}(s)$ and debited by the amount of taxes, σ_{t-1}. Therefore, the constraint on the bank account for a household s is,

$$B_t(s) \leqslant (1-\gamma)W_{t-1}h_{t-1}(s) \ + (1+i_{t-1})B_{t-1}(s) - \sigma_t, \ t \neq T_j(s) \tag{4}$$

$$B_t(s) \leqslant (1-\gamma)W_{t-1}h_{t-1}(s) \ + (1+i_{t-1})B_{t-1}(s) - X_t(s) - \sigma_t, \ t = T_j(s) \tag{5}$$

Now we turn to the sequence of the inter-temporal budget constraint. At the beginning of period t each household has to decide the amount of money and bonds it desires to hold, subject to its level of wealth at that period,

$$M_t(s) + B_t(s) \leq \mathbb{W}_t(s) \tag{6}$$

The wealth at the beginning of each period t will depend on the payments received by the hours of labour supplied in the previous period, $W_{t-1}h_{t-1}(s)$, the unspent money of the same previous period, $Z_{t-1}(s)$, the interest received on bonds, $(1+i_{t-1})B_{t-1}(s)$ and the amount of taxes paid in the previous period, σ_{t-1}. Therefore, the wealth for each period can be written as[4],

$$\mathbb{W}_t(s) = W_{t-1}h_{t-1}(s) + Z_{t-1}(s) \ + (1+i_{t-1})B_{t-1}(s) - \sigma_{t-1} \tag{7}$$

If we substitute (7) in (6) we get the period by period budget constraint. If we sum from $t = 0, 1, 2, \ldots$ and multiply by $Q_t = \dfrac{1}{(1+i_0) \times \ldots \times (1+i_t)}$, with $Q_0 = 1$, such that households make the decision at time 0, we get the present value of the

[4] The expression for the nominal wealth comes from the restrictions (1), (2), (4) and (5). Note that $X_t(s)$ is not wealth because it is just a transfer between the two accounts. For this reason, it does not take place in (7). Note also that the wealth of an active and inactive household is the same. The only difference is the proportion of wealth which is held in bonds and in money.

inter-temporal budget constraint of household s,

$$\sum_{t=0}^{\infty} Q_t(s)M_t(s) \leqslant \overline{\mathbb{W}}_0 + \sum_{t=0}^{\infty} Q_{t+1}(s)W_t h_t(s) + \sum_{t=0}^{\infty} Q_{t+1}(s)Z_t(s) - \sum_{t=0}^{\infty} Q_{t+1}\sigma_t(s) \quad (8)$$

$\overline{\mathbb{W}}_0$ is the exogenous initial nominal wealth of household s in terms of money, $\bar{M}_0 > 0$, and bonds, $\bar{B}_0 > 0$.

Following the developments of previous authors, such as Verheij (2012) and Silva (2012), we now look at the cash-in-advance constraint of households between each holding period, i.e., the period of time between each transfer - $T_{j+1}(s)$ and $T_j(s)$, $j = 0, 1, 2, \ldots$. In the first holding period, the household's expenditures will depend on the initial exogenous money holdings, $\bar{M}_0$, and on the earnings from labour of that period. However, the constraint for the first holding period, between $t = [T_0(s), T_1(s)-1]$, will be different from that of the sequent holding periods, for $j \geq 1$, because the amount of money available in the first holding period do not depend on the agent choices, so it can be optimal, given his pattern of consumption, not to spend all the money. As such, $Z_{t-1}(s)$ might be positive, for $t = T_1(s) - 1$.

However, one should have in mind that for the sequent holding periods the amount of unspent money that is carried to the next period is zero because it is not optimal. The reason is the fact that as long as the interest rate is positive, the agent is losing interest. This way, he will only have in the bank account exactly the quantity enough to pay the consumption. Thus, the holding period cash-in-advance constraint is given by,

$$\begin{aligned} &p_{T_j(s)}c_{T_j(s)}(s) + \cdots + p_{T_{j+1}(s)-1}c_{T_{j+1}(s)-1}(s) + Z_{T_1(s)-1} \\ &\leqslant \bar{M}_0 + \gamma(W_{T_j(s)}h_{T_j(s)} + \cdots + W_{T_{j+1}(s)-2}h_{T_{j+1}(s)-2}),\ j = 0, 1, 2, ... \end{aligned} \quad (9)$$

Now that we have described the sequence of cash-in-advance constraint we can introduce it into the inter-temporal budget constraint derived earlier (8). Doing this, we build the inter-temporal budget constraint that will be used in the households' problem, which is equal to the one used in Verheij (2012)[5],

$$\sum_{j=0}^{\infty} Q_{T_j} \sum_{t=T_j(s)}^{T_{j+1}(s)-1} p_t c_t(s) \leqslant \phi + \sum_{j=0}^{\infty} Q_{T_j(s)} \sum_{t=T_j(s)-1}^{T_{j+1}(s)-2} \gamma W_t h_t(s) + (1-\gamma) \sum_{t=0}^{\infty} Q_{t+1} W_t h_t(s)$$
$$+ (Q_{T_1(s)} - 1)(\bar{M}_0 + \sum_{t=0}^{T1(s)-2} \gamma W_t h_t(s) - \sum_{t=0}^{T1(s)-1} p_t c_t(s)) \quad (10)$$

$\phi = \overline{\mathbb{W}}_0 - \sum_{t=0}^{\infty} Q_{t+1} \sigma_t(s)$.

In each period, agents get utility from consumption, c_t, and from leisure, $(1-h_t)$. The period-by-period utility function is the same used in King et al (1988), and more recently by Verheij (2012) and Silva (2012), since its properties ensure an equilibrium state,

$$U(c_t(s), h_t(s)) = \frac{[c_t(s)(1-h_t(s))^{\alpha}]^{1-1/\eta}}{1-1/\eta} \quad (11)$$

The parameters α and $1/\eta$ represent the relative preference for leisure and the relative risk aversion, respectively.

The household's problem consists in choosing the optimal sequence of consumption, $\{c_t(s)\}_{t=0}^{\infty}$, and leisure, $\{h_t(s)\}_{t=0}^{\infty}$, such that its utility function is maximized,

$$max \sum_{t=0}^{\infty} \beta^t \frac{[c_t(s)(1-h_t(s))^{\alpha}]^{1-1/\eta}}{1-1/\eta} \quad (12)$$

subject to the its inter-temporal budget constraint, (10) and the constraint on the bank account for the first holding-period[6],

$$\sum_{t=0}^{T1(s)-1} p_t c_t(s)) \leqslant \bar{M}_0 + \sum_{t=0}^{T1(s)-2} \gamma W_t h_t(s) \quad (13)$$

The parameter β is the discount factor, $0 < \beta < 1$.

[5] See section "Inter-temporal budget constraint" of appendix to see the extended explanation.

[6] The constraint (13) is derived from (9) for $j = 0$. (9) is the general case. However, since the conditions for the first holding period are different, because at least one agent will make is first transfer, we must include it in the maximization process.

The first-order conditions are the following:

$$[c_t]: \quad \beta^t U_{c,t}(s) = P_t[\lambda(s) Q_{T_1(s)} + \mu(s)] \quad \text{M.1}$$

$t = T_0(s), ..., T_1(s) - 1$

$$[c_t]: \quad \beta^t U_{c,t}(s) = P_t \lambda(s) Q_{T_j(s)} \quad \text{M.2}$$

$t = T_j(s), ..., T_{j+1}(s) - 1$

$$[h_t]: \quad \beta^t U_{h,t}(s) = -W_t\{\lambda(s)[(1-\gamma)Q_{t+1} + Q_{T_1(s)}\gamma] + \mu(s)\gamma\} \quad \text{M.3}$$

$t = T_0(s), ..., T_1(s) - 2$

$$[h_t]: \quad \beta^t U_{h,t}(s) = -W_t\{\lambda(s)[Q_{T_j(s)}\gamma + (1-\gamma)Q_{t+1}(s)]\} \quad \text{M.4}$$

$t = T_j(s), ..., T_{j+1}(s) - 2$

For simplicity of exposition, we denote $U_{c,t}(s)$ and $U_{h,t}(s)$ the marginal utilities of consumption and labour of household s at time t. As in Verheij (2012), from the first order conditions, we can find *inter* and *intra-holding* marginal rate of substitution of consumption and *inter* and *intra-holding* marginal rate of substitution between leisure and consumption.

The *intra-holding* marginal rate of substitution of consumption, i.e. in the same holding period, is the following,

$$\frac{U_{c,t}(s)}{U_{c,t+1}(s)} = \frac{\beta P_t}{P_{t+1}}, \ t = T_j(s), ..., T_{j+1}(s) - 1 \quad (14)$$

For any pair $\left[T_j(s), T_{j+1}(s)\right]$, $j = 1, 2, 3, ...$, the *inter-holding* marginal rate of substitution of consumption is,

$$\frac{\beta^{T_j(s)} U_{c,T_j(s)}(s)}{\beta^{T_{j+1}(s)} U_{c,T_{j+1}(s)}(s)} = \frac{Q_{T_j(s)} P_{T_j(s)}}{Q_{T_{j+1}(s)} P_{T_{j+1}(s)}} \quad (15)$$

In the special case of the first and second holding periods, $t = T_0(s) = 0$ and $t = T_1(s)$, the *inter-holding* marginal rate of substitution depends on the Lagrange multiplier of the inter-temporal budget constraint and on the multiplier for the constraint of the first holding period,

$$\frac{U_{c,0}(s)}{P_0} = \frac{\beta^{T_1(s)} U_{c,T_1(s)}(s)}{P_{T_1(s)}} \left[1 + \frac{\mu(s)}{\lambda(s) Q_{T_1(s)}} \right] \tag{16}$$

Following the same reasoning, it will exist four marginal rates of substitution between leisure and consumption, two for the first holding period and two for the other periods.

