LISBON SCHOOL OF ECONOMICS & MANAGEMENT
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MASTER OF SCIENCE IN FINANCE

MASTER´S FINAL WORK

DISSERTATION

A STATIC APPROACH TO THE NELSON-SIEGEL-SVENSSON MODEL: APPLICATION TO SEVERAL NEGATIVE YIELDS CASES

VÍTOR HUGO FERREIRA CARVALHO

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ABSTRACT

The appearance of negative bond yields presents significant challenges to the fixed income markets, mainly to the adjust and forecasting methods. The Nelson-Siegel-Svensson model (NSS) is in most cases adopted by central banks to estimate the term structure of interest rates.

In this study, it was chosen the NSS model to fit the yield curves of a set of countries which registered negative sovereign bond yields. No changes or constraints were done to the model or its parameters. It was applied with friendly, widely available, and simple tools. The model adjusted well for all countries yield curves, even with partial bond yields data. A comparison between market instantaneous interest rate and interest rate for a very distant future, that the model can predict, was done, with good results for the instantaneous interest rate.

Since the main set of countries, included in the study, are within the Eurozone, an evaluation of a shared debt securities (i.e. Eurobonds) possible behaviour was analysed. The NSS model can be a valuable, easy to use and adaptable tool, to fit the yield curve with negative yields, for the use of the monetary policy institutions and market players, at least in a static way.

JEL Classification: C02; C18; E43; E47; G12; G17

Key Words: yield curve; negative bond yields; Eurobonds; Nelson-Siegel-Svensson model

RESUMO

O aparecimento de obrigações com taxas de juro negativas apresenta desafios significativos para os mercados de rendimento fixo, principalmente com os métodos de ajuste e previsão. O modelo de Nelson-Siegel-Svensson (NSS) é, na maioria dos casos, adotado pelos bancos centrais para estimar a estrutura a prazo das taxas de juros.

Neste estudo, foi escolhido o modelo NSS para ajustar as curvas de taxa de juro de um conjunto de países que registaram obrigações soberanas com taxas de juro negativas. Não foram feitas alterações ou restrições ao modelo ou aos seus parâmetros. Foi aplicado com ferramentas amigáveis, amplamente disponíveis e simples. O modelo ajustou-se bem para todas as curvas de taxa dos países, mesmo com dados parciais de taxas de juro das obrigações. Uma comparação entre a taxa de juros instantânea do mercado e a taxa de juro para um futuro muito distante, que o modelo pode prever, foi feito, com bons resultados para a taxa de juro instantânea.

Uma vez que o principal conjunto de países, incluído no estudo, está dentro da zona do Euro, foi analisado o possível comportamento de uma dívida partilhada (ou seja, Eurobonds).

O modelo NSS pode ser uma ferramenta valiosa, fácil de usar e adaptável, para ajustar a curva de taxas de juro negativas, para uso das instituições de política monetária e dos operadores do mercado, pelo menos de forma estática.

Classificação JEL: C02; C18; E43; E47; G12; G17

Palavras-chave: curva de taxas de juro; taxas de juro negativas de obrigações; Eurobonds; modelo de Nelson-Siegel-Svensson

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1 INTRODUCTION

The existence of negative bond yields presents significant challenges to the fixed income markets. Some, are related to the modelling and forecasting methods, others are due to the actual size of assets with negative yields ($13,4 trillion, Financial Times, 2016) and, finally, one can detect the impact on financial theory and the implications for bond holders and issuers.

As the Nelson and Siegel model (1987) and proposed extension of Svensson (1994) are in most cases adopted by central banks to estimate the term structure of interest rates (BIS, 2005), the Nelson-Siegel-Svensson (NSS) model was chosen in this study to fit the yield curves of a set of countries which registered negative, sovereign bond yields.

Negative yields are recent and can in some way be an outcome of some important aspects. The 2008 financial crisis lead Federal Reserve (Fed) to start quantitative easing programs[1] until October 29th, 2014, later followed by European Central Bank (ECB) (ECB, 2017a) in the aftermath of 2010/2011 European government debt crisis and the significant reduction in the directorate interest rate of ECB. Japan with its lost decades[2] (Hayashi & Prescott, 2002) and low rates, combined presently, with China and world GDP reduction growth, had lead the fixed income markets to search for "safe heavens". These "safe heavens" issuers are the ones that have higher ratings and, so can provide higher certainty that will service entirely their debts. In a certain way, the high debt

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[1] Available at: https://www.thebalance.com/what-is-quantitative-easing-definition-and-explanation-3305881 | https://www.ecb.europa.eu/explainers/show-me/html/app_infographic.en.html Accessed date: August 7th, 2017

[2] Hayashi and Prescott, used the expression "Lost decade" to refer to Japan 1990s economic stagnation, due to low growth productivity, although this term is referring to the 1990s, the real wages fallen, low growth and deflation persisted in time, conducting Japan to economic stimulus, creating fiscal deficits and the highest debt in the world.

levels of European Union countries, and the highest debt of the world, like in Japan (234% of the GDP in 2015) (OECD, 2017), should demand greater yields for these issuers, but ratings (that seem to be more favourable to developed countries (Cantor & Packer, 1996)) and the lack of possibility for the emerging countries to capture the fixed income markets with intensity, have conduct to the present situation characterized by higher debts issuers related to their GDP with, in some cases, the lowest yields, and, awkwardly, cases of negative yields, that are something not so predictable and common.

Given that the market players (e.g. insurance companies, pension funds, banks) have the need to estimate and model the term structure of interest rates with these recent negative bond yields, this work contributes to solve this matter, proposing the use of NSS model, by means of friendly, widely available, and simple tools. Hence, the objectives of this study are:

- to evaluate the adequacy of the NSS model through the fit of the yield curve, at a certain date, with at least one negative yield value and, through the interest rates values that one can deduct from the model, compared with market data, with a easy to use approach; and,
- to evaluate the results of the model with partial market bond yields data (short, intermediate and long-term).

Thus, the present work is composed, in addition to Introduction, by the literature review, the methodology, the results and the conclusion chapters, regarding the two main objectives above mentioned.

The literature review chapter intends to present and describe the model chosen, it´s application and importance, as well as the approaches done to fit negative yields market data.

In the methodology chapter, the NSS model and parameters are described in detail, as well as the calibration method, the analysis procedure, the data and software definitions to accomplish data analysis.

The results chapter collects the main outcomes of the work and leads the way to new studies. Indeed, given that the greater set of countries are European and in the Eurozone, it will be taken the opportunity to see a comparison between their yield curves and some effects of a possible future shared Eurozone debt security (i.e. Eurobonds). Conclusion chapter presents the main findings.

2 LITERATURE REVIEW

The term structure of interest rates, or yield curve, is a key variable of economics and finance (Büttler, 2007). The direct relation between term structure of interest rates and yield curve, should be clarified. Málek (2005), in Hladíková & Radová (2012), places the distinction to three equivalent descriptions of the term structure of interest rates:

- the discount function, which specifies zero-coupon bond prices as a function of maturity;
- the spot yield curve, which specifies zero-coupon bond yields (spot rates) as a function of maturity;
- the forward yield curve, which specifies zero-coupon bond forward yields (forward rates) as a function of maturity.

The discount function, entails some undesirable conditions. Bond prices are insensitive to yields changes for shorter maturities, and minimizing price errors, sometimes, results in large yield errors for bonds for those shorter maturities (Svensson, 1994). Also, monetary policy makers and economic discussions, generally, focus on interest rates rather than prices (Geyer & Mader, 1999). For these reasons, the discount function, could not be a suitable description of the term structure of interest rates.

To the purpose of an entire evaluation of the yield curve (maturities can be as high as 30, 50 and even 100 years), the forward market products are not adequate since they have a short time limit, so the forward yield curve can be a proper description of the yield curve for the shorter maturities.

In case of the spot yield curve, the market has no zero-coupon bonds for all maturities, and only a few set of countries issue those instruments, so coupon government bonds should be considered. The use of coupon bond, with different coupon rates, instead of zero-coupon bonds, have negligible impact, in accordance to Kariya et al (2013, in Inui, 2015). Svensson (1994) mentioned that to get implied forward interest rates from yield to maturity (YTM) on coupon bonds is more complicated than on zero coupon bonds. The YTM obtained from market data will give, implied spot rates, instead of real spot rates, since one cannot compute, the entire yield curve with all maturities (i.e. spot yield curve), from zero-coupon bond yields. Although, Cox et al (1985) stated that "the expectations hypothesis postulates that bonds are priced so that the implied forward rates are equal to the expected spot rates". Synthesising, the term structure of interest rates or yield curve, is computed through YTM of government coupon bonds, and through that, one will get the implied rates.

One of the objectives and usefulness to fit the yield curve is to provide the monetary policy institutions with indicators of rates evolution and expectations (e.g. inflation). The need for the monetary policy institutions to have these indicators, increased when flexible exchange rates have replaced fixed exchange rates (Svensson, 1994). Other significant purpose is related to fixed income market participants (e.g. hedging strategies, assets allocation for pension funds).

To fit the yield curve there are several methods. Based on Sundaresan (2009) compilation, these include:

- the Vasicek (1977) model is a mean reversion process. Allows negative rates, but doesn´t calibrate with market data;
- the Rendleman and Bartter (1980) model follows a simple multiplicative random walk. Rates are assumed to be lognormally distributed, which invalidates its use in the case of negative yields;

- the Cox, Ingersoll and Ross (CIR) model (Cox et al, 1985) it´s a mean reversion model, but doesn´t allow negative interest rates and doesn´t calibrate with market data;
- the Ho and Lee (1986) model, is calibrated to market yields. Assumes a normal distribution of interest rates and interest rates can become negative;
- the Black, Derman and Toy (1990) (BDT) model can be calibrated through market equity options data, but assumes that rates follow a lognormally distribution, which invalidates its use in the case of negative yields. Combines mean reversion and volatility;
- the Black and Karasinski (1991) model is calibrated to market yields and volatilities. Separates mean reversion and volatility;
- the Nelson and Siegel (1987) and Svensson (1994) extension is an exponential function to approximate the unknown forward rate function;
- the Bootstrapping method will generate a zero-coupon yield curve from existing market data such as bond prices, but lacks robustness (Martellini et al, 2003).

