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MASTER'S FINAL WORK
DISSERTATION

CASE FOR A CLOSER LOOK AT MIGRATION: ANALYSIS OF THE PUERTO RICAN EXPERIENCE

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Dedication

This thesis is dedicated to Professor Alexandra and my family. Thank you all for providing support, patience, and understanding throughout this very emotionally challenging year. Particularly, I'd like to thank my sister Cristina for serving as proofreader and editor of this work.

Abstract

In this thesis, migration in Puerto Rico is modeled, both in the short-term and long-term, using time series analysis and the Ornstein Uhlenbeck Stochastic Process. Results show that, in agreement with previous works, migration continues to be a very difficult and volatile variable to model. However, it is shown that contrary to most of the literature, migration is a variable that does have a long-term impact on Puerto Rico's economy, demography, and actuarial industry. In the end, a numerical example with insurance applications is provided as well as future ideas on how this subject can be further explored in Puerto Rico.

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Introduction

In this thesis, net migration to/from Puerto Rico is examined both in the short-term and the long-term. Migration is presented as an important variable for economic development, demographic change, and long-term government funding and planning in Puerto Rico. Furthermore, it is shown that, in Puerto Rico, migration in the last decade has had more impact on demographics than deaths and births.

Any change in population can be considered a function of the amount of (1) births, (2) deaths, (3) immigrations, and (4) emigrations in a period of time. In general, births and deaths have taken center stage in most actuarial, demographic, and health studies. However, the experience in Puerto Rico, as shall be seen throughout this work, has brought about a case for a closer look at migration.

In Puerto Rico, migration started to become an observable and quantifiable variable by 2014 with the publication of The Migrant's Profile (*Perfil del Migrante)* [52 – 57]. Notwithstanding the relative youth of the data, it is known, from Passenger data and the decennial Census, that Puerto Rico has had a long history of mass migrations; the biggest, prior to the one seen in the last decade, being in the 1950s [28]. These facts serve as motivation for this work.

This Thesis is organized into four (4) chapters and a conclusion. The chapters and their contents are as follows:

- In Chapter 1, a brief introduction of Puerto Rican history post-U.S. invasion, a summary of the pension crisis in Puerto Rico, and a small numerical example on the economic impact of migration are outlined. Particular attention is given to the four (4) public laws and reforms that have set the benefits currently being paid in the Government's pay-as-you-go system, as well as the reasons that brought about these changes.
- In Chapter 2, a literary review on migration models is presented. The core of the review is centered around four (4) main models: Probabilistic Extrapolation of Time Series, Expert Opinion Modelling, Bayesian Forecasting, and Econometric Models with Covariate Information. The chapter closes with a literary review of migration from the point of view of Puerto Rico.
- In Chapter 3, the different sources of data are described. Three official institutions gather and publish data on Puerto Rican population movements. These are: (1) The U.S. Census Bureau, (2) The U.S. Bureau of Transportation Statistics' Research and Innovative Technology Administration (RITA), and (3) The Port Authority of Puerto Rico (La Autoridad de los Puertos de Puerto Rico).
- In Chapter 4, Puerto Rico's net migration is estimated both in the short-term and long-term. Short-term forecasts of monthly net passengers are represented as an ARIMA process, while the long-term behavior of net migration is modeled in terms of an Ornstein-Uhlenbeck process.

The calculations were done using R statistical software.

1 – Historical Background

In this chapter, a summary of the minimum information required to understand the government-sponsored pension crisis in Puerto Rico, the economic problems that have led to this, and the role migration plays in the scenario is presented. This chapter covers a brief history of the Island, the different pension schemes instituted by the government, and the current difficulties the system faces. At the end of this chapter, the role migration plays in this crisis and the population is stated.

1.1 – Brief History of Puerto Rico

Puerto Rico is a Caribbean island discovered and colonized by the Spanish from 1493 to 1898. According to Curet-Cuevas (2003, p. 16) [13], during the Spanish occupation, the island was an agricultural society with overall slow economic and demographic growth rates. This stagnant outlook began to change by 1898 when the United States (U.S.) obtained the Island of Puerto Rico as part of the spoils of the Spanish-American War.

Since the U.S. occupation, four (4) types of organic acts have been enacted as a form of local government. These are (1) the Military Government, (2) The Foraker Act, (3) The Jones Act, and (4) *El Estado Libre Asociado* (ELA) or the Commonwealth of Puerto Rico. Curet-Cuevas (2003, p. 17 – 18 & 195) [13] provides a brief, but concise description of each of the aforementioned system's structure and these are:

1) Military Government (1898) – A provisional government set up immediately after the U.S. acquired the island.
2) The Foraker Act (1900) – Instituted civil government in Puerto Rico.
    a. Appointed a President nominated Governor and Instituted a bicameral senate with the majority of its members also directly appointed by the President of the United States of America.
    b. Inducted Puerto Rico into the U.S.'s Tariff, Monetary, and Judicial System.
3) The Jones Act (1917) – Organized Puerto Rico as a Non-incorporated Territory of the U.S.
    a. Legislation was now composed of members elected by popular vote.
    b. Governor and members of the Supreme Court continued to be appointed by the President, but the Governor was given the right to appoint his staff.
    c. U.S. citizenship was granted to all people born in Puerto Rico.
4) ELA (1952) – Established the modern-day Puerto Rican style Commonwealth.
    a. Gave Puerto Rico the right to draft their own Constitution and appoint all members of local government by popular vote.

The ELA also gave birth to the government-sponsored pension system which, is the main topic of the following sections.

1.2 –The Retirement System of The Government of Puerto Rico Before July 1, 2017

Government-sponsored pensions for Puerto Rican public sector employees were implemented on January 1, 1952, to promote the following long-term objectives (CESPR, 2013, p. 4) [10] (1) economic security for the remaining lifetime of retired government employees, (2) benefits that adjust to the pensioners' needs (medical included), and (3) attraction and retention of

skilled employees for the public sector. The system, as mentioned in Article 4-101 (3 L.P.R.A § 775) [61], is a trust fund, and, as is stipulated, any change in the structure of the benefits or costs within the scheme must be supported by actuarial reports with a clear description of the costs. Participants in the system, as mentioned in Article 1-105 (3 L.P.R.A § 764) [61], include posts of confidence (*puestos de confianza*), regular career employees, and transitory employees in any agency, administration, board, office, or branch of the central government, as well as municipal employees and mayors (transitory employees are excluded at the municipal level), and any employee from existing or future public corporations. According to Article 1-104 (3 L.P.R.A § 763) [61], a public corporation is any government instrument or institution in which the government or its branches serve as an employer.

The pension system, before August 2017, obtained its funds from three (3) main sources (CESPR, 2013, p. 4; Santiago-Rivera, 2016, p. 380) [10, 38]: (1) individual mandatory contributions automatically deducted from paychecks (member contributions), (2) employer contributions (*Aportaciones Patronales* as known in Spanish) and (3) the fund's return on investments. According to financial statements for June 30, 2017, presented in Bowen, Nugent & Warren (2017, p. 14) [6], 54.49% of all contributions to the fund came from employer contributions (the Government), 23.99% from member contributions, 6.91% from investments, and the remaining percentage from miscellaneous sources.

According to Santiago-Rivera (2016, p. 381) [38], Puerto Rico's public pension scheme can be divided into four (4) Acts. Namely, these are (1) Act No. 447, (2) Act No. 1, (3) Act No. 305 (also known as Reform of 2000), and (4) Act No. 3. Each Act sets a different pension scheme and benefits payout. Therefore, the Acts' implementation dates serve as a broad base into which active pensioners can be classified. That is to say that employees fall under one scheme or the other based on the year they entered or left the system. The schedule is as follows:

1) Government Employees that were employed between 1951 and April 1, 1990, fall under Act No. 447.
2) Government Employees that were employed between April 1, 1990 and December 31, 1999 fall under Act No. 1.
3) Government Employees that were employed between January 1, 2000, and July 1, 2013. fall under Act No. 305.
4) Government Employees that were employed from July 1, 2013 fall under Act No. 3.

1.3 – Pension Benefits

As was previously mentioned, government-sponsored pension schemes and benefits are defined according to the year the employee joined the system. Currently, pensioners receive the benefits that were stipulated in their original contract either completely or with some amendments based on the year of retirement (Act No. 3 reduced benefits). According to Act No. 447 Article 2-118 (3 L.P.R.A § 786) [61] payments are made monthly throughout the lifetime of the pensioner. The base benefits determined under Act No. 447 Article 2-101 (3 L.P.R.A § 766) to Article 2-114 (3 L.P.R.A § 774) [61] are as follows:

1) Retirement age
    a. 55 years of age with 25 years of service.
    b. 58 years of age with 10 years of service.
    c. 50 years of age with 25 years of service for high-risk employees such as policemen, firefighters, electric workers, etc.

2) Benefits
    a. The original 447 defined benefits are equal to the maximum between a fixed annuity equal to 1.5% the average annual salary of the first 20 years of service times the years of service plus, if service exceeds 20 years, 2% the average annual salary of the subsequent years times the number of service years after the first 20 years or $500 monthly.
    b. Upon having 30 years of public service, the pensioner is entitled to 65% of their average salary if he retires before the age of 55 and 75% if he retires after 55 years of age. This benefit was known as the Merit Pension. If the pensioner receives full social security, then the fixed rate benefits are adjusted (0.5% basis point reduction).
    c. Due to Act 305 amendments, if a person withdraws from the system, he/she can request a refund of their invested funds plus accumulated interest.
    d. In the case of non-work-related incapacitation, the employee can request the annuity described in (a), and in case of work-related death, the widow is entitled to an annuity equal to 50% of the average salary of the deceased until the widow remarries.

The system described under Act No. 447 was initially financed by the employer (Government) and has continuously led to actuarial deficits (CESPR, 2013, p. 9) [10]. Based on these deficits, the system has been subject to several amendments and new Acts. Below, **Table 1** summarizes some of these changes (Act No.447 Chapters 2 – 5; CESPR, 2013, p. 12 - 23; Santiago-Rivera, 2016, p. 379 – 386) [61, 10, 38].