The marginal rate of substitution between leisure and consumption for the first holding period are,

$$-\frac{U_{h,t}(s)}{U_{c,t}(s)} = \frac{W_t\{\lambda_t(s)[(1-\gamma)Q_{t+1} + Q_{T_1(s)}\gamma] + \mu_t(s)\gamma\}}{P_t[\lambda_t(s)Q_{T_1(s)} + \mu_t(s)]}, \ t = T_0(s), ..., T_1(s) - 2 \tag{17}$$

$$-\frac{U_{h,t}(s)}{U_{c,t}(s)} = \frac{W_t\{\mu_t(s)\gamma - \lambda_t(s)Q_{t+1}\}}{P_t[\lambda_t(s)Q_{T_1(s)} + \mu_t(s)]}, \ t = T_1(s) - 1 \tag{18}$$

And for the other periods it is,

$$-\frac{U_{h,t}(s)}{U_{c,t}(s)} = \frac{W_t\{Q_{T_j(s)}\gamma + (1-\gamma)Q_{t+1}(s)\}}{P_t Q_{T_j(s)}}, \ t = T_j(s), ..., T_{j+1}(s) - 2 \tag{19}$$

$$-\frac{U_{h,t}(s)}{U_{c,t}(s)} = \frac{W_t\{Q_{T_j(s)}\gamma + (1-\gamma)Q_{T_{j+1}}(s)\}}{P_t Q_{T_j(s)}}, \ t = T_{j+1}(s) - 1 \tag{20}$$

3.2 Firms

Contrary to the endowment economies in the models of Alvarez et al (2012), Chiu (2007), amongst others, the production in our model will be endogenous. As in the case of Silva (2012), Adao & Silva (2012), Verheij (2012), Enders (2015) and Edmond (2003), firms will operate in perfectly competitive markets and hire labour supplied by households as the unique input of production. This input is traded in a competitive market and prices are flexible.

The production technology will be given by,

$$Y_t = A_t L_t \tag{21}$$

$$A_t = \varphi_a A_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2), \quad 0 < \varphi_a < 1 \tag{22}$$

A_t is the parameter that controls the total factor productivity. Firms will maximize production subject to,

$$C(w) = W_t L_t \tag{23}$$

From the the firms' problem, we know that the real remuneration of the production factor is equal to its marginal physical productivities,

$$W_t / P_t = A_t \tag{24}$$

3.3 Government

The government issues bonds, $\bar{B}_t$, and prints money, $\bar{M}_t$. For simplicity, in this exercise we will not consider public expenditures, and the unique spending will concern the debt service, both interest and debt amortization (as in Alvarez et al (2009) and Edmond (2003)). Revenues come from taxes, $\bar{\sigma}_t$, collected from households and new issued debt, $\bar{B}_t$. This way, the period-by-period budget constraint states that the spending, $(1 + i_{t-1})\bar{B}_{t-1}$, should not be higher than the revenue side,

$$(1 + i_{t-1})\bar{B}_{t-1} \leqslant \bar{M}_t - \bar{M}_{t-1} + \bar{B}_t + \bar{\sigma}_t \tag{25}$$

Multiplying by Q_t and summing for $t = 0, 1, 2, ...$ we get the present value of the inter-temporal government budget constraint,

$$\mathbb{G}_0 \leqslant \sum_{t=0}^{\infty} Q_{t+1}\bar{M}_t i_{t+1} + \sum_{t=1}^{\infty} Q_t \bar{\sigma}_t \tag{26}$$

$\mathbb{G}_0$ are the initial public obligations on money and bonds, and $\sum_{t=0}^{\infty} Q_{t+1}\bar{M}_t i_{t+1}$ is the present value of future seigniorage revenues.

4 Stationary Equilibrium

4.1 Competitive Equilibrium

A competitive equilibrium is a collection of allocations $\{c_t(s), h_t(s), M_t(s), B_t(s)\}_{t=0}^{\infty}$ and prices $\{Q_t, \pi_t\}_{t=0}^{\infty}$ such that households and firms maximize their utility and production functions, the government satisfies its budget constraint and markets clear.

The economy will be in equilibrium when the goods market is in equilibrium, $\frac{1}{N}\sum_{s=0}^{N-1} c_t(s) = Y_t$, the labour is in equilibrium, $\frac{1}{N}\sum_{s=0}^{N-1} h_t(s) = L_t$, the bonds market is in equilibrium, $\frac{1}{N}\sum_{s=0}^{N-1} B_t(s) = \bar{B}_t$, and the money market is also in equilibrium, $\frac{1}{N}\sum_{s=0}^{N-1} M_t(s) = \bar{M}_t$.

4.2 Solving the Model

The equilibrium will be characterized by constant inflation, nominal interest rate and productivity. As such, the endogenous variables of the model, consumption, labour, production and the real interest rate will be constant.

As in Alvarez et al. (2009), Silva (2012) and Verheij (2012), from now on households will be indexed by the number of periods since they made the last transfer, denoted by $s = 0, 1, 2, ..., N-1$. For instance, a household of type $s = 0$ is active in the current period. A household of type $s = 1$ was active in the previous period, and one of the type $s = N-1$ will be active in the next period.

From the *intra-holding* marginal rate of substitution of consumption (14) we get the following relationship in the steady state,

$$\frac{U_c(s)}{U_c(s+1)} = \frac{\beta}{1+\pi}, \quad s = 0, ..., N-2 \tag{27}$$

From the *inter-holding* marginal rate of substitution of consumption (15) we get the well-known relationship between nominal interest rate and inflation rate,

$$\frac{1}{\beta} = \frac{1+i}{1+\pi} \tag{28}$$

From the marginal rate of substitution between leisure and consumption we get the following,

$$-\frac{U_h(s)}{U_c(s)} = A[\gamma + (1-\gamma)(1+i)^{-(s+1)}], \; s = 0, ..., N-2 \tag{29}$$

$$-\frac{U_h(s)}{U_c(s)} = A(1+i)^{-N} \qquad , \; s = N-1 \tag{30}$$

To solve the steady state system of equations we used the $N-1$ *intra-holding* optimal conditions of the marginal rate of substitution between leisure and consumption for $s = 0, ..., N-2$, plus that for $s = N-1$, the goods market equilibrium condition and the labour market equilibrium condition. This way, we have to solve a system of $2N+1$ equations. These $2N+1$ equations are used to find the N optimal levels of consumption, $N-1$ optimal labour supply, the equilibrium output and the equilibrium inflation. Thus, we have $2N+1$ unknowns.

The remaining unknowns of the households, $M_t(s)$, $Z_t(s)$, $X_t(s)$ and $B_t(s)$ will be characterized using the conditions (1), (2), (3), (4) and (5). Note that the real wage, from the firm's maximization problem, is equal to A in the steady state. From now on, lower case letters correspond to the variables in real terms.

Using (1) and (2), the real money holdings in the steady state will be,

$$m(0) = z(N-1) + \gamma A h(N-1) + x(0), \quad s = 0 \tag{31}$$

$$m(s) = z(s) + \gamma A h(s), \quad s = 1, ..., N-1 \tag{32}$$

By condition (3), the real unspent money is defined as,

$$z(0) = m(0) - c(0), \quad s = 0 \tag{33}$$

$$z(s) = m(s) - c(s), \quad s = 1, ..., N-1 \tag{34}$$

The real money transfers are defined using conditions (1), (3) and the fact that in equilibrium real money supply is constant, i.e., $m_t - m_{t-1} = 0$. Thus, we have that,

$$x(0) = c(N-1) - \gamma A h(N-1), \quad s = 0 \tag{35}$$

The equations (4) and (5) give us the conditions to solve for bond holdings[7]. It can be shown that real bond holdings are given by,

$$\begin{bmatrix} b(0) \\ b(1) \\ \cdots \\ b(N-1) \end{bmatrix} = \begin{bmatrix} 1 & -1/\beta & \cdots & -1/\beta \\ -1/\beta & 1 & \cdots & -1/\beta \\ \vdots & \vdots & \ddots & \vdots \\ -1/\beta & -1/\beta & \cdots & -1/\beta \end{bmatrix}^{-1} \begin{bmatrix} (1-\gamma)Ah(0) - x(0) \\ (1-\gamma)Ah(1) \\ \cdots \\ (1-\gamma)Ah(N-1) \end{bmatrix}$$

5 Results

5.1 Steady State

5.1.1 Calibration

The model will be calibrated so that it adjusts to the U.S. economy in the period between 1974 and 2014. The parameters that have to be calibrated are $\beta, \alpha, \eta, \gamma$,

[7]We aggregated equations (4) and (5) in matrix denomination. The first row corresponds to the active agent.

