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MASTER

ACTUARIAL SCIENCE

MASTERS FINAL WORK

INTERNSHIP REPORT

WORKERS' COMPENSATION BEST ESTIMATE

JOÃO PEDRO PINHEIRO DE CARVALHO

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Abstract

This work presents an analysis to the Workers' Compensation best estimate under the Solvency II regime that came into force in January 1st 2016, modelling the liabilities based on the applicable legislation, mainly the Law 98/2009.

Within the scope of Solvency II, the best estimate of non-life liabilities are calculated separately under claims provision (concerning claims that have already happened) and premium provision (concerning future claims that are covered by the existing contractual obligations). The best estimate of life liabilities should be calculated separately for each policy.

Workers' Compensation presents the particularity of being composed of different natured liabilities, which provides in its modeling the application of life and non-life actuarial methodologies. Under Solvency II, these liabilities are split into two lines of business: Workers' Compensation insurance using non similar to life techniques (NSLT) and annuities stemming from non-life insurance contracts and relating to health insurance obligations using similar to life techniques (SLT).

The approach to this report was conducted separately considering the breakdown of the best estimate under Solvency II and the Workers' Compensation liabilities divided into non similar and similar to life techniques.

Due to the diversification of existing literature, this work has been developed focusing on the methodologies that are most frequently applied in the insurance market.

Keywords: Solvency II, Workers' Compensation, best estimate, claims provision, premium provision.

Sumário

O presente trabalho apresenta uma análise às melhores estimativas de acidentes de trabalho sob o regime de Solvência II que entrou em vigor a 1 de janeiro de 2016, apresentando uma modelização das responsabilidades com base na legislação existente, principalmente a Lei n.º 98/2009.

No âmbito de Solvência II, as melhores estimativas das responsabilidades de seguros não vida são calculadas separadamente em provisão para sinistros (respeitantes a sinistros que já ocorreram) e provisão para prémios (relativamente a sinistros futuros que são cobertos pelas responsabilidades abrangidas pelos limites dos contratos existentes). No que diz respeito a seguros vida, as melhores estimativas devem ser calculadas separadamente para cada apólice.

As responsabilidades de acidentes de trabalho apresentam a particularidade de serem compostas por diferentes naturezas, o que proporciona na sua modelização a aplicação de metodologias atuariais não-vida e vida. Em Solvência II, estas responsabilidades são divididas em duas classes de negócio: acidentes de trabalho utilizando bases técnicas não semelhante a técnicas de vida (NSTV) e rendas decorrentes de contratos de seguro de natureza não vida e relacionados com responsabilidades de seguro de acidentes e doença utilizando bases técnicas semelhantes a técnicas de vida (STV).

A abordagem ao tema foi realizada de forma separada tendo em consideração a desagregação da melhor estimativa em Solvência II, e nas diferentes responsabilidades de acidentes de trabalho: não semelhantes e semelhantes a técnicas de vida.

Devido à literatura existente para provisionamento ser bastante diversificada, o trabalho foi desenvolvido com foco nas metodologias que mais frequentemente são aplicadas no mercado segurador.

Palavras-chaves: Solvência II, seguro acidentes de trabalho, melhor estimativa, provisão para sinistros, provisão para prémios.

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Last but not the least, I would like to express my sincere gratitude to my family and friends, for all the support and encouragement throughout my master final work and my life in general.

Contents

List of Abbreviations

ALAE – Allocated Loss Adjustment Expenses

APA – Average Pensioner Age

ASF – *Autoridade de Supervisão de Seguros e Fundos de Pensões* (Portuguese Insurance and Pension Funds Supervisory authority)

BE – Best Estimate

CR – Combined Ratio

EALAV – Expected Annual Lifetime Assistance Value

EAPV – Expected Annual Pension Value

EGP – Earned Gross Premium

EIOPA – European Insurance and Occupational Pensions Authority

EPLA – Expected Proportion Lifetime Assistance

EPP – Expected Proportion Pensions

FAT – *Fundo de Acidentes de Trabalho* (Workers' Compensation Fund)

FDKW – Fully Disabled for Any Kind of Work

FDUW – Fully Disabled for the Usual Work

IBNER – Incurred But Not Enough Reported

IBNR – Incurred But Not Reported

LA – Lifetime Assistance

LoB – Line of Business

LR – Loss Ratio

NSLT – Non-Similar Life Techniques

OLR – Opening Loss Ratio

PDW – Partially Disabled for Work

PPV – Pensioner Present Value

PVFP – Present Value of Future Premiums

PVFPen^{I} – Present Value of Future Pensions with the possibility to be redeemable

PVFPen^{II} – Present Value of Future Pensions that cannot be redeemable

RPV – Redemption Present Value

SLT – Similar Life Techniques

UDD – Uniform Distribution of Deaths

ULAE – Unallocated Loss Adjustment Expenses

UP – Unexpired Premium

VA – Volatility Adjustment

VM – Volume Measure

WC – Workers' Compensation

List of Tables

List of Figures

Introduction

This report is the result of an internship that I carried out from February 2016 to June 2016 at EY Portugal. As a member of the actuarial team, I had the opportunity to apply what I have learned in my master's degree in real projects, working alongside experienced actuaries and facing the diversified challenges and dynamics of a global organization.

During my internship program, I was integrated in several tasks concerning audit assignments and actuarial valuation of the technical provisions for several insurers under Solvency II. Initially, I started to take knowledge of the particularities specific at each line of business and the Workers' Compensation line of business attracted my attention in particular. This was due to three main reasons:

I. Its diversified and complex risk exposures;
II. It is a mandatory policy that is highly regulated;
III. It is composed of life and non-life techniques.

With this in mind, I consider that an analysis of the Workers' Compensation line of business represents a challenge and the possibility to explore a theme that gives me an overview of both life and non-life actuarial methodologies.

Therefore, this report presents a practical analysis of the Workers' Compensation best estimate under Solvency II and its particularities strongly regulated and specified at the Law 98/2009.

Firstly, an overview of the Workers' Compensation liabilities and the best estimate under Solvency II is presented. Afterwards, the Workers' Compensation best estimate under Solvency II is analysed and divided into three categories:

I. BE for WC NSLT claims provision;
II. BE for WC SLT;
III. BE for premium provision.

In the last chapter, it is presented a practical application of the analysed methodologies, concluding with an impact analysis to the discount effect under Solvency II.

1. Overview

1.1. Workers' Compensation

Beginning in the late 1800s, many countries in Europe adopted laws to protect the employees from work-related accidents. Under Chancellor Otto von Bismark's command, Germany was the pioneer country and became a model for the industrialized world. An example is the "Workman's Compensation Act." model adapted in 1897 by the UK. Since 1913, employers in Portugal have the legal obligation to take over work-related accidents costs.

Nowadays, Workers' Compensation (WC) program provides coverage for two types: work-related injuries and occupational illnesses (contracted as a natural incident of a repeated exposure that the worker is subject to). This program differs substantially from voluntary to mandatory systems and in most European countries (the exceptions are Belgium, Denmark, Finland, Norway and Portugal) these liabilities are managed by their national social security. In Portugal, WC program is mandatory for the employers however its liabilities are covered by nature distinct entities: the work-related injuries are insured by undertakings while the occupational illnesses are managed by the national social security. It is strongly regulated (Law 98/2009) to safeguard the beneficiaries, with a fully detailed specification of all benefits and liabilities.

WC is designed to cover medical expenses for workers injured in their professional activities (or while travelling from/to home) and to compensate them for lost wages and/or their dependents if they die in work-related accidents. These benefits come in two types, in cash or in kind, and can result on a temporary or permanent liability for the undertaking.

Under Solvency II, WC is composed of workers' compensation insurance LoB, using non similar to life techniques (NSLT), and annuities stemming from non-life insurance contracts and relating to health insurance obligations LoB, using similar to life techniques (SLT). WC NSLT liabilities are mainly temporary medical and pharmaceutical expenses, nurse care and temporary compensation of the salary. Concerning WC SLT liabilities, disability pensions (classified into three categories based on the degree of disability) and pensions for dependants (spouse, ascendants and descendants) on death of the worker represent the most important benefits in cash. Lifetime assistance, such as permanent medical assistance or prostheses replacement/maintenance over the lifetime of the injured worker, constitute the SLT liabilities in kind.

Despite having liabilities similar to life insurance, WC is present in the Portuguese market as a non-life business, representing 15,4% of the total with an impressive 107,3% loss ratio in 2016.

WC insurance is one of the most interesting lines of business to analyse due to its liabilities' diversity and the high loss ratio currently in the Portuguese market. These levels of loss ratios are higher than normal and stem from a situation of technical imbalance that has persisted since the financial crisis, driven by the contraction of demand (due to the reduction of national economic activity) and strong competition between insurance companies (aggressive pricing practices in order to avoid losing market share).

1.2. Best estimate under Solvency II

Solvency II's best estimate represents the expected present value of all the future cash flows related to the past, present and future exposure of the existing contractual obligations. These cash flows are composed of all claims payments, allocated expenses to the claims, unallocated expenses and the expected future premiums related to policies in force. The best estimate should be obtained by taking into account the uncertainty and the variability of future cash flows, calculated without any prudence margin, using realistic assumptions and applying relevant actuarial methods complemented with an adequate level of expert judgment.

Regarding the appropriateness of the data, the actuary should apply necessary adjustments to the historical data in order to increase the credibility and the quality of the projections.

The time value of money should be taken into account by applying the risk free interest rates that EIOPA prescribes. The discount factor is no longer a constant value over time and it should also be applied to the WC NSLT claims provision.

The best estimate for non-life insurance obligations is composed of the sum of the best estimate for claims provision and the best estimate for premium provision:

I. The best estimate for claims provision is composed of the reserve related to incurred claims, including the IBNR and IBNER. This is the expected future cash flows related to the payment of claims that have already occurred.

II. The best estimate for premium provision is the expected present value of the cash flows related to future claims events of the existing contracts and, subject to some conditions (related to the definition of the contract boundary, i.e. where the undertaking has the unilateral right to cancel the contract or to reject/change the premium), renewals of existing contracts may also be included in the projection. This provision should take into account the expected profits in the future premiums and, consequently, the best estimate for premium provision might be negative.

The best estimate for life insurance obligations should be calculated separately for each policy, projecting the cash flows separately. Whenever the calculation policy by policy represents an undue burden on the insurer, projections can be made by grouping policies only if there are homogeneous risk groups, with no significant differences in the nature, and insurer obtains approximately the same results for the best estimate.

Moreover, the best estimate is calculated gross of reinsurance. The reinsurance best estimate is calculated separately under the same principles applied to the gross best estimate and it is included on the assets side of the balance sheet after having taken into account the counterparty default adjustment (expected losses due to the default of the counterparty).

The best estimate should also include provisions for loss adjustment expenses (LAE). Typically, LAE are split into two parts: allocated loss adjustment expenses (ALAE) and unallocated adjustment expenses (ULAE). ALAE are those that are directly attributed to a specific claim (such as costs to investigate the veracity of claims, lawyers' fees, among others) and therefore are included in the claims figures as a part of claims payments. ULAE are those costs which the undertaking occurs to the settlement of the claims and that are not possible to attribute directly to each claim (such as salaries at the claims handling department, IT costs, property costs, among others). Therefore, ULAE should be estimated separately.

2. Best estimate for WC NSLT claims provision

Workers' Compensation NSLT liabilities are mainly composed of medical expenses and temporary compensation of the salary. In this chapter, it is presented and explained the Thomas Mack's model, one of the most recognised methodologies to estimate the claims provision in respect to liabilities non similar to life techniques. There has been quite a number of literatures about this method therefore we will provide only a brief description of it. A more detailed analysis of this methodology and its respective proofs can be found in Mack's papers mentioned in our references.

Mack's model appears for the first time in 1993 with the main goal of measuring the variability of the Chain-Ladder estimation. A confidence interval of the estimation is of great importance for the actuary to understand the existing error under the projection made. Chain-Ladder is an intuitive methodology that does not have a probability distribution for the data, calculated using the average of the individual development factors between *j-1* and *j*, with *j=0,1,…,n*. Therefore, for the development year *j*, the modification rate of the payments settlement is given by,

$$\hat{f}_j = \frac{\sum_{i=0}^{n-j-1} C_{i,j+1}}{\sum_{i=0}^{n-j-1} C_{i,j}}, j = 0,1,\ldots,n-1$$

where $C_{i,j}$ represents the accumulated claims paid amount (including allocated loss adjustment expenses) after *j* years regarding to accidents occurred in year *i*:

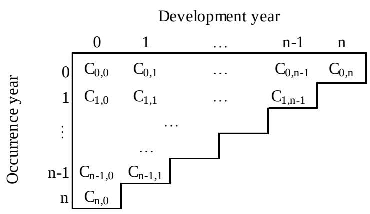

Figure 1 Run off triangle

A tail factor should be implemented by the actuaries when they expect that the development years considered in the run-off triangle end before all the claims are settled. Thomas Mack suggested that the tail factor might be a linear extrapolation of $\ln(\hat{f}_k - 1)$ by straight line $a * k + b, a < 0,$ together with $\hat{f}_{tail} = \prod_{k=n}^{\infty} \hat{f}_k$. Alternative methodologies to estimate the tail factor and its standard error can be found at CAS Tail Factor Working Party, "The estimation of loss development tail factors: A summary report" in *Casualty Actuarial Society E-Forum*, 2013. It is important to mention that the tail factor selected and its errors must be chosen taking into account a personal assessment of the future development factors by the actuary.

Using the development factors presented above and the historical data available, the future expected payments are:

$$\hat{C}_{i,j} = C_{i,n-i} * \prod_{h=n-i}^{j-1} \hat{f}_h \quad, 0 \le i \le n, 0 \le j \le n, i+j > n \text{ and } \hat{C}_{i,tail} = \hat{C}_{i,n} * \hat{f}_{tail} \; if\ a\ tail\ factor\ is\ applied$$

Under Solvency II, claims provision is also discounted. For this reason, the payments must be converted to incremental values with the aim of applying the EIOPA risk free interest rates to obtain the present value of the payments.

