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ECONOMETRIA APLICADA E PREVISÃO

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Dissertação

STRUCTURAL CHANGES IN DURATION OF BULL AND BEAR MARKETS AND THEIR CONNECTION WITH BUSINESS CYCLES

JOÃO ANTÓNIO MENDES DA CRUZ

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ORIENTAÇÃO:

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Structural Changes in Duration of Bull and Bear Markets and their Connection with Business Cycles

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Abstract

The present work analyses relations between finance and macroeconomics, aiming to answer how structural changes in duration of bull and bear markets are connected with business cycles. In order to do so, we review the structural change test proposed by Nicolau (2016) and introduce two similar alternatives, which through a Monte Carlo simulation study, show less over-rejection for some data generating processes, proving to be useful in obtaining robust results.

We apply the tests to a database consisting on adjusted market capitalization stock market indexes of 38 developed and emerging markets, constructed by Morgan Stanley Capital International. In our results we find several structural changes that seem to be linked to macroeconomic events, furthermore, there is statistical evidence that decreases in duration of bull market cycles anticipate the peak of business cycles.

Keywords: Bull and Bear Markets, Business Cycles, Duration of Bull and Bear Market Cycles, Economic Crisis, MSCI, Structural Change Test.

JEL Codes: C12, E32, E44, F44, G01, G15.

Structural Changes in Duration of Bull and Bear Markets and their Connection with Business Cycles

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Resumo

O presente trabalho analisa relações entre finanças e macroeconomia, procurando responder a como quebras de estrutura na duração dos mercados bull e bear estão ligadas aos ciclos económicos. Para tal, é revisto o teste de quebras de estrutura proposto por Nicolau (2016) e são introduzidos dois testes alternativos, que através de um estudo de simulação Monte Carlo, evidenciam menos sobre-rejeição para alguns processos geradores de dados, provando ser úteis na obtenção de resultados robustos.

Aplicamos os testes a uma base de dados composta por índices bolsistas de 38 mercados desenvolvidos e em emersão, ajustados à capitalização de mercado, construídos pela Morgan Stanley Capital International. Nos resultados obtidos encontramos várias quebras de estrutura que revelam estar ligadas a eventos macroeconómicos, além disso, existe evidência estatística de que decréscimos na duração dos ciclos de mercado bull antecedem o pico dos ciclos económicos.

Palavras-Chave: Ciclos Económicos, Crises Económicas, Duração de Ciclos de Mercados Bull e Bear, Mercados Bull e Bear, MSCI, Teste de Quebra de Estrutura.

Códigos JEL: C12, E32, E44, F44, G01, G15.

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1 – Introduction

One can characterize the financial markets' behaviour as bull and bear (henceforth BB) markets: When studying a financial time series, it is possible to recognize prolonged periods of time where an underlying trend seems to be involved, these periods are related with the BB markets.

Although this feature has been quite studied by academics, there is not a clear definition for it, nevertheless, the descriptions provided by Chauvet & Potter (2000) and Sperandeo (1990) are useful for their simplicity and insight. The former describe bullish (bearish) markets as periods of generally increasing (decreasing) market prices, while the latter uses a similar description, yet more precise, by defining a bull market as "a long-term (...) upward price movement characterized by a series of higher intermediate (...) highs interrupted by a series of higher intermediate lows", and a bear market as a "long-term downtrend characterized by lower intermediate lows interrupted by lower intermediate highs".

One of the main reasons for the popularity of BB markets in the last years is its importance in analysing and predicting financial markets, in this sense, a great amount of work has been developed in identifying, modelling and predicting BB markets. Lunde & Timmermann (2004), Maheu et al. (2012), and Kole & Van Dijk (2017) are just some examples of the research made in this area.

Less work has been achieved in analysing the BB markets duration, and studying its possible structural changes. The importance of this subject is directly linked with the applications of BB markets as key components of stock markets: If a structural change in the cycle duration is wrongly left unconsidered, then an analysis based on BB markets will most likely be compromised. For testing these structural changes only the test proposed by Nicolau (2016) is known to date. Throughout the current work, this statistical test is introduced, along with two simple alternatives derived from the former, with a Monte Carlo simulation study being additionally carried out in order to analyze the statistical properties of these tests.

This work's empirical application intends to be a valid contribution in the study of links between finance and macroeconomics, a field of research that became especially active after the crisis of 2008 that affected economies worldwide. To achieve so, the present work focuses in the analysis of the connections between structural changes in the duration of BB markets and the business cycles.

The upper mentioned statistical tests are applied to a database consisting on adjusted market capitalization stock market indexes of 38 developed and emerging markets, and the breakpoints found are then compared with the peaks and troughs verified in the business cycles as well as periods of other global macroeconomic events. In this comparison, the structural changes are then explained and justified considering both financial and macroeconomic frameworks.

The remainder of this work is organized as follows: Section 2 introduces some preliminary concepts on BB markets, specifically stationary first order Markov Chain Processes and the expected time of permanency in a given state. Section 3 presents several methods for the identification of BB markets. Section 4 revises the existent structural change test in duration of BB markets and presents two alternative tests. Additionally a simulation study is carried out in this section, in order to ascertain the empirical size and power of these tests, along with the critical values of the alternatives. Section 5 introduces the empirical study where the tests are applied to adjusted market capitalization stock market indexes and the connections between structural changes in duration of BB markets and the business cycles are analyzed. Section 6 presents extensions and possible further research to this work. Section 7 revises the obtained results and concludes.

2 – Preliminary Concepts on BB Markets

BB markets conveniently suit the probabilistic model proposed by Andrey Markov. Let $\{S_t\}$ be an indicator variable, which verifies $S_t = 1$ if the stock market is in a bull state and $S_t = 0$ if the stock market is in a bear state at period t. The evolution of S_t is assumed throughout this work to be governed by a stationary and ergodic first order Markov Chain Process. The transition probabilities capture the temporal dependence of BB markets and are presented as:

$$P(S_t = j|I_{t-1}) = P(S_t = j|S_{t-1} = i) = p_{ij} \ \forall i, j = 0, 1 \tag{2.1}$$

Where I_{t-1} is the σ-algebra generated by the available information at $t - 1$. The above probabilities imply that, S_t is independent of S_{t-k} with $k > 1$.

Since the transition probabilities are constant over time given $\{S_t\}$ stationary, the following one step probability transition matrix completely describes the Markov Chain Process:

$$\mathbf{S} \coloneqq \begin{bmatrix} P(S_t = 1|S_{t-1} = 1) & P(S_t = 0|S_{t-1} = 1) \\ P(S_t = 1|S_{t-1} = 0) & P(S_t = 0|S_{t-1} = 0) \end{bmatrix} \tag{2.2}$$

In order to introduce the concept of BB market durations, it is pertinent to consider the following random variables:

$$T_{Bull} \coloneqq \min(t > 0: S_t = 0|S_0 = 1)\,;\ T_{Bear} \coloneqq \min(t > 0: S_t = 1|S_0 = 0) \tag{2.3}$$

The variable T_{Bull} represent the time of first passage to the bear state given that the market started at a bull state, T_{Bear} has an analogous interpretation. Since the Markov chain is assumed to be stationary, the expected value of the variables mentioned above is constant and given by:

$$\theta_1 \coloneqq E(T_{Bull}) = \frac{1}{1 - p_{11}};\ \ \theta_0 \coloneqq E(T_{Bear}) = \frac{1}{1 - p_{00}} \tag{2.4}$$

See, for example, Taylor & Karlin (1998). Equation (2.4) shows the expected time of permanency in the BB states related to a certain series, that is, the duration of the BB market cycles. Intuitively, to test the hypothesis of whether the BB market durations are constant over time or not, is to test if the equality $\theta_{i,t} = \theta_i$ holds for all t. Such is a major focus of this current work, as will be presented in the next sections.

3 – Identification of BB Markets in a Time Series

Before introducing the structural change tests in duration of BB markets and their applications, it is first necessary to present methods for the identification of bullish and bearish states. To do so, there are two distinguishable main approaches, one nonparametric and other based on a parametric statistical model. The latter uses regime-switching models (see, for example, Maheu et al. (2012)) which have their own advantages since they give more depth into the process under study and allow a direct statistical inference. However, these models and their results are dependent on a correct specification, which is something not desirable in the present work. It is preferred the use of a transparent and robust method of identification, that solely reflects the tendency of the market. Such leads to the use of nonparametric rules-based methods.

The two main algorithms in the literature are presented by Pagan & Sossounov (2003) and Lunde & Timmerman (2004). The former's approach is based on the algorithm developed by Bry & Boschan (1971) for dating business cycles and consists in the identification of peaks and troughs as well as the adoption of duration censoring rules that restrict the minimal lengths of any phase. Seeing that the method proposed by Lunde & Timmerman (2004) does not impose such restrictions in the cycle's durations, it will be selected throughout this work for the identification of BB markets since it is preferred over the one presented by Pagan & Sossounov (2003).

To identify bullish ($S_t = 1$) and bearish ($S_t = 0$) states, the chosen method uses two parameters, λ_1 and λ_2, as a way of measuring the dimension of a rise (drop) in the time series to be considered a peak (trough).

Let the stock market be in a bullish state at $t = t_0$, with $P_{t_0}^{Max}$ equal to its value at that period (P_{t_0}) and consider the stopping time variables τ_{Max} and τ_{Min} defined by:

$$\begin{aligned} \tau_{Max} &= \inf\{t_0 + \tau : P_{t_0+\tau} \geq P_{t_0}^{Max}\} \\ \tau_{Min} &= \inf\{t_0 + \tau : P_{t_0+\tau} < (1 - \lambda_2) P_{t_0}^{Max}\} \end{aligned} \tag{3.1}$$

If $\tau_{Max} < \tau_{Min}$, then set $P_{t_0+\tau_{Max}}^{Max} = P_{t_0+\tau_{Max}}$ and $S_t = 1, \forall \ t \in \{t_0 + 1, \dots, t_0 + \tau_{Max}\}$, otherwise set $P_{t_0+\tau_{Min}}^{Min} = P_{t_0+\tau_{Min}}$ and $S_t = 0, \forall \ t \in \{t_0 + 1, \dots, t_0 + \tau_{Min}\}$.

