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MASTER'S FINAL WORK

DISSERTATION

ON ROBO ASSESSMENT OF RISK PROFILES

MADALENA MENDES DE ALMEIDA ESTEVES DE OLIVEIRA

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SUPERVISION:
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ABSTRACT

Nowadays, the technological world has been growing at a very fast rate, which means there has to be a quick adaptation and companies feel the need to reinvent themselves. Technological innovations also reached the asset management service industry with the so-called the Robo-Advisors. These are platforms that provide financial advice or automated investment management. Robo-Advisors collect information about their clients' financial situation and future goals through questionnaires, then recommending ETF based portfolios supposed to fit investor's risk profile. However, questionnaires seem to be vague, and robos do not reveal the methods used in asset allocation. This study aims at contributing to the understanding the effectiveness of these platforms. It relies on expected utility theory, and, for various levels of relative risk aversion we propose optimal mean-variance portfolios. We then compare our portfolios with the portfolios proposed by the Riskalyze platform, for three different types of investors: conservative, moderate and aggressive. By evaluating their in-sample and out-of-sample performance. We conclude, that in the long run, the methodology used by robo-portfolios, according to the investor's risk profile, can be effective for investors who have a higher level of risk aversion, however for investors with relatively lower risk aversion the mean-variance portfolios tend to perform better.

Keywords: Robo-Advisors; financial advisory services; risk profile; mean variance theory; expected utility theory; relative risk aversion; risk tolerance function; portfolios; assets; sharpe ratio

Jel Classification: C61, G11

RESUMO

Nos tempos que correm, o mundo tecnológico tem crescido a um ritmo muito acelerado, o que significa que tem de haver uma rápida adaptação, e as empresas sentem a necessidade de se reinventar. As inovações tecnológicas também alcançaram a indústria de serviços de gestão de ativos com os chamados Robo-Advisors. Estas são as plataformas que fornecem aconselhamento financeiro ou gestão automatizada de investimentos. Os Robo-Advisors coletam informações sobre a situação financeira e os objetivos futuros de seus clientes através de questionários, recomendando carteiras baseadas em ETFs, supostamente adequadas ao perfil de risco do investidor. No entanto, os questionários parecem vagos e os robôs não revelam os métodos usados na alocação de ativos. Este estudo visa contribuir para a compreensão da eficácia dessas plataformas. Baseia-se na teoria da utilidade esperada e, para vários níveis de aversão relativa ao risco, propomos carteiras de média-variância ótimas. Em seguida, comparamos as nossas carteiras com as carteiras propostas pela plataforma Riskalyze, para três tipos diferentes de investidores: conservador, moderado e agressivo. Avaliando o seu desempenho in-sample e out-of-sample. Concluímos que, a longo prazo, a metodologia utilizada pelos robo-portfolios, de acordo com o perfil de risco do investidor, pode ser eficaz para investidores que apresentam um maior nível de aversão ao risco, porém para investidores com aversão ao risco relativamente menor os portfólios de média-variância tendem a ter melhor desempenho.

Palavras-Chave: Robo-Advisors; serviços de assessoria financeira; perfil de risco; teoria da média variância; teoria da utilidade esperada; aversão relativa ao risco; função de tolerância ao risco; carteiras; ativos; sharpe ratio

Classificação Jel: C61, G11

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LIST OF ABBREVIATIONS

A – Aggressive portfolio

C – Conservative portfolio

EF – Efficient Frontier

EUT – Expected Utility Theory

H – Homogeneous portfolio

M – Moderate portfolio

MV – Minimum Variance portfolio

MVT – Mean-Variance Theory

RRA – Relative Risk Aversion

RTF – Risk Tolerance Function

SR – Sharpe Ratio

T – Tangent portfolio

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1. INTRODUCTION

Nowadays, technological innovations are becoming more frequent, and they cover the various business areas, as they aim to keep up with the new needs of consumers. Thus, it is easy to perceive that financial institutions had to hold on with this growth and therefore adjust their services to remain competitive. According to some studies, since 2013 this technological revolution has been increasing by 24% on average each year (Executive Summary World Robotics 2019 Industrial Robots, 2018; Zakamouline & Koekebakker, 2009), and it is expected that they will replace 47% of the current jobs in the next 20 years (Acemoğlu & Restrepo, 2020). With the current COVID pandemic tackled to rapid adaptation to technology and digital platforms, this may happen even faster.

Technological innovations reached the wealth management services industry, with automated financial advisors, the so called Robo-Advisors. Robo-Advisors aim to automate and improve the process of creating diversified portfolios fit to each investor's risk profile, at a low cost. In addition to wanting to remain competitive, financial institutions also aim to save costs and reduce the workload for their employees, using artificial intelligence as an ally for achieving these goals. These platforms are a big phenomenon, specially in the United States where, for instance, more than 1 millions clients of Bank of America trust in the online platform "Erica" to give them financial advices (Crosman, 2018).

Given that Robo-Advisors are starting to gain market share, it is easy to understand the importance of studying them and see their credibility, both in terms of their ability to risk profile investors and their ability to build efficient portfolios.

This study focus on Riskalyze Platform, a robo-advisor in the United States that claims to be transforming the advisory industry, by quantitatively measuring the risk tolerance of their clients. Although Riskalyze advertise the best performance of their portfolios, taylor-made to each profile, there are no studies evaluating the out-of-sample performance of their portfolios, nor their ability to match investor's risk profile.

There is still not much information regarding robo's methods of asset allocation, as these platforms do not reveal the methodology used, for strategic reasons, but there is even less information on the risk profile evaluation method. One thing is clear they tend to classify investors in just three broad classes – conservative, moderate, aggressive – which seems very low fish-tech at the best. In this study we use the mean-variance approach and expected utility theory and propose optimal portfolios for investors with various levels of relative risk aversion. By comparing our portfolios with those provided by Riskalyze, this study contributes to understand the effectiveness of these platforms. While all other studies have analyzed the viability of these questionnaires or what these platforms focus on, this study is the first to analyze and to look at the risk profiles in terms of the different levels of RRA.

The remaining of the text is organized as follows. Chapter 2 presents Literature Review, in which you will find what has already been said in other studies, and what is the revelation of this work. Chapter 3 explains the methodology used. Chapter 4 presents the data related details. Chapter 5 reports and discusses the results, as the name indicates, this describes the results obtained through the analysis performed. Finally, Chapter 6 summarizes the main conclusions and discusses future research.

2. LITERATURE REVIEW

Despite being a very recent topic, there are already several studies on robo-advisors, since it is a growing market. This chapter provides an overview of the literature up to now.

Much of the existing literature is more directed towards the industry, with focus on the digitalization of financial advisory. According to Jung et al (2017) in the first step of the digitalization of the wealth management industry, brokers provided their financial advisory services at a much more affordable price than financial advisors, which has meant that the target audience has segmented itself into a new niche. The downsides of the issue, are the lack of personal financial advice, and the low number of products available. Nowadays, however, bank account management and other banking services can be performed entirely digitally. Customers, still prefer hybrid solutions, as on the one hand these allow them to search for information and compare products available online, but on the other hand, they do not need to make their final decisions without first consulting an (human) investment manager.

Also from a business model point of view the robo-advisors service is easily scalable, which makes it to analyse business model from the perspective of the service provider.

Recent studies have also looked at the impact that artificial intelligence has on financial technology. Belanche et al (2019) proposes a research framework to understand robo-advisor adoption and how personal and sociodemographic variables impact the main relationships. With this study, we can conclude that the main key determinants for adoption are the consumers' attitude regarding robo-advisors, mass-media and the interpersonal subjective norms. In this study, it is also determined that customers who have a greater knowledge about these platforms tend to value personal utility and attitudes rather than relying on subjective norms that are based on the opinions of others. It was also observed that the basic demographic variables are not that important, which means that the robo-advisors can have as possible users, people of any gender and age, although there may be the need to use specific strategies for different geographic regions.