N and A. We will define β, the discount factor, equal to $\left(\frac{1}{1.03}\right)^{1/12}$, such that 2% annual inflation implies an annual real interest rate of 3 %. The preference for leisure parameter, α, will be set to 1.2, such that agents spend around 40 percent of their time working. We set η, the degree of risk aversion, equal to the value usually used in the literature, which is one (some examples are Silva (2012), Adão & Silva (2012) and Verheij (2012)). The paycheck parameter, γ, will be setted equal to 0.6, accordingly to the most recent data on the U.S. households[8]. The technology parameter, A, will be setted such that the output of the benchmark case (when $\pi = 2\%$) is one.

To calibrate N, the duration of the holding period measured in months, we used the formula proposed by Alvarez et al (2009) to define N under the assumption of an exogenous market segmentation. Their formula relates aggregate velocity, $\bar{v}$, to N and γ, as $\bar{v} = 2/[(N+1)(1\text{-}\gamma)]$. However, to use it, we must have data on the velocity of money for each level of inflation, in order to find the correspondent N.

Considering the Quantitative Theory of Money (QTM), where $PY = MV$, P is the price level, Y the real output, M the money demand, and V the velocity of money, we can see that the money-income ratio, $\frac{M}{PY}$, is equal to the inverse of the velocity of money, $\frac{1}{V}$. We will estimate the relation between the money-income ratio and the nominal interest rate to find the velocity of money for each level of interest rates. After this, we use the level of the nominal interest rate which is compatible with each inflation regime, given the conditions of our model. In the final, we get a relation between the velocity and the inflation level.

Figure 3 plots the estimates for the relation between the money-income ratio and the nominal interest rate[9]. From condition (28), when the inflation level is 1%, 2%

[8]As in Alvarez et al (2009) in order to calibrate γ we had to analyse the microeconomic data to see what has been the percentage of the personal income that has been received as wages and salaries of the US households. We observed that this percentage has been around 60% on average over the period from 1974 to 2014. The data was collected from the US National Bureau of Economic Analysis.

[9]Following Teles & Zhou (2005), amongst others, we use the MZM aggregate to compute the money-income ratio, since the traditional measures (M1, M2, and M3) have been losing its stable relation with interest rates - highly proportioned by the deregulatory measures taken in the 90's.

an 4%, the nominal interest rate is 1.25%, 2.25% and 4.26%, respectively. Then, we used this values of interest rates to see the corresponding money-income ratio. Our findings led us to use N equal to 21, 18 and 14 when the inflation is 1%, 2% and 4%, respectively[10].

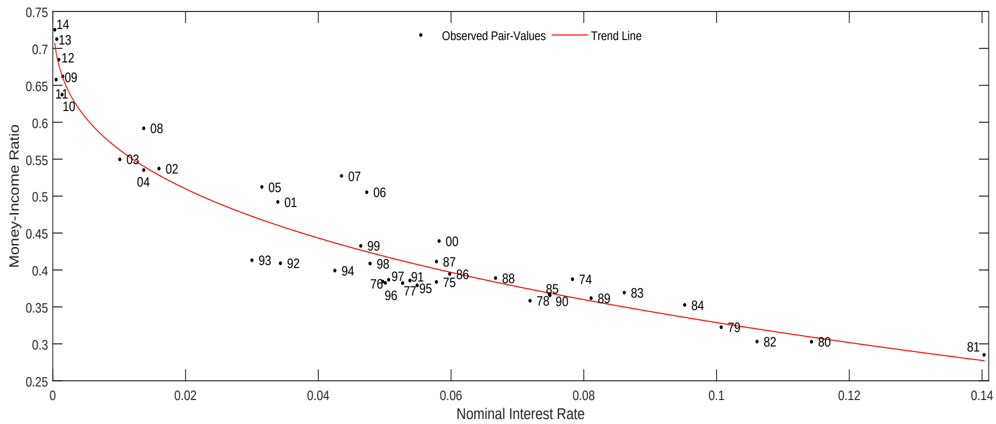

Figure 3: Money-income ratio and nominal interest rates. The curve of figure 3 is of the type $ax^b + c$, with $a = -0.9396$, $b = 0.3233$ and $c = 0.7751$. The coefficients a, b and c were estimated using the non-linear least squares estimator, with confidence intervals of 95%. These calculations were made using the *curve fitting* tool in MATLAB. The nominal interest rate is the 3-month treasury bill rate.

Nonetheless, if one were to use the calibration suggested by some authors such as Silva (2012), where the segmentation is endogenous, N would not be very different from these values. It would be 24, 21 and 18 for 1%, 2% and 4% inflation rate, respectively.

5.1.2 The Benchmark Case

To facilitate the exposition, the case of 2% inflation will be considered the benchmark case. Figure 4 shows the typical *saw-tooth* pattern of money holdings in this type of model. As we can see, the pattern of the real money holdings is decreasing as the agent is closer to the transaction moment (which occurs when $N = 18$). This happens because the amount they have in the bank account must be sufficient to

Also Lucas and Nicolini (2015) used a new aggregate, the "NewM1".

[10]Figure A.1 of the appendix shows the evolution of N as the inflation increases.

pay their consumption expenditures throughout the holding period. As such, they have incentives to spend it steadily.

Given the decreasing pattern of money holdings, their consumption is decreasing as well, since money is the only good which can be used to pay consumption (figure 5). Figure 6 shows us the labour supply curve. As we can see, it is increasing in s, i.e. as the time of the transaction gets closer the labour supplied is higher - as in the case of the velocity. This fact is expressed in the marginal rate of substitution between consumption and labour (equations 29 and 30) - whenever consumption is decreasing, labour supply is increasing.

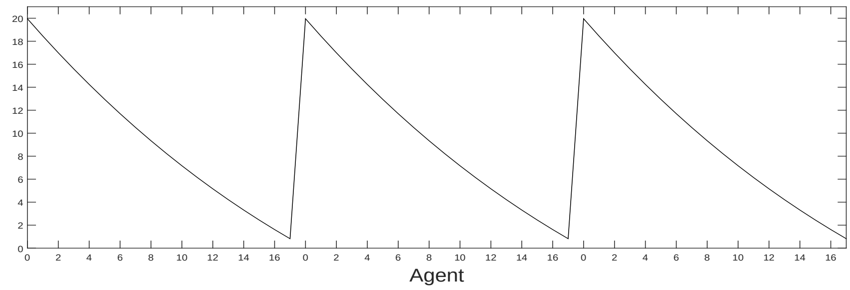

Figure 4: Real Money Holdings

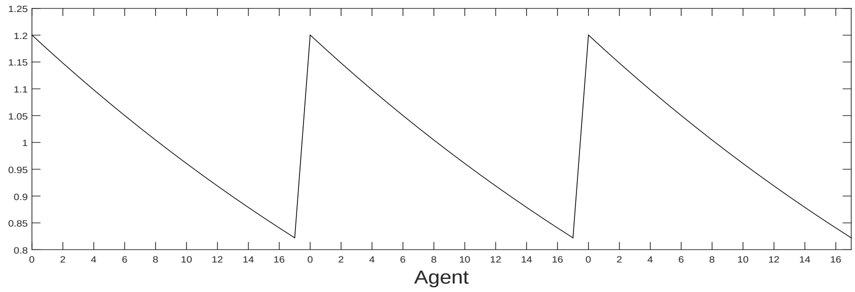

Figure 5: Real Consumption

Figure 7 shows the pattern of the velocity of money. As we can see, it shows that the velocity is increasing in s. The reason why it is increasing in s has to do with the amount of money the household s has in its account. A household of the type $s = 0$ has the largest amount of money and so it tends to spent it relatively slowly when compared to one of the type $s = N - 1$, because that amount of money must be sufficient to pay the consumption expenditures until the next transfer. The non-constant velocity is a characteristic of this type of models.

Note that each household has the same wealth, i.e. $\mathbb{W}_t(s)$ is equal for the household of type $s = 0$ and for the type $s = N - 1$. What differs is how this wealth is split between money and bonds. A household of type $s = 0$ holds more money and less amount of bonds compared to one of the type $s = N - 1$, which holds less money but more bonds. Therefore, while money holdings are decreasing in s, bond holdings are increasing in s, such that the level of wealth remains unchanged.