It was beyond our purpose to use all models. It was used the NSS since the purpose of this study is to get a model calibrated with market data and to evaluate the interest rates, from the model, without evaluate volatilities for yields or bond prices, as is required on some other models. In fact, several curve fitting spline methods have been criticized for having undesirable economic properties and for being 'black box' models (Seber & Wild, 2003 in Annaert et al, 2010).

The NSS model is parsimonious and has been widely used in academia and in practice, but is sensitive to the starting values of the parameters ($\beta_{1,2,3,4}$ and $\lambda_{1,2}$) (Annaert et al, 2010).

The NSS model respects the restrictions imposed by the economic and financial theory (rates take real numbers and not complex ones and are higher for long-terms than for shorter ones) and can take any yield curve form empirically observed in the market (Diebold & Rudebusch, 2013, in Ibáñez, 2015). Moreover, if the NSS could work in a negative yield market, this would be of most importance to hedging strategies (mainly for market participants, to hedge against flattening or steepening of the yield curve) and get forecasts for interest rates levels (very useful for monetary policy makers).

Another purpose is to fit the yield curve and get a static value of instantaneous interest rate (IIR) and interest rate of a very distant future (IRVDF), and check if the values given by the model are in accordance with the market ones. Additionally, one objective is to use a friendly, widely available tool for a not so in-depth user of math tools or software. It´s not a purpose to evaluate time evolution of the interest rates, based on the NSS parameters ($\beta_{1,2,3,4}$ and $\lambda_{1,2}$) found.

3 METHODOLOGY

The yield curve (term structure of interest rates), that can be estimated from bond yields of a certain economic region, is of utmost importance to monetary and economic authorities to support decisions processes and stablish policies, as well as to market participants for their investments and actions (Martellini et al, 2003).

In this work, it was chosen a curve fitting statistical model (like NSS model) to check if it works with negative yields and all along the yield curve. In some cases, oscillating in signal (i.e. positive and/or negative) yield values. The NSS model is a curve fitting model that can provide us with values for instantaneous and distant future interest rates.

The approach taken does not add more factors, parameters nor terms to the current NSS model. It computes all yield curves for each of the selected countries and tries to get economic and financial data to evaluate the forecast adequacy of the model, even in cases of issuers with few negative yields. Hence, it is not an objective of this work to study the NSS model parameters time series nor forecast its values to get a yield curve evolution. It was adopted a static fitting to check how NSS model works with negative yields at some part of the yield curve.

The Nelson Siegel model and Svensson extension, Equation (1), is a parametric curve-fitting method procedure, is statistical in their approach, and generally do not have a sound economic foundation.

(1) $$\boldsymbol{\gamma(\theta) = \beta_1 + \beta_2 \left[\frac{1-e^{-\frac{\theta}{\lambda_1}}}{\frac{\theta}{\lambda_1}}\right] + \beta_3 \left[\frac{1-e^{-\frac{\theta}{\lambda_1}}}{\frac{\theta}{\lambda_1}} - e^{-\frac{\theta}{\lambda_1}}\right] + \beta_4 \left[\frac{1-e^{-\frac{\theta}{\lambda_2}}}{\frac{\theta}{\lambda_2}} - e^{-\frac{\theta}{\lambda_2}}\right]}$$

Svensson (1994) extension adds a new factor, with a new decay parameter, Equation (2), to improve a better fit. As clearly described by Guedes (2008), for the Nelson Siegel model, there can be an economic interpretation of the parameters.

(2) $$\beta_4\left[\frac{1-e^{-\frac{\theta}{\lambda_2}}}{\frac{\theta}{\lambda_2}}-e^{-\frac{\theta}{\lambda_2}}\right]$$

In this work, and since Svensson extension is an added component to the Nelson Siegel model, to better fit the yield curve, it was considered to interpret the parameters as they are defined for Nelson Siegel model:

- $\gamma(\theta)$ is the yield to maturity value (spot rate) at the time of data collection with maturity θ;
- β_1 is the IRVDF;
- $\beta_1+\beta_2$ is the initial value of the curve and can be interpreted as the IIR;
- $-\beta_2$ is the spread between the interest rates of long and short times (i.e. average slope of the curve);
- $\beta_{1,2}$ and β_3 determine how short and long interest rates interchange and are responsible for the hump (≅inclination) that the yield curve shows.
- β_4 is the extension of the model proposed by Svensson in 1994, that can be interpreted as an independent decay parameter, that will introduce a new hump to better fit the model;
- θ is the maturity of the bond;
- λ_1 and λ_2 are parameters responsible for how inclination and curvature behave, don´t have an economic interpretation although determine the interchange between short and long interest rates.

Until negative bond yields appearance in some markets, NSS model did not present much difficulties on its application and was widely used.

Guedes (2008) stated that $\beta_1+\beta_2>0$, which for the paradigm of that time and until then appeared to be a very reasonable economic and financial condition. The general perception that rates, at least nominal ones (real rates, that consider other effects as inflation, can be lower than zero) would always be positive, lead to the definition of limits that the models should work. Central banks, like ECB for deposit facility rate, have presently, nominal negative rates. Although, more frequent, cases of negative yields and negative interest rates is even somehow something strange and awkward.

Time and markets have showed that $\beta_1+\beta_2$ (interpreted as the IIR) can be lower than zero. This study tries to show that there is an economical and real-world interpretation for $\beta_1+\beta_2<0$.

At a first approach, it is expected that the yield curve fitting, with some negative bond yields, would be more difficult, due to the calibration process, which usually calculates the minimum value of the sum of squared residuals (SSR). As stated by Svensson (1994), the parameters are then chosen, so, as to minimize the sum of squared yield errors between estimated and observed yields. The analysis done used the NSS model and SSR without any kind of changes in the formulas. Gilli et al (2010) stated that one possibility for the calibration is to use Equation (3) to calculate the SSR, being y the NSS model calculated yield and y^M the market yield value.

(3) $$\min_{\beta,\lambda} \sum (y - y^M)^2$$

In this study, the market values were the bond yields for each maturity, for each country. Then the NSS model would, through Microsoft Excel Solver (Frontline System, 2017a) function, compute the residuals minimum value, and so, getting the values for the parameters ($\beta_{1,2,3,4}$ and $\lambda_{1,2}$). The parametrization of Solver for the data used in this thesis, will be detailed on the Analysis chapter.

For the forecasting purposes, only a few market bond yields maturities where given to test and the NSS model ran to check if it could adjust the curve for the missing maturities. Partial market data was considered following the classification of the beginning of 1990´s, that bond markets used to bond maturities: short, intermediate and long term (Martellini et al, 2003), being the most usual time frame for each division as follow: bonds with maturities until 5 years are called short term bonds, from 5 to 10/12 years they are called intermediate bonds and higher than 10/12 years are called long bonds.

When forecasting the NSS model for short term maturity bonds, the 5 years’ time frame, couldn´t be considered as a fixed period, because the model didn´t produced good fitting date. The NSS model seems to need at least one negative yield market data to proceed with proper calibration and so provide parameters that could provide a reasonable fitting curve. Taking this into consideration the short-term time frame was different for every country and ranged from 2 to 5 years.

The intermediate period had its inferior limit defined by the higher value found from short term forecast (STF). The upper limit was defined by the best observed fitting depending on the mix of; always as possible to no more than 10 years (Lithuania is a special case because has no bond maturities higher than 7 years), and the wider period that could be considered with no market data to calibrate the model (Switzerland is a special case where the limit is 25 years). This way it was defined the widest period, with no market data, and that the NSS model produced parameters that result in a very good fitting.

The adequacy of NSS model to get parameter values ($\beta_{1,2,3,4}$ and $\lambda_{1,2}$) accurate enough, with partial market data was evaluated for 3 sectors of the yield curve: short, intermediate and long term.

For STF, the model calibrated, only, with market yields for intermediate and long-term maturities, thus getting different values for the parameters as the ones obtained when all the market data was used to calibrate the model. The parameters values and countries yields curves with the lower forecast can be assessed in appendix II. Similarly, the same was done when calculating the intermediate and long-term maturities forecasts. For each of the forecast maturities, the model had access to only the other maturities, for which computed the factors values that best fitted the curve. The Solver function ran as much times as possible to get the best forecast fit values.

3.1 DATA

The study considers 304 different government bonds, from a group of 20 countries (Austria, Belgium, Bulgaria, Czech Republic, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Lithuania, Luxembourg, Netherlands, Portugal, Slovakia, Slovenia, Spain, Sweden and Switzerland) with at least one negative yield to maturity government bond, at the date of data collection, March 15th, 2017 for the following issuer countries: Austria, Denmark, Finland, France, Netherlands, Sweden and Switzerland, March 16th, 2017, for the following issuer countries: Germany, Japan, and May 5th, 2017 for the following issuer countries: Belgium, Bulgaria, Czech Republic, Ireland, Italy, Lithuania, Luxembourg, Portugal, Slovakia, Slovenia and Spain. Bloomberg was the data source used to get bonds information used in the study, through a Bloomberg Terminal. Inflation indexed bonds were not considered.

After a first approach to evaluate NSS model for the entire yield curve of countries whose data was gathered at March 15^{th} and 16^{th} 2017, the set of issuers were extended to incorporate the other 11 countries that presented negative yield to maturity at May 5^{th}, 2017. The extent of study countries was made taking into consideration two main purposes: first, to try to get more issuers to evaluate model adequacy to a wider set of data; second, and because from the second set of countries they are mainly from Europe and under ECB monetary policy, to try to get a wider, detailed and if possible, some conclusion that could apply to Europe and/or Eurozone area.

From the actual 19 countries that compose Eurozone (European Union, 2017) (use the Euro as their official currency and are under ECB monetary policy), 14 of them are included in this study. The other 5 Eurozone countries (Cyprus, Estonia, Greece, Latvia and Malta) were not included in the study, since they did not have any fixed income security with a negative yield, at the study period; March 15^{th} and 16^{th}, 2017 and May 5^{th}, 2017.

Presently, European Union has 28 members (European Union, 2017), so, half of the members had, at study dates, negative bond yields. Croatia had negative yields for the period of 2016 end to beginning of 2017, although by the time of data collection (May 5^{th}, 2017) yields for all maturities were positive.

Table I and Figure 1 show how many different securities were used for each country as well as the denomination currencies.

