Instituto Superior de Economia e Gestão
UNIVERSIDADE TÉCNICA DE LISBOA
DESDE 1911

MESTRADO

ECONOMIA MONETÁRIA E FINANCEIRA

TRABALHO FINAL DE MESTRADO

DISSERTAÇÃO

RESPONSES OF INFLATION AND OUTPUT TO MONETARY SHOCKS IN A BAUMOL-TOBIN MODEL

THOMAS JOEL VERHEIJ

SETEMBRO - 2012

Instituto Superior de Economia e Gestão
UNIVERSIDADE TÉCNICA DE LISBOA
DESDE 1911

MESTRADO EM
ECONOMIA MONETÁRIA E FINANCEIRA

TRABALHO FINAL DE MESTRADO
DISSERTAÇÃO

RESPONSES OF INFLATION AND OUTPUT TO MONETARY SHOCKS IN A BAUMOL-TOBIN MODEL

THOMAS JOEL VERHEIJ

ORIENTAÇÃO:

BERNARDINO ADÃO, BANCO DE PORTUGAL

SETEMBRO - 2012

Responses of Inflation and Output to Monetary Shocks in a Baumol-Tobin Model

Thomas Verheij*†

Instituto Superior de Economia e Gestão,

Universidade Técnica de Lisboa

Banco de Portugal

Supervisor: Bernardino Adão

Banco de Portugal

27th September 2012

*I am very grateful to Bernardino Adão for all his support and comments. I am also very grateful to Banco de Portugal for providing all the necessary conditions, within the program "Estágios MEMF – Banco de Portugal", to make this dissertation possible. I thank Isabel Horta Correia, André Silva and Pedro Teles for their comments. I also thank Miguel Seixas and Gerrit Verheij for their very usefull suggestions.

†G8

Abstract

The question of how monetary policy affects the main economic variables remains one of the most important questions of the economic literature. With this dissertation I will try to contribute to the literature to answer this question. I will create a general equilibrium model with market segmentation based on the model of Alvarez et al (2009). The agents of the model will make transactions between money and bonds every N periods. The money is needed to buy goods but does not receive interest. The novelty of my model is that production will be endogenous. I will introduce a shock to the nominal interest rate and obtain the responses of inflation and output. The main conclusions are twofold. In the first place, I obtain that the shock to the nominal interest rate has real effects because inflation responds sluggishly. In the second place, I obtain that the response of inflation changes significantly when production is endogenous instead of exogenous.

JEL Codes: E3, E4, E5

Keywords: cash-in-advance models, market segmentation, interest rate shocks

Contents

1 Introduction

The question of how monetary policy affects the main economic variables remains one of the most important questions of the economic literature. Since Hume (1752) and Wicksell (1898) has there been written about money, prices and the effect of monetary policy on these variables. Today it is widely accepted that monetary policy is neutral in the long run. This means that it does not affect real economic variables in the long run. There also exist some consensus that monetary policy can affect real variables in the short run. However the way the main marcoeconomic variables respond in the short run to monetary policy is a question to which still does not exist one unique answer.[1] With this dissertation I will try to contribute to the literature to answer this question.

The conventional way to try to understand the response of the main macroeconomic variables to monetary policy is to create a general equilibrium model where monetary policy is represented by a shock to some monetary variable. Today it is usually to identify a monetary policy shock as a shock to the nominal interest rate, instead of using a shock to the money supply. To be able to obtain some short run real response of the economy mainly two types of models are used in the literature. The first type of models is based on nominal rigidities. The second type are models including market segmentation. This dissertation will be part of the second type of literature.

I will create a general equilibrium model with market segmentation based on the model of Alvarez et al (2009). The main goal of my dissertation is to analyze the

[1] For a review of this issue and others issues related to monetary policy see Walsh (2010).

strength of the results presented by them. The main difference between the model I use and the model of Alvarez et al (2009) is that here production will be endogenous, while they use an endowment economy. This way I will try to understand in which way making production endogenous changes the response of inflation to monetary policy in this type of models. I also analyze how output is affected by monetary policy in a model with market segmentation. Further, I use a different and simple, non linear way to solve the equilibrium response of the model to the shock of the nominal interest rate.

My dissertation will be organized in the following way. In the rest of the first section I will introduce the literature used for my dissertation and present the main results of this dissertation. In the second section I introduce the model, explaining the behavior of the agents of the model; the households, the firms and the government. In the thirth section I present the steady state equilibrium of the model. I will explain the calibration used and show the behavior of the agents in the steady state of the economy. The forth section is about the monetary policy shock. I will explain the method used to solve the equilibrium response of the model to the shock and present the response of the main variables in an endowment economy and in a production economy. Finally, I will conclude in section five.

1.1 Literature

The literature of market segmentation starts around 1950 with Baumol (1952) and Tobin (1956). They create a partial equilibrium model with two financial assets, cash and bonds. The agents in their models need cash to be able to buy goods for consumption. So the agents need to withdraw cash, which has a transaction cost. This way, the main conclusion of these papers is that because financial transactions have a cost it will not be optimal for the agents to withdraw cash every period, i.e, financial transactions will be made infrequently.

Later, Grossman and Weiss (1983) and Rotemberg (1984) create a general equilibrium model based on the Baumol-Tobin framework. In their models their exist two types of agents; one that makes financial transactions in the even periods, and another that only makes financial transactions in the odd periods. Their market segmentation is exogenous and not a result of an endogenous optimization process because they simply assume the existence of two types of agents. They analyze the steady state effects of open-market operations in an endowment economy. The main result of their work is that in their models an open-market operations has real effects.

The main reference of my dissertation, as mentioned above, is Alvarez et al (2009).They create a more general version of the Grossman-Weis-Rotemberg model where agents make transactions between their financial assets every N periods. They use an endowment economy to obtain the response of the main monetary variables to monetary shocks. Their main conclusion is that prices respond sluggishly to a exogenous increase of the money supply because in this type of models aggregate velocity of money is not constant but endogenous and responds to the shock. They also conclude that inflation responds sluggishly to an exogenous shock of the nominal interest rate and that monetary policy affects the real interest rate. I my dissertation I will focus on shocks of the nominal interest rate.

In my model the market segmentation will be exogenously imposed, as in the Grossman-Weiss-Rotemberg framework. A next step would be to introduce the time between two transactions of the financial assets into the optimization decision, this means making the market segmentation endogenous. This creates additional difficulties to solve the model and lies behind the goal of this dissertation. However their exist some literature using general equilibrium models with endogenous market segmentation. Silva (2012), for example, creates a model where agents can choose when they make a transaction between their financial assets. He analyses what happens to the welfare cost of inflation and concludes that exogenous market segmentation underestimates the welfare cost of inflation.

1.2 Main results

The main results of my dissertation are twofold. In the first place, I obtain the well known result that inflation responds sluggishly to an exogenous shock of the nominal interest rate. This way monetary policy can affect the real interest rate in the short run and, consequently, consumption, labor supply and output. So, I can conclude that market segmentation can be important to explain the way monetary policy affects output and inflation. It is important to point out that prices are fully flexible in my model and that all the real effects of the monetary policy shock result from the market segmentation.

In the second place, another important result of my dissertation is that making production endogenous changes the response of the model to the monetary policy shock. The numerically response of inflation in a production economy is quite different from the response of inflation in an endowment economy. I first analyze the response of inflation in an endowment economy and I obtain the same results as Alvarez et al (2009). During the first months after the shocks inflation decreases and only after around six months inflation starts to increase, turning back to his steady state value after around four years. Now by letting the agents reallocate their labor supply after the shocks, which means making production endogenous, I obtain a different response of inflation to the interest rate shock. Now, instead of decreasing, inflation starts to increase right after the shock.

2 The model

I will create a simple general equilibrium Baumol-Tobin model. Time is discrete and at any moment there will be an asset market and a market for the good and for labor. This two markets will be physically separated. I assume that their will only exist two financial assets; money and nominal bonds. I assume that money is hold on the bank account and that the bonds are hold on the brokerage account. The money demand results from the fact that the agents need money to buy goods for consumption, but money on the bank account does not receive interest. On the other hand, bonds can not be used to buy goods but do receive interest. I further assume that the asset market opens before the good market. This way, the agents first visit the asset market to make a transaction between the brokerage account and the bank account and only after that the good market opens. Here each household divides between a shopper and a worker; the shopper uses his money on the bank account to buy goods and the worker offers his labor to the firms and receives a payment.[2]

As mentioned before, the market segmentation in my model is exogenously imposed. This means that I impose that each agent only makes a transaction between his financial assets every N periods. Again, this is not a result of an endogenous optimization process so that the agents can not rearrange the time between two financial transactions after the interest rate shock. However this is an ad-hoc assumption, one could argue that small monetary shocks will not have much effect on the time between two financial transactions. So, in the model their will exist N types of agents and every period only a fraction of $\frac{1}{N}$ of the agents will visit the asset market to make a transaction between his financial assets.