TABLE 1: Summary of Changes in Pension Scheme and Benefit

	Act No. 1 (1990)	Act No. 305 (2000)	Act No. 3 (2013)
Merit Pension	Remains	Eliminated	Eliminated
Financing	8.275% of employee's paycheck deducted automatically and 9.275% employer contributions.	8.275% of employee's paycheck deducted automatically and 9.275% employer contributions.	10% of employee's paycheck deducted automatically and increasing employer contributions.
Retirement Age	65 years old	60 years old	61 years old for Employees under Act No. 447. 65 years old for Act No. 1 and Act. No 305 67 years old for Act No. 3
Participation in Savings account.	Can transfer to savings account	Mandatory	Not Applicable
Participation in Hybrid program for employees as of June 30, 2013	Mandatory	Mandatory	Mandatory
Incapacity Benefit	No change from Act. 447.	Money invested in fund can be used for any of the options listed in (3) of savings account rights.	Incapacity insurance must be paid separately.

TABLE 1 (continued): Summary of Changes in Pension Scheme and Benefit

	Act No. 1 (1990)	Act No. 305 (2000)	Act No. 3 (2013)
Death Benefit	40% of average salary	Money invested in savings account will be paid as a lump sum to the pensioner's dependents.	Money invested in savings account will be paid as a lump sum to the pensioner's dependents.

The table includes two systems that have not yet been mentioned: (1) Savings account and (2) Hybrid Program. These two systems were replacements for the fixed annuity and merit pension benefits established under Act No. 447. These replacements only applied to the people that did not retire before the new Act was in place and will be discussed in the following paragraphs.

The savings account introduced as part of Act No. 305 required a minimum contribution of 8.275% (Santiago-Rivera, 2016, p. 382) [38] of the employee's monthly paycheck and gave the pensioner the following rights:

1) The right to change once a year the proportion of money put into alternative investments in 10% increments.
2) Investment opportunities include fixed flow investments, or investments in U.S. Treasury Bills, the Funds investment portfolio with a 90% claim on the return or other investments with Board (Pension Fund's Board) approval.
3) At the age of retirement, married pensioners can either opt purchase a joint-life annuity from a Board approved (Pension Fund's Board) insurer or receive a lump sum of the money invested in the fund (not net of taxes). Single pensioners, on the other hand, can purchase a whole life annuity from a Board approved insurer or receive the lump sum payment of their investment in the fund.

Finally, Act. No. 3 put in effect what is known as the hybrid program (Article 5-105, 3 L.P.R.A. § 787e) [61]. The hybrid program is financed by deducting 10% off of the employees' paychecks and employer contributions (EC) based on the following formula, which is an interpretation of the law made for this work (Article 5-106, 3 L.P.R.A. § 787f) [61]:

$$EC_t = X(0.12275 + 0.01(\min(t - 2013, 3))I_{t\geq 2013} + 0.0125(\min(t - 2016, 5))I_{t\geq 2016}) \quad (1)$$

Where

- X is the monthly pay of the employee
- t is the year.
- I is an indicator function.

The benefits are similar to those stipulated under the savings account, with the main difference being that upon retiring from the system the accumulated amount will be divided by a factor and then used to purchase a whole life annuity (Article 5-110, 3 L.P.R.A. § 787j) [61]. According to Santiago-Rivera (2016, p. 384) [38], under this new system, people that leave the public sector before the age of retirement cannot withdraw their invested money, though they can purchase a deferred annuity with whatever amount has been accumulated over $10,000 (Article 5-110, 3 L.P.R.A. § 787j) [61].

1.4 – Reasons for The System's Decline and The New System

On August 23, 2017, after a series of economic downturns, the Government's defined benefits and merit pension system was on the verge of becoming insolvent. This brought about ratification of Act 106-2017 which resulted in the creation of a new pay-as-you-go pension scheme for all employees hired after July 1, 2017 (Bowen et al., 2017, p. 6) [6]. It is stated within the preface of Act 106-2017 that the law is a direct result of the guidelines set by the Puerto Rico Oversight, Management, and Economic Stability Act (PROMESA). To better understand the situation, the following outline on PROMESA and bankruptcy is provided:

1) PROMESA was established by U.S. Public Law No. 114-187 as part of the agreement that allowed the Puerto Rican Government to declare bankruptcy in early 2017.
2) According to an article on BBC Mundo (2017) [2], the Government of Puerto Rico filed for bankruptcy because of their inability to pay the outstanding debt (part of this debt was associated with lack of liquidity in the Pension Fund).
3) PROMESA was tasked with leading the process of debt restructuring and approving infrastructure-related projects (Public Law No. 114-187, section 1 2016) [60].

Before describing the new pension scheme, it is important to mention some of the reasons the Government went into bankruptcy and some of the reasons why the Pension system was close to insolvency. Since this work is set in the context of the pension system, we will not dwell too long on the reasons for bankruptcy and instead just briefly mention them as they appear in Act 106-2017.

According to Act 106-2017 *Exposición de Motivos* (Outline of Motives) the reasons for the Government's bankruptcy are: (1) lack of transparency with stakeholders, (2) continuous deficits, (3) lack of proper financing in COFINA (a public corporation created for debt repayment) and Banco de Fomento (Public Corporation responsible for financing projects), (4) continuous increase in Government's Payroll (lack of funds from the Government), and (5) insolvency of Pension Scheme. According to Santiago-Rivera (2016, p. 399 – 410) [38], the reasons for the pension system reaching a state of insolvency are:

1) Lack of actuarial studies regarding the benefits scheme – Benefits of the system were originally set as fixed, and not proportional to the amount the person contributes.
2) Benefits and their changes did not have a stable and continuous financing source. One must recall that the Government was responsible for most of the system's financing.
3) The Pension system covers expenses that are not legally its responsibility – Benefits under law 447 include medical expense incentives and seasonal bonuses.
4) Misuse of funds – Lack of increase in employer contributions by the local government has made the system dependent on contributions by the employees and the central government (not the local government branch which in effect is the true patron). Additionally, the Government has borrowed from the Pension System's fund and repaid without interests.
5) High quantity of incapacity beneficiaries.
6) Increase in life expectancy – The average Puerto Rican life expectancy was 61.9 years at the time Act. 447 (1951) was approved while, currently the average life expectancy is around 77.2 - 79 years (Santiago Rivera, 2016, p. 407) [38].
7) Bad investment strategies – Employer contribution backed bonds were emitted to invest in high-risk investments pre-financial crash of 2008. This led to a current debt repayment of approximately 9 billion out of a loan that was of approximately 3 billion (Santiago Rivera, 2016, p. 408) [42].

8) Reduction in government employees as a strategy to preserve the country's credit rating and reduce payroll – All employees that left the system before June 30, 2013, were entitled to withdraw their contributions to the system.
9) Early retirement windows used to lower the number of people on the government's payroll.

The pay-as-you-go system established under Act 106-2017 has three components. These are: (1) Outstanding Pensions will be covered by the Puerto Rican Government's General Fund (*Fondo General*), (2) employer contributions will cease, and (3) a new and independent contribution plan for active government employees will be drafted (Act 106-2017, *Los Sistemas de Retiro*, p. 7 – 9 of printout) [62]. According to Article 1.3 of Act 106-2017, to pay the outstanding active pension benefits, the previous scheme's fund was ordered to liquidate its assets and transfer all of its available funds to the General Fund of the Government. Finally, the new system functions as a savings account with 8.5% mandatory monthly contributions and the employee has the right to say how he/she wishes this money to be invested (Article 3.6 of Act 106-2017, p. 19 of printout) [62]. The employee also has the right to withdraw his/her money without penalties and any fees associated with the account are covered by the Government.

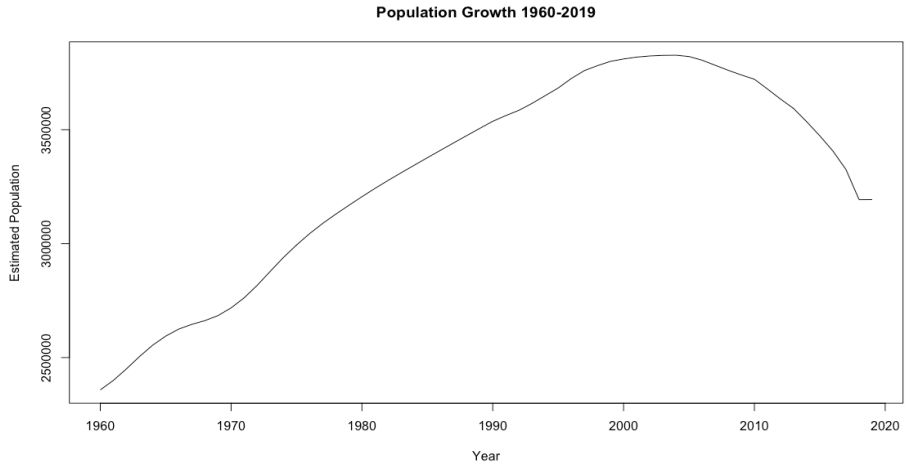
1.5 – Link Between Migration and the Pension Crisis

From the previous sections, it should be clear that the benefits paid were not sustainable and, regardless of the pension scheme, the role of the Government as a co-contributor (source of income) for Government-sponsored pensions in Puerto Rico is essential. Under Act No. 447 (1951), the Government funded the entire program, under Acts No. 1 (1990), 305 (2000), and 3 (2013) the Government functioned as a co-contributor to the fund, and under Act No. 106 (2017) the Government functions as the main benefactor for all existing active pensioners. This means that the General Fund of the Government is, in essence, the driving force behind the solvency of the entire outstanding pension scheme. This leads to the following claim: the pension crisis is a direct result of the Government's poor financial position and loss of wealth, and one factor affecting national wealth is migration.