Tables II and III show the countries target of the study, their date of data collection, correspondent monetary policy institution, currency, if they belong to the European

Union, the β_1 and $\beta_1+\beta_2$ theoretical values (obtained from the fitting process) it´s observed values, and notes.

Table I - Number of bonds per country

Countries	Number of bonds	Situation	Currency
Austria	16	Included	EUR
Belgium	14	Included	EUR
Bulgaria	9	Included	BGN
Croatia	9	Excluded	HRK
Czech Republic	12	Included	CZK
Denmark	6	Included	DKK
Finland	12	Included	EUR
France	26	Included	EUR
Germany	38	Included	EUR
Ireland	12	Included	EUR
Italy	15	Included	EUR
Japan	18	Included	JPY
Lithuania	11	Included	EUR
Luxembourg	6	Included	EUR
Netherlands	14	Included	EUR
Portugal	13	Included	EUR
Slovakia	12	Included	EUR
Slovenia	13	Included	EUR
Spain	15	Included	EUR
Sweden	16	Included	SEK
Switzerland	17	Included	CHF
Total	304		

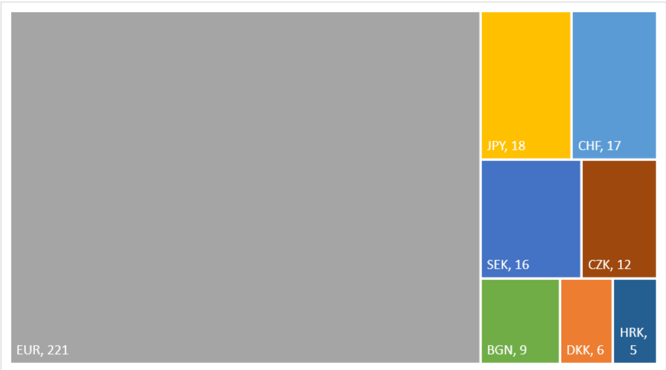

Figure 1 – Number of bonds per currency

Table II collects all countries under ECB monetary policy, so use Euro as their currency. These countries share and are in the same currency risk and in the same rates referential so they compose an important subset of the data sample. Table III collects all the other cases. It wasn´t consider the European Union countries do to the fact that Bulgaria, Czech Republic, Denmark and Sweden can determine their rates independently from the

ECB and can control their currency exchange rate (Bulgaria has a fixed exchange rate to the Euro). Tables IV and V are, respectively, to countries under ECB and the ones outside ECB monetary policy. These tables show the data if it is considered the theoretical value, for the IRVDF, the yield to maturity of the lowest maturity bond (1 year). Tables VI and VII are, respectively, to countries under ECB and the ones outside ECB monetary policy. These tables show the data considering the observed value, for the IRVDF, the yield to maturity of the highest maturity bond.

3.2 Analysis

The application of Solver function, to all bonds of the countries described in Data chapter, took into consideration the following conditions: a GRG Non-linear algorithm for the resolution method[3], restriction precision value of 10^{-8} (the standard value used by Solver is 10^{-6}, a lower value will provide a more precise value, although increases the time Solver spends to get to a solution), it was used the default selection for Solver to Use automatic rounding, the value chosen for the Convergence (value between 0 and 1) was 10^{-8}, it defines the upper limit for the relative change in destiny cell, for the last 5 iterations, to define when Solver should stop (i.e. if in the last 5 iterations the relative change in the value of the destination cell in less than 10^{-6}%, then Solver stops to try to converge even more) (Microsoft, 2017a).

Since the results obtained with direct differentiation (default on Solver) were very good considering that there wasn´t observed difference from the ones obtained for central differentiation (for a few countries tested), also there were no message from Solver

[3] Generalized Reduced Gradient algorithm for optimizing nonlinear problems.

mentioning it couldn´t improve, and because direct differentiation is much faster, it was used direct differentiation for all yield curves fitting computation.

Solver uses a Generalized Reduced Gradient Algorithm for optimizing nonlinear problems (Microsoft, 2017b), which, provides a locally optimal solution to a reasonably well-scaled, non-convex model (Frontline System, 2017b). A function *f* is convex if the function *f* is bellow any line segment between two points on *f*. Figure 2 is an adaptation from Tomioka (2012) and shows an example of convex and non-convex function.

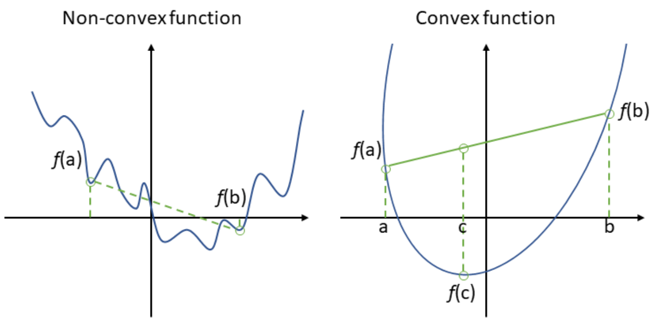

Figure 2 - Convex and non-convex function

The starting values for ($\beta_{1,2,3,4}$ and $\lambda_{1,2}$) should be in or as near as possible of the order of magnitude of the expected values. Values near or below 0.01 for β_i and 1 to λ_j were used. After first solution provided by Solver, the parameters values were submitted to small changes and the Solver function ran again, to get a SSR as low as possible. Only when Solver provided the message that after 5 iterations the fitting curve hasn't changed, that solution was considered as the final one. It wasn´t applied any restrictions to the values that ($\beta_{1,2,3,4}$ and $\lambda_{1,2}$) could take.

When modelling the entire yield curve, having NSS model access to all market yields collects from the countries, to use in SSR, or when modelling the entire yield curve, with part of the market data available (i.e. the cases of short-term, intermediate and long-term, bonds maturities), the parameters ($\beta_{1,2,3,4}$ and $\lambda_{1,2}$) could take any value, and no restriction was applied to them. The parameters values obtained for each country can be accessed on Table VIII (NSS model used all market yields available), Table IX (short term maturities forecast or simply STF), Table X (intermediate term maturities forecast or simply, intermediate term forecast (ITF)) and Table XI (long term maturities forecast or simply, long term forecast (LTF)).

It has been referred that β_1 can be interpreted as the IRVDF and $\beta_1+\beta_2$ as the IIR. It was considered in this study, the IIR, as the overnight rate (in practice, instantaneous rate can be identified with an overnight forward rate (Svensson, 1994)) supervised by the countries monetary policy institution. For countries under ECB rules the rate considered is the unsecured overnight lending rate, Eonia®[4] (Euro OverNight Index Average). Eonia® is the observed value that compares with the theoretical obtained from the NSS model. The definition of a very distant future and its correspondent interest rate for that time horizon is, in a certain way, a not concrete date. Due to market present situation of ECB monetary easing policy that are intended to run until the end of December 2017, or beyond, if necessary (ECB, 2017b), and considering the most time distant rate at which Euro interbank term deposits are offered Euribor®[5] 12 months, this was the rate chosen as the observed value to compare with β_1.

In Table III, due to the uniqueness of each country monetary policy institution the rates considered to be the benchmark for β_1 (IRVDF) and $\beta_1+\beta_2$ (IIR) are diversified. For the $\beta_1+\beta_2$ it was chosen the corresponding overnight rate or then the repo rate with the shorter time horizon (a repo rate is the rate at which banks can borrow from their Central bank). Hladíková & Radová (2012) also used the repo rate to compare with the

[4] Available at: https://www.emmi-benchmarks.eu/euribor-eonia-org/eonia-rates.html Accessed date: August 6th, 2017

[5] Available at: https://www.emmi-benchmarks.eu/euribor-org/about-euribor.html Accessed date: August 6th, 2017

starting value of estimated forward rate. These two rates are very close to one another (Martellini et al, 2003). Similarly, for β_1 (IRVDF), it was chosen the corresponding rate equivalent to the country´s Euribor.

Theoretical and observed IRVDF and IIR can be compared on Figures 3 and 4 and Tables II and III. As mentioned before, the definition of very distant future is not concrete, so it was considered the following two changes when evaluating the data and analysis;

- theoretical value, considered as the YTM of the lowest maturity bond (1 year). Data can be assessed on Tables IV and V, and Figure 5.
- observed value, considered as the YTM of the highest maturity bond. Data can be assessed on Tables VI and VII, and Figure 6.

A descriptive statistical analysis (with the calculation of: mean, median, standard deviation, kurtosis, asymmetry, minimum and maximum) was done to the differences of the theoretical and observed values. This, alongside with comparison with theoretical and observed values, can help to get more substantiated conclusions. This analysis was applied to two sets of countries data (all study countries and then to the subset of countries supervised by ECB) for both the IIR and the IRVDF.

Since the main set of countries in the study are from Europe, it was compared all yield curves (Figure 7) for these issuers. The spectrum of maturities that each country chooses, or can access to, in the market, is very different, as well as the yields that each one has, are very wide. The differences for the yield curves are related to the premiums required by the market and are dependent on: ratings, political risk, GDP growth, debt levels, economic development, among other variables.

The 10-year maturity bonds yield is one of the most used and compared in financial markets. For the set of European countries, only Lithuania hadn´t maturities higher than 7 years, so it cannot be compared with its fellow European countries.