Further, the only uncertainty in the model will be the interest rate shock at $t = 0$. The agents can not anticipate the shock and do not expect other shocks in the future. By using this assumptions I am able to solve the model and I can analyze the results of the interest rate shock isolated from other shocks. This way I will not use any notation related to uncertainty.

2.1 Households

There will exist a continuum of infinitely lived households with measure one. Each household will maximize their intertemporal utility function subject to the budget constraints. They will face a intertemporal budget constraint, constraints on the bank account and the brokerage account and cash-in-advance constraints. Each household sells hours of labor, $h_t(s)$, to the firms and receives a payment, $W_t h_t(s)$.

[2] See Alvarez et al (2002) for a more detailed description.

The index $t = 0, 1, 2, ...$ represents the time and the index $s = 0, 1, .., N-1$ represents the type of household. The nominal quantity of money hold on the bank account will be $M_t(s)$ and the quantity of nominal bonds hold on the brokerage account will be $B_t(s)$. Each bond has a maturity of one period, a price equal to one and will payoff R_t at the end of the period. This way, if a agent has $B_t(s)$ at the beginning of t on his brokerage account, then he will have $R_t B_t(s)$ at the end of t on his brokerage account. So R_t denotes the interest rate from the beginning of period t to the end of period t.

I will start to write the intertemporal budget constraint of a household of type s. At the beginning of t each household needs to decide between the quantity of money and the quantity of nominal bonds, subject to his wealth at that moment, $\Omega_t(s)$. This means

$$M_t(s) + B_t(s) \leq \Omega_t(s)$$

The wealth at the beginning of t will be equal to the payment received for the hours of work of the previous period, the quantity of bonds hold on the brokerage account at the end of the previous period, the quantity of money hold on the bank account at the end of the previous period, $Z_{t-1}(s)$, and minus some lump-sum tax paid by the household to the government, τ_{t-1}, at the end of $t-1$. This way I can write the budget constraint for each moment t as

$$M_t(s) + B_t(s) \leq W_{t-1}h_{t-1}(s) + R_{t-1}B_{t-1}(s) + Z_{t-1}(s) - \tau_{t-1}$$

If I multiply the constraint for t by Q_t, where $Q_t = Q_{t-1}\frac{1}{R_{t-1}}$ and $Q_0 = 1$, and sum them all for $t = 0, 1, 2, ...$, then I can obtain the intertemporal budget constraint for a household of type s:

$$\sum_{t=0}^{\infty} Q_t M_t(s) \leq \Omega_0(s) + \sum_{t=0}^{\infty} Q_{t+1} W_t h_t(s) + \sum_{t=0}^{\infty} Q_{t+1} Z_t(s) + - \sum_{t=0}^{\infty} Q_{t+1} \tau_t \tag{1}$$

where $\Omega_0(s)$ is the initial wealth in money and bonds of the household.

Now I will write the constraints on the bank account faced by each household. First, as in Alvarez et al (2009), I will assume that a part of the payment received for the hours of labor goes to the bank account, $\gamma W_t h_t(s)$, and the other part goes to the brokerage account, $(1-\gamma) W_t h_t(s)$.[3] This way, the quantity of money on the bank account at the beginning of t will be equal to the part of payment received on the bank account, $\gamma W_{t-1} h_{t-1}(s)$, the money on the bank account at the end of $t-1$ and carried to t, $Z_{t-1}(s)$, and, in the case the household visits the asset market, a transaction made from the brokerage account to the bank account, $X_t(s)$, at the beginning of t. So the evolution of the bank account can be written as

$$M_t(s) = \gamma W_{t-1} h_{t-1}(s) + Z_{t-1}(s) + X_t(s)$$

Every household only makes a transaction between his brokerage account and his bank account every N periods. I will assume that household s makes a transaction at $t = T_1(s), T_2(s), ..., T_j(s), ...$, where $T_{j+1}(s) - T_j(s) = N$. Further I also assume that $T_0(s) = 0$, however this does not mean that the household makes a transaction

[3] Alvarez et al. (2009) refer to γ as the *paycheck parameter* and interpret $(1-\gamma)$ as "*the fraction of total income that households receive in the form of interest and dividends paid on assets held in their brokerage accounts*". Once one of my main goals is to analyze the strength of their result I will just fix γ equal to them.

at $t = 0$, the household will only make a transaction at $t = 0$ if we have $T_1 = 0$.[4] This way we only have $X_t(s) \neq 0$ when $t = T_j(s)$, for $j = 1, 2, ...$ Introducing this notation into the bank account evolution constraints I obtain

$$M_t(s) = \gamma W_{t-1} h_{t-1}(s) + Z_{t-1}(s), \text{ for } t \neq T_j(s) \tag{2}$$

$$M_t(s) = \gamma W_{t-1} h_{t-1}(s) + Z_{t-1}(s) + X_t(s), \text{ for } t = T_j(s) \tag{3}$$

Further, the household can use the money on the bank account at t, $M_t(s)$, to buy goods, $P_t c_t(s)$, or to carry on the bank account to the next period, $Z_t(s)$. So the cash-in-advance constraint of the bank account will be

$$P_t c_t(s) + Z_t(s) \leq M_t(s) \tag{4}$$

Note here that by definition I also have to impose that $Z_t(s) \geq 0$. This because the households can not carry a negative quantity of money on their bank account to the next period, or in other words, they can not borrow money on the bank account.

The constraints on the brokerage account are the following. The quantity of bonds hold on the brokerage account at the beginning of $t \neq T_j(s)$, moment at which the household does not make a transaction, will be equal to the part of the payment received on the brokerage account and the quantity of bonds hold on the brokerage account at the end of $t - 1$. Further, for simplicity, I also assume that the lump-sum tax paid to the government is made from to the brokerage account. This means that the evolution of the brokerage account at $t \neq T_j(s)$ can be written as

$$B_t(s) = (1 - \gamma) W_{t-1} h_{t-1}(s) + R_{t-1} B_{t-1}(s) - \tau_{t-1} \tag{5}$$

[4] The idea of this notation is token from Silva (2012).

At the moment of a transaction to the bank account, $t = T_j(s)$, the constraint on the brokerage account will be

$$B_t(s) + X_t(s) \leq (1-\gamma) W_{t-1} h_{t-1}(s) + R_{t-1} B_{t-1}(s) - \tau_{t-1} \tag{6}$$

Next I will write the cash-in-advance constraints of the household for each holding period. By holding period I mean the period between two transactions of the financial assets; the first holding period will be the period until the first transaction is made; the second holding period will be the period between the first and the second transaction; and so on. At the beginning of the first holding period the household has some initial, and exogenously fixed, money holdings, $\bar{M}_0(s)$, on his bank account. Further, he will also receive a part of his salary on the bank account. So, during the first holding period he can use the initial money holdings and the payments received during his first holding period for his consumption expenditures during that period. Beside that, it can be optimal for the household to leave a positive quantity of money on his bank account at the end of the holding period, $Z_{T_1(s)-1}(s)$. If this is the case will depend on the initial fixed money holdings and on the magnitude of the shock at $t = 0$. So, the cash-in-advance constraint for the first holding period will be

$$\begin{aligned} & P_0 c_0(s) + P_1 c_1(s) + \ldots + P_{T_1(s)-1} c_{T_1(s)-1}(s) + Z_{T_1(s)-1}(s) \\ \leq \quad & \bar{M}_0(s) + \gamma \left(W_0 h_0(s) + W_1 h_1(s) + \ldots + W_{T_1(s)-2} h_{T_1(s)-2}(s) \right) \end{aligned} \tag{7}$$

The cash-in-advance constraints for the following holding periods are not very different from the cash-in-advance constraint of the first holding period. The only differences are that from now on $Z_{T_j(s)-1}(s)$ will be equal to zero and the money holdings at the beginning of the period are not longer exogenous. $Z_{T_j(s)-1}(s)$ will be equal to zero because it will never be optimal for the household to carry a positive quantity of money on his bank account to the next holding period because, without any uncertainty, he will always be better of if he just transfers that money to his brokerage account at the beginning of the holding period. In that case the household holds more money in bonds on his brokerage account and so he will also receive more interest. This way the cash-in-advance constraints for the following holding periods will become

$$\begin{aligned} & P_{T_j(s)}c_{T_j(s)}(s) + ... + P_{T_{j+1}(s)-1}c_{T_{j+1}(s)-1}(s) \\ \leq \; & M_{T_j(s)}(s) + \gamma\left(W_{T_j(s)}h_{T_j(s)}(s) + ... + W_{T_{j+1}(s)-2}h_{T_{j+1}(s)-2}(s)\right), \\ \text{for } j = \; & 1, 2, ... \end{aligned} \tag{8}$$

