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INTERNSHIP REPORT

RESERVE RISK – AN APPLICATION TO ORSA

VÂNIA ISABEL RAMOS ELIAS

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PREFACE

This work is the result of a six-month curricular internship taken in the position of junior actuary at Companhia de Seguros Tranquilidade, S.A., for the completion of the Master in Actuarial Sciences.

During the internship I was engaged in different activities of the Actuarial Department, such as the ratemaking process for a new product, reserves estimation and ad-hoc data analysis.

Furthermore, I had contact with the Risk Management Department regarding Solvency II issues involving a high degree of quantitative assessment, which the actuarial team is well placed to perform. In this context, the opportunity of working in a specific topic of Solvency II arose and I decided to work on the estimation of one of the undertaking specific parameters - the reserve risk - for two lines of business (LoB): Motor Vehicle Liability and Motor Others.

I would like to thank Tranquilidade for making my internship possible and for providing all the necessary conditions for my work. To João Barata for sharing his knowledge and experience and helping me with the transposition of the literature to the business reality, and to my colleagues in the Actuarial and Risk Management Departments, who were always keen on helping me with everything I could need.

A special thanks to my supervisor in ISEG, João Andrade e Silva, for his help and support along this internship. His orientations and suggestions made it possible to get as far as I did, and his formal corrections were fundamental in writing this report.

ABSTRACT

Under Solvency II, insurance undertakings must have, as part of their risk management system, a regular practice of assessing their overall solvency needs with a view to their specific risk profile, known as 'Own Risk and Solvency Assessment' (ORSA). ORSA aims to identify whether the particular risk profile of an undertaking deviates from the assumptions underlying the regulatory capital calculation (i.e. European Standard Formula).

In this context, this work aims at estimating the undertaking specific parameters (USP) for reserve risk, for Motor Vehicle Liability and Motor Others. In a long term perspective, alternative models were applied to the estimation of the ultimate reserve risk. For Solvency Capital Requirements, a short-term perspective, it is necessary to estimate the one-year reserve risk factors, which was done by applying the three different methods presented and allowed by the European Insurance and Occupational Pensions Authority (EIOPA). The results for the different models and methods in both perspectives were compared and the impact of the USP was assessed in terms of capital gains.

KEYWORDS

Solvency II, ORSA, USP, Solvency Capital Requirement, Reserve Risk, Mack, Bootstrap, Munich Chain Ladder, Merz-Wüthrich.

LIST OF ACRONYMS AND ABREVIATIONS USED

BI – Bodily-Injury

CEIOPS – Committee of European Insurance and Occupational Pensions Supervisors

EIOPA - European Insurance and Occupational Pensions Authority

EU – European Union

IBNR – Incurred But Not Reported

IBNER – Incurred But Not Enough Reported

IDS – accidents that follow the direct compensation to the policy holder system

LoB – Line of Business

MCL – Munich Chain Ladder

MCR – Minimum Capital Requirement

MD – Material Damage

MSEP - Mean Squared Error of Prediction

ORSA – Own Risk and Solvency Assessment

SCL – Separate Chain Ladder

SCR – Solvency Capital Requirement

SLT – Similar to Life Techniques

USP – Undertaking Specific Parameter(s)

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1. INTRODUCTION

Solvency II project aims to review the prudential regime for insurance and reinsurance undertakings in the European Union (EU), and in particular to ensure that they can survive difficult periods, thus protecting policyholders and the stability of the financial system as a whole.

The need for this prudential regime becomes more evident in this new, globalized world of closely interdependent economies, where the recent financial crisis has affected almost every part of the world and the recovery from this global financial crisis remains fragile.

The Solvency II Directive 2009/138/EC, that codifies and harmonizes the EU insurance regulation, introduces a new requirement concerning risk and capital management activities. At the core of the Directive, Article 45 requires that: «as part of its risk-management system every insurance undertaking and reinsurance undertaking shall conduct its own risk and solvency assessment. »

One of the purposes of the own risk and solvency assessment (ORSA) is to identify whether the particular risk profile of an undertaking deviates from the assumptions underlying the regulatory capital calculation (e.g. European Standard Formula). Its framework leads undertakings towards a better understanding and management of their risk profiles, in accordance with their strategic choices.

The ORSA Issues Paper – CEIOPS (2008) - gives a crisp definition:

> The ORSA can be defined as the entirety of the processes and procedures employed to identify, assess, monitor, manage, and report the short and the long term risks a (re)insurance undertaking faces or may face and to determine the own funds necessary to ensure that the undertakings overall solvency needs are met at all times.

Underlying this definition, one of the aspects that must be taken into consideration in the ORSA is the degree to which the undertakings risk profile deviates from the assumptions underlying the Solvency Capital Requirement (SCR), calculated with the standard formula or with its specific risk parameters or internal model.

Furthermore, in its Article 48, the Directive describes the actuarial role as follows:

> 1. Insurance and reinsurance companies shall provide for an effective actuarial function to: (…) contribute to the effective implementation of the risk-management system referred to in the article 44, in particular with respect to the risk modelling underlying the calculation of the capital requirements set out in Chapter VI, Sections 4 and 5, and to the assessment referred to in Article 45.

This document will start with a brief framework on Solvency II, the ORSA and the role of the undertaking specific parameters, the so-called USP, followed by a detailed presentation of the risk reserve parameter, its specificities and its interaction with the remaining components of the Standard Formula.

To estimate the ultimate reserve risk, a set of methods was selected and applied to the Motor data of the company, and their results are analyzed and compared.

A short-term approach for each of the selected methods is then taken in order to capture the one-year risk required for SCR purposes.

2. THE SOLVENCY II REGIME

Solvency II is not just about capital, but it is rather a comprehensive programme based on three pillars:

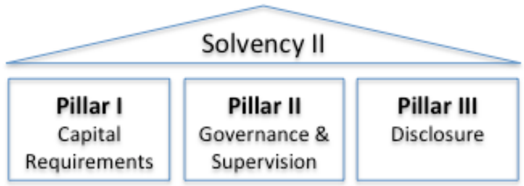


Figure 1: Three pillars structures of Solvency II

Pillar I defines the financial resources that a company needs to hold in order to be considered solvent, in particular it defines two thresholds: Solvency Capital Requirement (SCR) and Minimum Capital Requirement (MCR). SCR is calculated using either a standard formula or, with regulatory approval, an internal model, while the MCR is calculated as specified in CEIOPS (2009c) and it cannot fall below 25% or exceed 45% of the SCR.

Pillar II deals with the qualitative requirements for the (re)insurers: the system of governance and the risk management system, as well as the requirements for the effective supervision of (re)insurers.

Finally, the focus of Pillar III is on disclosure requirements, both to the regulator and to the general public.

2.1. *Own Risk Solvency Assessment (ORSA)*

The ORSA, introduced in pillar II, is a key element of Solvency II. It is the (re)insurer's own assessment of the capital required to run his business, reflecting the company's risk profile and tolerances. It must be produced at least annually, and will be subject to external assessment, but not public disclosure. Likely, it will produce a different result from the regulatory capital requirement imposed by pillar I, but a deviation between the ORSA and the SCR calculation doesn't automatically lead to an increase of capital.

When performing the ORSA exercise, together with many other activities and evaluations, the undertaking will evaluate:

1. How well does the standard formula capture its specific risks?
2. How sensitive are the results of the standard formula to changes in the mix of risks, and the impact of reinsurance and other risk mitigation methods?
3. How do the results differ between the standard SCR and the SCR calculated using Undertaking Specific Parameters (USP)?

2.2. Undertaking Specific Parameters (USP)

Companies using the Solvency II standard formula should consider using undertaking specific parameters in calculating their risk capital as they allow for better assessment of undertaking specific risk profiles in the standard formula, which in turn leads to a more accurate calculation of SCR. Even though USP require additional and complex work, for many companies they may be worth the extra effort.

2.2.1. Legal background to USP

The Solvency II Directive, in its article 104 (Design of the basic Solvency Capital Requirements), states:

> Subject to approval by the supervisory authorities, insurance and reinsurance **undertakings may**, within the design of the standard formula, **replace a subset of its parameters by parameters specific to the undertaking concerned**, when calculating the life, non-life and health underwriting risk modules.

Additionally, in article 110 (Significant deviations from the assumptions underlying the standard formula calculation), we can read:

> Where it is inappropriate to calculate the Solvency Capital Requirement in accordance with the standard formula (...) because **the risk profile** of the insurance or reinsurance undertaking concerned **deviates significantly from the assumptions underlying the standard formula calculation**, the supervisory authorities may, by means of a decision stating the reasons, require the undertaking concerned to replace a subset of the parameters specific to that undertaking when calculating the life, non-life and health underwriting risk modules, as set out in Article 104 (7). Those specific parameters shall be calculated in such a way to ensure that the undertaking complies with Article 101(3).

The referred article 101(3) defines the following:

> The Solvency Capital Requirement shall be calibrated so as to ensure that all quantifiable risks to which an insurance or reinsurance undertaking is exposed are taken into account. It shall cover existing business, as well as the new business expected to be written over the following 12 months. With regard to

existing business it shall cover only unexpected losses. It shall correspond to the **Value-at-Risk** of the basic own funds of an insurance or reinsurance undertaking subject to a **confidence level of 99,5% over a one-year period**.

2.2.2. Usefulness of USP

There are some reasons for an undertaking to use USP :

- to better adjust and reflect a company's risk profile - if historical data or appropriate external data show different volatility on premium and reserve risk, replacing the market-average parameters with the company-specific parameters based on its USP will lead to a lower SCR.

- if a new (re)insurance programme cannot be adequately reflected in the standard formula, an undertaking can use USP. The new structure can be applied to the historical gross book on an as-if basis for the reserve risk as well as for the premium risk. This way, the company can derive USP which better reflect the undertaking's situation.

- USP are an input to ORSA: «That assessment shall include (...) the overall solvency needs taking into account the specific risk profile, approved risk tolerance limits and the business strategy of the undertaking» (Article 45).

Furthermore, if approval for USP fails, it can be an input for a partial internal model or it can form a part of the validation of results emerging from internal model.

2.2.3. CEIOPS' Advice on USP

CEIOPS' advice on USP - CEIOPS (2010a) - identifies the subset of standard parameters that may be replaced by USP. For all other parameters, undertakings shall use the values considered for the standard formula. There are four sub-modules of the standard formula in which parameters can be replaced:

i. Non-life premium and reserve risk;
ii. Non-SLT (Similar to Life Techniques) health premium and reserve risk;
iii. SLT health revision risk;
iv. Life revision risk.

The sub-module of interest for this work is the first one, which includes three possible USP: standard deviation for premium and for reserve risk and adjustment factor for non-proportional reinsurance, being the **standard deviation for reserve risk** the one to be estimated as defined in CEIOPS' advice on the SCR non-life underwriting risk module - CEIOPS (2009a).

In order to be able to use the USP, undertakings must obtain supervisory approval and must demonstrate that standard parameters do not better reflect their risk profile. Supervisors must also be satisfied that "cherry-picking" to give the lowest SCR has not taken place.

A credibility mechanism is required when applying USP. Depending on the number of years for which data are available, and in the use solely of internal data or the use of external data, more or less weight is given to the undertaking versus the standard parameter, by applying a credibility factor (*c*):

$$\sigma_{(res,lob)} = c \times \sigma_{(U,res,lob)} + (1-c)\sigma_{(S,res,lob)}$$

where:

$\sigma_{(res,lob)}$=final undertaking specific parameter, after applying the credibility factor;

$\sigma_{(U,res,lob)}$= undertaking specific parameter;

$\sigma_{(S,res,lob)}$= standard parameter;

For the two LoBs of interest, full credibility is only given with fifteen or more years of internal historical data for Motor Vehicle Liability and with at least ten years for Motor Others. If external data is used, the maximum credibility in both cases is 63%.

