SCHOOL OF ECONOMICS & MANAGEMENT LISBON

MASTER OF SCIENCE IN ACTUARIAL SCIENCE

MASTERS FINAL WORK

INTERNSHIP REPORT

MODELLING LONG-TERM WORKER´S COMPENSATION - AN APPLICATION TO A GENERAL INSURANCE COMPANY

MIGUEL COLBURN HERCULANO

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Acronyms

APS: Associação Portuguesa de Seguradoras

BE: Best Estimate

CR: Capital Requirement

FAT: Fundo de Acidentes de Trabalho

IAA: International Actuarial Association

IAS: Indexante de Apoios Sociais

ISP: Instituto de Seguros de Portugal

KS: Kolmogorov-Smirnov

LoB: Line of Business

L&S: Life and Savings

LTGA: Long Term Guarantees Assessment

PV: Present value

P&S: Property and Casualty

QIS: Quantitative Impact Study

ROE: Return on Equity

SCR: Solvency Capital Requirement

STJ: Supremo Tribunal de Justiça

TS: Technical Specifications

VaR: value at risk

WsC: Worker´s Compensation

Acknowledgments

This report results from the work developed during a Co-op internship at Allianz Portugal, leading to the award of the MSc in Actuarial Science. The project was undertaken at the Actuarial Department, under supervision of the company´s actuaries and professors at University. The objective was to develop an internal model for life underwriting risks to which Worker´s Compensation is subject, with a view of complying with the new solvency requirements set by Solvency II.

I wish to thank Allianz Portugal for the opportunity to develop this stimulating project which would not be feasible without the data that was kindly provided. Special thanks go to Carina, Tânia and Prof. Lourdes Centeno for the thorough review of the work and constant support throughout my time in the Company. I thank ISEG for sponsoring this degree. I am also grateful to all other colleagues and professors for their time and help.

Modelling Long-Term Worker´s Compensation
An Application to a General Insurance Company

Miguel Colburn Herculano

Abstract

This paper resumes the main findings from modeling life underwriting risks to which Worker´s Compensation is exposed. Models presented aim to shorten the path between ad hoc procedures in place and the new capital requirements foreseen by Solvency II. The legal framework of this line of business is primarily explained as it is determinant for modeling purposes. We then provide a discussion about risk models in use, major options, assumptions and other relevant issues that were regarded when modeling this line of business.

Keywords: Internal Model, Worker´s compensation, Solvency II, SCR, Elasticity, CIR model, VaR, Geometric Mortality Model, life underwritings risks, stochastic modeling, longevity risk, interest rate risk, expense risk, revision risk.

1. Introduction

Possessing an adequate amount of capital is a major concern for insurance companies. In order to assure continuity of their business, it is critical to make sure they are capable of meeting obligations assumed. Furthermore, a strong capital position is important in providing a *buffer* against business volatility and gaining the Market´s confidence, including investors, supervisors and policyholders.

The level of capital held must be such that it not only complies with regulatory requirements but also ensures high ratings and maximizes ROE, Return on Equity. There is thus a *trade-off* in determining the appropriate level of capital of an insurance undertaking. High capital levels provide confidence and an increased solvency capacity. However, that will also decrease the attractiveness of the business, lowering the ROE and may result in the discontinuance of less profitable products.

The Solvency II project, the new regime that will replace existing regulation and establish more consonant rules across the EU, will introduce economic *risk-based* requirements that aim to be more *risk sensitive* and *entity-specific*. One important feature of the new regime is the establishment of quantitative requirements regarding own funds, in particular SCR, the Solvency Capital Requirement, defined as:

"The level of capital that enables an insurance undertaking to absorb significant unforeseen losses and that gives reasonable assurance to policyholders and beneficiaries."

The SCR accounts for all quantifiable risks[1], reflecting the risk of the total balance sheet, and is based on the Value-at-Risk calibrated at a 99,5% level over a 1-year time horizon. The idea is that ruin only occurs once in every 200 years. The SCR may be calculated using either the *European Standard Formula* or an *Internal Model* that must be previously validated by the Supervisor.

According to the IAA, the International Actuarial Association, an internal model may be regarded as *"(...) a mathematical representation of the insurer's business operations."* This is a rather broad definition, illustrating the wide variety of purposes of this tool. This work suggests an approach to quantify life underwriting risks to which Worker´s Compensation is exposed. Despite being classified as P&C, Property and Casualty, this LoB, line of business, has an important L&S, Life and Savings, component that must be modeled using life actuarial techniques.

The use of Internal Models within insurance companies is becoming increasingly popular due to a number of factors amongst which are the recent developments of *risk-based* insurance regulatory requirements.

Our objective is to propose a model for Worker´s Compensations that accurately captures major life underwriting risks involved in this LoB in an entity specific manner, contributing thereby to a more efficient risk management, approximating current practice to Solvency II standards. Revision, Longevity and Expense risks will be

[1] Underwriting, market, credit and operational risks

modeled in separate models and aggregated through appropriate methodology combining individual risks into a single capital number.

Parallel to the main purpose of this project we develop a long term longevity study and explore market interest rate risk modeled with a view to analyzing its impact when discounting Best Estimate Cash Flows. These modules are however a complement to the core purpose and quantification of regulatory capital requirements is not assessed for these risks.

For the sake of confidentiality and data protection, calculations presented are based on modified data.

2. Worker´s compensation insurance and its legal framework

WsC is an important LoB for the Portuguese Insurance industry, representing approximately 14% of non-life business and almost 5% of total business of the Industry. Latest statistics disclosed by the ISP reveal that, for the whole insurance industry, the loss ratio of this LoB is 1,016. It is out of this study´s scope to analyze this figure. Nevertheless, this clearly indicates the need to focus on this line of business.

Despite being classified as General insurance, this LoB features important Life assurance characteristics, and its mathematical treatment requires use of life actuarial techniques. This kind of insurance is compulsory for both regular employees and self-employed workers according to Law nº 98/2009.

Legislation regarding Worker´s Compensation dates back to the *Industrial Revolution* era, as Industrialization increased the frequency and severity of accidents at work. At first, legislation recognized that benefits were due in case of work casualties but they were only payable if the victims could prove the employer was to blame for, whatever happened. This principle changed throughout Europe at the end of the nineteenth century. From then on, employers were liable for any damage occurred during their employees work shifts, regardless of guilt. In Portugal this principle entered into force from the beginning of the twentieth century, during the first Republic with the Law nº83 of 24 of July 1913. Since then, our legislation has shifted towards a system that ensures more rights to workers. The broader definition of work accident and the extensions to the set of benefits that the workers are entitled to, recently introduced, are examples of this. An increase in frequency and severity of claims is expected as a direct consequence of this new law.

According to latest statistics, 47,2% of the total numbers of accidents of this nature in Portugal were registered amongst workers of the Manufacturing Industry and Construction[2]; these sectors employ approximately 26% of the total employed population. Statistics regarding the number of these accidents that lead to death are more spread out. Still, 45,2% of those occurred in the Construction & Manufacturing industry sectors. Evidence also shows that the total number of occupational accidents is decreasing. During the decade 2000-10, this figure dropped nearly 8%.

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[2] CAE Rev.3, Table 3 Gabinete de Estratégia e Planeamento (2010). Estatísticas em Síntese – Acidentes de Trabalho pág. 2.

According to Portuguese law, an occupational accident is one that occurs in the workplace, during working time, that causes direct or indirectly prejudice, leading to reduction of work, earning capacity or death[3]. Some extensions to this definition were made[4]. For instance, casualties that may occur during the employees' lunch break or on the way to/from work may also be considered an occupational accident.