Now using the bank account constraints and the cash-in-advance constraints, I can write the intertemporal budget constraint as (see the appendix for more details)

$$\sum_{j=0}^{\infty} Q_{T_j(s)} \sum_{t=T_j(s)}^{T_{j+1}(s)-1} P_t c_t(s) \leq$$

$$\begin{aligned} & \bar{\Omega}_0(s) + \left(Q_{T_1(s)} - 1\right) Z_{T_1(s)-1}(s) + \sum_{t=0}^{T_1(s)-2} \gamma W_t h_t(s) + \\ & + \sum_{j=1}^{\infty} Q_{T_j(s)} \sum_{t=T_j(s)-1}^{T_{j+1}(s)-2} \gamma W_t h_t(s) + \\ & + \sum_{t=0}^{\infty} Q_{t+1}(1-\gamma) W_t h_t(s) - \sum_{t=0}^{\infty} Q_{t+1}\tau_t \end{aligned}$$

Finally, I will substitute $Z_{T_1(s)-1}(s)$ for $Z_{T_1(s)-1}(s) = \bar{M}_0(s) + \sum_{t=0}^{T_1(s)-2} \gamma W_t h_t(s)$ $- \sum_{t=0}^{T_1(s)-1} P_t c_t(s)$, and I obtain the intertemporal budget constraint that I will use in the optimization problem of the household

$$\sum_{j=0}^{\infty} Q_{T_j(s)} \sum_{t=T_j(s)}^{T_{j+1}(s)-1} P_t c_t(s) \leq$$

$$\begin{aligned} &\bar{\Omega}_0(s) + \left(Q_{T_1(s)} - 1\right)\left(\bar{M}_0(s) + \sum_{t=0}^{T_1(s)-2} \gamma W_t h_t(s) - \sum_{t=0}^{T_1(s)-1} P_t c_t(s)\right) + \\ &+ \sum_{t=0}^{T_1(s)-2} \gamma W_t h_t(s) + \sum_{j=1}^{\infty} Q_{T_j(s)} \sum_{t=T_j(s)-1}^{T_{j+1}(s)-2} \gamma W_t h_t(s) + \\ &+ \sum_{t=0}^{\infty} Q_{t+1}(1-\gamma) W_t h_t(s) - \sum_{t=0}^{\infty} Q_{t+1}\tau_t \end{aligned} \tag{9}$$

The optimization problem of the household will be to choose consumption, $\{c_t(s)\}_{t=0}^{\infty}$, and labor supply, $\{h_t(s)\}_{t=0}^{\infty}$, that maximizes his intertemporal utility function, subject to the intertemporal budget constraint (9) and to the bank account constraint for the first holding period

$$\sum_{t=0}^{T_1(s)-1} P_t c_t(s) \leq \bar{M}_0(s) + \sum_{t=0}^{T_1(s)-2} \gamma W_t h_t(s) \tag{10}$$

Beside this, I also need to impose, by definition, that $Z_t(s) \geq 0$.[5]

The momentary utility function used will be the KPR utility function (King, Ploser & Rebelo 1987):

$$u[c_t(s), h_t(s)] = \frac{[c_t(s)(1-h_t(s))^{\chi}]^{1-1/\eta}}{1-1/\eta}$$

I use this utility function because it is a well known utility function in the literature and it also allows me to compare easily the results of an endowment economy with a production economy. If I set the elasticity of intertemporal substitution of labor,

[5] In practice I will only check, at the end, if these constraints hold with the solution obtained.

χ, closely to zero, then labor supply will be constant, consequently output will be constant, and I will have an endowment economy. If I increase χ then I will obtain the results of a production economy because labor supply and output will respond to the shock.

The first order conditions of

$$\max_{\{c_t(s)\}_{t=0}^{\infty},\{h_t(s)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{\left[c_t(s)\left(1-h_t(s)\right)^{\chi}\right]^{1-1/\eta}}{1-1/\eta}$$

subject to (9) and (10) are the following

$$\begin{aligned}
\left[\partial c_t(s)\right] &: \quad \beta^t u'\left[c_t(s)\right] = \lambda Q_{T_1(s)} P_t + \mu P_t, \\
\text{for } t &= 0, ..., T_1(s) - 1 \\
\left[\partial c_t(s)\right] &: \quad \beta^t u'\left[c_t(s)\right] = \lambda P_t Q_{T_j(s)}, \\
\text{for } t &= T_j(s), ..., T_{j+1}(s) - 1 \text{ and } j = 1, 2, ... \\
\left[\partial h_t(s)\right] &: \quad \beta^t u'\left[h_t(s)\right] = -\lambda\left[Q_{T_1(s)}\gamma + Q_{t+1}(1-\gamma)\right] W_t - \mu\gamma W_t, \\
\text{for } t &= 0, ..., T_1(s) - 2 \\
\left[\partial h_t(s)\right] &: \quad \beta^t u'\left[h_t(s)\right] = -\lambda\left[Q_{T_j(s)}\gamma + Q_{t+1}(1-\gamma)\right] W_t \\
\text{for } t &= T_j(s) - 1, ..., T_{j+1}(s) - 2 \text{ and } j = 1, 2, ...
\end{aligned}$$

where λ is the Lagrange multiplier of the intertemporal budget constraint and μ the Lagrange multiplier of the bank account constraint. I use $u'\left[c_t(s)\right]$ for the derivation of $u\left[c_t(s), h_t(s)\right]$ in order of $c_t(s)$ and $u'\left[h_t(s)\right]$ for the derivation of $u\left[c_t(s), h_t(s)\right]$ in order of $h_t(s)$.

I obtain two types of intertemporal optimal conditions for consumption from these first order conditions. The optimal conditions of consumption between two periods of the same holding period I will call intra-holding optimal conditions. On the other hand, the optimal conditions of consumption between two periods of different holding periods I call inter-holding optimal conditions. The intra-holding optimal conditions are the same for all holding periods

$$\frac{1}{P_t}u^{'}\left[c_t\left(s\right)\right]=\beta\frac{1}{P_{t+1}}u^{'}\left[c_{t+1}\left(s\right)\right] \tag{11}$$

$$\text{for } t=T_j\left(s\right),...,T_{j+1}\left(s\right)-2 \text{ and } j=0,1,...$$

The inter-holding optimal condition between the first and second holding period depends on the ratio of the Lagrange multiplier of the bank account constraint and the intertemporal budget constraint, $\frac{\mu(s)}{\lambda}$,

$$\frac{1}{P_0}u^{'}\left[c_0\left(s\right)\right]=\beta^{T_1(s)}\frac{1}{P_{T_1(s)}}u^{'}\left[c_{T_1(s)}\left(s\right)\right]\left(1+\frac{\mu\left(s\right)}{\lambda}\frac{1}{Q_{T_1(s)}}\right) \tag{12}$$

while the inter-holding optimal conditions between the other holding periods do not depend on the Lagrange multipliers

$$\frac{1}{P_{T_j(s)}}u^{'}\left[c_{T_j(s)}\left(s\right)\right]=\beta^{N}\frac{1}{P_{T_{j+1}(s)}}u^{'}\left[c_{T_{j+1}(s)}\left(s\right)\right]\frac{Q_{T_j(s)}}{Q_{T_{j+1}(s)}} \tag{13}$$

$$\text{for } j=1,2,...$$

Notice that the inter-holding optimal conditions are written as optimal condition between consumption of the first period of each holding period but could be written as optimal conditions between any period of two different holding periods.

The equations of the marginal rate of substitution between leisure and consumption during the first holding period are the following

$$\begin{aligned}\frac{-u^{'}\left[h_t\left(s\right)\right]}{u^{'}\left[c_t\left(s\right)\right]} &= \frac{W_t}{P_t}\frac{\gamma Q_{T_1(s)}+\left(1-\gamma\right)Q_{t+1}+\gamma\frac{\mu(s)}{\lambda}}{Q_{T_1(s)}+\frac{\mu(s)}{\lambda}} \\ \text{for } t &= 0,...,T_1\left(s\right)-2\end{aligned} \tag{14}$$

$$\frac{-u'[h_t(s)]}{u'[c_t(s)]} = \frac{W_t}{P_t}\frac{Q_{T_1(s)}}{Q_{T_1(s)}+\frac{\mu(s)}{\lambda}} \tag{15}$$
$$\text{for } t = T_1(s)-1$$

And the equations of the marginal rate of substitution between leisure and consumption for the other holding periods are

$$\frac{-u'[h_t(s)]}{u'[c_t(s)]} = \frac{W_t}{P_t}\frac{\gamma Q_{T_j(s)}+(1-\gamma)Q_{t+1}}{Q_{T_j(s)}} \tag{16}$$
$$\text{for } t = T_j(s),...,T_{j+1}(s)-2 \text{ and } j=1,2,...$$

$$\frac{-u'[h_t(s)]}{u'[c_t(s)]} = \frac{W_t}{P_t}\frac{Q_{T_{j+1}(s)}}{Q_{T_j(s)}} \tag{17}$$
$$\text{for } t = T_{j+1}(s)-1 \text{ and } j=1,2,...$$

Here also only the optimal conditions between leisure and consumption in the first holding period depend on the ratio of the Lagrange multipliers.