CEIOPS (2010a) presents a detailed description of the methods and assumptions that undertakings should apply to calculate their USP for reserve risk, but it «does not consider one method to be perfect and proposes that undertakings apply a variety of methods to estimate their volatility». Undertakings will have the onus of explaining how and why they have selected the final factor, taking into consideration their risk profile.

3. THE STANDARD FORMULA

In Solvency II regime, the SCR is, by definition, the «level of capital that enables an insurance undertaking to absorb significant unforeseen losses and that gives reasonable assurance to policyholders and beneficiaries» and it shall take account of all quantifiable risks and the net impact of all possible risk mitigation techniques.

The standard formula was built in order to provide a harmonized way of calculating this level of capital for all the undertakings and it was calibrated to achieve the target criteria of 99.5% Value-at-Risk for 1 year of time-horizon.

It presents a modular structure as shown in the figure bellow:

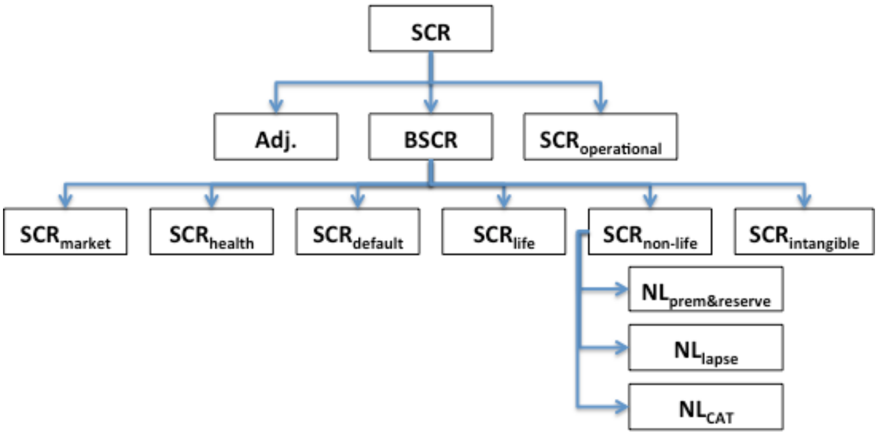


Figure 2: SCR modular structure

3.1. *Non-life underwriting risk*

The underwriting risk can be defined as «the risk of loss, or of adverse change in the value of insurance liabilities, due to inadequate pricing and provisioning.»

CEIOPS (2009a) provides advice and guidance on the methods, assumptions and standard parameters to be used in the design of the non-life underwriting risk module, as required in Article 111(c) of Directive 2009/138/EC and has its legal support in Articles 111, 101, 104 and 105 of the Directive.

Although the risk of interest for the purpose of this report is the reserve risk, the calculations for the combined premium risk and reserve risk will be presented in order to understand how the two risks interact in the standard formula. Premium risk calculations will be detailed only when necessary to understand the reserve risk calculations.

3.2. *Non-life premium and reserve risk sub-module*

The capital charge for premium and reserve risk (NL_{pr}) is given by:

$$NL_{pr} = \rho(\sigma) \cdot V$$

where:

V = volume measure

σ = combined standard deviation, resulting from the combination of the reserve and premium risk standard deviations, and

$$\rho(\sigma) = \frac{\exp(N_{0.995} \cdot \sqrt{\log(\sigma^2 + 1)})}{\sqrt{\sigma^2 + 1}} - 1 \ .$$

$N_{0.995}$ = 99.5% quantile of the standard normal distribution

Note that $\rho(\sigma)$ is computed assuming a log-normal distribution of the underlying asset and an expected value of 1 in order to be consistent with the VaR 99.5%. It can be approximated by: $\rho(\sigma) \approx 3 \cdot \sigma$.

To calculate the volume measure and the combined standard deviation it is necessary to calculate them for each individual LoB and for both premium risk and reserve risk, and then aggregate them using the formulae bellow.

For the volume measure we have

$$V = \sum_{LoB} V_{(prem,LoB)} + \sum_{LoB} V_{(res,LoB)}$$

$$V_{(res,LoB)} = PCO_{lob}$$

Where:

$V_{(prem,LoB)}$, $V_{(res,LoB)}$= volume measure for premium and for reserve risk.

$\sigma_{(prem,LoB)}$, $\sigma_{(res,LoB)}$=standard deviation for premium and for reserve risk.

PCO_{lob} = best estimate for claims outstanding for each LoB.

As the Advice states, «the standard deviation for premium and reserve risk for each LoB is defined by aggregating the standard deviations for both sub-risk under the assumption of a correlation coefficient of **α=0.5**.»

$$\sigma_{(lob)} = \frac{\sqrt{(\sigma_{(prem,lob)}V_{(prem,lob)})^2 + 2\alpha\sigma_{(prem,lob)}\sigma_{(res,lob)}V_{(prem,lob)}V_{(res,lob)} + (\sigma_{(res,lob)}V_{(res,lob)})^2)}}{V_{(prem,lob)} + V_{(res,lob)}}$$

Finally, the overall standard deviation is given by:

$$\sigma = \sqrt{\frac{1}{V^2} \cdot \sum_{r \times c} CorrLob_{r,c} \cdot \sigma_r \cdot \sigma_c \cdot V_r \cdot V_c}$$

where

r,c = all indices of the form (LoB)

$CorrLob_{r,c}$ = the correlation coefficient between LoB r and LoB c

The correlation matrix *CorrLob* structure is presented and explained in QIS3 - CEIOPS (2007) - which the CEIOPS' Advice on non-life underwriting risk calibration – CEIOPS (2010c) – considers to be «appropriate». The values considered are the ones embedded in QIS4 and QIS5 (Annex 1).

3.3. Standard Parameter for Reserve Risk

CEIOPS (2010b) explains how the reserve risk calibration was performed, identifying the data used, the assumptions considered and detailing the six different methods applied to calibrate the reserve risk to each LoB.

For the LoB Motor, vehicle liability, the data sample included data from 327 undertakings and from 106 undertakings for LoB Motor, other classes, in both cases gross of reinsurance. The different methods were applied to the collected data.

For **Motor, vehicle liability** methods 1 and 2 provided a relatively poor fit but with some credibility in the tail, while method 4 gave significantly lower factors than all the other methods. Therefore, the technical factor was chosen as the average of methods 1, 2, 3, 5 and 6, leading to the standard parameter for reserve risk of **11%**.

For **Motor, others** methods 1 and 2 provided a relatively poor fit and again method 4 gave significantly lower factors than all the other methods. Therefore the technical factor was chosen as the average of methods 1, 5 and 6, leading to the standard parameter for reserve risk of **20%**.

In the next chapters, different methods will be used to estimate the undertaking specific parameters for these two LoB, that would eventually substitute the 11% and the 20% in the standard formula, for vehicle liability and other classes, respectively.

In order to do so, the CEIOPS' Advice on technical provisions – CEIOPS (2009b) – was taken into consideration to ensure that data complies with the standards for data quality, in terms of appropriateness, completeness and accuracy of data.

Furthermore, the same applicable assumptions considered for reserve risk calibration in CEIOPS (2010b) were applied in this analysis, namely:

- Expenses are not considered in the run-off triangles used to derive the reserve risk standard deviation but are included in the reserve best estimate in the standard formula. Expenses are expected to be less volatile than the claims and as result applying the estimate for reserve risk to both claims and expenses reserves is being conservative;
- No explicit allowance was made for inflation; it was assumed that inflationary experience in the period 2000 to 2012 was representative of the inflation that might occur.

In order to obtain 100% credibility for the parameters estimates, the number of years of historical data to be used is at least 15 for vehicle liability and 10 for other classes – see CEIOPS (2010a). However, It must be assured that data from each year is coherent and comparable; otherwise the results may prove meaningless.

Due to relevant changes in the company's claims handling and settling processes, only the last 13 years of historical data were considered, which means that a credibility factor will be applied to the USP for Motor, Vehicle Liability, as explained in section 2.2.3..

4. METHODS FOR RESERVE RISK ESTIMATION

When performing the ORSA exercise, (re)insurers are expected to define their overall solvency needs, which implies the choice of a relevant time horizon.

While the quantitative requirements are related to the first pillar of the directive and therefore to a 1-year time horizon, the forward looking perspective within ORSA requires to look beyond this period.

Accordingly we can consider:

- Single-period solvency – in the regulatory sense, having enough own funds to avoid economic bankruptcy over 1 year with a 99.5% threshold;
- Multi-year solvency – having enough own funds to avoid economic bankruptcy over the whole time horizon with a *p* threshold.

The reserve risk estimation was first approached in a multi-year perspective, by applying different stochastic methods to estimate the ultimate reserve risk parameter.

Then the analysis was shortened to the one-year time horizon for the specific purpose of obtaining the USP.

4.1. Setting up the model

Let us consider that annual data is available. Different periods of time may be considered with the respective adjustments in the notation and formulae.

C_{ik} – accumulated total claims amount of accident year i, i=1, 2, …, I , either paid or incurred up to development year k, k=1, 2, …, I-i+1.

The set of available data can be grouped as follows:

Accident year	Development year						
	1	2	…	k	…	I-1	I
1	C_{11}	C_{12}	…	C_{1k}	…	$C_{1,I-1}$	C_{1I}
2	C_{21}	C_{22}	…	C_{2k}	…	$C_{1,I-1}$	
…	…	…	…	…	…		
i	C_{i1}	C_{i2}	…	C_{ik}			
…	…	…	…				
I-1	$C_{I-1,1}$	$C_{I-1,2}$					
I	C_{I1}						

Table 1 - Triangle of accumulated claims

For the purpose of estimating the reserve risk, tails in the run-off triangles were not considered. It was assumed that the tail has the same estimated variability. The chain-ladder method is the basis for the methods that will be considered, therefore let us summarize the method for obtaining a deterministic estimation of the reserves:

i. Estimate the chain ladder development factors:

$$\hat{f}_k = \frac{\sum_{i=1}^{I-k} C_{i,k+1}}{\sum_{i=1}^{I-k} C_{i,k}} \text{ , } k=1,\ldots,I\text{-}1$$

ii. Obtain the estimation for the accumulated claims in the lower triangle of the claims data:

$$\hat{C}_{i,I-i+2} = C_{i,I-i+1}\hat{f}_{I-i+1} \text{ , } i=2,\ldots,I$$

$$\hat{C}_{i,k+1} = \hat{C}_{ik}\hat{f}_k \text{ , } i=3,\ldots, I \text{ and } k=I\text{-}i+2,\ I\text{-}i+3,\ldots,\ I\text{-}1$$

iii. Estimate the outstanding claim reserve for accident year i=2,…,I:

$$\hat{R}_i = \hat{C}_{iI} - C_{i,I+1-i}$$

iv. The total outstanding claim reserve is given by:

$$\hat{R} = \sum_{i=2}^{I} \hat{R}_i$$

4.2. Stochastic methods for ultimate reserve risk

This chapter presents three stochastic models for claims reserving and reserve risk estimation: Mack's model, Munich Chain Ladder model with an appropriate bootstrap simulation technique and the bootstrap simulations for the pure Chain Ladder algorithm.

4.2.1. Mack's Model

Mack (1993) presents a distribution-free formula for the standard error of the chain ladder estimates, by considering the first two moments for the cumulative payments.