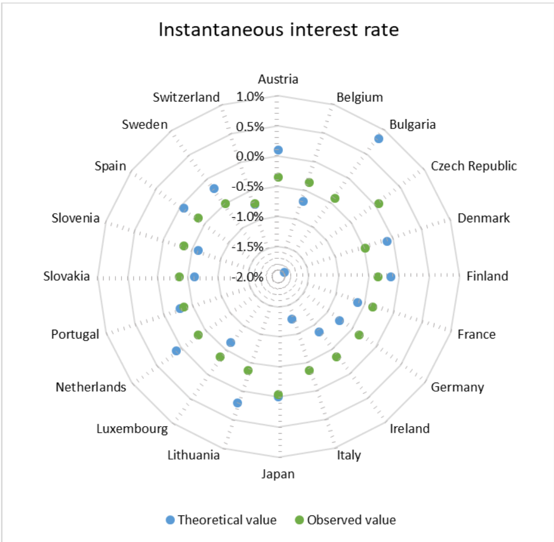

Figure 3 - Theoretical and observed IIR

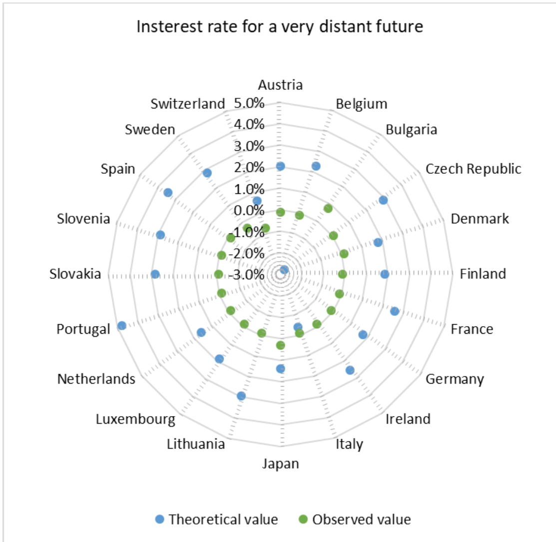

Figure 4 – IRVDF (with observed value considered as Euribor 12 M)

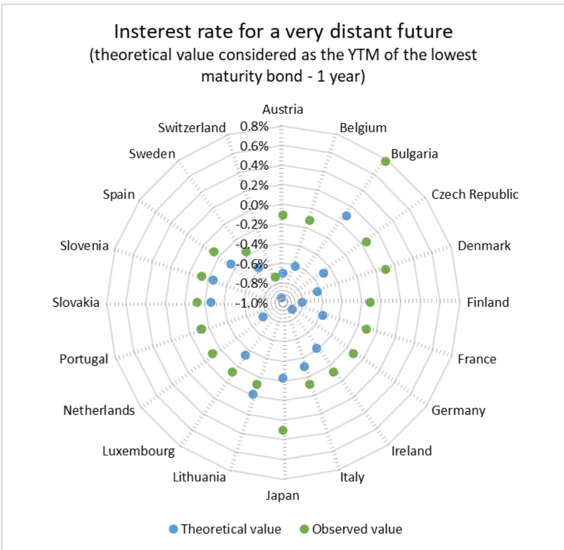

Figure 5 - IRVDF (with theoretical value considered as the YTM of the lowest maturity bond (1 year))

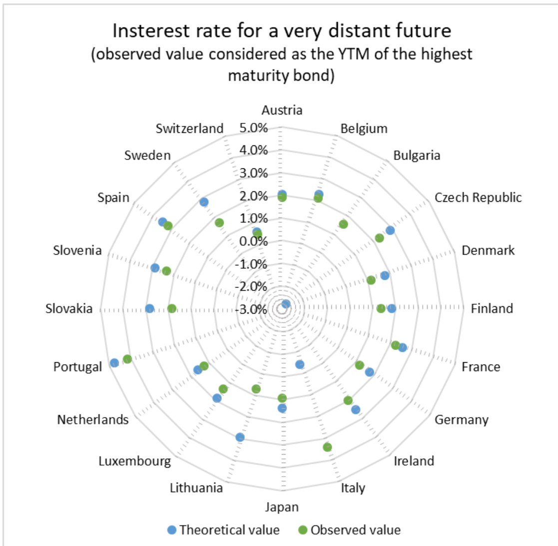

Figure 6 - IRVDF (with observed value considered as the YTM of the highest maturity bond)

As a theoretical exercise, if European countries on the Eurozone agreed eventually on a shared debt security (i.e. Eurobonds), it could, on an initial phase, be issued bonds with maturities at 10 years, being the higher maturities (>10 years) only by own country´s choice. On Figure 8, one can see this set of countries (without Lithuania) and their yield curves.

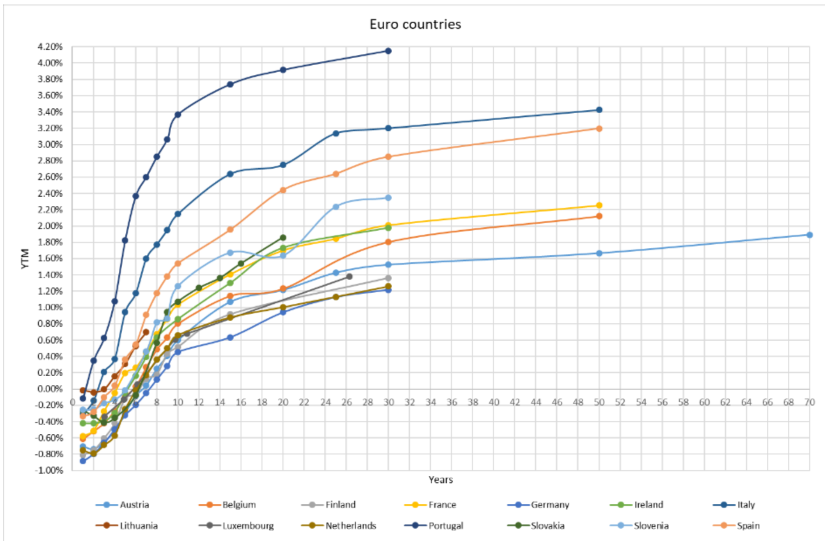

Figure 7 - European countries yield curves

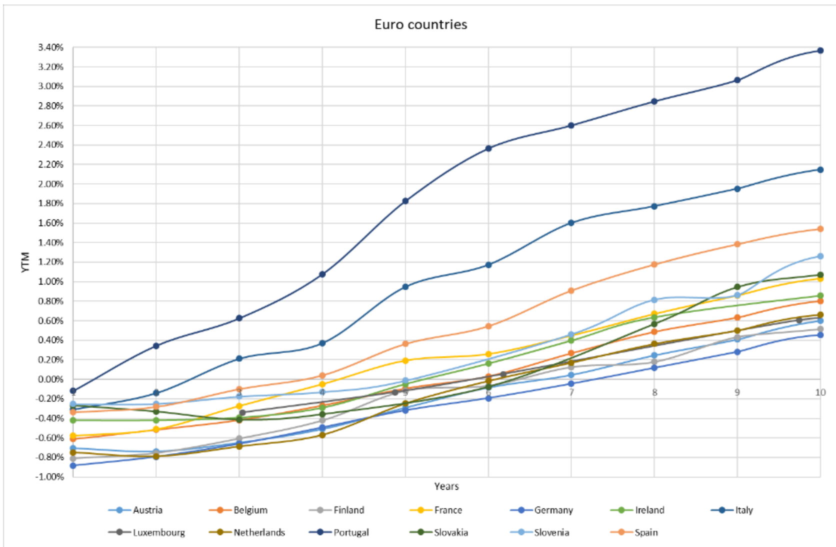

Figure 8 - European countries yield curves (maturities until 10 years)

It was evaluated, for the Eurozone countries, if the differences seen, for the theoretical and observed rates values, for β_1 (IRVDF), could be explained by the excess rate, that each country has related to Germany (Germany has the highest credit rating and their

Sovereign CDS, net of US is 0.00%)[6]. It´s mentioned also the Moody´s credit ratings, for each country.

Figure 9, shows the differences, for the theoretical and observed rates values, for β_1 (IRVDF), for two interpretations of the very distant future. One, the comparison between Sovereign CDS, net of US (or net of Germany, since both have the same value) (blue bar) and the Observed value, for β_1, considered as the YTM of the highest maturity bond (green bar). So, in case of Portugal, the excess rate that Portugal has related to Germany is 2.9342%, this means that the YTM of its highest maturity bond is 2.9342% higher than the YTM of the highest maturity bond of Germany, and the relation with the Sovereign CDS, net of US is of the same sign and similar value.

The second set of comparisons that can be accessed on Figure 9, are between: observed value for the β_1 parameter (considered as Euribor 12 Months) and its difference with Germany observed value (also, Euribor 12 Months) (red bar); and the difference between, the theoretical value for β_1 (considered as YTM of the lowest maturity bonds, 1 year) for each country and Germany (yellow bar).

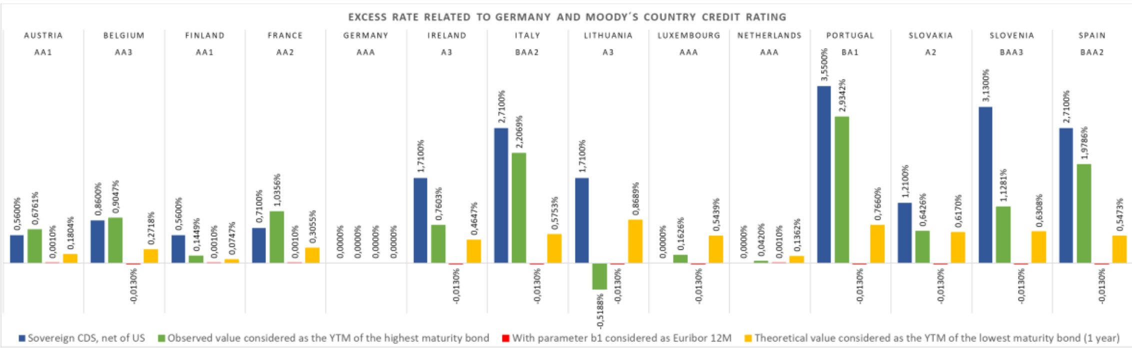

Figure 9 - Excess rate related to Germany

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[6] Available at: http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/ctryprem.html
Accessed date: June 10th, 2017

4 RESULTS AND DISCUSSION

The NSS model fitting process, with no restrictions on the parameters values, adjusts well the yield curve for the wide variety of countries and maturities range (appendix II). It was pointed out that Nelson-Siegel model was not appropriate to be applied to Japan Government Bonds market because it might show negative interest rate and abnormal shape in short term region (Kikuchi & Shintani, 2012, in Inui, 2015). In this work, and using NSS model, the curve shows a good fitting (Figure 20) and the difference between the short interest rate, chosen as the observed value, and the theoretical one, is 0.044%, which is a low value (Table III).

The values obtained for β_1 and $\beta_1+\beta_2$, interpreted, as IRVDF (Figure 4 and Tables II and III) and IIR respectively (Figure 3 and Tables II and III), show that theoretical and observed values are closer to each other for the IIR, than for the IRVDF, who presents a wider difference.