According to the financial statements published by Hacienda de Puerto Rico for the fiscal year 2019-2020 (2020, p. 1) [20], the General Fund gets its income from six (6) broad sources: (1) income taxes, (2) sales tax, (3) tariffs, (4) lottery, (5) licenses, and (6) fines. Out of these, income and sales tax are the highest contributors to the general fund's health, and these depend largely on the contributions of the people; especially high-skilled, high-income bracket people. Not taking into account proper use of funds or good administration (a topic that without a doubt has an immense impact on the crisis), the government's financial position is largely affected by the overall composition of its people and their incomes. Therefore, the Government's financial position depends on the wealth of its citizens.

Migration shifts the composition of a nation's population, and in the case of Puerto Rico, these effects are substantial. To illustrate this point, population data since 1960 will be presented. **Figure 1** shows how the overall population of Puerto Rico has changed during the years. Historical population estimates were taken from Trading Economics (Trading Economics, accessed September 27, 2020) [44].

FIGURE 1: Population Growth 1960 – 2019



To better assess the driving factors behind these population changes, the following table is provided. **Table 2** shows yearly population estimates (Population) (Trading Economics, accessed September 27, 2020) [44], the yearly rate of change for the population (Change), crude deaths per year (Crude.D), crude births per year (Crude.B) (both taken from Macro Trends, Accessed September 27, 2020) [30, 31], the difference between these two rates (Crude difference), crude migration estimate (Crude.M), migration estimate in absolute terms (Migration Est), and the actual migration estimate provided by the Puerto Rico Community Survey (PRCS; see chapter 3 for further information) [56]. Migration estimate series was obtained from the population change equation (U.S. Census Bureau, 2010) [46]:

$$Population_t = Population_{t-1} - Deaths_{t-1} + Births_{t-1} + Net\ Migration_{t-1} \quad (2)$$

TABLE 2: Select Indicators of Population Change

	Population	Change	Crude.D	Crude.B	Crude.Diff	Migration.Est	PRCS
2018	3,195,153	-129,848	-31,075	24,529	-6,547	-123,301	
2017	3,325,001	-81,494	-30,992	27,361	-3,631	-77,863	-77,321
2016	3,406,495	-66,671	-30,734	30,175	-559	-66,112	-67,480
2015	3,473,166	-61,708	-30,403	33,026	2,623	-64,331	-64,238
2014	3,534,874	-58,203	-30,009	35,927	5,918	-64,121	-64,073
2013	3,593,077	-41,411	-29,454	38,722	9,268	-50,679	-49,194
2012	3,634,488	-44,244	-29,761	40,775	11,014	-55,258	-54,456
2011	3,678,732	-42,793	-30,059	42,853	12,795	-55,588	-53,569
2010	3,721,525	-18,885	-30,159	44,679	14,520	-33,405	-28,153
2009	3,740,410	-20,456	-30,275	46,544	16,270	-36,725	-29,966
2008	3,760,866	-22,129	-30,400	48,445	18,045	-40,174	-34,128
2007	3,782,995	-22,219	-30,590	49,639	19,049	-41,268	-31,252

TABLE 2 (Continued): Select Indicators of Population Change

	Population	Change	Crude.D	Crude.B	Crude.Diff	Crude.M	PRCS
2006	3,805,214	-16,148	-30,731	50,767	20,035	-36,183	-36,603
2005	3,821,362	-5,516	-30,787	51,755	20,967	-26,483	-12,444
2004	3,826,878	783	-30,792	52,662	21,870	-21,087	
2003	3,826,095	2,394	-30,785	53,543	22,759	-20,365	
2002	3,823,701	4,927	-30,764	54,509	23,745	-18,818	
2001	3,818,774	8,169	-30,717	55,425	24,708	-16,539	

Table 2 reveals the following facts:

1) Crude death rates have remained fairly constant throughout the last two decades.
2) Crude birth rates have gradually been decreasing.
3) The difference between population change, births, and deaths (used for net migration) is a close match for the actual migration estimates provided by the PRCS. A Notable exception is found in 2005 which is the starting year of the program (data was an estimate from Public Use Micro Sample (PUMS) not from the PRCS directly; Rodríguez Ayuso et All, 2011) [37].
4) The effects of migration exceed births since 2011.

Table 2 supports the following claim: in Puerto Rico net migration is an important factor when analyzing demographics.

In summary, the pension scheme of the Government of Puerto Rico (originally a fully funded system and then recently a pay-as-you-go system; more on chapter 2) was designed with a strong assumption on the sustained wealth of the Puerto Rican Government. It supposes a Government with plentiful resources and consequently a large pool of people with good incomes. In Puerto Rico and throughout the last two (2) decades, migration has shown how quickly the composition of a nation's demographics can change, sometimes exceeding death and or any other demographic variable.

2 – Migration in the World and Puerto Rico

In this chapter, a literary review of migration models is conducted. The core of the review is centered around analyses done in a post ratification of the European Union environment and the four most common types of migration models employed by researchers and institutions. From the four (4) main models that will be presented, particular importance is given to three (3): Probabilistic Extrapolation of Time Series, Bayesian Forecasting, and Econometric Models with Covariate Information. A literary review on Puerto Rican migration papers is also presented, though the material on this area is rather scarce. At the end of this section, some closing remarks are provided and the direction this work takes is stated.

2.1 – Literary Review

The study of Migration is by no means a simple task. According to Disney et al. (2015, p. 5) [15], migration estimates, when compared to fertility and mortality, are far more volatile and uncertain. Sources of uncertainty they identify are (1) inherit uncertainty of future events, (2) uncertainty in migration data, and (3) uncertainty in forecasting models (Disney et al., 2015, p. 9 – 10; see also Bijak et al., 2019, p. 473) [15, 4]. These sources of uncertainty will be commented on in the following paragraphs.

The error in migration forecasting due to uncertainty of future events can be attributed to the fact that migration is "particularly susceptible to events which are difficult to predict in terms of timing as well as the scale of impact, such as fluctuations in the economic cycle, the incidence of armed conflict [including riots and wars], and changes in policies or political circumstances" (Disney et. al., 2015 p. 5 & 9) [15]. Vanella & Deschermeier (2019, p. 2) [51] contextualize inherent uncertainty in terms of policies that are either pro or against emigration/immigration. Examples against open migration policy include fear of terrorism and cheap labor. Conversely, open migration policies are centered around the rejuvenation of demographics. Another interesting factor, attributed to the inherent uncertainty of future events, is the establishment of networks of migrants (communities) in foreign nations that further facilitate the migration process of future migrants (Disney et al., 2015, p. 10) [15]. In the case of Puerto Rico, the network effect is very large since, according to estimates from the *Hispanic or Latino Origin by Specific Origin* for year 2018 from the U.S. Census Bureau's American Community Survey [47] (see chapter 3 for more information), the Stateside population of Puerto Ricans is approximately 5.8 million people while the population (including other ethnicities) in the island is approximately 3.1 million (Census Bureau, 2018) [46].

The second source of uncertainty in migration data, according to Disney et al. (2015, p. 10) [15], stems from two (2) components: (1) different definitions regarding migration (particularly those related to the length of stay), and (2) inaccuracy, inconsistency or incompleteness of data. As examples of the former claim, the following definitions of migration are provided:

1) Migration in terms of Migrants according to the United Nations (UN) is defined as "any person who is moving or has moved across an international border or within a State away from his/her habitual place of residence, regardless of (1) the person's legal status; (2) whether the movement is voluntary or involuntary; (3) what the causes for the movement are; or (4) what the length of the stay is" (United Nations, 2020) [45].
2) Encyclopedia Britannica defines migration as "the permanent change of residence by an individual or group" (Bauer, 2020) [8].

3) The U.S. Census Bureau defines migration as "[movement across] a boundary, such as a county or a state line, and is either domestic migration (movement within the U.S.) or international migration (movement between the U.S. and other countries). Mobility includes both short and long-distance moves" (U.S. Census Bureau, 2020b) [49].

On the other hand, an example of inconsistencies in the data can be found in *Perfil del Migrante 2017* [56] since the official migration estimates, gathered in a document known as the Puerto Rico Community Survey (PRCS; see chapter 3 for more information) [56], was interrupted in 2017 due to the damages caused by Hurricanes Irma and Maria (Velázquez Estrada, 2019, p. 10) [56]. Vanella & Deschermeier (2019, p. 3) [51] provide another example of how the data may be incomplete or biased in terms of undercounting or double counting. Explicitly, the researchers argue that a source of the aforementioned bias can be found in German migration since estimates result from extrapolating voluntary registering and unregistering of the migrant's address.

According to Disney et all (2015, p. 11) [15], data uncertainty should be subject to a four (4) criteria analysis that includes (1) definition or adherence to the official definition of long-term migration (at least a year) by the UN, (2) coverage or proportion of migrant population that is identified or surveyed by the measurement, (3) bias in the data, and (4) accuracy (see also Bijak et al., 2019, p.472) [4].

Model uncertainty is the third and final source of uncertainty. It is based on the principle that different models can output different results and uncertainty estimates even when applied to the same datasets. The models used in migration forecasting will be further examined throughout this section.

Disney et al. (2015) [15], in their report *Evaluation of existing migration forecasting methods and models,* identify two (2) broad categories for migration forecasting, which are (1) deterministic models and (2) probabilistic models. Deterministic models include judgement-based scenarios that do not aim to predict how migration will be impacted by policies or economic events but rather relies on using past observations to make deterministic evaluations (Disney et. al., 2015, p. 19) [15]. Probabilistic models, on the other hand, are constituted by (1) Probabilistic Extrapolation of Time Series, (2) Probabilistic Expert-Based Forecasts, (3) Bayesian Forecasting, and (4) Econometric Models with Covariate Information (Disney et al., 2015, p. 20 – 25) [15].

Probably the most common and favored probabilistic model for migration analyses is Probabilistic Extrapolation of Time Series. According to Bijak et al. (2019) [4], the main reason Time Series analysis is a favored model is because "even if credible explanations of past migration flows existed, their tenets would be difficult to extrapolate into the future" (Bijak et al., 2019, p. 470 - 471) [4]. Moreover, Bijak et al. (2019, p.473) [4] argue that ARIMA models tend to be the standard approach for migration analysis and that these tend to not exceed a parametrization of (1, 1, 1) (Bijak et al., 2019, p.474) [4]. An ARIMA (p, q, d) type model has the following equation (Hyndman & Athanasopoulos, 2018) [23]:

$$\phi(L)\Delta^d X_t = c + \theta(L)\varepsilon_t \quad (3)$$

Where:

1) $\phi(L)$ lagged AR coefficient and $\theta(L)$ lagged MA coefficient
2) X_t Stochastic Process representing migration at t.
3) Δ^d number of differencing on time series.
4) c a constant.
5) ε_t error term distributed as white noise.

