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INTERNSHIP REPORT

EVALUATION OF TECHNICAL PROVISIONS AND SOLVENCY CAPITAL REQUIREMENT FOR LIFE INSURANCE

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Abstract

In the pursuit of assuring the financial soundness of an insurance undertaking, a fundamental step is to make sure that the Technical Provisions and Solvency Capital Requirement calculated by the undertaking correspond properly to the obligations and risks it is subjected to.

The new European solvency regime, Solvency II, brought new challenges and implied a deep analysis to the values hold by the undertakings.

This report follows an internship at the Portuguese Insurance and Pension Funds Supervisory Authority whose main objective was to create a tool capable of calculating the amount of Technical Provisions and Solvency Capital Requirement related to the most representative types of Life insurance products. The chosen types are annuity contracts, whole life and term insurance and finally endowment policies. The necessary background studies for each one were made. Noteworthy is the study made for endowment contracts including profit-sharing clauses. Given its inherent relationship with the undertaking's investments and thus with the financial market, economic scenarios were simulated to reproduce the possible behaviour of such investments. A hybrid Heston-Gaussian two-factor model was used to reproduce the behaviour of the interest rate and a stock index.

Resumo

Na prossecução do objetivo de garantir a solidez financeira de uma empresa de seguros, é um passo fundamental assegurar que as Provisões Técnicas e o Requisito de Capital de Solvência calculados pela seguradora correspondem adequadamente às suas obrigações e riscos a que se encontra exposta.

O novo regime Europeu de solvência, Solvência II, trouxe novos desafios e implicou uma análise profunda aos valores detidos pelas seguradoras.

Este relatório decorre de um estágio na Autoridade de Supervisão de Seguros e Fundos de Pensões cujo objetivo principal foi a criação de uma ferramenta capaz de calcular o valor das Provisões Técnicas e do Requisito de Capital de Solvência correspondente aos produtos do ramo Vida mais relevantes. Os tipos de produtos escolhidos foram as anuidades, os seguros de vida inteira e temporários e ainda os capitais diferidos. Os estudos de base necessários para cada um dos referidos tipos foram feitos. Um caso relevante é o estudo efetuado para os produtos com participação de resultados. Dada a sua inerente relação com os investimentos da seguradora e portanto com o mercado financeiro, foram simulados cenários financeiros na tentativa de reproduzir o possível comportamento dos referidos investimentos. Um modelo híbrido Heston-Gaussian dois fatores foi utilizado para reproduzir o comportamento da taxa de juro e de um índice acionista.

Acknowledgements

By the end of this project I feel the necessity to express my sincere gratitude to my supervisor Sofia Frederico for her incessant support and commitment during my internship. I also want to thank my supervisor Professor Hugo Borginho, for the opportunity of integrating ASF, and Professor Lourdes Centeno for playing an important role in making the internship possible. Further I want to mention Professor Onofre Simões that inspired me to choose the field of study.

Finally but not less important I'm deeply grateful to my parents and sister, for their unconditional support during my entire life, and to Marta Tavares for her daily inspiration and strength.

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Acronyms and Abbreviations

ASF	Autoridade de Supervisão de Seguros e Fundos de Pensões
BE	Best Estimate
DRS	Departamento de Análise de Riscos e Solvência
EIOPA	European Insurance and Occupational Pensions Authority
FFT	Fast Fourier Transform
MSE	Mean Square Error
QE	Quadratic Exponential
RM	Risk Margin
RMSE	Root Mean Square Error
SCR	Solvency Capital Requirement
TP	Technical Provisions
VBA	Visual Basic for Applications

1-Introduction

This work was developed during an internship at the Risk Analysis and Solvency Department (DRS) of the Portuguese Insurance and Pension Funds Supervisory Authority (ASF).

DRS, whose responsibilities include monitoring of the solvency and financial solidity of the insurance and pension funds market, on a macroprudential perspective, always working in alignment with the European Insurance and Occupational Pensions Authority (EIOPA), plays a prominent role in what concerns the implementation of the new European solvency regime, Solvency II.

Of great relevance under any regime is assuring that both technical provisions and capital requirements are being adequately calculated and covered by the insurance undertakings, this is the motivation behind this work. I was requested to develop a tool that would be capable of calculating the amount of Technical Provisions (TP) as well as Solvency Capital Requirement (SCR) for the main types of Life insurance products. This tool would also allow users to perform sensitivity analysis by testing different assumptions and evaluating its impact on TP and SCR.

The main program that comprises the deterministic calculations for each type of product considered in this work was built in *Microsoft Excel* using *Visual Basic for Applications (VBA)* to allow for more flexibility with regard to the inputs that can be introduced by the users. As outputs, the program not only computes the main results in terms of TP and SCR, but also makes cash flows projections, allowing users to analyse the liabilities profile and make the necessary comparisons.

As referred above, assumptions, such as mortality tables, financial variables and others, are needed as inputs for this program. The estimations for some of these assumptions were based on previous projects, namely Pateiro (2013), applied with the purpose of updating dynamic mortality tables for the Portuguese pension funds' population to the most recent available data using the software *R*. These tables are taken as reference for the annuitants' population. For participating insurance, Frederico (2010) and Barker (2015) were taken as reference in order to create economic scenarios that would enable the projection of the investment returns for a portfolio composed of bonds and stocks. This was achieved through the use of *GNU Octave*.

In chapter 2, a brief introduction to the main specifications of Solvency II in what concerns the valuation of TP and SCR, with focus on Life insurance business, is given. Following this, in chapter 3 a description of the types of Life insurance products that were considered and the main assumptions and calculations made by the program is presented. This being done, in chapter 4 the chosen method to generate economic scenarios is explained, the need to create such tool to simulate the behaviour of financial variables is justified later in this report. In chapter 5 the application of the previously described techniques is presented along with its numerical results. Finally a brief conclusion of this work is done in chapter 6.

2-Solvency II framework

Now a brief introduction to the relevant Solvency II concepts will be made. It can be found, as a main principle of this regime, the fact that assets and liabilities should be valued based on economic principles. From this we reach the definition of TP, the current amount the undertakings would have to pay if they were to transfer their insurance obligations to another undertaking. This being stated, valuation should rely as much as possible on market information. When the valuation of TP as a whole is not applicable (i.e. when the future cash-flows associated with insurance obligations cannot be reliably replicated using financial instruments), insurance undertakings should separately calculate a Best Estimate (BE) and a Risk Margin (RM). The calculations for TP should take account for the time value of the money, considering for that effect the relevant risk-free interest rate term structure.

BE, defined as the average of all possible outcomes weighted by the respective probabilities, should be calculated using a market consistent approach and applying the appropriate actuarial and statistical methods, including stochastic, deterministic and analytical techniques.

As explained in Baldvisnsdóttir & Palmborg (2011) an appropriate management of financial guarantees by the insurance undertakings has earned extra importance with the fall in the return rates obtained by the undertakings. This problem is still affecting undertakings today in the context of the low interest rate environment. Keeping this in mind, the use of simulation methods for contracts where cash-flows depend of investment returns, such as participating contracts that give rise to discretionary benefits usually leads to more robust results.

Under this approach, economic scenario generators play a key role in modelling the behaviour of financial variables, namely interest rate and equity indexes. For a market consistent valuation, scenarios are generally projected under a risk neutral probability measure.

With regard to the valuation of BE, also noteworthy is the concept of contract boundaries, that defines which cash flows should be included in this calculation. As a general rule, all obligations relating to the existing contracts should be included in it. However, after some defined points in time, as referred in article 18 of the Commission Delegated Regulation (EU) 2015/35, of 10 of October 2014, premiums do not belong to the contract, excluding obligations related to the referred premiums. Those points in time correspond to the dates when the undertaking has the possibility of exercising unilateral rights that would allow it not to assume future risks. This is based in the concept that if the undertaking can avoid incoming cash-flows, outgoing cash-flows will also not occur, preventing the undertaking's exposure to any related risks.

Now for the second part of TP, RM has the aim to ensure that TP represents the amount another undertaking would be expected to require to accept and meet the existing insurance obligations. It is defined as the cost of providing eligible own funds that would match the necessary SCR. This is achieved through the Cost-of-Capital approach.

SCR is the amount of capital necessary to ensure that the undertaking can withstand a relevant volume of unexpected losses. It is calculated under a 99,5% confidence level and a 1 year time horizon. This calculation can be done using the standard formula or internal models developed by the undertaking itself, which require the approval of the supervisor.

The standard formula comprises individual risk modules which are aggregated using correlation matrices. The calculation of the SCR should also reflect, when applicable, the adjustment for the loss absorbing capacity of TP. This adjustment takes into account the potential compensation of unexpected losses by a concurrent reduction in TP. The referred adjustment accounts for the risk mitigating effect of future discretionary benefits, benefits subject to a potential reduction by the undertaking as a mean to cover unexpected losses.

Given the scope of this work the main focus falls on the life underwriting risk module (SCR Life). According to the Commission Delegated Regulation (EU) 2015/35, of 10 of October 2014, in the standard formula the shocks for each of the SCR Life sub-modules are:

- **Mortality risk-** it will be the loss in own funds, as for all the categories, resulting from a permanent increase of 15% in mortality rates, used for the calculations of BE.
- **Longevity risk-** it is calculated for a decrease of 20% in mortality rates.
- **Disability/morbidity risk-** it is calculated based on a combination of events, an increase of 35% of the disability/morbidity rates for the following 12 months, an increase of 25% of the rates after those 12 months, accompanied by a permanent decrease of 20% of the recovery rates from disability/morbidity status.
- **Life-expense risk-** the calculations are based on a scenario of a 10% increase in the amount of the considered expenses combined with a 1% increase on the accounted inflation rates.
- **Revision risk-** applying only for annuities where the benefits can increase as a result of a change in the legal environment or in the health status of the insured person, this SCR is computed considering a permanent 3% increase in those benefits.
- **Lapse risk-** lapse risk is divided into three scenarios, SCR Lapse is the largest of the three. The first and second are opposite scenarios, a permanent increase or decrease of 50% in the options exercise rates for the relevant options. The third scenario is the mass lapse risk, with exception of some situations not considered in this work, is calculated with a discontinuance of 40% of the insured people.
- **Life-catastrophe risk-** for this final component SCR is reached through a scenario where an increase of 0,15 percentage points in the considered mortality rates for the next 12 months occurs.

3-Implementation of the calculation tool

In this chapter a description of the calculation methods and associated hypothesis will be presented. As written above, a tool was developed in Excel to evaluate the TP and SCR. Easily comprehendible is the fact that, in the perfect scenario, all calculations should be done policy per policy or at least by homogeneous risk groups. This statement is backed up by the Solvency II general valuation principles. However, when combining the complexity and diversity of the Life insurance contracts that can be found in the market with the scope and granularity of information that is available to supervisors, one is forced to assume some simplifying hypothesis, such as aggregating the contracts into more standardized types of insurance or using average inputs. Logically, this limitation has to be taken into account in the interpretation of the results.

Three broad groups were considered in this work: annuity contracts, whole life and term insurance (in this category a separate approach was taken for annual renewable term contracts) and endowment insurance (participating and non-participating). Away from this works' scope are the unit-linked contracts.

The following figure provides an overview of the weight of each of these categories in the Portuguese Life insurance market.

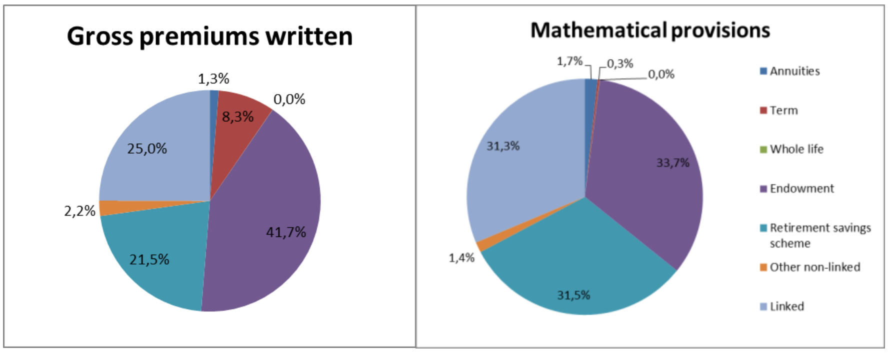

Figure 1: Portuguese Life insurance market (2015)

In what concerns the Life insurance market, non-linked insurance represented, in 2015, 75% of the total life insurance premiums. This compares to a share of close to 80% in 2014, showing an increase of the weight of unit-linked contracts during 2015.

Within the category of term insurance, annual renewable term contracts corresponded to 88% of premiums. The profitability of this line of business has remained stable over the years due to a continuous increase in pricing that compensated the decline in the number of insured persons and capital sums.