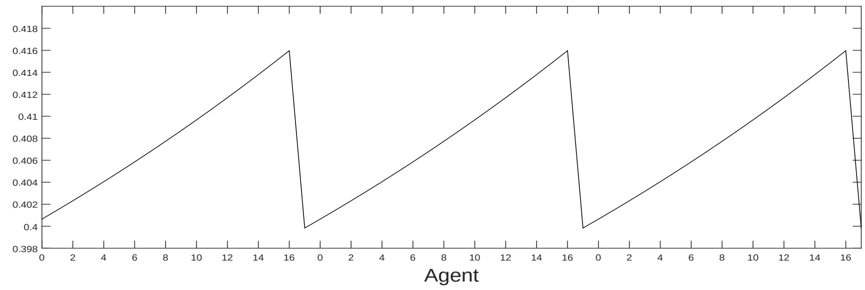

Figure 6: Labour Supply

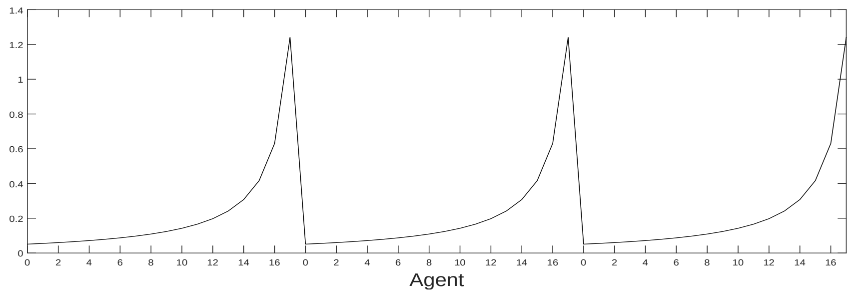

Figure 7: Velocity of Money

5.1.3 Long Run Implications of Different Inflation Regimes

Before studying the short-run effects of changing the inflation regime, we take a brief look on what happens in the economy in the long-run. Changing the average level of inflation makes agents readjust their choices on how to use their money. One immediate consequence is the change in the opportunity cost of present and future consumption. Thus, money holdings, consumption, labour supply and bond holdings will change in response to these readjustments.

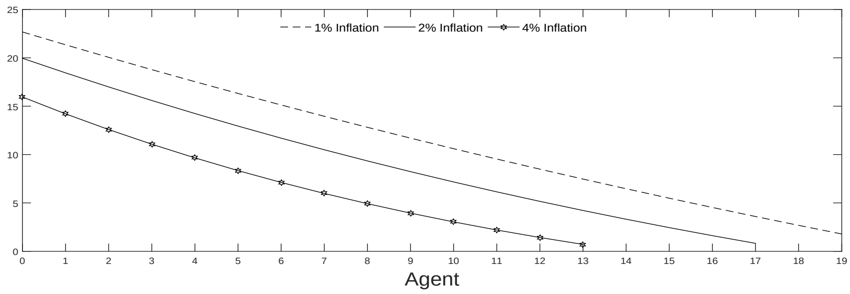

Figure 8: Real Money Holdings

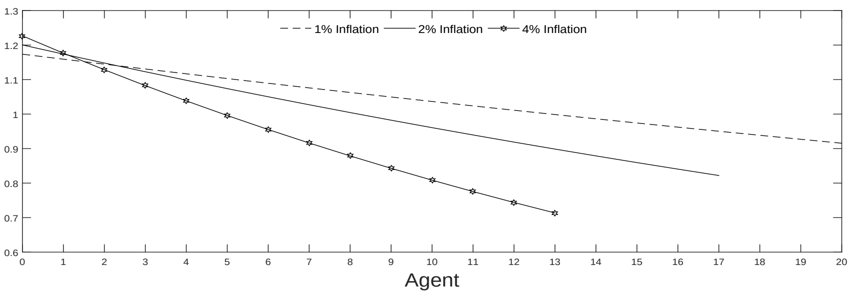

Figure 9: Real Consumption

The level of real money holdings decreases as the inflation level increases (figure 8). This happens because rational agents choose to hold less money today and invest more in bonds today (their bond holdings increases), in order to consume relatively more in the future, due to the higher nominal interest rates income. In other words, they choose to postpone a part of their actual level of consumption (figure 9). In fact, for the majority of the different types of households, the level of consumption decreases.

Velocity increases as a result of the fact that the effect of inflation is higher in the money holdings than in the level of consumption (figure 11). Hence, if the decrease in the amount of money held is higher than the decrease of present consumption, velocity increases (also described by Edmond(2003)). Figure 10 plots the changes in the labour supply. We can see that when the inflation target increases, the amount of labour supplied decreases. This happens due to the lower real wage, since nominal

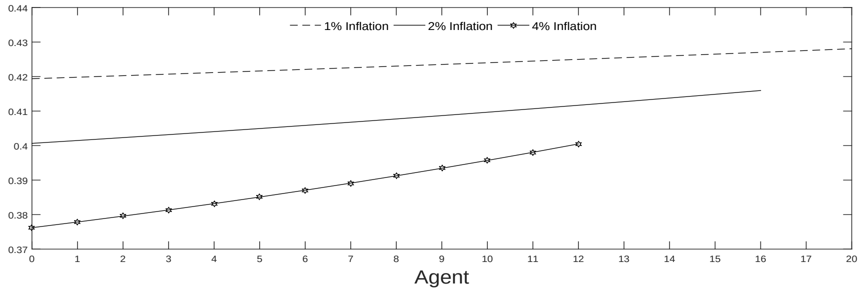

Figure 10: Labour Supply

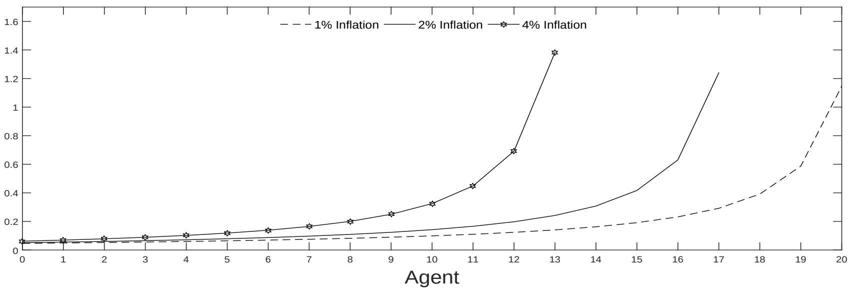

Figure 11: Velocity of Money

wages do not adjust to the level of inflation. Another reason pointed by Cooley & Hansen (1989) is that households increase the consumption of leisure and decrease the consumption of goods due to the *inflation tax* charged in the latter. Thus, the labour supply decreases as the inflation level (and so the *inflation tax*) increases.

Table 1 summarizes the effects of different levels of inflation on steady state real output Y_{ss}, consumption C_{ss}, labour supply L_{ss}, money holdings M_{ss} and velocity V_{ss}[11]. The deviations are in respect to the benchmark model (when $\pi^* = 2\%$).

One can observe that whenever changes in inflation take place, either positive or negative, all the variables change. When we increase the average level of inflation production, labour supply and money held by households decreases. On the opposite, velocity and bond holdings increases (results are very similar to the ones obtained by Cooley & Hansen (1989)). We can also see that money holdings are more sensible to changes in inflation than the output. This happens due to the response of the velocity - when money holdings are increasing, the velocity is decreasing, and

[11]See section "Other equations" of appendix to see how aggregated velocity is computed.

vice-versa.

Table 1: Long Run Impact of Changing Inflation Target

	Inflation Regime		
	1%	2%	4%
		Levels	
Y_{ss}	1.04	1.00	0.95
C_{ss}	1.04	1.00	0.95
L_{ss}	0.42	0.41	0.39
M_{ss}	11.0	9.4	7.2
V_{ss}	1.1	1.3	1.6
		Variation, %	
ΔY_{ss}	3.9	-	-5.12
ΔC_{ss}	3.9	-	-5.12
ΔL_{ss}	3.9	-	-5.12
i	1.25%	2.25%	4.26%

The heterogeneity of our model allows us to see the equilibrium response of each type household to the change in the inflation regime (see table 2)[12]. For instance, when the target increases from 2% to 4% the decrease in the money holdings is increasing in s. This means that those households that will have to wait less time to make a transfer are more willing to give up a higher amount of money than those that, for instance, have made a transfer in the previous period. Recall that consumption goods can only be purchased using money. Thus, and given the limit access households have to their interest income, their money stock has to be sufficient until the next transfer.

Going from 2% to 1% inflation has similar results, but with different directions. We have seen that decreasing the inflation, increases the general money holdings. However, this increase in money holdings is increasing in s. This means that a household of type "5 months" increases less its money holdings compared to one of the type $s = N - 1$. The reason can be the fact that the first one has made a transfer more recently, and thus it holds a larger amount of money, compared to the

[12]Since for each different type of inflation, there is a different N, and so a different number of households, to make this comparison reasonable we indexed households by the number of periods they will still wait to make a transfer. The type of agent "5 months" is the agent that will make a transfer 5 months from now. But this agent corresponds to the agent $s = 16$ in the case of 1% inflation, $s = 13$ in the case of 2% inflation, and to the agent $s = 4$ in the case of 4% inflation. The same reasoning applies to the other cases.

type $s = N - 1$. Thus, this household has little incentive to have a higher amount of money.