Overall, and according to *judge-made law*[5], an occupational accident must include a *spatial element*, the workplace, a *temporal element*, during working time, and a *causality element*, the accident must cause injuries and those injuries must result in disability or death. Moreover, the event must be considered an accident, meaning it must be *occasional, sudden* and caused by an *external source*.

Another important feature of this Lob is the existence of a Workers´ Compensation Fund, known as FAT managed by the ISP. This fund´s purpose is to reimburse compensations payable to victims, whenever the responsible entity is unable to do so. The fund is also responsible for updating pensions and assuring risks that have been denied by insurance companies. This fund is financed by all insurance companies that must pay 0,85% of redemption value of each pension and thrice the value of the annual pension payable to victims if no beneficiaries exist.

2.1. Benefits

In case of occupational accident, victims are entitled to two sorts of benefits, *in kind* and *in cash*. The former are diverse in nature and intend to reestablish the victim's health

[3] Art. 6 law nº100/97, 13 of September
[4] See Art. 9 law 98/2009, 4th of September
[5] Court Judgment, STJ 14/04/2010

and working capacity. The later, include allowances, pensions and subsidies of a pecuniary nature. A non-comprehensive list of benefits prescribed by law is shown below[6].

Benefits in kind	Benefits in cash
Medical assistance	Temporary allowance
Medication assistance	Permanent incapacity pension
Nursing assistance	Subsidy for funeral related expenses
Hospitalization and thermal treatment	Subsidy for severe permanent incapacity
Transport & accommodation	Subsidy for housing reconversion
Professional reintegration	Professional rehabilitation subsidy
Medical and functional rehabilitation	Death pension
Psychological assistance to victims family	Death Subsidy
	Pension for third party assistance

Table 1: Benefits payable in case of occupational accident

Amongst cash benefits, one may distinguish between those that are payable once and others that give rise to a series of payments that may be regarded as annuities. This is important in the modeling process because the later are treated as life liabilities giving rise to Mathematical Provisions. It is common to classify benefits as *lifetime assistance, Pensions* or *general claims.* The first two represent long term liabilities and the later have a short term nature.

2.2 Benefits scheme by severity

The law states the framework of possible forms of disability[7], establishing that disabilities may be *permanent* or *temporary*. In case disability is *temporary*, it may be considered *partial* or *absolute*. On the other hand, if it is considered *permanent* it may be considered *absolute*, *absolute for regular work* and *partial*.

[6] For a comprehensive list of legal benefits see articles 25 and 47 of law 98/2009, 4th of July
[7] This classification is based on the national table of incapacities due to occupational accidents disclosed through DL 352/2007 of 23th October

Classification is based upon a *disability coefficient* that is also used in calculating benefits. Legislation also uses IAS in establishing formulae for compensation calculation. This value is set to 419,22 € for 2013 however it is revised on a yearly basis.

The set of benefits to which a certain beneficiary is entitled will depend upon the severity of the associated casualty.

2.2.1 Permanent or temporary incapacity

The set of benefits and calculation formula used in case of permanent or temporary disability is summarized in Table 7 in Appendix 3. Another important thing to bear in mind is the redemption of pensions that may be compulsory or facultative. We will deal with this later.

2.2.2 Death

If the occupational accident results in death, benefits are payable to the victim's family. Benefits formulae are summarized in Table 8 in Appendix 3. Other relatives may also be eligible to receive benefits. The interested reader may confer further details regarding death benefits in Section III, Law nº 98th/2009.

2.3 Redemption, expiry and revision

It is important to bear in mind that in some situations pensions may be *redeemed*. These situations are described by the law[8]. A *lifelong pension*, payable to a victim is *mandatorily redeemable* if his or her *partial permanent disability coefficient* is less than

[8] See section VII, law 98/2009, 4th September

30% or if the pension´s value does not exceed six times the value of the minimum wage. A *lifelong pension* may also be *partially redeemable* under legal conditions. *Redemption capital* is calculated taking into account the beneficiaries' age and pension or gross wage, depending on the casualty.

Pensions may *expire* due to a variety of circumstances. Death pensions expire if the widow or widower *remarries* or if the orphan attains age limit and/or education limit.

According to the current law, victims may require revision of their disability once per calendar year.

3. Modelling Worker´s Compensations

WsC is a very particular line of business as it encompasses life and non-life liabilities. We have seen that benefits payable may be divided in:

i) General Claims;

ii) Lifetime Assistance;

iii) Pensions.

The first class of benefits give rise to non-life liabilities as they represent lump sum payments of random amount. Pensions configure regular life insurance liabilities as their amount is fixed and may last for many years. Lifetime assistance have a more hybrid nature. Claims give rise to mathematical provisions; however, their amount is unknown. Lifetime assistance payments may last for many years just as pensions but annual payments are highly irregular and depend on the health condition of beneficiaries.

Our goal is to consider life underwriting risks as presented in SCR.7.1 of the Long Term Guarantee Assessment, Technical Specifications pp.177.

4. Risk Models in Use

Our aim is to model revision, longevity, expense and interest rate risks, the most concerning for this line of business for the company. We use a simple mortality stochastic model to describe short term longevity risk and a model for revision risk involving two random variables describing revision severity and frequency. A mortality projection model was used to estimate long term mortality improvements in our portfolio and interest rates were modeled by fitting the CIR, Cox Ingersoll Ross model to historical data. Goodness of fit was assessed using the KS, Kolmogorov-Smirnov test where applicable. The methodology adopted follows closely the approach suggested in Rosa, C. (2012).

4.1 Longevity Risk

Longevity risk is defined as the risk of loss due to an increase in the value of insurance liabilities resulting from unfavorable changes in mortality rates[9]. This risk is particularly relevant in the long run, in a realistic scenario of persistently low mortality, but its impact must also be assessed in the short term, accounting for sudden changes in mortality or in its volatility pattern, considering an extreme scenario. Therefore, it is

[9] See SCR.7.3. Technical Specification on the Long Term Guarantee Assessment

important to study the impact of longevity on those benefits that are payable until the death of the beneficiary (ie, pensions and Lifetime Assistance benefits).

4.1.1 Long term Impacts

To analyze the long term impact of longevity on our Best Estimate, we need to verify the extent to which the present value of our liabilities subject to mortality risk shift as a result of a permanent change in the mortality pattern. To this end we need to predict future mortality levels, which is far from easy. One approach involves considering a mortality projection model. The Portuguese Association of Insurers developed a study that involved the Construction of Annuity Tables for Portugal. This project´s finding may be found in Maeder (2008). The author used a simple mortality projection model that we adopt with a view to applying mortality improvements to our own portfolio. This model considers heterogeneous mortality developments amongst ages and countries' own specificities making it particularly interesting for our purposes. According to Bravo (2009), the model was developed by Nolfi (1959) and has been adopted in constructing mortality tables in Spain (PERM/F 2000) and Austria (AVÖ 2005R) among others. It may be described by the following equation:

$$q_{x,t} = q_{x,t_0} e^{-\lambda_x (t-t_0)}$$

More details about this model are given in Appendix 1.

The equation implies that mortality decreases exponentially with time and that the difference between the logs of the coefficients is linear (ie, $\ln q_{x,t} - \ln q_{x,t_0} = -\lambda_x(t - t_0)$). Note that λ_x plays an important part in the model representing the annual intensity of mortality decrease. The larger λ_x, the more rapidly mortality decreases with time.

Our interest is to use the set of estimated values for the improvement mortality factors λ_x to project mortality developments in our annuity portfolio in 10 years' time, our long-term benchmark. We will use the results found in Maeder (2008) to do so. Our results are summarized in the figure below.