Mack's assumptions:

- $E(C_{i,k+1} \mid C_{i1},...,C_{i,k}) = C_{i,k} f_k$, i=1,…,I and k=2,…,I;
- there is no dependency between accident years;
- $Var(C_{i,k+1} \mid C_{i1},...,C_{i,k}) = C_{i,k} \sigma_k^2$, i=1,…,I and k=1,…,I-1, where σ_k^2 can be

  estimated as follows: $\hat{\sigma}_k^2 = \frac{1}{I-k-1} \sum_{i=1}^{I-k} C_{ik} \left(\frac{C_{i,k+1}}{C_{ik}} - \hat{f}_k \right)^2$, k=1,…,I-2.

For the last development year there is not enough data, therefore $\hat{\sigma}_{I-1}^2$ is computed in a different way. For the purpose of this work, it was used the approximation from Mack (1999). Alternatively, the log-linear regression could be used or an appropriate numeric value could be assumed.

Under the assumptions above, it can be shown that the mean squared errors are:

$$\widehat{mse(\hat{R}_i)} = \hat{C}_{iI}^2 \sum_{k=I+1-i}^{I-1} \frac{\hat{\sigma}_k^2}{\hat{f}_k^2} \left(\frac{1}{\hat{C}_{ik}} + \frac{1}{\sum_{j=1}^{I-k} C_{jk}} \right)^2 \text{ , } i=2,\ldots,I \text{ and}$$

$$\widehat{mse(\hat{R})} = \sum_{i=2}^{I} \left\{ \left(\left(s.e.(\hat{R}_i)\right)^2 + \hat{C}_{iI} \left(\sum_{j=i+1}^{I} \hat{C}_{jI} \right) \sum_{k=I+1-i}^{I-1} \frac{2 \hat{\sigma}_k^2 / \hat{f}_k^2}{\sum_{n=1}^{I-k} C_{nk}} \right) \right\}.$$

4.2.2. Bootstrap for Chain Ladder

The Bootstrap technique presented in Efron and Tibshirani (1993), is a simple but powerful technique to obtain information from one single sample of data. Assuming the observable data to be independent and identically distributed, the generated sets of pseudo-data are consistent with the underlying distribution of the observed data. Therefore, statistics of interest can be obtained.

Generically, the methodology consists in sampling with replacement from the observed data sample, in order to obtain a sufficient number of sets of pseudo-data.

England and Verral (1999) present a relevant application of the bootstrap to obtain the estimation error of reserve estimates from the Chain-Ladder model.

Very often, data are not identically distributed, since the means and/or variances may depend on covariates, therefore it is common to resample residuals instead, which are usually independent and identically distributed or can be adjusted for that purpose.

England and Verral (2002) suggest the following bootstrap procedure:

- Obtain the standard chain-ladder development factors from cumulative data;
- Obtain cumulative fitted value for the past triangle;
- Obtain incremental fitted values, $\hat{m}_{ik}$, for the past triangle by differencing;
- Calculate the unscaled Pearson residuals for the past triangle:

$$r_{ik}^{(P)} = \frac{C_{ik} - \hat{m}_{ik}}{\sqrt{\hat{m}_{ik}}}$$

- Estimate the Pearson scale parameter ϕ, by:

$$\hat{\phi} = \frac{\sum_{i,kI-i+1} \left(r_{ik}^{(P)}\right)^2}{\frac{1}{2}I(I+1) - 2I + 1}$$

- Adjust the Pearson residuals using:

$$r_{ik}^{adj} = \sqrt{\frac{I}{\frac{1}{2}I(I+1) - 2I + 1}} \times r_{ik}^{(P)}$$

- Begin the iterative loop, to be repeated *N* times:
  i. Resample the adjusted residuals with replacement, creating a new past triangle of residuals;
  ii. For each cell in the past triangle, obtain a set of pseudo-incremental data by solving the unscaled Pearson residuals in order to C_{ij}, i.e.

$$C_{ik} = \hat{m}_{ik} + r_{ik}^{(P)} . \sqrt{\hat{m}_{ik}} \; ;$$

  iii. Create the corresponding set of pseudo-cumulative data;
  iv. Apply the standard chain-ladder method to the pseudo-cumulative data to obtain a future triangle of cumulative payments;

v. Obtain from iv) the future triangle of incremental payments by differencing. This values will be used as the mean, $\tilde{m}_{ij}$, when simulating the process distribution;

vi. For each future payment cell (*i,j*), simulate a payment from the process distribution with mean $\tilde{m}_{ij}$ and variance $\hat{\phi}\tilde{m}_{ij}$;

vii. Sum the simulated payments in the future triangle by origin year and overall to give the origin year and the total reserve estimates respectively;

viii. Store the results, and return to the start of the iterative loop.

The standard deviation of the stored results gives an estimate of the prediction error.

Sometimes, the residuals after adjustment may still have inherited skewness of the original data. In these situations the bootstrap procedure presented above can be misleading since it uses an approximation to the normal distribution. Pinheiro et al. (2003) deal with this situation by introducing an extra step to the bootstrap procedure.

4.2.3. Bootstrap for Munich Chain Ladder

Quarg and Mack (2004) present a new approach to claims reserving methodologies, that aims at reducing the gap between IBNR and IBNER (Incurred But Not Reported and Incurred But Not Enough Reported, respectively) projections based on paid losses and based on incurred losses, which are often far different from each other. For practical reasons, companies tend to choose one of the run-off triangles, ignoring the result that would be obtained if the other run-off triangle would be used.

This approach assumes that Mack's model is applicable to both paid and incurred losses triangles and it shows that commonly there are positive correlations between paid and incurred losses that are ignored and that should be taken into account in the reserving process.

Instead of performing two separate chain ladder calculations (SCL), the Munich Chain Ladder (MCL) combines the paid-loss (*P*) and incurred-loss (*I*) data types by taking (*P/I*) and (I/P) ratios into account when doing projections.

The point of MCL is to estimate individual development factors f_{ik} that are different for each origin and development year, as an alternative for a common factor for each development year f_k. Using the observed correlations between the two run-off triangles, the first diagonal is projected for both triangles. The next diagonals are projected with the implicit projected correlation of the last diagonal and the process is repeated recursively until the last cell of each triangle has been projected.

The application of this method to different data sets, including the data sets used for this work, shows evidence that the MCL projections for paid and incurred losses result in closer values than the SCL projections, which is to say that using MCL we obtain P/I ratios closer to 100% (in the long run we expect to pay all and not more than the incurred losses).

However this method is only applicable if we assume that initial reserves are correctly estimated, otherwise the run-off triangle will reflect systematic corrections, either increases in the incurred loss as a result of underestimated case reserves or reductions as a result of overestimated case reserves. These systematic corrections will inadequately influence the P/I and the I/P ratios, thus applying the method will result in meaningless projections.

Steps for MCL application:

i. As initial data, consider the triangles of paid and incurred data with the same structure as presented in Table 1;

ii. For each run-off triangle, calculate the development factors and the standard deviation parameters as in Mack's Model;

iii. Calculate for each development year the observed P/I and I/P ratios;

iv. Adjust the observed paid losses with the observed I/P ratios and the incurred losses with the observed P/I ratios, for the respective development year and then obtain their standard deviations (ρ_P and ρ_I respectively);

v. Compute the conditional residuals for P, I, P/I and I/P, using the parameters σ_P, σ_I, ρ_P and ρ_I;

vi. Using the residuals of the P and I/P triangles draw the paid residual plot and obtain the correlation (λ^P), similarly, with the I and the P/I triangle draw the incurred plot and obtain it's correlation (λ^I);

vii. Recursively, using the estimated correlations, correct the development factors for the next development year and project the next diagonal of paid and incurred losses, until the ultimate losses are projected for all origin years.

For detailed explanation and formulae see Quarg and Mack (2004).

The steps above allow us to obtain deterministic projections for the ultimate reserves, using paid and incurred losses. However, for the purpose of the present work, it is still necessary to estimate the risk implicit in this method.

Liu and Verral (2010) present a bootstrap approach to estimate the predictive distributions of reserves produced by the MCL, by applying bootstrapping methods to dependent data and consequently taking correlations into account.

Considering the categorization of the models introduced by England and Verral (2007) into recursive and non-recursive, since the MCL is a recursive model, Liu and Verral follow their approach of bootstrap for recursive models.

However, since we are dealing with two sets of correlated data, independence assumption is not met and therefore the normal bootstrap technique cannot be used. The correlation observed in the data represents real dependence between the paid and incurred claims and it should remain unchanged within any resampling procedure.

Bootstrap Algorithm for MCL

After applying the MCL method to obtain the residuals for the four data sets, adjust the Pearson residual estimates to correct the bootstrap bias and group all four adjusted residuals together.

Then start the iterative loop to be repeated N times ($N \geq 10000$), consisting of the following steps:

i. Randomly select from the grouped residuals with replacement, so that a pseudo sample of the grouped residuals is created. This is the key step of this bootstrap methodology as it allows to generate pseudo samples of the four residuals that reflect the same correlation structure from the observed data;

ii. Calculate the pseudo samples of the four triangles by inverting the Pearson residuals;

iii. Compute the $C^P{}_{i,j}$ and $C^I{}_{i,j}$ weighted averages of the bootstrap paid and incurred development factors, where $C^P{}_{i,j}$ and $C^I{}_{i,j}$ are the paid and incurred losses for origin year *i* and development year *j*, respectively;

iv. Obtain the corresponding correlation coefficient for the resampled data using the pseudo residuals;

v. Calculate the variances for the bootstrap data;

vi. Compute the bootstrap development factors adjusted by the correlation coefficient between the pseudo data for the resampled bootstrap paid and incurred run-off triangles;

vii. Recursively, simulate a future payment for each cell in the lower triangle for both paid and incurred claims, from the process distribution with the mean and the variance obtained in vi), assuming a normal distribution;

viii. Sum the simulated payments in the future triangle by origin and overall, to obtain the origin year and the total reserve estimates, respectively;

ix. Store the results and return to the start of the iterative loop;

The standard deviation of the stored results gives an estimate of the prediction error. For detailed explanation and formulae see Liu and Verral (2010).

4.3. Methods for one-year reserve risk

For the 1-year reserve risk, CEIOPS (2010a) details the three different methods that are currently accepted for purposes of USP estimation.

4.3.1. Method 1

This method assumes that the variance of the best estimate for claims outstanding in one year plus the incremental claims paid over the one year is proportional to the current best estimate for claims outstanding.

It essentially consists in reviewing the run-off of the claims reserves based on the undertaking's data of historical claims provisions and payments, requiring at least five years of covered data, in order to compare the claims provision at the start of a financial year with the sum of the undertaking's own claims provision at the end of the financial year plus claims paid during that same year, and from there obtain an estimate for the constant of proportionality.

Lets first consider the following relationships:

$$V_{Y,lob} = \sum_{i+j=Y+1} PCO_{lob,i,j} \ ,$$

$$R_{Y,lob} = \sum_{\substack{i+j=Y+2 \\ j\neq 1}} PCO_{lob,i,j} + \sum_{\substack{i+j=Y+2 \\ j\neq 1}} I_{lob,i,j} \ ,$$

where:

$V_{Y,lob}$ = Volume measure by calendar year Y and LoB.

$PCO_{lob,i,j}$ = Best estimate for claims outstanding by LoB for accident year i and development year j.

$R_{Y,lob}$ = Best estimate for outstanding claims and incremental paid claims for the exposures covered by the volume measure, but in one year's time by calendar year and LoB.

$I_{lob,i,j}$ = Incremental paid claims by LoB, for accident year i and development year j.

The behaviour of losses is formulated as:

$$R_{Y,lob} = V_{Y,lob} + \sqrt{V_{Y,lob}}\beta_{lob}\varepsilon_{Y,lob},$$

where:

β^2_{lob} = Constant of proportionality for the variance of the best estimate for claims outstanding in one year plus the incremental claims paid over the one year by LoB.

$\varepsilon_{Y,lob}$ = An unspecified random variable with distribution with mean zero and unit variance.