Similarly, with the stock market in a bearish state at $t = t_0$, the stopping time variables τ_{Max} and τ_{Min} are defined by:

$$\begin{aligned} \tau_{Min} &= \inf\{t_0 + \tau : P_{t_0+\tau} \leq P_{t_0}^{Min}\} \\ \tau_{Max} &= \inf\{t_0 + \tau : P_{t_0+\tau} > (1 + \lambda_1) P_{t_0}^{Min} \end{aligned} \tag{3.2}$$

If $\tau_{Min} < \tau_{Max}$, then set $P_{t_0+\tau_{Min}}^{Min} = P_{t_0+\tau_{Min}}$ and $S_t = 0, \forall \ t \in \{t_0 + 1, \dots, t_0 + \tau_{Min}\}$, otherwise set $P_{t_0+\tau_{Max}}^{Max} = P_{t_0+\tau_{Max}}$ and $S_t = 1, \forall \ t \in \{t_0 + 1, \dots, t_0 + \tau_{Max}\}$.

By repeating the above set of rules the BB states are identified. Notice that the algorithm simply defines bullish cycles as the movements of a time series between two local maximums without significant drops in the middle, or as the movements between a local minimum and a local maximum, and so, no duration restrictions are implied. The bearish cycles are analogously defined.

The identification of BB markets through this algorithm depends on the choice of (λ_1; λ_2). If this parameters are set too low, then small downward (upward) movements during a bull (bear) cycle are considered a sign of a transition to a bear (bull) state. Moreover, it is intended that the upward drift in stock prices is considered, this is achievable by setting $\lambda_1 > \lambda_2$. Knowing this concerns, throughout this work the values ($0{,}20$; $0{,}15$) are chosen for these parameters.

4 – Structural Change Tests in Duration of BB Markets

4.1 – A Revision on Structural Change Tests in Duration of BB Markets

In this section, the structural change test introduced by Nicolau (2016) is presented.

This test aims to detect differences in the duration of BB markets between two subsamples, that is, if the transition probabilities inherent to the BB states have changed from one period to another.

When applied to the stock market, evidence of a structural change in a specific date may prove that some sort of phenomenon led to an increase or decrease in the cyclicity of the time series. This analysis can have interesting applications in understanding the relations linking some economical or financial events and the stock market in study.

The test compares the estimated duration of the bull (bear) market cycle using all T observations, with the estimated durations using the first $\mathrm{t} \in]\mathrm{w}, \mathrm{T}[$ observations, where w is a start-up value to be explained during this section and T the sample size. This way, a great deviation between durations represents an evidence of a structural change in the stability of the bull (bear) cycle.

To estimate the duration of these cycles, consider the already presented equation (2.4), with the transition probabilities $\mathrm{p_{ii}}\ \forall \mathrm{i} = 0, 1$ replaced by their maximum likelihood estimates $\hat{\mathrm{p}}_{\mathrm{ii}}$:

$$\hat{\theta}_1 = \frac{1}{1 - \hat{\mathrm{p}}_{11}};\ \hat{\theta}_0 = \frac{1}{1 - \hat{\mathrm{p}}_{00}} \tag{4.1.1}$$

Therefore, given the functional invariance propriety, $\hat{\theta}_{\mathrm{i}}$ is the maximum likelihood estimate of θ_{i}, $\mathrm{i} \in \{0, 1\}$.

To obtain $\hat{\mathrm{p}}_{\mathrm{ii}}$, first consider the following initial probabilities:

$$\mathrm{p}_{\mathrm{u}}^{(0)} := \mathrm{P}(\mathrm{S}_0 = \mathrm{u}), \mathrm{u} \in \{0,1\} \tag{4.1.2}$$

With $\mathbf{S_{T+1}} = \{S_0, S_1, \dots, S_T\}$ a realization of length $\mathrm{T} + 1$ of the stochastic process $\{\mathrm{S_t}\}$, the likelihood function based on $\mathbf{S_{T+1}}$ is specified by:

$$\mathrm{L} = \mathrm{p}_{\mathrm{u}}^{(0)} \prod_{\mathrm{k}=1}^{\mathrm{T}} \mathrm{p}_{\mathrm{S_{k-1}S_k}} = \mathrm{p}_{\mathrm{u}}^{(0)} \prod_{\mathrm{i,j}=0}^{1} \mathrm{p_{ij}}^{\mathrm{n_{ij}}} \tag{4.1.3}$$

Where $\mathrm{n_{ij}}$ is the number of times that a transition from i to j is verified in the sample. The log-likelihood function is given by:

$$\log(\mathrm{L}) = \log(\mathrm{p_u^{(0)}}) + \sum_{\mathrm{i,j=0}}^{1} \mathrm{n_{ij}} \log(\mathrm{p_{ij}}) \qquad (4.1.4)$$

The maximum likelihood estimator of the transition probability $\mathrm{p_{11}}$ is obtained through:

$$\frac{\partial \log L}{\partial \mathrm{p_{11}}} = 0 \Leftrightarrow \frac{\mathrm{n_{11}}}{\hat{\mathrm{p}}_{11}} - \frac{\mathrm{n_{10}}}{1 - \hat{\mathrm{p}}_{11}} = 0 \Leftrightarrow \frac{\mathrm{n_{11}} - \mathrm{n_1}\hat{\mathrm{p}}_{11}}{\hat{\mathrm{p}}_{11}(1 - \hat{\mathrm{p}}_{11})} = 0 \Leftrightarrow \hat{\mathrm{p}}_{11} = \frac{\mathrm{n_{11}}}{\mathrm{n_1}} \quad (4.1.5)$$

Notice that $\mathrm{n_1} = \mathrm{n_{11}} + \mathrm{n_{10}}$, which is the number of 1's found in the realization $\mathbf{S_{T+1}}$. In general, the estimates of $\hat{\mathrm{p}}_{\mathrm{ij}}$ can analogously be obtained by $\hat{\mathrm{p}}_{\mathrm{ij}} = \frac{\mathrm{n_{ij}}}{\mathrm{n_i}}$, $\mathrm{i, j} \in \{0, 1\}$. Consequently, equation (4.1.1) can be written as:

$$\hat{\theta}_1 = \frac{\mathrm{n_1}}{\mathrm{n_1} - \mathrm{n_{11}}};\ \hat{\theta}_0 = \frac{\mathrm{n_0}}{\mathrm{n_0} - \mathrm{n_{00}}} \qquad (4.1.6)$$

Under the stationary hypothesis specified in Section 2, and given that $\hat{\theta}_\mathrm{i}$ is the maximum likelihood estimator of θ_i, its asymptotic behavior for $\mathrm{i} \in \{0, 1\}$ is:

$$\hat{\theta}_\mathrm{i} \xrightarrow{\mathrm{p}} \theta_\mathrm{i} \qquad (4.1.7)$$

$$\sqrt{\mathrm{T}}\left(\hat{\theta}_\mathrm{i} - \theta_\mathrm{i}\right) \xrightarrow{\mathrm{d}} \mathrm{N}\left(0, \frac{\mathrm{p_{ii}}}{(1 - \mathrm{p_{ii}})^3 \pi_\mathrm{i}}\right) \qquad (4.1.8)$$

Where $0 < \mathrm{p_{ii}} < 1$ and $\pi_\mathrm{i} := \mathrm{P}(\mathrm{S_t} = \mathrm{i})$. To prove equation (4.1.8) notice that $\sqrt{\mathrm{T}}(\hat{\mathrm{p}}_{\mathrm{ii}} - \mathrm{p_{ii}}) \xrightarrow{\mathrm{d}} \mathrm{N}\left(0, \frac{\mathrm{p_{ii}}(1-\mathrm{p_{ii}})}{\pi_\mathrm{i}}\right)$ (see, for example, Basawa & Rao (1980)).

As mentioned before, the goal is to test whether the durations of the BB markets are constant over time or if its structure has changed. To do so, let $\theta_{1,t}$ and $\theta_{0,t}$ be the durations of a bull cycle and bear cycle at t, respectively, and focus on observations $t = \lfloor rT \rfloor$, for $r \in R$, a pre-specified compact subset of $(0, 1)$, where $[x]$ is the integer part of x.

To test H_0: $\theta_{i,\lfloor rT \rfloor} = \theta_i \;\; \forall r \in R$ (i.e. parameter constancy) against its alternative H_1: $\theta_{i,\lfloor rT \rfloor} \neq \theta_i$ for some $r \in R$, the following statistic is crucial:

$$Q_{i,T}(\lfloor rT \rfloor) = \sqrt{\frac{\lfloor rT \rfloor - w}{T - w} * \frac{\lfloor rT \rfloor}{\hat{\sigma}_i^2}} * \left(\hat{\theta}_{i,\lfloor rT \rfloor} - \hat{\theta}_{i,T}\right) \tag{4.1.9}$$

For $i \in \{0, 1\}$, with $\hat{\sigma}_i^2$ the maximum likelihood estimate of $\mathrm{Var}_a\left(\hat{\theta}_{i,T}\right) = \frac{p_{ii}}{(1-p_{ii})^3 \pi_i}$. For the computation of $Q_{i,T}(\lfloor rT \rfloor)$ it will also be considered the case where $\hat{\sigma}_i^2$ is the maximum likelihood estimate of $\mathrm{Var}_a\left(\hat{\theta}_{i,\lfloor rT \rfloor}\right)$ (the asymptotic variance of the estimated duration using the first $\lfloor rT \rfloor$ elements in the sample), with its results compared to the previous situation[1]. For the distribution of $Q_{i,T}(\lfloor rT \rfloor)$ let:

$$Z_{i,t} = \frac{(1 - \theta_i)S_t + \theta_i S_t S_{t-1}}{\sqrt{\mathrm{Var}_a\left(\hat{\theta}_{i,T}\right)\pi_i(1 - p_{ii})}} \tag{4.1.10}$$

Then, under the stationary assumption mentioned at the beginning of Section 2, one can prove (see Nicolau (2016)):

[1] Differences should occur only for finite samples, given that asymptotically the use of $\widehat{\mathrm{Var}}_a\left(\hat{\theta}_{i,T}\right)$ and $\widehat{\mathrm{Var}}_a\left(\hat{\theta}_{i,\lfloor rT \rfloor}\right)$ is equivalent.

$$X_{i,T}(1) = \frac{1}{\sqrt{T}} \sum_{t=1}^{T} Z_{i,t} \xrightarrow{d} W(1) \qquad (4.1.11)$$

$$X_{i,T}(r) = \frac{1}{\sqrt{T}} \sum_{t=1}^{\lfloor rT \rfloor} Z_{i,t} \xrightarrow{d} W(r) \qquad (4.1.12)$$

Finally, the distribution of $Q_{i,T}(\lfloor rT \rfloor)$ is given by:

$$Q_{i,T}(\lfloor rT \rfloor) \stackrel{d}{=} X_{i,T}(r) - rX_{i,T}(1) \xrightarrow{d} W(r) - rW(1) \qquad (4.1.13)$$

The test statistic and respective asymptotic distribution are obtained through the application of the continuous mapping theorem to equation (4.1.13). This theorem states that for a function $g: \mathbb{R}^m \rightarrow \mathbb{R}^p$ continuous in its domain and independent of T, the following applies:

$$X_T \xrightarrow{d} X \Rightarrow g(X_T) \xrightarrow{d} g(X) \qquad (4.1.14)$$

Therefore, equation (4.1.13) can be transformed as:

$$\mathrm{Sup}_{r \in R} {Q^2}_{i,T}(\lfloor rT \rfloor) \xrightarrow{d} \mathrm{Sup}_{r \in R}[W(r) - rW(1)]^2 \qquad (4.1.15)$$

Which holds under H_0. To test for possible structural changes in the BB markets stability, consider the result in equation (4.1.15).