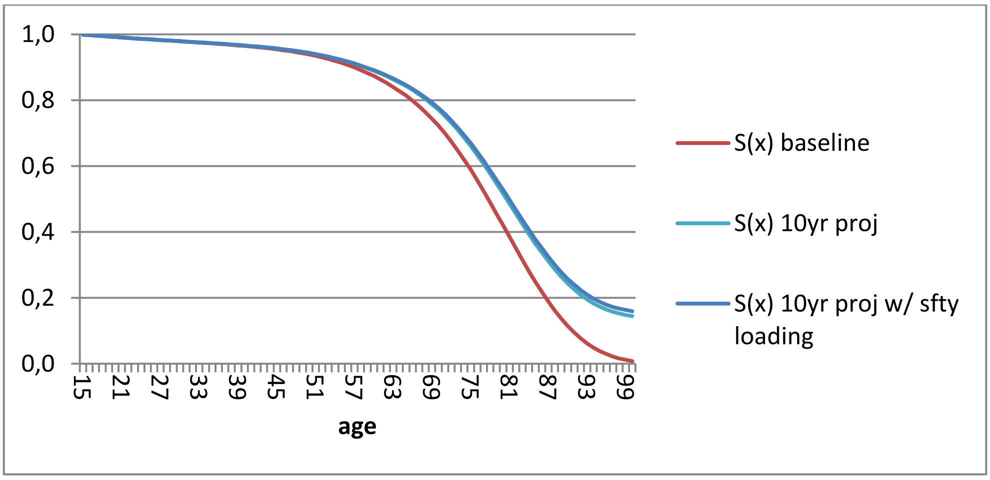

Figure 1: Survival Functions corresponding to Baseline and Projected mortality rates

The graph shows the survival functions according to mortality tables we currently use (the baseline) and our projected 10 year survival functions with and without a safety loading[10]. Lowering mortality is particularly evident for older ages.

We now examine the impact of the consideration of the projected mortality on the present value of our liabilities and in particular the capital requirements that arise from longevity risk on a long-term view.

[10] Further details on the safety loading referred may be found in Appendix 1

The results are shown in the table below.

		Pensions	Lifetime Assistance	Total
(1)	**BE baseline**	53.027.892	14.771.863	**67.799.755**
(3)	**BE proj mort**	53.539.812	15.395.517	**68.935.329**
(3)-(1)	**impact**	511.919	623.654	1.135.573
(4)	**BE proj mort w/ sfty load**	53.801.547	15.481.874	**69.283.420**
(4)-(1)	**impact**	773.654	710.011	1.483.665

Table 2: Best Estimate values of Pensions and Lifetime Assistance in euros

The table resumes the Best Estimate of liabilities by kind of liability. Note that the line of the table that calculates the impact of the consideration of low mortality represents additional costs that must be regarded in the long-run view.

Overall, we can see that considering mortality rates in the long-run we can realistically expect an increase of 1.135.573€ on our Best Estimate figures for Lifetime assistance and pensions in the face of lifetime improvement.

4.1.2 Short term impacts

As mentioned, in the short term we are interested in studying the effect of sudden changes in mortality, in particular it is of interest to assess the impact in terms of the capital requirements of the occurrence of an extreme worst case scenario in a one year time horizon, in accordance with Solvency II. Our previous analysis focuses on projecting the longevity impact in order to suggest a realistic scenario. Now we are interested in studying the impact of an extreme scenario and thus the analysis is different.

The QIS 5 and more recently the LTGA exercise suggest considering a 20% instantaneous, permanent shock to mortality to all ages. This value must be regarded when using the European Standard Formula; however it does not take into account entity specific characteristics.

By constructing a stochastic model to predict payments during the following year and reserves at the end of the year we take into consideration our exposure to risk in the way that simulated figures account for the pensioners age and type of beneficiary.

We used a stochastic model by simulating 1000 replicas of mortality scenarios for the forthcoming year. We considered the random variables:

$$I_k = \begin{cases} 1 & if\ pensioner\ k\ dies\ in\ the\ forthcoming\ year \\ 0 & otherwise \end{cases}, k = 1,2,\dots,N \qquad ,$$

where N is the number of pensioners in our portfolio. We assume $I_k \sim Bernoulli(q_x)$, where x is the age of pensioner k, and generate N variables with this distribution for each simulation. We are then able to simulate the Reserve associated to each pensioner at the end of the year and the payments to each pensioner occurring during the year in the following way:

$$P_{[0,1]} = \sum_{k=1}^{N} \frac{P_k}{12} \left[\ddot{a}_{\overline{12}|i} (1 - I_k) + \ddot{a}_{\overline{6}|i} I_k \right]$$

where 0 and 1 are the opening and closing accounting year dates;

$P_{[0,1]}$ are the aggregate payments during account year 1;

P_k is the annual payment due to pensioner k and $\quad a_{\bar{k}|i} = \dfrac{1 - v^k}{i}$

Note that we have assumed that all deaths occur at the middle of the year.

We also need to simulate aggregate reserves at the end of the year. To do this we need to compute individually the reserve associated to each pensioner and after this, sum the individual reserves for each replica.

In doing this we must take into account the nature of each pensioner. The most common pensioner´s reserves are calculated regarding the following formulae:

i. Victims: $\ddot{a}_x^{(12)} \approx a_x + \frac{13}{24}$

ii. Partners & Parents: $\ddot{a}_{x:65-x}^{(12)} + \frac{4}{3}\, {}_{65-x}E_x\, \ddot{a}_{65}^{(12)}$

iii. Orphans: $\ddot{a}_{x:25-x}^{(12)}$

where:

${}_{65-x}E_x = v^n\, {}_n p_x$ is the expected present value of the pure endowment;

$a_x = \sum_{i=1}^{\infty} v^i\ {}_i p_x$

${}_n p_x = P(T_x > n) = S_x(n)$ where T_x is the future lifetime of an individual aged x.

${}_n q_x = P(T_x \leq n) = 1 - S_x(n)$

Before proceeding, it is important to explain the way we deal with lifetime assistance benefits. Unlike pension benefits, lifetime assistance payments are random and thus, we need to estimate payments for the forthcoming year in order to proceed with our analysis. The way reserves are calculated is also different. For these benefits we calculate reserves taking into account the average payment of the kind of injury the victim suffered from, and multiply this quantity by an annuity that depends on the age of the victim.

By looking at historical data of payments for each pensioner of this nature we find that it exhibits high volatility and has no defined trend. However, considering the aggregate amount of payments for all pensioners year by year, there seems to be a pattern of downward regular trend in payments. For each pensioner, we generated 1000 pseudo-payments from a Normal random variable with mean equal to the mean cost of the associated injury of that particular pensioner, and standard deviation assumed equal to historical data. Done this, we calculated the average simulated payment and treated these benefits as pensions thereafter.

It is important to bear in mind that a given pensioner k may be benefiting from a pension and a lifetime assistance annuity. In these cases one must simulate one unique value for I_k.

Having done this we obtain the sequence $\{r_k, k = 1,2, \dots, N\}$, the reserves for each pensioner at the end of the year.

We must then compute the aggregate reserve taking into account the simulated deaths by considering the matrix:

$$A = \begin{bmatrix} r_{11} & \cdots & r_{1J} \\ \vdots & \ddots & \vdots \\ r_{N1} & \cdots & r_{NJ} \end{bmatrix}$$

The matrix has got N rows and J=1000 columns. One line for each pensioner and one column for each simulation. By running the sum $R_j^1 = \sum_{i=1}^{N} r_{ij}$ for each column j, we get the total reserve for each simulation at the end of the year. Thus, for each replica j we may compute the capital requirements for the year given by:

$$CR_j^1 = vR_j^1 - R^0 + P_{[0,1]}$$

We are now interested in analyzing the impact of the occurrence of a worst case scenario. This means we are interested in comparing the expected capital requirements for longevity short-term risk to an extreme, worst case scenario that is calculated considering the VaR at a 99,5% level.