The estimator for β_{lob} is then:

$$\hat{\beta}_{lob} = \sqrt{\frac{1}{N_{lob}-1}\sum_{Y}\frac{(R_{Y,lob}-V_{Y,lob})^2}{V_{Y,lob}}},$$

where:

N_{lob} = The number of data points available by LoB where there is both a value of $V_{Y,lob}$ and $R_{Y,lob}$.

Note that in this formulae $R_{Y,lob}$ refers to the past observed values.

Finally, the one-year reserve risk is estimated using:

$$\sigma_{(U,res,lob)} = \frac{\hat{\beta}_{lob}}{\sqrt{PCO_{lob}}},$$

where:

PCO_{lob} =The best estimate for claims outstanding by LoB.

This method tends to produce a higher USP factor when observed claims run-off is different from that initially expected. Moreover, it produces a USP factor that will be applied to future reserves, therefore it's only valid if the claims reserving methodology (implicit in the historical data use for the estimation) was the same along the years used for the estimation and if it remains the same in the future.

4.3.2. Method 2

The second method is based on the mean squared error of prediction (MSEP) of the claims development result over a one-year time horizon using the Merz-Wüthrich method presented by Merz and Wüthrich (2008).

In this method, the reserve risk factor is calculated as the square root of the estimated mean squared error, divided by the undertaking's own claims provision:

$$\sigma_{(U,res,lob)} = \frac{\sqrt{MSEP}}{PCO_{lob}}$$

Where the MSEP is the one obtained with Merz-Wüthrich method.

The use of this method is only possible when the claims triangle is consistent with the Merz-Wüthrich model assumptions.

4.3.3. Method 3

This method is similar to method 2, except that the square root of the estimated mean squared error is now divided by the outstanding claims reserve estimated using a chain-ladder projection method:

$$\sigma_{(U,res,lob)} = \frac{\sqrt{MSEP}}{CLPCO_{lob}}$$

Where $CLPCO_{lob}$ is the best estimate for claims outstanding estimated using the chain ladder method applied to paid claim developments.

Method 3 produces a higher risk factor than method 2 when the undertaking's own claims provision is higher than the provision implied by a chain-ladder projection. Conversely, if the undertaking's own claims provision is lower than the provision implied by a chain-ladder projection, then method 2 is the one that produces a higher risk factor.

From a theoretical perspective Method 3 is more adequate then 2, because it applies an estimate of the MSEP that was developed specifically for the pure chain ladder method, therefore, applying it to a different model might not reflect correctly the actual reserve risk. However, the final use of the reserve risk factor is to be applied to the best estimate, which usually is not obtained with the pure chain ladder.

5. APPLICATION TO MOTOR, VEHICLE LIABILITY AND MOTOR, OTHERS

The different models and methods presented in the previous chapters were applied to the company's data for these two lines of business.

Information on claims payments and case reserves, both net of reimbursements, by origin and development year, for accidents occurred from 2000 to 2012, was collected and treated using the software *SAS Enterprise Guide* and *Microsoft Excel*.

Due to the different behaviors of sub-lines of business and in order to obtain more accurate values to the ultimate reserve risk, data was collected separately. LoB Motor, Vehicle Liability was separated in Bodily Injury (BI), Material Damages (MD) and IDS – accidents that follow the direct compensation to the policy holder system. LoB Motor, others was separated in Own Damage and Passengers.

For the one-year reserve risk, data is to be considered grouped by the two lines of business, for consistency with the approach used in CEIOP's documents.

The application of the theoretic models present in the previous chapters was performed using the statistical software *R* for *Windows GUI* front-end. Additional calculations and the implementation of the capital charge calculations were performed using *Microsoft Excel*.

In order to preserve the confidentiality of the data, this chapter will only present the final results in terms of reserve risk estimation.

5.1. Results: Ultimate Reserve Risk

For the estimation of the ultimate reserve risk, a great use of the R® package '*ChainLadder'* was done.

This package has implemented several functions for claims reserving, namely the Mack' Model, the MCL Model and the Bootstrap for Chain Ladder:

- MackChainLadder – based on Mack (1993) and Mack (1999);
- BootChainLadder – based on England and Verral (2002);
- MunichChainLadder – based on Quarg and Mack (2004).

The use of these functions is exemplified in Annex 2.

The bootstrap procedure used for projections obtained via the MCL method was fully implemented as can be seen in Annex 3.

When applying these methods to the data available, two relevant situations were detected:

- Bodily injury: the correlation between payments and incurred claims was negative. This is due to prudent case reserve estimates that are later reduced, resulting in reduction of claims incurred while payments continuously increase. As a consequence, the MCL method is not applicable, since it assumes and uses the positive correlation between the two data sets to approximate the resulting projections.
- Own Damage: a couple of cells in the incremental payments, in the last developments years are negative due to some reimbursements. Whereas there is no problem for Mack and MCL methods, for the Chain Ladder Bootstrap it resulted in a very high reserve risk factor.

The residuals distribution and the correlations obtained by the MCL method can be found in annexes 4 to 8.

The overall results obtained are presented in the tables bellow.

Reserve Risk ($\sigma_{\%}$)	Mack	MCL	Bootstrap
Bodily Injury	13.25%	n.a.	15.23%
Material Damage	12.97%	13.56%	11.28%
IDS	22.22%	25.05%	18.83%
Own Damage	18.05%	18.69%	78.51%
Passengers	36.68%	48.38%	28.26%

Table 2 - Ultimate reserve risk factors

The Bootstrap procedure shows the lower factors, except for Bodily Injury and Own Damage for the reasons mentioned above.

On the other hand, as expected, the MCL produces higher factors. The MCL ultimate projections are usually lower than the ones obtained in the other methods and the standard deviation is not lower enough to compensate, so the ratio – our reserve risk – is higher.

The Passengers data has a smaller size therefore the respective run-off triangle may not be sound enough to produce meaningful results.

Furthermore, most of the accidents result in low costs for the company, while some result in very high costs, consequently the run-off triangle is expected to show more variability.

5.2. Results: One-year Reserve Risk

The one-year reserve risk estimation should be performed using the claims payments run-off triangles for two data sets: vehicle liability and other.

Method 1 was computed as indicated in CEIOPS (2010a), using 8 years of historical data grouped for the two data sets. An example of the data used is illustrated in Annex 9.

However, for Method 2 and 3 a different approach was taken.

Given two claims run-off triangles, their chain-ladder projections only add up to the projections of the combined triangle under some specific conditions discussed by Ajne (1994). Additionally, Ajne (1994) presents sufficient conditions for inequality between the combined projection vector and the sum of the two original projections vectors.

For Motor, others, considering the combined run-off results in approximately the same reserves as adding up the reserves for Own Damage and Passengers. However the same doesn't apply to Motor, vehicle liability, mainly due to the different patterns of Bodily Injury data when compared to Material Damage and IDS data.

Bodily Injury has a longer tail and the payments volume has a smaller weight in the first developments years. This is sufficient condition for the chain-ladder projections of the combined portfolio to be significantly less than the sum of the corresponding projections of the individual data sets (Theorem 2 in Anje (1994)).

For this reason, calculations were performed separately and correlations for the three components of this LoB were estimated in a best approximation possible basis, in order to obtain the overall standard deviation.

The correlations were estimated from the ultimate reserves for each origin year, resulting in correlations of 0.62, 0.07 and -0.43 for BI and MD, BI and IDS and MD and IDS, respectively.

To apply method 2 and 3 it was necessary to calculate the MSEP using the Merz-Wüthrich method (see Annex 10).

While method 3 uses the best estimate for claims outstanding obtained via Chain Ladder – which is to say via Mack's model –, method 2 considers the best estimate obtained using other methods, namely the MCL.

The results follow calculations in section 2.2.3. and are presented below.

Motor, Vehicle Liability	**Method 1**	**Method 2 (MCL)**	**Method 3 (Mack)**
$\sigma_{(U,res,MotorVehicleLiability)}$	9.84%	9.41%	9.30%
credibility factor	0.59	0.92	
$\sigma_{(res,MotorVehicleLiability)}$	**10.32%**	**9.54%**	**9.44%**

Table 3 - One-year reserve risk factors for LoB Motor, Vehicle Liability

Motor, Others	**Method 1**	**Method 2 (MCL)**	**Method 3 (Mack)**
$\sigma_{(U,res,MotorOthers)}$	20.40%	16.88%	15.64%
credibility factor	0.81	1.00	
$\sigma_{(res,MotorOthers)}$	**20.32%**	**16.88%**	**15.64%**

Table 4 - One-year reserve risk factors for LoB Motor, Others

The estimates obtained are to be compared with the standard parameters of 11% and 20% for Motor, Vehicle Liability and Motor, Others, respectively. Since the number of historical data available (8 years for method 1 and 13 for methods 2 and 3) is not yet fully met for all methods and LoBs, the respective credibility factors in CEIOPS (2009a) must be applied.

Method 1 produces higher factors than the other methods, in both cases very close to the standard parameters.

Method 2 and Method 3 origin factors are close and significantly better that the standard parameters, being slightly higher when using MCL best estimates.

As a reference the Portuguese Regulator estimates for the Portuguese undertakings 13.2% for Motor Vehicle Liability and 16.9% for Motor Other, considering a simple average of the estimates obtained for each undertaking, or 10.0% and 12.9% respectively, if considering a weighted average.

In terms of capital requirements (section 3.2.), the impacts are as follows:

	Reserve Risk M.T.L.	Reserve Risk M.Others	**Capital gains**
Standard Parameter	11.00%	20.0%	**-**
USP - Method 1	10.32%	20.3%	**3.3%**
USP - Method 2 (MCL)	9.54%	16.9%	**7.7%**
USP - Method 3 (Mack)	9.44%	15.6%	**8.4%**

Table 5 - Impact of USP in capital charges for the Non-Life module

USP calculated with method 1 has a smaller impact. The factor for Motor, Other is slightly higher than the standard but the one for Motor, Vehicle liability is lower than the standard and the best estimate for claims outstanding weights more in the total best estimate, resulting in a gain of capital of 3.3%.

The gains with method 2 and 3 are considerably higher, representing around 8.4% for method 3 and 7.7% for method 2.

Different correlations were tested. In a pessimistic scenario (0.8 for BI and MD, 0.15 for BI and IDS and independence for MD and IDS) the capital gains were 6.2% and 6.9% for Method 2 and Method 3, respectively. An optimistic scenario of independence results in capital gains of 11.2% and 11.8%, respectively.

6. CONCLUSIONS AND FURTHER DEVELOPMENTS

The aim of this work was to understand the impact that the undertaking specific parameters may have in Solvency II capital requirements for LoBs Motor Vehicle Liability and Motor Others.

The Directive and all the documentation supporting ORSA and the USP confirmed the complexity and extent of this project.

The literature on claims reserving is much diversified but due to time limitations, it was necessary to focus on a restrict number of methods. Therefore the methods more commonly used and more explored were the ones selected.

The results obtained seem to support the intuitive idea that using USP actually results in capital gains for the company, however the use of an USP has to go over an approval process from the regulator. The company must select the method that believes to be more adequate to its own data and the selection of a particular method has to be explained to the regulator. The regulator needs evidence that the USP better reflects the company's risk profile.

Across this work some aspects had to be simplified, however they should be analysed more carefully in the future. Further developments for this work would be:

1. To consider a tail in the run-off triangles and in the reserve risk estimation. In this work, it was assumed that the tail has the same estimated variability;
2. To develop methodologies similar to Merz-Wüthrich method, but considering claims projections with models other than Chain Ladder.