The implementation of this test is straightforward: First, one needs the series of 1's and 0's relative to the BB states inherent to the series in study (consider the Lunde & Timmerman (2004) rules-based method, already discussed in Section 3). The statistics ${Q^2}_{i,T}([rT])$ for $r \in R$ are obtained from this series by using the subsamples $\{1, \dots, w\}, \{1, \dots, w+1\}, \dots, \{1, \dots, T\}$, which leads to the question of what value for w should be chosen. While a small value is associated with the impossibility of obtaining the statistics ${Q^2}_{i,T}([rT])$ for some $r \in R$, for a large value of w the breakpoint may be missed. The choice of w suggested is one that allows for the sample $\{1, \dots w\}$ to have at least two state transitions as a way of avoiding the impossibility to calculate the statistics ${Q^2}_{i,T}(w)$, and the subsequent to it. With w chosen, obtain $\{\hat{\theta}_{i,w}, \hat{\theta}_{i,w+1}, \dots, \hat{\theta}_{i,T}\}$ using the subsamples $\{1, \dots, w\}$, $\{1, \dots, w+1\}, \dots, \{1, \dots, T\}$, respectively. $\hat{\sigma}_i^2$ can be estimated either one time, using the whole sample, yielding $\hat{\sigma}_i^2 = \widehat{\mathrm{Var}}_a\left(\hat{\theta}_{i,T}\right)$ or multiple times using the above subsamples, yielding $\hat{\sigma}_i^2 = \widehat{\mathrm{Var}}_a\left(\hat{\theta}_{i,\lfloor rT \rfloor}\right)$. Finally, after the computation of $\{{Q^2}_{i,T}(w), {Q^2}_{i,T}(w + 1), \dots, {Q^2}_{i,T}(T)\}$, the maximum value of this sequence is compared to the corresponding critical value and in case $H_0\colon \theta_{i,\lfloor rT \rfloor} = \theta_i\ \forall r \in R$ is rejected, the estimate for the breakpoint is given by the period in which ${Q^2}_{i,T}(\lfloor rT \rfloor)$ achieves its maximum.

The critical values obtained through simulation are 1,46, 1,78 and 2,54, for test sizes of 10%, 5% and 1%, respectively.

The tests to be introduced in the next section are based on equation (4.1.13) and are also obtained through the continuous mapping theorem, being very similar to the one previously presented. These tests essentially differ from each other in the application of the theorem, which yields different test statistics and asymptotic distributions.

4.2 – Alternative Structural Change Tests in Duration of BB Markets

The test presented previously uses the supremum squared value of $Q_{i,T}(\lfloor rT \rfloor)$ as its test statistic, analogously, other test statistics can be obtained from $Q_{i,T}(\lfloor rT \rfloor)$ to test H_0: $\theta_{i,\lfloor rT \rfloor} = \theta_i \ \forall r \in R$, against its alternative H_1: $\theta_{i,\lfloor rT \rfloor} \neq \theta_i$ for some $r \in R$.

In this section two alternatives are presented: Based on the mean-score test of Andrews & Ploberger (1994), instead of using the supremum value of ${Q^2}_{i,T}(\lfloor rT \rfloor)$, one may use an integral instead. Furthermore, in place of using the squared $Q_{i,T}(\lfloor rT \rfloor)$, the absolute value of this statistic will also be studied.

To obtain the alternative test statistics notice that $Q_{i,T}(\lfloor rT \rfloor)$ is a step function:

$$\int_0^1 {Q^2}_{i,T}(\lfloor rT \rfloor)\, dr = \sum_{j=w+1}^{T-1} {Q^2}_{i,T}(j) * \frac{1}{T} \qquad (4.2.1)$$

$$\int_0^1 |Q_{i,T}(\lfloor rT \rfloor)|\, dr = \sum_{j=w+1}^{T-1} |Q_{i,T}(j)| * \frac{1}{T} \qquad (4.2.2)$$

Given the result s expressed in equation (4.1.13), through the continuous mapping theorem the asymptotic distributions associated with the previously mentioned test statistics follow:

$$\int_0^1 {Q^2}_{i,T}(\lfloor rT \rfloor)\, dr \xrightarrow{d} \int_0^1 [W(r) - rW(1)]^2\, dr \qquad (4.2.3)$$

$$\int_0^1 |Q|_{i,T}(\lfloor rT \rfloor)\, dr \xrightarrow{d} \int_0^1 |W(r) - rW(1)|\, dr \qquad (4.2.4)$$

The implementation of the alternative tests is again, similar to the one discussed in the previous section. The greatest difference is that one does not use the supremum values of ${Q^2}_{i,T}(\lfloor rT \rfloor)$ or $|Q|_{i,T}(\lfloor rT \rfloor)$, but instead its sum, consequently it is not possible to give an estimate for the breakpoint directly.

The critical values are obtained through Monte Carlo simulation, which takes place in the next section, alongside with a simulation of the three structural change tests' real size and statistical power.

4.3 – Monte Carlo Simulation Study

4.3.1 – Procedure and Design

In this section, the main goals are to determine the real size and statistical power of the structural change tests in duration of BB markets[2]. In addition, the simulation study will also aim to find the asymptotic critical values associated with the alternative tests.

Throughout the simulation study, the number of replications used is $N = 10\,000$ and the tests' statistical properties are obtained by taking into account a nominal test size of 0,05.

For the alternative tests' asymptotic critical values, consider the following data generating process (henceforth DGP) specified for the variable indicator of the BB markets, S_t:

[2] In the present work, the Monte Carlo simulation study is achieved using the TSP software.

$$\begin{cases} p_{11} \coloneqq P(S_t = 1|S_{t-1} = 1) = 0{,}95; \; t = \{1,2,\dots,T\} \\ p_{00} \coloneqq P(S_t = 0|S_{t-1} = 0) = 0{,}95; \; t = \{1,2,\dots,T\} \end{cases} \tag{4.3.1}$$

Where the sample size considered is $T = 25\,000$ and $\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,T})$. Notice that the results using $\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,T})$ or $\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,\lfloor rT \rfloor})$ should be asymptotically equivalent.

To develop the simulation study applied to the statistical properties inherent to the three structural change tests, it is highly important to realize that the statistical tests depend on a series of factors, including the DGP selected[3], the number of observations and whether to use $\widehat{Var}_a(\hat{\theta}_{i,T})$ or $\widehat{Var}_a(\hat{\theta}_{i,\lfloor rT \rfloor})$, for that reason, it is pertinent to perform multiple simulations followed by a comparison of results.

In order to simulate the real size, consider:

$$\begin{cases} p_{11} \coloneqq P(S_t = 1|S_{t-1} = 1) = \alpha; \; t = \{1,2,\dots,T\} \\ p_{00} \coloneqq P(S_t = 0|S_{t-1} = 0) = \beta; \; t = \{1,2,\dots,T\} \end{cases} \tag{4.3.2}$$

The choices for $(\alpha;\ \beta)$ are: $(0{,}996;\ 0{,}996), (0{,}99;\ 0{,}99)$ and $(0{,}95, 0{,}95)$, three sample sizes are used, $T = 3\,000$, $T = 6\,000$ and $T = 15\,000$, as well as both $\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,T})$ and $\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,\lfloor rT \rfloor})$.

To simulate the statistical power of the three tests, consider the DGP's:

$$\begin{cases} p_{11}^1 \coloneqq P(S_t = 1|S_{t-1} = 1) = \gamma; \; t = \{1,2,\dots,T_{break} - 1\} \\ p_{00}^1 \coloneqq P(S_t = 0|S_{t-1} = 0) = \delta; \; t = \{1,2,\dots,T_{break} - 1\} \\ p_{11}^2 \coloneqq P(S_t = 1|S_{t-1} = 1) = \lambda; \; t = \{T_{break}, T_{break} + 1,\dots,T\} \\ p_{00}^2 \coloneqq P(S_t = 0|S_{t-1} = 0) = \psi; \; t = \{T_{break}, T_{break} + 1,\dots,T\} \end{cases} \tag{4.3.3}$$

[3] The DGPs are chosen having in mind the characteristics of the database studied in the next sections.

The choices for $(\gamma;\ \delta;\ \lambda;\ \psi)$ are: $(0{,}996;\ 0{,}99;\ 0{,}99;\ 0{,}99)$, $(0{,}99;\ 0{,}99; 0{,}996;\ 0{,}99)$, $(0{,}99;\ 0{,}98;\ 0{,}99;\ 0{,}95)$, $(0{,}99;\ 0{,}95;\ 0{,}99;\ 0{,}98)$, $(0{,}98;\ 0{,}95;\ 0{,}95;\ 0{,}95)$ and $(0{,}95;\ 0{,}95;\ 0{,}98;\ 0{,}95)$. Again, three sample sizes are used, $T = 3\,000$, $T = 6\,000$ and $T = 15\,000$, as well as both $\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,T})$ and $\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,\lfloor rT \rfloor})$. Additionally, the breakpoint at T_{break} is set on the 50^{th} and 80^{th} percentiles of the sample.