If the YTM of the highest maturity is considered the observed value for the IRVDF, the values are very close to the theoretical ones. Specifically, the excess rate related to Germany, can be almost fully explained.

The difference between theoretical and observed IIR, for the all sets of countries, has an almost normal distribution (kurtosis=3.14) with: mean of -0.055%, median of 0.019%, standard deviation of 0.644%, minimum of -1.926%, and a maximum of 1.233%. These results show a very wide range, probably influenced by different monetary policies. The same values, for the countries under ECB monetary policy, show a platykurtic distribution (kurtosis=-067) with: mean of -0.081%, median of -0.251%, standard deviation of 0.429%, minimum of -0.906% and a maximum of 0.564%, which represents a shorter range, suggesting the same monetary policy.

The difference between theoretical and observed IRVDF, for the all set of countries, has a leptokurtic distribution (kurtosis=5.92), with: mean of 2.058%, median of 2.274%, standard deviation of 1.688%, minimum of -3.501% and a maximum of 4.888%, showing significant dispersion. The same values for the countries under ECB monetary policy, show a platykurtic distribution (kurtosis=2.69), with: mean of 2.470%, median of 2.524%, standard deviation of 1.154%, minimum of -0.288% and a maximum of 4.888%, which also shows a wide range.

NSS model theoretical values for β_1 (IRVDF) are generally the value of the yield of the longest maturity in the yield curve (except for the extreme cases of Bulgaria, Italy, Lithuania and Sweden). In a certain way, this is the most very distant future that is available for each country, so, if the highest maturity for each country is the market interpretation of very distant future, then the model provides good values. Otherwise if for "very distant future" one considers the one-year time frame, then the model is not a proper one.

The results for short, intermediate and long-term forecasts, can be assessed, respectively in appendices II, III and IV. The short-term forecast shows the model difficulties to fit the yield curve, given that the beginning of the yield curves is less smooth than the intermediate and long terms. Also, it is in the shorter term that negative yields appear.

The intermediate and long-term forecasts show very acceptable fittings, in some cases with very few maturities the NSS model can adjust the entire curve.

Considering the subset of countries and yield curves that can be seen on Figure 8, and if eventually a shared debt security (i.e. Eurobonds) issue was done, the market would, theoretically, lower the risk premium and the yields for the most stressed countries (the ones that show higher yields). For the lower risk premium issuers, it will increase the yields. Since, all countries would share the risk, these risk premiums reflected on yields, could be a price to pay to get a more equal and more distressful financial system in the Eurozone.

The evaluation of rate differences related to excess rate to Germany show that there is a clear relation, Figure 9, between excess rate observed and the Sovereign CDS, net of US, at least for the main set of countries considered. Only for Ireland, Lithuania and Slovenia, the differences are higher than 1%. If it´s taken into consideration that bonds data was gathered at least 3 months after the Sovereign CDS considered. The excess rate related to Germany is well explained.

5 CONCLUSIONS

The NSS model was applied to 20 countries and it shows that fits well the entire yield curves, even for negative yields. It is a very friendly methodology and can be used with a simple and widely available tool.

The forecast of the IIR can be improved, although the differences between theoretical and observed values, appear to be small.

If, the IRVDF is considered the rate at the highest bond maturity, then the model, presents good values.

The interpretation of the parameters for the NSS model as they are interpreted to Nelson Siegel (NS) model, seems to be adequate.

In the case of countries under ECB monetary policy, the rate is defined by ECB, but, in practice, European countries in the Eurozone are very different in essence (e.g. economic models, debt levels, financial history, weight and importance on financial markets). So, expecting them all to have the same rates, from the model, seems not be a realistic hypothesis. It can be concluded that rates shouldn´t be all the same, since the market is requesting a country risk premium (CPR) for each one, related to their ratings, debt level, GDP, national budgets and deficits, political risk, among other factors. If European countries under the Eurozone had the same debt securities, like Eurobonds, then rates would be the same and yield curve would be only one, so the expected rates values given by the NSS model could be more precise and a good proxy for the market participants. Despite these results, Eurozone same debt securities could be target for further investigation.

The excess rate related to Germany, by the Moody´s ratings and corresponding Sovereign CDS, net of US (or Germany, since both countries share the same value) for countries under ECB monetary policy, can be explained from the model parameters, when considering the IRVDF to be the yield to maturity of highest maturity for that country. The countries that presented a difference higher than 1%, are Ireland, Lithuania and Slovenia.

The forecast outputs show good fitting data to real values for both intermediate term and long-term maturities. On the other hand, short term forecasted values aren´t as accurate as expected which leads to conclude that, in this case, it isn´t a good model. The reasons for this can be the instability of monetary policy and the volatility of short term interest rates.

Hence, the NSS model can be a valuable, easy to use and adaptable tool, to fit the yield curve with negative yields, for the use of the monetary policy institutions and market players, at least in a static way.

6 REFERENCES

Annaert, J., Claes, A., Ceuster, M., Zhang, H. (2010). *Estimating the Yield Curve Using the Nelson-Siegel Model – A Ridge Regression Approach*, Universiteit Antwerpen.

BIS (2005). *Zero-coupon yield curves: technical documentation*, BIS Papers, 25, Bank for International Settlements.

Büttler, H. (2007). *An Orthogonal Polynomial Approach to Estimate the Term Structure of Interest Rates,* Swiss National Bank Working Papers, 8.

Cantor, R. & Packer, F. (1996). *Determinants and Impact of Sovereign Credit Ratings*, Federal Reserve Bank of New York, 37-53.

Cox, J. C., Ingersoll, J. E., Ross, S. A. (1985). *A Theory of the Term Structure of Interest Rates*, Econometrica, 53 (2), 385-407.

ECB (2017a). *Asset purchase programmes*. Available at: https://www.ecb.europa.eu/mopo/implement/omt/html/index.en.html [Accessed date: August 7th, 2017].

ECB (2017b). *Monetary policy decisions*. Press Release, 20 July 2017. Available at: https://www.ecb.europa.eu/press/pr/date/2017/html/ecb.mp170720.en.html [Accessed date: August 6th, 2017].

European Union (2017). *EU monetary cooperation*. Available at: https://europa.eu/european-union/about-eu/money/euro_en [Accessed date: March 15th, 2017].

Financial Times (2016). *Value of negative-yielding bonds hits $13.4tn*. Available at: https://www.ft.com/content/973b6060-60ce-11e6-ae3f-77baadeb1c93?mhq5j=e3 [Accessed date: July 2nd, 2017].

Frontline Systems (2017a). *Excel Solver Online Help*. Available at: https://www.solver.com/excel-solver-online-help [Accessed date: August 8th, 2017].

Frontline Systems (2017b). *Excel Solver – GRG Nonlinear Solving Method Stopping Conditions*. Available at: https://www.solver.com/excel-solver-grg-nonlinear-solving-method-stopping-conditions [Accessed date: June 15th, 2017].

Geyer, A. & Mader, R. (1999). *Estimation of the Term Structure of Interest Rates – A Parametric Approach,* Oesterreichische Nationalbank Working Paper 37.

Gilli, M., Große, S. and Schumann, E. (2010). *Calibrating the Nelson-Siegel-Svensson model*, COMISEF Working Paper Series 31.

Guedes, J. (2008). *Modelos Dinâmicos da Estrutura de Prazo das Taxas de Juro*, IGCP Instituto de Gestão da Tesouraria e do Crédito Público, I. P..

Hayashi, F. & Prescott, E. C. (2002). *The 1990s in Japan: A Lost Decade*, Review of Economic Dynamics, 5 (1), 206-235.

Hladíková, H. & Radová, J. (2012). *Term Structure Modelling by Using Nelson-Siegel Model*, European Financial and Accounting Journal, 7 (2), 36-55.

Ibáñez, F. (2015). *Calibrating the Dynamic Nelson-Siegel Model: A Practitioner Approach*, Central Bank of Chile.

Inui, K. (2015). *Improving Nelson-Siegel term structure model under zero / super-low interest rate policy*, Meiji University.

Martellini, L., Priaulet, P. and Priaulet, S. (2003). *Fixed-Income Securities Valuation, Risk Management and Portfolio Strategies*, Wiley.

Microsoft (2017a). *Função SolverOptions*. Available at: https://msdn.microsoft.com/pt-br/library/office/ff195446.aspx [Accessed date: June 15th, 2017].

Microsoft (2017b). *Solver Uses Generalized Reduced Gradient Algorithm*. Available at: https://support.microsoft.com/en-us/help/82890/solver-uses-generalized-reduced-gradient-algorithm [Accessed date: July 2nd, 2017].

Nelson, C. & Siegel, A. (1987). Parsimonious Modelling of Yield Curve, *Journal of Business*, 60, 473-489.

OECD (2017). *General government debt (indicator)*. doi: 10.1787/a0528cc2-en; Available at: https://data.oecd.org/gga/general-government-debt.htm [Accessed date: June 17th, 2017].

Sundaresan, S. (2009). *Fixed Income Markets and Their Derivatives*, Academic Press, Third Edition.

Svensson, L. E. (1994). Estimating and Interpreting Forward Interest Rates: Sweden 1992-1994, *Centre for Economic Policy Research* Discussion Paper 1051.

Tomioka, R. (2012). *Convex Optimization: Old Tricks for New Problems*, The University of Tokyo, DTU PhD Summer Course.