Robo-Advisors had a great growth, since digitalization is beginning to prevail in our world, this resulted in a threat in the traditional fund and wealth management industry, so Phoon et al (2018) examines the postulation that robo-advisors have an edge over traditional wealth managers, since these platforms combine the judgement and computing resources of "man and machines" or "bionic power" in order to create alternative wealth management services. They believe that in the long run, robo-advisors will commercialize the simplest and most technical aspects of wealth management. They confirm this tendency, however they also show these services will be more used by investors with simpler needs, since in traditional service market they do not find personal customization, and digital services are cheaper, more accessible and customer-centered. As for investors with more sophisticated needs, they will continue to prefer traditional portfolio managers. Fisch et al (2018) defends the same as Phoon et al (2018), however, in addition, the study shows that robo-advisors may be less likely to lead to problems of conflict of interest related to the products they sell, even if these platforms are being increasingly integrated into traditional full-service banks, brokers, and asset management companies.

Despite the extensive literature, not many details are known about the methods of assessing the risk profile of the investors, with few exceptions. Gai & Vause (2005) propose a method for measuring the risk appetite based on variation in the ratio of risk neutral to subjective probabilities used by investors when evaluating possible future returns for an asset, with plausible responses to major economic events (e.g. crises). This method presents advantages when compared with other methods, since it does not rely on restrictive assumptions and it uses all the available information regarding risk-neutral and subjective probability distributions. Grable (1997) studied "whether the variables gender, age, marital status, occupation, self-employment, income, race and education could be used individually or in combination to both differentiate among levels of investor risk tolerance and classify individuals into risk-tolerance categories", by taking into consideration Leimberg, Statinsky, LeClair, and Dyle (1993) financial management model as the theoretical basis. The authors concluded that the two demographic characteristics that proved to be most effective in determining the differentiation and classification of respondents in various risk tolerance categories are the educational level and gender. Tertilt and Scholz (2018) analyzed the process of assessing investors' individual risk preferences, through a questionnaire of 10 questions, where approximately only 60% have an impact on the risk categorization.

The fact that these questionnaires are not very accurate, causes meaningful errors. For example, customers, in the long run, may lose return when the risk assessment is very conservative, at the other extreme, if the portfolio recommendations are too risky, customers will feel uncomfortable if the risk materializes.

From a more statistical view point Alsabah et al (2019) propose an alternative exploration-exploitation algorithm intended to negotiate expensive requests for portfolio choices by the investor with autonomous trading decisions based on obsolete estimates of investor risk aversion. According to the authors, this algorithm allows robo-advisors to provide the investor with a portfolio that is close to the optimal policy. They show that the learning speed of platforms is related to the consistency of investor decisions and the necessary forecast of risk aversion estimation. In terms of portfolio allocations, most robo-advisors claim to rely on mean-variance theory, Perrin & Roncalli (2019) show that the allocation of portfolios could benefit from if large-scale optimization algorithms, as the old methods end up being limiting. Boreiko et al (2020) analyze how the risk profiles of investors affects robo-advised portfolios. They consider a set of 53 advising platforms from US and Germany and use the ordinary least square (OLS) regression model with corrections for autocorrelation and heteroskedasticity base on the Newey-West method. They regress equity allocations against the risk profile of the investors and other explanatory variables, and conclude these algorithms are able to identify various types of risk profiles, however it is observed that substantial variability is evident within the same groups of investors.

Specifically, with regard to the risk profile of the investor, the emphasis of their literature has been more on the questionnaires that this robo-advisors execute in order to get to the risk profile of the investor. In this study we take a different approach by relying on expected utility theory (EUT) and mean-variance theory (MVT) to find optimal portfolios for investors with different levels of relative risk aversion (RRA), from a methodological point of view, we follow the approach in Gaspar & Silva (2020), and to consider a realistic range of RRA levels.

In this study we capture the in-sample and out-of-sample performance of real portfolios proposed by Riskalyze with optimal portfolios based upon MVT and EUT.

3. METHODOLOGY

The aim of this study is to use mean-variance theory (MVT) and expected-utility theory (EUT) to identify optional portfolios, for investors with different levels of relative risk aversion (RRA).

We then compare the out-of-sample performance of these theoretically optimal portfolios, with that of Riskalyze actual portfolio proposals for conservative, moderate and aggressive investors.

3.1 MEAN-VARIANCE PORTFOLIOS

Given a set of risky assets, Markowitz (1987) MVT allows to find all efficient portfolios. That is, all portfolios that the biggest reward at a given level of risk, or the least risk at a given level of return.

MVT is still the "standard" portfolio building method, widely used, not only by academics, but also by practitioners.

Given a set of n risky assets with individual expected returns R_i, for i = 1,…,n, the expected return of any portfolio p is given by:

$$\bar{R}_p = \sum_{i=1}^{n} x_i \, \bar{R}_i \tag{1}$$

where, x_i shows the weight of each individual asset in a portfolio n and we have:

$$\sum_{i=1} x_i = 1 \tag{1.1}$$

The risk of a portfolio, as evaluated by the variance is given by:

$$\sigma_p^2 = Var\left(R_p\right) = Var\left(\sum_{i=1}^{n} x_i\, R_i\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i\, x_j \sigma_{ij} \tag{2}$$

where σ_{ij} denotes the covariance between the returns of asset I and j.

In vector notation we can use

$$\bar{R} = \begin{pmatrix} \bar{R}_1 \\ \bar{R}_2 \\ \vdots \\ \bar{R}_n \end{pmatrix} \tag{3}$$

$$X = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{pmatrix} \tag{4}$$

$$V = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_n^2 \end{pmatrix} \tag{5}$$

and easily write the expected return of any portfolio as

$$\bar{R}_p = X'\bar{R} \tag{6}$$

and its variance as

$$\sigma_p^2 = X'VX \tag{7}$$

In this study we focus all same MVT efficient portfolios: the tangent (T) portfolio, the minimum variance (MV) portfolio, as well as optimal portfolios for various levels of relative risk aversion (RRA).

3.1.1 TANGENT (T) PORTFOLIO

The tangent (T) portfolio is the one with the highest Sharpe ratio, and since we are considering that short selling is not allowed, it is the one solving the following maximization problem:

$$max\ \theta = \frac{\bar{R}_p - R_f}{\sigma_p} \tag{8}$$

s.t.

$$\bar{R}_p = X'\bar{R} \tag{8.1}$$

$$\sigma_p^2 = X'VX \tag{8.2}$$

$$X'1 = 1 \tag{8.3}$$

where 1 is a vector of ones and we also impose $x_i \geq 0$ to $\forall\ i = 1, \dots , n$.

The inequality restrictions guarantee short selling is not allowed and that MVT portfolios are more directly comparable with robo-portfolios, that also do not consider short selling. Due to short selling restrictions there are no close form solutions for the weights on these portfolios.

3.1.2 MINIMUM VARIANCE (MV) PORTFOLIO

As the risk-free decreases, the investment slope becomes more steeper. Thus, if we consider that the risk-free is tending to less infinite, we will find the minimum variance portfolio, so we have to optimize the following expression:

$$\underset{X}{min}\ \sigma_p^2 = X'VX \tag{9}$$

s.t.

$$X'1 = 1 \tag{9.1}$$

and $x_i \geq 0$ for all $i = 1, \dots , n$

3.1.3 RRA OPTIMAL PORTFOLIOS

Besides the classical tangent (T) and minimum variance (MV) portfolios described above, in this study we also consider optimal MVT portfolios for investors with different levels of relative risk aversion (RRA). So we take the investor´s perspective, and analyze preferences regarding the set of risk options.