Thus, the best estimate for the claims provision is the sum of all expected present value of future cash flows:

$$E[R] = \sum_{i=1}^{n} \sum_{j=n+1-i}^{n} \left(\frac{\hat{C}_{i,j} - \hat{C}_{i,j-1}}{(1 + r_{j-n+i})^{j-n+i-0,5}} \right) + I * \sum_{i=0}^{n} \frac{\hat{C}_{i,tail} - \hat{C}_{i,n}}{(1 + r_{i+1})^{i+0,5}}$$

Where $I = \begin{cases} 1, if\ a\ tail\ factor\ is\ applied \\ 0 \qquad\qquad\qquad\quad , otherwise \end{cases}$

As mentioned before, the Mack's model appears with the main goal of obtaining a confidence interval for the projection presented above. This model is based in the following three assumptions:

1. $E[C_{i,j+1}|C_{i,j}] = f_j * C_{i,j}, i = 0, \dots, n\ and\ j = 0, \dots, n-1$
   And $E[C_{i,tail}|C_{i,n}] = f_{tail} * C_{i,n}, i = 0, \dots, n\ if\ a\ tail\ factor\ is\ applied$
2. $\{C_{i,1}, C_{i,2}, \dots, C_{i,n}, I * C_{i,tail}\}\ and\ \{C_{j,1}, C_{j,2}, \dots, C_{j,n}, I * C_{j,tail}\}\ are\ independents\ when\ i \ne j$
3. $Var[C_{i,j+1}|C_{i,j}] = C_{i,j} * \sigma_j^2\ ,\ i = 0, \dots, n\ and\ j = 0, \dots, n-1$

   where $\sigma_j^2 = \frac{1}{n-j-1} * \sum_{i=0}^{n-j-1} C_{i,j} * \left(\frac{C_{i,j+1}}{C_{i,j}} - \hat{f}_j \right)^2, j = 0,1, \dots, n-2$

   $$\sigma_{n-1}^2 = min\left(\frac{\sigma_{n-2}^4}{\sigma_{n-3}^2}, \min(\sigma_{n-3}^2, \sigma_{n-2}^2) \right)$$

   And $Var[C_{i,tail}|C_{i,n}] = C_{i,n} * \sigma_{tail}^2\ if\ a\ tail\ factor\ is\ applied$

   Where $\sigma_{tail}^2\ might\ be\ an\ approximation\ taking\ into\ account\ that\ if\ \hat{f}_{k-1} \ge \hat{f}_{tail} \ge \hat{f}_k\ therefore\ \sigma_{k-1}^2 \ge \sigma_{tail}^2 \ge \sigma_k^2, 0 \le k \le n-1.$

In order to check if the assumptions above are verified and to decide if Mack's model makes sense to apply, a test for each assumption is proposed:

1. The 1st assumption implies that there is no correlation between the development factors. To verify this condition, it is applied the Spearman test that consists firstly in sort by ascending order, for a fixed year j, the development factors and denote their order number by $r_{i,j}, 1 \leq r_{i,j} \leq n-j$. After that, similarly, it is sorted the precedent development factors ($\frac{C_{i,j}}{C_{i,j-1}}$), where the last value is disregarded, and then denoting by $s_{i,j}, 1 \leq s_{i,j} \leq n-j$ the respective order number.

   The Pearson's coefficient, T_j, is given by

   $$T_j = 1 - 6 * \sum_{i=0}^{n-j-1} \frac{(r_{i,j} - s_{i,j})^2}{((n-j)^3 - n + j)} \quad, 1 \leq j \leq n-2 \text{ and } -1 \leq T_j \leq 1$$

   If there is no correlation between the development factors, then $E[T_j] = 0 \; and \; Var(T_j) = \frac{1}{n-j-1}$.

   In order to apply Mack's model, the aim is to know if the assumption is verified for the triangle as a whole and not for every pair. Thus, the formal test for the overall triangle is given by,

   $$T = \sum_{j=1}^{n-2} \frac{n-j-1}{\frac{(n-1)*(n-2)}{2}} * T_j$$

   Intuitively there is no correlation when $E[T] = 0 \; and \; Var(T) = \frac{1}{\frac{(n-1)*(n-2)}{2}}$.

   As the distribution of each $T_j, n-j \geq 10$ can be approximated to the Normal distribution and because T results from aggregating several uncorrelated T_k's, we can assume that T might be approximated to the Normal distribution. Due to the test be only an approximation and the goal is to detect correlations in substantial parts, a 50% confidence interval instead of the 95% normally applicable is used:

   $$-\frac{0{,}67}{\sqrt{\frac{(n-1)*(n-2)}{2}}} \leq T \leq +\frac{0{,}67}{\sqrt{\frac{(n-1)*(n-2)}{2}}}$$

   When T is not within the interval, the assumption is not verified and might be better to use alternative methods to calculate the best estimate for claims provision.

2. The 2nd assumption represents the independency between different accident years. A closer look to this condition reveals that the development factors $\hat{f}_j, 0 \leq j \leq n-1$ should be unbiased.

   To test whether the development factors are unbiased, a stochastic test is proposed. An occurrence in a certain year affects its diagonal $D_j = \{C_{j,0}, C_{j-1,1}, C_{j-2,2}, \dots, C_{0,j}\}, 0 \leq j \leq n$, and consequently the adjacent development factors $C_j = \left\{\frac{C_{j,1}}{C_{j,0}}, \frac{C_{j-1,2}}{C_{j-1,1}}, \dots, \frac{C_{0,j+1}}{C_{0,j}}\right\}$ and

$C_{j-1} = \left\{\frac{C_{j-1,1}}{C_{j-1,0}}, \frac{C_{j-2,2}}{C_{j-2,1}}, \dots, \frac{C_{0,j}}{C_{0,j-1}}\right\}$. After that, we split the development factors in two groups (smaller and larger) and then check if one of the groups prevails. For this purpose, it is order for every j,$0 \le j \le n-1$, $F_j = \left\{\frac{C_{i,j+1}}{C_{i,j}} | 0 \le i \le n-1-j\right\}$ that contains all development factors between the years j and j+1. Once each F_j is formed, it is subdivided into two types: the larger factors, LF_j, and the smaller factors, SF_j that contains the elements greater/smaller than the median of F_j, respectively. When F_j has an odd number of elements, there is a value equal to the median which is excluded. With the subdivision made, all development factors are associated to the smaller set $S = SF_0 + \dots + SF_{n-2}$, to the larger set $L = LF_0 + \dots + LF_{n-2}$ or to the eliminated set. Intuitively, every not eliminated development factor has a probability of 50% to belong to S or L. Once classified, it should be analysed if there are diagonals where smaller/larger factors prevail. If there is a relation between accident years, it is expected that each diagonal will have approximately the same number of S and L. However, if $Z_j = \min(L_j, S_j)$ is significantly smaller than $\frac{L_j+S_j}{2}$ there is an influence between different accident years and the assumption is not verified. To test this, it is proposed that Z_j follows a probability distribution where each development factor has 50% of probability of belonging to L or S. Each $L_j$and S_j follows a binomial distribution with parameters $\grave{n} = L_j + S_j$ and $p = 0{,}5$. Thus, assuming that $m = \frac{\grave{n}-1}{2}$ denotes the largest integer $\le \frac{\grave{n}-1}{2}$,

$$E[Z_j] = \frac{\grave{n}}{2} - \binom{\grave{n}-1}{m} * \frac{\grave{n}}{2^{\grave{n}}}$$

$$Var(Z_j) = \frac{\grave{n} * (\grave{n}-1)}{4} - \binom{\grave{n}-1}{m} * \frac{\grave{n} * (\grave{n}-1)}{2^{\grave{n}}} + E[Z_j] - E^2[Z_j]$$

Our goal is to test the overall $Z = Z_1 + \dots + Z_{n-1}$. As under the null-hypothesis the different $Z_j's$ are uncorrelated[1] and assuming that Z is approximately a Normal distribution, the assumption is not verified with a 95% confidence interval if Z is not contained within the interval

$$E[Z] - 1{,}96 * \sqrt{Var(Z)} \le Z \le E[Z] + 1{,}96 * \sqrt{Var(Z)}$$

3. Interpreting the 3rd assumption, the conditional variance of $C_{i,j+1}$ is directly proportional to $C_{i,j}$ with a constant factor σ_j^2. Therefore, if the assumption is verified, the weighted residual is given by,

$$\left(C_{i,j+1} - C_{i,j} * \hat{f}_j\right)^2 \approx C_{i,j} * \sigma_j^2 \text{ which is equivalent to } \sigma_j \approx \frac{C_{i,j+1} - C_{i,j} * \hat{f}_j}{\sqrt{C_{i,j}}}$$

[1] $E[Z] = E[Z_1] + \dots + E[Z_{n-1}]$ and $Var(Z) = Var(Z_1) + \dots + Var(Z_{n-1})$

Under the assumption, it is expected that there is no relation between the weighted residual and $C_{i,j}$. In order to conclude if the assumption is verified, we should plot all pairs $\left(\frac{C_{i,j+1}-C_{i,j}*\hat{f}_j}{\sqrt{C_{i,j}}}, C_{i,j}\right)$ and observe if they are purely random without a specific trend. When a trend is observable, it is recommendable to use alternative development factors or even not to apply this method.

With the assumptions now explained, it is time to present the variability associated to the estimation. Based in the assumptions above, Thomas Mack has derived that the standard error of $\hat{C}_{i,j}$ is gathered in the following recursive formula with a starting point $\left(s.e.(\hat{C}_{i,n-i})\right)^2 = 0$,

$$\left(s.e.(\hat{C}_{i,j+1})\right)^2 = (\hat{C}_{i,j})^2 * \left(\left(s.e.(\hat{F}_{i,j})\right)^2 + \left(s.e.\left(\dot{\hat{f}}_j\right)\right)^2\right) + \left(s.e.(\hat{C}_{i,j})\right)^2 * \dot{\hat{f}}_j^{\,2}$$

Where

$$\left(s.e.(\hat{f}_j)\right)^2 = \frac{\hat{\sigma}_j^2}{\sum_{k=0}^{n-j-1} C_{k,j}} \ and \ \left(s.e.(\hat{F}_{i,j})\right)^2 = \frac{\hat{\sigma}_j^2}{\hat{C}_{i,j}}, \qquad 0 \le i \le n \ and \ 0 \le j \le n-1$$

Intuitively the standard error for the occurrence year i, simultaneously equivalent to the standard error of the estimated reserve $\hat{R}_i = \hat{C}_{i,n} - C_{i,n-i}$, is given by

$$\left(s.e.(\hat{C}_{i,n})\right)^2 = (\hat{C}_{i,n})^2 * \sum_{k=n-i}^{n-1} \frac{\left(s.e.(\hat{F}_{i,k})\right)^2 + \left(s.e.\left(\dot{\hat{f}}_k\right)\right)^2}{\hat{f}_k^2}$$

If a tail factor is included the equation is automatically extended to

$$\left(s.e.(\hat{C}_{i,tail})\right)^2 = (\hat{C}_{i,tail})^2 * \left(\left(s.e.(\hat{F}_{i,tail})\right)^2 + \left(s.e.\left(\dot{\hat{f}}_{tail}\right)\right)^2\right) + \left(s.e.(\hat{C}_{i,n})\right)^2 * \dot{\hat{f}}_{tail}^{\,2}$$

Where $\left(s.e.(\hat{F}_{i,tail})\right)^2$ and $\left(s.e.\left(\dot{\hat{f}}_{tail}\right)\right)^2$ might be approximated taking into account that if $\hat{f}_{k-1} \ge \hat{f}_{tail} \ge \hat{f}_k$ therefore $\left(s.e.(\hat{F}_{i,k-1})\right)^2 \ge \left(s.e.(\hat{F}_{i,tail})\right)^2 \ge \left(s.e.(\hat{F}_{i,k})\right)^2$ and $\left(s.e.\left(\dot{\hat{f}}_{k-1}\right)\right)^2 \ge \left(s.e.\left(\dot{\hat{f}}_{tail}\right)\right)^2 \ge \left(s.e.\left(\dot{\hat{f}}_k\right)\right)^2, 0 \le k \le n-1$

Moreover, it is equally important to determine the standard error of the overall ultimate $\sum_{i=0}^{n} \hat{C}_{i,ntail}$ where ntail $= \begin{cases} tail, & \text{if a tail factor is applied} \\ n, & \text{otherwise} \end{cases}$. In this case, we cannot simply sum all $\left(s.e.(\hat{C}_{i,ntail})\right)^2, 0 \le i \le n$, because they are correlated through common factors $\dot{\hat{f}}_j$ and $\hat{\sigma}_j^2$. Therefore, using the same reasoning applied before, the standard error of the overall reserve can be estimated when $j = \begin{cases} n \; if \; ntail = tail \\ n-1 \; if \; ntail = n \end{cases}$, where n+1=tail and $\hat{f}_n^2 = \hat{f}_{tail}^2$ in the following recursive formula with a starting point j=0,

$$\left(s.e.\left(\sum_{i=n-j}^{n} \hat{C}_{i,j+1}\right)\right)^2 = \sum_{i=n-j}^{n}\left(\hat{C}_{i,j}\right)^2 * \left(s.e.\left(\hat{F}_{i,j}\right)\right)^2 + \left(\sum_{i=n-j}^{n} \hat{C}_{i,j}\right)^2 * \left(s.e.\left(\hat{f}_j\right)\right)^2 +$$
$$+\left(s.e.\left(\sum_{i=n-j+1}^{n} \hat{C}_{i,j}\right)\right)^2 * \hat{f}_j^2$$

When the volume of the outstanding claims is large enough, it is possible to construct a confidence interval taking into account the central limit theorem. The symmetric 95%-confidence interval is given by $]E[R] - 1{,}96 * s.e.\left(\sum_{i=0}^{n} \hat{C}_{i,ntail}\right); E[R] + 1{,}96 * s.e.\left(\sum_{i=0}^{n} \hat{C}_{i,ntail}\right)[$

The best estimate for claims provision should also include the future loss adjustment expenses (LAE) of incurred claims. As previously mentioned, ALAE are generally included in the run-off triangle (due to be sufficiently homogeneous and objective) and hence they are already estimated. In this regard, to complete the best estimate for claims provision, the only missing part is the provision for ULAE (more information on the type of expenses that compose the ULAE can be found in "Delegated Regulation (EU) 2015/35 of 10 October 2014 supplementing Directive 2009/138/EC of the European Parliament and of the Council on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II)", European Comission, 10 October 2014). Due to our main goal is not to focus on methodologies for ULAE provision, we will not explain them. More information about it can be found in E. Ohlsson, "Unallocated Loss Adjustment Expense Reserving" Mathematical Statistics Stockholm University, 2013 and in N. Rietdorf and A. H. Jessen, "Provisions for Loss Adjustment Expenses" Astin, 2011.