Regarding financial insurance, the uncertainty in financial markets has been causing a significant volatility in the financial profitability of the assets underlying these contracts. In recent years, life insurance undertakings have been offering lower guaranteed rates in new products as response to the current low interest rate environment. But due to the limited representativeness of these products in the overall portfolio of existing contracts, the global average guaranteed rate has not decrease significantly. Therefore, it is predictable that this sector will continue to face difficulties in the next years.

BE is calculated as the expected present value of all future cash-flows and, for each risk, SCR as the difference between an hypothetical BE applying each specific SCR's shock to the relevant assumptions and the real BE. Provided that Life insurance is being analysed, the main focus is on the calculation of SCR Life. Keeping this in mind, only RM based on SCR Life is calculated. From the formula of the RM one can conclude that the calculation of future SCRs is necessary. The value for such capital requirements for future moments is obtained assuming that all variables behave according to the assumptions made for the calculation of the BE until the point in time when the SCR is being calculated considering the relevant shocks from that moment on.

Due to the importance of the discount effect on the value of the BE, the capital requirement for the interest rate risk, considering the up and down scenarios in the standard formula, is also calculated. Although interest rate risk affects both assets and liabilities, only the effect on the liabilities side is considered, as on the valuation of assets is outside of the scope of this work.

This internship had the goal to create a tool, as flexible as possible, that could be used in practice to analyse the impact of different assumptions and with that gain some additional insight on how outputs such as TP or the risk profile would vary under different scenarios. Bearing that in mind, all the values that one could think of as inputs instead of fixed constants were indeed left as open variables. Some obvious examples of these inputs are discount rates, mortality tables, age of the insured person, benefits, premiums, expenses, term of the contract, SCRs' shocks, among others.

Although each group of contracts has its own specifications, some general assumptions were made.

- **Unit of time –** in this work the unit of time considered is always the year.
- **Expenses –** expenses were assumed to be a cash-out-flow at the beginning of the year, at the same time premiums are received (when applicable). With the way the program is constructed, an average expense per insured person is considered as an input.
- **Disability/Morbidity –** the wide variety of complementary coverages offered in the market combined with the available information makes it complicated to model this type of benefits. So it was decided to exclude this segment. However, one can comprehend its relevance for some of the types of contracts considered, as these benefits are often part of whole life, term and participating insurance. Consequently, the disability/morbidity sub-module of the standard formula is also not applicable.
- **Revision risk –** this risk is not applicable for the products that are being considered.

3.1-Annuities

The existence of several variants of annuities makes it challenging to analyse every possible contract. Taking that into consideration, some simplifying assumptions were made. A fixed individual annuity is considered, though the possibility of growth of benefits is included in the program. Also noteworthy is the fact that no future premiums are considered, which implies the assumption that the contract is fully paid by the annuitant. The previous assumption eliminates the concern with the way the annuity was sold, i.e. lump-sum or regular payments, which is a reasonable assumption as there are no premium payments for annuities already in force. For the sake of flexibility, the user is given the option to choose between lifelong and temporary annuity and also between immediate and deferred.

In what concerns SCR Life, the longevity risk pointedly stands out. The reason for this is the fact that the longer the life of beneficiaries the larger the number of payments made by the insurer and so if the mortality is lower than the one predicted by the mortality table it can cause long-term financing problems. Apart from this risk, only expense risk is relevant, provided that the remaining components are either not considered or not applicable to these products.

As referred above, the user is given the choice of which mortality table to use. However, when dealing with annuities, an additional option is given, dynamic mortality tables. The reason behind this possibility only being available for this type of contract is the importance of reflecting future evolutions in mortality on the calculation of the best estimate for these products.

3.1.1-Dynamic mortality tables

In accordance with what was written in the introduction, in Pateiro (2013) dynamic mortality tables for the Portuguese pension funds' population were built. As can be read in the paper, the use of these tables allows the calculation of TP considering a future trend on mortality. It is stated in Pateiro (2013) that the best way to achieve longevity increments in the tables is to use models that would extrapolate mortality tendencies. The previous sentence implies that substantial historical data is required to apply such models, this is why at first general population is analysed and afterwards a relational model is applied to obtain the results for the pension funds' population. The fact that these tables are built for the pension funds' population is not deterrent of using them to the studied annuities due to the similarity between the mortality profile of the annuitant of the studied contracts and the pension funds' beneficiaries.

The work's main model was Poisson-Lee-Carter. However, given the shortage of data for ages above 90, a different approach was required. The choice fell on the Denuit and Goderniaux method (Denuit & Goderniaux 2005).

Stating the base hypothesis to be the assumption that all forces of mortality are constant between time and age intervals, in accordance with the primary model,

$\mu_x(t)$: force of mortality of an individual aged x at year t.

(1) $\mu_{x+u}(t+s) = \mu_x(t)$ for $0 \leq u < 1$ and $0 \leq s < 1$.

From here one can arrive to the conclusion that forces of mortality of individual aged x year t can be obtained by the ratio of the number of deaths of individuals aged x at year t and the number of individuals aged x exposed at year t.

The main model, Poisson-Lee-Carter (Brouhns et al. 2002a) results of the original Lee-Carter Model (Lee & Carter 1992),

(2) $ln\, m_x(t) = \alpha_x + \beta_x k_t + \varepsilon_x(t)$

for $x = \{x_1, x_2, \dots, x_n\}$ and $t = \{t_1, t_2, \dots, t_k\}$, where $\varepsilon_x(t)$ is the random part of the model ($\varepsilon_x(t) \sim N(0, {\sigma_\varepsilon}^2)$).

Parameter estimation from observed mortality is done with singular value decomposition method, thus obtaining a minimum squares solution.

Once the parameters are obtained, ARIMA models can be used to project the time trend k_t.

The innovation of the used model was introduced in Brouhns et al. (2002), claiming that the number of deaths can be accurately represented by a Poisson random variable. The authors replace the random term $\varepsilon_x(t)$ by a random variation derived from the inclusion of a Poisson regression,

(3) $D_{xt} \sim Poisson\big(E_{xt}\mu_x(t)\big)$, with $\mu_x(t) = \exp\,(\alpha_x + \beta_x k_t)$.

Now the parameters are estimated by maximizing the logarithm of the likelihood function.

As stated in Pateiro (2013), an alternative method had to be chosen to extrapolate death probabilities of ages above 90. This method needed to be adequate taking into consideration the characteristics of this age group that exhibits a deceleration in the growth of death probabilities. Now a brief description of the chosen method will be made.

The Denuit and Goderniaux method (Denuit & Goderniaux 2005) is based on a log quadratic regression,

(4) $$\ln q_x(t) = a_t + b_t x + c_t x^2 + \varepsilon_{xt}, \qquad \varepsilon_{xt} \sim N(0, \sigma^2),$$

this model will demand that the user decides a value for the limit of the human age, both in Pateiro (2013) and in this work the decision was to consider 125 as the maximum value.

Finally the relational method, to adjust the results to the pension funds' population, two different models were used, Brass relational model (Brass 1974) and a relational model based on Cox proportional hazards model (Cox 1972).

The first type of models relates the two populations using a function $f(\mu_x)$,

(5) $$f\left(\mu_{x,t}^{est}\right) = a + b \times f(\mu_{x,t}^{ref}).$$

This relationship is assumed to hold in time and for all ages.

Again the decision of which function to use was the same in Pateiro (2013) and in this work, the choice fell on the logarithmic function.

Using the alternative, Cox proportional hazards, a base assumption will be that the force of mortality of the study group is proportional to the one of the reference population,

(6) $$\mu_{x,t}^{est} = a \times \mu_{x,t}^{ref},$$

as it can be observed, the proportionality factor, a, is independent of age. Therefore, the relationship will hold for all ages and in time.

3.2-Whole life and term insurance

This type of contract is purely life protection insurance, with a lump-sum being paid in case of death of the insured person. When considering term insurance, logically, the payment is only made if the death occurs during the term. Given the structure of these products, the relevant SCR's components are different. Mortality risk is now important, which implies the addition of the life catastrophe risk. It was decided to separate the group and to analyse annual renewable term contracts in a different worksheet. These contracts have a higher relevance in the current market (in 2015 they represented around 88% of the total premiums for term insurance) and require a distinct programming code.

3.2.1-Classical whole-life and term insurance

For this group of contract the user can first choose whether it is a whole-life or term insurance, in which case the remaining years until maturity is needed as input. Similarly to what was done for the annuity contracts, the hypothesis that all premiums are already paid was established. Thus, there is no need to calculate premiums.

3.2.2-Annual renewable term insurance

As the denomination implies these are annual independent contracts, which in the Portuguese market are commonly associated with mortgages. This justifies the choice to leave an input that defines the variation of the insured capital over time. For these products, BE may be negative, due to the way the products are constructed.

- **Premiums-** there is a need to calculate the premiums for these contracts. They are computed in such a way that the insured capital, amount to be paid in case of death, is included as an input and the program returns the pure premium. To this pure premium, calculated as the present value of the mortality benefit, a load is applied, resulting in the commercial premium, which is assumed to be paid as a lump sum in the beginning of each year. The option of using mortality assumptions and discount rates to calculate the premiums different from the ones applied to calculate BE is included.
- **Lapse-** in this case it is also relevant to consider the possibility that the policyholder ceases to pay the premiums, implying that the contract will not be renewed. If, in particular, the premiums beyond the next renewal date is included in the calculation of the BE, there is exposure to lapse risk. Whether these future premiums and the associated obligations can be considered in practice by insurance undertakings, it will depend on the application of the contract boundaries.

3.3-Endowment

This type of contract, also commonly named deferred capital, consists, as easily interpreted by the second denomination, on the payment of a lump-sum at maturity, corresponding to the accumulated invested capital, if the beneficiary is alive. However, there is a lot more than what meets the eye, there are multiple variations of these products that complicates its analysis. A particular case of these products are the retirement saving schemes, which have some specific characteristics such as longer maturity and withdrawal conditions defined by law.

These products are closer to a financial investment, they are many times preferred because they are perceived as safer than other kinds of investments, namely direct equity investments. One very common variant of this type of product is the participating or with-profits insurance, where part of the interest rate used to accumulate the capital every year depends on the results of the company. In this work, for simplification purposes, the profit sharing mechanism considered depends only on financial results. Although these are mainly contracts of a financial nature, endowment insurance frequently offers life protection too, as well as a withdrawal option.

For the calculation of the SCR, the capital requirements for all the risks that are within the scope of this work are computed, although mortality and longevity risks are not simultaneously applicable.

- **Premiums-** left as inputs there are two variables that will define how much the policyholder will invest, which can also be considered as the previously accumulated amount, the initial capital, and the annual premiums. The number of premiums is left open for input.
- **Guaranteed rate-** the minimum rate at which the capital will be accumulated each year is also left as an input and it can vary with time.
- **Profit-sharing-** as introduced above, profit-sharing will depend on the financial results of the insurance company. A participating rate is left as an input for each year, corresponding to the percentage of the results that will be distributed to policyholders. The extra return rate, in addition to the guaranteed rate, is defined as the participating rate multiplying for the

difference between the profit rate of the company's investments and the guaranteed rate.

(7) $$ps_t = d_t \times (r_t - g_t)$$

Where $ps_t$is the profit sharing rate for year t, $d_t$is the participating rate, $r_t$is the return rate of the company's investments underlying these contracts and $g_t$the guaranteed rate. The resulting rate will then be multiplied by the accumulated capital, it is assumed in this work that these computations are made at the end of each contract year. Logically, if $ps_t$is negative the value of the profit-sharing in that year will be 0.

To calculate BE and SCR for moment 0, the program uses simulations to find r_t, but given the complexity of SCR projections, necessary to obtain the RM, values for BE and SCR for future moments are calculated under a central scenario, according to which investments' return would follow the relevant risk free interest rate term structure.

As the profit-sharing mechanism only depends on financial results, the SCR Life shocks will not have an impact on the amount that is distributed to policyholders. This is also based on the hypothesis that the participation rate $d_t$is the minimum percentage that is contractually defined and therefore cannot be reduced following a shock scenario.

In what concerns the interest rate risk, changes to the relevant risk free interest rate term structure would have an impact on future investments' return. However, as this in an additional sensitivity analysis, only the net calculation is performed, taking into account the impact of the scenarios in the future profit sharing.

- **Options and financial guarantees evaluation-** the time value of options and financial guarantees is calculated. The method behind it was inspired by what is done in the industry and by the CFO Forum Market Consistent Embedded Value Principles, according to which the time value is the difference between the BE calculated using stochastic scenarios and the BE calculated based on the central scenario.
- **Death Benefits-** a very usual feature of this type of contract is the inclusion of a death clause, according to which, in case of death during the contract, a benefit is paid to a beneficiary. To model this benefit, inspiration was again

found on what is done in the market. It is assumed that the lump-sum paid in case of death is the accumulated capital itself. It is accumulated until the end of the year of death as it was assumed that the benefit is paid at the end of the referred year. An also important assumption is that this benefit is calculated right before profit-sharing amount is added to the account. The provision for this situation is calculated adding for each contract year the present value of the total death benefits paid in that year. Logically, this total is reached calculating the total number of deaths in each year. To find that number one has not only to consider the mortality rates but also withdrawal rates, to be explained shortly.