ANNEXES

Annex 1. CorrLob – Matrix of correlations between LoBs

CorrLob	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
1: Motor vehicle liability	*100%*	*50%*	*50%*	*25%*	*50%*	*25%*	*50%*	*25%*	*50%*	*25%*	*25%*	*25%*
2: Other motor	*50%*	*100%*	*25%*	*25%*	*25%*	*25%*	*50%*	*50%*	*50%*	*25%*	*25%*	*25%*
3: MAT	*50%*	*25%*	*100%*	*25%*	*25%*	*25%*	*25%*	*50%*	*50%*	*25%*	*25%*	*50%*
4: Fire	*25%*	*25%*	*25%*	*100%*	*25%*	*25%*	*25%*	*50%*	*50%*	*50%*	*25%*	*50%*
5: 3rd party liability	*50%*	*25%*	*25%*	*25%*	*100%*	*50%*	*50%*	*25%*	*50%*	*25%*	*50%*	*25%*
6: Credit	*25%*	*25%*	*25%*	*25%*	*50%*	*100%*	*50%*	*25%*	*50%*	*25%*	*50%*	*25%*
7: Legal exp.	*50%*	*50%*	*25%*	*25%*	*50%*	*50%*	*100%*	*25%*	*50%*	*25%*	*50%*	*25%*
8: Assistance	*25%*	*50%*	*50%*	*50%*	*25%*	*25%*	*25%*	*100%*	*50%*	*50%*	*25%*	*25%*
9: Miscellaneous.	*50%*	*50%*	*50%*	*50%*	*50%*	*50%*	*50%*	*50%*	*100%*	*25%*	*25%*	*50%*
10:Np reins. (property)	*25%*	*25%*	*25%*	*50%*	*25%*	*25%*	*25%*	*50%*	*25%*	*100%*	*25%*	*25%*
11:Np reins. (casualty)	*25%*	*25%*	*25%*	*25%*	*50%*	*50%*	*50%*	*25%*	*25%*	*25%*	*100%*	*25%*
12:Np reins. (MAT)	*25%*	*25%*	*50%*	*50%*	*25%*	*25%*	*25%*	*25%*	*50%*	*25%*	*25%*	*100%*

Source: QIS5 Calibration Paper – CEIOPS (2010c) – page 354/384[1]

[1] CEIOPS has also published a calibration paper which includes a description on the derivation of these correlations, which is available on CEIOPS' website under http://www.ceiops.eu/media/files/consultations/QIS/QIS3/QIS3CalibrationPapers.pdf

Annex 2. Using the R package 'ChainLadder'

```
#################### ChainLadder Package#########################
suppressPackageStartupMessages(library(ChainLadder))
#READING PAYMENTS
#In order to preserve the confidentiality of the data the paid and incurred triangles here
#presented correspond to the data use by Quarg and Mack (2004)
PAID=matrix(c(576,1804,1970,2024,2074,2102,2131,
        866,1948,2162,2232,2284,2348,NA,
        1412,3758,4252,4416,4494,NA,NA,
        2286,5292,5724,5850,NA,NA,NA,
        1868,3778,4648,NA,NA,NA,NA,
        1442,4010,NA,NA,NA,NA,NA,
        2044,NA,NA,NA,NA,NA,NA),nrow=7,ncol=7,byrow=TRUE)
#READING INCURRED COSTS
INC=matrix(c(978,2104,2134,2144,2174,2182,2174,
        1844,2552,2466,2480,2508,2454,NA,
        2904,4354,4698,4600,4644,NA,NA,
        3502,5958,6070,6142,NA,NA,NA,
        2812,4882,4852,NA,NA,NA,NA,
        2642,4406,NA,NA,NA,NA,NA,
        5022,NA,NA,NA,NA,NA,NA),nrow=7,ncol=7,byrow=TRUE)
#Mack Model
Mack <- MackChainLadder(Triangle=PAID, est.sigma="Mack")
write.table(Mack$FullTriangle , file="MackFullTriangle.csv" , sep=";")       #Ultimate projections
write.table(Mack$Mack.S.E , file="MackSE.csv",sep=";")                       #Standard Error
plot(Mack, lattice=TRUE)
#Munich Chain Ladder Model
MCL <- MunichChainLadder(PAID,INC, est.sigmaP="Mack",est.sigmaI="Mack")
write.table(MCL$MCLPaid , file="MCLDCPaid.csv" , sep=";")              #Ult. projections Paid
write.table(MCL$MCLIncurred , file="MCLDCIncurred.csv" , sep=";")    #Ult. projections Incurred
plot(MCL)        #Results and paid/incurred residuals regression
#Bootstrap for Chain Ladder
Boot <- BootChainLadder(PAID,R=10000,process.distr=c("od.pois"))
write.table(Boot$IBNR.Totals , file="BootIBNR.csv" , sep=";")                #IBNR projections
plot(BootDC)
```

Annex 3. Bootstrap for Munich Chain Ladder

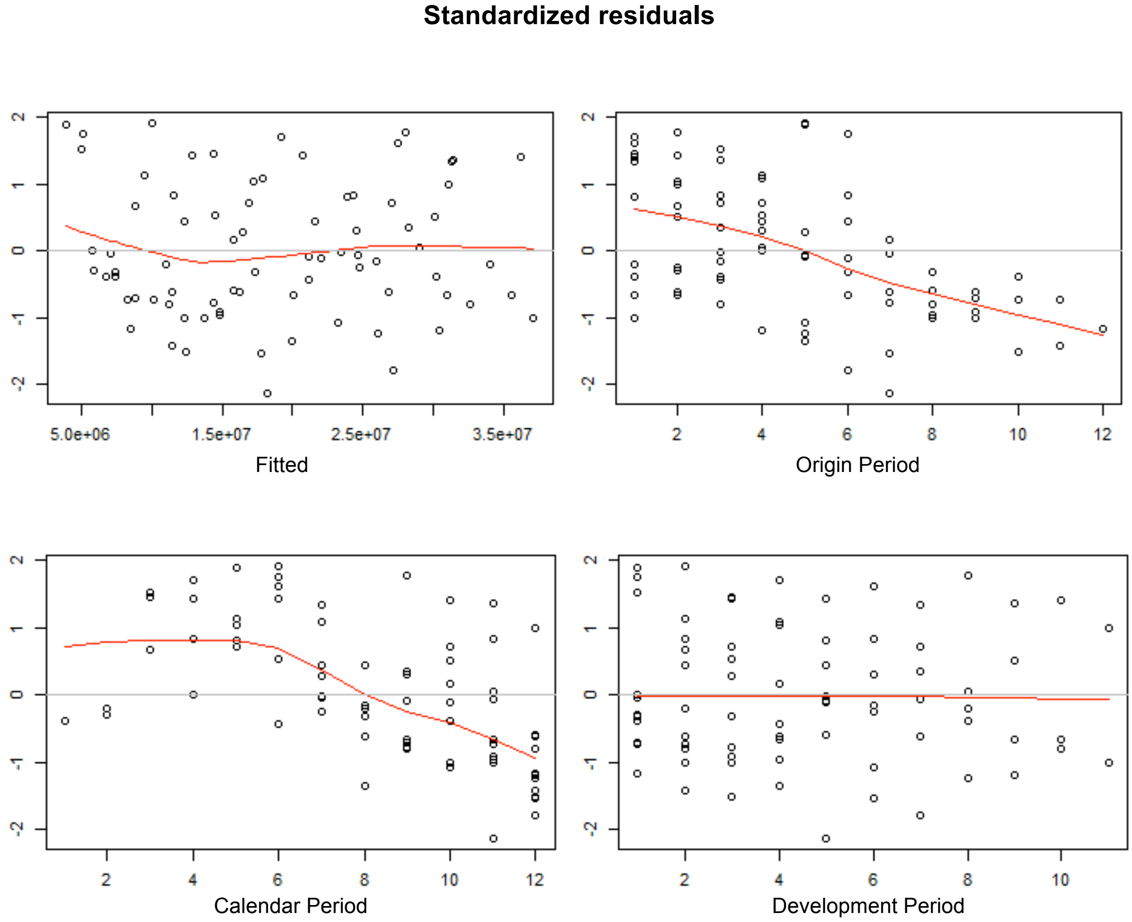
```
#################### Bootstrap for Munich Chain Ladder ##########################
#Continuing from Annex 2
nr=nrow(PAID);nc=ncol(PAID)
#1-period factors, Q and Qinverse
FP<-matrix(NA,nrow=nr,ncol=nc)
for(j in 1:nc) FP[,(j-1)]=PAID[,j]/PAID[,(j-1)]
FI<-matrix(NA,nrow=nr,ncol=nc)
for(j in 1:nc) FI[,(j-1)]=INC[,j]/INC[,(j-1)]
Q<-PAID/INC
QInv<-1/Q
############## Bootstrap  ####################
##### 0. Obtaining the 4 residuals #####
PRes<-t(matrix(MCL$PaidResiduals,ncol=ncol(PAID),byrow=TRUE))
IRes<-t(matrix(MCL$IncurredResiduals,ncol=ncol(PAID),byrow=TRUE))
QRes<-t(matrix(MCL$QResiduals,ncol=ncol(PAID),byrow=TRUE))
QInvRes<-t(matrix(MCL$QinverseResiduals,ncol=ncol(PAID),byrow=TRUE))
##### 1. Adjust the Pearson Residuals #####
#Calculating the adjustmet factors
Adjust <- matrix(0,ncol=nr, nrow=nc)
for( j in 1:(nr-2) ) Adjust[,j] <- sqrt((nc-j)/(nc-j-1))
#Obtaining the adjusted residuals
AdjPRes <- PRes*Adjust
AdjIRes <- IRes*Adjust
AdjQRes <- QRes*Adjust
AdjQInvRes <- QInvRes*Adjust
for (i in 1:nr){AdjQRes[i,(nr-i+1)]<-NA ;AdjQInvRes[i,(nr-i+1)]<-NA}
##### 2. Grouping the residuals #####
auxP <-matrix(AdjPRes[(AdjPRes!=0 & !is.na(AdjPRes)) ],ncol=1,byrow=TRUE)
auxI <-matrix(AdjIRes[(AdjIRes!=0 & !is.na(AdjIRes))],ncol=1,byrow=TRUE)
auxQ <-matrix(AdjQRes[(AdjQRes!=0 & !is.na(AdjQRes))],ncol=1,byrow=TRUE)
auxQInv <-matrix(AdjQInvRes[(AdjQInvRes!=0 & !is.na(AdjQInvRes))],ncol=1,byrow=TRUE)
AllRes<-cbind(auxP,auxI,auxQInv,auxQ)
##### 3. LOOP #####
Nboot<-10000
originP<-matrix(NA,nrow=Nboot,ncol=nc)
originI<-matrix(NA,nrow=Nboot,ncol=nc)
totalP<-matrix(NA,nrow=Nboot,ncol=1)
totalI<-matrix(NA,nrow=Nboot,ncol=1)
for (N in 1:Nboot){
        #Obtaining bootstrap residuals
 nres <- nrow(AllRes)
        nsam <- (nc-1)*nc/2
        auxiliar <- sample(1:nres,nsam,replace=T)
        ItRes<-matrix(0, nrow=nsam, ncol=4)
 for(i in 1:nsam){
                ItRes[i,]<-AllRes[auxiliar[i],]
        }
 #transforming bootstrap residuals vectors in matrixs
        MatResP<-matrix(NA, nrow=nr, ncol=nc)
        MatResI<-matrix(NA, nrow=nr, ncol=nc)
        MatResQ<-matrix(NA, nrow=nr, ncol=nc)
        MatResQInv<-matrix(NA, nrow=nr, ncol=nc)
        for(j in 1:nc){
```