2.2 Firms

I will assume a very simple production side of the economy. Total production, Y_t, will be linear in total labor, L_t, used, so

$$Y_t = AL_t$$

where A is a technological parameter. The firms will maximize their profits and therefore the real wage per hour, w_t, paid to the households will be equal to the constant marginal productivity

$$w_t = \frac{W_t}{P_t} = A$$

2.3 Government

The government issues nominal bonds, B_t^g, and prints money, M_t^g. For simplicity I assume that their do not exist government spending but only a lump-sum tax paid by the households to the government. The policy instrument of the government is the nominal interest rate. This way, the government will satisfy the demand for money and bonds at the exogenously fixed interest rate.

The budget constraint faced by the government at t will be

$$R_{t-1}B_{t-1}^g + M_{t-1}^g \leq M_t^g + B_t^g + \tau_t$$

By multiplying the budget constraint for t by Q_t and sum them for $t = 0, 1, 2, ...$ I obtain the intertemporal budget constraint of the government

$$\Omega_0^g \leq \sum_{t=0}^{\infty} Q_{t+1} \left(R_t - 1\right) M_t^g + \sum_{t=0}^{\infty} Q_t \tau_t \tag{18}$$

where Ω_0^g are the initial obligations in money and bonds of the government and $\sum_{t=0}^{\infty} Q_{t+1} \left(R_t - 1\right) M_t^g$ is the present value of the future inflation taxes.

3 Steady State Equilibrium

3.1 Clearing Conditions

The competitive equilibrium of this economy will be defined as a sequence of allocations, prices and policies such that: (i) the private agents, households and firms, solve their optimization problems given the sequence of prices and policies; (ii) the budget constraints of the government are satisfied; and (iii) all markets clear. The market clearing conditions are the following

$$\frac{1}{N}\sum_{s=0}^{N-1} c_t(s) = Y_t$$
$$\frac{1}{N}\sum_{s=0}^{N-1} h_t(s) = L_t$$
$$\frac{1}{N}\sum_{s=0}^{N-1} M_t(s) = M_t^g$$
$$\frac{1}{N}\sum_{s=0}^{N-1} B_t(s) = B_t^g$$

3.2 Steady State

The steady state of the economy of the model will be defined by a constant nominal interest rate and a constant inflation rate. This way consumption, labor supply and output will also be constant. Further, I will set the initial conditions such that all the households, in steady state, behave the same during their holding period. This means, for example, that the amount transferred from the brokerage account to the bank account, in steady state, will be the same for all household, but the transactions will be made at different moments. This way the only heterogeneity along the households results from the market segmentation.

From now on I will use an specific way to index the households. The household will still be indexed by $s = 0, 1, ..., N - 1$, but now s will mean the position of the household in his holding period. This means that a household that makes a transaction at t will be of type $s = 0$ at t. At $t + 1$ he will be of type $s = 1$, at $t + 2$ of $s = 2$ and so on. This way, the moment before a new transaction the household will be of type $s = N - 1$.

From (11) I obtain the steady state version of the intra-holding optimal condition for consumption

$$u^{'}\left[c\left(s\right)\right]=\frac{\beta}{\pi}u^{'}\left[c\left(s+1\right)\right], \text{ for } s=0,...,N-2 \tag{19}$$

where $\pi_t=\frac{P_t}{P_{t-1}}$ is the gross inflation rate between t and $t-1$, and π is the constant steady state inflation rate. From the inter-holding optimal condition, (13), I obtain that the real interest rate will be equal to the intertemporal discount rate

$$\frac{R}{\pi}=\frac{1}{\beta}$$

Further, the steady state versions of the marginal rate of substitution between leisure and consumption, (16) and (17), are

$$\frac{-u'\left[h\left(s\right)\right]}{u'\left[c\left(s\right)\right]}=A\left[\gamma+R^{-(s+1)}\left(1-\gamma\right)\right], \text{ for } s=0,...,N-2 \tag{20}$$

$$\frac{-u'\left[h\left(s\right)\right]}{u'\left[c\left(s\right)\right]}=AR^{-N}, \text{ for } s=N-1 \tag{21}$$

Now I will use these $N-1$ intra-holding optimal condition, N optimal conditions between leisure and condition and the clearing condition for the good market and the labor market to obtain the steady state values of consumption, labor supply and output, $\left\{c\left(s\right),h\left(s\right),Y\right\}_{s=0}^{N-1}$.This way I have a non-linear system of $2N+1$ equations and $2N+1$ unknowns.[6]

The other unknowns of the households, $M_t\left(s\right)$, $Z_t\left(s\right)$, $X_t\left(s\right)$ and $B_t\left(s\right)$, can be easily obtained using the cash-in-advance constraints, (8) and (4), the constraints on the evolution of the bank account, (2) and (3), and the constraints of the brokerage account, (5) and (6).

[6] I solve this non-linear system using a default function of the program MATLAB named *fsolve*.

3.3 Calibration

The calibration used in this model is based on Alvarez et al (2009) as one of the main purposes of this dissertation is to compare the results with the results obtained by them. Each period in the model corresponds to a month. The annual steady state inflation rate will be set equal to 5 per cent and the intertemporal annual discount rate equal to $\frac{1}{1.04}$, this means

$$\begin{aligned} \pi &= (1.05)^{1/12} \\ \frac{1}{\beta} &= (1.04)^{1/12} \end{aligned}$$

The degree of risk aversion will be set equal to one, $1/\eta = 1$, and the technological parameter will be set such that output equals one. In the benchmark case I will set the elasticity of intertemporal substitution of labor, χ, equal to 1.75 because in that case the households will spent around 35 per cent of their time working. Further, the number of periods between two financial transactions will be equal to $N = 38$ and the *payment check* parameter equal to $\gamma = 0.6$. The choice of these parameters is based on microeconomic data about the trade frequency of households (Alvarez et al 2009) and set such that the annual average velocity of money equals 1.5.

3.4 Results

The steady state behavior of the households in this model is very similar to the behavior in conventional inventory models of money demand. The agents need enough money on their bank account to be able to buy goods during the whole holding period. So they transfer money from their brokerage account to their bank account and use that money, and the money they receive as salary on their bank account, for consumption during the holding period. This way the real money holdings are decreasing during the holding period and at the moment of a new transaction their will be no money left on the bank account. In Figure 1 we can see the real money holdings of a household at the beginning of each period during the holding period.

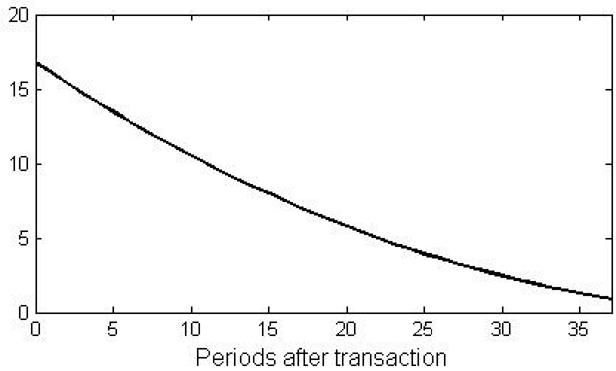

Figure 1: Real Money Holdings

In Figure 2 and 3 we see the steady state behavior of consumption and labor supply during a holding period. We see that consumption of the households will be decreasing during the holding period. This happens for two reasons. Because the opportunity cost of consumption at the end of the holding period is higher than the opportunity cost of consumption at the beginning of the holding period. This cost is higher at the end of the holding period because the agents need to safe the money at the bank account during more periods, which, in the case of a positive inflation rate, reduces the real value of the money. On the other hand, due to the intertemporal discount rate agents prefer consumption today instead of consumption later. We can see this in equation (19)

While consumption is decreasing during the holding period, labor supply will be increasing. Also for this behavior there are two main reasons. In the first place, from the marginal rate of substitution between leisure and consumption, (20) and (21), we know that when consumption is lower, labor supply will be higher. So once consumption is decreasing during the holding period, labor supply will be increasing. In the second place, the marginal revenue of one hour work is higher in the beginning and lower at the end of each holding period.