- **Withdrawal benefits-** being investment type insurance, these products include the possibility of withdrawing the capital before reaching the term of the contract, although insurance companies commonly apply a penalty. In this work, this penalty was only applied to the return of the year of withdrawal. Again, the hypothesis that the payment is made at the end of the year, right before profit-sharing is calculated, is followed. From that it can be concluded that the penalty will only affect the guaranteed return. With a similar process to the one followed to obtain total deaths in each year, total annual withdrawals are found and used to reach the best estimate for the withdrawal benefits provision.

  As inputs for this specific part annual withdrawal rates and the annual return penalty are required.

4-Economic Scenario Generator

As previously explained, the use of an economic scenario generator is a key element for the valuation of participating contracts. From here it was decided to use an economic scenario generator to simulate the behaviour of bonds and stocks. The main inspiration for this work was Barker (2015), where hybrid models are used to perform such simulations. In concrete, a Heston Gaussian two-factor interest rate model is analysed. In this approach the two models are calibrated separately and several simulation schemes are attempted, one of them is the Quadratic Exponential (QE) scheme. The main innovation in this approach is considering the correlation between the interest rate and the stock processes.

4.1-Heston model

The choice fell on this model as it is one of the most common stochastic volatility models and it was decided to follow the stochastic volatility approach. In addition, the referred model also allows us to price European options using semi-analytical formulas. Under a risk neutral probability measure this model can be represented by the following system of equations,

$$
\begin{aligned}
&dS(t) = rS(t)dt + \sqrt{v(t)}\,S(t)dW_1(t) \\
&dv(t) = \kappa(\theta - v(t))dt + \xi\sqrt{v(t)}\,dW_2(t) \qquad (8) \\
&dW_1(t)dW_2(t) = \rho_{HE}\,dt.
\end{aligned}
$$

From $\{S(t): t \geq 0\}$ and $\{v(t): t \geq 0\}$ is found, respectively, the representation of the evolution of the price and volatility of a given stock or index, (W_1, W_2) is a bi-dimensional Brownian motion with an instant correlation ρ_{HE}. The parameter r represents the risk free interest rate, with the volatility of the asset price being given by $\sqrt{v(t)}$. From the equations one can see that $v(t)$ follows a mean reverting process, θ is the long term mean and κ defines the reversion speed and the volatility clustering. The volatility of volatility or the strength of the volatility smile is given by ξ.

Bearing in mind the concerns with the speed and accuracy of the computations, the pricing of financial options is done based on the Fast Fourier Transform (FFT). Using the characteristic function of the model this technique obtains an analytic expression for the option price inverting the Fourier transform. The formula for the price of an European call with maturity T and strike price K as well as the explanation and description of its components can be found in Frederico (2010),

$$C_T(k) = \int_k^{\infty} e^{-rT}\left(e^{x_T} - e^k\right) f_T(x_T) d \tag{9}$$

where $x_T = log[S_T]$, $K = e^k$ and $f_T(x)$ is the density function of x under a risk neutral probability measure.

To calibrate the Heston model one can use several objective functions. In Barker (2015) it can be found the justification to use a quadratic norm. Bearing that in mind it was decided to minimize the Mean Square Error (MSE) in accordance to what was done in Frederico (2010). The model is calibrated to call options of the Eurostoxx 50 Index at the 29th of December of 2015. An aspect to take into consideration is to find instruments with a relevant maturity date. As this work is related to Life insurance, longer maturities are more critical.

4.2-Gaussian two-factor model

Being an arbitrage free model, it is designed to exactly match the relevant interest rate term structure, which is one of the main conditions for a market consistent valuation.

This model also has the advantage of presenting analytical formulas to price bonds and interest rate derivatives, making it an attractive option to price swaptions. It is a two-factor model, which allows the modelling of the slope of the interest rate term structure as it can capture more information from the swap volatility. Under a risk neutral probability measure the short rate's dynamics can be represented by

$$r(t) = x(t) + y(t) + \varphi(t), \quad r(0) = r_0. \tag{10}$$

The processes $\{x(t): t \geq 0\}$ and $\{y(t): t \geq 0\}$ satisfy the equations

$$(11) \qquad \begin{aligned} dx(t) &= -ax(t)dt + \sigma dW_{\mathrm{x}}(t), \quad x(0) = 0 \\ dy(t) &= -by(t)dt + \eta dW_{\mathrm{y}}(t), \quad y(0) = 0 \end{aligned}$$

where $(W_{\mathrm{x}}, W_{\mathrm{y}})$ is a bi-dimensional Brownian motion with instant correlation ρ_{G2}, $dW_{\mathrm{x}}(t)dW_{\mathrm{y}}(t) = \rho_{G2}dt$. The parameters r_0, a, b, σ, η are positive constants. The final parameter $\varphi(t)$ is the term which fits the term structure, meaning $\varphi(0) = r_0$.

Provided that the G2++ model does not have an exact solution to price swaptions an alternative approximated formula is used in Frederico (2010), assuming that under the model the swap rate follows a Normal distribution. The formula for a payer swaption is the following,

$$(12) \qquad P_{payer,N}(0) = N_{T_0,T_N}(0)\left[\left(S_{T_0,T_N}(0) - K\right)\Phi\left(\frac{S_{T_0,T_N}(0) - K}{\sigma_{T_0,T_N}}\right) + \sigma_{T_0,T_N}\,\varphi\left(\frac{K - S_{T_0,T_N}(0)}{\sigma_{T_0,T_N}}\right)\right]$$

where the formulas for all the components can be found in Frederico (2010), $S_{[T_0,T_n]}(t)$ is the forward swap rate over time t for a payer interest rate swap with maturity in T_0 and payments in $T_1 < \ldots < T_n (T_0 < T_1)$, K is the strike price, Φ and φ are respectively the cumulative distribution function and the probability distribution function of a standard normal variable and σ_{T_0,T_N} is the square root of the average integrated variance of the swap rates S_{T_0,T_N} in the interval $[0, T_0]$.

In accordance to the procedure for the Heston model the objective is to minimize the differences between the market prices of a derivative and the ones produces by the model. The model was calibrated to swaption prices calculated by the normal model, the pricing formula for a payer swaption with strike K at time 0 was found in a research report from Milliman[1],

$$(13) \qquad P_{payer,N} = \sigma_N\sqrt{T_0}\,\left(\hat{d}_1\Phi\left(\hat{d}_1\right) + \varphi\left(\hat{d}_1\right)\right)\sum_{i=1}^{n} P(0, T_i),$$

$$(14) \qquad \hat{d}_1 = \frac{S_{[T_0,T_n]}(0) - K}{\sigma_N\sqrt{T_0}},$$

[1] The new normal – Using the right volatility quote in times of low interest rates for Solvency II risk factor modelling, September 2015

$$(15) \qquad S_{[T_0,T_n]}(t) = \frac{P(t,T_0) - P(t,T_n)}{\sum_{i=1}^{n} P(t,T_i)}.$$

Where $P(t,T)$ is the value of a zero-coupon bond with nominal 1, maturity at time T and the parameter σ_N is the normal volatility, obtained from the market observed implied volatility matrices.

The previously stated choice is justified by the low interest rates environment that emerged after the 2008 financial crisis extending itself for almost 10 years so far, it is driven by both cuts to the base rates and the use of Quantitative Easing by central banks. Rates have approached, in some cases even crossed, the zero line, this is a major obstruction to the use of the usual Black volatilities. Black's formula, assuming that the rates follow a lognormal distribution, becomes infinitely sensitive to price changes as rates approach zero, calculations are impossible for negative strikes or forward rates.

4.3-Correlation between interest rate and equity

As explained in Barker (2015) a Monte Carlo approach is needed to combine the two previous models. Since this is being done the correlation between interest and equity models $\rho_{r,S}$ has to be considered, it's found from historical estimation. A 2 year rolling window of the 10 years correlation between the returns from Eurostoxx 50 and a proxy for the short term interest rate, EURIBOR 3M was chosen.

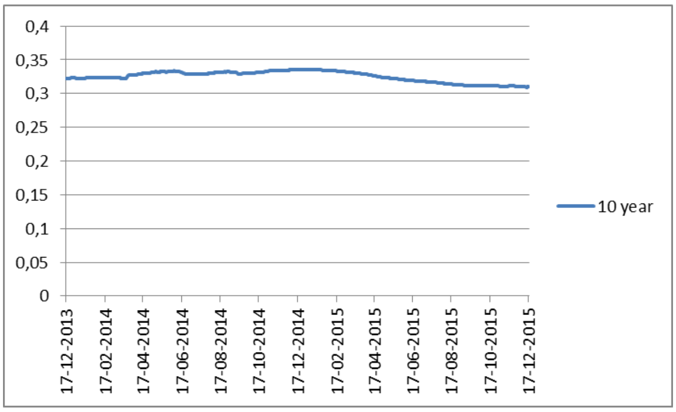

Figure 2: Rolling correlations between the first difference of EURIBOR 3M and log Eurostoxx 50

4.4-Monte Carlo simulation

Considering now the hybrid Heston Gaussian two-factor model, the stock price is a function of both the interest rate and the volatility, from that one needs to find a separate discretization scheme for the interest rate and volatility before considering the asset process. In addition to the stock the behaviour of bonds needs also to be simulated.

First, the interest rate model, from the Cholesky decomposition applied to the correlation matrix in accordance with Barker (2015), a formula to simulate the paths of x and y is obtained,

(16)
$$x(t+dt)=x(t)e^{-adt}+\sqrt{\frac{\sigma^2}{2a}\left(1-e^{-2adt}\right)}Z_{\mathrm{x}}$$
$$y(t+dt)=y(t)e^{-bdt}+\sqrt{\frac{\eta^2}{2b}\left(1-e^{-2bdt}\right)}\left[\rho_{G2}Z_{\mathrm{x}}+\sqrt{1-\rho_{G2}^2}Z_{\mathrm{y}}\right]$$

From here one can reach the simulated prices for zero-coupon bonds, priced under the following formula, in accordance to Barker (2015),

(17)
$$P(t,T)=$$
$$=\frac{P^M(0,T)}{P^M(0,t)}exp\left[\frac{1}{2}\left(V(t,T)-V(0,T)+V(0,t)\right)-\frac{1-e^{-a(T-t)}}{a}x(t)-\frac{1-e^{-b(T-t)}}{b}y(t)\right]$$

(18)
$$V(t,T)=\frac{\sigma^2}{a^2}\int_t^T\left[1-e^{-a(T-u)}\right]^2du+$$
$$+\frac{\eta^2}{b^2}\int_t^T\left[1-e^{-b(T-u)}\right]^2du+$$
$$+2\rho_{G2}\frac{\sigma\eta}{ab}\int_t^T\left[1-e^{-a(T-u)}\right]\left[1-e^{-b(T-u)}\right]du$$

where $P^M(0,t)$ denotes the market zero discount factor maturing at time t.

For the volatility process a moment matching approximation is done in order to approximate $v(t+dt)$ by a Gaussian variable, a QE scheme was followed,

(19)
$$v(t+dt)=a(b+Z_v)^2$$

where $a=\frac{m}{1+b^2}$, $m=\theta+(v(t)-\theta)e^{-\kappa dt}$, $b^2=z-z+\sqrt{z}\sqrt{z-1}$, $z=2\frac{m^2}{s^2}$,

$s^2=\frac{v(t)\xi^2 e^{-\kappa dt}}{\kappa}(1-e^{-\kappa dt})+\frac{\theta\xi^2}{2\kappa}(1-e^{-\kappa dt})^2$.

An alternative function is also proposed for small values of $v(t)$ because the moment matching will not work in this situation,

(20) $$v(t+dt)=\begin{cases}0 & if\, 0\le u\le p\\ \beta^{-1}\ln(\frac{1-p}{1-u}) & ifp<u\le 1\end{cases}$$

where $p=\frac{\frac{s^2}{m^2}-1}{\frac{s^2}{m^2}+1}$, $\beta=\frac{1-p}{m}$ and u is a uniform random number.

Finally, the asset prices are simulated combining both processes, formulas for the simulation can be found in Barker (2015).