In modelling choice under uncertainty we consider Expected Utility Theory (EUT) first created by Bernoulli (1738) in order to solve the St Petersburg Paradox, where he came up to the distinction between expected value and expected utility, and further developed by Von Neumann and Morgenstern (1947) to model economic agents' decisions. The EUT consists on the analysis of situations where individuals must make a decision without knowing the outcome, so it is easy to perceive that the individuals will choose the outcome that has the highest expected utility. In this way, throughout the years there was an attempt to model financial risk-taking behavior, in order to be able to use this tool in the financial services context, or by policy makers who are interested in the results associated with risk-taking.

When considering the EUT it is generally assumed that the person's relative risk aversion will influence the utility function of wealth, because an investor's risk-taking preferences are shaped by some factors that usually are not examined as a component of expected utility analysis, such as demographic and socioeconomic factors (Grable, Britt, & Webb, 2008). When analyzing the RRA, we know that the aversion is in terms of fractions or proportions of current wealth that might be lost instead of absolute amounts. It is important to bear in mind that our investor may present three different risk profiles, where afterwards their degree of risk aversion varies. As we only look at levels of risk aversion, not their derivative, so for this analysis we will not be interested in the demographic and socioeconomic factors. Therefore we understand when the RRA < 0, we are towards a risk seeker investor, when RRA = 0, this presents a risk neutral investor, and when the RRA > 0, this means the investor is risk averse.

Through the EUT we realize that investors do not have as main concern the monetary results resulting from the outcomes, but rather the utility that money provides, with this, and through rational axioms, the problem of investment choice focuses only on maximization of the final expected utility, E[U(W)], known in finance as risk tolerance function (RTF). Risk Tolerance is a fundamental factor in investing, since it is the degree of variability that an investor is whiling to take in its financial planning. Investors will have different profiles, since some of them are willing to take more risk than others. If an investor presents low tolerance, he will have more conservative investments in comparison with an investor that presents a higher tolerance towards risk.

Regarding utility functions, there are some functions that are more common in finance due to their mathematical treatability, which are: exponential, logarithmic, power and iso-elastic. However, in most of the mentioned cases there is an obstacle, which makes it difficult to obtain the RTF, E[U(w)], in closed form, which is the non-linearity of this functions. Therefore, there are two possible situations for solving this problem, namely, the numerical numeration of the RTF (via Monte-Carlo simulation), or the second-order Taylor approximation, where we can always get a viable approach of the RTF in closed form. Following Gaspar and Silva (2020), in this study we should rely on a common approximation thar results from the second-order Taylor approximation, which can be represented as follows:

$$\boldsymbol{E}\,[\boldsymbol{U}(\boldsymbol{w})] \approx\ \overline{\boldsymbol{R}} - \frac{\mathbf{1}}{\mathbf{2}}\,\boldsymbol{RRA}\,(\boldsymbol{w_0})\,(\overline{\boldsymbol{R}}^2 +\ \boldsymbol{\sigma^2}) \tag{10}$$

where, w_0 denotes the initial wealth, $\bar{R} = \frac{w-w_0}{w_0}$ the expected final return on the investment and σ is the associated volatility, while $RRA(w_0)$ represents the coefficient of relative risk aversion evaluated at initial wealth,

$$RRA(w_0) = -w_0\,\frac{U''(w_0)}{U'(w_0)} \tag{11}$$

To calculate the RTF for the various types of investors, we look at both approaches mentioned above. So we start by scrutinizing three concrete utility functions, where each represent the three basic investor profiles: $U(w) = w^2$, $U(w) = w$ and $U(w) = \ln(w)$. Subsequently, we use the approximation in (11) to analyze the various levels of RRA that range from -1 to 6, and find optimal portfolios for each value of RRA.

The level -1 represents our risk loving utility, $U(w) = w^2$, level 0 concerns to risk neutral utility, $U(w) = w$, and level 1 remits to risk averse utility, $U(w) = \ln(w)$. By considering RRA from -1 to 6, we take a range of risk aversion coefficients consistent with the empirical literature.

After evaluate the in-sample data, we move on to the out-of-sample data analysis. In this period, we aim to observe the performance of the investment for the following years. This analysis is done for the portfolios we created with the different levels of RRA. To be able to make the comparisons we consider that aggressive investors are the investors that present RRA coefficients from -1 to 1. For those considered to be conservative, we consider the RRA levels between 4 and 6. The remaining RRA levels, will be compared with the values obtained in the portfolio provided by the platform for moderate investors.

3.2 ROBO PORTFOLIOS

For this study, we have only three portfolios on the Riskalyze platform, one for each broad classification of investors. The data for these portfolios was provided by the authors Gill et al (2017), that on the 31st March 2017 simulated three portfolios, through real investments. For most robos, there are only three broad classifications for investor profiles, which are: conservative, moderate and aggressive, on which we will focus our study.

The aggressive risk investors are the ones that are enthusiastic in taking large amounts of risk and do not settle back when observe downward movements in their portfolios. They usually go for the risky asset classes, and when the market is performing well, they invest in the assets that present higher returns. Moderate risk investors are willing to take some risk, and they can handle until a certain percentage a downward in their portfolio before taking their money. They usually invest part of their money in riskier assets and the other part in safer assets (50/50). Conservative risk investors are the ones that are hardly able to take any risk, so they always go for the safest assets, the ones that offer them capital protection, since they do not want to suffer losses. The risk tolerance of each investor is influenced by some determining factors, such as the financial situation, asset class preference, time horizon and the purpose of the investment. This being the methodology used by the Riskalyze platform to categorize the risk type of its investors.

3.3 PERFORMANCE MEASURES

We make a comparative analysis of the performance of all portfolios previously mentioned, where we start by estimate the amount that is allocated to each asset. For this, we consider we will invest \$100 in the portfolio, and from there see how it involves until the end of time. In addition to these considerations, we also found it relevant to consider a monthly rebalancing, in order to realign the weightings of the portfolio, and considering the H portfolio as a naïve benchmark.

We opt for the monthly rebalancing, because according to Almady, Rapach & Suri (2014), the monthly (annual) rebalancing presents the best outperformances when unit transaction costs are below (above) approximately 50 basis points, when we speak of dynamic portfolios, we realize that the annual rebalancing is what outperform unit transaction costs exceeds 400 basis points.

In addition to the evolution of performance, we calculate the Sharpe ratio (SR) for each portfolio both in-sample and out-of-sample, since it is commonly used as a performance measurement. According to Zakamouline & Koekebakker (2009), we can understand that the performance measure is related to the "level of maximum expected utility provided by the asset", which means when an asset present a greater performance measure, the asset will provide a higher level of maximum expected utility. However, the Sharpe Ratio is only considered a meaningful measure of the portfolio performance, when we are able to measure the risk through the Standard Deviation. We decided to choose Sharpe Ratio as the best indicator to make this comparison since this ratio evaluates the portfolio manager on the basis of both the rate of return and diversification. So we have:

$$SR = \frac{\bar{R}_p - R_f}{\sigma_p} \tag{12}$$

where, $\bar{R}_p$ is the expeted return of the portfolio, σ_p is the volatility of the portfolio and R_f is the risk-free interest rate of the market.

After computing the RTF for several portfolios, we decided to calculate the amounts for the Return, Standard Deviation and Sharpe Ratio for each of the portfolios created by the Robo-Advisor. With this, we are able to analyze the portfolios that we have created with the ones from the online platform, so as to achieve our conclusions we need to look to the Sharpe Ratio values, since this ratio evaluates the portfolio manager on the basis of both the rate of return and diversification. We find that the superior portfolio is the one that has the superior risk-adjusted return.

4. DATA

To carry out this study, we use data from three portfolios compositions proposed by the Riskalyze platform, for the three diferent types of investors, on the 31st March 2017 for an investment horizon of 5 years.

We collected daily prices for all 15 ETFs in the three portfolio compositions for our sample period.