We will calculate the difference between the mean scenario and an extreme scenario given by:

$$SCR = VaR^{99,5\%}\left(CR_j^1\right) - \overline{CR}.$$

Model results are summarized below:

average CR	2.756.283
extreme CR	3.101.821
impact	345.538

Table 3: Capital Requirements for Longevity Risk in euros

We found that the difference between the average capital requirements and an extreme scenario for capital consumption in a 1-year view is 345.538€. This is the solvency capital the company must have to cope with in an extreme scenario in the forthcoming year. This figure compares with 382.054€ required when applying the longevity shock of 20% suggested in the LTGA.

4.2 Revision Risk

Liabilities are exposed to revision risk if there is the possibility that benefits payable increase in such a way that these liabilities' amounts are higher than foreseen. We must then consider this risk module for our portfolio of annuities. To access this risk we consider the same methodology used in the longevity short term model where we quantified the capital requirements for each replica and then calculated the empirical extreme scenario for our set of simulations.

To address revision risk we must consider the possibility of the fixed pensions payable to each pensioner in our portfolio being revised. Remember that each pensioner has the right to claim benefit revision once a year if his/her health status worsens. To find the distribution of capital requirements for the current year we will need to project reserves to the end of the accounting year and simulate payments during the year. For this we need to fit a distribution to historical data on revision amount and simulate the number of revisions.

To find the distribution that best describes the relative revision amount we fitted three possible distributions to the dataset on the severity of relative revision amounts during the three last years. Parameters of the distributions used were found using the Maximum Likelihood Estimators.

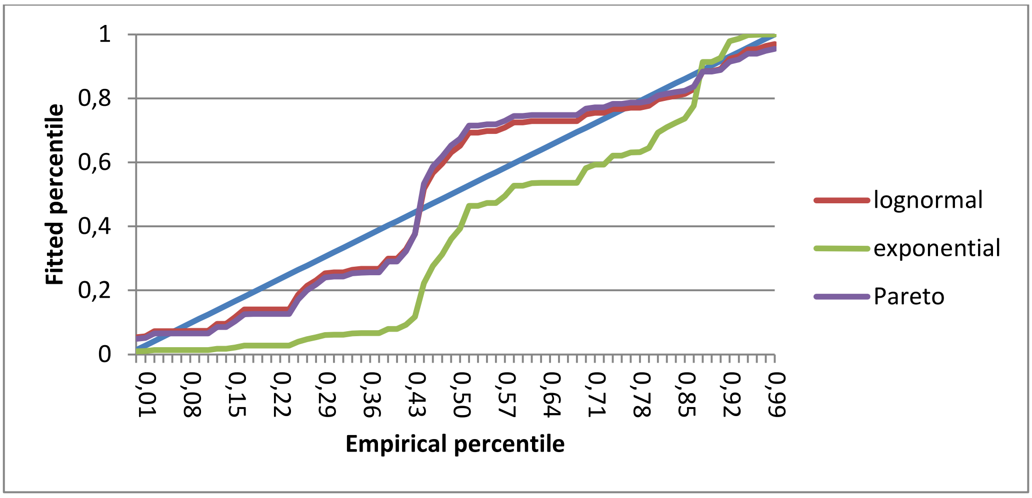

Figure 2: P-P plot: Fitting Statistical Distributions to historical data of relative revision amounts

The straight line represents the empirical distribution function of data and the adjusted curves depict three distribution functions chosen to describe historical data. By looking at the graph we notice that heavier tailed distributions describe data better and we may discard the exponential function as the lognormal and Pareto seem to provide a better fit. To choose from the former two and to ensure the adjustment is satisfactory we may run the KS test. According to this test, under H_0 the model describes data adequately. Evidence suggests that the lognormal distribution fits data better at a significance level of $\alpha = 0{,}05$. We thus adopt the lognormal distribution to describe the relative revision amount in our annuities portfolio.

We perform a two-step simulation. First we simulate a Bernoulli variable taking the values:

$$I_j = \begin{cases} 1 & if\ pensioner\ j\ sees\ his\ pension\ revised\ in\ the\ forthcoming\ year \\ 0 & otherwise \end{cases},$$

$j = 0,1,2,\dots,N$

afterwards, provided that $I_j = 1$, we simulate the relative revision amount and apply this to the individual pensions. We may then calculate the mathematical provision relative to liabilities for each pensioner at the end of the year. And thus compute:

$$CR_j^1 = vR_j^1 - R^0 + P_{[0,1]}$$

Following the same approach used in our longevity short-term model we may compute a realistic and an extreme scenario.

average CR	3.602.989
extreme CR	5.500.266
impact	1.897.278

Table 4: Capital Requirements for Revision Risk in euros

The table resumes the final results from the simulation exercise that was based on 1000 replicas. The model suggests that in a worst case scenario, we should be prepared to cover 1.897.278€ in revision processes payable to beneficiaries. The revision shock figure now takes into account the company´s internal reality and lies below the 2.459.473€ capital number necessary to comply with the 3% revision shock suggested by the LTGA when adopting the Standard Formula.

4.3 Expense Risk

Expense risk according to the TS SCR.7.6 *"arises from the variation in the expenses incurred in servicing insurance contracts"*.

We will consider therefore expenses allocated to contracts individually and attempt to model expenses payable and expense reserve in one year´s time. Outside of our analysis remain expenses of other nature such as unallocated expenses that charge the whole line of business and are not related to policies individually.

To do this we need to analyze the expense pattern for each policy as reserves are set case by case. This makes expense risk difficult to model, as ad hoc reserving means we cannot simulate it in a systematic way. Moreover, expenses for each policy throughout the years exhibit a volatile behaviour with no defined trend.

We opted to carry forward reserves assuming the amount deconstituted in the year corresponds solely to expense amount payable in the year. It is a simplifying assumption that is needed to deal with subjectivity inherent to the way expense reserves are managed.

We model expense payment during the year by generating a Normal random variable with parameters calculated from historical data. In this way we may find the extreme scenario as required by the SCR definition. Following the same procedure as in the Longevity risk model we arrive to the results:

realistic scenario	21.260	impact	25.154
extreme scenario	46.414		

Table 5: Capital Requirements for Expense Risk in euros

In a realistic scenario 21.260 € are needed in the forthcoming year to address expense risk. The difference between this realistic and an extreme scenario that is based on the VaR at 99,5% is 25.154€. This amount should be considered with a view of accounting for unforeseeable losses.

4.4 Interest Rate Risk

Interest rate risk is quite different from other risks covered so far. It is independent from policies and impacts capital requirements through the discount rate to which cash flows are subject throughout the years. It is out of this works scope to put forward a capital requirements figure for interest rate risk. We will instead analyze the impact of the projected, realistic yield curve on capital requirements for life underwriting risks studied so far.

To do this, let us consider a stochastic term structure model. Models of this kind represent the yield curve as a stochastic process $\{r(t): t > 0\}$, where $r(t)$ is the instantaneous risk free rate for maturity t also designated as the short rate.

Historical values for the yield curve are based on the EURO swap rates that reflect the interest rate charged on interbank loans, assumed risk free. This has been a popular alternative to the usage of sovereign bonds that are no longer considered riskless.