(21) $$\begin{aligned}&\ln S(t+dt)=\ln S(t)+\int_t^{t+dt} r(u)du-\delta dt+K_0+K_1v(t)+K_2V(t+dt)\\&+\rho_{xS}\sqrt{K}Z_x+\frac{\rho_{yS}-\rho_{xS}\rho_{xy}}{\sqrt{1-\rho_{xy}^2}}\sqrt{K}Z_y\\&+\sqrt{(1-\rho_{vS}^2-\rho_{xS}^2-\frac{\rho_{yS}-\rho_{xS}\rho_{xy}}{\sqrt{1-\rho_{xy}^2}}K}Z_S.\end{aligned}$$

Where the values above are also defined in Barker (2015), and

(22) $$\begin{aligned}&\int_t^{t+dt} r(u)du\approx\frac{dt}{2}(x(t+dt)+x(t)+y(t+dt)+y(t))+\ln P^M(0,t)\\&-\ln P^M(0,t+dt)+\frac{1}{2}[V(0,t+dt)-V(0,t)].\end{aligned}$$

5-Results

This is a more practical chapter where the application of the theory will be made, considering the reference date the end of 2015. We will first present the model calibration whose results will enable the simulation of the representative investments of the undertakings in what concerns profit-sharing contracts.

5.1-Model Calibration

5.1.1-Heston model

To obtain the model parameters, the model needs to be calibrated to market information. In this case, the implied volatilities associated to options over the Eurostoxx 50 index are used. Given that we are comparing prices, a theoretical one is obtained from the implied volatilities. In conformity to what was done in Frederico (2010) these prices are determined by the Black-Scholes formula.

In contemplation of performing a wide analysis without making the calculation too much time consuming, 7 different strikes, representing between 90% and 110% of the stock price at the valuation date, were chosen. Concerning maturity, given the small liquidity of options with high maturities and given our concern with those longer maturities, implied volatilities were extrapolated for maturities until 9 years by the following formula that can be found in Baldvisnsdóttir & Palmborg (2011),

$$IV_t = \sqrt{IV_\infty^2 + \frac{1}{at}(1-e^{-at})(IV_0^2 - IV_\infty^2)} \tag{23}$$

where all the parameters, IV_0^2, IV_∞^2 and a are obtained using Excel's Solver to minimize the Root Mean Square Errors (RMSE) between the values obtained by the formula and the market observed values.

As stated in the previous chapter the calibration was done by minimizing the MSE. In order to ensure that the optimization procedure would not return a local minimum, one applied a global minimum search procedure, Simulated Annealing, which is implemented in Octave through the function "samin". The results were the following,

	29-12-2015
κ	0,03948
θ	0,45078
v_0	0,04334
ξ	0,52852
ρ_{HE}	-0,25788
MSE	214,41

Table 1: HE model parameters

5.1.2-G2++ model

The calibration of the G2++ model was based on the relevant risk free interest rate term structure and on swaptions implied volatility, recalculating as explained before, a theoretical price based on Normal volatilities, collected from at-the-money swaptions.

Again, the logical goal is to minimize the differences between the model prices and the prices obtained from the implied volatility, the objective function is the following,

$$\text{(24)} \qquad min \sum_{i=1}^{N} \left(\frac{model\ price_i - theoretical\ price_i}{theoretical\ price_i} \right)^2 ,$$

with N as the total number of observations.

A similar procedure to the one used for the Heston model was followed to calibrate this model. The table below expresses the result of the calibration. The swaptions with shorter, until 4 years, expiries and tenors were disregarded as its inclusion affected the calibration for longer expiries and tenors. As this work is focused in Life Insurance, longer maturities emerge as more relevant.

	30-12-2015
a	0,499019
b	0,056903
σ	0,028109
η	0,013509
ρ_{G2}	-0,954405

Table 2: G2++ model parameters

This being done, and to evaluate the quality of the adjustment the relative differences between the two prices were calculated, the results are presented in the figure below.

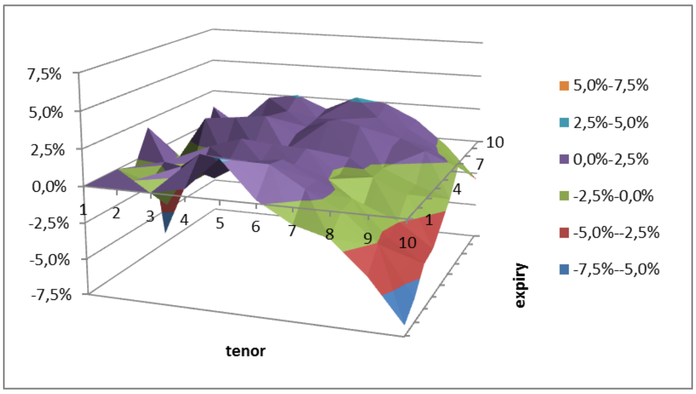

Figure 3: Adjustment errors for the G2++ model

5.2-Portfolio simulation

Regarding the simulation it was decided to consider a portfolio with 75% of the investment in bonds with non-relevant credit risk and 25% in stocks. This conservative approach was defined considering the typical undertaking's investments.

The evolution of the portfolio's value is assumed to have the following behavior, in accordance with Andreatta & Corradin (2003),

$$F(t) = pA(t) + (1-p)G(t)\,,\, 0 \le p \le 1, \tag{25}$$

where p is the proportion of the investment in the stock component, whose value is represented by $A(t)$. $G(t)$ has a similar meaning but for the bond index.

It is considered that the bond index value at any point in time is the accumulated results considering a negotiation strategy of zero coupon bonds with duration D strategy with a fixed transaction horizon δ.

Hereafter the percentiles of the simulated evolution of the portfolio of assets are presented. The parameters obtained for each model as well as $\delta = 0{,}25$ and $D = 5$ were used as inputs.

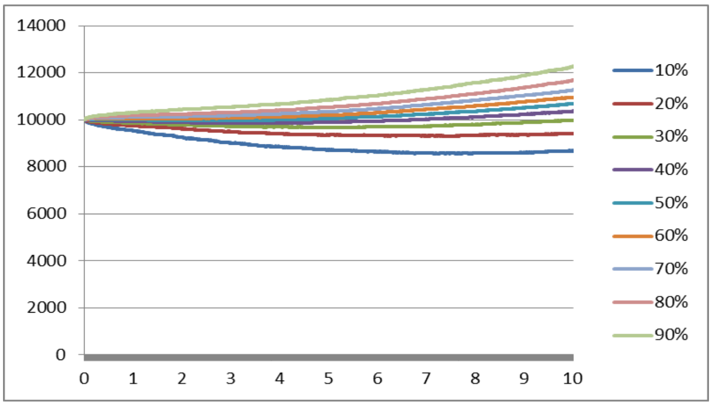

Figure 4: Simulated evolution of a portfolio investing 25% in stocks and 75% in bonds

Percentiles										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
10%	-4,715%	-2,936%	-2,599%	-1,739%	-1,513%	-0,854%	-0,642%	-0,067%	0,254%	0,867%
20%	-2,510%	-1,338%	-1,336%	-0,966%	-0,502%	-0,252%	-0,142%	0,312%	0,240%	0,545%
30%	-1,403%	-0,780%	-0,659%	-0,231%	-0,227%	0,274%	0,242%	0,694%	0,992%	0,795%
40%	-0,641%	-0,499%	-0,320%	0,124%	0,227%	0,504%	0,792%	0,936%	1,197%	1,293%
50%	-0,041%	-0,211%	0,012%	0,172%	0,537%	0,841%	1,096%	1,152%	1,465%	1,613%
60%	0,518%	0,029%	0,234%	0,419%	0,678%	1,048%	1,404%	1,404%	1,695%	1,750%
70%	1,095%	0,301%	0,415%	0,667%	0,943%	1,251%	1,583%	1,710%	1,945%	2,042%
80%	1,824%	0,567%	0,674%	0,940%	1,272%	1,423%	1,868%	2,025%	2,350%	2,707%
90%	3,039%	1,295%	0,978%	1,242%	1,653%	1,684%	2,303%	2,578%	2,677%	3,215%

Table 3: Percentiles for the simulated return rates of the portfolio

5.3-Application of the calculation tool

For each of the types of contract considered, a case study was developed to exemplify the application of the calculation tool. Whenever possible, average market values were taken into account as inputs (e.g. average age of insured persons, average benefit per insured person). For other inputs, additional assumptions were made. For each case, an initial population of 1000 individuals was specified. With regard to the projection of expenses, due to data limitation issues, an amount of 10 euros per insured person was introduced as input. This estimation, based on market average, was obtained by allocating the total administrative expenses per main type of product and then per insured person. It is also worth to remind that the SCR Interest rate indicated in the following tables only reflects the variation of the BE.

5.3.1-Annuities

For this type of contract, two sub-cases of a lifelong immediate annuity were analysed, in order to compare the results between the use of static and dynamic mortality tables. For the static table the choice fell on TV 88/90, as it is quite used for longevity products in the Portuguese market. On the other hand, the male dynamic table 2012 was applied.

Nº of annuitants	1.000,00	
Average annuity	2.971,33	
Average age	74	
Mortality table	**TV 88/90**	**Male Dynamic Table 2012**
Best Estimate	33.546.352,94	35.946.022,72
Risk Margin	2.062.106,26	2.325.701,32
RM/BE	6,15%	6,47%
SCR Life/TP	9,68%	9,73%
SCR Life	3.447.483,41	3.725.136,29
SCR Longevity	3.441.288,97	3.718.165,37
SCR Life-Expense	24.452,53	27.502,94
SCR Interest rate (down)	746.448,11	936.538,10

Table 4: Results for annuity contracts

From the table above one can conclude, for instance, that the use of the dynamic table leads to an increase of 7,15% in the BE and a higher SCR as well as RM.

In what concerns the composition of SCR Life, for this type of contract longevity risk represents almost the total of the SCR Life. The fact that longevity risk is so relevant for these products reinforces the importance of using the most appropriate mortality tables.

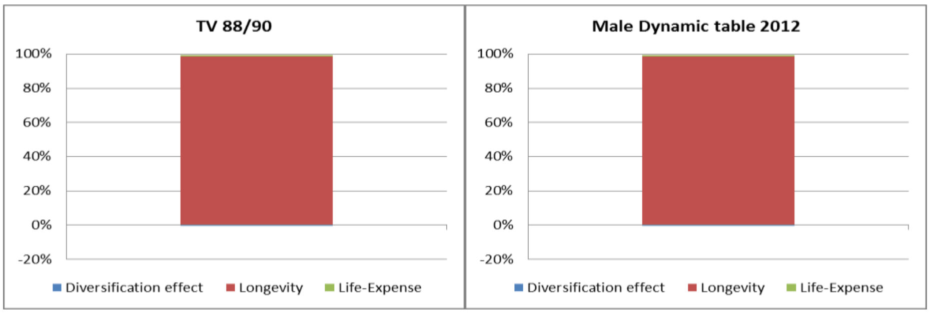

Figure 5: SCR Life composition for annuity contracts

5.3.2-Whole life and term insurance

5.3.2.1-Classical whole life and term insurance

For this case study it was decided to compare the results for a whole life contract and a term contract with a remaining maturity of 11 years, maintaining all the other assumptions equal.

Insured people	1.000,00	
Average death benefit	14.432,69	
Average age	46	
Mortality table	50% GKM 95	
Term	**Whole life**	**11**
Best Estimate	6.210.977,68	487.241,29
Risk Margin	549.197,71	31.508,14
RM/BE	8,84%	6,47%
SCR Life/TP	5,10%	13,71%
SCR Life	344.600,08	71.127,19
SCR Mortality	298.224,94	55.406,42
SCR Life-Expense	106.297,16	17.387,41
SCR Life-Catastrophe	12.399,93	20.998,23
SCR Interest rate (down)	1.422.652,48	6.888,41

Table 5: Results for whole life and term contracts

From this analysis some noteworthy observations emerge related to the length of the contract and the risk profile reflected in the SCR Life. In fact, as shown in the figure below, for shorter term contracts the SCR Life tends to be more diversified and the Life catastrophe risk represents a higher proportion of the total SCR Life. It is also worth to highlight the large dimension of the SCR Interest Rate for the whole life contract when compared to the SCR Life.

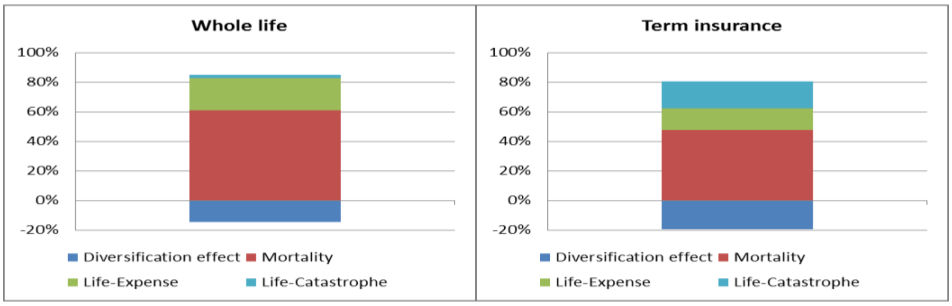

Figure 6: SCR Life composition for whole life and term contracts

5.3.2.2-Annual renewable term insurance

Insured people	1.000,00	**Average age**	46
Average death benefit	31.464,14	**Premia mortality**	70% GKM 95
Mortality table	50% GKM 95	**Pure premium charge**	10%
Term		**14**	**1**
Best Estimate		-299.069,80	-15.138,50
Risk Margin		82.744,78	2.990,52
RM/BE		-27,67%	-19,75%
SCR Life/TP		-87,34%	-409,65%
SCR Life		188.948,55	49.763,68
SCR Mortality		109.852,59	6.982,92
SCR Life-Expense		15.097,49	1.000,00
SCR Lapse (mass)		113.572,52	-
SCR Life-Catastrophe		47.696,95	47.270,42
SCR Interest rate (up)		13.260,96	-

Table 6: Results for annual renewable term contracts

For this type of contracts a relevant analysis is the impact of the application of contract boundaries, by comparing the results between the projection until the next date of renewal (assumed to be one year) and the projection for a longer period of time. As it could be expected, for these contracts a negative BE emerges. This value results from the fact that higher mortality values are being used for premia calculations than for BE calculations. In addition, a charge is applied to the resulting pure premium. Both the previous assumptions are common in the insurance market.