To develop the Monte Carlo simulations, examine the following steps, which are identical among the applications here presented:

1. Generate a sample with size equal to T, related to the continuous variable $U \sim \mathrm{Uniforme}(0,1)$.
2. Initialize the process $\{S_t\}$ with regard to the initial probabilities specified, taking into account that:

$$\begin{cases} p_1^{(1)} \coloneqq P(S_1 = 1) = \dfrac{p_{01}}{1-(p_{11}-p_{01})} \\ p_0^{(1)} \coloneqq P(S_1 = 0) = \dfrac{p_{10}}{1-(p_{00}-p_{10})} \end{cases} \qquad (4.3.4)$$

   And:

$$\begin{cases} S_1 = 1 \text{ if } U_1 \leq p_1^{(1)} \\ S_1 = 0 \text{ if } U_1 > p_1^{(1)} \end{cases} \qquad (4.3.5)$$

3. Considering the specified transition probabilities, generate $\{S_{2,}S_3, \dots, S_T\}$ through:

$$\begin{cases} S_t = 1 \text{ if } (S_{t-1} = 1\ \wedge\ U_t \leq p_{11}) \vee (S_{t-1} = 0 \wedge\ U_t > p_{00}) \\ S_t = 0 \text{ if } (S_{t-1} = 1\ \wedge\ U_t > p_{11}) \vee (S_{t-1} = 0 \wedge\ U_t \leq p_{00}) \end{cases}, \forall t \in \{2,3, \dots, T\} \quad (4.3.6)$$

4. With knowledge of $\{S_{1,}S_2, \dots, S_T\}$, obtain the statistics[4] n_i and n_{ii} $\forall t \in \{w, w+1, \dots, T\}$, which allow the computation of $\hat{\sigma}_i^2$ and $\hat{\theta}_{i,\lfloor rT \rfloor}$ $\forall r \in R$. Then, proceed to assemble $\{Q_{i,T}(w), Q_{i,T}(w+1), \dots, Q_{i,T}(T)\}$.
5. Calculate the test statistic.
6. Repeat the previous steps for every one of the N replications.

A final step that differs between the simulation of the critical values and the statistical properties must be introduced. For the former:

7A. After obtaining the N test statistics associated with a given test, sort them in ascending order. The 90^{th}, 95^{th} and 99^{th} percentiles give the critical values for test sizes of 10%, 5% and 1%, respectively.

And for the latter:

7B. After obtaining the N test statistics associated with a given test, count the number of rejections of H_0: $\theta_{i,\lfloor rT \rfloor} = \theta_i$ $\forall r \in R$, considering the specified nominal test size. The arithmetic mean of the number of rejection yields the empirical size/power of the test (depending on whether H_0: $\theta_{i,\lfloor rT \rfloor} = \theta_i$ $\forall r \in R$ holds or not).

4.3.2 – Discussion and Results

The results derived from the Monte Carlo experiments are presented during this section. Essentially, the simulation study is divided in the computation of asymptotic critical values related to the alternative tests and

[4] In order to obtain the critical values associated with the alternative tests, through the given DGP there is no need for the computation of the statistics $\{Q_{i,T}(w), Q_{i,T}(w+1), \dots, Q_{i,T}(T)\}$ for both $i = 1$ and $i = 0$ given its redundancy. To this end, consider only the case where $i = 1$.

analysis of the real size and power of the structural change tests in duration of BB markets.

Through the Monte Carlo simulation study, the critical values obtained considering test sizes of 90%, 95% and 99% are respectively 0,34, 0,45 and 0,75, for the test associated with equation (4.2.3) and 0,49, 0,58 and 0,76 for (4.2.4).

Tables 1 – 3 gather the information relative to the simulation results of the tests' empirical size. It is observable that the test size is influenced in multiple extents, whether it is the sample size, the usage of $\widehat{\sigma}_i^2 = \widehat{\mathrm{Var}}_a\left(\widehat{\theta}_{i,T}\right)$ against $\widehat{\sigma}_i^2 = \widehat{\mathrm{Var}}_a\left(\widehat{\theta}_{i,\lfloor rT \rfloor}\right)$ or the transition probabilities specified in the DGP.

It seems that the choice of $\widehat{\sigma}_i^2$ is quite significant, noticing that $\widehat{\sigma}_i^2 = \widehat{\mathrm{Var}}_a\left(\widehat{\theta}_{i,\lfloor rT \rfloor}\right)$ is associated with size distortions up to four times the nominal size for the test introduced by Nicolau (2016), and two times for the alternative tests. These distortions are mitigated considering $\widehat{\sigma}_i^2 = \widehat{\mathrm{Var}}_a\left(\widehat{\theta}_{i,T}\right)$, with the real size fairly approximating the nominal.

The sample size and DGP also play an important role in the tests' real size, with less over-rejection being detected for smaller transition probabilities $(p_{11};\ p_{00})$ and larger sample sizes. Therefore, one may extrapolate that the number of state transitions influences the size properties of the given tests, with these exhibiting less over-rejection the more state transitions verified in the sample.

Overall, the alternative tests show less over-rejection, comparing with the existing structural change test.

Likewise the previous case, the statistical power of the tests is influenced by the same extents, with addition to the characteristics of the structural change considered.

Through tables 4 – 9 it is clear that the sample size and transition probabilities affect the statistical power in a way that the more state transitions (larger sample sizes and smaller transition probabilities, p_{11} and p_{00}) the more statistical power.

As it should be expected, the tests evidence more statistical power when the structural change is in the middle of the sample, comparatively to a more extreme position (say, in the 80^{th} percentile). Such yields the difficulty for the tests to detect structural changes in duration of BB markets when these are verified at the beginning or at the end of the sample.

A rather interesting outcome is obtained when comparing the results for $\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,T})$ against $\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,\lfloor rT \rfloor})$. It seems that the tests using $\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,T})$ have strictly better power when the structural change is associated with a decrease in the duration of the cycles, and conversely, the tests using $\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,\lfloor rT \rfloor})$ show more power when there is an increase in duration of the cycles. This justifies the use of both $\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,T})$ and $\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,\lfloor rT \rfloor})$ when applying the structural change tests, since for finite samples, one is suitable for the detection of increases and the other of decreases in duration of BB markets.

Comparing the three tests' statistical power, one verifies that the test introduced by Nicolau (2016) has generally more statistical power than the alternatives. Such power may be alarmingly low when working with small samples sizes and high duration BB markets but reaches admissible values otherwise, with the tests appearing to be consistent.

In sum, the tests' statistical properties improve in function of larger sample sizes and lower transition probabilities which translate in more state transitions, with the alternative tests showing less problems of size distortion but also less power in the simulation experiments. The usage of $\hat{\sigma}_i^2 = \widehat{\mathrm{Var}}_a\left(\hat{\theta}_{i,[rT]}\right)$ is justified by its better results in detection of increases in duration of BB cycles, while $\hat{\sigma}_i^2 = \widehat{\mathrm{Var}}_a\left(\hat{\theta}_{i,T}\right)$ show better results in detection of decreases in duration of BB cycles.

5 – How Are Structural Changes in Duration of Bull and Bear Markets Connected with the Business Cycle

"[Economists] will have to do their best to incorporate the realities of finance into macroeconomics"

Paul Krugman in *New York Times Magazine, 2 September 2009*

The links between macroeconomics and finance became an active field of research especially after the crisis of 2008 that affected economies worldwide, as economic recessions seem to be accompanied by several financial disruptions.

Claessens et al. (2009) show that recessions regularly coincide with periods of contractions in domestic credit and declines in asset prices, moreover, recessions linked with credit crunches and house price busts are deeper and last longer in comparison with other recessions.

Other authors such as Estrella & Mishkin (1998) and Avouyi-Dovi & Matheron (2005) had previously tried to relate finance and macroeconomics. While the former conclude that financial variables such as stock prices have predictive power over economical recessions in the United States, the latter show that the stock market cycle and the business cycle verify a significant concordance in that country, with the start of stock market contractions preceding contractions in real GDP.

Claessens et al. (2012) addressed the question of "how does the nature of business cycles vary across different phases of financial cycles?" having concluded the presence of strong interactions and synchronization among these cycles, with the financial cycles affecting the duration and strength of recessions and recoveries in the economy.

The present study aims to be a valid contribution to further understanding the links between finance and macroeconomics, by exploring the possible relations of structural changes in duration of BB markets and the business cycle, a research field never considered to date.

To analyze the connections between structural changes in duration of BB markets and the business cycle, the statistical tests presented are applied to adjusted market capitalization stock market indexes of several countries and the information regarding breakpoints crossed with the peaks and troughs verified in business cycles and further macroeconomic events. In this sense, the main goals of this empirical study are to detect and describe the periods where statistical evidence of structural changes is common among countries and to perceive how these increases and decreases in duration of BB cycles are connected with the business cycles.

5.1 – Data and Methodology

The database comprises adjusted market capitalization stock market indexes of 37 developed and emerging markets, constructed by Morgan Stanley Capital International (MSCI) and downloaded from DataStream.

The classification of the market follows three essential criteria: Economic development, market accessibility and size/liquidity[5]. The adjusted market capitalization stock market indexes are derived from the equity universe, precisely the investable market index. This index is then divided by the size of the companies with respect to their full market capitalization, resulting in the large, mid and small cap indexes[6]. For each market (country) considered, the structural change tests are applied to the bull and bear markets identified from the three size indexes.

From the 37 markets considered, 21 are classified as developed and 16 as emerging markets. The developed markets are: Canada and United States of America from the Americas; Belgium, Denmark, Germany, Finland, France, Ireland, Israel, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and United Kingdom from Europe and Middle East; Australia, Hong Kong, Japan and Singapore from the Pacific. The emerging markets

[5] See: www.msci.com/documents/1296102/1330218/MSCI_Market_Classification_Framework+2017.pdf/21f360a0-930c-4ca6-9864-d981820dfa0a.

[6] See: www.msci.com/eqb/methodology/meth_docs/MSCI_June2017_GIMIMethodology.pdf.

are: Brazil, Chile, Mexico and Peru from the Americas; Hungary, Russia, South Africa, Turkey, Qatar and United Arab Emirates from Europe, Middle East and Africa; China, India, Indonesia, Korea, Malaysia and Philippines from Asia.

The sample size varies from 3 556 to 6 482 for the daily index prices, due to restrictions in the availability of its source (DataStream), nevertheless, the vast majority of the index prices considered have more then 5 900 daily observations and only for two markets the sample size is less than 5 000. The last observed period is identical among the elements of the database considered and it corresponds to the 3rd of April 2017 (see tables 10 and 11).

After the identification of the BB markets inherent to the large, mid and small cap indexes considered for each country, through the algorithm suggested by Lunde & Timmerman (2004), the application of the structural change tests follows. During the course of this analysis, it is admitted that the estimated dates of breakpoint given by the structural change tests are consistent. This assumption is supported by Bai (2000) since as specified in Section 2, the bull and bear markets are assumed to be governed by a stationary and ergodic first order Markov Chain Process, which has a first order vector autoregressive representation holding the same asymptotic proprieties.