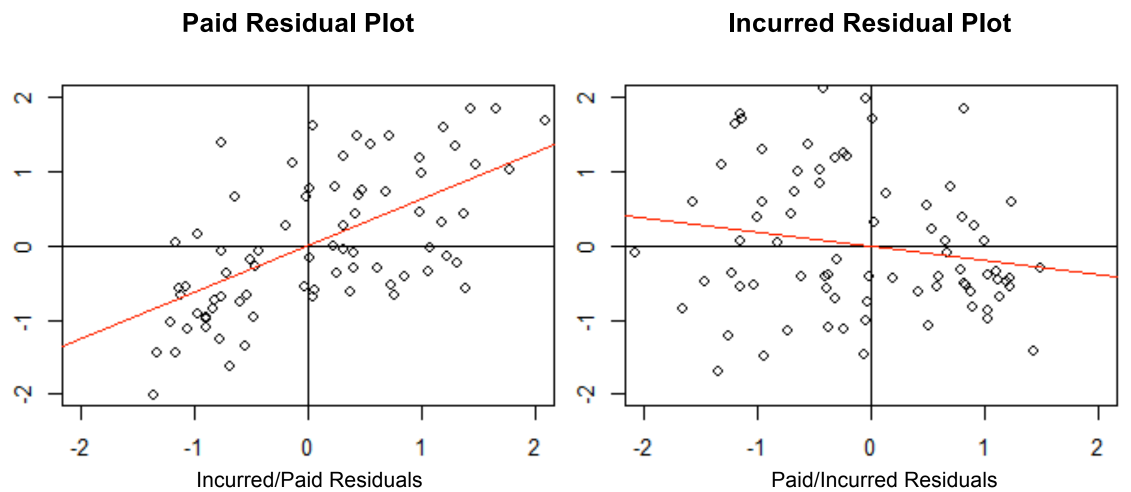
```
		i<-1
		while(i<=nc-j){
		MatResP[i,j]<-ItRes[nsam-((nc+1-j)*(nc-j)/2)+i,1]
		MatResI[i,j]<-ItRes[nsam-((nc+1-j)*(nc-j)/2)+i,2]
		MatResQInv[i,j]<-ItRes[nsam-((nc+1-j)*(nc-j)/2)+i,3]
		MatResQ[i,j]<-ItRes[nsam-((nc+1-j)*(nc-j)/2)+i,4]
		i<-i+1
		}
	}
	#obtaining boostrap increment ratios
	MatRatiosP<-matrix(NA, nrow=nr, ncol=nc)
	MatRatiosI<-matrix(NA, nrow=nr, ncol=nc)
	MatRatiosQ<-matrix(NA, nrow=nr, ncol=nc)
	MatRatiosQInv<-matrix(NA, nrow=nr, ncol=nc)
for(j in 1:(nc-1)){
	i<-1
	while(i<=nc-j){
	MatRatiosP[i,j]<-(MatResP[i,j]*Mack$sigma[j])/sqrt(PAID[i,j])+Mack$f[j]
	MatRatiosI[i,j]<-(MatResI[i,j]*MackInc$sigma[j])/sqrt(INC[i,j])+MackInc$f[j]
	MatRatiosQ[i,j]<-(MatResQ[i,j]*MCL$rhoI.sigma[j])/sqrt(INC[i,j])+MCL$q.f[j]
MatRatiosQInv[i,j]<- (MatResQInv[i,j]*MCL$rhoP.sigma[j])/sqrt(PAID[i,j])+MCL$qinverse.f[j]
	i<-i+1
	}
 }
 #obtaining bootstrap development factors
BootfP<-rep(0,nc);BootfI<-rep(0,nc);BootfQ<-rep(0,nc); BootfQInv<-rep(0,nc)
	sumP<-rep(NA,nc);sumI<-rep(NA,nc)
	for(j in 1:(nc-1)){
		sumP[j]<-colSums(PAID,na.rm=TRUE)[j]-PAID[nc-j+1,j]
		sumI[j]<-colSums(INC,na.rm=TRUE)[j]-INC[nc-j+1,j]
		i<-1
		while(i<=nc-j){
			BootfP[j]<- BootfP[j]+(PAID[i,j]/sumP[j])*MatRatiosP[i,j]
			BootfI[j]<- BootfI[j]+(INC[i,j]/sumI[j])*MatRatiosI[i,j]
			BootfQ[j]<- BootfQ[j]+(INC[i,j]/sumI[j])*MatRatiosQ[i,j]
			BootfQInv[j]<- BootfQInv[j] + (PAID[i,j]/sumP[j])*MatRatiosQInv[i,j]
			i<-i+1
		}
	}
#obtaining the bootstrap CORRELATION COEFFICIENTS
LambdaP<-  sum(MatResQInv*MatResP,na.rm=TRUE)/sum(MatResQInv^2,na.rm=TRUE)
LambdaI<-sum(MatResQ*MatResI,na.rm=TRUE)/sum(MatResQ^2,na.rm=TRUE)
	#Obtaining the VARIANCES
	VarP<-rep(0,nc); VarI<-rep(0,nc);
 for(j in 1:(nc-2)){
		i<-1
		while(i<=nc-j){
			VarP[j]<-VarP[j]+(PAID[i,j]*(MatRatiosP[i,j]-BootfP[j])^2)/(nc-j-1)
			VarI[j]<-VarI[j]+(INC[i,j]*(MatRatiosI[i,j]-BootfI[j])^2)/(nc-j-1)
			i<-i+1
		}
	}
	VarQ<-rep(0,nc); VarQInv<-rep(0,nc)
	for(j in 1:(nc-1)){
		i<-1
		while(i<=nc-j){
			VarQ[j]<-VarQ[j]+(INC[i,j]*(MatRatiosQ[i,j]-BootfQ[j])^2)/(nc-j)
VarQInv[j]<-VarQInv[j]+(PAID[i,j]*(MatRatiosQInv[i,j]-BootfQInv[j])^2)/(nc-j)
```

```
                    i<-i+1
            }
      }
sigmaP<-sqrt(VarP)
      sigmaI<-sqrt(VarI)
      tauI<-sqrt(VarQ)
      tauP<-sqrt(VarQInv)
      #estimating the last ratio sigma/tau
sigtauP<-sigmaP/tauP
      sigtauI<-sigmaI/tauI
      period<-c(1:nc)
      fitP<-lm(log(sigtauP) ~ period,na.action=na.exclude)
      fitI<-lm(log(sigtauI) ~ period,na.action=na.exclude)
      sigtauP[nc-1]<-exp((nc-1)*fitP$coefficients[2]+fitP$coefficients[1])
      sigtauI[nc-1]<-exp((nc-1)*fitI$coefficients[2]+fitI$coefficients[1])
      alphaP<-pmax(0,pmin(LambdaP*(sigtauP),0.99))
      alphaI<-pmax(0,pmin(LambdaI*(sigtauI),0.99))
#Obtaining the bootstrap ADJUSTED DEVELOPMENT FACTORS
      BlambdaP<-matrix(NA, nrow=nr, ncol=nc)
      BlambdaI<-matrix(NA, nrow=nr, ncol=nc)
 BPAID<-PAID; BINC<-INC; BRatiosQ<-MatRatiosQ; BRatiosQInv<-MatRatiosQInv
      for(k in 1:(nc-1)){
            j<-k
            while(j<=(nc-1)){
BlambdaP[nr-j+k,j]<-BootfP[j]+alphaP[j]*(BINC[nr-j+k,j]/BPAID[nr-j+k,j]-BootfQInv[j])
BlambdaI[nr-j+k,j]<-BootfI[j]+alphaI[j]*(BPAID[nr-j+k,j]/BINC[nr-j+k,j]-BootfQ[j])
                    BPAID[nr-j+k,j+1]<-BlambdaP[nr-j+k,j]*BPAID[nr-j+k,j]
                    BINC[nr-j+k,j+1]<-BlambdaI[nr-j+k,j]*BINC[nr-j+k,j]
                    j<-j+1
            }
      }
#Obtaining normal distributed observations
      BPAIDfinal<-PAID; BINCfinal<-INC
      for(k in 1:(nc-1)){
            j<-k
            while(j<=(nc-1)){
                    meanP<-BlambdaP[nr-j+k,j]*BPAIDfinal[nr-j+k,j]
      sdP<-sqrt(VarP[j]*BPAIDfinal[nr-j+k,j])
                    meanI<-BlambdaI[nr-j+k,j]*BINCfinal[nr-j+k,j]
                    sdI<-sqrt(VarI[j]*BINCfinal[nr-j+k,j])
                    BPAIDfinal[nr-j+k,j+1]<-rnorm(1,mean=meanP,sd=sdP)
                    BINCfinal[nr-j+k,j+1]<-rnorm(1,mean=meanI,sd=sdI)
                    j<-j+1
            }
      }
#Obtaining the origin year and the total amounts
      originP[N,]<-BPAIDfinal[,nc]
originI[N,]<-BINCfinal[,nc]
totalP[N] <- colSums(BPAIDfinal)[nc]
      totalI[N] <- colSums(BINCfinal)[nc]
}
#averages
averP<-colMeans(originP)
averI<-colMeans(originI)
taverP<-mean(totalP)
taverI<-mean(totalI)
#reserves
reservesP<-averP-c(2131,2348,4494,5850,4648,4010,2044)
```

```
reservesI<-averI-c(2131,2348,4494,5850,4648,4010,2044)
totreserveP<-sum(reservesP)
totreserveI<-sum(reservesI)
#standard deviations
stdP<-rep(NA,nrow=1,ncol=(nc-1))
for(j in 1:nc-1)stdP[j]=sd(originP[,j+1])
stdI<-rep(NA,nrow=1,ncol=(nc-1))
for(j in 1:nc-1)stdI[j]=sd(originI[,j+1])
totstdP<-sd(totalP)
totstdI<-sd(totalI)
```