Appendices

Appendix I – Countries data

Table II - Countries data under ECB monetary policy

Country	Date of data aquisition		Theoretical value	Observed value	Δ	Notes	Monetary Policy Institution	Currency	European Union
Austria	03/15/2017	β1	2.0225%	-0.1100%	2.1325%	Euribor 12 M	ECB	Euro	Yes
		β1+β2	0.0980%	-0.3540%	0.4520%	Eonia			
Belgium	05/05/2017	β1	2.2917%	-0.1240%	2.4157%	Euribor 12 M	ECB	Euro	Yes
		β1+β2	-0.6863%	-0.3570%	-0.3293%	Eonia			
Finland	03/15/2017	β1	1.8388%	-0.1100%	1.9488%	Euribor 12 M	ECB	Euro	Yes
		β1+β2	-0.1306%	-0.3540%	0.2234%	Eonia			
France	03/15/2017	β1	2.5685%	-0.1100%	2.6785%	Euribor 12 M	ECB	Euro	Yes
		β1+β2	-0.6198%	-0.3540%	-0.2658%	Eonia			
Germany	03/16/2017	β1	1.7462%	-0.1110%	1.8572%	Euribor 12 M	ECB	Euro	Yes
		β1+β2	-0.7518%	-0.3540%	-0.3978%	Eonia			
Ireland	05/05/2017	β1	2.5090%	-0.1240%	2.6330%	Euribor 12 M	ECB	Euro	Yes
		β1+β2	-0.8571%	-0.3570%	-0.5001%	Eonia			
Italy	05/05/2017	β1	-0.4124%	-0.1240%	-0.2884%	Euribor 12 M	ECB	Euro	Yes
		β1+β2	-1.2632%	-0.3570%	-0.9062%	Eonia			
Lithuania	05/05/2017	β1	2.9534%	-0.1240%	3.0774%	Euribor 12 M	ECB	Euro	Yes
		β1+β2	0.2070%	-0.3570%	0.5640%	Eonia			
Luxembourg	05/05/2017	β1	1.8750%	-0.1240%	1.9990%	Euribor 12 M	ECB	Euro	Yes
		β1+β2	-0.6429%	-0.3570%	-0.2859%	Eonia			
Netherlands	03/15/2017	β1	1.5679%	-0.1100%	1.6779%	Euribor 12 M	ECB	Euro	Yes
		β1+β2	0.0971%	-0.3540%	0.4511%	Eonia			
Portugal	05/05/2017	β1	4.7638%	-0.1240%	4.8878%	Euribor 12 M	ECB	Euro	Yes
		β1+β2	-0.2846%	-0.3570%	0.0724%	Eonia			
Slovakia	05/05/2017	β1	2.8222%	-0.1240%	2.9462%	Euribor 12 M	ECB	Euro	Yes
		β1+β2	-0.6126%	-0.3570%	-0.2556%	Eonia			
Slovenia	05/05/2017	β1	2.8705%	-0.1240%	2.9945%	Euribor 12 M	ECB	Euro	Yes
		β1+β2	-0.6078%	-0.3570%	-0.2508%	Eonia			
Spain	05/05/2017	β1	3.4861%	-0.1240%	3.6101%	Euribor 12 M	ECB	Euro	Yes
		β1+β2	-0.0656%	-0.3570%	0.2914%	Eonia			

Table III - Countries data not under ECB monetary policy

Country	Date of data aquisition		Theoretical value	Observed value	Δ	Notes	Monetary Policy Institution	Currency	European Union
Bulgaria	05/05/2017	β1	-2.7195%	0.782%	-3.5015%	SOFIBOR (Sofia Interbank Offered Rate)	Bulgarian National Bank	BGN	Yes
		β1+β2	0.8329%	-0.4000%	1.2329%	LEONIA (LEv OverNight Index			

						Average) Reference Rate			
Czech Republic	05/05/2017	$\beta1$	2.8872%	0.0500%	2.8372%	Deposit Facility	Czech National Bank	CZK	Yes
		$\beta1+\beta2$	-1.8763%	0.0500%	-1.9263%	2W repo rate			
Denmark	03/15/2017	$\beta1$	1.7728%	0.0950%	1.6778%	CIBOR 12M	Denmark National Bank	DNK	Yes
		$\beta1+\beta2$	-0.1107%	-0.4857%	0.3750%	Tomorrow/next (T/N) Rate			
Japan	03/16/2017	$\beta1$	1.3822%	0.3000%	1.0822%	Basic Discount Rates and Basic Loan Rates	Bank of Japan	Yen	No
		$\beta1+\beta2$	0.0010%	-0.0430%	0.0440%	Average value of Uncollateralized Overnight Call Rate for Mar. 16			
Sweden	03/15/2017	$\beta1$	2.8118%	-0.3650%	3.1768%	STIBOR Fixing 6M	Sweden National Bank	SEK	Yes
		$\beta1+\beta2$	-0.1883%	-0.5000%	0.3117%	Repo rate			
Switzerland	03/15/2017	$\beta1$	0.5743%	-0.7300%	1.3043%	3-month LIBOR CHF	Swiss National Bank	CHF	No
		$\beta1+\beta2$	-0.7354%	-0.7300%	-0.0054%	SARON (formerly repo overnight index (SNB))			

Table IV - Countries data under ECB monetary policy (theoretical value considered as the YTM of the lowest maturity bond - 1 year) – IRVDF

Country	Date of data aquisition	Theoretical value (considered as the YTM of the lowest maturity bond, 1 year)	Observed value	Δ	Notes	Monetary Policy Institution	Currency	European Union
Austria	03/15/2017	-0.7037%	-0.1100%	-0.5937%	Euribor 12M	ECB	Euro	Yes
Belgium	05/05/2017	-0.6123%	-0.1240%	-0.4883%	Euribor 12M	ECB	Euro	Yes
Finland	03/15/2017	-0.8094%	-0.1100%	-0.6994%	Euribor 12M	ECB	Euro	Yes
France	03/15/2017	-0.5786%	-0.1100%	-0.4686%	Euribor 12M	ECB	Euro	Yes
Germany	03/16/2017	-0.8841%	-0.1110%	-0.7731%	Euribor 12M	ECB	Euro	Yes
Ireland	05/05/2017	-0.4194%	-0.1240%	-0.2954%	Euribor 12M	ECB	Euro	Yes
Italy	05/05/2017	-0.3088%	-0.1240%	-0.1848%	Euribor 12M	ECB	Euro	Yes
Lithuania	05/05/2017	-0.0152%	-0.1240%	0.1088%	Euribor 12M	ECB	Euro	Yes
Luxembourg	05/05/2017	-0.3402%	-0.1240%	-0.2162%	Euribor 12M	ECB	Euro	Yes
Netherlands	03/15/2017	-0.7479%	-0.1100%	-0.6379%	Euribor 12M	ECB	Euro	Yes
Portugal	05/05/2017	-0.1181%	-0.1240%	0.0059%	Euribor 12M	ECB	Euro	Yes
Slovakia	05/05/2017	-0.2671%	-0.1240%	-0.1431%	Euribor 12M	ECB	Euro	Yes
Slovenia	05/05/2017	-0.2533%	-0.1240%	-0.1293%	Euribor 12M	ECB	Euro	Yes
Spain	05/05/2017	-0.3368%	-0.1240%	-0.2128%	Euribor 12M	ECB	Euro	Yes

Table V - Countries data not under ECB monetary policy (theoretical value considered as the YTM of the lowest maturity bond - 1 year) – IRVDF

Country	Date of data aquisition	Theoretical value (considered as the YTM of the lowest maturity bond - 1 year)	Observed value	Δ	Notes	Monetary Policy Institution	Currency	European Union
Bulgaria	05/05/2017	0.0960%	0.7820%	-0.6860%	SOFIBOR (Sofia Interbank	Bulgarian National Bank	BGN	Yes

					Offered Rate)			
Czech Republic	05/05/2017	-0.4917%	0.0500%	-0.5417%	Deposit Facility	Czech National Bank	CZK	Yes
Denmark	03/15/2017	-0.6339%	0.0950%	-0.7289%	CIBOR 12M	Denmark National Bank	DNK	Yes
Japan	03/16/2017	-0.2303%	0.3000%	-0.5303%	Basic Discount and Basic Loan Rates	Bank of Japan	Yen	No
Sweden	03/15/2017	-0.5647%	-0.3650%	-0.1997%	STIBOR Fixing 6M	Sweden National Bank	SEK	Yes
Switzerland	03/15/2017	-0.9485%	-0.7300%	-0.2185%	3-month LIBOR CHF	Swiss National Bank	CHF	No

Table VI - Countries data under ECB monetary policy (observed value considered as the YTM of the highest maturity bond - 1 year) – IRVDF

Country	Date of data aquisition		Theoretical value	Observed value (considered as the YTM of the highest maturity bond)	Δ	Monetary Policy Institution	Currency	European Union
Austria	03/15/2017	β1	2.0225%	1.8931%	0.1294%	ECB	Euro	Yes
Belgium	05/05/2017	β1	2.2917%	2.1217%	0.1700%	ECB	Euro	Yes
Finland	03/15/2017	β1	1.8388%	1.3619%	0.4769%	ECB	Euro	Yes
France	03/15/2017	β1	2.5685%	2.2526%	0.3158%	ECB	Euro	Yes
Germany	03/16/2017	β1	1.7462%	1.2170%	0.5291%	ECB	Euro	Yes
Ireland	05/05/2017	β1	2.5090%	1.9774%	0.5317%	ECB	Euro	Yes
Italy	05/05/2017	β1	-0.4124%	3.4239%	-3.8363%	ECB	Euro	Yes
Lithuania	05/05/2017	β1	2.9534%	0.6983%	2.2551%	ECB	Euro	Yes
Luxembourg	05/05/2017	β1	1.8750%	1.3796%	0.4953%	ECB	Euro	Yes
Netherlands	03/15/2017	β1	1.5679%	1.2591%	0.3088%	ECB	Euro	Yes
Portugal	05/05/2017	β1	4.7638%	4.1513%	0.6125%	ECB	Euro	Yes
Slovakia	05/05/2017	β1	2.8222%	1.8597%	0.9625%	ECB	Euro	Yes
Slovenia	05/05/2017	β1	2.8748%	2.3451%	0.5297%	ECB	Euro	Yes
Spain	05/05/2017	β1	3.4861%	3.1956%	0.2904%	ECB	Euro	Yes

Table VII - Countries data not under ECB monetary policy (observed value considered as the YTM of the highest maturity bond - 1 year) – IRVDF

Country	Date of data aquisition		Theoretical value	Observed value (considered as the YTM of the highest maturity bond)	Δ	Monetary Policy Institution	Currency	European Union
Bulgaria	05/05/2017	β1	-2.7195%	1.6040%	-4.3235%	Bulgarian National Bank	BGN	Yes
Czech Republic	05/05/2017	β1	2.8872%	2.3068%	0.5804%	Czech National Bank	CZK	Yes
Denmark	03/15/2017	β1	1.7728%	1.1336%	0.6391%	Denmark National Bank	DNK	Yes
Japan	03/16/2017	β1	1.3822%	0.9289%	0.4533%	Bank of Japan	Yen	No
Sweden	03/15/2017	β1	2.8118%	1.7023%	1.1095%	Sweden National Bank	SEK	Yes
Switzerland	03/15/2017	β1	0.5743%	0.4627%	0.1116%	Swiss National Bank	CHF	No