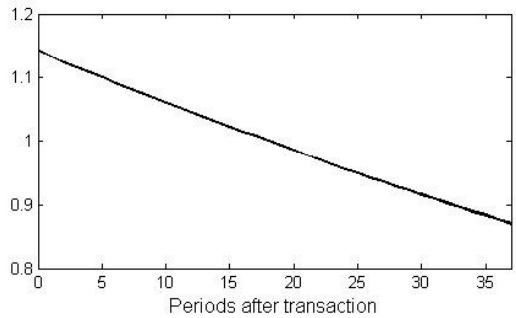

Figure 2: Consumption

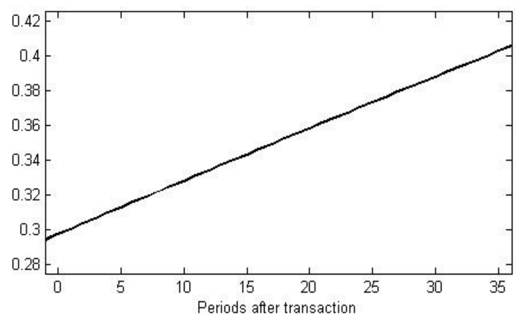

Figure 3: Labor Supply

4 Monetary Policy Shock

As mentioned before I will identify the monetary policy shock as an exogenous shock to the nominal interest rate. I assume that the deviation of the nominal interest rate of his steady state value follows a AR(1) process: $\tilde{R}_t = \rho\tilde{R}_{t-1} + \varepsilon_t$, where I fix $\rho = 0.87$, $\varepsilon_0 = 0.01$ and $\varepsilon_t = 0$, for $t = 1, 2, ...$, as in Alvarez et al (2009). I assume that the agents of the model do not anticipated the shock and only observe it at the beginning of $t = 0$. After the shock I assume that their does not exist any uncertainty to be able to isolate the effects of the interest rate shock.

In the steady state the government can choose either to fix the nominal interest rate or the money supply. Here I assume that the interest rate is exogenously fixed and that the money supply will be determined by the equilibrium conditions.

However, at the moment of the shock, $t = 0$, to fully describe the monetary policy one needs to impose some initial condition about the price level or about the nominal money supply. I will assume that the government controls the nominal money supply at $t = 0$. More, I assume that the nominal money supply, at $t = 0$, continues to grow at its steady state rate. In practical terms, this is equivalent to assuming, as in Alvarez et al (2009), that the shock to the nominal interest rate does not affect the price level at $t = 0$.

4.1 Solution Method

Unlike Alvarez et al (2009), I will solve the equilibrium response of the economy of the model in a simple non-linear way. I start to assume that the economy, after the interest rate shock, will be back to its initial steady state after a sufficiently high number of periods, t^*. This way I can solve the response of inflation, output, consumption and labor supply backwards. I will start to solve the response for $t = t^* - 1$, then for $t = t^* - 2$ and so on. Note that the path of the nominal interest rate, $\{R_t\}_{t=0}^{\infty}$, is exogenous and therefore known.

So, the economy will be back to its initial steady state at $t = t^*$. This means that inflation, output, consumption and labor supply will be back to their initial steady state values

$$\begin{aligned}
\pi_{t^*} &= \pi \\
Y_{t^*} &= Y \\
c_{t^*}(s) &= c(s), \text{ for } s = 0, ..., N-1 \\
h_{t^*}(s) &= h(s), \text{ for } s = 0, ..., N-1
\end{aligned}$$

Now I will obtain inflation, output, consumption and hours of labor for $t = t^* - 1$. This way I have $2N+2$ unknowns and will also need $2N+2$ equations. The equations will be the following:

- $N - 1$ intra-holding optimal conditions for consumption between $t = t^* - 1$ and $t = t^*$

$$\begin{aligned} u'\left[c_{t^*-1}\left(s-1\right)\right] &= \frac{\beta}{\pi_{t^*}} u'\left[c_{t^*}\left(s\right)\right], \\ \text{for } s &= 1, 2, ..., N-1 \end{aligned}$$

- 1 inter-holding optimal condition for consumption between $t = t^* - 1$ and $t = t^*$

$$u'\left[c_{t^*-1}\left(N-1\right)\right] = \frac{\beta}{\pi_{t^*}} u'\left[c_{t^*}\left(0\right)\right] R_{t^*-N} R_{t^*-N+1} ... R_{t^*-1}$$

- N marginal rate of substitution conditions between leisure and consumption at $t = t^* - 1$

$$\begin{aligned} \frac{-u'\left[h_t\left(s\right)\right]}{u'\left[c_t\left(s\right)\right]} &= A\left[\gamma + \frac{1}{R_{t^*-1-s} ... R_{t^*-1}}\left(1-\gamma\right)\right] \\ \text{for } s &= 0, 1, ..., N-2 \end{aligned}$$

$$\frac{-u'\left[h_t\left(N-1\right)\right]}{u'\left[c_t\left(N-1\right)\right]} = A \frac{1}{R_{t^*-N} ... R_{t^*-1}}$$

- 1 market clearing condition for the labor market at $t = t^* - 1$

$$\frac{1}{N} \sum_{s=0}^{N-1} h_{t^*-1}\left(s\right) = L_{t^*-1}$$

- and 1 market clearing condition for the good market at $t = t^* - 1$

$$\frac{1}{N} \sum_{s=0}^{N-1} c_{t^*-1}\left(s\right) = Y_{t^*-1}$$

By solving this nonlinear system of $2N + 2$ equations and unknowns I obtain π_{t^*-1}, Y_{t^*-1}, $\left\{c_{t^*-1}\left(s\right)\right\}_{s=0}^{N-1}$ and $\left\{h_{t^*-1}\left(s\right)\right\}_{s=0}^{N-1}$.[7]

[7] Again, to solve this non-linear system I will use the MATLAB function *fsolve*.

Once I know the values of the main variables for $t = t^* - 1$, I can use the same method to obtain inflation, output, consumption and hours of labor for $t = t^* - 2$. Using the same equations as above, but now for $t = t^* - 2$, I have again a system of $2N + 2$ equations and unknowns. By repeating this method I obtain the response of the main variables of the model to the interest rate shock for $t = N-1, N, ..., t^*-1, t^*$.

To obtain the response of the economy for the first $N - 1$ periods the equations used change a little. The reason therefore is that for the first $N - 1$ periods at least one of the types of households will be in their first holding period and, hence, the inter-holding optimal conditions and the marginal rate of substitution between leisure and consumption will depend on the ratio of the Lagrange multipliers, as we can see in (12), (14) and (15). This way, for the first $N - 1$ periods, I have $N - 1$ additional unknowns, $\left\{\frac{\mu(s)}{\lambda}\right\}_{s=0}^{N-1}$, and therefore will also need $N - 1$ additional equations. The equations used will be the bank account constraints of each type of households for the first holding period, see (10). Note that for the household which makes a transaction at $t = 0$ the first holding period does not exist and so he also does not have a bank account constraint for that period. That is why I have $N - 1$, instead of N, additional equations. Further, I will assume that these constraints hold with equality. This means that the households do not leave money on the bank account at the end of the holding period, but decide to spent it all in consumption. Obviously, while solving the model I will always check if the solution I obtain is the right solution.[8] So, this way I will add the following $N-1$ equations

$$\begin{aligned}\sum_{t=s}^{N-1} P_{t-s}c_{t-s}(t) &= \bar{M}_0(s) + \sum_{t=s}^{N-2} \gamma W_{t-s}h_{t-s}(t), \\ \text{for } s &= 1,2,...,N-1\end{aligned}$$

These equations contain consumption and labor supply until the first transaction is made, therefore, for the first $N-1$ periods I solve the response of the economy as one only system. This way, the system will contain $(N-1)(2N+2)+(N-1)$ unknowns: $\pi_t, Y_t, \{c_t(s)\}_{s=0}^{N-1}$ and $\{h_t(s)\}_{s=0}^{N-1}$ for $t = 0,1,...N-2$ and $\left\{\frac{\mu(s)}{\lambda}\right\}_{s=0}^{N-1}$. The equations of the system will be the following:

for $t = 0,1,...,N-2$,

- $N-1$ intra-holding optimal conditions for consumption between t and $t+1$

$$\begin{aligned}u'[c_t(s-1)] &= \frac{\beta}{\pi_{t+1}}u'[c_{t+1}(s)], \\ \text{for } s &= 1,2,...,N-1\end{aligned}$$