Table 2: Changes to the case $\pi^* = 2\%$, %

	Money Holdings		Consumption	
Type of Agent	$\pi^* = 1\%$	$\pi^* = 4\%$	$\pi^* = 1\%$	$\pi^* = 4\%$
8 months	4.18	-0.69	3.9	-0.57
5 months	7.4	-6.49	7.07	-6.19
1 months	11.3	-13.2	11.3	-13.2

5.2 Impulse Response Dynamic

5.2.1 Methodology

The model will be solved using non-linear methods using the function *fsolve* of MATLAB. First, we assume that after the exogenous shock in the nominal interest rate the economy will be back to its steady state equilibrium after a sufficient number of months, t^*. We will assume that both the interest rate and the productivity shock follows a first-order autoregressive process (AR(1)), and therefore their path are known. Since both paths are known we can solve the model backwards, i.e., we find the values for consumption, inflation, hours of labour and production at $t = t^* - 1$ using the values at t^* - note that at $t = t^*$ the economy is in equilibrium. There is one value for the equilibrium output, one equilibrium value for the inflation, and N equilibrium values for consumption and hours of work - since we have defined the existence of N agents. Therefore there are 2N+2 unknowns. To solve this we need to use the optimality conditions we have derived earlier: the $N - 1$ *intra-holding* optimal conditions between t^* and $t = t^* - 1$,

$$\frac{U_{c,t^*-1}(s)}{U_{c,t^*}(s+1)} = \frac{\beta}{1+\pi_{t^*}}, \quad s = 0, ..., N-2 \tag{36}$$

The *inter-holding* optimal condition,

$$\frac{U_{c,t^*-1}(N-1)}{U_{c,t^*}(0)} = \frac{\beta}{1+\pi_{t^*}}\Big[(1+i_{t^*-N})\dots(1+i_{t^*-1})\Big], \quad s=0,...,N-2 \tag{37}$$

N marginal rate of substitutions between leisure and consumption at $t = t^* - 1$,

$$-\frac{U_{h,t^*-1}(s)}{U_{c,t^*-1}(s)} = A_t\left[\gamma + \frac{(1-\gamma)}{(1+i_{t^*-1-s})\dots(1+i_{t^*-1})}\right], \ s=0,...,N-2 \tag{38}$$

$$-\frac{U_{h,t^*-1}(s)}{U_{c,t^*-1}(s)} = \frac{A_t}{(1+i_{t^*-N})\dots(1+i_{t^*-1})}, \ s=N-1 \tag{39}$$

The market clearing condition for the goods market,

$$\frac{1}{N}\sum_{s=0}^{N-1} c_{t^*-1}(s) = Y_{t^*-1} \tag{40}$$

And the market clearing condition for the labour market,

$$\frac{1}{N}\sum_{s=0}^{N-1} h_{t^*-1}(s) = L_{t^*-1} \tag{41}$$

Summing up, we have to solve a system of $2N$+2 equations to find the $2N$+2 unknowns. Now that we have the values of inflation, consumption, hours of work and production we can apply the same method, using the same equations, to solve for $t = t^* - 2$, and so on.

However, for the first $N-1$ months there are some households which will be in their first holding period. As we have seen before, for the first holding period the *inter-holding* optimal conditions and the marginal rate of substitution of leisure and consumption depend on the ratio of the Lagrange multipliers (see 16, 17 and 18). Note that, as mentioned by Verheij (2012), the household which is of the type $s = 0$ at $t = 0$ does not have a first holding period in the first $N-1$ months. Thus, we have only $N-1$ additional unknowns (there can be $N-1$ households which are in the first holding period) for the first $N-1$ periods, and so we need

$N-1$ additional equations. These equations are the bank account constraints for the $N-1$ households that are in the first holding period (see 13).

Then, our system of equations become: the $N-1$ *intra-holding* optimal conditions between t and $t=t+1$, for $t=0,1,...,N-2$

$$\frac{U_{c,t}(s)}{U_{c,t+1}(s+1)} = \frac{\beta}{1+\pi_t}, \quad s=0,...,N-2 \tag{42}$$

The *inter-holding* optimal condition between t and $t+1$,

$$\frac{U_{c,t}(N-1)}{U_{c,t+1}(0)} = \frac{\beta}{1+\pi_{t+1}}\left[1+\frac{\mu(s)}{\lambda(s)Q_{t+1}}\right], \quad s=0,...,N-2 \tag{43}$$

N marginal rate of substitutions between leisure and consumption at t, if the household has made a first transaction,

$$-\frac{U_{h,t}(s)}{U_{c,t}(s)} = A_t\left[\gamma+\frac{(1-\gamma)}{(1+i_{t^*-1-s})\dots(1+i_{t^*-1})}\right], \; s=0,...,N-2 \tag{44}$$

if the household has not yet made a transaction at t,

$$-\frac{U_{h,t}(s)}{U_{c,t}(s)} = A_t\frac{\{\lambda_t(s)[(1-\gamma)Q_{t+1}+Q_{t+N-s}\gamma]+\mu_t(s)\gamma\}}{[\lambda_t(s)Q_{t+N-s}+\mu_t(s)]}, \; s=0,...,N-2 \tag{45}$$

$$-\frac{U_{h,t}(s)}{U_{c,t}(s)} = A_t\frac{\{\mu_t(s)\gamma-\lambda_t(s)Q_{t+1}\}}{[\lambda_t(s)Q_{t+N-s}+\mu_t(s)]} \tag{46}$$

The market clearing condition for the goods market at t, for $t=0,1,\dots,N-2$,

$$\frac{1}{N}\sum_{s=0}^{N-1} c_t(s) = Y_t \tag{47}$$

The market clearing condition for the labour market at t, for $t=0,1,\dots,N-2$,

$$\frac{1}{N}\sum_{s=0}^{N-1} h_t(s) = L_t \tag{48}$$

And the bank account constraint for t, for $t = 0, 1, \ldots, N-2$,

$$\sum_{t=0}^{T1(s)-1} p_t c_t(s) \leqslant \bar{M}_0 + \sum_{t=0}^{T1(s)-2} \gamma W_t h_t(s) \tag{49}$$

For simplicity all equations will be solved assuming they hold with equality. This means that in practice consumers will not leave any money available in their bank account from one holding period to the another. As this exercise will be done using the interest rate channel, as Alvarez et al (2009) pointed out, this creates an indeterminacy because there are more than just one path of the money growth which is consistent with an exogenous path of the nominal interest rate (present in the bank account constraint). Since the empirical results show that, on impact, the change of the inflation is little or absent (see Christiano et al (1998)) we solve this choosing the path of the money growth that remains inflation constant in the first period after the shock (as in Alvarez et al (2009) and Verheij (2012)).

5.2.2 Nominal Interest Rate Shock

As previously stated we intend to study how does the inflation regime influence the effectiveness of the nominal interest channel used by the monetary authorities to stimulate the economy. To study this, we will observe what are the responses of real rates and production to an unanticipated persistent temporary shock of one-percentage-point in the nominal interest rate. The nominal rate is assumed to follow an AR(1) process with a persistence coefficient, ρ, equal to 0.87 as in Alvarez et al (2009).

Changing the inflation regime has effects on the real rate adjustment to a shock in the nominal interest rate. The model used suggests that as the average level of inflation increases, the response of the real interest rate to that shock is smaller. This response is measured as deviations from its steady state level, in percentage-points (pp). Figures 12, 13 and 14 express this ideas. In the case of 1% inflation, on impact, the real rate falls by 0.82 pp as a response to a temporary decrease of the nominal rate of 1 pp, while in the case of 2% and 4% this immediate effect is 0.76 pp and 0.70 pp.

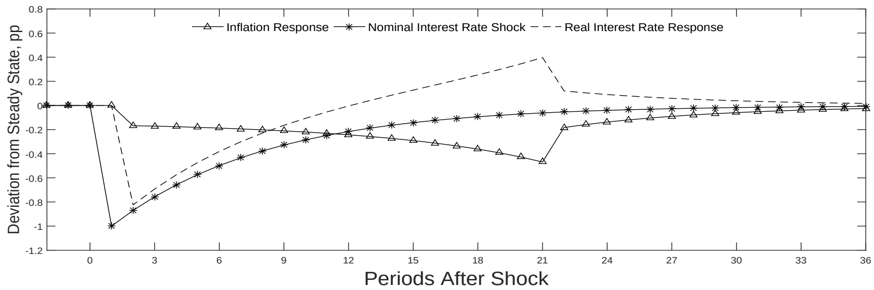

Figure 12: Case of 1% Inflation

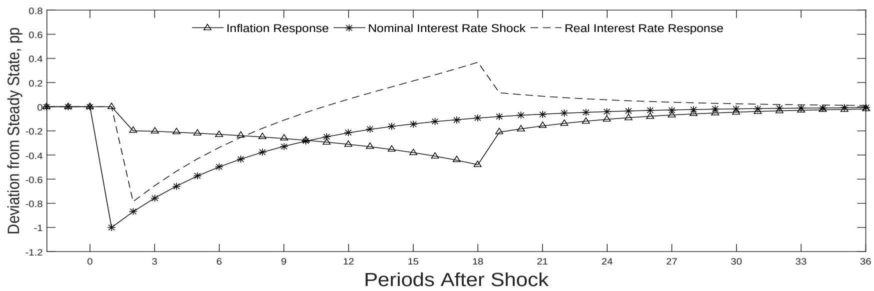

Figure 13: Case of 2% Inflation

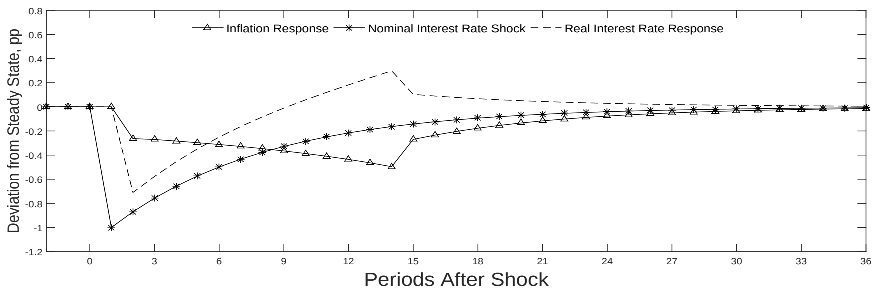

Figure 14: Case of 4% Inflation

The same figures allows us to see the reason why real rates do not stay constant when nominal rate changes. The reason is the fact that inflation adjusts sluggishly to the shock in the nominal interest rate, which allows the real interest rate to vary and produce a persistent liquidity effect (Alvarez et al(2009)). In fact, for such a thing occur, it would have to be the case that, on impact, the inflation rate increased by 1pp.