Bijak et al. (2019, p. 477) [4] found that a general ARIMA (1, 0, 0) produced overall low errors and good calibration when the series was long and had stationary features. They also found that for long series and stationary features, ARIMA (1, 0, 1), ARIMA (0, 1, 0), and ARIMA (1, 1, 0) produced reasonable results. Conversely, results were unsatisfactory for any model with non-stationary behavior. A typical weakness found in this type of analysis is that ARIMA models do not incorporate the effects of known shock events into the model or its prediction and errors (Bijak et al., 2019, p. 473; Disney et al., 2015, p. 21) [4, 15].

Expert-based approaches are typically characterized by a simple equation of the form:

$$m_t = \bar{m}_t + X_t \tag{4}$$

which states that migration flow (m_t) is a function of the expert given average trajectory of the migration process $(\bar{m}_t)$ and a stochastic process (X_t) (Disney et al., 2015, p. 21) [15]. A pure expert approach does not use the complete data and, as such, can become a big source of bias when forecasting (Bijak et al., 2019, p. 473; Disney et al., 2015, p. 21) [4, 15].

Bayesian Forecasting, according to Bijak et al. (2019, p. 473) [4] can be considered a mix of both expert opinions and statistical models (see also Disney et al., 2015, p.21 – 22) [15]. This type of model has the advantage of producing reasonable results even when the amount of observations is small by incorporating expert opinion (Bijak et al., 2019, p. 473; Disney et al., 2015, p. 22) [4, 15]. Ways in which expert judgment can be included, according to Disney et al. (2015, p. 22) [15], are by providing prior distributions of the different parameters based on expert judgment. Bijak & Wiśniowski (2010a) [5] used a Bayesian Approach for seven (7) European countries to estimate total immigration. Within this study, expert opinion was used for the trend component of the AR model by providing a deterministic prior function $g(t)$ for five (5) out of the seven (7) countries being studied (Bijak & Wiśniowski, 2010a, p. 779) [5]. The authors found that immigration was barely predictable and almost always a non-stationary Random Walk (Bijak & Wiśniowski, 2010a, p. 793) [5]. A few years later Bijak, et al. (2019, p. 477) [4] further strengthened these results by showing that a Bayesian approach on a short series (less than 20 observations) produces middle-sized errors for random walks and large errors for any other ARIMA parametrization.

Finally, econometric models have the distinction of being able to predict migration while verifying economic theories (Disney et al., 2015, p. 24; Bijak et al., 2019, p. 473) [15, 4]. Models that fall within this category usually include country-specific factors, time-specific factors, and cross-sectional effects as well as pure economic factors such as income, employment rates, and population size (Disney et al., 2015, p. 24) [15]. According to de Beer (2000, p. 12 - 14) [14], there are seven (7) main economic theories used to explain migration. These are:

1) Geographical differences in supply and demand for labor.
2) Rational cost-benefit analysis of migration.
3) Diversification of household incomes.
4) Demand for cheap labor in developed economies.
5) Globalization and the flow of capital and goods.
6) Effect of networks that reduce the costs and risks of migration.
7) Private organization outsourcing labor (expats).

General econometric migration models are of the following form (Brücker & Siliverstovs, 2006, p. 39 - 40) [7]:

$$m_{it} = \sum_{p=1}^{n} \phi_p X_{pft} + \sum_{q=1}^{m} \theta_q X_{qit} + \rho mst_{i,t-1} + \mu_i + \varepsilon_{it} \quad (5)$$

Where

1) m_{it} is the migration amount of a country i at time t.
2) X_{pft} and X_{qit} are observable time varying characteristic (usually wages and employment rates for the standard model). Here *f* and *i* refer to countries.
3) ϕ_p and θ_q are the regression's coefficients.
4) μ_i are time invariant unobservable variables.
5) $\varepsilon_{it} \sim N(0, \sigma)$ is the error.
6) $mst_{i,t-1}$ is the lagged migration stock.

Brücker & Siliverstovs (2006, p. 39) [7] used several econometric models when estimating migration to Germany from 18 European sources and found that the standard migration econometric model was not properly specified since migration flow exhibited stationary behavior while the explanatory variables exhibited non-stationary behavior. Čajka et al. (2014) [9] used a Seemingly Unrelated Regression on data from 2008 to 2012 to study the effects of visa abolition on Eastern European countries and found that the effects of this institutional policy did not greatly affect migration tendencies. Disney et al. (2015, p. 34; see also Bijak et al., 2019) [15] [4] tested an Autoregressive Distributed Lag (ADL) model with perfect foresight (the authors use actual realizations of exogenous variables) and found that errors in the model were of mid-size for most of their parametrizations. A particular problem that one can find in this type of model is determining what can be considered appropriate values for the forecast of covariates (Disney et al., 2015, p. 26) [15].

Besides the models that were previously mentioned, some other models such as the Principal Component Analysis (Vanella & Deschermeier, 2019) [51], Lee Carter Model (Wiśniowski et al., 2015) [58] and Utility Theory (Hatton, 2016; Bijak & Wiśniowski, 2010) [21, 3] have been applied to large data sets with different migrant subgroups. The Utility Theory migration approach by Bijak & Wiśniowski (2010b) [3] is particularly interesting since the analysis is conducted under a Bayesian framework and in terms of utility functions based on migration policies and other qualitative information.

2.2 – Migration Literature in Puerto Rico

In Puerto Rico, migration studies have mostly been a topic addressed by sociologists, anthropologists, and economists. These studies tend to be of a qualitative nature with the occasional supplement of descriptive statistics (examples include Duany, 1997 and 2003; León-López & Dávila, 2015; Santiago & Rivera-Batiz, 1996) [16, 17, 29, 39]. An exception to this type of research can be found in *Economic factors behind Migration between Puerto Rico and the United States* (translated from Spanish) by Pol (2001) [33]. In this paper, the researcher used a Linear Regression to assess the impact of selected economic variables on migration. Variables found to be significant include: unemployment, wages, and the age group of 25 to 39 years old.

Beside the studies by independent researchers, the Institute of Statistics of Puerto Rico also conducts a yearly analysis of migration (usually two (2) years delayed). The reports published by the Institute are, like most of the available literature, limited to descriptive statistics and interpretations of the information gathered by the PRCS (Velázquez Estrada, 2014 – 2019) [52 –

56]. Taken all together, it seems that a probabilistic approach to migration has not been thoroughly explored in the context of Puerto Rico.

Notwithstanding the lack of probabilistic analysis, some important findings have been made throughout the existing literature. León-López & Dávila (2015) [29] conducted a descriptive analysis on Puerto Rican Migration based on data from the PRCS and found that the driving force behind the changes in population in the XXI century (since 2004) has been migration. Pol (2001) [33] found that migration seems to be cyclical with a 20-year lapse. This lapse usually implies that people leave the island during their productive years and return once retired (Pol, 2001, p. 15) [33]. A final important finding is the lack of agreement between existing migration data in Puerto Rico (Velázquez Estrada, 2014 – 2019) [52 – 56].

As the years pass, the need for a thorough analysis of Puerto Rican migration is becoming more and more pressing. As previously shown in the closing section of chapter 1 and as also mentioned by León-López & Dávila (2015, p. 71) [29], Puerto Rico faces an unprecedented shift in its population due to the Island's low birth rate, high death rate, high life expectancy (approximately 79yr.), and high negative migration balance. This work aims to promote a probabilistic approach to migration in Puerto Rico.

2.3 – Closing Remarks

Once again, it must be stated that migration is a very complex phenomenon filled with a considerable amount of uncertainty. To deal with uncertainty, Disney et al. (2015, p. 7) [15] recommend a three-stage process for migration forecasting. This process is:

1) Identify whether migration trends are stable or the result of known shocks.
2) Assess the data with their relative strengths and weaknesses.
3) Choose an appropriate model that conforms to the overall behavior of the data.

Despite the validity of this three-stage process, the authors warn that the former does not lead to forecasts with small errors, but rather leads to forecasts that are not either unjustifiably precise or unreasonably uncertain (Disney et al., 2015, p. 7) [15]. After all, as stated by Bijak et al. (2019, p. 10) [4]: "[t]here is no perfect model and choosing which model to apply is a matter of judgement."

Given the literature and available data (more information available in chapter 3), the analysis of Puerto Rican migration will be done from a Time Series analysis point of view. Particularly, as done by Disney et al. (2015) [15] and Bijak et al. (2019) [4], an ARIMA model will be used to estimate the parameters and provide short term forecasts. In addition, this work provides a transformation of the short-term estimates to an Ornstein-Uhlenbeck process. This process is considered the continuous-time equivalent of the ARIMA (1, 0, 0) process and is proposed as a way to describe the long-run monthly base distribution of migration.

Computations are done in R and use the packages *stats* (R Core Team, 2019) [34], *forecast* (Hyndman et al., 2019) [24], and *tseries* (Trapletti & Hornik, 2019) [42]. For more information on these packages, refer to the Cran library. Some modifications were done when using the *forecast* package as recommended by Shumway & Stoffer (2017, online) [41].

3 – The Data

In this chapter, the different sources of data will be examined and presented. Currently, three official institutions gather and publish data on Puerto Rican population movements. These are: (1) The U.S. Census Bureau, (2) The U.S. Bureau of Transportation Statistics' Research and Innovative Technology Administration (RITA), and (3) The Port Authority of Puerto Rico (La Autoridad de los Puertos de Puerto Rico).