- 1 inter-holding optimal condition for consumption between t and $t+1$

$$u'[c_t(N-1)] = \frac{\beta}{\pi_{t+1}}u'[c_{t+1}(0)]\left(1+\frac{\mu(s)}{\lambda}\frac{1}{Q_{t+1}}\right)$$

- N marginal rate of substitution conditions between leisure and consumption at

t

- if the household has already made a transaction at t

[8] To do this I use the optimal conditions for the first holding period and the fact that if $Z_t(N-1) > 0$, for $t = 0,1,...,N-2$, then the bank account constraint for the first holding period will not be binding and we have $\mu = 0$.

$$\begin{aligned}\frac{-u^{'}\left[h_t\left(s\right)\right]}{u^{'}\left[c_t\left(s\right)\right]} &= A\left[\gamma+\frac{1}{R_{t^*-1-s}...R_{t^*-1}}\left(1-\gamma\right)\right] \\ \text{for } s &= 0,1,...,N-2\end{aligned}$$

- if the household has not made yet a transaction at t

$$\begin{aligned}\frac{-u^{'}\left[h_t\left(s\right)\right]}{u^{'}\left[c_t\left(s\right)\right]} &= A\frac{\gamma Q_{t+N-s}+\left(1-\gamma\right)Q_{t+1}+\gamma\frac{\mu(s)}{\lambda}}{Q_{t+N-s}+\frac{\mu(s)}{\lambda}} \\ \text{for } s &= 0,1,...,N-2\end{aligned}$$

$$\frac{-u'\left[h_t\left(N-1\right)\right]}{u'\left[c_t\left(N-1\right)\right]}=A\frac{Q_{t+1}}{Q_{t+1}+\frac{\mu(s)}{\lambda}}$$

- 1 market clearing condition for the labor market at t

$$\frac{1}{N}\sum_{s=0}^{N-1}h_{t^*-1}\left(s\right)=L_{t^*-1}$$

- 1 market clearing condition for the good market at t

$$\frac{1}{N}\sum_{s=0}^{N-1}c_{t^*-1}\left(s\right)=Y_{t^*-1}$$

- and $N-1$ bank account constraints

$$\sum_{t=0}^{T_1(s)-1}P_tc_t\left(s\right)=\bar{M}_0\left(s\right)+\sum_{t=0}^{T_1(s)-2}\gamma W_th_t\left(s\right)$$

So, solving this system of $\left(2N+2\right)\left(N-1\right)+\left(N-1\right)$ equations and unknowns I obtain $\pi_t, Y_t, \left\{c_t\left(s\right)\right\}_{s=0}^{N-1}$ and $\left\{h_t\left(s\right)\right\}_{s=0}^{N-1}$ for $t=0,1,...N-2$.[9]

[9] Note that for the bank account constraints I need to use the assumption that the nominal money supply grows at its steady state rate at $t=0$. In practice, I assume that the inflation rate from $t=-1$ to $t=0$ is equal to the steady state inflation rate, $\pi_0=\pi$.

4.2 Results

The main goal of this dissertation is to analyze the robustness of the results obtained by Alvarez et al (2009). They obtain the response of inflation in a model very similar to the one presented here. The main difference between their model and my model is that in their model output is exogenous and constant, while in my model output is endogenous and responds to the interest rate shock. To be able to obtain the results of an endowment economy I will fix the elasticity of labor, χ, very close to zero. This way labor supply will be constant and will not respond to the interest rate shock. By doing this I replicate the results of Alvarez et al (2009). Next, to be able to analyze the difference between the two types of models I increase χ and, consequently, labor supply will respond to the shock and output will be endogenous.

Figure 4 shows the exogenous shock to the nominal interest rate. As mentioned before, at $t = 0$ the interest rate increases with 100 basis point and returns then slowly back to his initial steady state value.

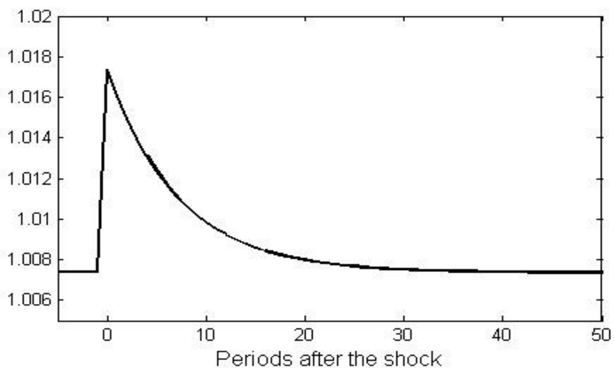

Figure 4: Gross Nominal Interest Rate

The following figures show the response of total output to the interest rate shock. The first case is when output is exogenous and constant ($\chi = 0$), i.e. the case when the economy is an endowment economy. For the following cases I increase the elasticity of labor until $\chi = 1,75$, which is the benchmark case; in which the agents spent around 35 per cent of their time working. As we can see, output starts to respond to the shock. At the moment of the shock output rises, until at most 0.65 per cent above its steady state value, but then starts to decrease and around three months after the shock output is below its steady state value. Then after around one year output starts to recover and goes slowly back to its steady state.

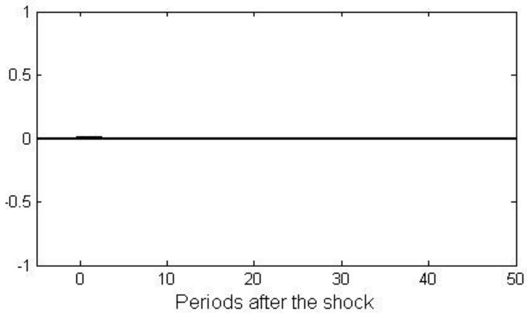

Figure 5: Output - $\chi = 0$

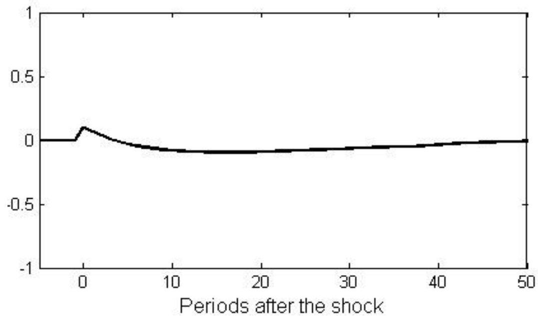

Figure 6: Output - $\chi = 0.1$

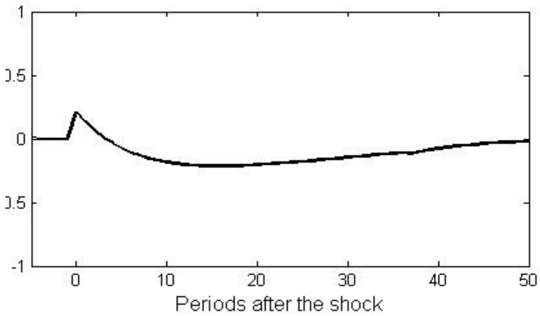

Figure 7: Output - $\chi = 0.25$

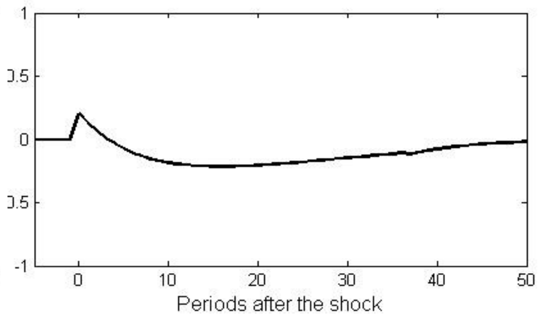

Figure 8: Output - $\chi = 0.5$

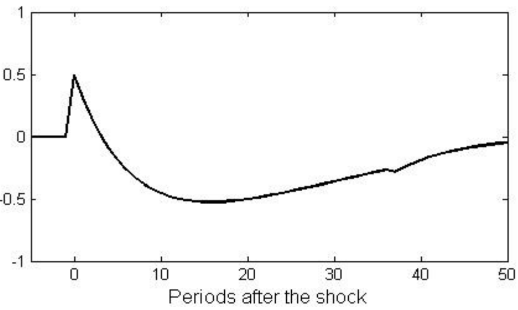

Figure 9: Output - $\chi = 1$

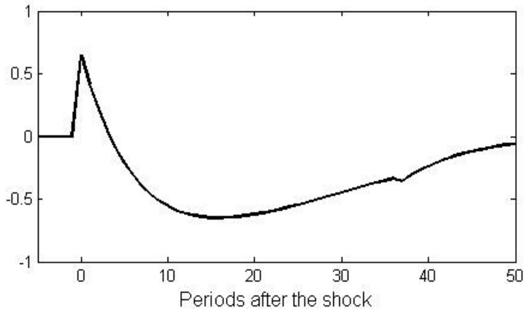

Figure 10: Output - $\chi = 1.75$

If we look at the graphics of the response of inflation to the shock of the nominal interest rate, I can conclude that making production endogenous changes the response of inflation. The first chart shows us the response of inflation in the case of an endowment economy. This result is the same as the one obtained by Alvarez et al (2009). Then by again increasing the elasticity of labor we can see what happens to the response of inflation if I let the agents reallocate their labor supply. In that case, we see that, instead of a reduction of the inflation rate, I obtain that the inflation rises after the interest rate shock. After this initial jump, inflation turns slowly back to its initial steady state value. So, hereby I conclude that letting the agents reallocate their labor supply after the interest rate shock changes the numerically response of the inflation rate significantly.