Both inflation and real rate responses produced by the model have different patterns from those presented by homogeneous agent models. This occurs because every N periods in our model agents readjust their money inventories, increasing the general liquidity of the economy. This could explain why the inflation has the jump after N periods - the liquidity of agents could create an inflation pressure. The real rate starts by decreasing as a first result of the shock in the nominal interest. After this, it stays around 12 months (in the three cases) below its steady state level. However, as the moment of a new transaction is soon the real rate increases slightly above its steady state value - but never offsets the immediate fall at the beginning. In the period after the transaction the real rate drops significantly and then returns back slowly to its steady state value.

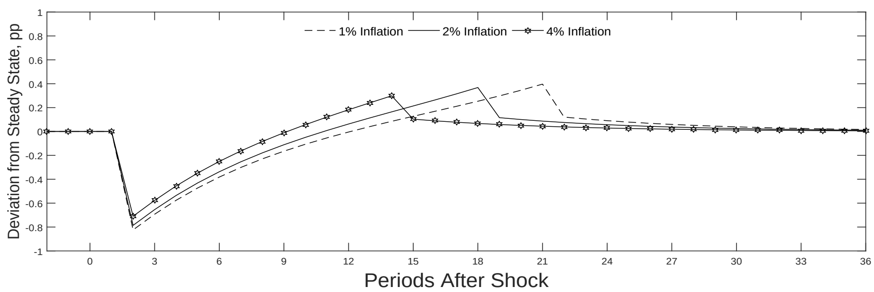

Figure 15: Real Rate Response

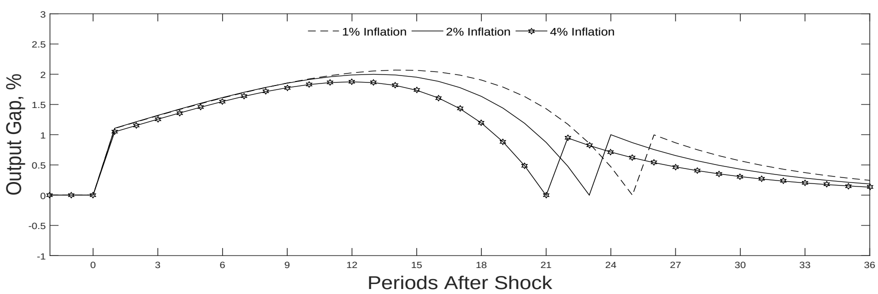

Figure 16: Production Response

Figure 16 shows the evolution of the output after the shock. As expected, monetary incentives through interest rate cuts have a positive effect over the production.

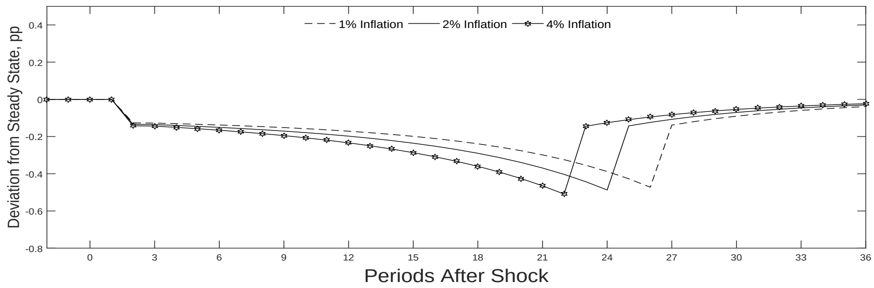

Figure 17: Inflation Response

In fact, such interest rate stimulus can increase the production above the potential at most to around 2%, depending on the inflation level. It seems to be the case that for higher inflation levels the (positive) output gap is smaller and less long standing.

As we said before, the effects of the increase in the liquidity in the economy have significant effects on the transition path of the output. As figure 16 suggests, at the middle of the holding period the output starts to go back to its steady state level. In the period after the transaction it jumps to 0.8% (around this value in the three cases) above its steady state level and then the deviation returns slowly to zero. The reason is that whenever there is a transaction, agents have more money and so they are able to consume more. This increase in consumption forces the production to increase. This explains the jump in both output, inflation and real interest rate.

Our model is able to meet the empirical findings about the time monetary policy measure lag to have their maximum effects on the output. Through figure 16 we can see that the maximum effect of the monetary policy measure is not reflected in the output in the first months after it has been applied. It seems to be the case that the effects of monetary policy in our model have a lag of about 12 to 18 months. Taylor and Wieland (2012), Batini and Nelson (2002), Gruen et al (1999), Brown and Santoni (1983) have found empirical evidence that this lag can be between 12 and 24 months.

Figure 17 illustrates the response of inflation for each level of inflation target. It can be seen that inflation falls more, the higher the target. As a consequence, given the pattern of the nominal interest rate, the real rate falls less [13]. In other words, this means that the price stickiness decreases for higher levels of inflation target. This is somehow connected to the time that agents have to readjust their portfolios to the shock in the nominal interest rate. Table 3 demonstrates this. For instance, 6 months after the shock in the nominal rate, when the target is 1% the inflation rate is 0.19 pp below its steady state level, while in the case of a target of 4% it is 0.32 pp below.

Table 3: Degree of price stickiness

Time	1	2	3	4	5	6
$\pi = 1\%$	0	-0.169	-0.171	-0.176	-0.182	-0.188
$\pi = 2\%$	0	-0.199	-0.204	-0.211	-0.22	-0.229
$\pi = 4\%$	0	-0.263	-0.271	-0.283	-0.297	-0.312

Although our model has little uncertainty, the aforementioned fact reflects some ideas of Friedman (1977). In his paper, the author emphasized the negative effects of higher levels of inflation on output growth, due to firms' capacity to extract information from the market. On empirical grounds, more recently, Judson and Orphanides (1999) found the same relationship.

5.2.3 The productivity shock

When we add the shock of the productivity factor[14] (figure 18), the responses of the real rate do not change significantly. In the previous case, we noticed that the real rate fall would be higher, the lower the inflation level. In this case, this relation is verified again (figure 19). As we can see, the real interest rate declines more, the lower the average level of inflation[15].

[13]Our model produced inflation responses such that the higher the target, the higher the volatility of the inflation adjustment. For instance, when the target is 4% the volatility, measured by the standard deviation, is 0.1158, while in the case of 1% is 0.106. Okun (1971) and Caporale et al. (2010), for the Euro Area, have found this empirical relationship as well.

[14]We assume that the productivity shock follows an AR(1) process, $A_t = \varphi_a A_{t-1} + \epsilon_t$ with a persistence coefficient, $\varphi_a = 0.95$, as in Benk et al (2005) and Cooley & Hansen (1989).

[15]The impulse-responses for output, consumption, labour and inflation can be seen in the appendix - figures A.2 to A.7.

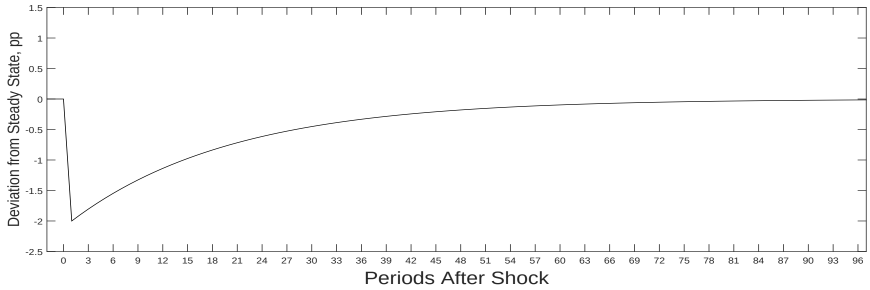

Figure 18: Productivity Shock

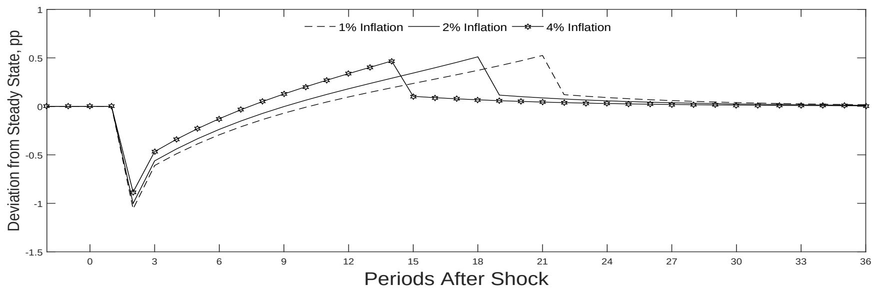

Figure 19: Real Interest Rate Response

However, in this case, when the inflation is equal to 1% and 2%, the real interest rate falls more when compared to the previous one, due to the response of inflation - the shock in the productivity, on impact, decreases the production but increases the consumption, creating a demand-pull inflation. Thus, supporting the previous relationship, increasing the target would not increase the effectiveness of monetary policy to invert the business cycle and stimulate the economic activity.