3.1 – The Puerto Rico Community Survey (PRCS)

The Puerto Rico Community Survey (PRCS) by the U.S. Census Bureau is a Puerto Rico specific adaptation of the American Community Survey (ACS). Both the PRCS and ACS share the same scope and overall content. In terms of scope, the PRCS serves to (1) manage change by providing accurate data for municipal leaders, and (2) assess whether a community is currently receiving the appropriate amount of federal aid (federal funds) (U.S. Census Bureau, 2020a, online information) [48]. Content-wise, the PRCS gathers data corresponding to social categories such as the respondent's age, sex, race, family composition, and marital status, as well as economic categories such as income, benefits, health insurance, education, occupation, means of transportation, housing, and expenses (United States Census, retrieved 2020a) [48].

Three (3) main differences between these surveys should be highlighted (U.S. Census Bureau, 2020c, p. 1) [50]. The first difference is the sample size. The PRCS has a sample size of 36,000 addresses, whereas the ACS has a sample size of approximately 3.5 million addresses. Despite this absolute difference, the ratio remains equal and lies around 1% and 2% of the population. The second main difference is that the PRCS can only be submitted via mail or in-person; while the ACS allows for online submissions as well. The final difference is that the PRCS only gathers information on sex and age, whereas the ACS also includes race.

If we refer to the previously mentioned contents, it becomes evident that the PRCS does not, in any direct way, ask a respondent whether he/she has migrated before, is in the process of migrating, or plans to migrate in the future. Since no question is directly related to migration, both emigration and immigration estimates in the PRCS are address-based. As explained by Velázquez-Estrada (2019, p. 9) [56], an address-based migration estimate is obtained by comparing the sample respondent's previous address with his/her current address. Therefore, the PRCS defines migration in terms of permanent and legal change in address or residence within the United States and its territories. **Table 4** presents the PRCS migration estimates.

Table 4: PRCS Yearly Migration Estimates

Year	Emigrants	Immigrants	Net
2005	47,208	34,764	-12,444
2006	67,110	30,507	-36,603
2007	60,388	29,136	-31,252
2008	67,862	33,734	-34,128
2009	62,074	32,108	-29,966
2010	59,885	31,732	-28,153
2011	76,218	22,649	-53,569
2012	74,500	20,044	-54,456
2013	73,846	24,652	-49,194
2014	83,844	19,771	-64,073

Table 4 (continued): PRCS Yearly Migration Estimates

Year	Emigrants	Immigrants	Net
2015	89,000	24,762	-64,238
2016	88,676	21,196	-67,480
2017	97,488	20,167	-77,321

The official migration estimates show a clear increasing trend in negative net migration. The overall effects are the result of a rapidly increasing emigration and a decreasing rate of immigration.

As a final note, Velázquez-Estrada (2019, p. 8) [56] points out that data recollection by the PRCS was put on hold after Hurricane Maria and Hurricane Irma passed in September 2017. Due to this atmospheric phenomenon and the damage it caused on the island, the data for 2017 only takes eight months into account.

3.2 – Air Travel Passenger Data

Though not a direct estimate of either immigration or emigration, air travel passenger data, given an appropriate aggregation, can be used as a proxy for net migration (Velázquez-Estrada, 2014 - 2019; Pol, 2001; Rayer, 2018) [56, 33, 35]. It should be stated that aggregation does not remove the underlying problems that are found in air passenger travel, but it does minimize it to a certain degree. Velázquez-Estrada (2019, p. 13) [56] identified the following problems regarding the use of this data:

1. Insensitive to changes in means of transportation – Passenger data does not register if passengers are using more than one mode of transportation when traveling. This means that if a person leaves from Puerto Rico on a plane and returns on a Boat or Cruise, this person would be, on a net migration basis, considered an emigrant. However, since the main form of transportation to and from Puerto Rico is via the International Airport Luis Muñoz Marin, one would not expect this effect to be large.
2. Highly sensitive to atmospheric events – The effect of Hurricane Irma and Maria led to huge differences between estimates by the PRCS and estimates using transportation data (for more information refer to **Table 5**).
3. Sudden changes in tourism – Puerto Rican air travel tends to have high peaks and lows during the summer and winter.
4. Failure to differentiate between extended periods of Vacation/Business travel and an actual change in location – Depending on the length and the booking time of a trip, a person might count as an "emigrant" in one month and then an immigrant in the next. Lack of information regarding the net deficiency/surplus makes travel and migration difficult to differentiate.
5. Other miscellaneous – Besides the aforementioned reasons, many other isolated cases may affect the data's quality. These include people that arrive or leave the country illegally or move out for extended periods for reasons other than leisure and business.

To circumvent these problems, Velázquez-Estrada (2019, p. 13) [56] proposes the yearly sum of the monthly net balances as an acceptable correction for net migration estimates.

Air travel passenger data has been provided by the official transportation statistics authorities of Puerto Rico and the United States since the 90s. On the stateside, The Bureau of Transportation Statistics (BTS) under the Research and Innovative Technology Administration

(RITA) has published monthly transportation data to and from the U.S. (territories like Puerto Rico included) since January 1990. On the local side, The Port Authority of Puerto Rico (PAPR) has been collecting similar data since January of 2000. In **Table 5**, an excerpt from the yearly estimates of net migration is presented for both of these institutions and the PRCS (Velázquez-Estrada, 2019, p. 42) [56].

Table 5: Different Migration Estimates

Year	BTS Net migration (in thousands)	PAPR Net migration (in thousands)	PRCS Net
2010	-46	-31	-28,153
2011	-38	-55	-53,569
2012	-47	-62	-54,456
2013	-49	-58	-49,194
2014	-83	-90	-64,073
2015	-93	-93	-64,238
2016	-84	-98	-67,480
2017	-294	-342	-77,321

3.3 – Closing Remarks

From the discussions in this chapter, it should be clear that migration is a variable that is very difficult to observe. Address-based estimates generally undermine the magnitude of migration in Puerto Rico by (1) only taking into account the movements within the U.S. and (2) considering financially committed shifts (i.e. the migrant has an address of his own). On the other hand, transportation-based estimates can be inflated by noise such as leisure and other short-term driven movements.

Despite the weaknesses and problems previously mentioned, monthly transportation data provided by BTS's RITA from January 1991 to October 2019 has been chosen for the development of this work. The main reasons for this choice are the data's rich history, and its capability to include migration movements outside of the United States.

4 – The Model

In this chapter, Puerto Rico's net migration will be estimated using two models. The first proposed model is an ARIMA for short-term forecasts of monthly net passengers, and the second model is an Ornstein Uhlenbeck transformation of the estimated ARIMA model for the long-term behavior of net migration.

4.1 – The Model's Data

It was stated in Chapter 3 that Monthly Passenger Air Travel data from January 1991 to October 2019 [25] would be used to approximate the net migration distribution of Puerto Rico. It was also stated in Chapters 2 and 3 that Migration can be measured and interpreted in several ways (address-based, movement-based, time-based, etc.) with the biggest difference between interpretations stemming from the concept of time. To eliminate this source of ambiguity, migration in this analysis will be formally defined in terms of the long-term deficit or surplus of people (at least 1 year) (Disney et al., 2015; Bijak et al., 2019) [15, 4] and will be approximated as the yearly sum balance of monthly air passenger movements (Velázquez-Estrada, 2019) [56]. The data for the entire period is presented in **Figure 2**.

Figure 2: Monthly Net Transportation

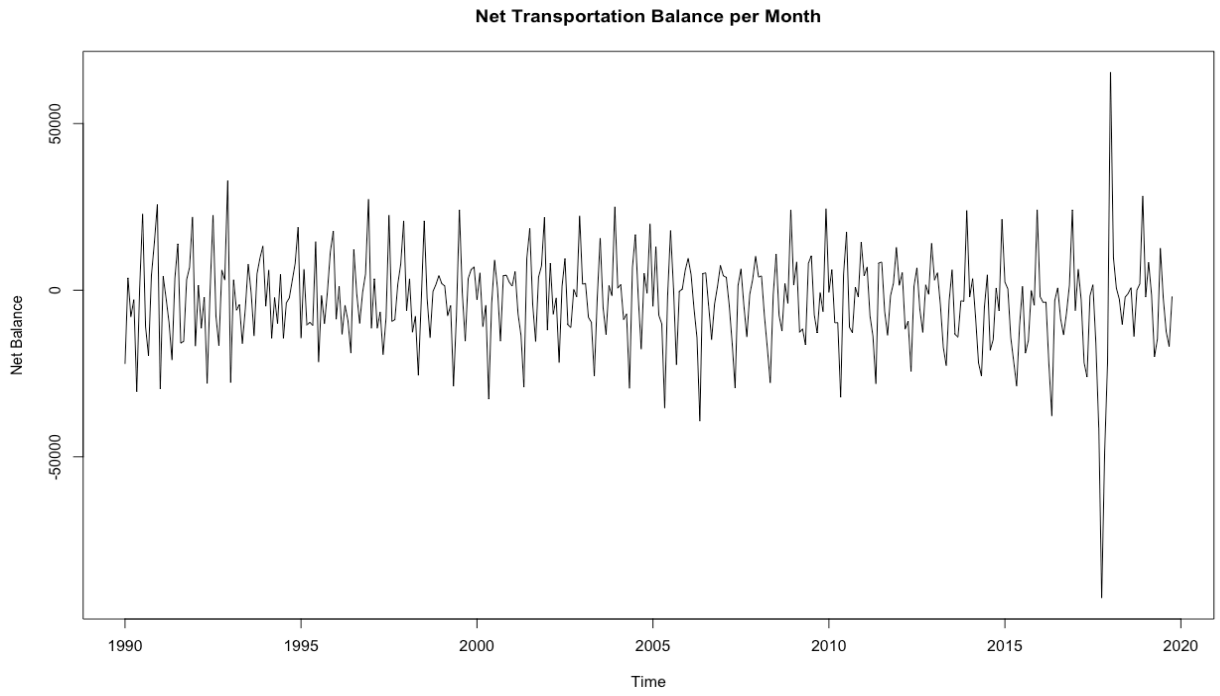


Figure 2 reveals some important insights on the overall behavior of Puerto Rican net passenger movements. The first immediate insight is that passenger movements are very seasonal and considerably volatile. Peaks in the series correspond to months 12, 1, 6, and 7. The second insight is that the overall time development of monthly net passenger balance seems to return to some sort of mean value relatively close to 0. The final insight is that the effects of natural

catastrophes are a considerable source of instability. The large spike seen towards the end of the series corresponds with the aftermath of Hurricane Maria and Hurricane Irma.