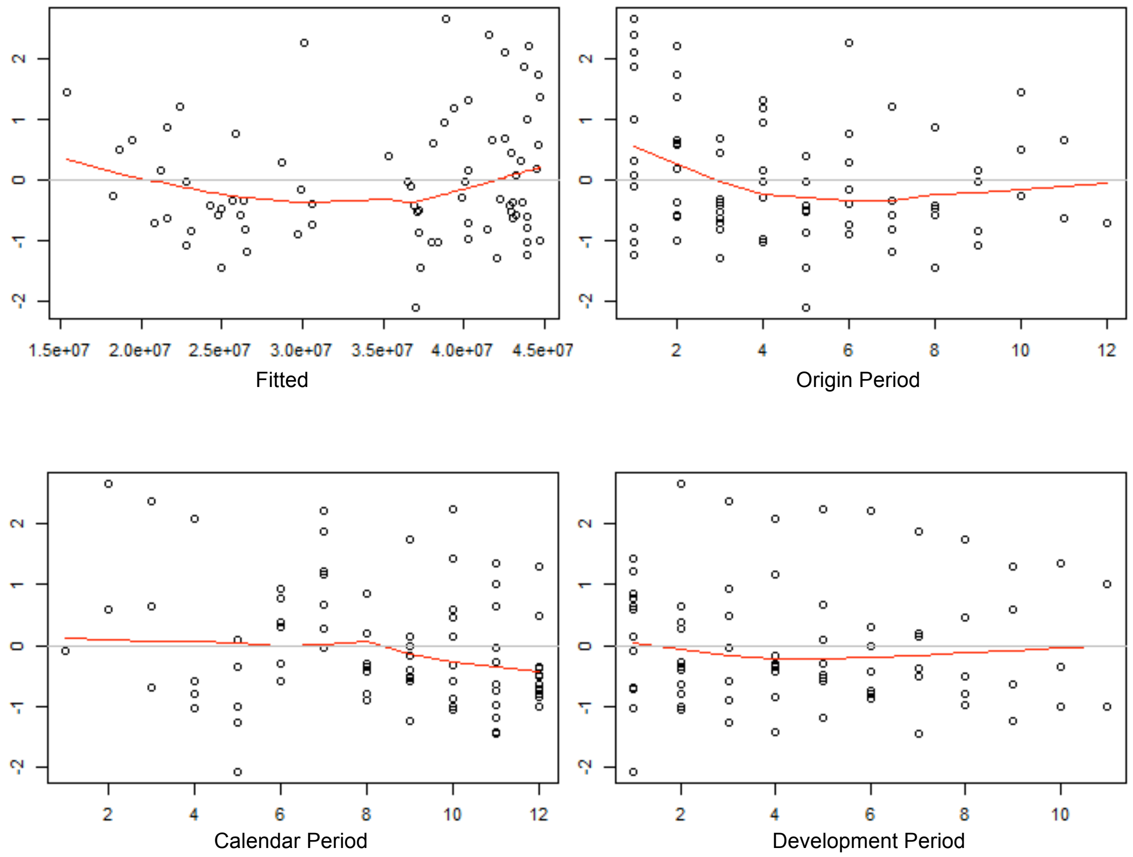
Annex 4. Plots for Bodily Injury



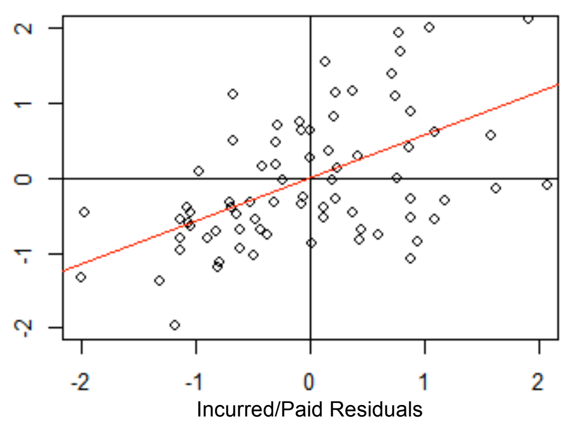


Annex 5. Plots for Material Damage

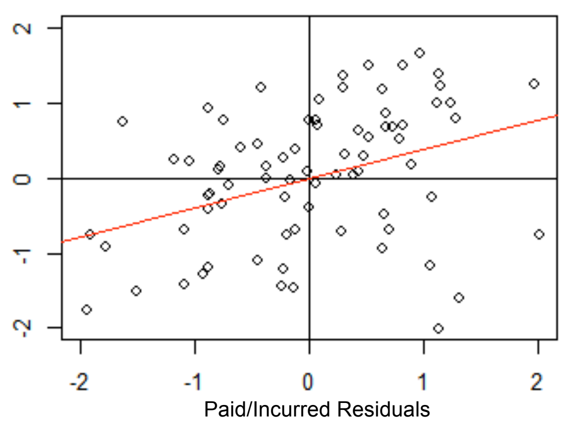
Standardized residuals



Paid Residual Plot

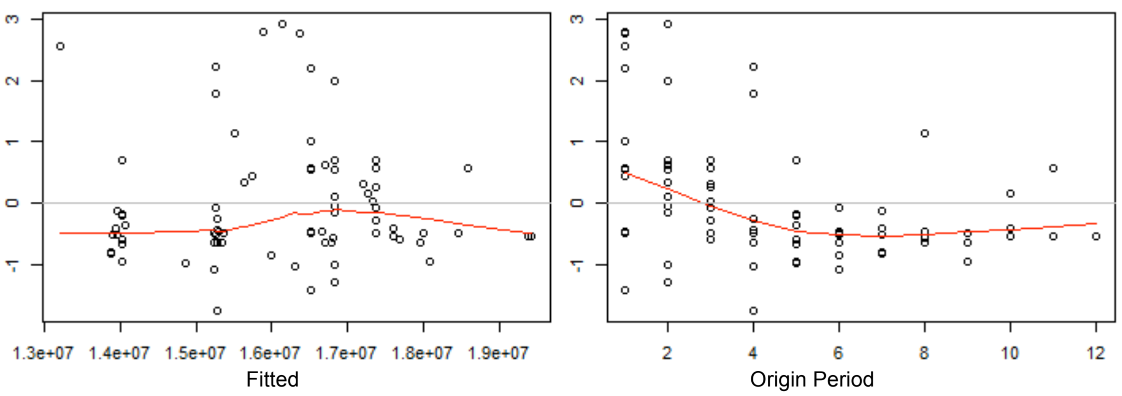


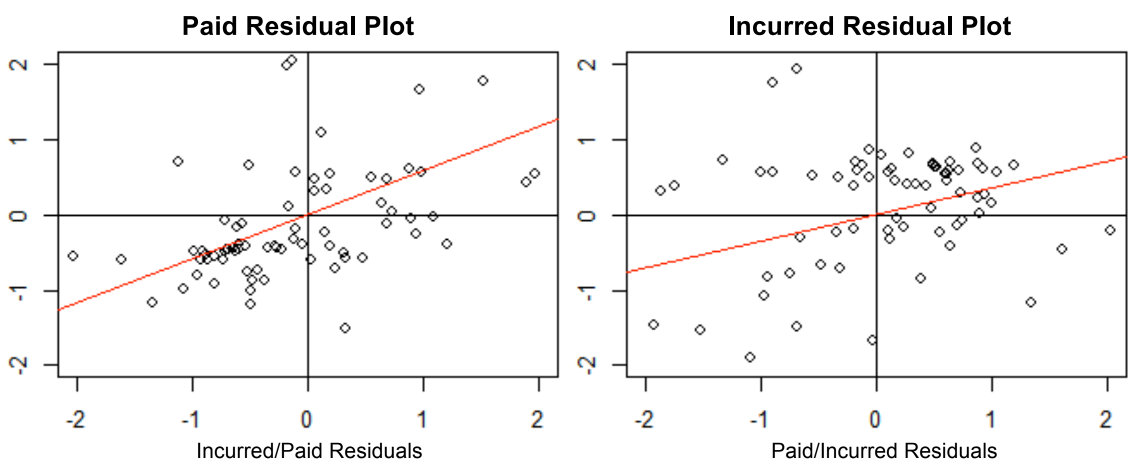
Incurred Residual Plot



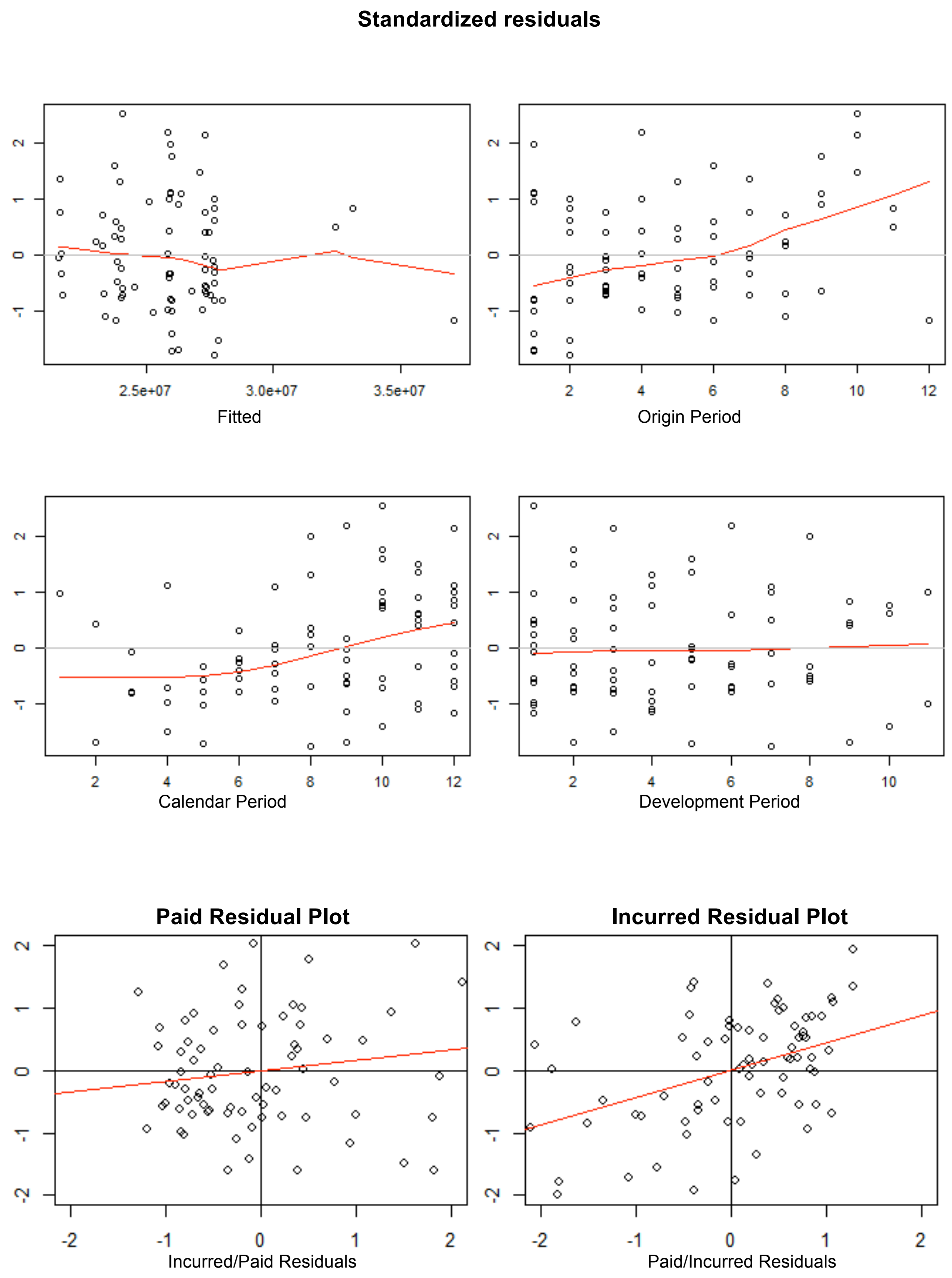
Annex 6. Plots for IDS

Standardized residuals

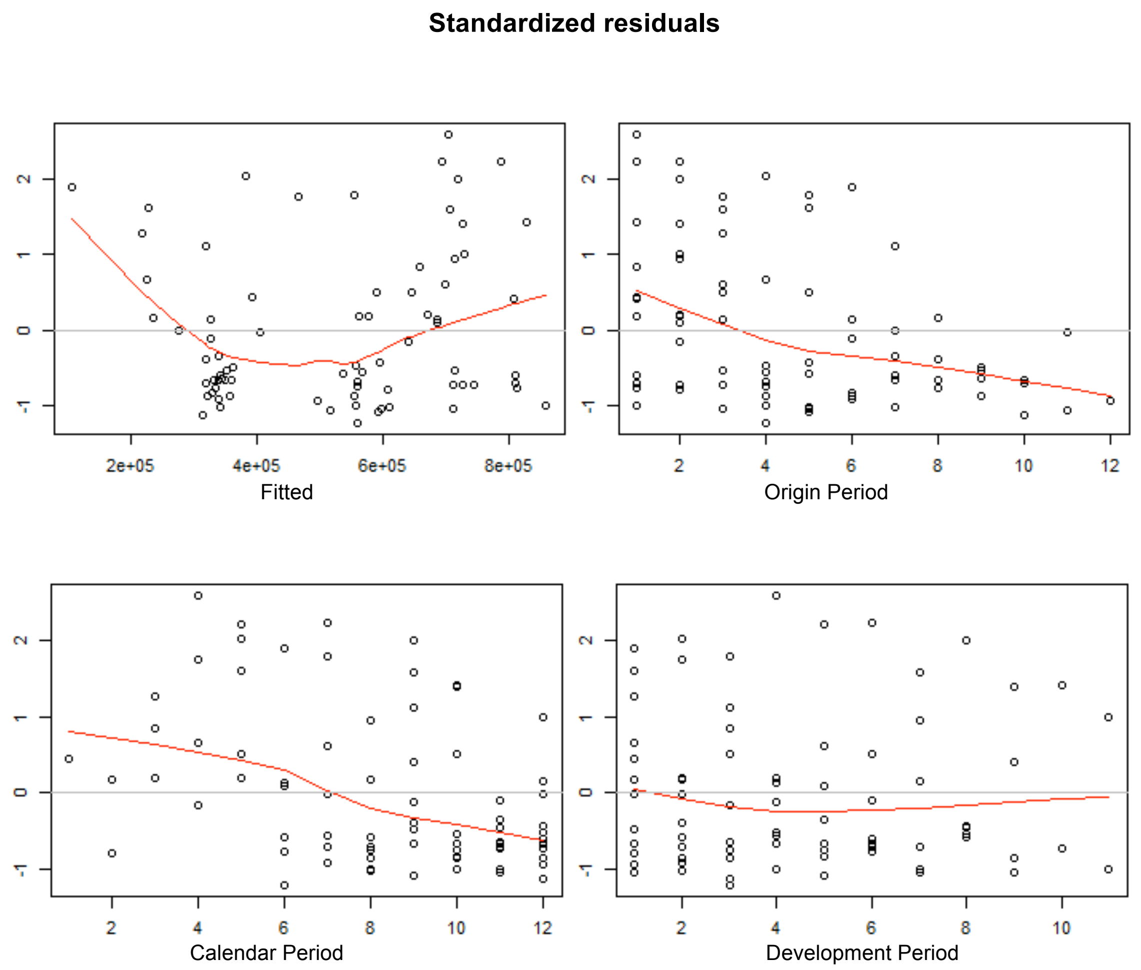


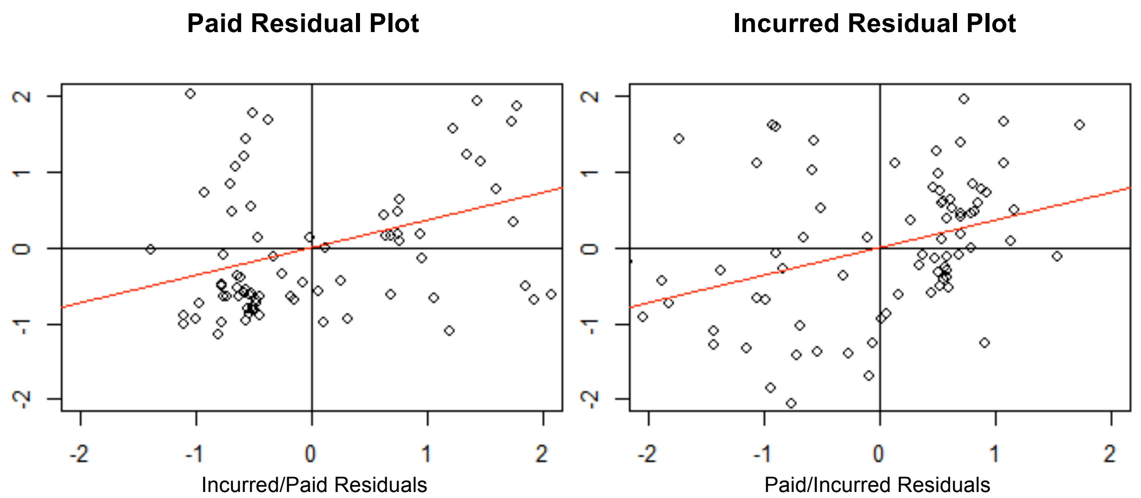


Annex 7. Plots for Own Damage



Annex 8. Plots for Passengers





Annex 9. Example for Method 1

Reserves Y

Origin \ Devel.	0	1	2	3	4	5	6	7	8	9	>
											19
1995										6	16
1996									13	9	15
1997								21	18	15	17
1998							31	22	15	10	44
1999						40	32	22	11	16	51
2000					55	44	32	20	28	23	59
2001				67	53	40	31	34	26	22	71
2002			106	83	73	63	46	37	33	22	
2003		132	111	92	79	52	43	26	21		
2004	177	132	106	87	73	65	37	26			
2005	188	121	97	72	63	33	21				
2006	197	143	99	67	38	36					
2007	198	135	102	77	54						
2008	191	120	92	51							
2009	140	86	57								
2010	126	84									
2011	123										
2012											

$V_{Y,lob}$

$R_{Y,lob}$

Reserves Y+1 + **Payments Y+1**

Reserves Y+1

Origin \ Devel.	0	1	2	3	4	5	6	7	8	9	>
											16
1996										9	15
1997									18	15	17
1998								22	15	10	44
1999							32	22	11	16	51
2000						44	32	20	28	23	59
2001					53	40	31	34	26	22	71
2002				83	73	63	46	37	33	22	57
2003			111	92	79	52	43	26	21	18	
2004		132	106	87	73	65	37	26	20		
2005		121	97	72	63	33	21	13			
2006		143	99	67	38	36	27				
2007		135	102	77	54	39					
2008		120	92	51	42						
2009		86	57	40							
2010		84	62								
2011		79									
2012											

Payments Y+1

Origin \ Devel.	0	1	2	3	4	5	6	7	8	9	>
											9
1996										3	10
1997									3	3	13
1998								10	7	5	18
1999							10	11	10	8	11
2000						13	13	12	6	4	12
2001					13	13	9	6	7	5	8
2002				19	10	10	8	8	5	5	8
2003			21	18	14	11	9	9	5	3	
2004		57	27	18	7	8	5	6	3		
2005		71	30	17	9	10	10	5			
2006		60	18	10	9	0	3				
2007		62	15	9	5	5					
2008		58	14	10	6						
2009		54	13	6							
2010		58	12								
2011		49									
2012											

Annex 10. Bootstrap for Merz-Wüthrich

```
################### Bootstrap for Merz-Wüthrich ##############################
diagonal<-function(M){
	nc<-ncol(M)
	diag<-rep(NA,nc)
	for(j in 1:nc) diag[j]<-M[nc-j+1,j]
	return(diag)
}
nr<-nrow(PAID);nc<-ncol(PAID); last<-diagonal(PAID)
#Obtaining individual factors and CL Factors
F<-matrix(NA,nrow=nr,ncol=nc)
for(j in 1:nc) F[,(j-1)]=PAID[,j]/PAID[,(j-1)]
sumcol <- colSums(PAID,na.rm=TRUE)
CLfactor<- rep(NA,(nc-1))
for(j in 1:(nc-1)) CLfactor[j]<-sumcol[j+1]/( sumcol[j]-last[j])
###Obtaining sigma and beta weights
nobs<-rep(NA,nc)
for(j in 1:nc) nobs[j]<-nc-j+1
auxsigma<-matrix(NA,nrow=nr,ncol=nc)
for(j in 1:(nc-1)) auxsigma[,j]<-PAID[,j]*(F[,j]-CLfactor[j])^2