Appendix II – Countries market and NSS model yield curves

Table VIII - NSS model $\beta_{1,2,3,4}$ and $\lambda_{1,2}$ factors (fitting the entire yield curve)

	β_1	β_2	β_3	β_4	λ_1	λ_2
Austria	0.020225	-0.019245	-0.120936	-0.076041	0.089915	1.791626
Belgium	0.022917	-0.029780	-0.885637	-0.074036	0.017113	1.960027
Bulgaria	-0.027195	0.035524	-0.090480	0.184881	2.357249	6.002618
Czech Republic	0.028872	-0.047634	-0.000031	-0.080808	0.587137	2.992772
Denmark	0.017728	-0.018835	-0.103959	-0.069599	0.071836	1.643519
Finland	0.018388	-0.019695	-0.028984	-0.044361	0.762873	2.345685
France	0.025685	-0.031882	0.002701	-0.038365	2.496532	2.566768
Germany	0.017462	-0.024980	-0.026692	-0.018188	1.736145	3.919552
Ireland	0.025090	-0.033661	-0.046439	-0.082346	0.172151	1.845687
Italy	-0.004124	-0.008508	-0.079507	0.128453	0.040498	19.122517
Japan	0.013822	-0.013812	-0.024702	-4.345566	4.373515	0.000334
Lithuania	0.029534	-0.027464	0.046821	-0.066374	5.818697	3.420608
Luxembourg	0.018750	-0.025178	-0.310594	-0.067273	0.019172	1.841284
Netherlands	0.015679	-0.014708	-0.064738	-0.069723	0.087805	1.341456
Portugal	0.047638	-0.050484	-0.219748	-0.125183	0.062279	1.106536
Slovakia	0.028222	-0.034348	-0.198596	-0.095590	0.054860	1.893719
Slovenia	0.028748	-0.034780	-0.213193	-0.088685	0.059622	1.919800
Spain	0.034861	-0.035517	-0.240889	-0.100771	0.068538	1.810130
Sweden	0.028118	-0.030002	-1.285713	-0.071285	0.026309	2.652613
Switzerland	0.005743	-0.013097	-0.026070	-0.000303	1.632627	0.002583

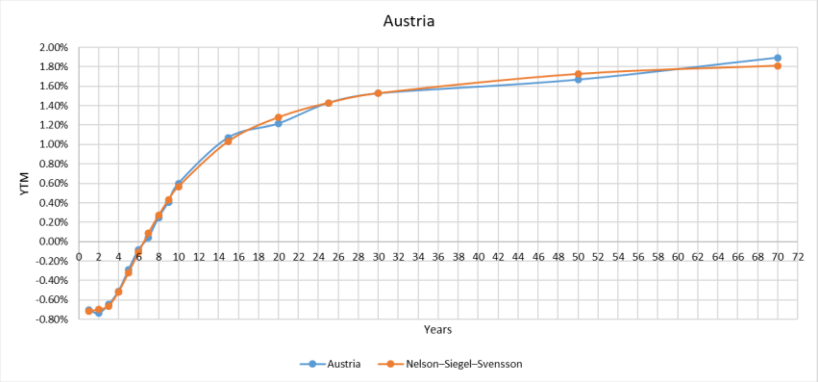

Figure 10 - Austria market and NSS yield curve (March 15, 2017)

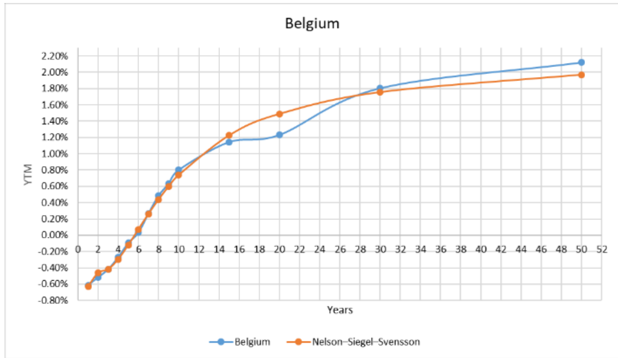

Figure 11 - Belgium market and NSS yield curve (May 5, 2017)

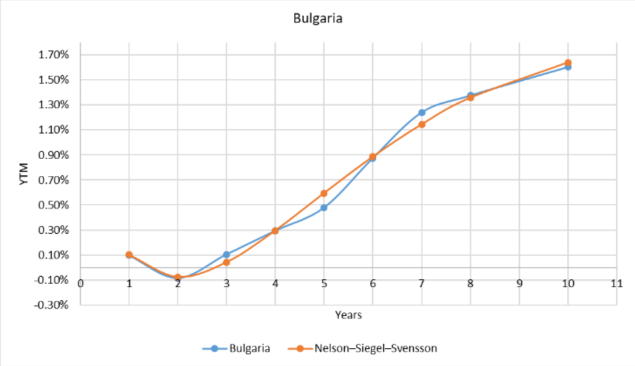

Figure 12 - Bulgaria market and NSS yield curve (May 5, 2017)

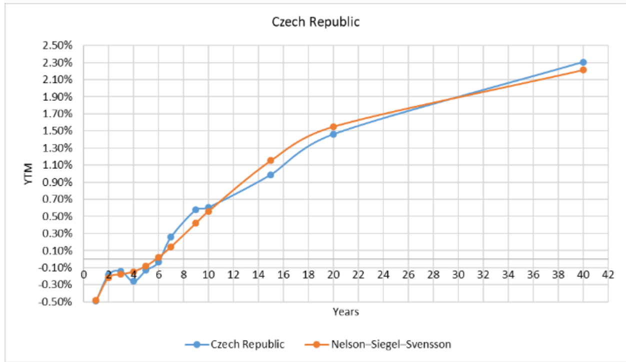

Figure 13 - Czech Republic market and NSS yield curve (May 5, 2017)

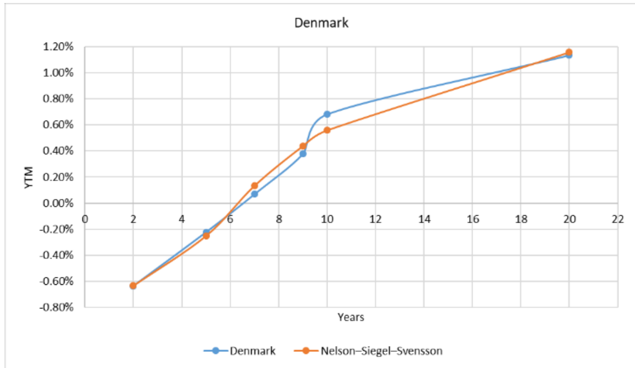

Figure 14 - Denmark market and NSS yield curve (March 15, 2017)

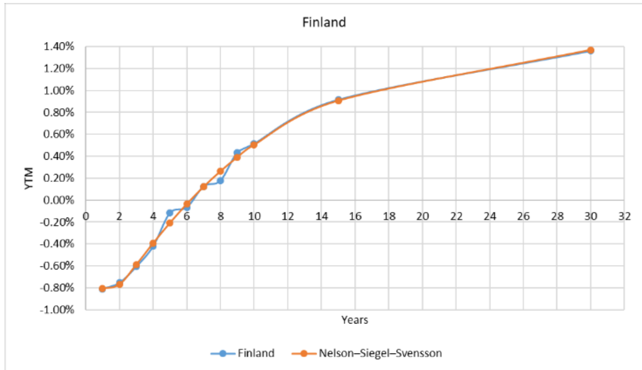

Figure 15 - Finland market and NSS yield curve (March 15, 2017)

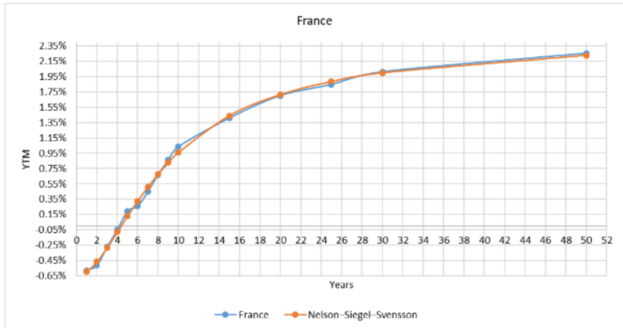

Figure 16 - France market and NSS yield curve (March 15, 2017)

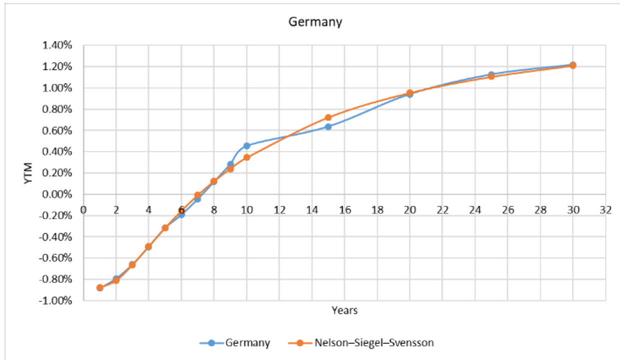

Figure 17 - Germany market and NSS yield curve (March 16, 2017)

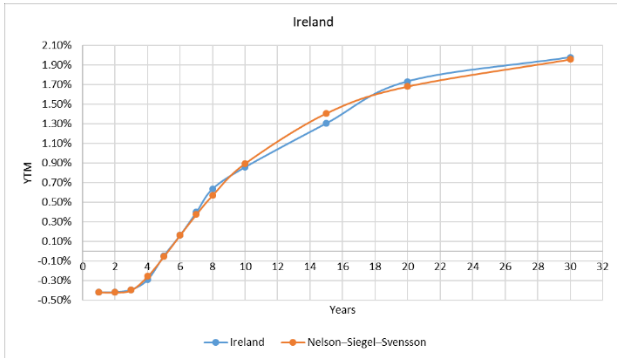

Figure 18 - Ireland market and NSS yield curve (May 5, 2017)