3. Best estimate for WC SLT

As previously mentioned, WC SLT liabilities are composed of pensions and lifetime assistance payments:

a. Pensions are split into disability pensions when the worker suffers an accident restricting his/her ability to work in a permanent way and pensions for dependants on the death of the insured. It is important to refer that some pensions can be redeemable if some conditions are fulfilled.
b. Lifetime assistance is composed mainly by permanent medical assistance payments or prostheses replacement over the lifetime of the injured worker.

Even though it is not included in our analysis, it is worth noting that insurers should also make an annual contribution based on the capital redemption relating to pensions in payment to the Workers' Compensation Fund managed by ASF.

WC SLT liabilities should be calculated case-by-case for the whole portfolio. Due to the diversity and the different exposure to risk, non-redeemable pensions, redeemable pensions and lifetime assistance should be analysed separately as presented below.

3.1. Non-redeemable pensions

Pensions are calculated on an annuity basis, payable monthly and adding two extra allowances: for holidays in July and for Christmas in November.

According to Portuguese law, the pension value can be revised one-time per civil year without time limit and can be requested by the policyholder or the undertaking. If the workers' incapacity has changed through the year, the pension value is adapted in accordance with the current incapacity.

The disability pension goes through three stages: provisional, defined and definitive. In the 1st stage there is not yet an agreement about the degree of incapacity, in the 2nd stage this degree is already determined and in the final step the pension value is completely defined from a legal point of view. Therefore, when the disability pension is in the 1st stage, its provision includes a correction rate calculated taking into account the past experience.

Moreover, for the pensions' calculation purposes, we have the following categories:

- **Fully disabled to perform any kind of work**

  Lifelong pension of 80% worker's wage plus 10% for each dependant limited to worker's wage.

- **Fully disabled for the usual work**

  Lifelong pension from 50% to 70% of the wage, depending on the workers' capability to execute other type of job.

- **Partial permanent incapacity**

  Lifelong pension equals to 70% of the reduction in the wage.

- **Pensions for the worker's dependants in case of death**

  The main beneficiaries of these pensions are spouses or equivalent, descendants and ascendants. A detailed description of which people are recognized in each category is found in Law 98/2009.

  The main pensions[2] paid in case of death are the following (all pensions are calculated proportionally to the worker's annual wage):

  - **Spouse or equivalent:** 30% until the normal retirement age and after that increases to 40%.
  - **Descendants:** 20% if there is one descendant, 40% if there are two descendants and 50% if there are three or more descendants. These pensions are paid until they complete 18 years old. The payment period must be extended until they complete 22 years old if they are at least at the secondary school level or 25 years if they are at the university level. All pensions should be adjusted if the number of descendants' beneficiaries changes. If a descendant suffers from any significant (75% or more) permanent disability

[2] It was excluded the case when the undertaking has to continue to pay a maintenance allowance for an ex-spouse when she/he had already been receiving before the worker's death. This was not considered because it is a rare occurrence with a residual impact.

to work or has a significant chronic disease, he/she receives the pension for the whole life. All of these pensions can double (with a max value of 80% of the annual worker's wage) if the other parent also dies.

- **Ascendants:** 10% for each ascendant until a maximum of 30%. If there are no others beneficiaries, they receive 15% each until retirement age or until a chronic disease appears and 20% after that.
- **FAT:** If the worker who died has no dependants, the undertaking should revert to the FAT three times his/her annual wage.

The sum of all retributions above cannot exceed 80% of the annual wage of the dead worker. If this happens, the retributions should be proportionally revised to not exceed this limit.

Therefore, below we present the best estimate for each process and methodologies to calculate the best estimate for each type of pension when it is not redeemable (explained posteriorly):

$$PPV = \begin{cases} 3 * w & if\ a = 0 \wedge d = 0 \\ a * (1 + cor) * (PPV_{worker}) + (1 - a) * (PPV_s + PPV_{desc} + PPV_{asc}) & otherwise \end{cases}$$

Where,

$$w = 12 * monthly\ wage\ of\ the\ worker\ when\ the\ accident\ happens * (1 + expense\ rate)$$

$$a = \begin{cases} 0\ if\ the\ worker\ died \\ 1\ if\ the\ worker\ is\ alive \end{cases}$$

$$d = number\ of\ dependants$$

$$cor\ is\ the\ correction\ rate.\ If\ the\ pension\ is\ defined \Rightarrow cor = 0$$

$$PPV_{worker} = Pension\ Present\ Value\ when\ the\ worker\ still\ alive$$

$$PPV_s = Pension\ Present\ Value\ of\ the\ spouse$$

$$PPV_{desc} = Pension\ Present\ Value\ for\ descendants$$

$$PPV_{asc} = Pension\ Present\ for\ ascendants$$

3.1.A. Worker is alive

$$PPV_{worker} = \begin{cases} \min(0{,}8+0{,}1*d;1)*w*\ddot{as}_x^{(12)} & if\ worker\ is\ fully\ disabled\ for\ any\ kind\ of\ work\ (fdkw) \\ m*w*\ddot{as}_x^{(12)} & ,if\ worker\ is\ fully\ disabled\ for\ the\ usual\ work\ (fduw) \\ 0{,}7*(w-w_{after\ accident})*\ddot{as}_x^{(12)} & ,if\ worker\ is\ partially\ disabled\ for\ the\ work\ (pdw) \end{cases}$$

Where,

$m \in [0{,}5; 0{,}7]$ *depending on the capacity to do another job*

$$\ddot{as}_x^{(12)} = \ddot{a}_x^{(12)} + \frac{\sum_{k=0}^{\infty_x} s_{k,x}}{12}$$

$$\ddot{a}_x^{(12)} \overset{UDD^3}{\approx} \frac{13}{24} + \sum_{k=1}^{\infty_x} \left(\frac{1+rev}{1+r_k}\right)^k * \frac{l_{x+k}}{l_x}$$

$$s_{k,y} = \left[v_{k+1}^{0{,}5+k} *_{0{,}5+k} p_y + v_{k+1}^{\frac{11}{12}+k} *_{\frac{11}{12}+k} p_y\right] * (1+rev)^{k+1}$$

$$_{e+k}p_x =_k p_x *_e p_{x+k} \overset{CFM^4}{\approx} {}_k p_x * (p_{x+k})^e \ , 0 \le e < 1$$

$$v_k = \frac{1}{(1+r_k)}, r_k\ from\ EIOPA\ risk\ free\ rate\ interest\ term\ structures$$

$$\infty_x = \max age\ in\ the\ mortality\ table - x$$

$$rev = revision\ rate$$

3.1.B. Pension for the spouse if the worker dies

Assumption: the spouse will not remarry (when this happens a lump sum retribution of three times the annual wage is paid and the pension is then cancelled)

$$PPV_s = \begin{cases} 0{,}3*w*\ddot{as}_y^{(12)} + 0{,}1*w*\frac{l_{RA}}{l_y}*v_{RA-y}^{RA-y}*\ddot{as}_{RA}^{(12)} & if\ spouse\ has\ not\ retired \\ 0{,}4*w*\ddot{as}_y^{(12)} & if\ spouse\ has\ already\ retired \\ 0 & if\ worker\ does\ not\ have\ spouse \end{cases}$$

Where,

$y = spouse's\ age$

$RA = normal\ retirement\ age$

[3] Under UDD approach: $\ddot{a}_x^{(m)} \approx \ddot{a}_x - \frac{m-1}{2m} = \sum_{k=0}^{\infty_x}(v^k *_k p_x) - \frac{m-1}{2m} = 1 - \frac{m-1}{2m} + \sum_{k=1}^{\infty_x}(v^k *_k p_x)$

[4] Constant Force of Mortality Assumption: for x integer and $0 \le e < 1$, $_e p_x = (p_x)^e$

3.1.C. Pensions for the descendants if the worker dies

Assumption: the descendant will not become significantly disabled to work and/or suffers from a chronic disease if he/she was healthy when he/she started receiving the pension, and will not become an orphan as well. It is also assumed that the descendants when entry to the secondary/university level, will not leave until they are 22/25 years old respectively. Moreover, when there are more than three descendants, the overall pension is allocated among the three youngest ones[5]: 20% for each of the two youngest descendants and the remaining 10% for the third youngest descendant (of course when both parents have died, the pension is only split for the two youngest descendants with 40% for each, due to the maximum limit retribution of 80%). This approach does not consider the case that an older descendant can remain longer than a young one if he/she studies at university level, and the younger does not. However, due to a high number of possible situations, modelling with no approaches could result in a very complex model and very difficult to apply in practice. Regarding this, the proposed methodology presents an adequate balance between the degree of complexity and the significance of all possible situations;

$$PPV_{desc}(c) = \begin{cases} 0 & ,c=0 \\ 0{,}4 * w * \frac{1}{h} * \left[\begin{array}{c} I_{c^d} * \sum_{i=1}^{\min(c^d,2)} \ddot{as}_{z_{(i)}}^{(12)} + \\ I_{c^{nd}} * \sum_{i=1}^{\min(c^{nd},2)} \left(d_{i1} * \ddot{as}_{z_{(i)}:\overline{18-z_{(i)}|}}^{(12)} + d_{i2} * \ddot{as}_{z_{(i)}:\overline{22-z_{(i)}|}}^{(12)} + d_{i3} * \ddot{as}_{z_{(i)}:\overline{25-z_{(i)}|}}^{(12)} \right) \end{array} \right] & ,c=1,2 \\ PPV_{desc}(2) + 0{,}1 * (h-1) * w * \left[\begin{array}{c} D_{z_{(3)}} * \ddot{as}_{z_{(3)}}^{(12)} + \\ \left(1 - D_{z_{(3)}}\right) \left(d_{31} * \ddot{as}_{z_{(3)}:\overline{18-z_{(3)}|}}^{(12)} + d_{32} * \ddot{as}_{z_{(3)}:\overline{22-z_{(3)}|}}^{(12)} + d_{33} * \ddot{as}_{z_{(3)}:\overline{25-z_{(3)}|}}^{(12)} \right) \end{array} \right] & ,c \geq 3 \end{cases}$$

Where,

$$h = \begin{cases} 1, \mathit{if\ they\ are\ orphans} \\ 2, \mathit{if\ they\ are\ not\ orphans} \end{cases}$$

$$I_x = \begin{cases} 0, \mathit{if}\ x = 0 \\ 1, \mathit{otherwise} \end{cases}$$

[5] When there are three or more descendants, it is used to split the 50% retribution uniformly for all descendants, provisioning independently for each descendant. However, this cannot be a reasonable approach when there is a big difference between the beneficiaries' age. For example, considering there are three descendants with 2, 4 and 24 years old: In this case, it would be provisioning 16,6(6)% with maturities of 23, 21 and 1 year respectively (assuming that all beneficiaries go to the university). However, if everyone survives until the maturity (it is the expected due to the high survival rates for early ages), this provision is underestimated because after the 1st year and for the following 20 years, the retribution will be 20% for each of the two youngest descendants and not 16,6(6)% due to the obligation of readjusting the retribution when the number of dependants changes (art. 64 Decree-Law 98/2009). Thus, in this particular case, the annual retribution is underestimated after the 1st year.

$$c^{nd} = number\ of\ descendants\ not\ disabled$$

$$c^{d} = number\ of\ descendants\ disabled$$

$$c = c^{d} + c^{nd} = number\ of\ descendants$$

$$z_{(i)} = age\ of\ the\ descendent\ i, in\ ascending\ order\ and\ firstly\ the\ disabled$$

$$d_{i1} = \begin{cases} 1\ ,\ \ z_{(i)} < 18 \\ 0,\ \ otherwise \end{cases}$$

$$d_{i2} = \begin{cases} pes * \frac{l_{18}}{l_{z_i}} * v_{18-z_i}^{18-z_i} * \ddot{as}_{18:\overline{4}|}^{(12)} * \frac{1}{\ddot{as}_{z(i):\overline{22-z(i)}|}^{(12)}}, z_i < 18 \\ 1\quad , 18 \leq z_i < 22 \\ 0\ \ , otherwise \end{cases}$$

$$d_{i3} = \begin{cases} puniv * \frac{l_{22}}{l_{z_i}} * v_{22-z_i}^{22-z_i} * \ddot{as}_{22:\overline{3}|}^{(12)} * \frac{1}{\ddot{as}_{z(i):\overline{25-z(i)}|}^{(12)}}, z_i < 22 \\ 1\quad , 22 \leq z_i < 25 \\ 0\ \ , otherwise \end{cases}$$

$$pes = probability\ of\ still\ studying\ with\ 18\ years\ old\ at\ secondary\ or\ higher\ level$$

$$puniv = probability\ of\ still\ studying\ with\ 22\ years\ at\ university$$

$$D_{z_{(3)}} = \begin{cases} 0, if\ the\ descendant\ with\ age\ z_{(3)} is\ not\ disabled \\ 1,\qquad otherwise \end{cases}$$

$$\ddot{as}_{x:\overline{n-x}|}^{(12)} = \ddot{a}_{x:\overline{n-x}|}^{(12)} + \frac{\sum_{k=0}^{n-x-1} s_{k,x}}{12}$$

$$\ddot{a}_{x:\overline{n-x}|}^{(12)} \overset{UDD^{6}}{\approx} \frac{13}{24} + \frac{11}{24} * v_{n-x}^{n-x} *_{n-x} p_x + \sum_{k=1}^{n-x-1} v_k^k *_k p_x$$

[6] Applying UDD approach: $\ddot{as}_{x:\overline{n}|}^{(12)} \overset{UDD}{\approx} \ddot{a}_{x:\overline{n}|} - \frac{11}{24} * \left(1 - v^n *_n p_x\right) = 1 + \sum_{k=1}^{n-1}\left(v^k *_k p_x\right) - \frac{11}{24} + \frac{11}{24} * v^n *_n p_x = \frac{13}{24} + \frac{11}{24} * v^n *_n p_x + \sum_{k=1}^{n-1} v^k *_k p_x$

3.1.D. Pensions for the ascendants if the worker dies

Assumption: When there are no other beneficiaries they will receive 20% each.

$$PPV_{asc} = \begin{cases} 0 & ,p = 0 \\ \sum_{i=1}^{min(3,p)} 0{,}1 * w * \ddot{as}_{p_{(i)}}^{(12)} & ,d - p \neq 0 \\ \sum_{i=1}^{p} min\left(0{,}2; \frac{0{,}8}{p}\right) * w * \ddot{as}_{p_{(i)}}^{(12)} & ,d - p = 0 \end{cases}$$

Where,

$p = number\ of\ ascendants$

$p_{(i)} = age\ of\ the\ ascendent\ i, in\ ascending\ order$

3.2. Redeemable pensions

However, some pensions can be redeemable. A pension is mandatorily redeemable, paid as a lump sum, to workers that reduced their capability up to 30% (that may have been quantified in the medical analysis immediately after the accident or changed after a review) and to beneficiaries receiving a lifespan pension when, in both cases, the annual pension value does not exceed six times the guaranteed monthly wage[7] on the day after the worker leaves the hospital or on the day that the worker dies. If the pensioner or the beneficiary requests, all lifespan pensions can also be partially redeemable when the remaining pension is greater than six times the guaranteed monthly wage and the redeemable capital is lower than a redeemable pension in the case of 30% incapacity. The mandatory redeemable pension is easily given by:

$RPV = Annual\ Pension\ value * redemption\ factor$

where the annual pension value is obtained according to the rules mentioned before and the redeemable factor[8] is published by the government in *portaria* no. 11/2000 for all ages and types of pension (Appendix C). In my opinion this redeemable factor should be

[7] Decree-Law 254-A/2015: Guaranteed monthly wage = 505€ from 1/10/2014 until 31/12/2015 and 530€ thereafter.