Before the data is suitable for analysis, some transformations must be made; such as removal of extreme values and seasonal decomposition. Seasonal adjustment of the data was done using *stl* (Seasonal Decomposition of Time Series By Loess) from the *stats* package (R Core Team, 2019) [34], and removal of extreme values within the data (particularly those seen towards the end of the series) were done using the *tsclean* function from the *forecast* package (Hyndman et al., 2019) [24]. The main reason behind removing the extreme values is because Hurricanes of the scale and power of Maria are considered a 1 in a 100 years event (previous Category 4 – 5 Hurricanes that have affected the Island occurred in 1851 and 1928; Fritz, 2017) [27].

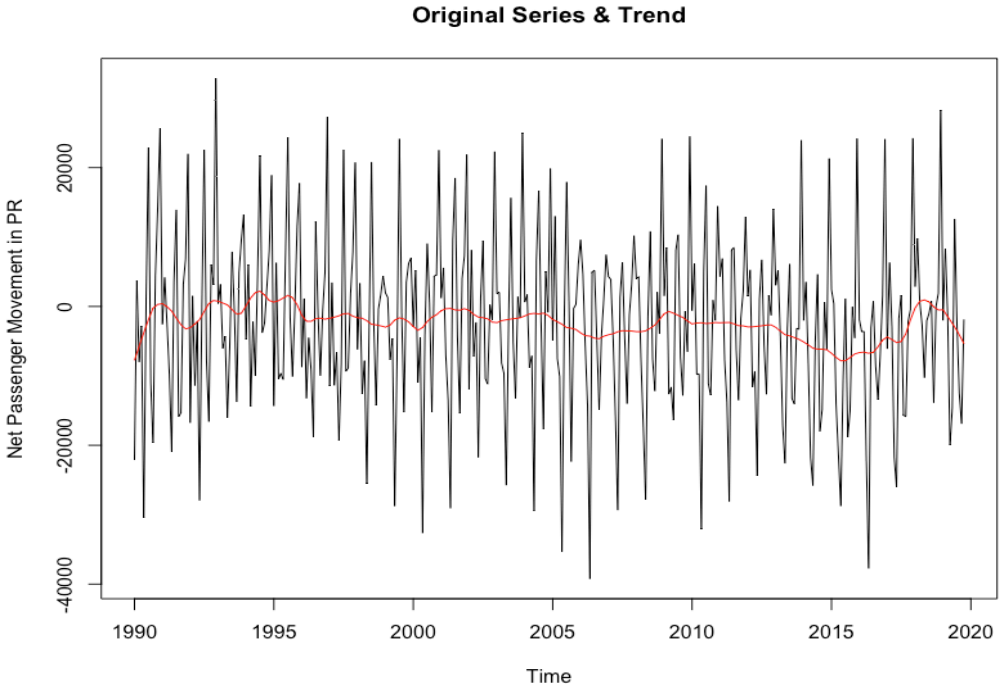
A proposition for dealing with the noise present in transportation data is to model the trend component of the series. As will be briefly shown, this modification leads to the same correlation and overall result as the sum of the 12-month net balance used by Velázquez Estrada (2014-2019) [52 – 56] and Pol (2001) [33]. The results are shown in **Table 6**.

Table 6: Correlation Between Corrected Passenger Movements and the PRCS Estimates

Estimates	Yearly Sum of Monthly Net Passengers	Trend Component of the series
Correlation Coeficient	0.635443	0.6338686
P-value	0.0196	0.02
95% Confidence Interval	(0.1299589, 0.8787578)	(0.1273662, 0.8781559)

In **Figure 3** the series with removed outlier values, and the corrected series are shown.

Figure 3: Outlier Removed Series and Corrected Series



The black series is the cleaned time series, while the red line is the trend component of the series. The trend component was selected as the model's data as a way to better deal with the noise problems that surround passenger data (leisure travel, business travel, long term vacations, etc. refer to chapter 3 for more information). The trend lets us see how, on average, monthly deficits are developing.

The trend component comes from an STL decomposition of the time series (Cleveland et al., 1990, p. 6) [11]. Specifically, it is a smoothing technique using locally fitted polynomials (more information on Cleveland et al., 1990) [11].

4.2 – The Model Choice and Parameter Estimation

Stochastic Differential Equations (SDEs) are continuous time processes composed of drift and diffusion terms. The general representation of an SDE is

$$dX_t = f(t, X_t)dt + g(t, X_t)dB_t \tag{6}$$

with $f(t, X_t)$ and $g(t, X_t)$ being functions that depend on time (t) and the unknown stochastic process X_t.

As was mentioned in chapter 2, the long-term migration of Puerto Ricans will be modelled as an Ornstein-Uhlenbeck (OU) stochastic process. In terms of the differential equation, the OU process satisfies the following expression.

$$dX_t = \theta(\mu - X_t)dt + \sigma dB_t \tag{7}$$

which by defining a process $Y_t = -(\mu - X_t)e^{\theta t}$ has analytical solution (Gardiner, 2004, p.106) [19]

$$Y_t = Y_0 + \sigma \int_0^t e^{\theta s} dB_s \tag{8}$$

and

$$X_t = X_0 e^{-\theta t} + \mu\left(1 - e^{-\theta t}\right) + \sigma e^{-t} \int_0^t e^{\theta s} dB_s \tag{9}$$

with a Normal distribution

$$X_t \sim N\left(X_0 e^{-\theta t} + \mu\left(1 - e^{-\theta t}\right); \frac{\sigma^2}{2\theta}\left(1 - e^{-2\theta t}\right)\right) \underset{t \to \infty}{\to} N\left(\mu, \frac{\sigma^2}{2\theta}\right) \tag{10}$$

The OU process has several interesting properties that can be applied to the migration topic. The first property is that the OU process is Markovian. According to Siegmund (2018, online) [40], "[a) stochastic process is called Markovian […] if at any time t the conditional probability of an arbitrary future event given the entire past of the process… equals the conditional probability of that future event given only *X(t)*" (direct quote). In mathematical terms, Markov processes in continuous time satisfy (Faris, 2001, p. 29) [18]

$$P(X_{t+h} = j | X_t = i) = q_{ij}h + o(h), \qquad for\ i \neq j \tag{11}$$

Where

1) $q_{ij}h$ is the transition rate from state i to j.
2) $o(h)$ is little o of the process

A second interesting property of the OU process is that the path moves towards a long-run mean. This property can be shown by taking the limit as *t* approaches ∞ on equation (10). In many actuarial analyses, the effects of Migration are considered to either be 0 or marginally positive (Plamondon et al., 2002) [32]. This implies that, in the long run, migration rates must reach a limiting value since, in practical terms, unbounded growth or contraction would lead to social and economic instability.

Estimation of the OU process and ARIMA models can be done by any suitable method. Popularly the equation's parameters have been estimated using Maximum Likelihood Estimator (MLE), Method of Moments, and/or Least Squares (LS) (Holý & Tomanová, 2020) [22].

The connection between the OU process and an ARIMA (1, 0, 0) can be shown by either taking the integrated process (9) and setting $\mu\left(1 - e^{-\theta t}\right) = c$, $e^{-\theta t} = \phi$ and $V_t = \sigma e^{-t} \int_0^t e^{\theta s} dB_s$ (Holý & Tomanová, 2020) [22] or applying the Euler-Maruyama discretization method to equation (7) (Ayranci, & Özgürel, 2014, p. 91) [1]. Either substitution will produce a simplified ARIMA (1, 0, 0) process of the form:

$$X_t = c + \phi X_{t-1} + V_t \tag{12}$$

To estimate equation (12) via MLE, a numerical solution to the following equation must be found (Zivot, E. 2005) [59]

$$\ln L(\phi | y) = -\frac{1}{2}\left(n \ln 2\pi + \ln \frac{\sigma^2}{1-\phi^2} + \frac{1-\phi^2}{\sigma^2}\left(y_1 - \frac{c}{1-\phi}\right)^2 + (n-1)\ln \sigma^2 + \frac{1}{\sigma^2}\sum_{t=1}^{n} U_t^2 \right) \tag{13}$$

Where $\sum_{t=1}^{n} U_t^2$ corresponds to the sum of errors squared that is minimized when obtaining the LS parameters.

The LS method of determining the parameters is achieved by minimizing the following expression (Ayranci, & Özgürel, 2014, p. 90) [1]

$$\sum_{t=1}^{n} U_t^2 = \sum_{t=1}^{n} \left(X_t - (c + \phi X_{t-1})\right)^2 \tag{14}$$

To obtain

$$\phi = \frac{\sum_{t=1}^{n} X_t X_{t-1} - n\bar{X}_t \bar{X}_{t-1}}{\sum_{t=1}^{n} X_{t-1}^2 - n\bar{X}_{t-1}^2} \tag{15}$$

$$c = \frac{\sum_1^n X_t - \phi \sum_1^n X_{t-1}}{n} \tag{16}$$

$$\gamma^2 = \sum_{t=1}^{n} X_t^2 - n\bar{X}_t^2 \tag{17}$$

Finally, the Method of Moments estimates is found by obtaining the sample's mean, variance, and covariance and then equating and simplifying these numbers to the ARIMA process's theoretical mean, variance and covariance (Holý & Tomanová, 2020, p. 4 – 5) [22].

For the short-term ARIMA model, four (4) configurations are proposed. These are: (1) ARIMA (1, 0, 0), (2) ARIMA (1, 0, 1), (3) ARIMA (1, 1, 0) and (4) ARIMA (1, 1, 1). The choice of short-term models corresponds to the recommendations made by Bijak et al. (2019, p.474) [4]. The results of the estimations are presented in **Table 7**.