For a 14 year projection, the SCR Life composition is quite diverse. One of the main risk components for these contracts is the Lapse risk, which plays no role in the one year contract. In this latter case, the Life-Catastrophe risk arises as the most important risk due to the short term of the contract.

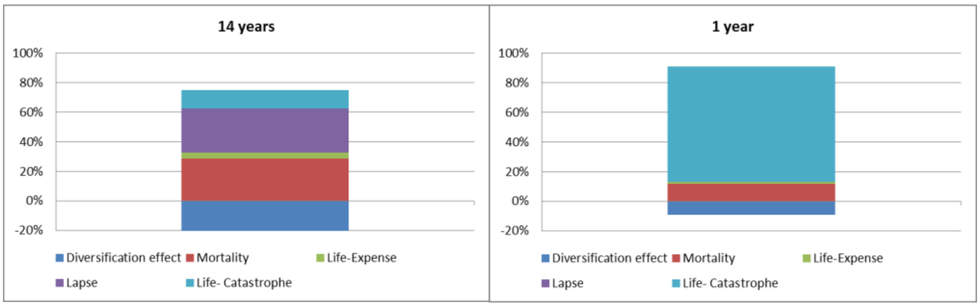

Figure 7: SCR Life composition for annual renewable term contracts

5.3.3-Endowment

Insured people	1.000,00	**Average age**	54
Average initial capital	13.049,03	**Average term**	8
Average premium	662	**Mortality table**	50% GKM 95
Average guaranteed rate	2,74%		
Profit-sharing %		**85%**	**0%**
Best Estimate		15.383.461,97	15.065.199,57
Risk Margin		62.023,97	62.023,97
RM/BE		0,40%	0,41%
SCR Life/TP		2,89%	2,35%
Profit-sharing component		318.262,40	-
Cost of options and guarantees		318.262,40	-
COG/BE		2,07%	-
SCR Life		446.088,76	355.745,25
SCR Longevity		5.261,20	3.914,10
SCR Life-Expense		7.947,79	7.947,79
SCR Lapse (decrease)		439.967,69	350.077,22
SCR Life-Catastrophe		2.899,24	2.420,30
SCR Interest rate (down)		226.364,39	226.364,39

Table 7: Results for endowment contracts

Comparing the results for the two sub-cases (i.e. with and without profit sharing), one can conclude from both the table above and the figure below that, for the assumptions made, no relevant structural differences emerge from the addition of a profit-sharing clause. Indeed, for the entire projection horizon, the guaranteed rate assumption is higher than the annual returns implicit in the relevant risk free interest rate term structure (i.e. in the central scenario no profit-sharing will occur) and also higher than the average simulated return rates for each year. It means that, on average, the portfolio's return would not even be sufficient to cover the guaranteed rates.

In both sub-cases Lapse risk represents the SCR Life almost entirely. The fact that the value for such component results from a decrease in the withdrawal rates may indicate that these are not profitable contracts. In fact, as referred above, the guaranteed rate assumption is on average higher than the return rates, indicating that the previous conclusion is not unreasonable.

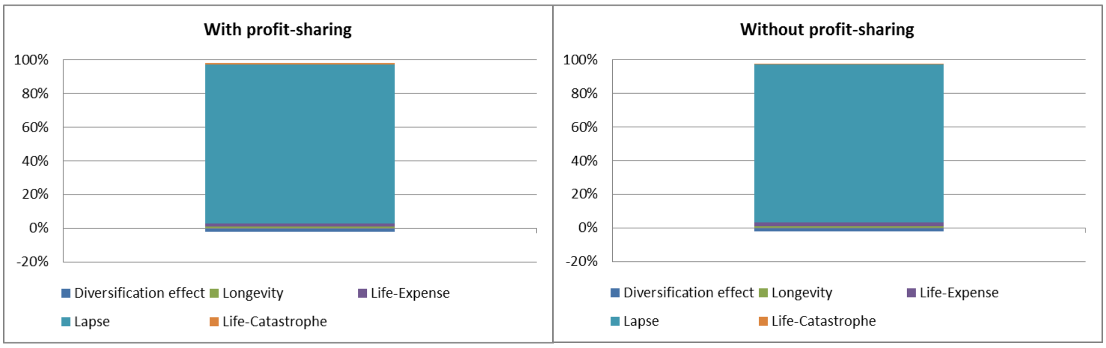

Figure 8: SCR Life composition for endowment contract

6-Conclusion

The internship behind this report has the goal, as stated before, of creating a tool capable of calculating TP and SCR for the main types of Life insurance products. Provided that the referred tool is meant to be used in practice, it is of great importance to follow a flexible approach and allow for the most recent economic and demographic conditions to be considered.

Following the previous paragraph, the chosen approach implied the update of previously developed work accompanied by the incorporation of some features that fitted the current situation better. This thought was behind the choice of the main types of contracts, as explained above, and the decision of leaving open inputs so that the user can adjust the calculations to the context of the time when the calculations are made. That is, for instance, why the dynamic mortality tables used were updated to the most recent data. In the future, this procedure can be repeated to make sure that the tables considered always include the most appropriate mortality expectations.

In what concerns the economic situation, the importance of simulating the future economic scenarios was already made clear. For this procedure, starting from the work that had been previously developed in Frederico (2010) an improvement has been introduced by incorporating the correlation between interest rate and stock behaviour. In the previous work, model limitations justify the fact that correlation between the short term interest rate and the net return of the stock index had not been considered before, thus this became a critical feature made possible by the incorporation of the approach followed in Barker (2015) in this work.

Another difference is the use of Normal implied volatilities instead of the most common Black volatilities. This is a temporary issue, justified by the low interest rate environment, thus becoming critical for the current market, where negative interest rates have a relevant probability.

Indeed, the market conditions have a significant impact on the calibration of an economic scenario generator. Taking into consideration that for some market conditions, when $\rho_{G2} = -1$, the use of the two factor model for the interest rate, as it is the case of the G2++, will be a prohibiting condition for the use of the chosen simulation procedure, a possible complement to this work would be creating the option of using a one factor interest rate model, a possible option would be the Hull-White model. This change would not be unrealistic given the fact that the referred value of ρ_{G2} is an indicator that the interest rate process could be defined by a one factor model.

In order to show the applicability of the created tool several examples were created. Bearing in mind the concern of making these examples relevant for analysis in the current market, average hypothesis about the specifications of the main types of products and its beneficiaries were assumed.

Finally, as a conclusion, it is my personal opinion that this internship can be seen as a success, not only for reaching the set objectives but also for the opportunity it gave me to apply my academic skills and develop them in the professional world.

A-Appendix

A.1- Dynamic mortality tables estimated parameters

A.1.1- Estimated parameters of the Poisson-Lee-Carter model

Male								Female							
x	α_x	β_x	x	α_x	β_x	t	k_t	x	α_x	β_x	x	α_x	β_x	t	k_t
0	-4,465693896	0,040617346	**46**	-5,344634711	0,005777231	**1970**	41,57716	**0**	-4,651463475	0,033136232	**46**	-6,114214391	0,008052852	**1970**	50,52255
1	-6,787376899	0,040786721	**47**	-5,257565685	0,006254884	**1971**	41,94014	**1**	-6,948687626	0,036090477	**47**	-6,040117779	0,006949227	**1971**	50,34294
2	-7,161957484	0,031292082	**48**	-5,157671292	0,00616208	**1972**	33,08041	**2**	-7,441452734	0,029399207	**48**	-5,947210643	0,007716377	**1972**	40,59595
3	-7,458314016	0,02993835	**49**	-5,093208712	0,006243808	**1973**	35,31969	**3**	-7,712674524	0,026625284	**49**	-5,909919573	0,00768473	**1973**	43,96526
4	-7,593324873	0,026932403	**50**	-5,011073762	0,006685453	**1974**	33,10525	**4**	-7,92636046	0,024167089	**50**	-5,809463662	0,008440875	**1974**	40,56473
5	-7,728506394	0,026196496	**51**	-4,949106739	0,006253296	**1975**	33,96641	**5**	-8,041579215	0,02133466	**51**	-5,737199305	0,007223791	**1975**	38,91795
6	-7,883170743	0,028104798	**52**	-4,873789205	0,00700293	**1976**	32,73712	**6**	-8,102094135	0,02028953	**52**	-5,657662801	0,00814518	**1976**	38,18311
7	-7,799676381	0,022965176	**53**	-4,771072936	0,006581352	**1977**	28,22098	**7**	-8,218699675	0,017909022	**53**	-5,591934964	0,008518655	**1977**	31,35393
8	-7,940531722	0,022274291	**54**	-4,713314573	0,007065458	**1978**	25,40292	**8**	-8,279250962	0,015976004	**54**	-5,538087951	0,008238399	**1978**	30,49522
9	-7,933891657	0,022690457	**55**	-4,633505464	0,007104311	**1979**	21,43656	**9**	-8,358085303	0,017864579	**55**	-5,456034367	0,008450981	**1979**	25,61436
10	-7,917723261	0,021034095	**56**	-4,567381563	0,007880694	**1980**	21,28489	**10**	-8,351853905	0,013376449	**56**	-5,368305186	0,008847246	**1980**	23,90436
11	-7,897300849	0,01873777	**57**	-4,472398483	0,007799669	**1981**	19,05345	**11**	-8,303077135	0,015106166	**57**	-5,305753759	0,008365548	**1981**	22,07418
12	-7,890882842	0,017773601	**58**	-4,40105111	0,008091019	**1982**	15,22381	**12**	-8,364174623	0,013826606	**58**	-5,192663357	0,009313501	**1982**	16,75448
13	-7,768502633	0,018717881	**59**	-4,318563679	0,008597389	**1983**	15,07086	**13**	-8,323397725	0,014917002	**59**	-5,122070015	0,009045371	**1983**	17,72186
14	-7,594888629	0,017955114	**60**	-4,223981549	0,008886133	**1984**	14,14198	**14**	-8,086610246	0,012569347	**60**	-5,017163375	0,009598391	**1984**	14,96016
15	-7,306141259	0,015010848	**61**	-4,147176187	0,008619951	**1985**	12,99295	**15**	-7,983747574	0,010360383	**61**	-4,941741688	0,009438626	**1985**	13,00356
16	-6,994131274	0,015004103	**62**	-4,05787496	0,009271925	**1986**	9,233307	**16**	-7,888076382	0,010538162	**62**	-4,848774402	0,010072626	**1986**	10,34815
17	-6,770974355	0,01374758	**63**	-3,964134219	0,009612783	**1987**	6,369743	**17**	-7,886759595	0,011815787	**63**	-4,747404358	0,010568161	**1987**	6,661865
18	-6,576390881	0,012304231	**64**	-3,892403138	0,009628237	**1988**	6,484576	**18**	-7,804730451	0,010481083	**64**	-4,64027069	0,010447367	**1988**	6,021713
19	-6,492357986	0,013652199	**65**	-3,78362347	0,009136956	**1989**	2,425178	**19**	-7,725669165	0,009667283	**65**	-4,540775875	0,010212225	**1989**	1,238734
20	-6,460463914	0,011999527	**66**	-3,705564963	0,009259239	**1990**	4,855987	**20**	-7,690185271	0,007483047	**66**	-4,439188869	0,010706592	**1990**	5,52395
21	-6,437738678	0,011185642	**67**	-3,604331695	0,009509084	**1991**	5,170994	**21**	-7,655792409	0,010324006	**67**	-4,326758647	0,010819231	**1991**	4,310826
22	-6,453216104	0,008651137	**68**	-3,508184056	0,010141386	**1992**	0,440481	**22**	-7,691489234	0,009763686	**68**	-4,203132049	0,011070093	**1992**	-2,13649
23	-6,452506705	0,009948945	**69**	-3,414666538	0,009962101	**1993**	2,176949	**23**	-7,643914818	0,009502736	**69**	-4,089969155	0,011359696	**1993**	-0,03784
24	-6,439549882	0,009679136	**70**	-3,306018173	0,010207831	**1994**	-5,85023	**24**	-7,561674006	0,009068446	**70**	-3,947163189	0,011185937	**1994**	-8,48836
25	-6,419308226	0,009325385	**71**	-3,216518745	0,009721254	**1995**	-4,87214	**25**	-7,598246112	0,009402562	**71**	-3,837888139	0,011115798	**1995**	-7,19334
26	-6,405604586	0,008073277	**72**	-3,109490572	0,010388571	**1996**	-3,68492	**26**	-7,525893783	0,010660358	**72**	-3,707298072	0,011357321	**1996**	-7,75259
27	-6,387294409	0,00697731	**73**	-3,00361027	0,010056832	**1997**	-8,11312	**27**	-7,472398662	0,010192209	**73**	-3,580686553	0,011371133	**1997**	-11,0067
28	-6,360852587	0,007170943	**74**	-2,899936309	0,010046869	**1998**	-9,03094	**28**	-7,479577847	0,010385199	**74**	-3,430161298	0,011556968	**1998**	-12,8113
29	-6,352119938	0,006755409	**75**	-2,794175748	0,009937124	**1999**	-10,4853	**29**	-7,409273947	0,009715872	**75**	-3,305874184	0,011781255	**1999**	-13,4557
30	-6,290117504	0,006695495	**76**	-2,684207284	0,009956992	**2000**	-14,6925	**30**	-7,337210104	0,008851745	**76**	-3,174258817	0,011292506	**2000**	-17,9576
31	-6,271437294	0,006234042	**77**	-2,579778905	0,00991769	**2001**	-17,8214	**31**	-7,250458455	0,009197451	**77**	-3,047409155	0,010835577	**2001**	-21,2984
32	-6,233931848	0,006844731	**78**	-2,46904674	0,009662187	**2002**	-19,2842	**32**	-7,222705642	0,009311172	**78**	-2,908193678	0,011188453	**2002**	-23,0519
33	-6,18701907	0,005898781	**79**	-2,363721488	0,009367962	**2003**	-21,2989	**33**	-7,104933849	0,008316259	**79**	-2,775917399	0,010581409	**2003**	-22,4947
34	-6,131977903	0,005572156	**80**	-2,312974485	0,007969583	**2004**	-28,2226	**34**	-7,067868753	0,008664614	**80**	-2,699480764	0,00885952	**2004**	-32,8106
35	-6,088600762	0,005713248	**81**	-2,21553259	0,007164344	**2005**	-27,1972	**35**	-6,964378758	0,007742308	**81**	-2,587633347	0,008040603	**2005**	-30,0296
36	-6,037373813	0,005289334	**82**	-2,114544029	0,007093316	**2006**	-33,0861	**36**	-6,927058415	0,008546452	**82**	-2,455481271	0,00792159	**2006**	-40,5111
37	-5,979728133	0,005694617	**83**	-2,010194144	0,006807176	**2007**	-36,33	**37**	-6,861437721	0,008221562	**83**	-2,336923802	0,007412302	**2007**	-40,327
38	-5,91954648	0,006165663	**84**	-1,911928138	0,006563217	**2008**	-38,7521	**38**	-6,757015572	0,009175068	**84**	-2,218602585	0,007175184	**2008**	-42,92
39	-5,854267323	0,005756036	**85**	-1,824125103	0,006225247	**2009**	-41,7344	**39**	-6,660859655	0,008075518	**85**	-2,104882045	0,00660786	**2009**	-45,1135
40	-5,773400052	0,005696781	**86**	-1,72243934	0,005509463	**2010**	-43,7175	**40**	-6,571913563	0,007597423	**86**	-1,985434917	0,006674895	**2010**	-48,0502
41	-5,715291918	0,005725634	**87**	-1,627806198	0,005754573	**2011**	-49,0837	**41**	-6,530833303	0,007691431	**87**	-1,882951393	0,006155026	**2011**	-53,5999
42	-5,62666058	0,0058547	**88**	-1,539809747	0,005273349	**2012**	-48,4547	**42**	-6,432146752	0,007042	**88**	-1,778658173	0,006312747	**2012**	-52,0331
43	-5,553561389	0,006076122	**89**	-1,448306718	0,005005953			**43**	-6,376502524	0,008196352	**89**	-1,671456424	0,005548594		
44	-5,482689227	0,005922588	**90**	-1,372256949	0,004584887			**44**	-6,299698697	0,00699033	**90**	-1,575649224	0,005350443		
45	-5,407979163	0,006616033						**45**	-6,205855578	0,006820976					