4.1 ETF MARKET DATA

Table I presents their description, abbreviations and categories. For each of the mentioned ETFs we have collected daily data from 1st March 2012 to 31st March 2020. We use the first 5-year period for the in-sample calculations. For out-of-sample performance we consider from 1st April 2017 to 31st March 2020. The out-of-sample period, finishes in the 31st March 2020, in order not to bias our analysis with the current pandemic crisis effect.

In Table I although 16 ETFs are mentioned, we will only consider the first 15, since the VMMXX has zero return and risk. For the computations, in addition to the 15 ETFs provided by the platform, we also have to consider a risk free asset, so for this study we decided to opt for the U.S. Treasury Bond, which presents a level of return in the amount of 0.16% for the 5-year horizon.

Figure 1 represents an overview of the evolution of the prices of each ETF, we have created a chart were we are able to observe that as of 2016 there was an increase in prices, however in February 2020 we could see that there was a big decrease, which can be linked to the situation of the Corona virus. This decrease will have an impact on ETFs' returns.

Table I - Riskalyze Platform Etfs

INDEX	DESCRIPTION	CATEGORY
BND	Vanguard Total Bond Market ETF	Intermediate-Term Bond
SHY	iShares 1-3 Year Treasury Bond	Short Government
SPY	SPDR® S&P 500 ETF	Large Blend
EFA	iShares MSCI EAFE	Foreign Large Blend
HYG	iShares iBoxx $ High Yield Corporate Bd	High Yield Bond
FLOT	iShares Floating Rate Bond	Ultrashort Bond
VNQ	Vanguard REIT ETF	Real Estate
QQQ	PowerShares QQQ ETF	Large Growth
DBC	PowerShares DB Commodity Tracking ETF	Commodities Broad Basket
DBL	Doubleline Opportunistic Credit Fund	Close-Ended Fixed Income Mutual Fund
EFR	Eaton Vance Senior Floating-Rate Fund	Close-Ended Fixed Income Mutual Fund
XLU	Utilities Select Sector SPDR® ETF	Utilities
EEM	iShares MSCI Emerging Markets	Diversified Emerging Markets
FPX	First Trust US IPO ETF	Exchange Traded Fund
FXI	iShares China Large-Cap	Exchange Traded Fund
VMMXX	Vanguard Prime Money Market Fund	Mutual Fund

Description, abbreviations and categories of the 16 ETFs provided by the Riskalyze platform, which will be used for the calculations in this study. The VMMXX is also an ETF handed over by the platform, however we will not use it in the computations, since it presents zero return and risk.

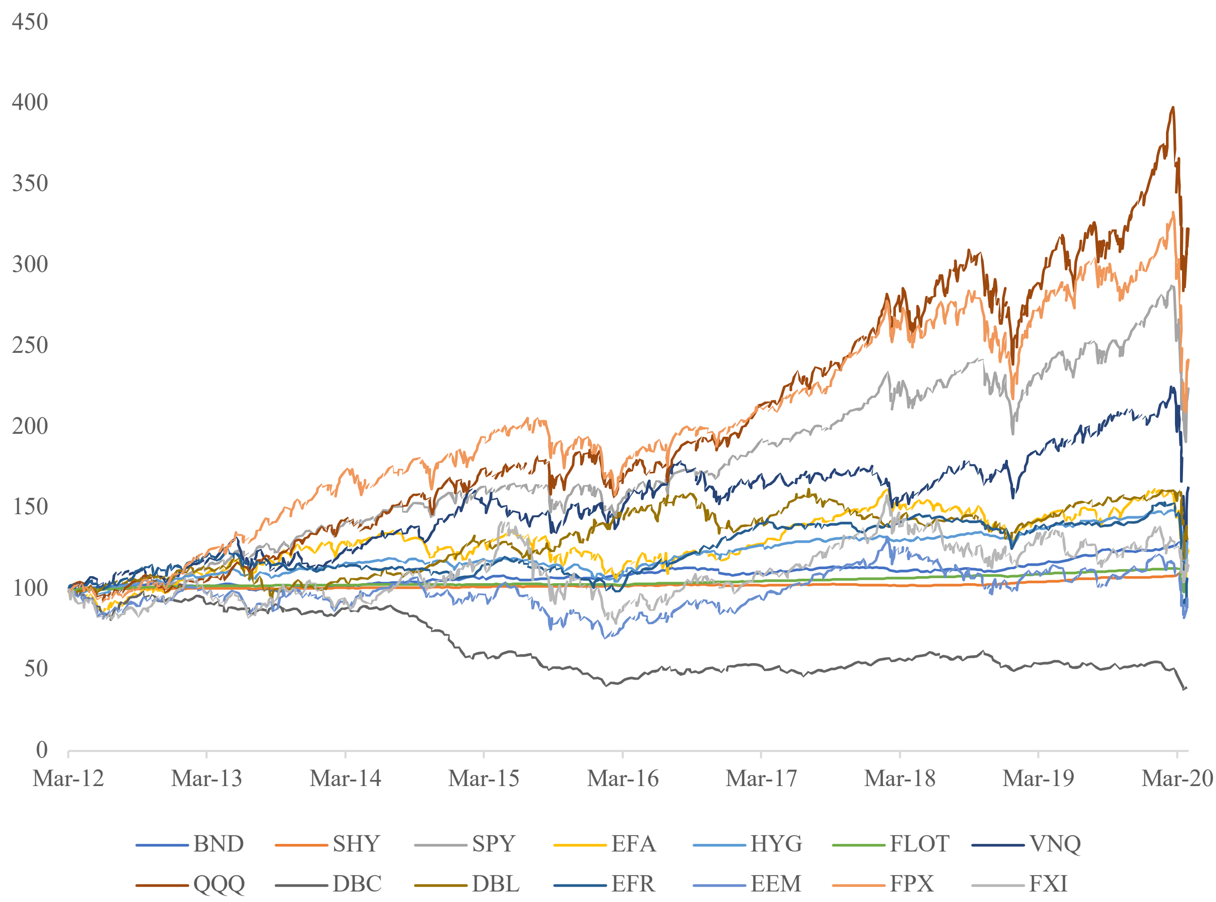

Figure 1 – Normalized ETFs evolution since March 2012

Normalized values of ETFs on a daily basis, starting with a notional value of $100 on 1st March 2012.

We use the first 5-years of data to estimate mean-variance inputs. Table II presents the historical expected returns and volatilities of each ETF, while Table III represents the variance-covariance matrix.

Table II - Expected Returns & Standard Deviations

INDEX	$\bar{R}$	σ
BND	1.95%	3.22%
SHY	0.48%	0.78%
SPY	13.40%	12.63%
EFA	6.53%	15.55%
HYG	4.91%	6.62%
FLOT	0.98%	0.97%
VNQ	11.04%	14.77%
QQQ	16.27%	14.96%
DBC	-12.05%	14.70%
DBL	8.31%	13.81%
EFR	7.16%	10.38%
XLU	12.21%	13.88%
EEM	1.39%	19.20%
FPX	16.00%	15.51%
FXI	4.51%	23.22%

Expected returns ($\bar{R}$) and volatility (σ) for each ETF, based upon daily prices from 31st March 2012 to 31st March 2020.

Figure 2 shows the mean-variance representation of the ETFs under analysis. As we can see, the ETF with the highest historical average return is the QQQ, which has a risk level in the order of 14.16%, the one that presents the lowest (and negative) level of return is the DBC ETF, as its price has been decreasing since 2014.