[8] The chosen redeemable factor must be the corresponding to the integer age that the beneficiary is closer to when the accident happens (for example, if a beneficiary, whose birthday is in January, is 38 years old and the accident happened in October, the chosen factor should be the corresponding to the age of 39).

refreshed because it is calculated using a 5,25% interest rate, unrealistic at present, resulting in an unfair redeemable value for the worker/beneficiary.

3.3. Lifetime assistance

Lifetime assistance represents the unlimited medical benefits that the injured worker requires through whole life due to the accident. There is a wide variety of benefits provided, such as hospitalization, surgeries, installation and replacement of devices, prostheses, periodical medical appointments or medicines.

Due to the different nature, the time interval between two payments may have several patterns (monthly, quarterly, annually, every two years, among others) or simply may not present a regular pattern.

The payment pattern may also vary during the lifespan: increasing/decreasing when the worker's status gets worse/better over the course of time. An illustration of this is given when a severe injury happens: the initial payments are higher due to initial hospitalizations, surgeries and adaption expenses. After the early years, they tend to decrease as the injured worker recovers.

For these reasons, predicting the cost of the lifetime assistance is particularly difficult. The payments uncertainty grows even more due to the medical inflation that should be considered to allow for a more appropriate estimation.

To have a proper estimation of the lifetime assistance reserve, we have followed the structure suggested at R. H. Snader, "*Reserving Long Term Medical Claims*" Casualty Actuarial Society, 1987, p. Volume LXXIV, that the analysis should be split into three stages: claim evaluation, medical evaluation and actuarial evaluation.

The claim evaluation aims to collect all possible accurate and useful information based on the latest medical report and on the amounts and timings of the medical expenses already paid. The claim evaluation should be performed annually.

The medical evaluation might be carried out by an expert in life underwriting or by someone able to express medical information in life expectancies, taking into account the medical report gathered in the 1st stage. This is particularly important because a worker who has suffered a very serious accident, resulting in deep injuries for life (paraplegic, quadriplegics, brain steam injuries, …), has a lower life expectancy when compared to the standard population reflected in the standard mortality table. When this happens, there is a high annual medical cost associated to the policyholder in which applying the standard mortality table will result in an extremely high reserve that could not reflect the reality. Therefore, the medical evaluation results in a revised mortality table, applying a multiplication factor f such that ${}_t\grave{p}_x = (1-f)^t * \frac{l_{x+t}}{l_x},\ 0 < f < 1,\ t = 1,2,\ldots$

Finally, taking into account all the relevant information of the previous steps, the EIOPA risk free interest rate term structures and considering the medical costs inflation, an actuarial evaluation for each beneficiary is given by

$$LA_x = \sum_{t=1}^{\infty_x} M_{x,t} * \left(\frac{1+med}{1+r_t}\right)^t * (1 + expense\ rate) * {}_t\grave{p}_x$$

where,

med *is the inflation rate for the medical costs*

$M_{x,t}$ *medical payment due to the worker aged* x *at end of the year* t

However, in practice, it is not often possible to have a detailed description of the medical payments due to the lack of historical data or due to the uncertainty of the worker's condition development. To fix this problem, all $M_{x,t}$ over time can take some approaches under an expert judgement. An accurate and duly justified expert judgement conducted by a life underwriting expert is crucial to obtain appropriate and confident estimations. Due to the diversity of life assistance payments, an estimation of future cash flows can be done in some ways, such as:

a) Average of the payments already made if they present a stable pattern, where the first payments are excluded when they resulted from initial payments that are not expected to be made again (such as hospitalization, surgeries, among others);
b) Average of the routine payments, such as medicines and medical appointments needed, adding up payments every *n* years for devices' replacements;
c) Average of the last payments, adding up an increasing/decreasing tendency through the lifespan if it is expected that the workers' health condition will improve/worsen;
d) Using a payment pattern of a similar accident when the accident under analysis is recent and/or there is no consistent medical feedback.

4. Best estimate for premium provision

As discussed before, the best estimate for premium provision includes the present value of the expected future cash flows, in or out, associated to existing contracts (taking into account the boundary of a contract as explained before):

- Future premiums (renewals and fractional premiums);
- Acquisition costs of the future premiums;
- Claim costs from claims occurred after the valuation date;
- Allocated and unallocated claims expenses from future claims events;
- Premiums already written but not yet earned (regarding to future exposure).

EIOPA ("Guidelines on the valuation of technical provisions" – Technical Annex III) suggests the following approach, based on the combined ratio, to calculate the best estimate for premium provision:

$$BE_{premium} = CR * VM + (CR - 1) * PVFP + AER * PVFP$$

where,

$$CR = \frac{Claims + Claims\ related\ expenses}{Earned\ premiums\ gross\ of\ acquisition\ expenses}$$

$$VM = Volume\ measure$$

$$PVFP = Present\ Value\ of\ Future\ Premiums$$

$$AER = Estimate\ of\ LoB\ acquisition\ expenses\ ratio$$

However, this simplification does not represent a robust methodology if the future exposure implies a significant level of complexity and diversity. WC is one of those cases (due to the existence of NSLT and SLT liabilities).

In order to calculate the WC best estimate for premium provision taking into account its diversified exposures, an alternative approach analysing the NSLT and SLT liabilities separately is presented:

$$BE_{premium} = disc.future\ claims_{n+1}^{NSLT} + disc.future\ claims_{n+1}^{SLT} + (ER + AER - 1) * PVFP + ER * VM$$

where,

$$PVFP = \left(\frac{Fractional\ Premiums + Renewal\ Premiums}{(1+r_1)^{0,5}}\right) * (1 - lapse\ rate)$$

$$AER = \frac{1}{n+1}\left(\sum_{i=0}^{n} \frac{Acquisitions\ costs_i}{Gross\ Written\ Premiums_i}\right)$$

$$ER = \frac{1}{n+1}\left(\sum_{i=0}^{n} \frac{Unallocated\ management\ expenses_i + Administrative\ expenses_i + Investment\ management\ expenses_i}{Earned\ Premiums\ gross\ of\ acquisitions_i}\right)$$

r_1 is the risk free interest rate established by EIOPA for the 1st year.

$disc.future\ claims_{n+1}^{NSLT}$ and $disc.future\ claims_{n+1}^{SLT}$ cover the liabilities directly related to claims, the same covered on claims ratio at EIOPA simplification.

Whenever possible it should be tested if the fractional and renewal premiums should consider distinct lapse rates.

In the next topics, intuitive methods for the NSLT and SLT future claims costs are developed.

The considered time horizon must be the boundary time until the conditions of the contract cannot be changed. According to EIOPA (topic 1.8 of "Guidelines on contract boundaries") the boundary of a contract is defined until the undertaking has the unilateral right to terminate or change the conditions of the policy. Combining it with the Portuguese law where the undertakings can change the premiums annually even if there is no change in the risk, the methodologies adopted have 1-year time horizon.

Furthermore, the alternative methods that are suggested and the formula above can be easily implemented to LoB's with no SLT liabilities using $disc.future\ claims_{n+1}^{SLT} = 0$

4.1. Future claims WC NSLT

In order to obtain the future costs directly related to claims, two approaches are suggested: an estimation of the future claims costs based on claims ratio presented at EIOPA simplification and an extension of Mack's model to the future exposure.

4.1.A. Approach based on the claims ratio – EIOPA simplification

In order to obtain the premium provision, it is proposed that the costs of the covered but not incurred claims are given by

$$Future\ claims_{n+1}^{NSLT} = [PVFP + UP] * (LR_{n+1})$$

where

$$LR_{n+1} = \frac{1}{n+1} * \left(\sum_{i=0}^{n} \frac{Ultimate\ Claims\ costs_i}{Earned\ Premiums\ gross\ of\ acquisitions_i} \right)$$

$UP\ (Unexpired\ Premium)\ is\ the\ Volume\ Measure$

The Unexpired Premium is composed of all premiums that the undertaking has already received but not yet earned which concern the exposure in the remaining time period of the contract at the valuation date. For instance, if it signed a 1-year contract with an annual premium of 1.200€ paid in advance at 30st November, the UP at 31st December will be 1.100€, at 31st January will be 1.000€ and so on.

However, if the simple average does not reflect a realistic ratio for the data under analysis (due to the existence of outliers or a tendency), the loss ratio should be calculated using alternative approaches. Some examples of these approaches are:

- Using the information available (last diagonal of reserve and payment triangle) for each year:

  $$LR_{n+1} = \frac{1}{n+1} * \left(\sum_{i=0}^{n} \frac{C_{i,n-i} + R_{i,n-i}}{Earned\ Premiums\ gross\ of\ acquisitions_i} \right)$$

- Taking into account the last years where the loss ratios are stable:

  $$LR_{n+1} = \frac{1}{n-k} * \left(\sum_{i=k}^{n} \frac{Ultimate\ Claims\ costs_i}{Earned\ Premiums\ gross\ of\ acquisitions_i} \right),$$

  $k = oldest\ year\ included$

- Weight average taking into account the tendency

  $$LR_{n+1} = \frac{1}{\sum_{i=1}^{n} i} * \left(\sum_{i=0}^{n} \frac{Ultimate\ Claims\ costs_i}{Earned\ Premiums\ gross\ of\ acquisitions_i} * i \right)$$

To estimate the proportion paid in the development years, the payment pattern estimated for the claims provision at year n is used. Finally, applying the EIOPA risk free interest rate term structures, the discounted future claims NSLT are given by

$$disc.future\ claims_{n+1}^{NSLT} = [UP + PVFP] * LR_{n+1} * \left(\frac{\hat{C}_{n,0}}{(1+r_1)^{0,5} * \hat{C}_{n,n}} + \sum_{i=1}^{n} \frac{\hat{C}_{n,i} - \hat{C}_{n,i-1}}{(1+r_{i+1})^{i+0,5} * \hat{C}_{n,n}}\right)$$

4.1.B. An extension of Mack's Model to the future exposure

In the present topic, an extension of the run-off triangle for the next year is developed in order to estimate the next year liability (yellow row):

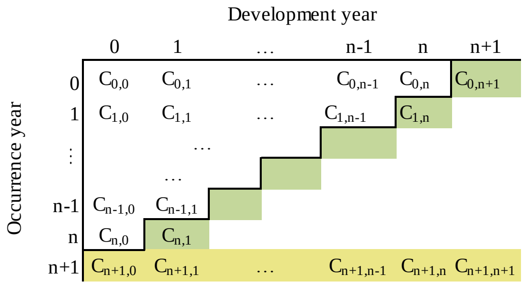

Figure 2 An extension of Mack's model - run off

Moreover, the following assumptions are made:

i) The payments made in *n+1* regarding to the previous accident years (green diagonal) are effectively the values estimated for claims provision

$$E[C_{n+1-i,i}] = C'_{n+1-i,i}\ and\ Var(C_{n+1-i,i}) = 0,\ i = 0, \ldots, n+1;$$

ii) The payments made in *n+1* of accidents occurred in the same year are

$$C_{n+1,0} = [UP + EP * (1 - lapse\ rate)] * (olr)$$

where,

$$EP = \frac{Fractional\ Premiums + Renewals\ Premiums}{(1 + r_1)^{0,5}}$$

$olr\ is\ the\ opening\ loss\ ratio$

As UP and EP are known values and lapse rate and olr are assumed to be independent random variables[9], the expected value and the variance of the $C_{n+1,0}$ are respectively:

$$E[C_{n+1,0}] = [UP + EP * (1 - E[lapse\ rate])] * E[olr]$$

$$\mathrm{Var}(C_{n+1,0}) = \mathrm{EP}^2 * \mathrm{Var(lapse\ rate)} * \mathrm{E}^2[\mathrm{olr}] + \mathrm{Var(olr)} * \left[\mathrm{EP}^2 * \mathrm{Var(lapse\ rate)} + \left(\mathrm{UP} + \mathrm{EP} * (1 - \mathrm{E[lapse\ rate]})\right)^2\right]$$

[9] $E[lapse\ rate] = \frac{1}{n+1}\left(\sum_{i=0}^{n} \frac{Number\ of\ cancelled\ policies_i}{Number\ of\ policies_i}\right)$ and $Var(lapse\ rate) = \frac{1}{n-1}\left(\sum_{i=1}^{n}\left(\frac{Number\ of\ cancelled\ policies_i}{Number\ of\ policies_i} - E[lapse\ rate]\right)^2\right)$