TABLE 7: ARIMA Model Estimates (with p-values)

Model	Intercept	ϕ_1	θ_1	σ
ARIMA (1, 0, 0)	-14.2069464 (0.0001786)	0.9860895 (<2e-16)		258.4381
ARIMA (1, 0, 1)	-14.3277695 (6.44e-05)	0.9807394 (<2e-16)	0.7660001 (<2e-16)	178.5659
ARIMA (1, 1, 0)	-34.554384 (0.539)	0.883134 (<2e-16)		124.3764
ARIMA (1, 1, 1)	-33.639511 (0.5366)	0.874843 (<2e-16)	0.035916 (0.4040)	124.3167

As can be seen from the results, despite the series being stationary (Dickey fuller 3.5609, p-value 0.03705) we cannot rule out the presence of a Unit Root a significance level of 5% for ARIMA (1, 0, 0) with a 95% confidence interval between 0.9711964 and 1.000983.

Setting $dt = 12$ for the OU process representation of an ARIMA (1, 0, 0), **Table 8** summarizes the estimates for the OU process.

TABLE 8: OU Process Parameter Estimates

Parameters	μ	θ	σ
Estimates	-2364.154	0.07223738	899.7836

The Parameters μ, θ, and σ were obtained by substituting and solving discrete equations (7) and (12). The step function was used instead of the integrated function since the paths for the equations were estimated using the Euler-Maruyama discretization method (Ayranci, & Özgürel, 2014, p. 91) [1]. Specifically, we have for increments

$$X_{t+1} = \underbrace{\theta\mu\Delta t}_{c} + \underbrace{(1 - \theta\Delta t)}_{\varphi} X_t + \sigma \underbrace{\Delta B_t}_{\sqrt{\Delta t}\varepsilon_k} = c + \varphi X_t + \sigma\sqrt{\Delta t}\varepsilon_t \tag{18}$$

which implies

$$\mu = \frac{c}{\theta \Delta t} \tag{19}$$

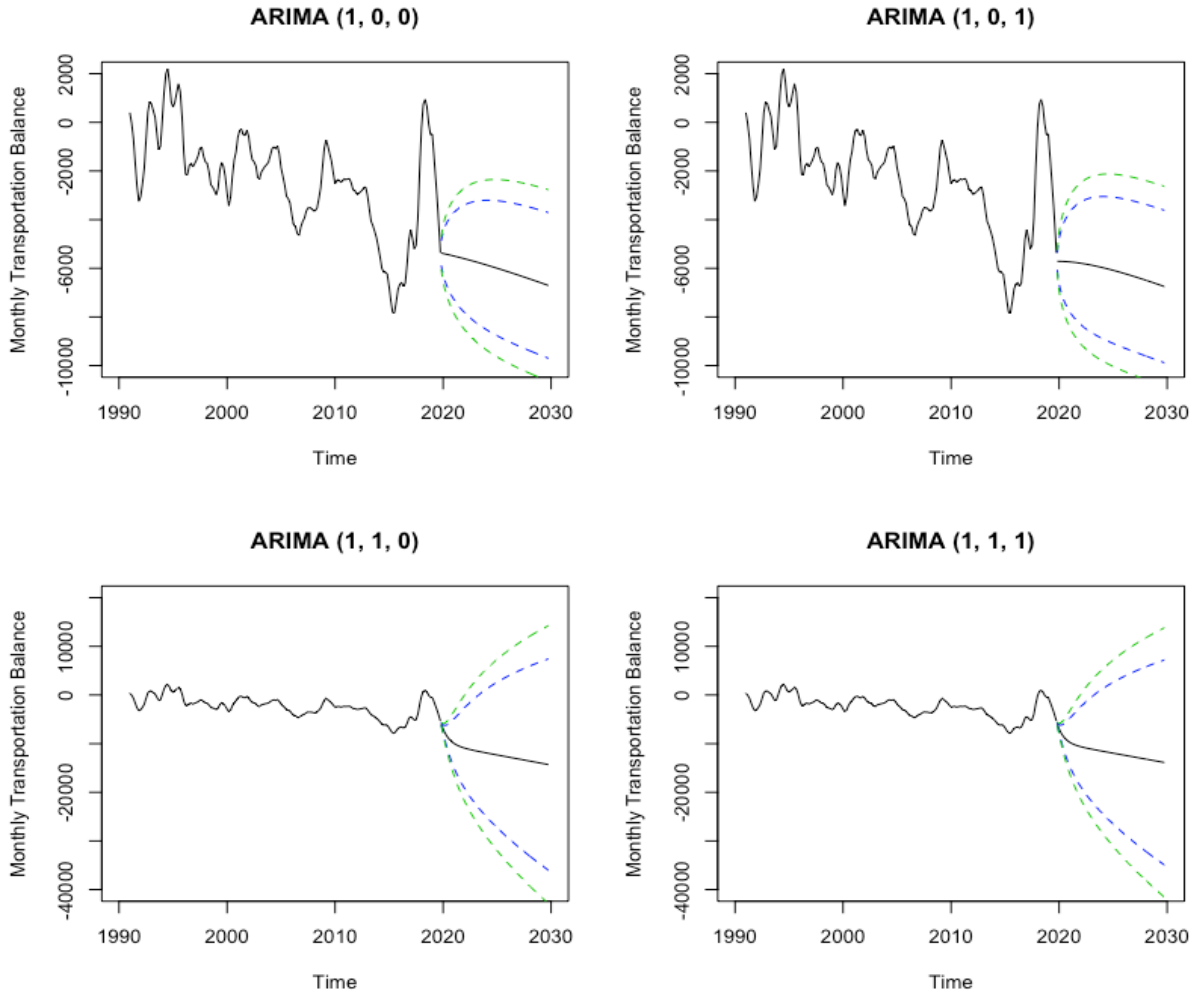
$$\theta = \frac{(1-\phi)}{\Delta t} \tag{20}$$

$$\sigma = \frac{\gamma^2}{\Delta t} \tag{21}$$

4.3 – Predictions and the Simulated Distribution

Using the predict functionality from the R package stats (R Core Team, 2019) [34], the forecasts in **Figure 4** were obtained.

FIGURE 4: ARIMA Model Short-Term Forecasts

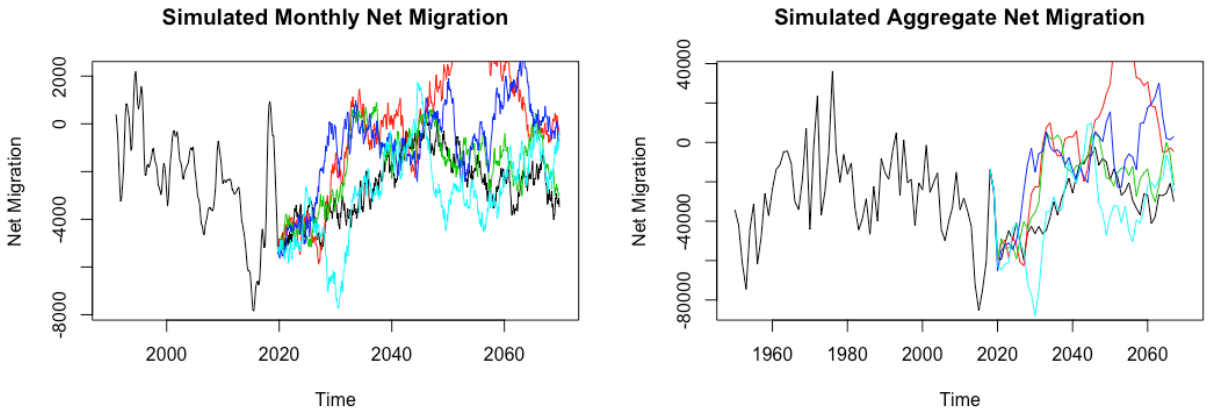


Where the blue dashed line corresponds to the 95% confidence interval, and the green dashed line corresponds to the 99% confidence interval.

The results seem to agree with the claims made by Disney et al. (2015) [15] and Bijak et al. (2019) [4]; as was mentioned in Chapter 2, an ARIMA (1, 0, 0) seems to exhibit overall good estimates when the series has stationary features and is long enough. Overall, all the models seem to suggest that the trend in negative net migration (deficit of people) will continue to increase for the next 5 to 10 years.

For the long-term migration, this work proposes an OU process simulation. The simulated paths are provided for both monthly estimates and the aggregate yearly path. The graphs, in **Figure 5**, show that as time passes monthly net migration decreases. The speed seems to correspond with the historical trend Pol (2001) [33] identified; i.e. a 20-year gap between large negative net migration or deficit followed by a brief 10-year period of positive net migration. The aggregated graph shows yearly population estimates since the 1950s (the data was compiled using several historical articles by Lehman College; accessed in 2020) [28].

FIGURE 5: Long-Term Net Migration OU Process Simulations



Taken all together, the models hint that out migration will continue for the next 5 to 20 years with stabilization being reached somewhere between 15 to 40 years. The model seems to hint that by 2050 migration should reach the long-run mean reverting value.

To observe what would happen if Puerto Rican migration were to reach 0 (value traditionally considered as the long-run impact of migration; see Plamondon et al., 2002) [36] at some time t, the following simulation is proposed.