Table III – Variance-Covariance Matrix for in-sample period

	BND	SHY	SPY	EFA	HYG	FLOT	VNQ	QQQ	DBC	DBL	EFR	XLU	EEM	FPX
BND	0,00104	0,00018	-0,00090	-0,00085	0,00006	-0,00001	0,00083	-0,00094	-0,00042	0,00094	-0,00007	0,00117	-0,00032	-0,00100
SHY	0,00018	0,00006	-0,00023	-0,00018	-0,00003	0,00000	0,00012	-0,00025	-0,00005	0,00011	-0,00006	0,00023	-0,00011	-0,00026
SPY	-0,00090	-0,00023	0,01595	0,01677	0,00577	0,00009	0,01189	0,01735	0,00726	0,00169	0,00352	0,00831	0,01874	-0,00026
EFA	-0,00085	-0,00018	0,01677	0,02419	0,00688	0,00013	0,01278	0,01780	0,01010	0,00203	0,00416	0,00869	0,02480	0,01814
HYG	0,00006	-0,00003	0,00577	0,00688	0,00439	0,00003	0,00508	0,00605	0,00407	0,00156	0,00205	0,00331	0,00834	0,00644
FLOT	-0,00001	0,00000	0,00009	0,00013	0,00003	0,00009	0,00008	0,00007	0,00010	0,00002	0,00006	0,00005	0,00018	0,00008
VNQ	0,00083	0,00012	0,01189	0,01278	0,00508	0,00008	0,02181	0,01195	0,00404	0,00442	0,00254	0,01284	0,01557	0,01258
QQQ	-0,00094	-0,00025	0,01735	0,01780	0,00605	0,00007	0,01195	0,02238	0,00653	0,00208	0,00384	0,00742	0,02005	0,02013
DBC	-0,00042	-0,00005	0,00726	0,01010	0,00407	0,00010	0,00404	0,00653	0,02159	0,00091	0,00234	0,00286	0,01352	0,00788
DBL	0,00094	0,00011	0,00169	0,00203	0,00156	0,00002	0,00442	0,00208	0,00091	0,01908	0,00231	0,00333	0,00327	0,00197
EFR	-0,00007	-0,00006	0,00352	0,00416	0,00205	0,00006	0,00254	0,00384	0,00234	0,00231	0,01077	0,00139	0,00453	0,00431
XLU	0,00117	0,00023	0,00831	0,00869	0,00331	0,00005	0,01284	0,00742	0,00286	0,00333	0,00139	0,01927	0,01108	0,00727
EEM	-0,00032	-0,00011	0,01874	0,02480	0,00834	0,00018	0,01557	0,02005	0,01352	0,00327	0,00453	0,01108	0,03685	0,02010
FPX	-0,00100	-0,00026	0,01729	0,01814	0,00644	0,00008	0,01258	0,02013	0,00788	0,00197	0,00431	0,00727	0,02010	0,02406
FXI	-0,00097	-0,00026	0,01897	0,02530	0,00783	0,00020	0,01395	0,02062	0,01202	0,00238	0,00462	0,00909	0,03779	0,02054

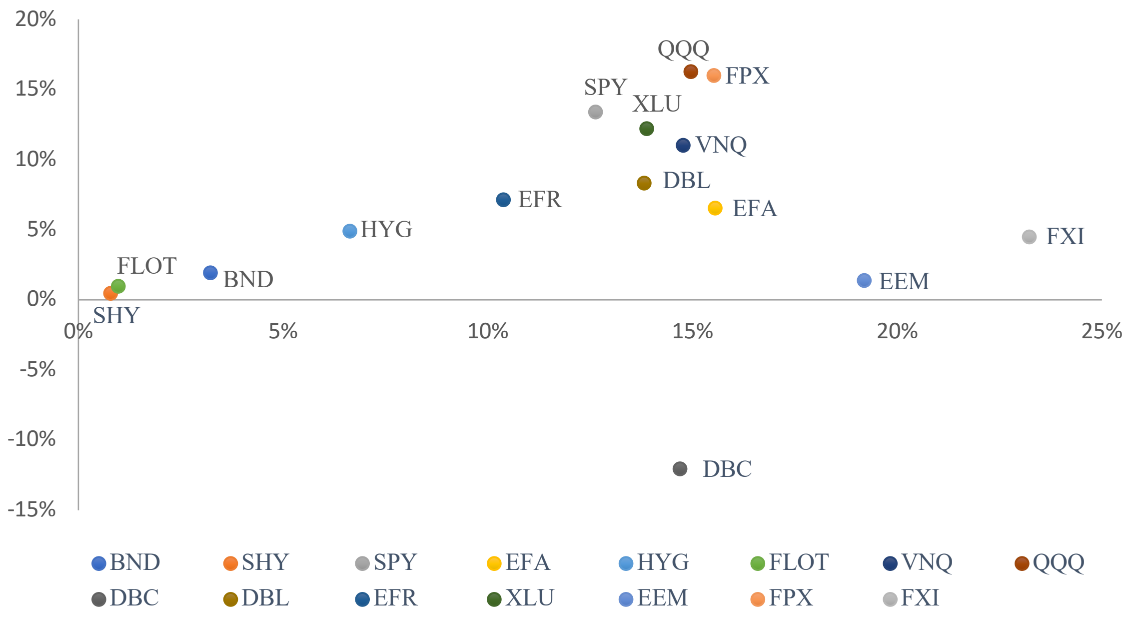

Figure 2 - Mean-Variance Representation of ETFs (March 2012 – March 2017)

Representation in the mean-variance plan of the ETFs used by the Riskalyze platform, so that we can understand the relationship between the return and the risk of each ETF, for the in-sample period.

4.2 MEAN-VARIANCE PORTFOLIOS

Through Figure 3, we can observe where the mean-variance portfolios stand regarding the efficient frontier. These portfolios correspond to the optimal portfolios when the weight invested in each ETF is equal (H), this one is used as naive benchmark, the portfolio with the highest SR (T), and the portfolio with the lowest possible risk level for the rate of the expected return (MV). Regarding its composition, we can see the weights assigned to each ETF through Table IV.

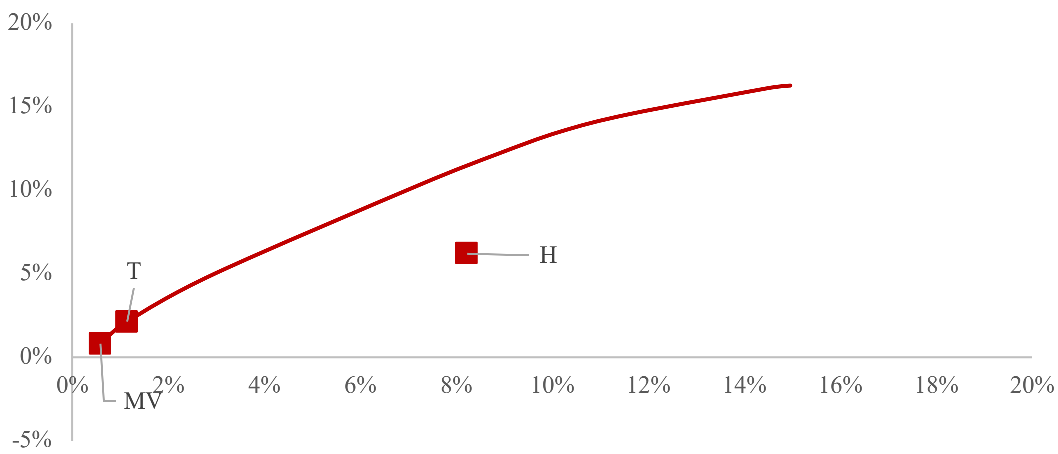

Figure 3 - Optimal Portfolios & EF

Based on the above inputs, and on the methodology described, the following compositions were obtained for the different optimal portfolios:

Table IV – Composition of Mean-Variance Portfolios

	T	MV	H
BND	13.03%	0.00%	6.67%
SHY	25.78%	61.81%	6.67%
SPY	4.20%	0.64%	6.67%
EFA	0.00%	0.00%	6.67%
HYG	0.00%	0.00%	6.67%
FLOT	51.37%	37.05%	6.67%
VNQ	0.00%	0.00%	6.67%
QQQ	0.00%	0.00%	6.67%
DBC	0.00%	0.00%	6.67%
DBL	1.13%	0.00%	6.67%
EFR	1.32%	0.05%	6.67%
XLU	0.00%	0.00%	6.67%
EEM	0.00%	0.00%	6.67%
FPX	3.17%	0.44%	6.67%
FXI	0.00%	0.00%	6.67%
$\bar{R}_p$	2.14%	0.82%	6.21%
σ_p	1.14%	0.59%	8.23%
SR	1.732	1.126	0.735

4.3 RISKALYZE PORTFOLIOS

The Riskalyze platform provides us with conservative, moderate, and aggressive portfolios. These will be the portfolios that we later use for our analysis since they are the bridge between our results and the robos, that is, we compare the results obtained with the portfolios created by us, with the portfolios provided by the Riskalyze platform, in order to understand the viability of Robo-Advisors.