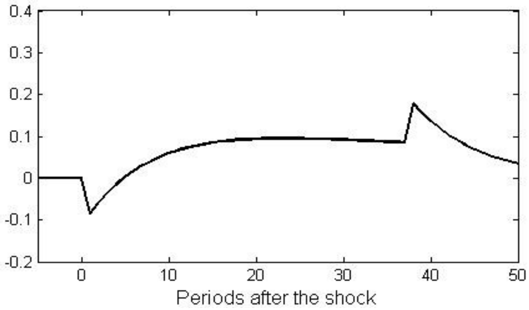

Figure 11: Inflation - $\chi = 0$

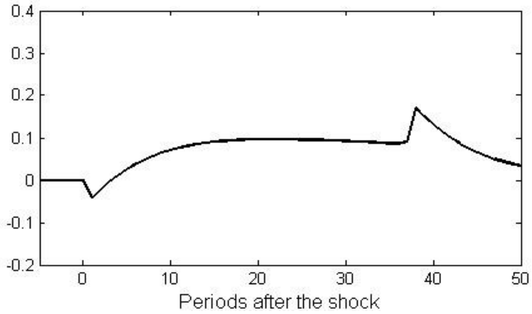

Figure 12: Inflation - $\chi = 0.1$

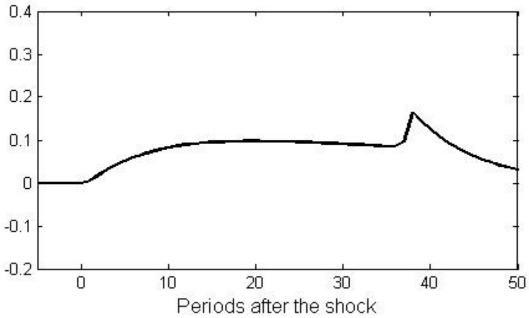

Figure 13: Inflation - $\chi = 0.25$

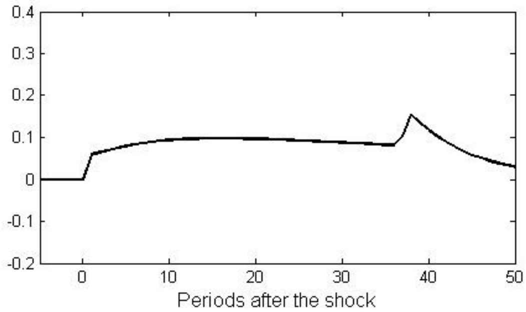

Figure 14: Inflation - $\chi = 0.5$

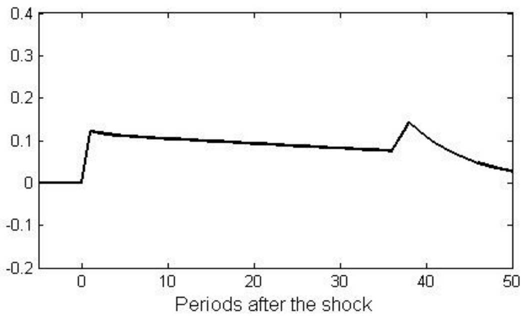

Figure 15: Inflation - $\chi = 1$

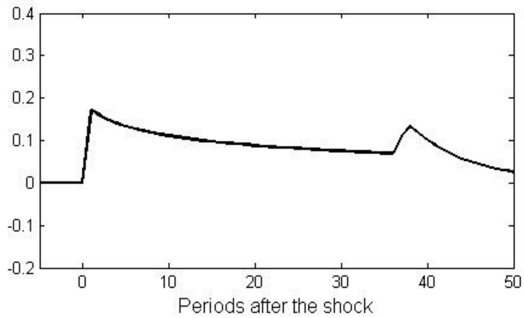

Figure 16: Inflation - $\chi = 1.75$

It is important to point out that when I increase the elasticity of labor the annual average velocity of money changes. As in Alvarez et al (2009) I set $N = 38$ and $\gamma = 0.6$ to obtain an annual average velocity of money equal to 1.5 in the case of an endowment economy. However in my benchmark case of endogenous production the velocity of money increases until around 1.65. To be able to obtain an average velocity of money equal to 1.5 I need to increase the number of periods between two financial transactions, N, or to decrease the part of the payment received on the bank account, γ. For example, if I increase N from 38 to 42 in the case of $\chi = 1.75$, the annual average velocity of money will decrease from 1.65 to 1.5. The results presented here are robust to this changes.

Now I will take a closer look at the response of output and inflation in case of a labor elasticity equal to 1.75. Below I show the behavior of three types of households after the interest rate shock: a household which makes a transaction at the moment of the shock, type $s = 0$ at $t = 0$; a household which will make a transaction 18 periods after the shock, type $s = 18$ at $t = 0$; and a household which has made a transaction the period before the shock and now only will make a transaction after 37 periods, type $s = 1$ at $t = 0$. By showing these three types of household I will try to understand what the behavior of the different types of households is.

As we can see, the household which makes a transaction at $t = 0$ (type $s = 0$ at $t = 0$) will lower his consumption and work more as response to the higher interest rate. By doing this other agents need to higher their consumption and lower their labor supply (see type $s = 1$ and $s = 18$ at $t = 0$). To stimulate the agents who do not have made a transaction yet to higher their consumption the inflation rate will rise in the periods after the shock. A higher inflation rate means that the opportunity cost of consumption rises and the households will antecipated consumption. After some periods, the agents who make a transaction at that moment will start to increase their consumption and work less after their transaction instead of doing the opposite. This is a consequence of the wealth effect resulting from the higher interest rate. When this effect starts to dominate, which will be after some months, output will come below its initial steady state value and the economy will get in a recession.