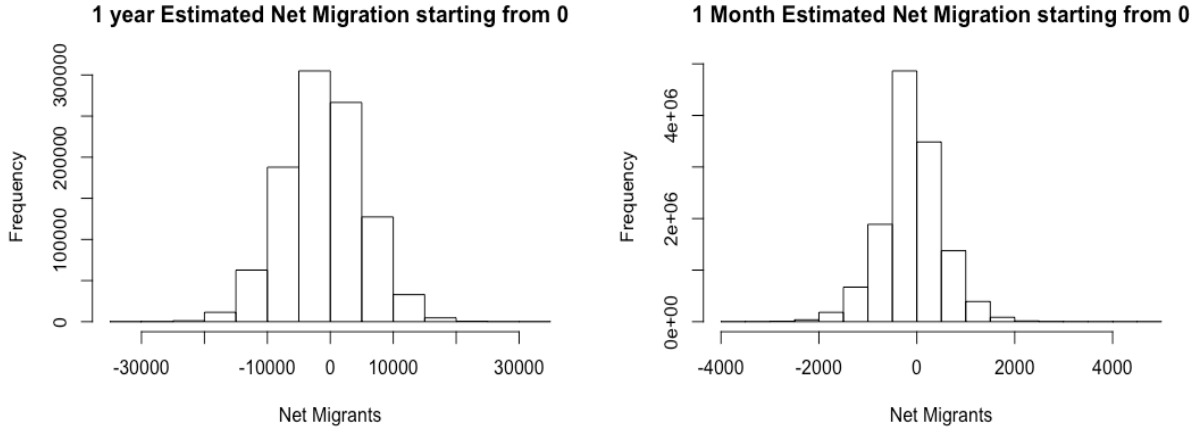
1. Set the initial starting point of the series at 0.
2. Define the length of the interval from the series using random sampling with replacement.
3. Generate 12 random $N(0,1)$ variables.
4. Estimate the following equation

$$X_{t+1} = X_t + \theta(\mu - X_t)\Delta t + \sigma\sqrt{\Delta t}\Phi^{-1}\big(U(0,1)\big) \tag{22}$$

5. Store values in a matrix of results.
6. Repeat process.

The previous simulation ran for 1,000,000 iterations and produced the following overall distribution; depicted in **Figure 6.**

FIGURE 6: Simulation of the Distribution of Puerto Rican Net Migration After Reaching Theoretical Long-run 0 Value



To determine just how sensitive the OU process's parameters (μ, θ and σ) are to the available data and how stable the estimates previously provided are, a second simulation was conducted. The steps used to estimate the parameter's distributions were the following:

1. Sample a random starting point within the time series with replacement.
2. Set condition that the resulting series must be at least of length 20 (Disney et al., 2015; Bijak et al., 2019) [15].
3. Estimate ϕ, c and γ^2 using an ARIMA (1, 0, 0). In case the series is determined to be non-stationary then a failure is recorded.
4. Using equations (19) – (21), obtain the implicit values of μ, θ, and σ and store them in a matrix.
5. Repeat process.

The aforementioned simulation ran for 1,000,000 iterations with a failure rate (times the time series was non-stationary) of 0.003093%. **Figure 7** shows the histograms for the parameters and **Table 9** shows the parameters' statistics.

FIGURE 7: θ, μ and σ Estimates

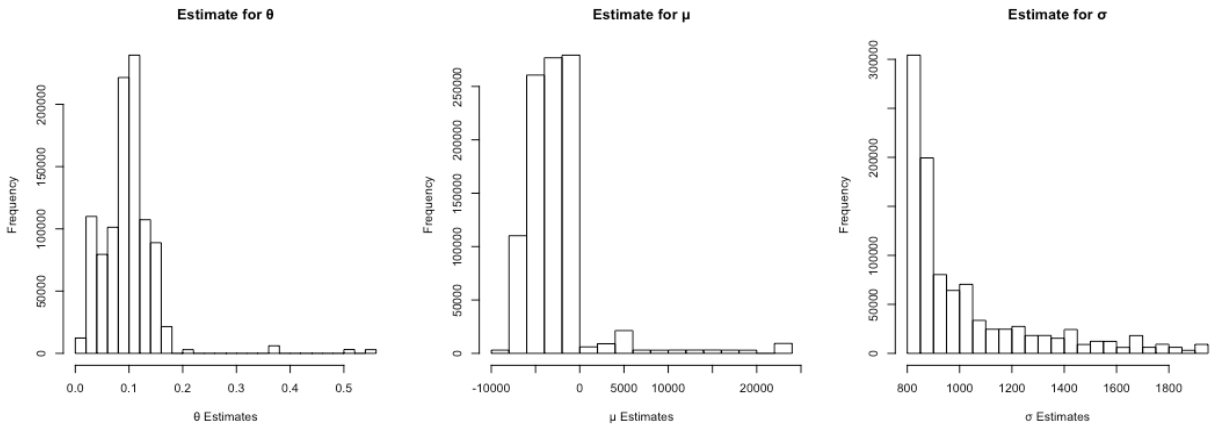


TABLE 9: θ, μ and σ Parameters' Statistics

Parameters	μ	θ	σ
Mean	-2580.609	0.09816899	1022.535
Standard Deviation	4119.072	0.0545081	261.4524
Standard Error	4.125457	5.459259e-05	0.2618577
95% Confidence Interval	(-2588.695, -2572.523)	(0.09806199, 0.09827599)	(1022.022, 1023.048)
99% Confidence Interval	(-2591.235, -2569.982)	(0.09802836, 0.09830961)	(1021.861, 1023.21)

4.4 – Application to Insurance

Migration may affect actuarial valuations in one of three ways based on the overall scheme of the pension plan. It can function as a correction on the withdrawal rate in cases where the pensioner is entitled to withdraw his/her money with or without interest upon leaving the scheme. That is to say that migration could be used as a constant or time-varying term added to the estimated withdrawal rate. Migration can also work as a correction in the number of premiums the institution can expect to receive (via taxes or direct contribution). Lastly, migration could be seen as a loss in future economic wealth and talent pool, which itself is a loss in economic terms. For Puerto Rico and the Government's pension scheme, migration affects the "premiums" or taxes of the people.

In the context of pay-as-you-go government-sponsored pensions, Migration affects three main areas: (1) Income Taxes, Property Taxes and Consumption Taxes (Government's Premiums) used to finance the system, (2) a decrement in the country's talent pool and human resource (loss in productive capacity or loss in future incomes), and (3) in cases where migration stems from a specific age group or profession, the nation experiences quick shifts in its demographic composition. In Puerto Rico, this last phenomenon is very relevant since researchers have theorized that the Puerto Rican migrants are young and educated individuals (Pol, 2002; Velazquez-Estrada, 2014 – 2019) [33, 56].

In this section, a simple example of how migration rates can impact government-sponsored pensions is presented. The exercise has the following assumptions:

1. Using the income intervals presented in the Perfil del Migrante (Velazquez-Estrada, 2019) [56], and the average composition of migrants from the PRCS [56], the monthly loss in income is estimated at $1,802.00.
2. Proportion of people in the income brackets used for the average salary shown in the previous point is assumed to be constant for all time and no deaths or retirements are considered.
3. Long-term mean, speed of reversion, and sigma in the OU process are equal to the estimates presented in **Table 9**.
4. Participation rate in the economy is 29.4%. This participation rate corresponds to the participation rate that the government of Puerto Rico presented in their yearly economic study for the Governor (Junta de Planificación, 2019) [26]**.**
5. Salaries in Puerto Rico grow at the rate of inflation of the U.S. economy, and the discount rate equals the growth rate of the Puerto Rican economy [43] [44].
6. The base population is the estimate for 2018 and the starting value for migration is the value of 2017 (see chapter 1) [56].
7. Migration (M) is a random variable distributed according to equation 10.

This simple exercise over the time horizon of 50 years (time that is taken from Plamodon et al., 2002) [32] leads to a direct loss of approximately $1.4 Billion dollars. However, this loss does not reflect the true cost of Puerto Rican Migration.

If one assumes that each migrant group represents a stream of income for the government and not just a single loss, then we would see that over the course of 50 years, the economic loss using a tax rate of 33%, is approximately $20 Billion or $400 million per year. To put this into perspective, this would imply an 8% yearly reduction in Government funds financed by personal taxes. That is to say that the effects of migration are felt as time passes and not immediately.

It's clear that this example has some strong assumptions due to data limitations and as such the values presented are not meant to be accurate. They do however serve the purpose of showing how migration affects national wealth.

Conclusions

Migration has been a part of humanity since the beginning of history. It is driven by both physical and psychological needs that, by construction, can only be satisfied via changes in geographic locations. In many ways, it is as quintessentially human as births and deaths.

In the 21st Century, migration has been promoted by several factors. First, advancements in technology that have made geographical shifts both cheap and quick, second, the promotion of globalization via English as the lingua franca and third, the establishment of economic communities such as the European Union. Other factors include (Castelli, 2018) [12]:

1) Inadequate human/economic development.
2) Effects of Climate Change
3) Education
4) Varying prices in housing

As seen in Chapter 1, in Puerto Rico these effects have been felt to a greater degree due to its territorial relationship with the United States. One must recall that Puerto Rico is officially recognized as a Commonwealth of the United States with free mobility between nations.

In chapters 2, 3, and 4 the complexity of migration was presented in terms of lack of a uniform definition, the noise surrounding the data, and the periodicity of measurements. Despite these complications, results were obtained for both short-term and long-term forecasts via Timeseries analysis and simple AR models. A particular proposition offered in this work is the use of the OU process as a long-term measurement of migration by promoting the concept of a long-run stable mean.

In terms of limitations, this work faced several important ones. The first and perhaps the biggest limitation was the lack of good quality data. In Chapter 3 it was shown that the best source of information for probabilistic analyses of migration in Puerto Rico is the estimates of monthly air travel passenger data. This data, in the context of long-term geographic mobility, is subject to a large amount of noise that, though as seen in Chapter 4 was corrected, could be avoided if there was good quality data. The second limitation faced throughout this work was the lack of scientific literature on the subject of Puerto Rican migration. In Chapter 2 it was shown that there are very few probabilistic studies on migration in Puerto Rico and that the study of migration mostly comes from first world countries, such as the United Kingdom (Disney et al. 2015) [15].

Finally, the following points are offered as recommendations for Puerto Rico particularly, but they also apply to any country:

1) Produce good quality and fairly frequent information on migration.
2) Gather information on migrants' age, sex, and other demographic variables consistently and more frequently.
3) Formulate policy regarding Puerto Rican migration. This includes strategies for long-term retention of the youth.
4) Include migration in their long-term estimates of government funds.

In the future, the information that would be gathered, given the recommendations, could facilitate the study of more complex events surrounding migration. One such event that could be explored is the study of the dynamics between migration and births and migration and death. A possible relationship that can be studied, which comes from the Puerto Rican experience, is whether a strong emigration of youth and low immigration of elderly retired people leads to serious long-term effects on both crude birth rates, crude death rates, the economy, and subsequently the insurance industry.

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