Thus, we start by calculating the in-sample expected return, the standard deviation and the Sharpe ratio for each portfolio, in order to be able to understand the investment performance of each one. Table V, shows that for the conservative (C) portfolio we obtain a SR equal to -0.0609, for the moderate (M) portfolio the SR is 0.9099, and for the aggressive (A) portfolio presents a SR in the amount of 0.7158. Through these values, we were able to understand which of the portfolios designed by Riskalyze has the best performance, in this case it is the moderate one.

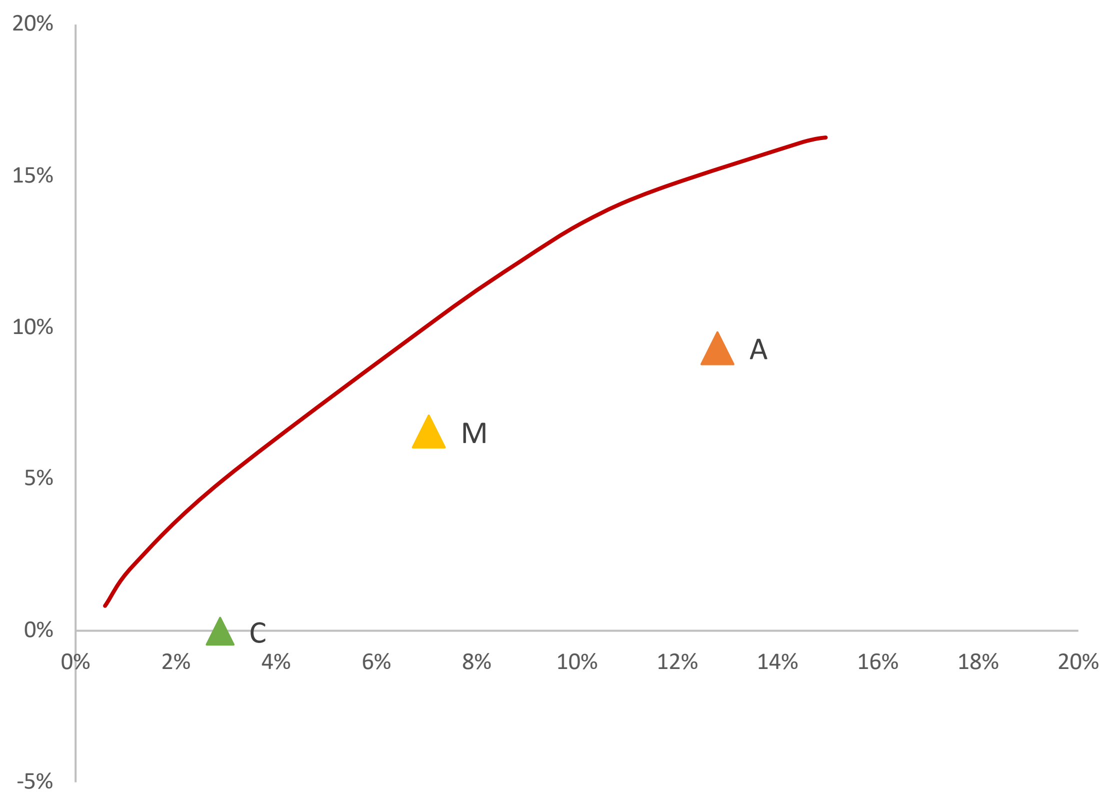

Figure 4 - Riskalyze Portfolios in a Mean-Variance Plan

Table V – Composition of Riskalyze Portfolios

	C	*M*	*A*
BND	35.00%	25.00%	0.00%
SHY	30.00%	1.00%	0.00%
SPY	13.00%	13.00%	26.00%
EFA	5.00%	15.00%	20.00%
HYG	5.00%	7.00%	0.00%
FLOT	5.00%	0.00%	0.00%
VNQ	2.00%	10.00%	12.00%
QQQ	0.00%	5.00%	17.00%
DBC	0.00%	5.00%	7.00%
DBL	0.00%	7.00%	0.00%
EFR	0.00%	7.00%	0.00%
XLU	0.00%	5.00%	0.00%
EEM	0.00%	0.00%	7.00%
FPX	0.00%	0.00%	6.00%
FXI	0.00%	0.00%	5.00%
VMMXX	5.00%	0.00%	0.00%
$\bar{R}_p$	-0.02%	6.57%	9.32%
σ_p	2.88%	7.04%	12.80%
SR	-0.0609	0.9099	0.7158

Calculation of the expected return and standard deviation, in order to determine the Sharpe Ratio for each of the portfolios provided by the Riskalyze platform, in order to be able to understand the investment performance of each of them, and later compare it with the portfolios created by us.

5. RESULTS

In order to make the analysis easier to understand, we decided to divide it into two parts, the in-sample results and the out-of-sample results.

5.1. IN-SAMPLE

Between 1st March 2012 and 31st March 2017, and for levels of relative risk aversion in the range of -1 to 1, we obtain the same portfolio and thus the same basic statistics (see the table VI).

As for the other levels of RRA portfolios differ. In Table VI, we can see the investment weights corresponding to each of the portfolios, and in addition it still gives us the value of the expected return, the covariance, the maximum value that can be obtained for the RTF for each RRA value, and the value of the Sharpe Ratio.

Table VI, reports the various Sharpe Ratio (SR) values for portfolios with different RRA levels. We notice then that as the RRA increases, the in-sample ratios also increases, inducting by the portfolio performance for more risk averse investors.

Table VI - RRA Portfolios - Investment Weights

RRA	BND	SPY	QQQ	DBL	EFR	XLU	FPX	$\bar{R}_p$	σ_p	**Max RTF**	**SR**
-1.00	0.00%	0.00%	100%	0.00%	0.00%	0.00%	0.00%	16.27%	14.96%	**0.1872**	**1.077**
0.00	0.00%	0.00%	100%	0.00%	0.00%	0.00%	0.00%	16.27%	14.96%	**0.1627**	**1.077**
0.25	0.00%	0.00%	100%	0.00%	0.00%	0.00%	0.00%	16.27%	14.96%	**0.1566**	**1.077**
0.50	0.00%	0.00%	100%	0.00%	0.00%	0.00%	0.00%	16.27%	14.96%	**0.1505**	**1.077**
0.75	0.00%	0.00%	100%	0.00%	0.00%	0.00%	0.00%	16.27%	14.96%	**0.1444**	**1.077**
1.00	0.00%	0.00%	100%	0.00%	0.00%	0.00%	0.00%	16.27%	14.96%	**0.1383**	**1.077**
1.25	0.00%	0.00%	91.63%	0.00%	0.00%	0.00%	8.37%	16.25%	14.85%	**0.1322**	**1.0836**
1.50	0.00%	0.00%	85.76%	0.00%	0.00%	0.00%	14.24%	16.23%	14.79%	**0.1262**	**1.0871**
1.75	0.00%	0.00%	81.56%	0.00%	0.00%	0.00%	18.44%	16.22%	14.75%	**0.1202**	**1.0889**
2.00	0.00%	0.00%	76.63%	0.00%	0.00%	2.79%	20.58%	16.10%	14.47%	**0.1142**	**1.1022**
3.00	0.00%	37.98%	0.00%	0.00%	0.00%	11.70%	50.32%	14.57%	11.62%	**0.0936**	**1.2400**
4.00	0.00%	35.41%	0.00%	13.38%	0.00%	14.33%	36.87%	13.51%	10.16%	**0.0779**	**1.3136**
5.00	2.94%	29.39%	0.00%	15.89%	10.35%	14.12%	27.31%	12.15%	8.84%	**0.0651**	**1.3569**
6.00	20.36%	25.70%	0.00%	12.47%	8.89%	9.65%	22.93%	10.36%	7.26%	**0.0556**	**1.4045**

Lists the investment weights for the RRA portfolios, and also show the Expected Return and Covariance of the portfolios, the maximum value that can be obtained for the RTF for each RRA value, and the value of the Sharpe Ratio.