To model the term structure, we need to adjust a model to the historical values we considered. There are many feasible alternatives. One possibility is to consider the Cox-Ingersoll-Ross (CIR) Model that is defined by the Stochastic Differential Equation:

$$dr_t = a(b - r_t)dt + \sigma\sqrt{r_t}dz_t$$

Further details on the model may be found in the Appendix 2.

This model was brought to data by fitting it to historical values of the yield curve. Parameters a and b were found numerically by Minimizing the objective function defined as:

$$\sum_{t=1}^{\omega} \frac{(r_{CIR}(t) - r_{empirical}(t))^2}{r_{empirical}(t)}$$

Where ω is the limiting age and thus the last year for which we need values for the short rate. The value for the standard deviation parameter was estimated through the Method of Moments.

Once the model is calibrated, we can use it to project the yield curve to the future through simulation. We considered a 1 year projection and 1000 replicas.

As depicted in the graph, the one year projected yield curve is above the current empirical curve for all maturities.

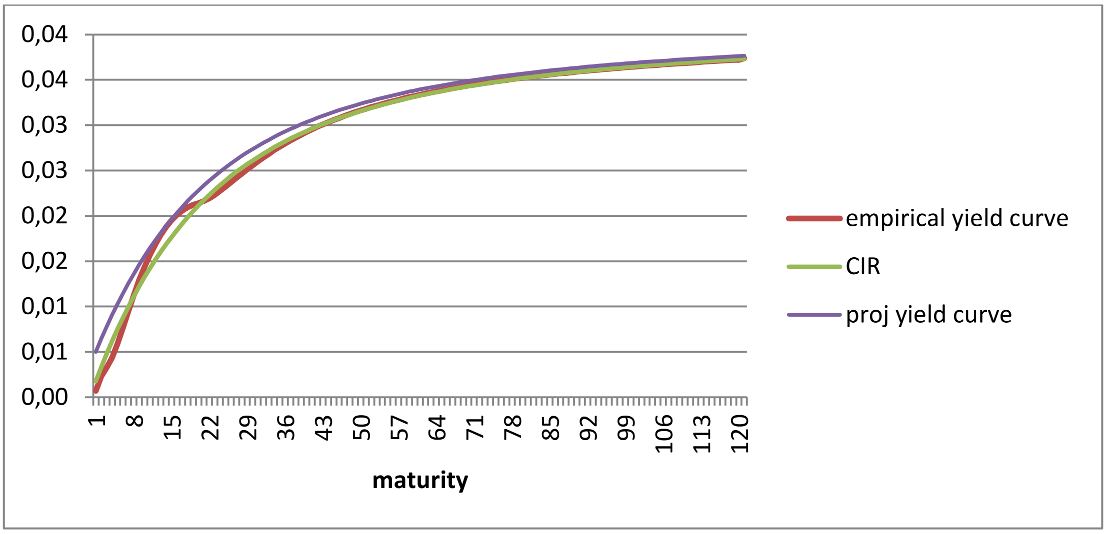

Figure 3: Empirical SWAP based yield curve, the CIR adjustment and the projected yield curve

The empirical yield curve is based on historical data. We have also plotted the adjusted CIR curve and the projected yield based on the CIR model on a one year basis that represents an average value of the 1000 simulations.

As mentioned, we will not quantify capital requirements for this risk. The projections found will be used to assess the sensitiveness of the Solvency Capital Requirement for life underwriting risks to interest rate in Chapter 6.

5. Correlations: Getting it all together

We modeled four risk typologies separately amongst which three are underwriting risks. It may be acceptable to consider market variables exogenous, but it is important we take into account correlations between underwriting risks. Disregarding dependence amongst capital requirements for different underwriting risks will result in a biased solvency capital figure.

SCR. 7.7 of the Technical Specifications of the LTGA suggest a standard correlation matrix between these risks, and a simple way of incorporating correlations in the model. In our application this results in considering the correlation matrix:

$$\sum = \begin{bmatrix} 1 & & \\ 0.25 & 1 & \\ 0.25 & 0.5 & 1 \end{bmatrix}$$

Where the first, second and third columns correspond to longevity, expense and revision risk.

The aggregate SCR results of computing $\sqrt{SCR^T \sum SCR}$ where SCR is the column vector in which each line entry corresponds to the individual SCR for longevity, expense and revision risks respectively.

Note that in applying this methodology we are tacitly accepting the assumption that capital requirements follow a normal distribution. If we simply sum up individual SCRs we would be assuming comonotonicity amongst capital requirements for the different risks. This would be equivalent to admitting, for instance, that the severity of an extreme scenario with respect to longevity would match revision and expense risks extreme scenarios, which would be way to pessimistic. Since correlations between risks are admitted to be smaller than one, we find diversification effects when aggregating individual SCRs.

The aggregation results in the total SCR figure for life underwriting risk to which the WsC LoB is exposed.

The table below summarizes overall results:

i=	Long risk	Exp risk	Revis Risk	Total	Diversif effect	SCR life underw risk
average CR	2.756.283	13.139	3.602.989	6.372.411	238.641	2.019.721
extreme CR	3.101.821	28.685	5.500.266	8.630.773		
SCR_i	345.538	15.546	1.897.278	2.258.362		

Table 6: Solvency Capital Requirements aggregation for life underwriting risk in euros

We arrive to a total capital number of 2.019.721€. This value accounts for major life underwriting risks studied for this line of business and should cover extreme worst case scenario in a 1-year view. The diversification gain previously discussed equals 238.641€.

6. Sensitivity Analysis

We will perform sensitivity analysis with a view to helping us understand the way our overall Solvency Capital Requirement shifts as a result of a small change in a key variable.

To help us with this task we will use the concept of Elasticity. Elasticities are popular tools for economic analysis. They are widely used by economists because they are simple to understand and calculate.

In mathematical terms, Elasticity may be defined in the following way:

$$E_{f(x),x} = \frac{d\ log\ f(x)}{d\ log\ x} \approx \frac{\%\Delta f(x)}{\%\Delta x}$$

The definition in continuous time is obviously only valid for differentiable functions. The discrete approximation is acceptable for small values of Δx.

An Elasticity may be interpreted as the percentual change in a given function as a result of a 1% change in an input variable, ceteris paribus. Note that the analysis assumes that every other variable remains fixed.

We found that:

i. $E_{SCR,rev} = 0.9263\%$

ii. $E_{SCR,long} = 0.0951\%$

iii. $E_{SCR,int\ rate} = -0.0016\%$

iv. $E_{SCR,exp} = 0.0011\%$

This means our SCR for life underwriting risk for this line of business increases 0.926%, -0.095%, 0.002% and 0.001% as a result of a 1% increase in the input variables considered. For revision the 1% increase was applied to the average relative revision amount, this means payments will increase and so will mathematical provisions calculated with the revised pension. Regarding longevity risk we applied a 1% increase to the probability of survival in the forthcoming year, for all ages. This impacted the simulation generating less deaths and thus more payments and higher reserves that mean higher capital requirements. The 1% increase in interest rates represents a timid decrease in the SCR because the rate in a 1-year view is small and thus, so is the shock applied. To calculate the expense elasticity we applied the 1% shock to the average expense amount that impacts the simulation of expenses for the forthcoming year.

7. Conclusions

WsC is an important line of business for the Portuguese Economy, not only because of the considerable business amount it represents for insurance companies, due to its mandatory nature, but also because of its influence in labour costs. As all employers need to insure their workers, premiums paid are in fact a component of total work compensation. Therefore, premiums charged by insurance companies are indirectly influencing Labor competitiveness of the Economy. This should be taken into consideration by authorities as should the increasing figures of the loss ratio of this LoB that may lead to increasing premiums and hence, decreasing labour competitiveness, without meaning better salaries and better living conditions for workers.