The application of the tests is rather simple, yet two remarks arise, the first one, concerning what start-up values to be used, the choice of w naturally differs between series, in order to follow the formerly mentioned strategy where the chosen value is the one that allows for the sample $\{1, \dots w\}$ to have at least two state transitions. Secondly, regarding $\hat{\sigma}_i^2$ being the maximum likelihood estimate of $\mathrm{Var}_a(\hat{\theta}_{i,T})$ or $\mathrm{Var}_a(\hat{\theta}_{i,\lfloor rT \rfloor})$, both cases are considered, since as seen previously, the former provides better statistical power when there is a decrease in duration of bull/bear markets, while the latter when there is an increase in its duration.

In terms of recognizing statistical evidence of structural changes, the following set of rules is considered:

a) Since the simulation study performed gave support that the test using the result in equation (4.1.15) with $\hat{\sigma}_i^2 = \widehat{\mathrm{Var}}_a(\hat{\theta}_{i,T})$ presents few over-rejection problems and more power than the respective tests that use (4.2.3) and (4.2.4), then if there is statistical evidence for the rejection of H_0 using (4.1.15) at a nominal test size of 0,05, a structural change associated with a decrease in the cycles is recognized.

b) Since the simulation study performed gave support that the test using the result in equation (4.1.15) with $\hat{\sigma}_i^2 = \widehat{\mathrm{Var}}_a(\hat{\theta}_{i,\lfloor rT \rfloor})$ presents considerable over-rejection but more power than the respective tests that use (4.2.3) and (4.2.4), a more cautious approach is taken: If there is statistical evidence for the rejection of H_0 using (4.1.15) at a nominal test size of 0,05, and using at least one of the tests associated with (4.2.3) and (4.2.4), at a nominal test size of 0,10, then a structural change associated with an increase in the cycles is recognized.

c) Since the tests that use the results in equations (4.2.3) and (4.2.4) have less over-rejection but also less power than the test that uses (4.1.15), then if there is statistical evidence for rejecting H_0 using (4.2.3) or (4.2.4) at a nominal test size of 0,05, a structural change associated with an increase or decrease in the cycles is recognized (depending on $\hat{\sigma}_i^2$).

It is also admissible that there might be more than one structural change in the duration of bull or bear markets in a given index. In this sense, after the application of the tests considering the whole sample, in case there is statistical evidence of a structural change in a given period, the tests are applied again using the two subsamples corresponding to the periods before and after the breakpoint. This simple process allows for the detection of multiple structural changes.

5.2 – Results

The current section focuses on presenting the results obtained from the application of the structural change tests in duration of BB markets to the database consisting on large, mid and small cap indexes constructed by MSCI.

This application led to the results evidenced in figures 1 and 2. A first analysis to the figures allows the identification of several structural changes in duration of BB markets in the periods summarized between 1996 and 2015. After a closer look, it is possible to identify that the structural changes follow some interesting patterns among the different markets. Consider the following phases regarding the mentioned patterns:

1. 1996 – 2001 (for developed markets) / 1996 – 1998 (for emerging markets): Period characterized by an increase in the cyclicity of BB markets, with decreases in the duration of these cycles registered in several markets.
2. 2002 – 2003 (for developed markets) / 1999 – 2003 (for emerging markets: Period with several increases in the duration of bull cycles associated with the given indexes.
3. 2004 – 2008: Numerous structural changes relative to decreases in the duration of bull cycles (henceforth DDBC) are observed, for both developed and emerging markets, especially during the period of 2006 – 2007. It is also noticeable that for some developed markets these structural changes are also accompanied by decreases in duration of bear markets. Interestingly, the DDBC usually occur first for the indexes associated with smaller companies and then for the larger.
4. 2009 – 2015: Increases in duration of bear markets are the main feature perceptible during these periods, with these structural changes noticeable for several developed and emerging markets.

Given these four phases, it is now intended to compare each one to the business cycle's behavior verified in the respective period and further macroeconomic events[7].

[7] For an extensive chronology of business cycles peaks and troughs presented by the Economic Cycle Research Institute (ECRI) see www.businesscycle.com/ecri-business-cycles/international-business-

Starting with the first phase perceived, one can observe that the bursting of the tech bubble, the global recession verified at the beginning of the XXI century and a gradual increase in the interest rates[8], which restrains the access of credit by companies, match this period of higher volatility with decreases of both BB markets' duration.

The increases verified during the second phase are easily explained by the expansionist period registered in worldwide economies and relatively low interest rates during that period.

Through the third phase, it is recognizable that the DDBC not only occur first for the indexes associated with smaller companies, but also seem to anticipate the recession period confirmed in business cycles worldwide. These two observations also happen during the first phase, although on a smaller scale.

To explain the pattern verified between smaller and larger companies, notice that Kim & Burnie (2002) show that smaller companies are more vulnerable to adverse changes in economic conditions given their lower productivity and higher financial leverage. Additionally, Ehrmann (2010) points that a monetary policy tightening, which leads to restricted access to credit for companies, is more likely to affect the smaller ones given the higher amount of collateral they have to pledge and their difficulties to access other forms of external finance, comparing with larger companies.

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cycle-dates-chronologies or see Fushing et al. (2010). For a detailed record of events that made an influence in global macroeconomics see www.businesscycle.com/ecri-about/track-record.

[8] See www.tradingeconomics.com/country-list/interest-rate for a detailed record of benchmark interest rates verified in the world economies.

Noticing that a monetary policy tightening actually happened during the third phase, with a progressive increase in interest rates worldwide during the period before the crisis, one concludes that the structural changes detected are therefore a combination between the vulnerability of smaller companies and the conditions verified throughout the pre-crisis period.

To explain the several increases in bear markets duration noticed in the fourth phase, consider the slowdown in the economic growth and the industrial slowdown, which provide evidence that although the crisis of 2008 is over, its effects are still present in the economy and in the financial markets.

One of the most interesting connections detected between structural changes in duration of BB markets and the business cycle was the fact that DDBC seem to anticipate periods of economic recession. Beside the use of visual inspection that allows for such conclusion, it is also desirable to perceive if there is statistical evidence that supports this statement. To this end, consider:

$$\mathrm{I_i(m)} = \mathrm{Max}\left\{\mathrm{I_i^{small}}, \mathrm{I_i^{medium}}, \mathrm{I_i^{large}}\right\} \tag{5.2.1}$$

Where

$$\mathrm{I_i^{small}} = \begin{cases} 1 & \text{if } \mathrm{A(m)} \\ 0 & \text{otherwise} \end{cases} \tag{5.2.2}$$

With $\mathrm{A(m)}$ the event where, for the i-th market, a DDBC in small companies occurs m months or less before a peak in the business cycle. $\mathrm{I_i^{medium}}$ and $\mathrm{I_i^{large}}$ are analogously defined.

If $I_i(m) = 0$, then either no structural changes/economic crisis were detected during the sample period, or the structural changes did not occur $s \leq m$ months before the crisis. In order to conduct this statistical application the first scenario is excluded, in this sense, only the markets where there is evidence in the sample of at least one DDBC and one economic crisis are included.

Under the H_0 stating that DDBC do not anticipate crisis in business cycles, $\{I_i(m)\}$ is a sequence of i.i.d random variables with Bernoulli distribution of parameter $p \coloneqq P(I_i(m) = 1)$, which is the probability of at least one DDBC occurring m months or less before an economic crisis, for a given market, with both events independent from each other. Then, the statistic that allows to test if these structural changes indeed anticipate periods of economic recession (H_1) is given by:

$$T(m) = \sum_{i=1}^{n} I_i(m) \sim \text{Binomial}(n, p) \qquad (5.2.3)$$

Where n is the number of markets in the sample verifying statistical evidence of DDBC and economic crisis. With $T(m)$ the sum of markets in the sample which verify at least one DDBC in less than m months before a crisis, then clearly the greater the $T(m)$, the greater the likelihood that DDBC anticipate economic crisis.

The question is now how to calculate p under H_0. Consider the formula:

$$\hat{p} = \sum_{J=1}^{k} \sum_{L=1}^{\infty} P(x = J \cap y = L) * \left[1 - \left(\frac{T - L * m * \frac{250}{12}}{T}\right)^{J}\right] \qquad (5.2.4)$$

x being a random variable relative to the total number of DDBC associated with the small, mid and large indexes of a given market, y a random variable relative to the number of economic crisis experienced in that market during $1996 - 2017$, and $k = \frac{T}{m * \frac{250}{12}}$. In this sense, $\left[1 - \left(\frac{T - L * m * \frac{250}{12}}{T}\right)^{J}\right]$ represents the probability of at least one of the J DDBC found in the size indexes of a given market anticipates in m months one of its L economic crisis. The estimation of $P(x = J \cap y = L)$ is done using the markets considered in the sample, by:

$$\widehat{P}(x = J \cap y = L) = \frac{\#\text{Markets verifying J DDBC and L crisis}}{\#\text{Markets veryfying DDBC and economic crisis}} \qquad (5.2.5)$$

The number of markets in the database verifying statistical evidence of DDBC is 26. The following problem arises: The sample size T is heterogeneous among markets and among the indexes. This way, the United Arab Emirates are removed from this analysis since its indexes' sample size is reasonably smaller comparing to the other markets. From the 25 left, the sample sizes are fairly similar, between 5 900 and 6 500 observations, with the majority verifying $T = 5\,960$. In this sense, for the present analysis the sample size is rounded to $T = 6\,000$.

To do the confrontation concerning the structural changes and the economic crisis' dates, one needs to have the information regarding both. The former were obtained directly from the application of the statistical tests, while the latter by considering the dates presented by ECRI[9] if available, and through Fushing et al (2010) otherwise[10]. Information regarding the business cycles of Hong Kong, Indonesia, Malaysia, Peru and Turkey was not found, while China evidenced no economic crisis during the period of $1996 - 2017$. In this sense, the number of n markets considered is 19 (see table 12).

Table 13 presents the results concerning the application of this statistical test considering two values for p, one estimated through the method discussed above and the other an overestimate of $P(I_i(m) = 1)$, $p = 0{,}5$, more favorable to the null hypothesis of no connection between DDBC and economic crisis.

The estimated probabilities inherent to the event in which DDBC occur m months or less before an economic crisis, with both events independent are 0,19 and 0,35 for $m = 12$ and $m = 24$, respectively. One concludes that for the 19 markets considered which show statistical evidence of at least one DDBC and one economic crisis, 14 have at least one DDBC preceding an economic recession in 12 months. The same number rises to 18 if the number of months considered is 24.

[9] See www.businesscycle.com/ecri-business-cycles/international-business-cycle-dates-chronologies.

[10] These two sources produce practical results that are relatively similar to each other, yet Fushing et al (2010) is used in a complementary way since it only considers the business cycles to 2010, while ECRI has that information until 2016.