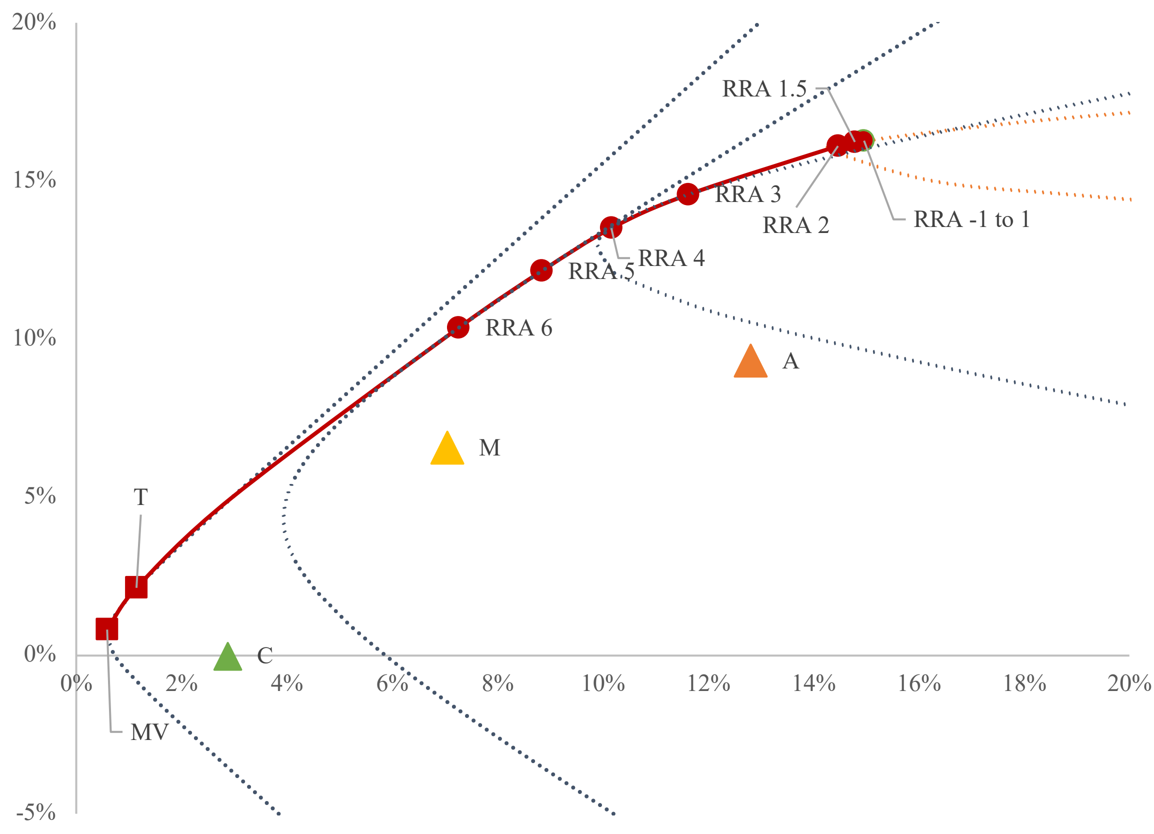

Figure 5 - Portfolios in a Mean-Variance Plan

From Figure 5 it is possible to see that the robo-portfolios, as well as the naive homogeneous portfolio, are inside the historical efficient frontier (EF), which tell us those portfolios must be selected according to criteria other than mean-variance efficiency or that the inputs used by the Robo-advisor substantially differ from the historical ones. The EF itself includes subsets of different hyperbolas, as expected in the case of no short selling and also evident from the portfolio compositions in Table VI, where it is clear the set of assets is not constant over the various mean-variance optimal portfolios.

Since Figure 5 presents the position of the RRA optimized portfolios with those proposed by the online platform, we perceive that the Riskalyze conservative portfolio is very close to a return level equal to zero. Table VI and Figure 5, demonstrates that in-sample RRA optimized portfolios are more efficient. Thus, we can say that if investors aspire to efficient portfolio, RRA optimization seems to do better. So with the data available on the market, and purely with the in-sample analysis, we realize that the proposed portfolios appear to be inefficient, or else they are not using MVT, at least with our inputs.

5.2 OUT-OF-SAMPLE

After the analysis of the in-sample data efficiency of the various portfolios, we move to the out-of-sample performance analysis.

We aim to observe the actual/forward performance of portfolios proposed based only on information up to March 2017. As previously mentioned the investment horizon of such portfolios would be 5 years, thus, until the end of March 2022. Unfortunately, our out-of-sample finishes in March 2020, so this out-of-sample analysis relies only on the first 3 years of investment.

We consider a notional investment of $100 in each portfolio, and from there see how they evolve. We assume monthly rebalancing, in order to realign the weightings of the portfolio.

From Figure 6, we can see how the portfolios created by us evolved from 31st March 2017 to 31st March 2020.