$E[olr] = \frac{1}{n+1}\left(\sum_{i=0}^{n} \frac{C_{i,0}}{PBA_i}\right)$ and $Var(olr) = \frac{1}{n-1}\left(\sum_{i=1}^{n}\left(\frac{C_{i,0}}{PBA_i} - E[olr]\right)^2\right)$

Proof.

$$E[C_{n+1,0}] = E[[UP + EP * (1 - lapse\ rate)] * olr] = [UP + EP * (1 - E[lapse\ rate])] * E[olr]$$

$$\begin{aligned} Var(C_{n+1,0}) &= Var([UP + EP * (1 - lapse\ rate)] * (olr)) \\ &= Var\left((UP + EP - EP * (lapse\ rate)) * (olr)\right) \text{ [10]} \\ &= EP^2 * Var(lapse\ rate) * E^2[olr] + Var(olr) \\ &* \left[EP^2 * Var(lapse\ rate) + \left(UP + EP * (1 - E[lapse\ rate])\right)^2\right] \end{aligned}$$

Therefore, it is possible to derive the development factors and consequently the expected accumulated payments for year n+1:

$$f_k = \frac{\left(\sum_{j=0}^{n-k-1} C_{j,k+1}\right) + C'_{n-k,k+1}}{\left(\sum_{j=0}^{n-k} C_{j,k}\right)}, 0 \leq k \leq n \text{ and } \hat{C}_{n+1,k} = E[C_{n+1,0}] * \prod_{j=0}^{k-1} f_j, 1 \leq k \leq n+1$$

To obtain the NSLT future liabilities discounted, it is necessary to convert the accumulated paid amounts into incremental paid amounts in order to discount applying the EIOPA free interest rate term structures and then sum all discounted cash flows:

$$disc.future\ claims_{n+1}^{NSLT} = \frac{E[C_{n+1,0}]}{(1+r_1)^{0,5}} + \sum_{j=1}^{n+1} \frac{\hat{C}_{n+1,j} - \hat{C}_{n+1,j-1}}{(1+r_{j+1})^{j+0,5}}$$

In 1999, Thomas Mack derived that the standard error of the $\hat{C}_{i,n+1}, i = 1, \dots, n+1$ can be rewritten as

$$\left(s.e.(\hat{C}_{i,n+1})\right)^2 = (\hat{C}_{i,n+1})^2 * \sum_{k=n+1-i}^{n} \frac{\left(s.e.\left(\hat{F}_{i,k}\right)\right)^2 + \left(s.e.\left(\hat{\hat{f}}_k\right)\right)^2}{\hat{f}_k^2}$$

where,

$$\left(s.e.\left(\hat{f}_k\right)\right)^2 = \frac{\hat{\sigma}_k^2}{\sum_{j=0}^{n-k} C_{j,k}} \text{ and } \left(s.e.\left(\hat{F}_{i,k}\right)\right)^2 = \frac{\hat{\sigma}_k^2}{\hat{C}_{i,k}}, \quad 0 \leq i \leq n+1 \text{ and } 0 \leq k \leq n$$

$$\hat{\sigma}_k^2 = \frac{\sum_{i=0}^{n-k} C_{i,k} * \left(\frac{C_{i,k+1}}{C_{i,k}} - \hat{f}_k\right)^2}{n-k}, 0 \leq k \leq n-1$$

$$\hat{\sigma}_n^2 = min\left(\frac{\hat{\sigma}_{n-1}^4}{\hat{\sigma}_{n-2}^2}, min(\hat{\sigma}_{n-2}^2, \hat{\sigma}_{n-1}^2)\right)$$

[10] According to *Goodman, L. (1960) "On the Exact Variance of Products", Journal of the American Stat. Assn., Vol. 5*: $Var(X * Y) = E[X]^2 * Var(Y) + E[Y]^2 * Var(X) + Var(X) * Var(Y)$, X and Y independents. $If\ X = olr, Y = UP + EP - EP * (lapse\ rate)\ where\ E[Y] = UP + EP - EP * E[lapse\ rate]\ and\ Var(y) = EP^2 * Var(lapse\ rate)$, the formula is deduced.

Thomas Mack has also derived the possibility to obtain the standard error for each $\hat{C}_{i,k}, k \leq n+1$, using a recursive process with the starting point $s.e.(C_{i,n+1-i}) = 0$:

$$\left(s.e.(\hat{C}_{i,k+1})\right)^2 = \hat{C}_{i,k}^2 * \left(\left(s.e.(\hat{F}_{i,k})\right)^2 + \left(s.e.(\hat{f}_k)\right)^2\right) + \left(s.e.(\hat{C}_{i,k})\right)^2 * \hat{f}_k^2$$

Based on the above and due to the fact that $\hat{C}_{n+1,0}$ is no longer a constant but an estimated value, it is suggested that the starting value of the recursion process should be equal to the square root of $\hat{C}_{n+1,0}$'s variance previously estimated.

Under this condition, the standard errors of the accumulated paid amounts in year n+1 are the following:

$$\left(s.e.(\hat{C}_{n+1,1})\right)^2 = (\hat{C}_{n+1,0})^2 * \left(\left(s.e.(\hat{F}_{n+1,0})\right)^2 + \left(s.e.(\hat{f}_0)\right)^2\right) + \left(s.e.(\hat{C}_{n+1,0})\right)^2 * (\hat{f}_0)^2$$

$$\left(s.e.(\hat{C}_{n+1,2})\right)^2 = (\hat{C}_{n+1,1})^2 * \left(\left(s.e.(\hat{F}_{n+1,1})\right)^2 + \left(s.e.(\hat{f}_1)\right)^2\right) + \left(s.e.(\hat{C}_{n+1,1})\right)^2 * (\hat{f}_1)^2$$

(…)

$$\left(s.e.(\hat{C}_{n+1,n+1})\right)^2 = (\hat{C}_{n+1,n})^2 * \left(\left(s.e.(\hat{F}_{n+1,n})\right)^2 + \left(s.e.(\hat{f}_n)\right)^2\right) + \left(s.e.(\hat{C}_{n+1,n})\right)^2 * (\hat{f}_n)^2$$

If a tail factor is applied, the recursion can be easily extended:

$$\left(s.e.(\hat{C}_{n+1,tail})\right)^2 = (\hat{C}_{n+1,n+1})^2 * \left(\left(s.e.(\hat{F}_{n+1,tail})\right)^2 + \left(s.e.(\hat{f}_{tail})\right)^2\right) + \left(s.e.(\hat{C}_{n+1,n+1})\right)^2 * (\hat{f}_{tail})^2$$

Following the same reasoning applied to the claims provision, the symmetric 95%-confidence interval is given by

$$]\hat{C}_{n+1,\grave{ntail}} - 1{,}96 * s.e.(\hat{C}_{n+1,\grave{ntail}}); \hat{C}_{n+1,\grave{ntail}} + 1{,}96 * s.e.(\hat{C}_{n+1,\grave{ntail}})[$$

where $\grave{ntail} = \begin{cases} tail, & if\ a\ tail\ factor\ is\ applied \\ n+1, & otherwise \end{cases}$

4.2. Future claims WC SLT

In order to estimate the discounted future claims WC SLT, the analysis will be disaggregated into: Pensions and Lifetime assistance. Therefore,

$$disc.future\ claims_{n+1}^{SLT} = disc.future\ claims_{n+1}^{pensions} + disc.future\ claims_{n+1}^{lifetime\ assist.}$$

4.2.A. Pensions

The expected pensions regarding the future exposure should be analysed separately for each type of pensioner, each one that might be redeemable also split into redeemable and non-redeemable.

The first part of the proposed approach consists in obtaining the annual average of the following variables:

a) Proportion of policies of type j per gross earning premium that resulted in pensions;
b) Pension value for each pensioner/beneficiary;
c) Pensioner age when he/she starts receiving the pension.

The annual average of the expected proportion policies of type j EPP_j might be easily obtained by,

$$EPP_j = \frac{\sum_{i=oldest\ year}^{n} \frac{Number\ of\ pensions\ of\ type\ j_i}{EGP_i}}{n - oldest\ year}$$ where, $EGP_i = Earned\ Gross\ Premium\ at\ year\ i$

Due to the law having been changed in 2009 and the beneficiaries took some time to realise that the revision rules had changed, we proposed that the average annual pension value for each pensioner/beneficiary should be limited to 2010 when there is an older historical data. Therefore, the expected annual pension value, $EAPV$, is given by,

$$EAPV_j = \frac{\sum_{i=max(2010;oldest\ year)}^{n} annual\ average\ value\ of\ pensions\ j_i}{Number\ of\ historical\ years\ considered\ \ where\ the\ number\ of\ pensions\ j > 0}$$

Regarding the average pensioner age, this should be performed using the ages when they had started receiving the pension and not the age that they are in the current year. Consequently, the average pensioner age for each type of pension j apa_j is equivalent to:

$$apa_j = \frac{\sum_{i=oldest\ year}^{n} annual\ average\ of\ pensioners\ j_i\quad start\ age}{Number\ of\ historical\ years\ considered\ \ where\ the\ number\ of\ \ pensions\ j > 0}$$

Moreover, the present value of the future expected pensions with the possibility to be redeemable is given by,

$$PVFPen^{I} = \sum_{j=pdw,s,asc,desc} PVFPen^{I}_{redeemable,j} + PVFPen^{I}_{non-redeemable,j}$$

where,

$$PVFPen^{I}_{k,j} = \begin{cases} EPP_{k,j} * EAPV_{k,j} * \ddot{as}^{(12)}_{apa_j} & , k = non\ red.\ and\ j = pdw, asc\ and\ spouse\ retired \\ EPP_{k,j} * EAPV_{k,j} * \left(\ddot{as}^{(12)}_{apa_j} + 0{,}1 * \frac{l_{RA-apa_j}}{l_{apa_j}} * v^{RA-apa_j}_{RA-apa_j} * \ddot{as}^{(12)}_{RA} \right) & , k = non\ red.\ and\ j = spouse\ not\ retired \\ EPP_{k,j} * EAPV_{k,j} * \left(d_1 * \ddot{as}^{(12)}_{\overline{apa_j:18-apa_j|}} + d_2 * \ddot{as}^{(12)}_{\overline{apa_j:22-apa_j|}} + d_3 * \ddot{as}^{(12)}_{\overline{apa_j:25-apa_j|}} \right) & , k = non\ red.\ and\ j = desc \\ EPP_{k,j} * EAPV_{k,j} * redeemable\ factor_{apa_j} & , k = redeemable\ and\ j = pdw, asc, spouse, desc \end{cases}$$

For the pensions that do not possess the characteristics to be redeemable, their overall estimation is given by $PVFPen^{II} = \sum_{j=fdkw,fduw} EPP_j * EAPV_j * \ddot{as}^{(12)}_j$

Finally, considering the exposure for the following year, the present value of the future costs regarding pensions is given by,

$$disc.\ future\ claims^{pensions}_{n+1} = (PVFP + UP) * (PVFPen^{I} + PVFPen^{II})$$

4.2.B. Lifetime assistance

The approach proposed for the lifetime assistance valuation follows the same reasoning presented above for pensions.

Due to the great diversity of payments regarding lifetime assistance, they are split into three categories taking into account their annual average payment: small, medium and high. We considered that all lifetime assistance policy is small if its annual mean payment is lower than 1.000€. When the annual mean payment is contained in the range [1.000;5.000] then the respective policy should be categorized as medium. Otherwise, it is classified as high.

Thus, it is necessary to compute for the three categories its annual average of:

a) Proportion of policies per gross earning premium that resulted in lifetime assistances;
b) Lifetime assistance payments for each beneficiary;
c) Beneficiary age when he/she starts receiving the lifetime assistance payments.

For all the formulas presented below, it is assumed that it represents the category under analysis (k=small, medium, high). So, the expected proportion of lifetime assistance policies for the category k ($EPLA_k$) can be obtained by,

$$EPLA_k = \frac{\sum_{i=oldest\ year}^{n} \frac{Number\ of\ k\ lifetime\ payments\ in\ year\ i}{EGP_i}}{n - oldest\ year}$$

Concerning the annual payments average for the lifetime assistance, it is not necessary to limit the oldest year to 2010 and then the expected annual lifetime assistance value for the category k ($EALAV_k$) is calculated as,

$$EALAV_k = \frac{\sum_{i=oldest\ year}^{n} annual\ average\ of\ the\ k\ \ payments\ in\ the\ year\ i}{Nr.\ historical\ years\ considered\ where\ the\ number\ k\ lifetime\ payments > 0}$$

Concerning the average beneficiary age, the same explanation presented for the pensions regarding the age when he/she starts receiving the pension is also applied for this case. Therefore, the annual average of the beneficiary age is defined as:

$$\overline{ala}_k = \frac{\sum_{i=oldest\ year}^{n} annual\ average\ of\ the\ k\ lifetime\ assistance\ age}{Number\ of\ historical\ years\ \ where\ the\ number\ of\ k\ lifetime\ payments\ \ > 0}$$

Finally, compiling all that was presented above and introducing the medical inflation rate, the discounted future costs regarding lifetime assistance for the following year n+1 are given by,

$$disc.future\ claims_{n+1}^{lifetime\ assist.} = (PVFP + UP) * \left(\sum_{all\ k} EPLA_k * EALAV_k * \left(\sum_{t=1}^{\infty \overline{ala}_k} \left(\frac{1 + med}{1 + r_t} \right)^t * \ {}_t\grave{p}_x \right) \right)$$

Naturally, taking into account the industrial sector that the worker belongs to, alternative approaches may be particularly interesting due to the variety of risks that each profession is exposed to (a construction worker or a metal worker are clearly more exposed to have a serious accident than an accountant or an actuary). A possible suggestion is to split the lifetime assistance beneficiaries into primary, secondary and tertiary sector. However, this or another alternative detailed analysis might be performed only if the historical data has a significant volume to make possible confident estimations.

5. Practical application

A practical application is done using data, which has been anonymised, of an undertaking as at December 31, 2015. In the following table it is presented the overall best estimate obtained, explained in detail in the following sub chapters.