Such result points to a strong statistical evidence that DDBC indeed anticipate economic crisis in the respective countries. It seems that most markets considered have at least one DDBC preceding an economic crisis. The P-Values obtained are significantly small even when using the overestimate $p = 0{,}5$, with the rejection of H_0: {DDBC do not anticipate economic crisis in the business cycle} verified for all the scenarios considered, for a test size of 0,01.

In summary, the results obtained allow to distinguish several patterns of structural changes in duration of BB markets, in the countries and indexes considered. Such structural changes seem to follow certain events occurred in the macroeconomic cycles, specifically, DDBC seem to anticipate economic crisis with those structural changes typically being first verified for smaller companies and then for larger. This way, monitoring the financial markets with respect to the duration of bull and bear markets may contribute to the identification and prevention of periods of economic recession.

6 – Extensions and Further Research

The application of the structural change tests in duration of BB markets to the database considered led to a better compression of the relation between the financial markets, specifically, structural changes in duration of its BB markets, and the business cycle. As mentioned during this work, such relation had never been studied to date, which makes this contribution a new approach in understanding the connections between finance and macroeconomics. Since so, the door is open for further investigation in this area: It would be interesting and relevant, for example, to apply the same tests to other databases consisting on financial time series and see if the conclusions of this work still hold, or to detect other possible relations worth of interest.

Furthermore, since the present work only intends to evidence the relations between the structural changes in duration of BB markets and the business cycle, such as DDBC anticipating economic crisis, it is still critical that one detects promptly these structural changes in order to anticipate relevant economic events, that is, the financial markets should be into close inspection for decreases and increases in duration of its BB markets and the investigation regarding BB markets duration should evolve in direction of providing methodologies to predict these structural changes.

7 – Conclusions

This work focused on the study and application of the structural change test in duration of BB markets proposed by Nicolau (2016) and two alternative tests computed from the former. These alternatives showed less size distortions but also less power than the existing test, which yield a great value in obtaining robust results when applied together with the first test.

The application of the studied tests to a database composed by large, mid and small cap indexes constructed by MSCI led to the detection of several relations between the BB markets and the business cycle, as several breakpoints seemed to be associated with the behavior verified in the business cycle and further events in the macroeconomic context worldwide.

Such relations shed more light in the association between macroeconomics and finance, an active field of research that gained a new impetus since the crisis of 2008.

The main breakthrough achieved during this work was the detection of a relation between DDBC and economic crisis. From inspection, one concludes that for 14 out of the 19 markets with evidence of both DDBC and economic crisis during $1996 - 2017$, DDBC anticipate at least one economic recession in 12 months. The same number rises to 18 if an anticipation of 24 months is considered. Statistically, there is strong evidence that this structural changes do not happen independently from economic crisis, which provides the conclusion that DDBC effectively seem to anticipate such macroeconomic events.

It is suggested that the duration of BB markets should be closely analyzed in the future, in order to detect possible changes that may be connected to macroeconomic events, such as the beginning of recession periods.

References

1. Andrews, D. W. K., Ploberger, W. (1994). Tests for Parameter Stability and Structural Change with Unknown Change Point. Econometrica 62, 1383 – 1414.

2. Avouyi – Dovi, S., Matheron, J. (2005). Interactions between Business Cycles, Financial Cycles and Monetary Policy: Stylised Facts. BIS Papers 22, 273 – 298.

3. Bai, J. (2000). Vector Autoregressive Models with Structural Changes in Regression Coefficients and in Variance-Covariance Matrices. Annals of Economics and Finance 1(2), 303 – 339.

4. Basawa, I.V., Rao, P. (1980). Statistical Inferences for Stochastic Processes: Theory and Methods. Academic Press, London.

5. Bry, G., Boschan, C. (1971). Cyclical Analysis of Time Series: Selected Procedures and Computed Programs. NBER: New York.

6. Chauvet, M., Potter, S. (2000). Coincident and Leading Indicators of the Stock Market. Journal of Empirical Finance 7, 87 – 111.

7. Claessens, S., Kose, M. A., Terrones, M. E. (2009). What Happens During Recessions, Crunches and Busts?. Economic Policy 24 (60), 653 – 700.

8. Claessens, S., Kose, M. A., Terrones, M. E. (2012). How do Business and Financial Cycles Interact?. Journal of International Economics 87 (1), 178 – 190.

9. Ehrmann, M. (2005). Firm Size and Monetary Policy Transmission—Evidence from German Business Survey Data. IFO Survey Data in Business Cycle and Monetary Policy Analysis. Heidelberg: Physica Verlag HD.

10. Estrella, A., Mishkin, F. S. (1998). Predicting US Recessions: Financial Variables as Leading Indicators. The Review of Economics and Statistics 80 (1), 45 – 61.

11. Fushing, H., Chen, S.C., Travis, J., Berge, T.J., Jordà, Ò. (2010). A Chronology of International Business Cycles through Nonparametric Decoding. Working Paper.

12. Kim, M.K., Burnie, D.A. (2002). The Firm Size Effect and the Economic Cycle. Journal of Financial Research 25 (1), 111 – 124.

13. Kole, E., Van Dijk, D.J. (2017). How to Identify and Predict Bull and Bear Markets?. Journal of Applied Econometrics 32, 120 – 139.

14. Lunde, A., Timmermann, A. (2004). Duration Dependence in Stock Prices: An Analysis of Bull and Bear Markets. Journal of Business & Economic Statistics 22, 253 – 273.

15. Maheu, J.M., McCurdy, T.H., Song, T. (2012). Components of Bull and Bear Markets: Bull Corrections and Bear Rallies. Journal of Business & Economic Statistics 30 (3), 391 – 403.

16. Nicolau, J. (2016). Structural Change Test in Duration of Bull and Bear Markets. Economics Letters 146, 64 – 67.

17. Pagan, A.R., Sossounov, K.A. (2003). A Simple Framework for Analysing Bull and Bear Markets. Journal of Applied Econometrics 18 (1), 23 – 46.

18. Sperandeo, V. (1990). Principals of Professional Speculation. New York: Wiley.

19. Taylor, H.M., Karlin, S., (1998). An Introduction to Stochastic Modeling. Academic Press.

A – Annex

A.1 – Figures

Figure 1 – Structural changes in duration of bull and bear markets associated with large, mid and small companies (1996 – 2005)

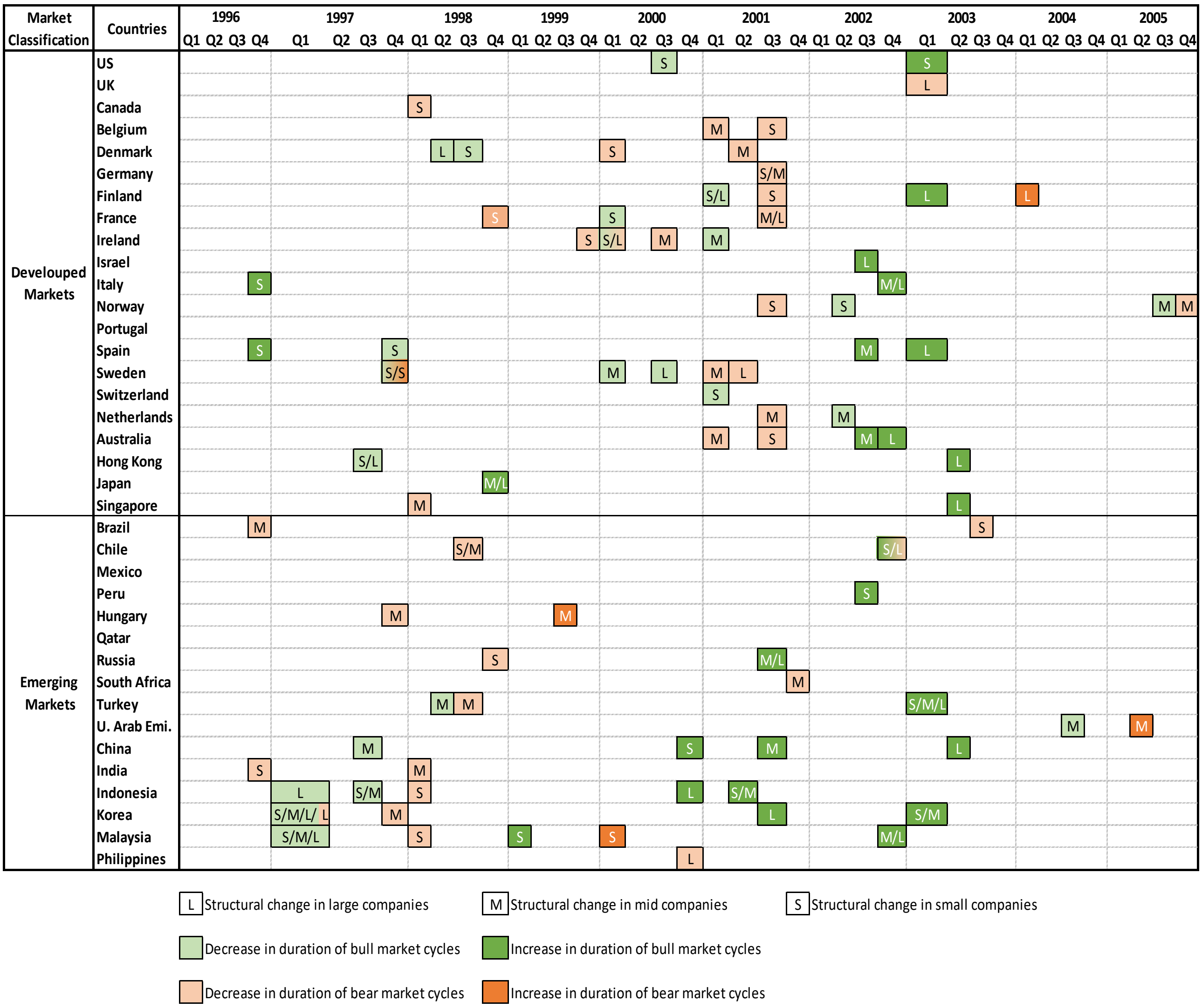
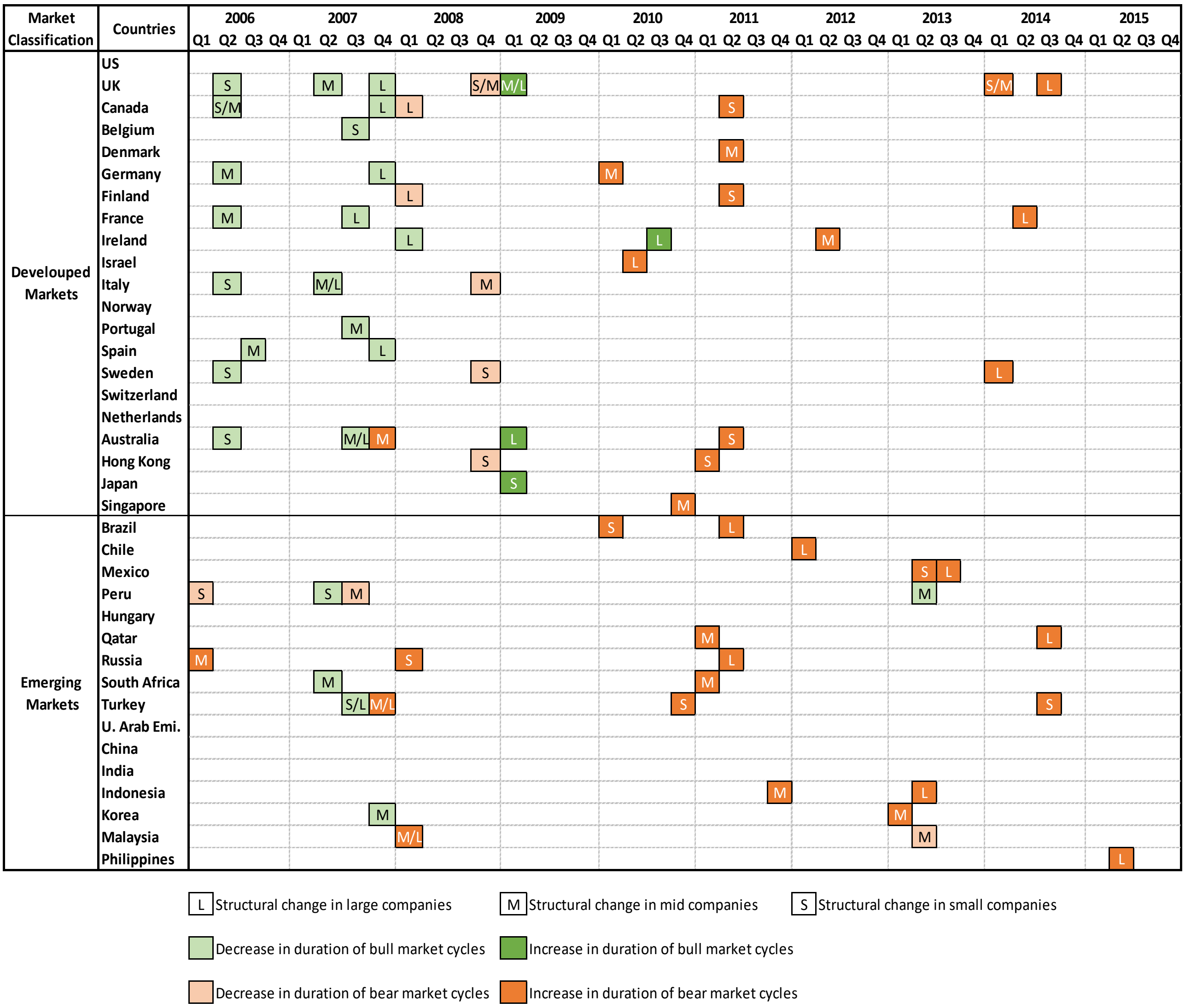