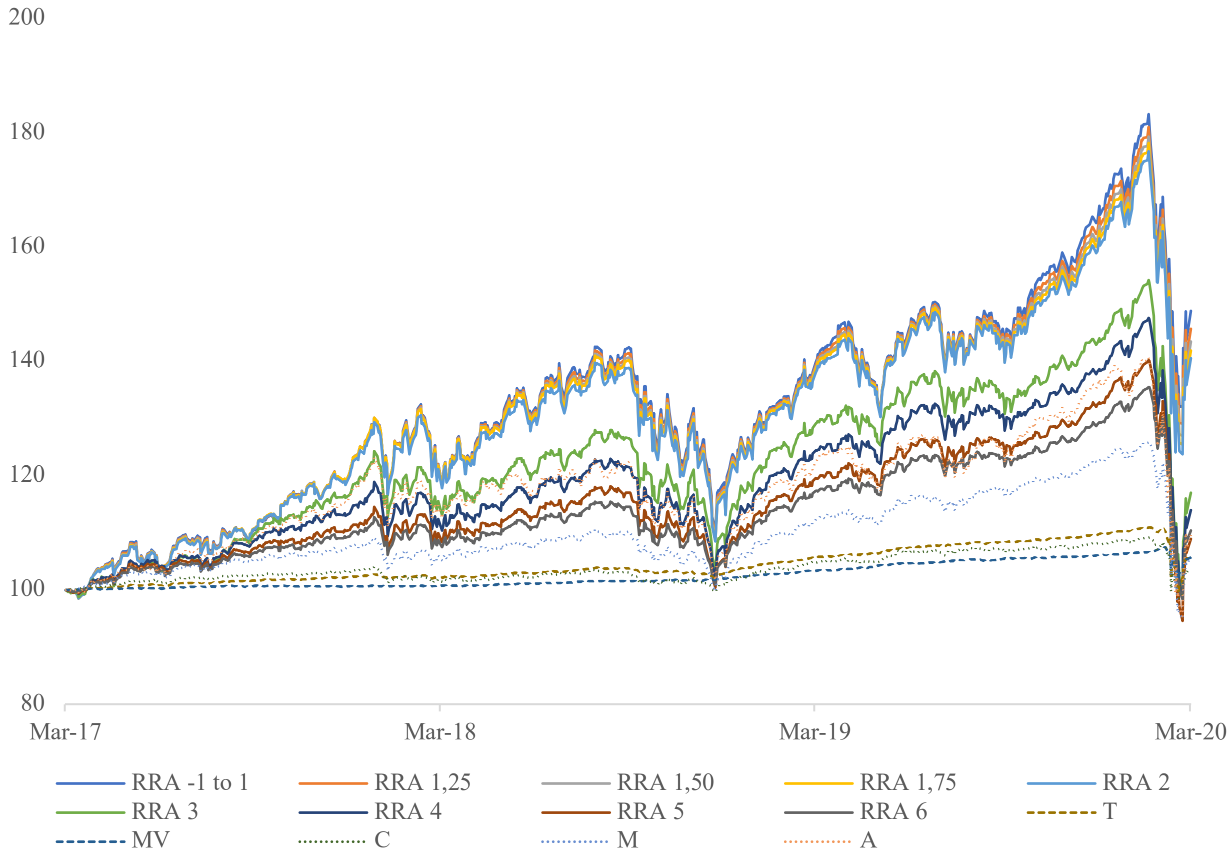

Figure 6 - Evolution of the Portfolios

Representation of the evolution of the Portfolios for the out-of-sample period.

From Table VII and Figure 6, we can see that if we invest $100 in the portfolio and look at the first two years of the out-of-sample period, the total value of the portfolio has been increasing, which represents a good evolution of the portfolio. However, starting in 2020, we notice that there is a decrease in the total amount of the portfolio's value, however this effect is explained by the pandemic phase that we are going through, the Corona virus, which is affecting the global economy.

Table VII - Investment Evolution

INVESTMENT = *$ 100*

DATE	03.04.2017	29.03.2018	29.03.2019	31.12.2019	30.03.2020
-1.00 – 1.00	*$ 99.94*	*$ 122.01*	*$ 138.07*	*$ 164.44*	*$ 148.85*
1.25	$ 99.90	$ 121.88	$ 137.62	$ 162.61	$ 145.68
1.50	$ 99.87	$ 121.79	$ 137.30	$ 161.32	$ 143.48
1.75	$ 99.85	$ 121.72	$ 137.07	$ 160.41	$ 141.93
2.00	$ 99.84	$ 121.11	$ 136.56	$ 159.30	$ 140.57
3.00	$ 99.66	$ 115.61	$ 128.34	$ 143.24	$ 117.02
4.00	$ 99.81	$ 112.39	$ 123.85	$ 138.15	$ 113.95
5.00	$ 99.84	$ 109.99	$ 118.97	$ 132.45	$ 108.96
6.00	$ 99.92	$ 108.75	$ 116.87	$ 128.98	$ 110.43
T	$ 99.99	$ 102.25	$ 105.69	$ 109.61	$ 105.93
MV	$ 100.01	$ 100.74	$ 103.43	$ 106.00	$ 105.70
H	$ 100.03	$ 109.14	$ 113.76	$ 123.22	$ 104.62
C	$ 100.12	$ 101.89	$ 104.96	$ 108.15	$ 103.71
M	$ 100.05	$ 106.04	$ 111.82	$ 122.57	$ 105.77
A	$ 99.96	$ 114.77	$ 121.19	$ 135.59	$ 109.61

Evolution of RRA portfolios..

After analyzing the evolution of investments in the various portfolios, we see how they also evolved in relation to the SR, this to confirm whether or not there was an improvement in their performance. Through table VIII, we realize that in the out-of-sample period the opposite of the in-sample period occurs, since as the RRA increases the value of the Sharpe Ratio decreases, that is, now the portfolios for investors who have a minor aversion to risk will get better results. When comparing our in-sample SR values with the out-of-sample, we can see that the out-of-sample SR have decreased, which means the performance of the portfolios have decreased in the following 3 years.

In Table VIII, we see the SR values for the portfolios created by the portfolio for the out-of-sample period. With these results, we understand that the value for the out-of-sample SR are higher than the in-sample values, which means we have an increase in the performance of this portfolios in the following 3 years.

When comparing, the SR for the out-of-sample values of the RRA portfolios and those suggested by Riskalyze, we realize that when our investor presents a moderate and aggressive profile, all our portfolios, for these types of investors, are more advantageous, since they have larger Sharpe Ratios. However, we found that when our investor has very high RRA levels (4, 5 and 6) the most advantageous portfolio would be the one suggested by the Riskalyze platform, conservative portfolio.

Table VIII - RRA Portfolios - Out-of-Sample Sharpe Ratio

PORTFOLIOS	$\overline{R}_p$	σ_p	SR
RRA -1.00 – 1.00	13.21%	0.23	0.5859
RRA 1.25	12.49%	0.23	0.5617
RRA 1.50	11.99%	0.23	0.5440
RRA 1.75	11.63%	0.23	0.5309
RRA 2.00	11.31%	0.22	0.5257
RRA 3.00	11.31%	0.22	0.5257
RRA 4.00	4.33%	0.18	0.2620
RRA 5.00	2.85%	0.17	0.1974
RRA 6.00	3.29%	0.14	0.2693
T	1.91%	0.05	0.4483
MV	1.84%	0.03	0.8493
H	1.50%	0.13	0.1520
C	1.21%	0.05	0.3315
M	1.86%	0.12	0.1975
A	3.05%	0.19	0.1871

Out-of-Sample Sharpe Ratio for the different portfolios.

6. CONCLUSION

The purpose of this study is to understand how Robo-Advisors analyze the risk profile of their investors, therefore, we compare the portfolios provided by the online platform with the portfolios created by us, which are calculated using analytical methods based on the mean-variance theory and the expected utility theory.

Through this comparison we can see if the methodology used by them is the same or not, and we can also see their performance over a period of 5 years, in order to understand what is the best way to calculate the investor's risk profile, and conclude if the method used by Robo-Advisors is viable.

Therefore, with our calculations, we can see that in the in-sample period if we want to optimize portfolios for different levels of RRA, we should invest in the portfolios we designed instead of investing in portfolios provided by the Riskalyze platform, since ours have a higher level of Sharpe Ratio, which means a better performance of these portfolios.

When we move to the out-of-sample period, if we compare the SR of the in-sample with the out-of-sample, we notice that they have decreased, which means that in these 3 years the performance of the portfolios has worsened. However, in the case of Riskalyze portfolios, we noticed that the Sharpe Ratio increased in the three suggested portfolios, thus showing that there was an improvement in their performance.

When comparing, the SR for the out-of-sample values of the RRA optimized portfolios and those suggested by Riskalyze, we notice investors with a more aggressive profile, that is, RRA less than zero, the method used by the RRA optimized portfolios is the most appropriate, and the same is observed when investors present a moderate level of risk. However, when the investor has a more conservative profile, that is, when we consider lower RRA levels, we realize that the best way to optimize portfolios is through the methodology used by the Riskalyze platform.

With the conclusions obtained, we can thus say that we do not recommend investing in robo-advisors, that is, if the investor wants to obtain a better performance from their portfolios, he should choose the methodology used in our study.

The realization of this study is very relevant, since it has never been studied how Robo-Advisors analyze the risk profile of their investors, however, during the development of this study, we faced several limitations, one of which is the fact that this topic is not yet very developed, since Robo-Advisors are a service that is emerging, so it is easy to perceive that there is not much information available. Another restriction is the fact that we are looking for a 5-year horizon, and only 3 of those 5 years have passed, which means that we are not aware of the complete performance of the portfolios under analysis. So, it would be interesting, that in 2 years from now we would do the calculations again, to see the real performance of these portfolios. However, despite these limitations, we managed to get an idea of how they will evolve from now on.

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