Table 1 Overall best estimate

BE incurred claims	**130.508.392**
NSLT	12.188.714
SLT	118.319.678
BE premium	**3.066.778**
Best estimate	**133.575.170**

5.1. BE for WC NSLT claims provision

Table 2 Accumulated paid amounts

	0	1	2	3	4	5	6	7	8	9	10
2005	5.563.684	8.687.314	9.077.949	9.146.775	9.221.794	9.307.884	9.270.912	9.221.082	9.252.337	9.257.235	9.258.856
2006	6.232.281	9.606.262	10.067.266	10.170.387	10.210.141	10.139.785	10.174.561	10.206.932	10.260.762	10.262.925	
2007	7.603.605	11.815.368	12.329.483	12.590.308	12.730.287	12.803.471	12.817.489	12.771.756	12.778.655		
2008	9.650.156	14.255.410	14.823.645	15.045.479	15.113.318	15.247.751	15.309.257	15.333.850			
2009	8.785.395	13.380.998	14.084.171	14.195.592	14.036.709	14.039.699	14.003.175				
2010	9.246.178	14.060.008	14.667.922	14.825.230	14.953.874	14.994.543					
2011	10.903.752	16.804.746	17.643.945	17.892.088	17.858.498						
2012	11.849.011	17.878.852	18.507.469	18.698.973							
2013	12.690.937	20.073.964	20.863.105								
2014	13.845.782	21.658.331									
2015	15.402.351										

Table 3 Development factors

0	1	2	3	4	5	6	7	8	9
1,538	1,043	1,012	1,003	1,004	1,001	0,999	1,003	1,000	1,000

As the development factors presented above are very close to 1 after the 5^{th} year and WC liabilities are split into NSLT and SLT, it is not expected to make temporary payments after 10 years and therefore no tail factor is applied.

Table 4 Full triangle projected

	0	1	2	3	4	5	6	7	8	9	10
2005	5.563.684	8.687.314	9.077.949	9.146.775	9.221.794	9.307.884	9.270.912	9.221.082	9.252.337	9.257.235	9.258.856
2006	6.232.281	9.606.262	10.067.266	10.170.387	10.210.141	10.139.785	10.174.561	10.206.932	10.260.762	10.262.925	10.264.722
2007	7.603.605	11.815.368	12.329.483	12.590.308	12.730.287	12.803.471	12.817.489	12.771.756	12.778.655	12.783.280	12.785.518
2008	9.650.156	14.255.410	14.823.645	15.045.479	15.113.318	15.247.751	15.309.257	15.333.850	15.377.653	15.383.219	15.385.913
2009	8.785.395	13.380.998	14.084.171	14.195.592	14.036.709	14.039.699	14.003.175	13.991.813	14.031.783	14.036.861	14.039.319
2010	9.246.178	14.060.008	14.667.922	14.825.230	14.953.874	14.994.543	15.003.510	14.991.337	15.034.162	15.039.603	15.042.236
2011	10.903.752	16.804.746	17.643.945	17.892.088	17.858.498	17.921.022	17.931.739	17.917.190	17.968.373	17.974.876	17.978.024
2012	11.849.011	17.878.852	18.507.469	18.698.973	18.750.520	18.816.167	18.827.420	18.812.144	18.865.883	18.872.711	18.876.016
2013	12.690.937	20.073.964	20.863.105	21.118.821	21.177.039	21.251.181	21.263.891	21.246.637	21.307.332	21.315.043	21.318.775
2014	13.845.782	21.658.331	22.599.878	22.876.881	22.939.945	23.020.260	23.034.027	23.015.337	23.081.084	23.089.438	23.093.481
2015	15.402.351	23.689.294	24.719.132	25.022.110	25.091.089	25.178.934	25.193.992	25.173.550	25.245.462	25.254.599	25.259.021

Table 5 Future payments discounted

	0	1	2	3	4	5	6	7	8	9	10	Overall
2005												0
2006											1.798	1.798
2007										4.628	2.243	6.871
2008									43.838	5.576	2.696	52.110
2009								-11.371	40.047	5.083	2.450	36.209
2010							8.975	-12.197	42.866	5.423	2.606	47.672
2011						62.573	10.738	-14.563	51.012	6.436	3.082	119.277
2012					51.588	65.774	11.264	-15.225	53.182	6.687	3.194	176.463
2013				255.916	58.331	74.213	12.667	-17.074	59.438	7.453	3.551	454.494
2014			942.286	277.540	63.125	80.045	13.624	-18.303	63.543	7.947	3.778	1.433.585
2015		8.293.456	1.031.834	303.266	68.747	86.934	14.747	-19.757	68.415	8.538	4.053	9.860.233
												12.188.714

Table 6 Standard errors of the BE for WC NSLT claims provision

s.e.($C_{i,j}$)	0	1	2	3	4	5	6	7	8	9	10
2005											
2006											17
2007										3.232	3.233
2008									38.474	38.760	38.774
2009								57.234	67.698	67.979	68.004
2010							53.002	79.871	88.350	88.677	88.709
2011						95.786	112.806	131.160	137.836	138.291	138.341
2012					135.009	167.490	178.748	191.672	196.539	197.147	197.219
2013				78.695	165.452	196.950	208.250	221.316	226.291	226.986	227.068
2014			109.624	141.002	208.566	236.985	247.630	259.948	264.658	265.461	265.557
2015		421.332	658.222	692.313	718.943	730.489	736.683	741.833	743.207	745.349	745.619
s.e.($\sum C_{i,j}$)		421.332	463.159	513.734	569.291	663.543	716.085	765.559	806.805	813.559	**813.751**

Table 7 Best estimate for WC NSLT claims provision

	Confidence Interval 95%		
	Lower bound	**Expected**	**Upper bound**
BE for WC NSLT claims provision	10.593.762	**12.188.714**	13.783.667

5.2. BE for WC SLT

5.2.A. BE for WC SLT – Pensions liabilities

WC SLT parameters established as well as their source and assumptions are presented as:

Table 8 Assumptions for WC SLT - Pensions

Parameter	Value	Source	Assumption
rev	0,5% for pensioner 0% for beneficiaries	Benchmark	-
cor	0,60%	Benchmark	-
expense rate	1,50%	Benchmark	-
pes	80,2%	Eurostat (June 2016): *Pupils in school with 18 years old - as % of corresponding 18 years old portuguese population*	All pupils in school with 18 years old are at least at secondary level.
puniv	39,7%	Eurostat (June 2016): *Pupils in school with 22 years old - as % of corresponding 22 years old portuguese population*	All pupils in school with 22 years old are at least at university level.
RA	66	It is the current portuguese retirement age	-
Mortality table	TV88-90	Benchmark	-

As the calculation is done case by case for the whole portfolio which is composed of more than 2.500 pensions, we will present the calculation of the provision for some distinct pensioners that were randomly chosen in each type and then present its overall provision value:

a. Type: fdkw (a=1); Status: not defined and non-redeemable; Age = 53; 12-monthly salary=3.959,11€; d=1:

$$PPV_{worker\ a} = (1{,}006) * PPV_{fdkw} = (1{,}0054) * 0{,}9 * 3.959{,}11 * \ddot{as}_{53}^{(12)} = 162.915€$$

b. Type: fduw (a=1); Status: defined and non-redeemable; Age= 63; 12-monthly salary=6.134,44€; m=0,7

$$PPV_{worker\ b} = PPV_{fduw} = 0{,}7 * 6.134{,}44 * \ddot{as}_{63}^{(12)} = 130.986€$$

c. Type: pdw (a=1); Status: defined and non-redeemable; Age = 60; reduction in the salary because of the accident=297,54€:

$$PPV_{worker\ c} = PPV_{pdw} = 0{,}7 * 297{,}54 * \ddot{as}_{60}^{(12)} = 7.252€$$

d. Type: death (a=0); Status: non-redeemable; Age = 56; 12-monthly salary=4.436€; dependents: 1 spouse (with age 58 and not retired), 2 descendants not orphans illegible to receive pension (with ages 10 and 24):

$$PPV_{worker\ d} = PPV_s + PPV_{desc}$$
$$= \left(0{,}3 * 4.436 * \ddot{as}_{58}^{(12)} + 0{,}1 * 4.436 * \frac{l_{66}}{l_{58}} * v_8^8 * \ddot{as}_{66}^{(12)}\right)$$
$$+ \left(0{,}4 * 4.436 * \frac{1}{2}\right.$$
$$* \left[\left(\ddot{as}_{10:\overline{8}|}^{(12)} + \left(0{,}802 * \frac{l_{18}}{l_{10}} * v_8^8 * \ddot{as}_{18:\overline{4}|}^{(12)} * \frac{1}{\ddot{as}_{10:\overline{12}|}^{(12)}}\right) * \ddot{as}_{10:\overline{12}|}^{(12)}\right.\right.$$
$$\left.\left.\left. + \left(0{,}397 * \frac{l_{22}}{l_{10}} * v_{12}^{12} * \ddot{as}_{22:\overline{3}|}^{(12)} * \frac{1}{\ddot{as}_{10:\overline{15}|}^{(12)}}\right) * \ddot{as}_{10:\overline{15}|}^{(12)}\right) + \left(\ddot{as}_{24:\overline{1}|}^{(12)}\right)\right]\right) = 55.115€$$

e. Type: pdw; Status: Redeemable; Age: 36; Annual pension value=550,59€;

$$PPV_{worker\ e} = 550{,}59 * 1{,}015 * 16{,}158 = 9.030€$$

The best estimate for WC SLT pensions is 88.869.321€ (including 9.912.809€ redeemable).

5.2.B. BE for WC SLT – Lifetime assistance

As there is a lack of proper analysis on which multiplication factor must be used for the mortality table regarding the WC SLT – lifetime assistance liabilities in the Portuguese market, it was assumed that f=0 and consequently a standard application of the TV88-90 table (the same chosen for pensions). It was also assumed that the medical inflation rate is equal to 1,981% (the average of medical costs' inflation in the last 15 years according *PORDATA* at 14/01/2016*)* and the mean average of the past payments for each case (in the portfolio under analysis it has not been possible to obtain a more detailed information than the average payments of each case). The expense rate used to pensions (1,5%) was equally assumed for lifetime assistance payments.

Due to the high number of cases (335) and that is provision is calculated case-by-case, we will present the calculation for a random case and after that the overall provision:

f. Age: 57; Annual mean payment: 1.936€:

$$LA_{57} = \sum_{t=1}^{\infty_{57}} 1.936 * (1{,}015) * \left(\frac{1{,}01981}{1 + r_t}\right)^t * \frac{l_{57+t}}{l_{57}} = 54.983€$$

The best estimate for WC SLT lifetime assistance liabilities is 29.450.357€.

5.3. BE for premium provision

5.3.A. Future claims WC NSLT

Table 9 Approach based on EIOPA simplification

UPR	Expected Future Cash-In		Exposure 2016	Loss ratio
	Fractional	Renewal		
4.874.186	834.900	2.024.000	**7.733.086**	51%

Table 10 Approach based on EIOPA simplification - Pattern

0	1	2	3	4	5	6	7	8	9	10
60,98%	32,81%	4,08%	1,20%	0,27%	0,35%	0,06%	-0,08%	0,28%	0,04%	0,02%

Table 11 Approach based on EIOPA simplification - Payments per development year

0	1	2	3	4	5	6	7	8	9	10	NSLT future claims discounted
2.411.406	1.298.905	161.259	47.238	10.679	13.458	2.277	-3.042	10.512	1.310	621	**3.954.624**

Table 12 Extension of Mack's model to future exposure - development factors

0	1	2	3	4	5	6	7	8	9
1,538	1,043	1,012	1,003	1,004	1,001	0,999	1,003	1,000	1,000

Table 13 Extension of Mack's model to future exposure - accumulated costs

	0	1	2	3	4	5	6	7	8	9	10
2016	2.322.555	3.572.161	3.727.452	3.773.139	3.783.541	3.796.787	3.799.058	3.795.975	3.806.819	3.808.197	3.808.864

Table 14 Extension of Mack's model to future exposure - $\left(s.e.\left(\widehat{C}_{11,k}\right)\right)^2$

0	1	2	3	4	5
34.750.020.811	103.406.167.800	114.132.929.193	117.786.083.800	121.153.375.381	123.368.478.112
6	**7**	**8**	**9**	**10**	**Standard error**
123.996.351.689	124.341.941.674	125.232.066.538	125.323.781.426	125.367.674.783	**354.073**

Table 15 Extension of Mack's model to future exposure - future claims undiscounted

	0	1	2	3	4	5	6	7	8	9	10
2016	2.322.555	1.249.606	155.292	45.687	10.401	13.246	2.271	-3.083	10.844	1.378	667

Table 16 Extension of Mack's model to future exposure - future claims discounted

	0	1	2	3	4	5	6	7	8	9	10	Expected future claims NSLT
2016	2.324.380	1.252.028	155.439	45.534	10.294	12.972	2.195	-2.933	10.133	1.263	599	**3.811.903**

5.3.B. Future claims SLT – Pensions

Table 17 Future claims SLT - Pensions

	Non-redeemable								
	FDKW			FDUW			PDW		
	EPP*(PVFP+UP)	EAPV	APA	EPP*(PVFP+UP)	EAPV	APA	EPP*(PVFP+UP)	EAPV	APA
	0,19	12.731	54	2,15	7.515	47	2,04	4.405	49
Discounted SLT Pensions	54.142			414.657			222.737		

	Non-redeemable (Beneficiaries in case of death)								
	Spouse			Descendants			Ascendants		
	EPP*(PVFP+UP)	EAPV	APA	EPP*(PVFP+UP)	EAPV	APA	EPP*(PVFP+UP)	EAPV	APA
	1,16	5.201	43	2,01	2.863	10	0,00	0	0
Discounted SLT Pensions	163.558			75.768			0		

	Redeemable								
	PDW			Beneficiaries in case of death					
				Spouse			Ascendants		
	EPP*(PVFP+UP)	EAPV	APA	EPP*(PVFP+UP)	EAPV	APA	EPP*(PVFP+UP)	EAPV	APA
	142,69	369	46	0,18	2.393	43	0,00	0	0
Discounted SLT Pensions	762.760			6.098			0		

Discounted future claims SLT – Pensions	**1.699.719**

5.3.C. Future claims SLT - Lifetime assistance

Table 18 Future claims SLT - Lifetime assistance

	EPLA*(PVFP+UP)	EALAV	ALA
Small	2	384	48
Medium	1	2.146	46
High	0	14.540	43

Discounted future claims SLT Lifetime assistance	**317.228**

Considering the Mack's Model applied to the future exposure, we have estimated that the best estimate for the premium provision is 3.066.778€.