Table 8: Estimated Poison-Lee-Carter parameters for male and female populations based on 1970-2012 data

A.1.2- Forecasted k_t values of the Poisson-Lee-Carter model

Male						Female					
t*	k_t^*	t*	*	t*	k_t^*	t*	k_t^*	t*	*	t*	k_t^*
2013	-50,5983	2055	-140,63	2097	-230,662	2013	-54,47491202	2055	-157,0305724	2097	-259,5862328
2014	-52,7419	2056	-142,774	2098	-232,806	2014	-56,91671346	2056	-159,4723739	2098	-262,0280343
2015	-54,8855	2057	-144,917	2099	-234,949	2015	-59,3585149	2057	-161,9141753	2099	-264,4698357
2016	-57,0291	2058	-147,061	2100	-237,093	2016	-61,80031634	2058	-164,3559767	2100	-266,9116372
2017	-59,1727	2059	-149,205	2101	-239,236	2017	-64,24211778	2059	-166,7977782	2101	-269,3534386
2018	-61,3163	2060	-151,348	2102	-241,38	2018	-66,68391922	2060	-169,2395796	2102	-271,79524
2019	-63,46	2061	-153,492	2103	-243,524	2019	-69,12572065	2061	-171,6813811	2103	-274,2370415
2020	-65,6036	2062	-155,635	2104	-245,667	2020	-71,56752209	2062	-174,1231825	2104	-276,6788429
2021	-67,7472	2063	-157,779	2105	-247,811	2021	-74,00932353	2063	-176,5649839	2105	-279,1206443
2022	-69,8908	2064	-159,923	2106	-249,954	2022	-76,45112497	2064	-179,0067854	2106	-281,5624458
2023	-72,0344	2065	-162,066	2107	-252,098	2023	-78,89292641	2065	-181,4485868	2107	-284,0042472
2024	-74,178	2066	-164,21	2108	-254,242	2024	-81,33472785	2066	-183,8903883	2108	-286,4460487
2025	-76,3216	2067	-166,353	2109	-256,385	2025	-83,77652928	2067	-186,3321897	2109	-288,8878501
2026	-78,4653	2068	-168,497	2110	-258,529	2026	-86,21833072	2068	-188,7739911	2110	-291,3296515
2027	-80,6089	2069	-170,641	2111	-260,672	2027	-88,66013216	2069	-191,2157926	2111	-293,771453
2028	-82,7525	2070	-172,784	2112	-262,816	2028	-91,1019336	2070	-193,657594	2112	-296,2132544
2029	-84,8961	2071	-174,928	2113	-264,96	2029	-93,54373504	2071	-196,0993954	2113	-298,6550559
2030	-87,0397	2072	-177,072	2114	-267,103	2030	-95,98553648	2072	-198,5411969	2114	-301,0968573
2031	-89,1833	2073	-179,215	2115	-269,247	2031	-98,42733791	2073	-200,9829983	2115	-303,5386587
2032	-91,3269	2074	-181,359	2116	-271,391	2032	-100,8691394	2074	-203,4247998	2116	-305,9804602
2033	-93,4706	2075	-183,502	2117	-273,534	2033	-103,3109408	2075	-205,8666012	2117	-308,4222616
2034	-95,6142	2076	-185,646	2118	-275,678	2034	-105,7527422	2076	-208,3084026	2118	-310,864063
2035	-97,7578	2077	-187,79	2119	-277,821	2035	-108,1945437	2077	-210,7502041	2119	-313,3058645
2036	-99,9014	2078	-189,933	2120	-279,965	2036	-110,6363451	2078	-213,1920055	2120	-315,7476659
2037	-102,045	2079	-192,077	2121	-282,109	2037	-113,0781465	2079	-215,633807	2121	-318,1894674
2038	-104,189	2080	-194,22	2122	-284,252	2038	-115,519948	2080	-218,0756084	2122	-320,6312688
2039	-106,332	2081	-196,364	2123	-286,396	2039	-117,9617494	2081	-220,5174098	2123	-323,0730702
2040	-108,476	2082	-198,508	2124	-288,539	2040	-120,4035509	2082	-222,9592113	2124	-325,5148717
2041	-110,619	2083	-200,651	2125	-290,683	2041	-122,8453523	2083	-225,4010127	2125	-327,9566731
2042	-112,763	2084	-202,795	2126	-292,827	2042	-125,2871537	2084	-227,8428141	2126	-330,3984746
2043	-114,907	2085	-204,939	2127	-294,97	2043	-127,7289552	2085	-230,2846156	2127	-332,840276
2044	-117,05	2086	-207,082	2128	-297,114	2044	-130,1707566	2086	-232,726417	2128	-335,2820774
2045	-119,194	2087	-209,226	2129	-299,258	2045	-132,612558	2087	-235,1682185	2129	-337,7238789
2046	-121,338	2088	-211,369	2130	-301,401	2046	-135,0543595	2088	-237,6100199	2130	-340,1656803
2047	-123,481	2089	-213,513	2131	-303,545	2047	-137,4961609	2089	-240,0518213	2131	-342,6074817
2048	-125,625	2090	-215,657	2132	-305,688	2048	-139,9379624	2090	-242,4936228	2132	-345,0492832
2049	-127,768	2091	-217,8	2133	-307,832	2049	-142,3797638	2091	-244,9354242	2133	-347,4910846
2050	-129,912	2092	-219,944	2134	-309,976	2050	-144,8215652	2092	-247,3772256	2134	-349,9328861
2051	-132,056	2093	-222,087	2135	-312,119	2051	-147,2633667	2093	-249,8190271	2135	-352,3746875
2052	-134,199	2094	-224,231	2136	-314,263	2052	-149,7051681	2094	-252,2608285	2136	-354,8164889
2053	-136,343	2095	-226,375	2137	-316,406	2053	-152,1469696	2095	-254,70263	2137	-357,2582904
2054	-138,486	2096	-228,518			2054	-154,588771	2096	-257,1444314		

Table 9: k_t forecasts of the Poisson-Lee-Carter model for male and female populations

A.1.3- Cox proportional hazards

	Male	Female
α	0,735572353	0,78056607
Standard Error	0,012817216	0,017276901

Table 10: Estimated parameters of the relational model based on Cox proportional hazards based on 1970-2012 data

A.2- Data used for financial models calibration

Maturity	16-12-2016	15-12-2017	21-12-2018	20-12-2019	18-12-2020	17-12-2021	27-12-2022	15-12-2023	20-12-2024
T	0,963888889	1,961111111	2,977777778	3,975	4,969444444	5,966666667	7	7,961111111	8,975
r	-0,157%	-0,131%	-0,040%	0,093%	0,228%	0,376%	0,526%	0,662%	0,798%
3000	23,03%	22,09%	21,69%	21,37%	21,19%	21,07%	20,98%	20,91%	20,86%
3100	22,28%	21,70%	21,33%	20,74%	20,27%	19,82%	19,37%	18,97%	18,56%
3200	21,49%	21,24%	20,99%	20,77%	20,59%	20,44%	20,30%	20,19%	20,08%
3300	20,85%	20,79%	20,68%	20,67%	20,63%	20,59%	20,55%	20,51%	20,47%
3400	20,23%	20,38%	20,39%	20,41%	20,42%	20,43%	20,43%	20,44%	20,44%
3500	19,65%	20,00%	20,11%	20,16%	20,19%	20,21%	20,23%	20,24%	20,25%
3600	19,12%	19,65%	19,86%	19,94%	19,99%	20,03%	20,06%	20,08%	20,09%

Source: Bloomberg

Table 11: Implied volatilities of options over Eurostoxx 50 index (29-12-2015)

Maturity	16-12-2016	15-12-2017	21-12-2018	20-12-2019	18-12-2020	17-12-2021	27-12-2022	15-12-2023	20-12-2024
T	0,963888889	1,961111111	2,977777778	3,975	4,969444444	5,966666667	7	7,961111111	8,975
r	-0,157%	-0,131%	-0,040%	0,093%	0,228%	0,376%	0,526%	0,662%	0,798%
3000	465,51	560,68	640,22	709,69	776,19	842,11	909,46	971,66	1036,69
3100	396,66	499,55	581,25	644,61	703,04	758,43	813,15	862,62	913,67
3200	332,04	440,82	525,70	598,66	666,19	731,89	798,31	859,34	923,07
3300	274,50	386,21	473,79	552,15	624,05	693,97	764,50	829,05	896,16
3400	222,75	336,02	425,49	504,42	577,58	648,88	720,98	787,11	855,98
3500	177,39	290,38	380,73	459,46	532,72	604,08	676,28	742,52	811,58
3600	138,63	249,21	339,42	417,95	491,12	562,42	634,61	700,89	770,05