Figure 2 – Structural changes in duration of bull and bear markets associated with large, mid and small companies (2006 – 2015)



A.2 – Tables

Table 1 – Real dimension associated with the structural change tests in function of transition probabilities and choice of $\hat{\sigma}_i^2$ $(T = 3\,000)$

Tests / $(P_{11};P_{00})$	$\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,T})$			$\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,[rT]})$		
	$Sup_{r\in R} Q_{i,T}^2([rT])$	$\int_0^1 Q_{i,T}^2([rT])dr$	$\int_0^1 \lvert Q_{i,T}([rT])\rvert dr$	$Sup_{r\in R} Q_{i,T}^2([rT])$	$\int_0^1 Q_{i,T}^2([rT])dr$	$\int_0^1 \lvert Q_{i,T}([rT])\rvert dr$
(0,996; 0,996)	0,08	0,06	0,06	0,24	0,12	0,11
(0,99; 0,99)	0,07	0,06	0,05	0,23	0,11	0,10
(0,95; 0,95)	0,07	0,05	0,05	0,15	0,07	0,06

Table 2 – Real dimension associated with the structural change tests in function of transition probabilities and choice of $\hat{\sigma}_i^2$ $(T = 6\,000)$

Tests / $(P_{11};P_{00})$	$\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,T})$			$\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,[rT]})$		
	$Sup_{r\in R} Q_{i,T}^2([rT])$	$\int_0^1 Q_{i,T}^2([rT])dr$	$\int_0^1 \lvert Q_{i,T}([rT])\rvert dr$	$Sup_{r\in R} Q_{i,T}^2([rT])$	$\int_0^1 Q_{i,T}^2([rT])dr$	$\int_0^1 \lvert Q_{i,T}([rT])\rvert dr$
(0,996; 0,996)	0,07	0,05	0,04	0,23	0,12	0,10
(0,99; 0,99)	0,07	0,05	0,05	0,20	0,09	0,08
(0,95; 0,95)	0,07	0,05	0,05	0,11	0,06	0,06

Table 3 – Real dimension associated with the structural change tests in function of transition probabilities and choice of $\hat{\sigma}_i^2$ $(T = 15\,000)$

Tests / $(P_{11};P_{00})$	$\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,T})$			$\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,[rT]})$		
	$Sup_{r\in R} Q_{i,T}^2([rT])$	$\int_0^1 Q_{i,T}^2([rT])dr$	$\int_0^1 \lvert Q_{i,T}([rT])\rvert dr$	$Sup_{r\in R} Q_{i,T}^2([rT])$	$\int_0^1 Q_{i,T}^2([rT])dr$	$\int_0^1 \lvert Q_{i,T}([rT])\rvert dr$
(0,996; 0,996)	0,06	0,05	0,04	0,21	0,10	0,08
(0,99; 0,99)	0,07	0,05	0,05	0,16	0,08	0,07
(0,95; 0,95)	0,05	0,05	0,05	0,08	0,06	0,06

Table 4 – Power associated with the structural change tests in function of transition probabilities and choice of $\hat{\sigma}_i^2$ (T = 3 000 and breakpoint at the 50^{th} percentile of the sample)

Tests / $(P_{11}^1; P_{00}^1)$ $(P_{11}^2; P_{00}^2)$	$\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,T})$			$\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,[rT]})$		
	$Sup_{r\in R} Q_{i,T}^2([rT])$	$\int_0^1 Q_{i,T}^2([rT])dr$	$\int_0^1 \lvert Q_{i,T}([rT])\rvert dr$	$Sup_{r\in R} Q_{i,T}^2([rT])$	$\int_0^1 Q_{i,T}^2([rT])dr$	$\int_0^1 \lvert Q_{i,T}([rT])\rvert dr$
(0,996; 0,99) (0,99; 0,99)	0,30	0,29	0,27	0,01	0,01	0,01
(0,99; 0,996) (0,99; 0,99)	0,01	0,07	0,09	0,47	0,44	0,42
(0,99; 0,99) (0,98; 0,95)	0,50	0,49	0,44	0,09	0,07	0,08
(0,99; 0,99) (0,95; 0,98)	0,35	0,36	0,35	0,88	0,69	0,65
(0,98; 0,95) (0,95; 0,95)	0,95	0,93	0,91	0,45	0,70	0,69
(0,95; 0,98) (0,95; 0,95)	0,72	0,81	0,79	1,00	1,00	1,00

Table 5 – Power associated with the structural change tests in function of transition probabilities and choice of $\hat{\sigma}_i^2$ (T = 3 000 and breakpoint at the 80^{th} percentile of the sample)

Tests / $(P_{11}^1; P_{00}^1)$ $(P_{11}^2; P_{00}^2)$	$\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,T})$			$\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,[rT]})$		
	$Sup_{r\in R} Q_{i,T}^2([rT])$	$\int_0^1 Q_{i,T}^2([rT])dr$	$\int_0^1 \lvert Q_{i,T}([rT])\rvert dr$	$Sup_{r\in R} Q_{i,T}^2([rT])$	$\int_0^1 Q_{i,T}^2([rT])dr$	$\int_0^1 \lvert Q_{i,T}([rT])\rvert dr$
(0,996; 0,99) (0,99; 0,99)	0,19	0,17	0,15	0,01	0,01	0,02
(0,99; 0,996) (0,99; 0,99)	0,03	0,06	0,05	0,31	0,27	0,26
(0,99; 0,99) (0,98; 0,95)	0,22	0,20	0,18	0,16	0,05	0,04
(0,99; 0,99) (0,95; 0,98)	0,28	0,26	0,26	0,70	0,48	0,45
(0,98; 0,95) (0,95; 0,95)	0,68	0,60	0,56	0,16	0,31	0,30
(0,95; 0,98) (0,95; 0,95)	0,42	0,49	0,47	0,78	0,69	0,67

Table 6 – Power associated with the structural change tests in function of transition probabilities and choice of $\widehat{\sigma}_i^2$ ($T =$ 6 000 and breakpoint at the 50^{th} percentile of the sample)

Tests / $(P_{11}^1; P_{00}^1)$ $(P_{11}^2; P_{00}^2)$	$\widehat{\sigma}_i^2 = \widehat{Var}_a(\widehat{\theta}_{i,T})$			$\widehat{\sigma}_i^2 = \widehat{Var}_a(\widehat{\theta}_{i,[rT]})$		
	$Sup_{r\in R} Q_{i,T}^2([rT])$	$\int_0^1 Q_{i,T}^2([rT])dr$	$\int_0^1 \lvert Q_{i,T}([rT])\rvert dr$	$Sup_{r\in R} Q_{i,T}^2([rT])$	$\int_0^1 Q_{i,T}^2([rT])dr$	$\int_0^1 \lvert Q_{i,T}([rT])\rvert dr$
(0,996; 0,99) (0,99; 0,99)	0,61	0,57	0,54	0,01	0,06	0,09
(0,99; 0,996) (0,99; 0,99)	0,08	0,26	0,28	0,83	0,74	0,71
(0,99; 0,99) (0,98; 0,95)	0,81	0,78	0,75	0,18	0,42	0,43
(0,99; 0,99) (0,95; 0,98)	0,73	0,69	0,67	0,97	0,84	0,83
(0,98; 0,95) (0,95; 0,95)	1,00	1,00	0,99	0,96	0,98	0,98
(0,95; 0,98) (0,95; 0,95)	0,99	0,98	0,98	1,00	1,00	1,00