Table 19 BE for premium provision

UP		4.874.186
PVFP		2.859.556
Discounted future claims	NSLT	3.811.903
	SLT	2.016.947
ER		4%
AER		2%
$BE_{premium}$		**3.066.778**

5.4. Discount rate impact

One of the biggest impacts of Solvency II is the discount effect. Under Solvency II, insurers have to use the risk free interest rate term structures published by EIOPA that changes over the time and presents a current low level when compared to the constant interest rate applied in the market for statutory purposes. In this way, we have proceeded the following sensitivity analysis comparing the best estimate for claims provision using EIOPA discount curve and using an interest rate of 3% for WC SLT provision and no interest rate applied to WC NSLT claims provision (it is not discounted under statutory approach). The best estimate for premium provision was not included in this discount impact analysis because Solvency II approach is not directly comparable to statutory approach.

Table 20 Discount rate - impact analysis

	SII	Statutory	Δ	Δ (%)
$BE_{incurred\ claims}$	**130.508.392**	**110.211.507**	**20.296.885**	**18,4%**
NSLT	12.188.714	12.188.618	96	0,0%
SLT	118.319.678	98.022.888	20.296.790	20,7%

There is no significant impact on the application of the discount effect to WC NSLT claims provision, due to the low interest rate environment for short maturities.

In what concerns to WC SLT liabilities, there is a high impact on the application of EIOPA risk free interest rates. As mentioned before, EIOPA risk free term structures present interest rates lower than the usual constant interest rate applied in the market for statutory purposes and as expected this effect has a considerable impact for long maturities, such as WC SLT liabilities. Due to this significant impact, EIOPA allowed the application of a transitional measure to the technical provisions in the implementation of Solvency II.

Conclusion

The aim of this report was to present an analysis of the WC best estimate under Solvency II, taking into account its particularities and analysing its liabilities separately for each type of exposure and homogeneity.

Through our research, we have faced several literature in actuarial methodologies to estimate the best estimate for WC NSLT claims provisions. Due to the high number of methodologies, which have been covered very comprehensively by colleagues, and as the main goal of our report is to cover all the WC liabilities, we only have discussed one of the most used methods by insurers for these type of liabilities.

Concerning WC SLT best estimate, we have analysed them using the traditional life techniques, selecting the more accurate assumptions to the Portuguese market, and taking into account the diversified risk exposures between non-redeemable pensions, redeemable pensions and Lifetime assistance liabilities.

Regarding the best estimate for premium provision, there is few literature about it due to be an add-on of the Solvency II, not directly compared to other provisions made for statutory purposes. Therefore, we have split the future claims costs into WC NSLT and WC SLT liabilities. In order to estimate the WC NSLT liabilities, we proposed an approach based on the claims ratio presented on the EIOPA simplified approach and we have also suggested an extension of the Mack's model to the future exposure. In respect to WC SLT future claims exposure, we have done an analysis mainly supported in rates, using historical data, and disaggregated into homogenous risk groups.

This report was performed taking into account the available inputs because one of the main goals was to estimate our provision based on a real case, and most of the times it is not possible to have all the inputs that the actuarial methodologies require to obtain the most accurate estimation.

Due to the extension of the research topic, we had to select and focus our analysis in the topics that we have considered to be the most significant and with the higher impact in the estimation. Whenever the categories presented an accurate analysis, we decided to not explore more alternatives and to limit the level of detail. However, we would like to present what we consider to be important for further developments of this report:

- Alternative methodologies for the WC NSLT best estimate for claims provision, including a comparative analysis to select the methodology between them that will reflect more accurately the data under analysis;

- A deeper analysis of ULAE expenses, including methodologies to its claims provision;

- An estimation of the annual contribution to Workers' Compensation Fund managed by ASF;

- A detailed analysis of the assumptions used for the WC SLT best estimate, including a comparative study to changes in the mortality table, revision and correction rate assumptions;

- Explore in detail the lifetime assistance liabilities, throughout a wide range of data;

- Improve the suggested approach to calculate the best estimate for the premium provision, with the determination of a confidence interval and including the uncertainty in a more structured form.

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Appendix A – Verification of Mack's model assumptions

A. 1st assumption

Table 21 1st assumption - Spearman test and Pearson's coefficient

	$r_{n,0}$	$s_{n,1}$	$r_{n,1}$	$s_{n,2}$	$r_{n,2}$	$s_{n,3}$	$r_{n,3}$	$s_{n,4}$	$r_{n,4}$	$s_{n,5}$	$r_{n,5}$	$s_{n,6}$	$r_{n,6}$	$s_{n,7}$	$r_{n,7}$	$s_{n,8}$	$r_{n,8}$
0	8	1	6	5	1	1	5	4	6	5	1	1	1	1	2	1	2
1	6	6	7	6	3	3	3	2	1	1	4	3	4	3	3	2	1
2	7	7	5	4	8	8	7	6	4	3	3	2	2	2	1		
3	1	1	3	2	7	7	4	3	5	4	5	4	3				
4	4	4	9	8	2	2	1	1	2	2	2						
5	3	3	4	3	5	5	6	5	3								
6	5	5	8	7	6	6	2										
7	2	2	1	1	4												
8	10	9	2														
9	9																

$(r_{i,j} - s_{i,j})^2$		**119**	**116**	**44**	**18**	**26**	**2**	**2**	**2**

K	0	1	2	3	4	5	6	7	8
Tk		0,008	-0,381	0,214	0,486	-0,300	0,800	0,500	-1,000
10-k-1		8	7	6	5	4	3	2	1
T_k*(10-k-1)		0,067	-2,667	1,286	2,429	-1,200	2,400	1,000	-1,000

Table 22 1st assumption verification

E[T]	Var(T)	Confidence Interval		Assumption verified?
		Lower	upper	
0,064	36	-0,112	0,112	**Yes**

B. 2nd assumption

Table 23 2nd assumption - order and classification of development factors

	Ascending order of development factors									
	0	1	2	3	4	5	6	7	8	9
0	8	6	1	5	6	1	1	2	2	1
1	6	7	3	3	1	4	4	3	1	
2	7	5	8	7	4	3	2	1		
3	1	3	7	4	5	5	3			
4	4	9	2	1	2	2				
5	3	4	5	6	3					
6	5	8	6	2						
7	2	1	4							
8	10	2								
9	9									

	Classification of development factors									
	0	1	2	3	4	5	6	7	8	9
0	L	L	S	L	L	S	S	*	S	*
1	L	L	S	S	S	L	L	L	L	
2	L	*	L	L	L	*	S	S		
3	S	S	L	*	L	L	L			
4	S	L	S	S	S	S				
5	S	S	L	L	S					
6	S	L	L	S						
7	S	S	S							
8	L	S								
9	L									

Table 24 2nd assumption - binomial distributions

					Binomial distribution				
	S_j	L_j	Z_j	$(L_j+S_j)/2$	N	P	m	$E(Z_j)$	$Var(Z_j)$
0	0	1	0	0,5	1	0,5	0	0,000	0,000
1	0	2	0	1	2	0,5	0	0,500	0,250
2	1	2	1	1,5	3	0,5	1	0,750	0,188
3	2	1	1	1,5	3	0,5	1	0,750	0,188
4	3	2	2	2,5	5	0,5	2	1,563	0,371
5	3	3	3	3	6	0,5	2	2,063	0,621
6	4	2	2	3	6	0,5	2	2,063	0,621
7	2	4	2	3	6	0,5	2	2,063	0,621
8	4	5	4	4,5	9	0,5	4	3,270	0,736
9	6	3	3	4,5	9	0,5	4	3,270	0,736
			18					16,289	4,331

Table 25 2nd assumption verification

$\sum E(Z_j)$	$\sum Var(Z_j)$	Confidence Interval		Assumption verified?
		lower	Upper	
16,29	4,33	12,210	20,368	Yes

C. 3rd assumption

The following graphics present the relation between the observed data and the estimated data using the development factor parameter.

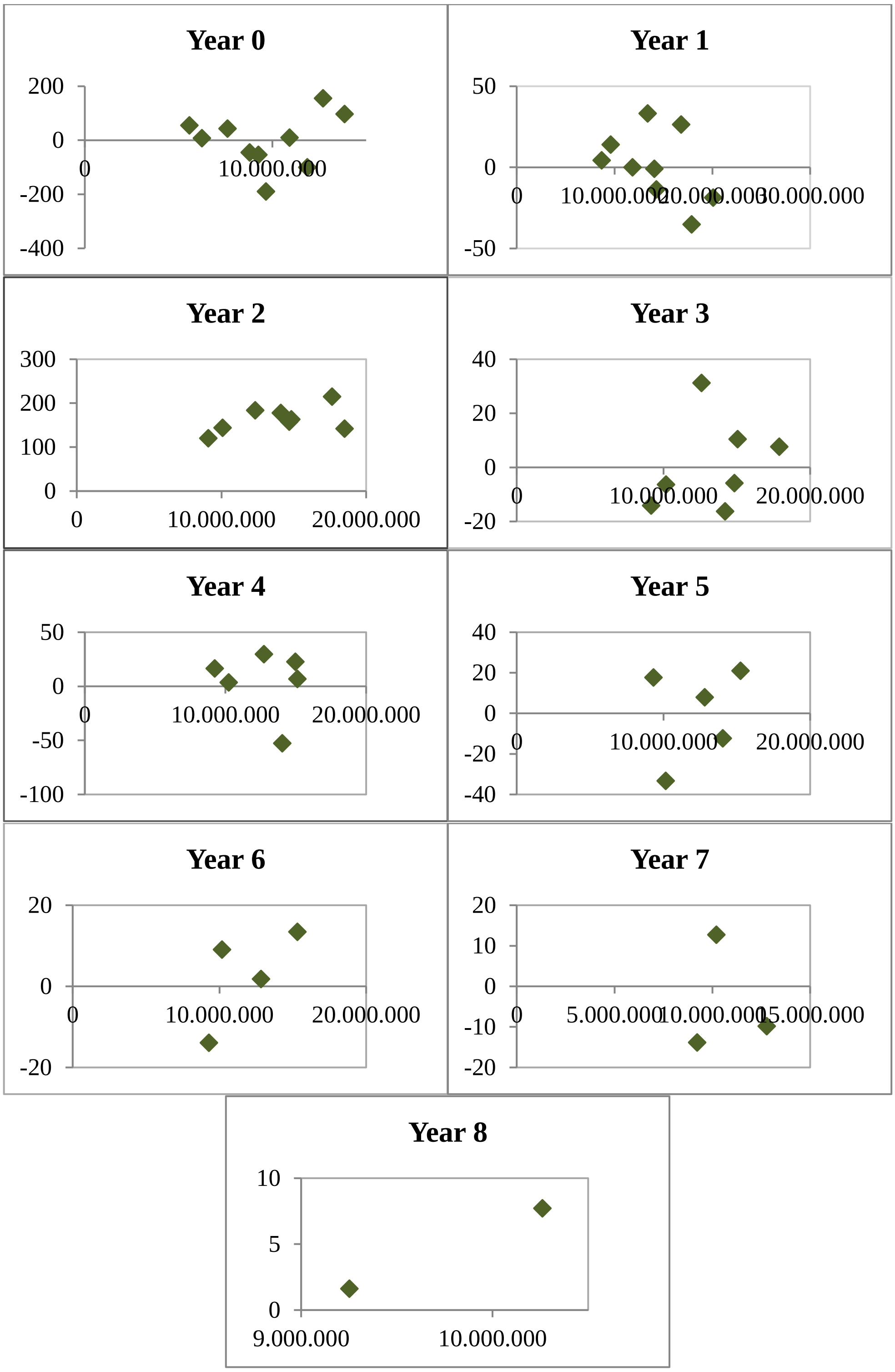

Figure 3 3rd assumption - Weighted residuals per year

After an accurate analysis of the graphics, it is possible to conclude that there are no relevant tendencies observed in the graphics above and thus the assumption is verified.