Table 12: Prices of options over Eurostoxx 50 index calculated by the Black-Scholes formula

		Tenor									
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Expiry	**1**	0,10%	12,84%	33,35%	40,20%	43,67%	49,13%	53,38%	56,38%	59,52%	62,62%
	2	29,83%	40,63%	47,01%	49,19%	53,71%	56,87%	60,18%	62,59%	64,95%	67,83%
	3	56,36%	53,22%	54,68%	58,61%	60,99%	63,19%	65,59%	67,27%	69,44%	70,57%
	4	61,25%	59,49%	62,34%	64,19%	66,35%	67,96%	69,51%	71,22%	71,78%	72,77%
	5	64,13%	64,85%	65,94%	67,43%	69,16%	70,26%	71,96%	72,00%	72,49%	73,38%
	6	73,92%	69,44%	69,96%	70,72%	71,89%	72,45%	72,70%	72,93%	73,23%	73,21%
	7	72,54%	69,56%	70,23%	70,99%	72,48%	71,93%	71,97%	72,59%	72,10%	72,09%
	8	74,99%	72,16%	71,88%	72,46%	72,87%	72,37%	72,49%	72,39%	72,20%	72,42%
	9	75,20%	73,16%	72,86%	72,29%	72,72%	72,46%	71,91%	71,85%	71,97%	72,46%
	10	73,21%	73,40%	71,64%	71,52%	72,16%	71,15%	70,91%	70,99%	71,44%	72,21%

Source: Bloomberg

Table 13: Swaptions Normal implied volatilities (30-12-2015)

		Tenor									
		1	2	3	4	5	6	7	8	9	10
Expiry	1	0,04	10,26	39,91	63,96	86,51	116,22	146,46	175,62	207,06	240,16
	2	16,85	45,78	79,19	109,99	149,30	188,50	231,07	272,53	315,54	362,98
	3	38,79	72,98	111,92	158,99	205,40	253,44	304,40	353,72	407,08	455,38
	4	48,31	93,31	145,72	198,60	254,54	310,17	366,79	425,49	477,79	532,89
	5	55,92	112,30	169,96	229,76	291,93	352,56	417,21	472,33	529,57	589,58
	6	69,62	129,74	194,32	259,46	326,50	390,94	453,01	513,98	574,59	631,73
	7	72,59	137,92	206,85	276,03	348,70	410,94	474,64	541,36	598,65	658,32
	8	78,73	150,02	221,89	295,16	367,11	432,81	500,39	565,12	627,61	692,42
	9	82,08	158,07	233,66	305,77	380,31	449,84	515,35	582,44	649,72	719,50
	10	82,50	163,67	236,99	311,97	389,18	455,62	524,33	593,86	665,55	739,85

Table 14: Swaptions prices calculated by the Normal model

A.3- Financial models calibration results

Maturity	16-12-2016	15-12-2017	21-12-2018	20-12-2019	18-12-2020	17-12-2021	27-12-2022	15-12-2023	20-12-2024
T	0,963888889	1,961111111	2,977777778	3,975	4,969444444	5,966666667	7	7,961111111	8,975
3000	-2,20%	-5,23%	-9,13%	-7,21%	-11,77%	-7,96%	-2,08%	-2,94%	-5,80%
3100	-9,00%	-7,96%	-11,31%	-9,57%	-7,04%	-3,13%	-5,20%	-7,64%	-6,90%
3200	-9,52%	-8,40%	-6,86%	-3,33%	-3,70%	-6,68%	-6,64%	-8,60%	-7,59%
3300	-6,40%	-4,02%	-2,81%	-6,30%	-6,70%	-8,29%	-7,38%	-6,38%	-5,36%
3400	-2,63%	-6,76%	-7,54%	-8,90%	-8,06%	-7,20%	-7,58%	-3,41%	-8,26%
3500	-9,35%	-10,58%	-9,84%	-9,12%	-23,76%	-19,83%	-25,05%	-26,57%	-28,20%
3600	-28,16%	-28,16%	-27,68%	-23,00%	-28,63%	-30,24%	-31,80%	-31,82%	-31,91%

Table 15: Relative differences between model and theoretical prices of options over Eurostoxx 50 index

		Tenor									
		1	2	3	4	5	6	7	8	9	10
Expiry	1	-	-	-	0,49%	2,67%	0,23%	-1,12%	-1,76%	-3,90%	-6,76%
	2	-	-	-1,46%	2,22%	1,51%	1,96%	0,54%	-0,59%	-2,56%	-5,88%
	3	-	-0,73%	0,62%	-0,01%	1,39%	1,71%	0,52%	-0,50%	-2,91%	-4,35%
	4	-0,63%	0,16%	-0,90%	0,56%	0,77%	0,79%	-0,01%	-1,74%	-2,43%	-4,10%
	5	1,95%	-0,07%	0,92%	1,70%	1,47%	1,37%	-0,30%	-0,24%	-1,25%	-3,06%
	6	-6,80%	-1,44%	-0,36%	0,60%	0,45%	0,50%	0,37%	-0,20%	-1,19%	-1,97%
	7	-1,62%	1,84%	2,08%	2,37%	1,18%	2,31%	2,13%	0,76%	0,68%	-0,29%
	8	-2,73%	0,17%	1,28%	1,33%	1,24%	1,97%	1,44%	0,92%	0,27%	-1,16%
	9	-1,89%	-0,22%	0,53%	1,80%	1,38%	1,56%	1,81%	1,08%	-0,16%	-2,10%
	10	1,18%	-0,28%	2,23%	2,61%	1,67%	2,75%	2,42%	1,33%	-0,52%	-2,97%

Table 16: Relative differences between model and theoretical swaptions prices

A.4- Octave code

A.4.1- Price determining algorithm for an European call option under the Heston model

```
function value=HE_Call_Price(kappa,theta,v0,psi,rho_HE,s0,strike,T)
global Dados_FD;
%Dados_FD=xlsread('Dados_FD.xls');
r=0
x0 = log(s0);
alpha = 0.75;
N = 4096;
c = 512;
eta_Fft = c/N;
b_Fft = pi/eta_Fft;
u = [0:N-1]*eta_Fft;
lamda = 2*b_Fft/N;
position = (log(strike) + b_Fft)/lamda + 1;
% For in-the-money and at-the-money options
if strike<=s0
v = u - (alpha+1)*1i;
zeta = -.5*(v.^2 +1i*v);
gamma = kappa - rho_HE*psi*v*1i;
PHI = sqrt(gamma.^2 - 2*psi^2*zeta);
%r=0;
A = 1i*v*(x0 + r*T);
B = v0*((2*zeta.*(1-exp(-PHI.*T)))./(2*PHI - (PHI-gamma).*(1-exp(-PHI*T))));
C = -(kappa*theta)/(psi^2)*(2*log((2*PHI - (PHI-gamma).*(1-exp(-PHI*T)))./ ...
(2*PHI)) + (PHI-gamma)*T);

charFunc = exp(A + B + C);
% Substitution of exp(-r*T) for P_0T
```

```
ModifiedCharFunc = charFunc* Dados_FD(round(T*360),1)./(alpha^2 + alpha -
u.^2 + ...
1i*(2*alpha +1)*u);
SimpsonW = 1/3*(3 + (-1).^[1:N] - [1, zeros(1,N-1)]);
FftFunc = exp(1i*b_Fft*u).*ModifiedCharFunc*eta_Fft.*SimpsonW;
payoff = real(fft(FftFunc));
CallValueM = exp(-log(strike)*alpha)*payoff/pi;
value = CallValueM(round(position));
% For out-of-the-money options
else
w1 = u-1i*alpha;
w2 = u+1i*alpha;
v1 = u-1i*alpha -1i;
v2 = u+1i*alpha -1i;
zeta1 = -.5*(v1.^2 +1i*v1);
gamma1 = kappa - rho_HE*psi*v1*1i;
PHI1 = sqrt(gamma1.^2 - 2*psi^2*zeta1);
% r=0;
A1 = 1i*v1*(x0 + r*T);
B1 = v0*((2*zeta1.*(1-exp(-PHI1.*T)))./(2*PHI1 - (PHI1-gamma1).*(1-exp(-
PHI1*T))));
C1 = -(kappa*theta)/(psi^2)*(2*log((2*PHI1 - (PHI1-gamma1).*(1-exp(-
PHI1*T)))./(2*PHI1)) ...
+ (PHI1-gamma1)*T);
charFunc1 = exp(A1 + B1 + C1);
% Substitution of  exp(-r*T) for P_0T
ModifiedCharFunc1 = Dados_FD(round(T*360),1)*(1./(1+1i*w1) - ...
Dados_FD(round(T*360),1)./(1i*w1) - charFunc1./(w1.^2 - 1i*w1));
zeta2 = -.5*(v2.^2 +1i*v2);
gamma2 = kappa - rho_HE*psi*v2*1i;
PHI2 = sqrt(gamma2.^2 - 2*psi^2*zeta2);

% r=0;
```

```
A2 = 1i*v2*(x0 + r*T);
B2 = v0*((2*zeta2.*(1-exp(-PHI2.*T)))./(2*PHI2 - (PHI2-gamma2).*(1-exp(-PHI2*T))));
C2 = -(kappa*theta)/(psi^2)*(2*log((2*PHI2 - (PHI2-gamma2).*(1-exp(-PHI2*T)))./(2*PHI2)) ...
+ (PHI2-gamma2)*T);
charFunc2 = exp(A2 + B2 + C2);
% Substitution of  exp(-r*T) for P_0T
ModifiedCharFunc2 = Dados_FD(round(T*360),1)*(1./(1+1i*w2) - ...
Dados_FD(round(T*360),1)./(1i*w2) - charFunc2./(w2.^2 - 1i*w2));
ModifiedCharFuncCombo = (ModifiedCharFunc1 - ModifiedCharFunc2)/2 ;
SimpsonW = 1/3*(3 + (-1).^[1:N] - [1, zeros(1,N-1)]);
FftFunc = exp(1i*b_Fft*u).*ModifiedCharFuncCombo*eta_Fft.*SimpsonW;
payoff = real(fft(FftFunc));
CallValueM = payoff/pi/sinh(alpha*log(strike));
value = CallValueM(round(position));
endif
endfunction
```

A.4.2- Optimization algorithm for the calibration of the Heston model

```
global Dados;
global Dados_FD;
global N_Obs;
global P_Call;
global Error;
% Dados [T s0 strike price]
% Dados_FD <- P(0,T)
Dados=xlsread('Dados_HEG2_2015.xlsx');
Dados_FD=xlsread('Dados_FD_2015.xlsx');
% Number of observations
```

```
Dim=size(Dados);
N_Obs=Dim(1);
input=[0.04;0.04;0.04;0.04;0.04];
lb=[0;0;0;0;-1];
ub=[5;5;1;1;1];
nt=10;
ns=5;
rt=0.75;
maxevals=1e10;
neps=5;
functol=1e-10;
paramtol=1e-3;
verbosity=2;
minarg=1;
control={lb,ub,nt,ns,rt,maxevals,neps,functol,paramtol,verbosity,minarg};
[x,obj,convergence,details]=samin("HE_Cost_Global",{input},control);
% Error calculation
x=[0.03948;0.45078;0.52852;-0.25788];
for i=1:N_Obs
P_Call(i)=HE_Call_Price(x(1),x(2),x(3),x(4),x(5),...
Dados(i,2),Dados(i,3),Dados(i,1));
Error(i)=(P_Call(i)-Dados(i,4))/Dados(i,4);
endfor
```

A.4.3- Price determining algorithm for a payer Swaption under the G2++ model

```
function value=Swaptions_Price(a,b,s,eta,rho,expiry,tenor,strike)
Vol=(s^2*(Func_C(a,expiry,tenor,strike))^2*((exp(2*a*expiry)-1)/(2*a))+...
eta^2*(Func_C(b,expiry,tenor,strike))^2*((exp(2*b*expiry)-1)/(2*b))+...
2*rho*s*eta*Func_C(a,expiry,tenor,strike)*...
Func_C(b,expiry,tenor,strike)*((exp((a+b)*expiry)-1)/(a+b)))^0.5;
```

```
Price=(Vol*Func_Sum_P0T(expiry,tenor))/(2*pi)^0.5;
value=Price;
endfunction
function value=Func_C(p,expiry,tenor,strike)
% Dados_P0T <- P(0,T)
global Dados_P0T;
% Dados_P0T=xlsread('Dados_Swap_P0T.xls');
value=(exp(-p*expiry)*(Dados_P0T(expiry,1)/Func_Sum_P0T(expiry,tenor)) ...
-exp(-p*tenor)*...
(Dados_P0T(tenor,1)/Func_Sum_P0T(expiry,tenor)) ...
-strike*Func_Sum(p,expiry,tenor))/p;
endfunction
function value=Func_Sum(x,expiry,tenor)
% Dados_P0T <- P(0,T)
global Dados_P0T;
% Dados_P0T=xlsread('Dados_Swap_P0T.xls');
sum=0;
for i=expiry+1:tenor
sum=sum+exp(-x*i)*(Dados_P0T(i,1)/Func_Sum_P0T(expiry,tenor));
endfor
value=sum;
endfunction
function value=Func_Sum_P0T(expiry,tenor)
% Dados_P0T <- P(0,T)
global Dados_P0T;
% Dados_P0T=xlsread('Dados_Swap_P0T.xls');
sum=0;
for i=expiry+1:tenor
sum=sum+Dados_P0T(i,1);
endfor
value=sum;
endfunction
```