Table 7– Power associated with the structural change tests in function of transition probabilities and choice of $\widehat{\sigma}_i^2$ ($T =$ 6 000 breakpoint at the 80^{th} percentile of the sample)

Tests / $(P_{11}^1; P_{00}^1)$ $(P_{11}^2; P_{00}^2)$	$\widehat{\sigma}_i^2 = \widehat{Var}_a(\widehat{\theta}_{i,T})$			$\widehat{\sigma}_i^2 = \widehat{Var}_a(\widehat{\theta}_{i,[rT]})$		
	$Sup_{r\in R} Q_{i,T}^2([rT])$	$\int_0^1 Q_{i,T}^2([rT])dr$	$\int_0^1 \lvert Q_{i,T}([rT])\rvert dr$	$Sup_{r\in R} Q_{i,T}^2([rT])$	$\int_0^1 Q_{i,T}^2([rT])dr$	$\int_0^1 \lvert Q_{i,T}([rT])\rvert dr$
(0,996; 0,99) (0,99; 0,99)	0, 34	0,29	0,27	0,04	0,05	0,05
(0,99; 0,996) (0,99; 0,99)	0,09	0,15	0,17	0,56	0,43	0,41
(0,99; 0,99) (0,98; 0,95)	0,35	0,34	0,31	0,17	0,11	0,12
(0,99; 0,99) (0,95; 0,98)	0,52	0,48	0,46	0,80	0,60	0,57
(0,98; 0,95) (0,95; 0,95)	0,92	0,88	0,83	0,67	0,73	0,70
(0,95; 0,98) (0,95; 0,95)	0,83	0,82	0,79	1,00	1,00	1,00

Table 8 – Power associated with the structural change tests in function of transition probabilities and choice of $\hat{\sigma}_i^2$ ($T =$ $15\ 000$ and breakpoint at the 50^{th} percentile of the sample)

Tests / $(P_{11}^1; P_{00}^1)$ $(P_{11}^2; P_{00}^2)$	$\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,T})$			$\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,[rT]})$		
	$Sup_{r\in R} Q_{i,T}^2([rT])$	$\int_0^1 Q_{i,T}^2([rT])dr$	$\int_0^1 \lvert Q_{i,T}([rT])\rvert dr$	$Sup_{r\in R} Q_{i,T}^2([rT])$	$\int_0^1 Q_{i,T}^2([rT])dr$	$\int_0^1 \lvert Q_{i,T}([rT])\rvert dr$
(0,996; 0,99) (0,99; 0,99)	0, 93	0,92	0,90	0,42	0,69	0,68
(0,99; 0,996) (0,99; 0,99)	0,71	0,81	0,80	0,97	0,94	0,93
(0,99; 0,99) (0,98; 0,95)	1,00	1,00	1,00	0,94	0,94	0,93
(0,99; 0,99) (0,95; 0,98)	0,99	0,92	0,92	1,00	1,00	1,00
(0,98; 0,95) (0,95; 0,95)	1,00	1,00	1,00	1,00	1,00	1,00
(0,95; 0,98) (0,95; 0,95)	1,00	1,00	1,00	1,00	1,00	1,00

Table 9 – Power associated with the structural change tests in function of transition probabilities and choice of $\hat{\sigma}_i^2$ ($T =$ $15\ 000$ breakpoint at the 80^{th} percentile of the sample)

Tests / $(P_{11}^1; P_{00}^1)$ $(P_{11}^2; P_{00}^2)$	$\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,T})$			$\hat{\sigma}_i^2 = \widehat{Var}_a(\hat{\theta}_{i,[rT]})$		
	$Sup_{r\in R} Q_{i,T}^2([rT])$	$\int_0^1 Q_{i,T}^2([rT])dr$	$\int_0^1 \lvert Q_{i,T}([rT])\rvert dr$	$Sup_{r\in R} Q_{i,T}^2([rT])$	$\int_0^1 Q_{i,T}^2([rT])dr$	$\int_0^1 \lvert Q_{i,T}([rT])\rvert dr$
(0,996; 0,99) (0,99; 0,99)	0, 66	0,59	0,65	0,16	0,29	0,29
(0,99; 0,996) (0,99; 0,99)	0,39	0,47	0,46	0,77	0,68	0,64
(0,99; 0,99) (0,98; 0,95)	0,76	0,69	0,63	0,44	0,52	0,50
(0,99; 0,99) (0,95; 0,98)	0,90	0,82	0,80	0,97	0,86	0,84
(0,98; 0,95) (0,95; 0,95)	1,00	1,00	1,00	1,00	0,99	0,99
(0,95; 0,98) (0,95; 0,95)	1,00	1,00	1,00	1,00	1,00	1,00

Table 10 – Data availability for developed markets

Market	Index	Number of daily observations	First observation	Last observation
United States of America	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	6 482	25/05/1992	03/04/2017
United Kingdom	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	6 333	31/12/1992	03/04/2017
Canada	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	6 333	31/12/1992	03/04/2017
Belgium	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	6 333	31/12/1992	03/04/2017
Denmark	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	6 333	31/12/1992	03/04/2017
Germany	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	6 333	31/12/1992	03/04/2017
Finland	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	6 333	31/12/1992	03/04/2017
France	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	6 333	31/12/1992	03/04/2017
Ireland	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	6 241	03/05/1993	03/04/2017
Israel	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	5 960	31/05/1994	03/04/2017
Italy	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	6 333	31/12/1992	03/04/2017
Norway	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	6 333	31/12/1992	03/04/2017
Portugal	Large	5 687	16/06/1995	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	5 047	28/11/1997	03/04/2017
Spain	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	6 333	31/12/1992	03/04/2017
Sweden	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	6 333	31/12/1992	03/04/2017
Switzerland	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	6 333	31/12/1992	03/04/2017
Netherlands	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	6 333	31/12/1992	03/04/2017
Australia	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	6 333	31/12/1992	03/04/2017
Hong Kong	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	6 333	31/12/1992	03/04/2017
Japan	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	6 333	31/12/1992	03/04/2017
Singapore	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	6 333	31/12/1992	03/04/2017

Table 11 – Data availability for emerging markets

Market	Index	Number of daily observations	First observation	Last observation
Brazil	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	5 960	31/05/1994	03/04/2017
Chile	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	5 960	31/05/1994	03/04/2017
Mexico	Large	5 437	31/05/1996	03/04/2017
	Mid	5 437	31/05/1996	03/04/2017
	Small	5 437	31/05/1996	03/04/2017
Peru	Large	5 699	31/05/1995	03/04/2017
	Mid	5 699	31/05/1995	03/04/2017
	Small	5 699	31/05/1995	03/04/2017
Hungary	Large	5 567	30/11/1995	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	5 960	31/05/1994	03/04/2017
Qatar	Large	3 556	25/08/2003	03/04/2017
	Mid	3 872	31/05/2002	03/04/2017
	Small	3 872	31/05/2002	03/04/2017
Russia	Large	5 437	31/05/1996	03/04/2017
	Mid	5 437	31/05/1996	03/04/2017
	Small	5 437	31/05/1996	03/04/2017
South Africa	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	5 960	31/05/1994	03/04/2017
Turkey	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	5 960	31/05/1994	03/04/2017
U. Arab Emi.	Large	3 742	29/11/2002	03/04/2017
	Mid	3 872	31/05/2002	03/04/2017
	Small	3 872	31/05/2002	03/04/2017
China	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	5 960	31/05/1994	03/04/2017
India	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	5 960	31/05/1994	03/04/2017
Indonesia	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	5 960	31/05/1994	03/04/2017
Korea	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	5 960	31/05/1994	03/04/2017
Malaysia	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	5 960	31/05/1994	03/04/2017
Philippines	Large	5 960	31/05/1994	03/04/2017
	Mid	5 960	31/05/1994	03/04/2017
	Small	5 960	31/05/1994	03/04/2017

Table 12 – Business cycles of markets with statistical evidence of DDBC

Market	Peak dates	Trough dates	Number of DDBC	Number of economic crisis
United States of America	Mar/01 Dec/07	Nov/01 Jun/09	1	2
United Kingdom	May/08	Jan/10	3	1
Canada	Jan/08	Jul/09	3	1
Belgium	Nov/97 Nov/99 Aug/08	Apr/98 Mar/01 Jan/09	1	3
Denmark	Nov/97 Nov/00 Aug/01 Aug/04 Aug/08	Apr/98 Apr/01 Oct/01 Jul/05 Apr/09	2	5
Germany	Jan/01 Apr/08	Aug/03 Jan/09	2	2
Finland	Feb/01 May/08	Jul/01 Dec/08	2	2
France	Aug/02 Feb/08 Apr/11	May/03 Feb/09 Nov/12	3	3
Ireland	Apr/99 Nov/00 Aug/03 Aug/06 Jul/07	Oct/99 Nov/01 Jan/04 Jan/07 May/09	3	5
Italy	Aug/07 Apr/11	Mar/09 Oct/14	3	2
Norway	Aug/02 May/08 Mar/09	Apr/03 Oct/08 Jul/09	2	3
Portugal	Aug/08	Jan/09	1	1
Spain	- Feb/08	Dec/93 Jul/13	3	1
Sweden	Apr/08	Mar/09	4	1
Switzerland	- Mar/01 May/08	Sep/96 Mar/03 May/09	1	2
Netherlands	Jan/97 Nov/99 Jul/08	Sep/97 Dec/01 Apr/09	1	3
Australia	May/96 Aug/00 Aug/08	Oct/96 Jan/01 Jan/09	3	3
Hong Kong	NA	NA	2	NA
Peru	NA	NA	2	NA
South Africa	Apr/97 Apr/08	Nov/98 Apr/09	1	2
Turkey	NA	NA	3	NA
U. Arab Emi.	NA	NA	1	NA
China	-	-	1	0
Indonesia	NA	NA	3	NA
Korea	Aug/97 Dec/02 Jul/08	Jul/98 Sep/03 Dec/08	4	3
Malaysia	NA	NA	3	NA

Source: ECRI
Source: Fushing et al. (2010)

Table 13 – Application of the Binomial test for evidence of dependence between DDBC and economic crisis

m months	p	n markets	$t_{obs}(m)$	P-Value
12	0,1943	19	14	0,0000
24	0,3516	19	18	0,0000
12	0,5000	19	14	0,0096
24	0,5000	19	18	0,0000