Even when we add a lag between the shock in the productivity and the decrease of the nominal interest rate, the results do not change[16]. From our calculations, only for lags greater than 10 months the results would be different. Since for more reasonable values, say 3 to 6 months, the results are qualitatively the same, we assumed the lag to be 0 months.

[16]This lag could be the time between the monetary authorities notice the shock and the time for them to decide which action to take and the dimension of that action.

6 Conclusions

Typically when central banks face economic slowdowns they use the interest rate channel to boost economies. However, we have seen that if the nominal interest rate is already at low levels, then their capacity to invert such economic slowdowns is little. The main objective of this dissertation was to study whether increasing the inflation target could increase the capacity of central banks to invert economic downturns. Specifically, we aimed to study whether the real interest rate decreases more when the inflation target is higher, as a response to a negative shock in the nominal interest rate.

To study this we used a general equilibrium model calibrated to the US economy for the period between 1974 to 2014. Our model contrasts the results obtained by Fuhrer & Madigan (1997) and Reifschneider & Williams (2000). It suggests that increasing the inflation target does not increase the real stimulus of central banks when they decrease the nominal interest rate by one percentage-point. In fact, the real interest rate declines more, the lower the target. This applies to both situations, when the interest rate shock is applied alone and when it occurs at the same time of the productivity shock. One possible explanation for this is the fact that for higher levels of inflation targets, the degree of price stickiness is lower. This makes the response of the real interest rate, on impact, to be smaller.

Another conclusion we can take from the impulse-response data is that our model is able to match the data on the lag between monetary policy actions and their maximum effects on the production. On average, the maximum impact is after 15 to 18 months.

What concerns the equilibrium state of the economy, increasing the inflation target decreases the real money holdings and consumption, since households have have an incentive to invest more in bonds due to the higher interest rates. But since the effects of inflation are higher in the money holdings than in their level of consumption, velocity of money increases. Our model also enabled us to see how each type of agent is affected by changes in inflation. For instance, when we increase the inflation target from 2% to 4% money holdings decrease more for those households which are more close to make a transfer, than for those that are more distant of it. On the other hand, when inflation decreases from 2% to 1%, the money holdings increase less for those households that have been active recently than for those which are close to a transfer moment.

One limitation of our model is that the endogenous variables are very sensitive to changes in the inflation rate. For instance, usually benefits from inflation to the output are below 1%. In our exercise we also abstracted from the importance of the *nominal anchor* that some economists claim, as well as the fact that most contracts existent in the economies are not indexed to the inflation - such as wages. We aimed to see only the benefits for monetary policy effectiveness.

Further research using this kind of models could introduce the input capital to the agents' decisions and to the firms' production process. For instance, the Cobb-Douglas production function could be a useful choice. This way it would be possible to see how the investment decisions would vary and whether it would change our results (see Silva(2012) and Cooley & Hansen (1989)). Another point could be instead of working in a closed economy, the model could be of an open-economy. That way it would be possible to account for the exchange rates importance in the agents' decisions (see Alvarez et al (2002)).

A final word should be given to the role of fiscal policy. If fiscal measures are too tight then this can have a significant effect on the inflation level, through its effects on the aggregated demand. In this case, even with low nominal interest rates the desired recovery probably will take time to happen, decreasing meanwhile the effectiveness of the monetary policy.

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A Appendix

Charts

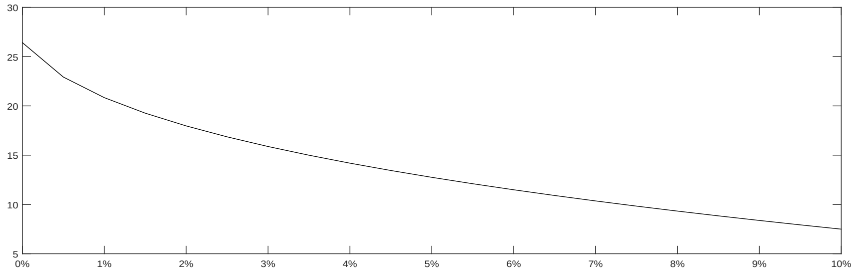

Figure A.1: Changes in the holding period, N, as the average level of inflation changes.

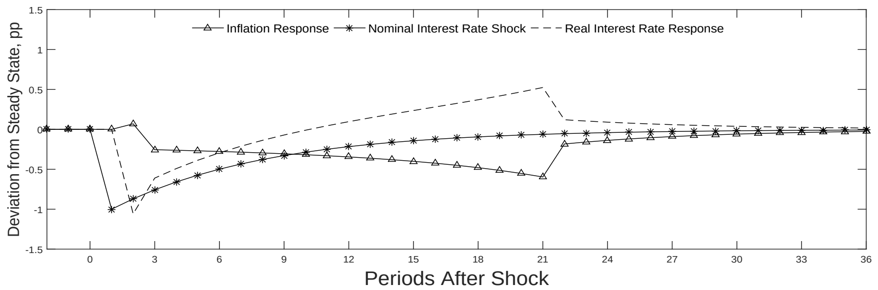

Figure A.2: Case of 1% Inflation

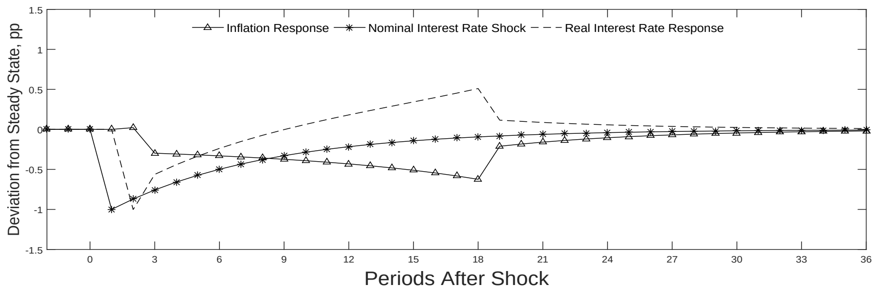

Figure A.3: Case of 2% Inflation

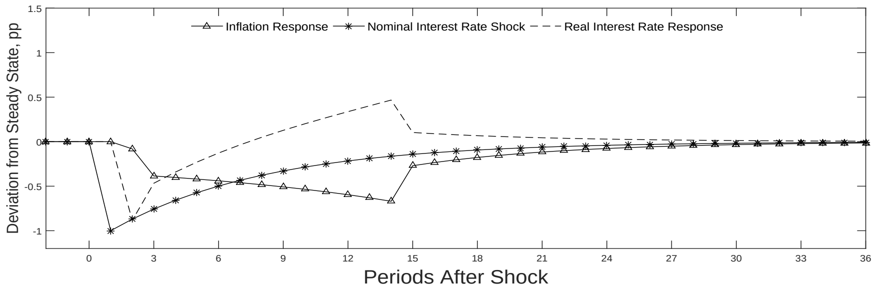

Figure A.4: Case of 4% Inflation

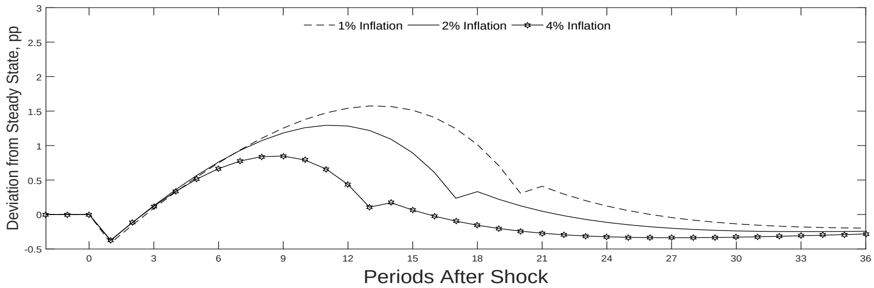

Figure A.5: Output Gap Response

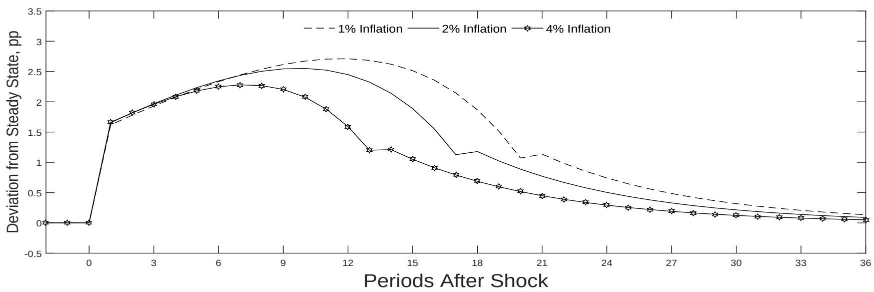

Figure A.6: Labour Response

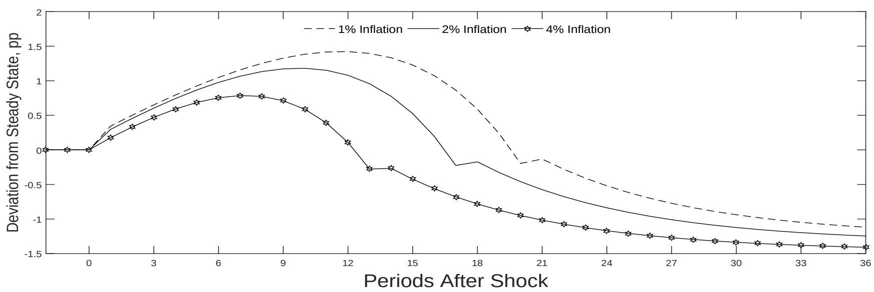

Figure A.7: Consumption Response

Algebra

Inter-temporal budget constraint

Recall that the cash-in-advance for each holding period is given by,

$$\begin{aligned} & p_{T_j(s)}c_{T_j(s)}(s) + \cdots + p_{T_{j+1}(s)-1}c_{T_{j+1}(s)-1}(s) + Z_{T_1(s)-1} \\ & \quad \leqslant \bar{M}_0 + \gamma(W_{T_j(s)}h_{T_j(s)} + \cdots + W_{T_{j+1}(s)-2}h_{T_{j+1}(s)-2}) \end{aligned} \tag{A.1}$$