Internal Models are important elements of the management system of an insurance company. Their utility goes far beyond the need to comply with regulatory regimes. They allow thorough analysis of risk exposure and provide an important contribution to informed decision making.

However they are models. Thus, one should bear in mind model risk and parameter risk.

When tackling longevity risk, in the long-run, we used the Geometric Mortality Model. This model assumes that the difference between the logs of the coefficients is linear. A strong assumption which, according to Bravo (2009) contradicts empirical studies and is not the most sophisticated model available. Other alternatives could provide more realistic results. A popular alternative is the Poisson-Lee-Carter Model.

We adopted the results found by Maeder (2008) relying on these to develop our own. Some points of his paper were unclear and rely on expert judgment (for instance the choice of safety factors $\delta = 15\%$ and $= 20\%$ were not justified by the author).

Notwithstanding, the model is simple to understand and estimate. It was successfully implemented with Portuguese data, captures Portuguese reality and was consonant with the company´s own models in place.

When dealing with short term longevity risk, we simulated deaths for all beneficiaries and taking these into account computed reserves one year later. We assumed independent lives in our portfolio and that deaths occurred in the middle of the year.

Interest rate risk was modeled through a CIR model. Other alternative approaches could have been used such as considering another more sophisticated two factor model introducing more randomness.

The risk free rate used in discounting is based on the EURO swap rate. This avoids considering sovereign bonds as riskless assets. However, as noted by Ford, N. (2012) one must bear in mind that interbank lending may contain a risk premium, especially for long term lending. Our yield curve for long term maturities is used in other contexts and this must be taken into account.

An alternative approach towards modeling expense risk would be advantageous. The fact that expense reserves are set on a case-by-case basis and that historical data on expenses is irregular makes it very difficult to model consistently. Modelling Lifetime Assistance benefits pensioner by pensioner is difficult for the same reasons. An econometric model using Panel Data would perhaps help attain more solid figures in this point.

When aggregating capital figures, we considered the aggregation methodology as suggested by the LTGA. An alternative approach would be to consider a copula, adjusting it to data and in this way embedding a dependence structure in the simulation process. A simple way of adjusting a Gaussian copula is provided by Borginho, H. (2005). This methodology is demanding in terms of data and for this reason we did not use it. However, it allows more flexibility in choosing the underlying statistical relation between the variables that are to be aggregated. It is one step further in making the model entity specific. For a pragmatic discussion on ways to deal with dependence in economic capital models in insurance see Spivak, G. (2009).

Sensitivity risk was performed using Elasticities. Although simple, these tools do not allow us to analyze scenarios in which multiple input variables change simultaneously.

References

Bravo, J. (2009) Comments on the Construction of Annuity Mortality Tables for Portugal, University of Évora – Department of Economics, CEFAGE-EU.

Borginho, H (2005) DFA Models as a tool for Solvency Assessment – An application to Portuguese General Insurance Companies, Faculty of Science and Statistics, Cass Business School, City University of London.

Cox, J. et al (1985) A Theory of the Term Structure of Interest Rates, Econometrica, Vol.53, Issue 2, pp.385-408.

Dickson, D. et al (2011) Actuarial Mathematics for Life Contingent Risks 3rd Ed., Cambridge University Press, International Series on Actuarial Science.

European Insurance and Ocuppational Pensions Authority (EIOPA) Long Term Guarantee Assessment, Technical Specifications. Available online: https://eiopa.europa.eu/consultations/qis/insurance/index.html

European Insurance and Ocuppational Pensions Authority (EIOPA) Quantitative Study Impact 5, Technical Specifications. Available online: https://eiopa.europa.eu/consultations/qis/insurance/index.html

Ford, N. (2012) The Solvency II Discount rate: Nothing is Simple, KPMG.

Available online:

http://www.kpmg.com/uk/en/issuesandinsights/articlespublications/pages/solvency-ii-discount-rate.aspx

IAA (2010) Notes on the use of Internal Models for Risk and Capital Management Purposes by Insurers. Available online: http://www.actuaries.org/CTTEES_SOLV/Documents/Internal_Models_EN.pdf

Klugman, S. A. et al (2008) Loss Models: From data to decision, 3rd Ed., Wiley Series in Probability and Statistics.

Maeder, P. (2008) Construction of Annuities Mortality Tables for Portugal, Technical Memorandum, Swiss Re, APS. Available online: http://www.apseguradores.pt/DocumentCatalog/DocumentLink2.aspx?DocumentId=5258

Portuguese Office for National Statistics (2011), Table 22.xls INE. Available online:

http://www.ine.pt/ngt_server/attachfileu.jsp?look_parentBoui=119688736&att_display=n&att_downloading=y

Portuguese Insurance Institute (2009) Industry Statistics: Claim costs; Premiuns earned. Available online: http://www.isp.pt/NR/exeres/5378CF72-A45A-41ED-AD50-0275C4DDFB4F.htm

Portuguese Insurance Institute (2009) Law number 98/2009, 4th September 2009. Available online: http://www.isp.pt/winlib/cgi/winlibimg.exe?key=&doc=18438&img=3008

Portuguese Insurance Institute (2009) Ordinance number 185/2007, 10th May 2007. Available online: http://www.isp.pt/winlib/cgi/winlibimg.exe?key=&doc=16422&img=2169

Portuguese Insurance Institute (2010) Regulation Nr. 15/2000 – Loss Reserving Worker´s Compensation.

Portuguese Insurance Institute (2010) Regulation Nr. 09/2008-R 25th September – Calculation of Technical Provisions based on Economic Principles.

Rosa, C. (2012) An Internal Model for Worker´s Compensation, Master´s Final Work Dissertation, ISEG, UTL.

Shaw. R. and Spivak, G. (2009) Correlations and Dependencies in Economic Capital Models, UK Actuarial Profession's Financial Insurance Risk Management, Conference of June 2009.

Appendix 1 – Description of the Geometric Mortality Model and its use for the purpose of projecting mortality in the future

The model may be described by the equation: $q_{x,t} = q_{x,t_0} e^{-\lambda_x (t-t_0)}$

Where: $q_{x,t}$ is the mortality rate for age x in year t;

λ_x represents the intensity of the annual mortality decrease and

t_0 is the base year upon which projections are applied to

The model is used to project mortality in the future, taking into account mortality improvements. It is based on the hypothesis that historical mortality patterns will prevail in the future.

We are concerned with annuity mortality that can differ from population mortality duo to issues such as adverse selection. This is dealt with by multiplying a discount factor to population mortality as follows:

$$q_x^{ann} = \theta_x q_x^{pop}$$

where: q_x^{ann} represents mortality experience for annuities;

q_x^{pop} represents mortality experience for the country's population and

θ_x accounts for the difference between population and annuity mortality capturing phenomena such as adverse selection.

This last factor was defined as:

$$\theta_x = \begin{cases} \dfrac{a_1 x + a_2}{a_3 x + 1}, & x \le x_l \\ b_0 + b_1 x + b_2 x^2, & x_l < x < x_h \\ \dfrac{c_1 x + c_2}{c_3 x + 1}, & x_h \le x \end{cases} \qquad (1)$$

Evidence shows small differences between annuity/population mortality for lower and higher ages and significant differences for middle age range. For these reasons a parabola is used to model the annuity/population mortality ratio; slow asymptotic decay towards the terminal age ω and the lowest age is guaranteed by the use of hyperbolic functions from above x_h and below x_l. These two former values are chosen in such a way that the function is continuous.

Upon this, a safety loading may be considered as a lower bound for the annuity/population mortality ratio defined as $\Psi_x = 1 - (1 - \theta_x)(1 + \delta)$. The project used $\delta = 15\%$.