A.4.4- Optimization algorithm for the calibration of the G2++ model

```
global Dados;
global Dados_P0T;
global N_Obs;
global P_Swaptions;
global Error;
% Data [expiry tenor price strike
Dados=xlsread('Dados_Swaptions_2015.xlsx');
Dados_P0T=xlsread('Dados_P(0,T)_2015.xlsx');
]
% Dados_P0T <- P(0,T)
% Number of observations
Dim=size(Dados);
N_Obs=Dim(1);
input=[0.9;0.9;0.9;0.9;0.9];
lb=[0;0;0;0;-1];
ub=[5;5;1;1;1];
nt=10;
ns=5;
rt=0.75;
maxevals=1e10;
neps=5;
functol=1e-10;
paramtol=1e-3;
verbosity=2;
minarg=1;
control={lb,ub,nt,ns,rt,maxevals,neps,functol,paramtol,verbosity,minarg};
[x,obj,convergence,details]=samin("Swaptions_Cost_Global",{input},control)
% Error calculation
fori=1:N_Obs
P_Swaptions(i)=Swaptions_Price(x(1),x(2),x(3),x(4),x(5),...
```

```
Dados(i,1),Dados(i,2),Dados(i,4));
Error(i)=(P_Swaptions(i)-Dados(i,3))/Dados(i,3);
endfor
```

A.4.5- Algorithm for the simulation of the portfolio's evolution

```
function value=V_tT(a,b,sigma,eta,rho_G2,t,T)
value=((sigma^2)/(a^2))*((T-t)+(2/a)*exp(-a*(T-t))-(1/(2*a))*exp(-2*a*(T-t))-
3/(2*a))+...
((eta^2)/(b^2))*((T-t)+(2/b)*exp(-b*(T-t))-(1/(2*b))*exp(-2*b*(T-t))-
(3/(2*b)))+...
(2*rho_G2*((sigma*eta)/(a*b))*((T-t)+((exp(-a*(T-t))-1)/a)+((exp(-b*(T-t))-
1)/b)-...
((exp(-(a+b)*(T-t))-1)/(a+b))));
endfunction
% Heston parameters;
kappa=xlsread('HE_Optimizar_Global_2015_Resumo.xlsx','Parametros','E5');
theta=xlsread('HE_Optimizar_Global_2015_Resumo.xlsx', 'Parametros','E6');
v0=xlsread('HE_Optimizar_Global_2015_Resumo.xlsx', 'Parametros','E7');
psi=xlsread('HE_Optimizar_Global_2015_Resumo.xlsx', 'Parametros','E8');
rho_HE=xlsread('HE_Optimizar_Global_2015_Resumo.xlsx', 'Parametros','E9');
% G2++ parameters;
a=xlsread('G2_Optimizar_Global_2015_Resumo.xlsx','Parametros','E5');
b=xlsread('G2_Optimizar_Global_2015_Resumo.xlsx', 'Parametros','E6');
sigma=xlsread('G2_Optimizar_Global_2015_Resumo.xlsx', 'Parametros','E7');
eta=xlsread('G2_Optimizar_Global_2015_Resumo.xlsx', 'Parametros','E8');
rho_G2=xlsread('G2_Optimizar_Global_2015_Resumo.xlsx', 'Parametros','E9');
% Correlation between interest rate and index return;
rho_rs=xlsread('Analise_correlacoes.xlsx', 'Calculos','Q7');
%global Dados_FD;
Dados_FD=xlsread('Dados_FD_2015.xlsx');
T=10;
N_Simul=4000;
```

```
Steps=360;
F0=10000;
prop_a=0.25;
D=5;
delta=0.25;
S0=3314
dt=1/Steps;
N_Steps=T*Steps;
phi_c=1.5;
% Calculation of factors independent of t
sigma_xt=sigma*((1/(2*a))*(1-exp(-2*a*dt)))^0.5;
sigma_yt=eta*((1/(2*b))*(1-exp(-2*b*dt)))^0.5;
K0=-dt*(rho_HE*kappa*theta)/psi;
K1=(dt/2)*(rho_HE*kappa/psi-0.5)-rho_HE/psi;
K2=(dt/2)*(rho_HE*kappa/psi-0.5)+rho_HE/psi;
%% Correlations and Cholesky decompositions
sigma1 = sqrt(sigma^2 + eta^2 + 2*rho_G2*sigma*eta);
sigma2 = eta*(a-b);
rhorv = (sigma*rho_G2+eta)/sigma1;
rho_sx = (sigma1*rho_rs)/sqrt(sigma1^2 + sigma2^2/((a-b)^2) + ...
    2*rhorv*sigma1*sigma2/(b-a));
rho_sy = 0.01;
% Vectors and matrices
xt=zeros(1,N_Steps+1);
yt=zeros(1,N_Steps+1);
Rt_v=zeros(1,N_Steps+1);
vt=zeros(1,N_Steps+1);
Log_St=zeros(1,N_Steps+1);
St=zeros(N_Simul,N_Steps+1);
Pt_v=zeros(N_Simul,T*delta+1);
Pt=zeros(N_Simul,N_Steps+1);
Bt=zeros(N_Simul,N_Steps+1);
Ft=zeros(N_Simul,N_Steps+1);
```

```
Ft(:,1)=F0;
S0=F0*prop_a;
B0=F0*(1-prop_a);
Z1=randn(N_Simul,N_Steps);
Z2=randn(N_Simul,N_Steps);
Z3=randn(N_Simul,N_Steps);
Z4=randn(N_Simul,N_Steps);
sum_v=zeros(N_Simul,1);
sum1_v=zeros(N_Simul,1);
perct_St=zeros(9,N_Steps+1);
perct_Bt=zeros(9,N_Steps+1);
perct_Ft=zeros(9,N_Steps+1);
% Simulation
for i=1:N_Simul
xt(1)=0;
yt(1)=0;
vt(1)=v0;
Log_St(1)=log(S0);
St(i,1)=S0;
for t=2:N_Steps+1
Zx=randn();
Zy=randn();
Mxt=((sigma^2)/(a^2)+rho_G2*((sigma*eta)/(a*b)))*(1-exp(-a*((t-1)*dt-(t-
2)*dt)))-...
((sigma^2)/(2*(a^2)))*(exp(-a*(T-(t-1)*dt))-exp(-a*(T+(t-1)*dt-2*(t-2)*dt)))-...
((rho_G2*sigma*eta)/(b*(a+b)))*(exp(-b*(T-(t-1)*dt))-exp(-b*T-a*(t-
1)*dt+(a+b)*(t-2)*dt));
Myt=((eta^2)/(b^2)+rho_G2*((sigma*eta)/(a*b)))*(1-exp(-b*((t-1)*dt-(t-2)*dt)))-
...
((eta^2)/(2*(b^2)))*(exp(-b*(T-(t-1)*dt))-exp(-b*(T+(t-1)*dt-2*(t-2)*dt)))-...
((rho_G2*sigma*eta)/(b*(a+b)))*(exp(-b*(T-(t-1)*dt))-exp(-b*T-a*(t-
1)*dt+(a+b)*(t-2)*dt));
xt(t)=xt(t-1)*exp(-a*dt) +sigma_xt*Zx;
```

```
yt(t)=yt(t-1)*exp(-b*dt)+sigma_yt*...
(rho_G2*Zx+((1-rho_G2^2)^0.5)*Zy);
integ_r(t)=(dt/2)*(xt(t)+xt(t-1)+yt(t)+yt(t-1))+log(Dados_FD(t-1,1))-
log(Dados_FD(t,1))+ ...
0.5*(V_tT(a,b,sigma,eta,rho_G2,0,t*dt)- V_tT(a,b,sigma,eta,rho_G2,0,(t-1)*dt));
m=theta+(vt(t-1)-theta)*exp(-kappa*dt);
s2=((vt(t-1)*psi^2*exp(-kappa*dt))/kappa)*(1-exp(-kappa*dt))+...
((theta*(psi^2))/(2*kappa))*(1-exp(-kappa*dt))^2;
phi=s2/m^2;
z=2*(phi^(-1));
if phi<=phi_c
b2=z-1+z^(0.5)*(z-1)^(0.5);
aa=(m/(1+b2));
Zv=randn();
vt(t)=aa*(b2^0.5+Zv)^2;
else
p=(phi-1)/(phi+1);
beta=(1-p)/m;
Uv=rand();
if Uv<=p
vt(t)=0;
else
vt(t)=(beta^-1)*log((1-p)/(1-Uv));
endif
endif
K=(dt/2)*(vt(t-1)+vt(t));
Zs=randn();
Log_St(t)=Log_St(t-1)+integ_r(t)+K0+K1*vt(t-1)+K2*vt(t)+rho_sx*sqrt(K)*Zx+...
(rho_sy-rho_sx*rho_G2)/sqrt(1-rho_G2^2)*sqrt(K)*Zy+...
sqrt(1-rho_HE^2-rho_sx^2-(rho_sy-rho_sx*rho_G2)/sqrt(1-rho_G2^2))*K*Zs;
St(i,t)=exp(Log_St(t));
endfor
Pt_v(i,1)=B0/Dados_FD(D*360,1);
```

```
Pt(i,1)=Dados_FD(D*360,1);
Bt(i,1)=Pt_v(i,1)*Pt(i,1);
for j=1:T*(1/delta)
for t=(j-1)*(360*delta)+1:j*(360*delta)
Pt(i,t+1)=((Dados_FD(round((D+(j-1)*delta)*360),1)/Dados_FD(t,1))*exp((1/2)*...
(V_tT(a,b,sigma,eta,rho_G2,t*dt,D+(j-1)*delta)-...
V_tT(a,b,sigma,eta,rho_G2,0,D+(j-1)*delta)+...
V_tT(a,b,sigma,eta,rho_G2,0,t*dt)-...
((1-exp(-a*(D+(j-1)*delta-t*dt)))/a)*xt(t+1)-...
((1-exp(-b*(D+(j-1)*delta-t*dt)))/b)*yt(t+1))));
endfor
for t=(j-1)*(360*delta)+1:j*(360*delta)
Bt(i,t+1)=Pt_v(i,j)*Pt(i,t+1);
endfor
Pt_v(i,j+1)=Bt(i,j*(360*delta)+1)/...
((Dados_FD(round((D+(j*delta))*360),1)/Dados_FD(j*(360*delta),1))*exp((1/2)*..
.
(V_tT(a,b,sigma,eta,rho_G2,j*(360*delta)*dt,D+(j*delta))-...
V_tT(a,b,sigma,eta,rho_G2,0,D+(j*delta))+...
V_tT(a,b,sigma,eta,rho_G2,0,j*(360*delta)*dt)-...
((1-exp(-a*(D+(j*delta)-j*(360*delta)*dt)))/a)*xt(j*(360*delta)+1)-...
((1-exp(-b*(D+(j*delta)-j*(360*delta)*dt)))/b)*yt(j*(360*delta)+1))));
endfor
sum=0;
for t=1:N_Steps
sum=sum+Rt_v(1,t+1);
endfor
sum_v(i)=exp(-sum);
sum1_v(i)=St(i,N_Steps+1)*exp(-sum);
for t=2: + N_Steps+1
if prop_a==0
Ft(i,t)=Bt(i,t);
else
```

```
if prop_a==1
Ft(i,t)=St(i,t);
else
Ft(i,t)=St(i,t)+Bt(i,t);
endif
endif
endfor
endfor

sum1=0;
sum2=0;
for i=1:N_Simul
sum1=sum1+sum_v(i);
sum2=sum2+sum1_v(i);
endfor
for t=1:N_Steps+1
perct_St(:,t)=prctile(St(:,t),[10 20 30 40 50 60 70 80 90]);
perct_Bt(:,t)=prctile(Bt(:,t),[10 20 30 40 50 60 70 80 90]);
perct_Ft(:,t)=prctile(Ft(:,t),[10 20 30 40 50 60 70 80 90]);
endfor
```

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Statistical data sources

Autoridade de Supervisão de Seguros e Fundos de Pensões.

Bloomberg.

EIOPA.

European Central Bank. Statistical Data Warehouse.

Human Mortality Database.