And the inter-temporal budget constraint by,

$$\begin{aligned} \sum_{t=0}^{\infty} M_t(s)Q_t(s) \leqslant \omega_0 + \sum_{t=0}^{\infty} W_t h_t(s)Q_{t+1}(s) \\ + \sum_{t=0}^{\infty} Z_t(s)Q_{t+1}(s) - \sum_{t=0}^{\infty} \sigma_t(s)Q_{t+1}(s) \end{aligned} \tag{A.2}$$

Summing (A.1) to $j = 0, ..., \infty$ and multiplying by Q_t we have:

$$\begin{aligned} \sum_{j=0}^{\infty} Q_{T_j} \sum_{t=T_j(s)}^{T_{j+1}(s)-1} p_t c_t(s) \; - \sum_{j=0}^{\infty} Q_{T_j} \sum_{t=T_j(s)}^{T_{j+1}(s)-2} \gamma W_t h_t(s) \\ -Q_{T_0(s)} Z_{T_1(s)-1} \;\; \leqslant \; \sum_{j=0}^{\infty} Q_{T_j} \sum_{t=T_j(s)}^{\infty} M_t \end{aligned} \tag{A.3}$$

To introduce (A.3) into (A.2) one have to have in mind that the the quantity of money held in all the periods, disregarding whether it is $t = T_j(s)$, must be equal to the sum of money holdings in the active and inactive periods. Thus, we can roughly use the following relationship:

$$\sum_{t=0}^{\infty} M_t(s)Q_t - \sum_{j=0}^{\infty} Q_{T_j(s)} \sum_{t=T_j(s)}^{\infty} M_t(s) = \sum_{t \neq T_j(s)}^{\infty} M_t(s)Q_t \tag{A.4}$$

Then, plugging (A.3) into (A.2), using the relationship of (A.4), it yields the intertemporal budget constraint,

$$\sum_{t\neq T_j(s)}^{\infty} M_t(s)Q_t+\sum_{j=0}^{\infty} Q_{T_j} \sum_{t=T_j(s)}^{T_{j+1}(s)-1} p_t c_t(s)+Q_{T_0(s)}Z_{T_1(s)-1}-\sum_{j=0}^{\infty} Q_{T_j} \sum_{t=T_j(s)}^{T_{j+1}(s)-2} \gamma W_t h_t(s) \leqslant$$
$$\omega_0+\sum_{t=0}^{\infty} W_t h_t(s)Q_{t+1}(s)+\sum_{t=0}^{\infty} Z_t(s)Q_{t+1}(s)-\sum_{t=0}^{\infty}\sigma_t(s)Q_{t+1}(s) \quad \text{(A.5)}$$

Nonetheless, $M_t(s)$ for $t \neq T_j(s)$ has already been described in (1)[17]. This way, we can substitute it into (A.5). Doing so, we get,

$$\sum_{t\neq T_j(s)}^{\infty} Q_t Z_{t-1}(s)+\sum_{t\neq T_j(s)}^{\infty} Q_t\gamma W_{t-1}h_{t-1}(s)+\sum_{j=0}^{\infty} Q_{T_j} \sum_{t=T_j(s)}^{T_{j+1}(s)-1} p_t c_t(s)+Q_{T_0(s)}Z_{T_1(s)-1}-$$
$$\sum_{j=0}^{\infty} Q_{T_j} \sum_{t=T_j(s)}^{T_{j+1}(s)-2} \gamma W_t h_t(s) \leqslant$$
$$\omega_0 + \sum_{t=0}^{\infty} W_t h_t(s)Q_{t+1} + \sum_{t=0}^{\infty} Z_t(s)Q_{t+1} - \sum_{t=0}^{\infty}\sigma_t(s)Q_{t+1} \quad \text{(A.6)}$$

Using the same relationship (A.4) but now for $W_t h_t(s)$ and $Z_{t-1}(s)$, and recalling that $Q_{T_0(s)} = 1$ it yields,

$$\sum_{j=0}^{\infty} Q_{T_j} \sum_{t=T_j(s)}^{T_{j+1}(s)-1} p_t c_t(s) \leqslant \omega_0+\sum_{j=0}^{\infty} Q_{T_j(s)} \sum_{t=T_j(s)-1}^{T_{j+1}(s)-2} \gamma W_t h_t(s)+(1-\gamma)\sum_{t=0}^{\infty} Q_{t+1}W_t h_t(s)$$
$$+ \sum_{j=0}^{\infty} Q_{T_j(s)}Z_{T_j(s)-1}(s) - Z_{T_1(s)-1} - \sum_{t=0}^{\infty}\sigma_t(s)Q_{t+1} \quad \text{(A.7)}$$

But as $Z_{T_j(s)-1} \neq 0$ only for $j=1$, then we know that $\sum_{j=0}^{\infty} Q_{T_j(s)}Z_{T_j(s)-1}(s)$ becomes only $Q_{T_1(s)}Z_{T_1}(s)$. Replacing into (A.7) we get,

$$\sum_{j=0}^{\infty} Q_{T_j} \sum_{t=T_j(s)}^{T_{j+1}(s)-1} p_t c_t(s) \leqslant \omega_0 + \sum_{j=0}^{\infty} Q_{T_j(s)} \sum_{t=T_j(s)-1}^{T_{j+1}(s)-2} \gamma W_{t-1}h_{t-1}(s) +$$
$$(1-\gamma)\sum_{t=0}^{\infty} Q_{t+1}W_t h_t(s) - (Q_{T_1(s)}-1)Z_{T_1(s)-1}(s) + - \sum_{t=0}^{\infty}\sigma_t(s)Q_{t+1} \quad \text{(A.8)}$$

[17] $M_t(s) = Z_{t-1}(s) + \gamma W_{t-1}h_{t-1}(s),\ t \neq T_j(s)$

$Z_{T_1(s)-1}(s)$ can also be substituted using the relationship given in (18) for the first holding period, since this quantity is reported to that period of time. As such, $Z_{T_1(s)-1}(s) = \bar{M}_0 \ + \sum_{t=0}^{T1(s)-2} W_t h_t(s) \ - \sum_{t=0}^{T1(s)-1} p_t c_t(s)$.

Substituting, one gets the sequence of the inter-temporal budget constraint that will be used in the maximization process:

$$\sum_{j=0}^{\infty} Q_{T_j} \sum_{t=T_j(s)}^{T_{j+1}(s)-1} p_t c_t(s) \leqslant \omega_0 + \sum_{j=0}^{\infty} Q_{T_j(s)} \sum_{t=T_j(s)-1}^{T_{j+1}(s)-2} \gamma W_{t-1} h_{t-1}(s) + (1-\gamma) \sum_{t=0}^{\infty} Q_{t+1} W_t h_t(s) - (Q_{T_1(s)} - 1) \left(\bar{M}_0 \ + \sum_{t=0}^{T1(s)-2} W_t h_t(s) \ - \sum_{t=0}^{T1(s)-1} p_t c_t(s) \right) \ - \sum_{t=0}^{\infty} \sigma_t(s) Q_{t+1} \quad \text{(A.9)}$$

Other equations

Aggregated velocity is given by

$$v_t = \frac{P_t y_y}{M_t} = \frac{1}{N} \sum_{s=0}^{N-1} \frac{P_t c_t(s)}{M_t} = \frac{1}{N} \sum_{s=0}^{N-1} v(s) \frac{M_t(s)}{M_t} \quad \text{(A.10)}$$

Tables

Table 4: Data, data source and periodicity

Variable	Period	Periodicity	Source
Fed Funds Rate	1974 - 2014	Monthly	Fred St. Louis
ECB Main Refinancing Operations Rate	1974 - 2014	Monthly	Fred St. Louis
Bank of England Official Rate	1974 - 2014	Monthly	Bank of England
MZM Stock	2000 - 2015	Annual	Fred St. Louis
Nominal GDP	2000 - 2015	Annual	Fred St. Louis
3M Treasury Bill Rate(secondary market rate)	2000 - 2015	Annual	Fred St. Louis
Personal Income and Its Disposition	1929 - 2014	Annual	BEA
USA Inflation Rate	2014 - 2016	Monthly	Fred St. Louis