With regard to the improvement factor λ_x, it is modeled in a similar way according to the expression:

$$\lambda_x = \begin{cases} 0 & x \leq 20 \\ \dfrac{a_1 x + a_2}{a_3 x + 1}, & 20 < x \leq x_l \\ b_0 + b_1 x + b_2 x^2 + b_3 x^3 + b_4 x^4, & x_l < x < x_h \\ \dfrac{c_1 x + c_2}{c_3 x + 1}, & x_h \leq x \end{cases}$$

Same arguments justify the choice of the expression. For ages below 20 no mortality improvements are considered. A 4th degree polynomial expression is now used to model mortality improvements in middle ages to assure smoothness of the expression.

Once again, improvements are different between population and insured mortality, thus, it is convenient to discount the rates by a factor φ_x representing the ratio of improvements in annuities and general population. Its expression is similar to (1). In this case we also should consider a contingency to ensure prudency by multiplying these rates by $\xi_x = 1 - (1 - \varphi_x)(1 - \varepsilon)$ with $\varepsilon = 20\%$.

The procedure allows the estimation of the improvement factors that embed the pattern of mortality observed in Portugal. Parameters for the expressions exhibited above were estimated and given in the original paper. The interested reader may refer to Maeder (2008) and Bravo (2009) for a thorough discussion of the methodology used. The Output for parameter estimation is given in Appendix 3, Table 10.

Appendix 2 – The Cox-Ingersoll-Ross Stochastic Term Structure Model and its use to project the yield curve

The Cox-Ingersoll-Ross Model is defined by the Stochastic Differential Equation:

$$dr_t = a(b - r_t)dt + \sigma\sqrt{r_t}dz_t$$

It was introduced by Cox, J.C. et al (1985).

The model describes the behavior of r_t, the short rate, modeled as a mean reverting quantity. The mean reverting speed is given by parameter $a > 0$. Thus, the short rate gravitates around the long term value b. $\{z_t, t > 0\}$ is a Brownian Motion and the term $\sigma\sqrt{r_t}dz_t$ guarantees that the short rate is never negative. The CIR model belongs to the one factor model family having only one random term z_t.

The Model benefits from a simple and easy to interpret expression. It does not allow for negative interest rates and adheres to historical data.

The term structure of interest rates can be defined by the process $\{P(t,T): T > t\}$ that is the market price of 1€ payable at time T > t. For the CIR model the former values may be determined in the following manner:

$P(t,T) = A(t,T)e^{-B(t,T)r_t}$

Where: $A(t,T) = \left[\frac{2\gamma e^{\frac{(a+\gamma)(T-t)}{2}}}{(\gamma+a)\left(e^{\gamma(T-t)}-1\right)+2\gamma}\right]^{\frac{2ab}{\sigma^2}}$ and $B(t,T) = \frac{2\left(e^{\gamma(T-t)}-1\right)}{(\gamma+a)\left(e^{\gamma(T-t)}-1\right)+2\gamma}$ in which $\gamma = \sqrt{a^2 + 2\sigma^2}$

Appendix 3- Disability and Death Benefits, Mortality rates and estimated parameters for the Geometric Mortality Model in use

Table 7: Disability Benefits

Incapacity	Eligible compensation	Calculation
Absolute Temporary Incapacity	Daily based allowance if incapacity does not last more than 30 days	Gross monthly wage * 70 % * number of days disable / 30
	Daily based allowance if incapacity term lasts between 30 days and 12 months	Gross annual wage * 70 % * number of days disable / 360
	Daily based allowance after 12 months disable	Gross annual wage * dc[11] * number of days disable / 360
Partial Temporary Incapacity	Daily based allowance if incapacity does not last more than 30 days	Gross monthly wage * dc * number of days disable / 360
	Daily based allowance if incapacity lasts more than 30 days	Gross annual wage * dc * number of days disable / 30
	Professional rehabilitation subsidy	Value of expenses covered until 1,1*IAS for programs lasting until 36 months
Absolute Permanent Incapacity	Annual lifelong pension	Gross annual wage * 80 % + 10 % for each dependent until max. 20%
	Pension for third party assistance	Monthly value until 1,1*IAS
	Subsidy for severe permanent incapacity	12 * 1,1*IAS
	Subsidy for housing reconversion	Expense value up to a ceiling of 12*1,1*IAS
Absolute Permanent Incapacity for regular work	Annual lifelong pension	Gross annual wage*x where 50% ≤ x ≤70% depending on working capacity
	Pension for third party assistance	Monthly value until 1,1*IAS
	Subsidy for severe permanent incapacity	Value of x%*12*1,1*IAS where 70% ≤ x ≤100%
	Subsidy for housing reconversion	Expense value until 12*1,1*IAS
	Professional rehabilitation subsidy	Value of expenses covered until 1,1*IAS for programs lasting until 36 months
Partial Permanent Incapacity	Annual lifelong pension	Gross annual wage * 70 % * dc
	Pension for third party assistance	Monthly value until 1,1*IAS
	Subsidy for severe permanent incapacity	Value of 12*1,1*IAS*dc
	Subsidy for housing reconversion	Expense value until 1,1*IAS
	Professional rehabilitation subsidy	Value of expenses covered until 1,1*IAS for programs lasting until 36 months

[11] Disability coefficient

Table 8: Death Benefits

Beneficiaries	Eligible compensation	Calculation
Partner	Lifelong annual pension in force until beneficiary retires	Gross annual wage * 30 %
	Lifelong annual pension in force after beneficiary retires or equivalent	Gross annual wage * 40 %
	Lump sum payable in case of remarriage or equivalent	3*annual pension
	Funeral expenses subsidy	Expense value until 4*1,1*IAS
	Death subsidy	Value of 12*1,1*IAS or half if it is also paid to descendant
Ex-Partner or legally separated	Lifelong annual pension in force until beneficiary retires	Gross annual wage * 30 % up to a maximum established for maintenance allowance
	Lifelong annual pension in force after beneficiary retires or equivalent	Gross annual wage * 40 % up to a maximum established for maintenance allowance
	Lump sum payable in case of remarriage or equivalent	Annual pensions*3
	Funeral expenses subsidy	Expense value until 4*1,1*IAS
	Death subsidy	Value of 12*1,1*IAS or half if it is also paid to descendant up to a maximum established for 12*maintenance allowance
Descendants	Lifelong annual pension if beneficiary is disable	Gross annual wage *x with x=20%, 40% or 50% depending on amount of descendants
	Temporary annual pension until eligible conditions hold (age and/or education)	Gross annual wage *x with x=20%, 40% or 50% depending on amount of descendants
	Funeral expenses subsidy	Expense value until 4*1,1*IAS
	Death subsidy	Value of 12*1,1*IAS or half if it is also paid to aforementioned beneficiaries
Ascendants	Lifelong annual pension if beneficiary is on low income	Gross annual wage * 10 %
	Lifelong annual pension if beneficiary is on low income and there are no other beneficiaries aforementioned until retirement age	Gross annual wage * 15 %
	Lifelong annual pension if beneficiary is on low income and there are no other beneficiaries aforementioned after retirement age or if beneficiary is disable	Gross annual wage * 20 %
	Funeral expenses subsidy	Expense value until 4*1,1*IAS