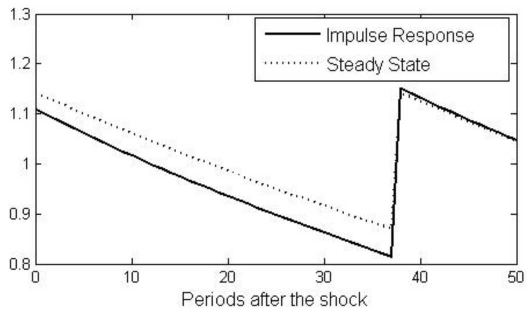

Figure 17: Consumption of a Household of type $s = 0$ at $t = 0$

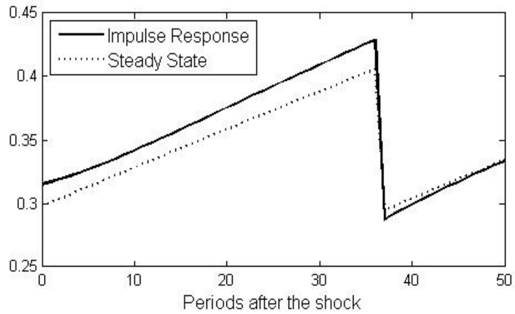

Figure 18: Labor Supply of a Household of type $s = 0$ at $t = 0$

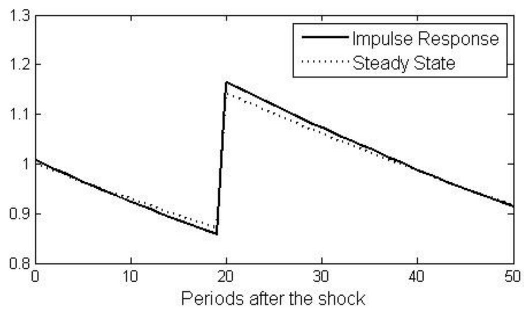

Figure 19: Consumption of a Household of type $s = 18$ at $t = 0$

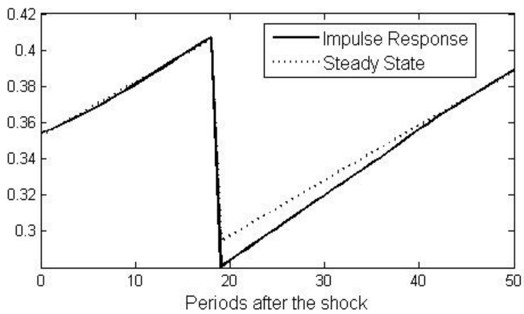

Figure 20: Labor Supply of a Household of type $s = 18$ at $t = 0$

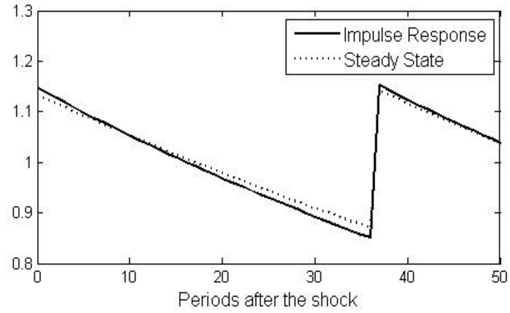

Figure 21: Consumption of a

Household of type $s = 1$ at $t = 0$

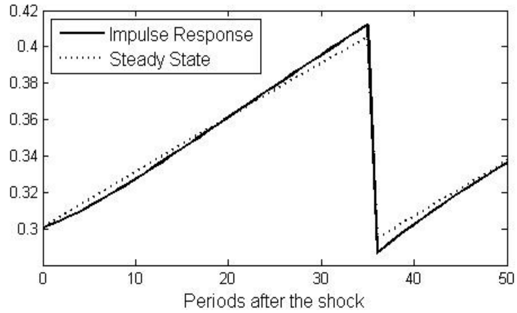

Figure 22: Labor Supply of a

Household of type $s = 1$ at $t = 0$

5 Conclusion

With this dissertation I try to contribute to the literature about monetary policy shocks in general equilibrium models with market segmentation. I use a very similar model to the one used by Alvarez et al (2009), but in my model production is endogenous. I introduce an exogenous shock to the nominal interest rate and calculate the response of the main variables of the model.

The main conclusion of my dissertation is that the introduction of production in the model changes the numerically response of inflation to a shock to the nominal interest rate. Instead of a initial drop of the inflation rate, in the case of an endowment economy, when production is endogenous the inflation rate increases at $t = 0$. However, beside this different response of inflation, one of the main conclusions in this type of models, that inflation responds sluggishly to monetary policy and that, therefore, monetary policy has real effects in the short run, remains valid. I also analyze the response of output to the monetary policy shock. At the moment of the shock there exist a positive effect on output but after a few months the economy gets into a recession and will slowly return to its steady state.

Further, in this dissertation I introduce a different and simple nonlinear way to solve the response of the model to the interest rate shock. By assuming that the economy will be back at its initial steady state at a sufficiently high t^*, I am able to solve the equilibrium response of the model backwards.

The following steps in this research should be in two directions. In the first place, one should introduce the market segmentation into the optimization process of the agents. This way the agents are able to adjust the number of periods between two financial transactions after the interest rate shock. It is important to analyze in which way turning the market segmentation endogenous changes the results obtained in this dissertation. A good example of how to introduce endogenous market segmentation into this type of models is Silva (2012). However, this lies behind the goals of this dissertation.

In the second place, it is important to understand in which way the results presented here are still valid when one uses a more complicated production function in the model. An important point would be to introduce capital into the production process because of the effects of the interest rate on capital acumulation.

A Appendix

First I will substitute the cash-in-advance constraints for each holding period, (7) and (8), into the intertemporal budget constraint, (1), and obtain

$$\begin{aligned}
&\sum_{t\neq T_j(s)}^{\infty} Q_t M_t(s) + \sum_{j=0}^{\infty} Q_{T_j(s)} \sum_{t=T_j(s)}^{T_{j+1}(s)-1} P_t c_t(s) + Z_{T_1(s)-1} - \\
&- \sum_{j=0}^{\infty} Q_{T_j(s)} \sum_{t=T_j(s)}^{T_{j+1}(s)-2} \gamma W_t h_t(s) \\
\leq\ & \Omega_0(s) + \sum_{t=0}^{\infty} Q_{t+1} W_t h_t(s) + \sum_{t=0}^{\infty} Q_{t+1} Z_t(s) - \sum_{t=0}^{\infty} Q_{t+1} \tau_t
\end{aligned}$$

Now I will substitute the bank account constraint for $t \neq T_j(s)$, (2), into the equations above and obtain

$$\begin{aligned}
&\sum_{t\neq T_j(s)}^{\infty} Q_t \gamma W_{t-1} h_{t-1}(s) + \sum_{t\neq T_j(s)}^{\infty} Q_t Z_{t-1}(s) + \sum_{j=0}^{\infty} Q_{T_j(s)} \sum_{t=T_j(s)}^{T_{j+1}(s)-1} P_t c_t(s) \\
&+ Z_{T_1(s)-1} - \sum_{j=0}^{\infty} Q_{T_j(s)} \sum_{t=T_j(s)}^{T_{j+1}(s)-2} \gamma W_t h_t(s) \\
\leq\ & \Omega_0(s) + \sum_{t=0}^{\infty} Q_{t+1} W_t h_t(s) + \sum_{t=0}^{\infty} Q_{t+1} Z_t(s) - \sum_{t=0}^{\infty} Q_{t+1} \tau_t
\end{aligned}$$

$$\begin{aligned}
&\sum_{j=0}^{\infty} Q_{T_j(s)} \sum_{t=T_j(s)}^{T_{j+1}(s)-1} P_t c_t(s) \leq \Omega_0(s) + \sum_{t=0}^{T_1(s)-2} \gamma W_t h_t(s) + \\
&+ \sum_{j=1}^{\infty} Q_{T_j(s)} \sum_{t=T_j(s)-1}^{T_{j+1}(s)-2} \gamma W_t h_t(s) + (1-\gamma) \sum_{t=0}^{\infty} Q_{t+1} W_t h_t(s) + \\
&+ \sum_{j=1}^{\infty} Q_{T_j(s)} Z_{T_j(s)-1}(s) - Z_{T_1(s)-1} - \sum_{t=0}^{\infty} Q_{t+1} \tau_t
\end{aligned}$$

Now knowing that $Z_{T_j(s)-1} = 0$ for $j = 2, 3, ...$ I obtain

$$\sum_{j=0}^{\infty} Q_{T_j(s)} \sum_{t=T_j(s)}^{T_{j+1}(s)-1} P_t c_t(s) \leq$$

$$
\begin{aligned}
&\bar{\Omega}_0(s) + \left(Q_{T_1(s)} - 1\right) Z_{T_1(s)-1}(s) + \\
&+ \sum_{t=0}^{T_1(s)-2} \gamma W_t h_t(s) + \sum_{j=1}^{\infty} Q_{T_j(s)} \sum_{t=T_j(s)-1}^{T_{j+1}(s)-2} \gamma W_t h_t(s) + \\
&+ \sum_{t=0}^{\infty} Q_{t+1}(1-\gamma) W_t h_t(s) - \sum_{t=0}^{\infty} Q_{t+1} \tau_t
\end{aligned}
$$

References

Alvarez, F., Atkeson, A. & Kehoe, P. (2002). Money, Interest Rates, and Exchange Ratess with Endogenously Segmented Markets. *Journal of Political Economy* 110 (1), 73-112.

Alvarez, F., Atkeson, A. & Edmond, C. (2009). Sluggish Responses of Prices and Inflation to Monetary Shocks in an Inventory Model of Money Demand. *Quarterly Journal of Economics* 124 (3), 911-967.

Baumol, W. (1952). The Transaction Demand For Cash: An Inventory Theoretic Approach. *The Quarterly Journal of Economics* 66 (4), 545-556.

Grossman, S. & Weiss, L. (1983). A Transactions-Based Model of the Monetary Transmission Mechanism. *The American Economic Review* 73 (5), 871-880.

King, R., Plosser, C. & Rebelo, S. (1987). Production, Growth and Business Cycles. *Journal of Monetary Economics* 21, 195-232.

Rotemberg, J. (1984). A Monetary Equilibrium Model With Transactions Costs. *Journal of Political Economy* 92 (1), 40-58.

Silva, A. (2012). Rebalancing Frequency and the Welfare Cost of Inflation. *American Economic Journal: Macroeconomics* 4 (2), 153-183.

Tobin, J. (1956). The Interest-Elasticity of Transactions Demand For Cash. *The Review of Economics and Statistics* 38 (3), 241-247.

Walsh, C. (2010). *Monetary Theory and Policy*, 3^{th} Ed. The MIT Press.