Appendix B – Summary: Methodologies for non-redeemable pensions

Pension Present Value (PPV)

$$PPV_i = \begin{cases} 3 * w & if\ a = 0 \wedge d = 0 \\ a * (1 + cor) * (PPV_{worker}) + (1 - a) * (PPV_s + PPV_{desc} + PPV_{asc}) & otherwise \end{cases}$$

I. Worker is alive

$$PPV_{worker} = \begin{cases} \min(0{,}8 + 0{,}1 * d; 1) * w * \ddot{as}_x^{(12)} & , if\ worker\ is\ fully\ disabled\ for\ any\ kind\ of\ work\ (fdkw) \\ m * w * \ddot{as}_x^{(12)} & , if\ worker\ is\ fully\ disabled\ for\ the\ usual\ work\ (fduw) \\ 0{,}7 * \left(w - w_{after\ accident}\right) * \ddot{as}_x^{(12)} & , if\ worker\ is\ partially\ disabled\ for\ the\ work\ (pdw) \end{cases}$$

II. In case of death

a) Spouse

$$PPV_s = \begin{cases} 0{,}3 * w * \ddot{as}_y^{(12)} + 0{,}1 * w * \frac{l_{RA}}{l_y} * v_{RA-y}^{RA-y} * \ddot{as}_{RA}^{(12)} & if\ spouse\ not\ retired \\ 0{,}4 * w * \ddot{as}_y^{(12)} & if\ spouse\ already\ retired \\ 0 & if\ worker\ does\ not\ have\ spouse \end{cases}$$

b) Descendants

$$PPV_{desc}(c) = \begin{cases} 0 & , c = 0 \\ 0{,}4 * w * \frac{1}{h} * \left[\begin{array}{c} I_{c^d} * \sum_{i=1}^{\min(c^d,2)} \ddot{as}_{z_{(i)}}^{(12)} + \\ I_{c^{nd}} * \sum_{i=1}^{\min(c^{nd},2)} \left(d_{i1} * \ddot{as}_{z_{(i)}:\overline{18-z_{(i)}|}}^{(12)} + d_{i2} * \ddot{as}_{z_{(i)}:\overline{22-z_{(i)}|}}^{(12)} + d_{i3} * \ddot{as}_{z_{(i)}:\overline{25-z_{(i)}|}}^{(12)} \right) \end{array} \right] & , c = 1{,}2 \\ PPV_{desc}(2) + 0{,}1 * (h-1) * w * \left[\begin{array}{c} D_{z_{(3)}} * \ddot{as}_{z_{(3)}}^{(12)} + \\ \left(1 - D_{z_{(3)}}\right)\left(d_{31} * \ddot{as}_{z_{(3)}:\overline{18-z_{(3)}|}}^{(12)} + d_{32} * \ddot{as}_{z_{(3)}:\overline{22-z_{(3)}|}}^{(12)} + d_{33} * \ddot{as}_{z_{(3)}:\overline{25-z_{(3)}|}}^{(12)} \right) \end{array} \right] & , c \geq 3 \end{cases}$$

c) Ascendants

$$PPV_{asc} = \begin{cases} 0 & , p = 0 \\ \sum_{i=1}^{min(3,p)} 0{,}1 * w * \ddot{as}_{p_{(i)}}^{(12)} & , d - p \neq 0 \\ \sum_{i=1}^{p} min\left(0{,}2; \frac{0{,}8}{p}\right) * w * \ddot{as}_{p_{(i)}}^{(12)} & , d - p = 0 \end{cases}$$

$cor\ is\ the\ correction\ rate.\ If\ the\ pension\ is\ defined \Rightarrow cor = 0$	$m \in [0{,}5; 0{,}7]\ depending\ on\ the\ capacity\ to\ do\ another\ job$
$a = \begin{cases} 0\ if\ the\ worker\ died \\ 1\ if\ the\ worker\ is\ alive \end{cases}$	$c^d = number\ of\ descendants\ disabled$
$c^{nd} = number\ of\ descendants\ not\ disabled$	$c = c^d + c^{nd} = number\ of\ descendants$
$I_x = \begin{cases} 0, if\ x = 0 \\ 1, otherwise \end{cases}$	$D_{z_{(3)}} = \begin{cases} 0, if\ the\ descendant\ with\ age\ z_{(3)} is\ not\ disabled \\ 1, \qquad otherwise \end{cases}$
$w = 12 * monthly\ wage\ of\ the\ worker\ when\ the\ accident\ happens * (1 + expense\ rate)$	$z_{(i)} = age\ of\ the\ descendent\ i, in\ ascending\ order\ and\ firstly\ the\ disabled$
$x = age\ of\ the\ worker\ when\ the\ accident\ happens$	$pes = probability\ of\ still\ studying\ with\ 18\ years\ old\ at\ secondary\ or\ higher\ level$
$d = number\ of\ dependants$	$puniv = probability\ of\ still\ studying\ with\ 22\ years\ at\ university$
$v_k = \frac{1}{(1 + r_k)}, r_k\ from\ EIOPA\ risk\ free\ rate\ interest\ curve$	${}_{e+k}p_y = {}_k p_x *_e p_{x+k} \overset{CFM}{\approxeq} {}_k p_x * (p_{x+k})^e, 0 \leq e < 1$ [11]
$\ddot{a}^{(12)}_{\overline{x:n-x\rceil}} \overset{UDD}{\approxeq} \frac{13}{24} + \frac{11}{24} * v^{n-x}_{n-x} *_{n-x} p_x + \sum_{k=1}^{n-x-1} v^k_k *_k p_x$ [12]	$\ddot{a}s^{(12)}_{\overline{x:n-x\rceil}} = \ddot{a}^{(12)}_{\overline{x:n-x\rceil}} + \frac{\sum_{k=0}^{n-x-1} s_{k,x}}{12}$
$s_{k,y} = \left[v^{0,5+k}_{k+1} *_{0,5+k} p_y + v^{\frac{11}{12}+k}_{k+1} *_{\frac{11}{12}+k} p_y \right] * (1 + rev)^{k+1}$	$h = \begin{cases} 1, if\ they\ are\ orphans \\ 2, if\ they\ are\ not\ orphans \end{cases}$
$\infty_x = \max age\ in\ the\ mortality\ table - x$	$d_{i1} = \begin{cases} 1, \ z_i < 18 \\ 0, \ otherwise \end{cases}$
$d_{i2} = \begin{cases} pes * \frac{l_{18}}{l_{z_i}} * v^{18-z_i}_{18-z_i} * \ddot{a}s^{(12)}_{\overline{18:4\rceil}} * \frac{1}{\ddot{a}s^{(12)}_{\overline{z(i):22-z(i)\rceil}}}, z_i < 18 \\ 1 \quad , 18 \leq z_i < 22 \\ 0 \ , otherwise \end{cases}$	$d_{i3} = \begin{cases} puniv * \frac{l_{22}}{l_{z_i}} * v^{22-z_i}_{22-z_i} * \ddot{a}s^{(12)}_{\overline{22:3\rceil}} * \frac{1}{\ddot{a}s^{(12)}_{\overline{z(i):25-z(i)\rceil}}}, z_i < 22 \\ 1 \quad , 22 \leq z_i < 25 \\ 0 \ , otherwise \end{cases}$
$\ddot{a}^{(12)}_x \overset{UDD}{\approxeq} \frac{13}{24} + \sum_{k=1}^{\infty_x} \left(\frac{1 + rev}{1 + r_k}\right)^k * \frac{l_{x+k}}{l_x}$ [13]	$\ddot{a}s^{(12)}_x = \ddot{a}^{(12)}_x + \frac{\sum_{k=0}^{\infty_x} s_{k,x}}{12}$
$y = spouse\ age$	$p = number\ of\ ascendants$
$RA = normal\ retirement\ age$	$p_{(i)} = age\ of\ the\ ascendent\ i, in\ ascending\ order$

[11] Constant Force of Mortality Assumption: for x integer and $0 \leq e < 1$, ${}_e p_x = (p_x)^e$

[12] Applying UDD approach: $\ddot{a}s^{(12)}_{\overline{x:n\rceil}} \overset{UDD}{\approxeq} \ddot{a}_{\overline{x:n\rceil}} - \frac{11}{24} * (1 - v^n *_n p_x) = 1 + \sum_{k=1}^{n-1}(v^k *_k p_x) - \frac{11}{24} + \frac{11}{24} * v^n *_n p_x = \frac{13}{24} + \frac{11}{24} * v^n *_n p_x + \sum_{k=1}^{n-1} v^k *_k p_x$

[13] Under UDD approach: $\ddot{a}^{(m)}_x \approx \ddot{a}_x - \frac{m-1}{2m} = \sum_{k=0}^{\infty_x}(v^k *_k p_x) - \frac{m-1}{2m} = 1 - \frac{m-1}{2m} + \sum_{k=1}^{\infty_x}(v^k *_k p_x)$

Appendix C – Pensions redeemable factors

Table 26 Pensions redeemable factors

Age	Redeemable factor			
	Orphans	Spouse	Ascendants	Other Pensioners
0	13,968	-	-	-
1	13,797	-	-	-
2	13,503	-	-	-
3	13,190	-	-	-
4	12,859	-	-	-
5	12,509	-	-	-
6	12,141	-	-	-
7	11,753	-	-	-
8	11,344	-	-	-
9	10,914	-	-	-
10	10,460	-	-	18,426
11	9,983	-	-	18,369
12	9,481	-	-	18,309
13	8,952	-	-	18,246
14	8,396	11,926	18,365	18,181
15	7,812	11,528	18,307	18,113
16	7,197	11,110	18,248	18,044
17	6,550	10,672	18,189	17,974
18	5,871	10,213	18,132	17,905
19	5,158	9,940	18,077	17,838
20	4,406	9,755	18,022	17,770
21	3,615	9,651	17,966	17,701
22	2,781	9,619	17,909	17,629
23	1,903	9,655	17,850	17,555
24	0,976	9,754	17,787	17,476
25	-	9,914	17,721	17,393
26	-	10,129	17,651	17,305
27	-	10,396	17,577	17,213
28	-	10,707	17,499	17,116
29	-	11,053	17,418	17,013
30	-	11,420	17,333	16,906
31	-	11,797	17,244	16,794
32	-	12,169	17,151	16,677
33	-	12,518	17,055	16,555
34	-	12,836	16,955	16,428
35	-	13,125	16,852	16,296
36	-	13,396	16,744	16,158
37	-	13,642	16,633	16,015
38	-	13,852	16,518	15,866
39	-	14,028	16,399	15,711
40	-	14,172	16,276	15,550

Age	Redeemable factor (cont.)			
	Orphans	Spouse	Ascendants	Other Pensioners
41	-	14,285	16,149	15,383
42	-	14,371	16,020	15,211
43	-	14,430	15,887	15,032
44	-	14,469	15,754	14,851
45	-	14,486	15,618	14,664
46	-	14,483	15,479	14,470
47	-	14,462	15,337	14,270
48	-	14,425	15,192	14,063
49	-	14,373	15,046	13,851
50	-	14,312	14,900	13,636
51	-	14,241	14,756	13,416
52	-	14,163	14,612	13,192
53	-	14,080	14,471	12,964
54	-	13,992	14,332	12,732
55	-	13,901	14,197	12,496
56	-	13,812	14,068	12,259
57	-	13,721	13,943	12,016
58	-	13,631	13,821	11,769
59	-	13,545	13,708	11,518
60	-	13,463	13,602	11,264
61	-	13,389	13,507	11,006
62	-	13,323	13,421	10,745
63	-	13,267	13,348	10,478
64	-	13,222	13,287	10,207
65	-	9,891	9,929	9,929
66	-	9,615	9,645	9,645
67	-	9,330	9,352	9,352
68	-	9,040	9,055	9,055
69	-	8,745	8,754	8,754
70	-	8,445	8,450	8,450
71	-	8,141	8,141	8,141
72	-	7,834	7,834	7,834
73	-	7,527	7,527	7,527
74	-	7,218	7,218	7,218
75	-	6,908	6,908	6,908
76	-	6,601	6,601	6,601
77	-	6,294	6,294	6,294
78	-	5,992	5,992	5,992
79	-	5,697	5,697	5,697
80	-	5,407	5,407	5,407
81	-	5,123	5,123	5,123
82	-	4,853	4,853	4,853
83	-	4,592	4,592	4,592
84	-	4,339	4,339	4,339
85	-	4,097	4,097	4,097
86	-	3,863	3,863	3,863

Age	Redeemable factor (cont.)			
	Orphans	Spouse	Ascendants	Other Pensioners
87	-	3,636	3,636	3,636
88	-	3,423	3,423	3,423
89	-	3,228	3,228	3,228
90	-	3,043	3,043	3,043
91	-	2,864	2,864	2,864
92	-	2,697	2,697	2,697
93	-	2,547	2,547	2,547
94	-	2,401	2,401	2,401
95	-	2,256	2,256	2,256
96	-	2,096	2,096	2,096
97	-	1,940	1,940	1,940
98	-	1,760	1,760	1,760
99	-	1,636	1,636	1,636
100	-	1,526	1,526	1,526
101	-	1,421	1,421	1,421
102	-	1,307	1,307	1,307
103	-	1,195	1,195	1,195
104	-	1,039	1,039	1,039
105	-	0,813	0,813	0,813
106	-	0,542	0,542	0,542

Appendix D – EIOPA Risk-free interest rates

Table 27 EIOPA Risk-free interest rates at 31/12/2015 without VA

Maturity	EIOPA Risk free interest rate 31/12/2015 without volatility adjustment	Maturity	EIOPA Risk free interest rate 31/12/2015 without volatility adjustment
1	-0,16%	47	2,78%
2	-0,13%	48	2,81%
3	-0,04%	49	2,83%
4	0,10%	50	2,86%
5	0,23%	51	2,89%
6	0,38%	52	2,91%
7	0,53%	53	2,93%
8	0,67%	54	2,96%
9	0,80%	55	2,98%
10	0,92%	56	3,00%
11	1,03%	57	3,02%
12	1,12%	58	3,04%
13	1,21%	59	3,06%
14	1,28%	60	3,08%
15	1,34%	61	3,10%
16	1,39%	62	3,12%
17	1,42%	63	3,13%
18	1,45%	64	3,15%
19	1,49%	65	3,16%
20	1,53%	66	3,18%
21	1,57%	67	3,20%
22	1,63%	68	3,21%
23	1,68%	69	3,22%
24	1,74%	70	3,24%
25	1,80%	71	3,25%
26	1,86%	72	3,26%
27	1,92%	73	3,28%
28	1,98%	74	3,29%
29	2,03%	75	3,30%
30	2,09%	76	3,31%
31	2,14%	77	3,33%
32	2,19%	78	3,34%
33	2,24%	79	3,35%
34	2,29%	80	3,36%
35	2,34%	81	3,37%
36	2,38%	82	3,38%
37	2,43%	83	3,39%
38	2,47%	84	3,40%
39	2,51%	85	3,41%
40	2,55%	86	3,42%
41	2,58%	87	3,43%
42	2,62%	88	3,43%
43	2,65%	89	3,44%
44	2,69%	90	3,45%
45	2,72%	91	3,46%
46	2,75%	92	3,47%

Maturity	EIOPA Risk free interest rate 31/12/2015 without volatility adjustment	Maturity	EIOPA Risk free interest rate 31/12/2015 without volatility adjustment
93	3,48%	122	3,65%
94	3,48%	123	3,65%
95	3,49%	124	3,66%
96	3,50%	125	3,66%
97	3,50%	126	3,66%
98	3,51%	127	3,67%
99	3,52%	128	3,67%
100	3,53%	129	3,68%
101	3,53%	130	3,68%
102	3,54%	131	3,69%
103	3,55%	132	3,69%
104	3,55%	133	3,69%
105	3,56%	134	3,70%
106	3,56%	135	3,70%
107	3,57%	136	3,70%
108	3,58%	137	3,71%
109	3,58%	138	3,71%
110	3,59%	139	3,71%
111	3,59%	140	3,72%
112	3,60%	141	3,72%
113	3,60%	142	3,72%
114	3,61%	143	3,73%
115	3,61%	144	3,73%
116	3,62%	145	3,73%
117	3,62%	146	3,74%
118	3,63%	147	3,74%
119	3,63%	148	3,74%
120	3,64%	149	3,75%
121	3,64%	150	3,75%