sigma<-sqrt((colSums(auxsigma,na.rm=TRUE))/(nobs-2))
sigma[nc-1]<-min(sigma[nc-2],sigma[nc-3],sigma[nc-2]^2/sigma[nc-3])
beta<-last/sumcol
###Obtaining ultimate and reserves
predPayments<-PAID
for(j in 2:nc){
	i<-nc-j+2
	while (i <=nc){
	predPayments[i,j]<-predPayments[i,j-1]*CLfactor[j-1]
	i<-i+1
	}
}
ultimate<-predPayments[,nc]
reserves<-rep(NA,nc)
for (j in 1:nc) reserves[j]<-ultimate[j]-last[nc-j+1]
###Obtaining process variance
auxvar<-matrix(0,nrow=nr,ncol=nc)
for(j in 2:nc){
	i<-nc-j+2
	while (i <=nc){
	auxvar[i,j]<-(sigma[j-1]/CLfactor[j-1])^2/predPayments[i,j-1]
	i<-i+1
	}
}
proc_var<-sqrt(rowSums(auxvar)*(ultimate^2))
proc_var_tot<-sqrt(sum(rowSums(auxvar)*(ultimate^2)))
###Obtaining estimation total error
auxerror<-matrix(0,nrow=nr,ncol=nc)
for(j in 2:nc){
	i<-nc-j+2
	while (i <=nc){
	auxerror[i,j]<-(sigma[j-1]/CLfactor[j-1])^2/(sumcol[j-1]-last[j-1])
	i<-i+1
	}
}
```

```
auxcov<-matrix(0,nrow=nr,ncol=nc)
for(j in 3:nc){
        i<-2
        while (i < j){
        auxcov[i,j]<-rowSums(auxerror)[i]*ultimate[j]*ultimate[i]
        i<-i+1
        }
}
sqrtCov<-sqrt(2*sum(auxcov))
est_error<-sqrt(rowSums(auxerror))*ultimate
est_err_tot<-sqrt(sum(est_error^2)+sqrtCov^2)
###Obtaining mse Mack
sqrmsep<-sqrt(proc_var^2+est_error^2)
sqrmseptot<-sqrt(proc_var_tot^2+est_err_tot^2)
mse<-sqrmsep^2
msetot<-sqrmseptot^2
###Obtaining process variance CDR
auxvarCDR<-matrix(0,nrow=nr,ncol=nc)
for(j in 2:nc) auxvarCDR[nc-j+2,j]<-auxvar[nc-j+2,j]
proc_varCDR<-sqrt(rowSums(auxvarCDR)*(ultimate^2))
proc_var_totCDR<-sqrt(sum(rowSums(auxvarCDR)*(ultimate^2)))
###Obtaining estimation total error CDR
auxerrorCDR<-matrix(0,nrow=nr,ncol=nc)
for(j in 2:nc) auxerrorCDR[nc-j+2,j]<-auxerror[nc-j+2,j]
for(j in 3:nc){
        i<-nc-j+3
        while (i <=nc){
        auxerrorCDR[i,j]<-auxerror[i,j]*beta[j-1]
        i<-i+1
        }
}
auxcovCDR<-matrix(0,nrow=nr,ncol=nc)
for(j in 3:nc){
        i<-2
        while (i < j){
        auxcovCDR[i,j]<-rowSums(auxerrorCDR)[i]*ultimate[j]*ultimate[i]
        i<-i+1
        }
}
sqrtCovCDR<-sqrt(2*sum(auxcovCDR))
est_error_CDR<-sqrt(rowSums(auxerrorCDR))*ultimate
est_err_tot_CDR<-sqrt(sum(est_error_CDR^2)+sqrtCovCDR^2)
###Obtaining mse_0
sqrmsep0<-sqrt(proc_varCDR^2+est_error_CDR^2)
sqrmseptot0<-sqrt(proc_var_totCDR^2+est_err_tot_CDR^2)
mse0<-sqrmsep0^2;msetot0<-sqrmseptot0^2
last<-union(last,sum(last))
ultimate<-union(ultimate,sum(ultimate))
reserves<-union(reserves,sum(reserves))
PrVarTot<-union(proc_var,proc_var_tot)
EsErrorTot<-union(est_error,est_err_tot)
MSEPTot<-union(sqrmsep,sqrmseptot)
PrVarCDR<-union(proc_varCDR,proc_var_totCDR)
EsErrorCDR<-union(est_error_CDR,est_err_tot_CDR)
MSEPCDR<-union(sqrmsep0,sqrmseptot0)
output<-
cbind(last,ultimate,reserves,PrVarTot,EsErrorTot,MSEPTot,PrVarCDR,EsErrorCDR,MSEPCDR
)
```

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