Table 9: Baseline, projected and LTGA Mortality rates

Age[12]	q_x	$q_{x\ LTGA}$	q_x'	q_x''	Age	q_x	$q_{x\ LTGA}$	q_x'	q_x''
0	0	0	0	0	**35**	0,001	0,001	0,001	0,001
1	0	0	0	0	**36**	0,001	0,001	0,001	0,001
2	0	0	0	0	**37**	0,002	0,001	0,001	0,001
3	0	0	0	0	**38**	0,002	0,001	0,001	0,001
4	0	0	0	0	**39**	0,002	0,001	0,002	0,002
5	0	0	0	0	**40**	0,002	0,001	0,002	0,002
6	0	0	0	0	**41**	0,002	0,002	0,002	0,002
7	0	0	0	0	**42**	0,002	0,002	0,002	0,002
8	0	0	0	0	**43**	0,002	0,002	0,002	0,002
9	0	0	0	0	**44**	0,002	0,002	0,002	0,002
10	0	0	0	0	**45**	0,003	0,002	0,002	0,002
11	0	0	0	0	**46**	0,003	0,002	0,002	0,002
12	0	0	0	0	**47**	0,003	0,002	0,003	0,003
13	0	0	0	0	**48**	0,003	0,003	0,003	0,003
14	0	0	0	0	**49**	0,004	0,003	0,003	0,003
15	0,0014	0,0011	0,0014	0,0014	**50**	0,004	0,003	0,003	0,003
16	0,0014	0,0012	0,0014	0,0014	**51**	0,004	0,004	0,004	0,004
17	0,0015	0,0012	0,0015	0,0015	**52**	0,005	0,004	0,004	0,004
18	0,0015	0,0012	0,0015	0,0015	**53**	0,005	0,004	0,005	0,005
19	0,0014	0,0012	0,0014	0,0014	**54**	0,006	0,005	0,005	0,005
20	0,0014	0,0011	0,0014	0,0014	**55**	0,007	0,005	0,006	0,005
21	0,0014	0,0011	0,0014	0,0014	**56**	0,007	0,006	0,006	0,006
22	0,0014	0,0011	0,0014	0,0014	**57**	0,008	0,006	0,007	0,006
23	0,0013	0,0011	0,0013	0,0013	**58**	0,009	0,007	0,007	0,007
24	0,0013	0,0011	0,0013	0,0013	**59**	0,01	0,008	0,008	0,008
25	0,0013	0,001	0,0013	0,0013	**60**	0,011	0,009	0,009	0,008
26	0,0013	0,001	0,0012	0,0012	**61**	0,012	0,009	0,009	0,009
27	0,0013	0,001	0,0012	0,0012	**62**	0,013	0,01	0,01	0,01
28	0,0013	0,001	0,0012	0,0012	**63**	0,014	0,011	0,011	0,011
29	0,0013	0,001	0,0012	0,0012	**64**	0,015	0,012	0,012	0,012
30	0,0013	0,001	0,0012	0,0012	**65**	0,017	0,014	0,013	0,013
31	0,0013	0,001	0,0012	0,0012	**66**	0,019	0,015	0,015	0,014
32	0,0013	0,001	0,0012	0,0012	**67**	0,021	0,017	0,016	0,016
33	0,0013	0,0011	0,0012	0,0012	**68**	0,023	0,019	0,018	0,018
34	0,0014	0,0011	0,0013	0,0013	**69**	0,026	0,021	0,021	0,02

[12] q_x :Baseline mortality; $q_{x\ LTGA}$ Baseline mortality with LTGA shock; q_x' projected mortality 10yrs; q_x''projected mortality 10yrs w/ safety loading

70	0,0289	0,0231	0,0231	0,0223	**106**	0,539	0,4312	0,5175	0,5168
71	0,0323	0,0258	0,026	0,0252	**107**	0,4016	0,3213	0,3866	0,3862
72	0,0359	0,0287	0,0292	0,0284	**108**	0,4181	0,3345	0,4036	0,4032
73	0,0399	0,0319	0,0328	0,0319	**109**	0,4353	0,3482	0,4213	0,4209
74	0,0442	0,0354	0,0367	0,0358	**110**	0,4524	0,3619	0,439	0,4387
75	0,0488	0,0391	0,0409	0,0399	**111**	0,471	0,3768	0,4582	0,458
76	0,0538	0,0431	0,0455	0,0445	**112**	0,4876	0,3901	0,4756	0,4754
77	0,0592	0,0473	0,0504	0,0494	**113**	0,5034	0,4028	0,4923	0,4921
78	0,0648	0,0519	0,0556	0,0546	**114**	0,5278	0,4222	0,5174	0,5172
79	0,0709	0,0567	0,0612	0,0602	**115**	0,5588	0,4471	0,5491	0,549
80	0,0773	0,0618	0,0672	0,0661	**116**	0,5333	0,4267	0,5254	0,5253
81	0,0841	0,0673	0,0735	0,0725	**117**	1	0,8	0,9875	0,9874
82	0,0913	0,073	0,0803	0,0792					
83	0,0988	0,0791	0,0874	0,0863					
84	0,1068	0,0854	0,0949	0,0938					
85	0,1152	0,0922	0,1029	0,1018					
86	0,124	0,0992	0,1112	0,1102					
87	0,1332	0,1066	0,12	0,119					
88	0,1429	0,1143	0,1293	0,1282					
89	0,153	0,1224	0,139	0,1379					
90	0,1635	0,1308	0,1491	0,1481					
91	0,1745	0,1396	0,1598	0,1587					
92	0,1859	0,1487	0,1708	0,1698					
93	0,1978	0,1582	0,1824	0,1815					
94	0,21	0,168	0,1944	0,1935					
95	0,2228	0,1782	0,2069	0,206					
96	0,2359	0,1888	0,2199	0,219					
97	0,2495	0,1996	0,2333	0,2325					
98	0,2635	0,2108	0,2472	0,2464					
99	0,2779	0,2223	0,2615	0,2607					
100	0,2927	0,2341	0,2762	0,2755					
101	0,3078	0,2462	0,2913	0,2907					
102	0,3232	0,2586	0,3069	0,3062					
103	0,339	0,2712	0,3228	0,3222					
104	0,3551	0,2841	0,339	0,3385					
105	0,3714	0,2971	0,3556	0,3551					

Table 10: Parameter estimation for Geometric Mortality Model[13]

θ_x	Best Estimates mortality portfolio	
coefficients	Males	Females
a_1	-0,0233100	-0,0194240
a_2	1,0000000	1,0000000
a_3	-0,0222870	-0,0184140
x_l	33	39
b_0	1,6833000	1,8066700
b_1	-0,0329330	-0,0346670
b_2	0,0002533	0,0002667
x_h	96	90
c_1	-0,0138910	-0,0153010
c_2	1,1520920	1,1480930
c_3	-0,0126240	-0,0141160

ϕ_x	Ratio of improv ann./pop.	
coefficients	Males	Females
a_1	-0,0072250	-0,0102713
a_2	1,2716635	1,1541660
a_3	0,0063581	-0,0025630
x_l	64	59
b_0	3,9800000	3,4992727
b_1	-0,0980000	-0,0859091
b_2	0,0007000	0,0006364
x_h	76	77
c_1	0,0038225	0,0399411
c_2	0,1376391	-1,3533832
c_3	-0,0033638	0,0211141

λ_x	Mortality improvements pop.	
coefficients	Males	Females
a_1	0,0003485	0,0003732
a_2	-0,0069693	-0,0074641
a_3	-0,0106774	-0,0086575
x_l	61	55
b_0	9,5340000E+00	-1,9203000E-01
b_1	-6,4262000E-01	1,6630000E-02
b_2	1,5869000E-02	-5,2730000E-04
b_3	-1,7020000E-04	7,5237000E-06
b_4	6,7095000E-07	3,8725000E-08
x_h	75	77
c_1	0,0001764	0,0043291
c_2	-0,0211633	-0,5411365
c_3	-0,0172102	-0,0887132

[13] These tables may be found in Maeder (2008)