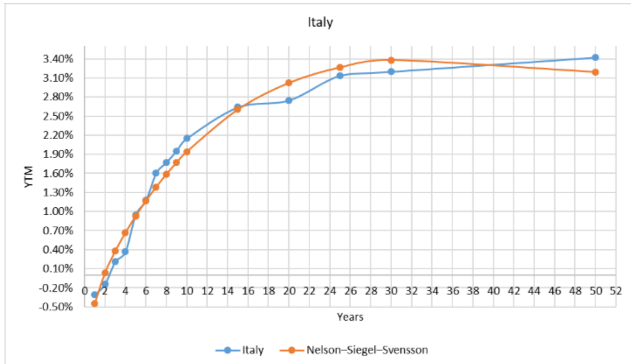

Figure 19 - Italy market and NSS yield curve (May 5, 2017)

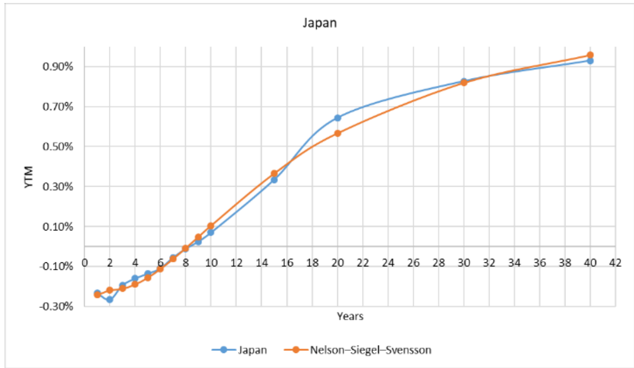

Figure 20 - Japan market and NSS yield curve (March 16, 2017)

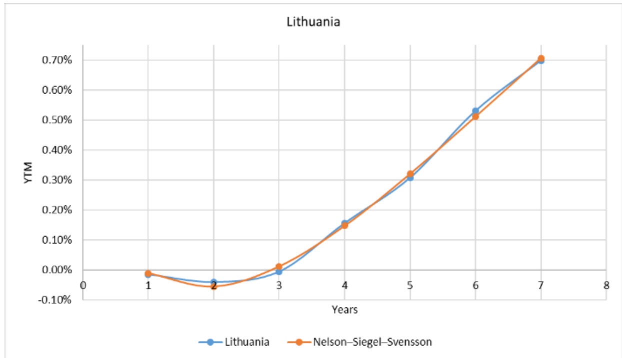

Figure 21 - Lithuania market and NSS yield curve (May 5, 2017)

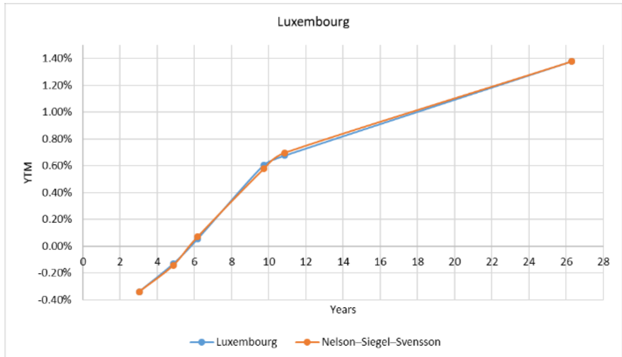

Figure 22 - Luxembourg market and NSS yield curve (May 5, 2017)

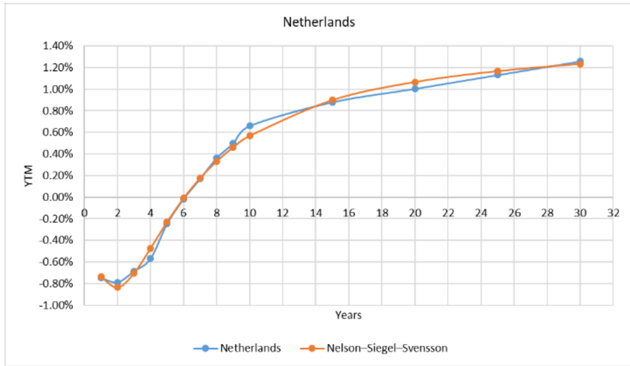

Figure 23 - Netherlands market and NSS yield curve (March 15, 2017)

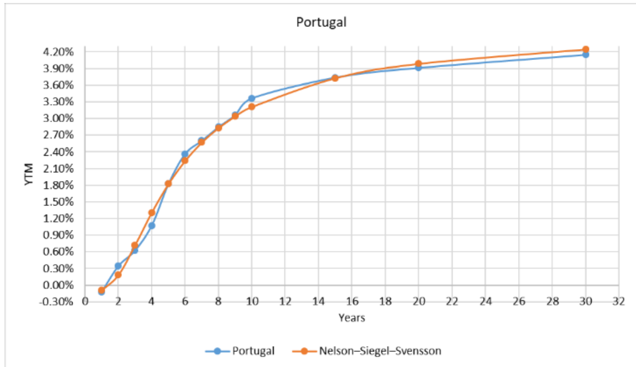

Figure 24 - Portugal market and NSS yield curve (May 5, 2017)

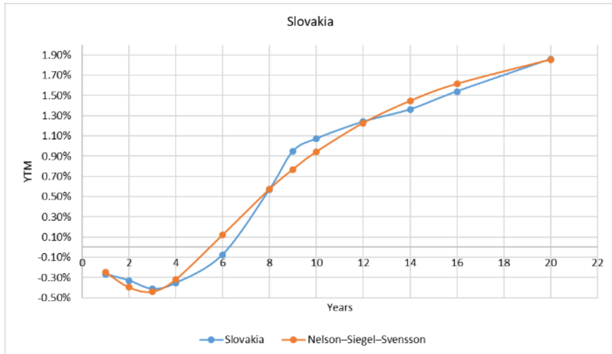

Figure 25 - Slovakia market and NSS yield curve (May 5, 2017)

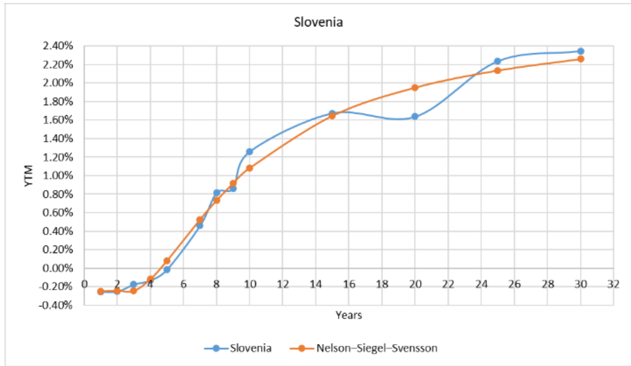

Figure 26 - Slovenia market and NSS yield curve (May 5, 2017)

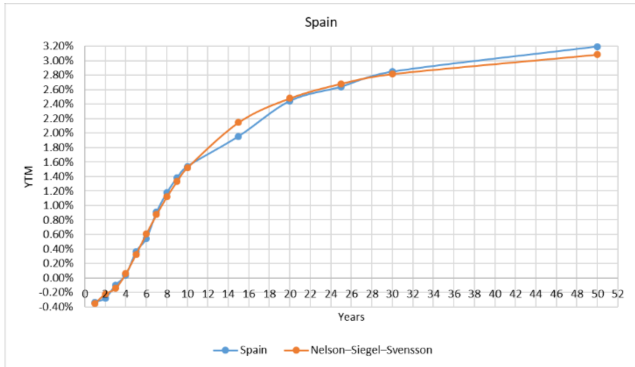

Figure 27 - Spain market and NSS yield curve (May 5, 2017)

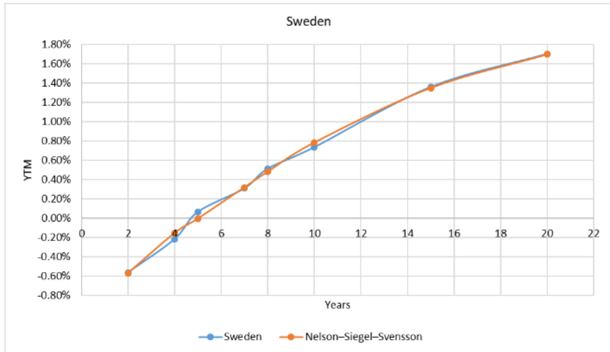

Figure 28 - Sweden market and NSS yield curve (March 15, 2017)

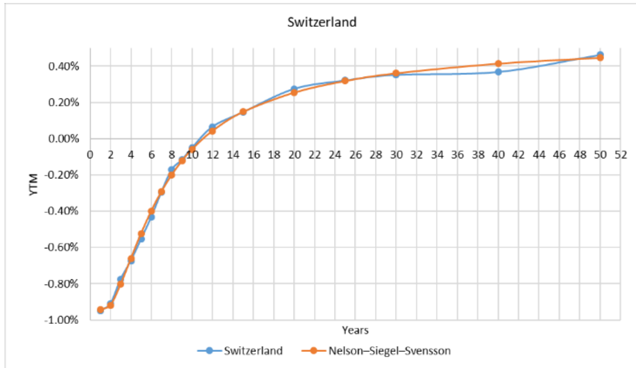

Figure 29 - Switzerland market and NSS yield curve (March 15, 2017)

Appendix III – Countries market and NSS model yield curves (short term forecast)

Table IX - NSS model $\beta_{1,2,3,4}$ and $\lambda_{1,2}$ factors (short term maturities forecast)

	β_1	β_2	β_3	β_4	λ_1	λ_2
Austria	0.020361	-0.008210	-0.066401	-0.029749	1.920081	0.304698
Belgium	0.025376	-0.037969	0.000077	0.000356	6.074789	0.000071
Bulgaria	0.030944	-0.013937	-0.000103	-0.082933	1.466186	1.491495
Czech Republic	0.011306	-0.017488	-0.026006	0.068027	8.156023	21.630683
Denmark	0.017778	-0.030000	-0.000051	-0.076807	0.009986	1.607286
Finland	0.020819	-0.033745	-0.017979	-2.010245	3.274978	19997.235701
France	0.025718	-0.030718	0.002344	-0.040094	2.463138	2.549057
Germany	0.016590	-0.026660	-0.000508	-0.027134	2.521240	2.616770
Ireland	0.025208	-0.032391	-0.069830	-0.078686	0.175415	1.908317
Italy	0.036929	-0.055425	-0.847133	0.806172	1.054440	0.987306
Japan	0.000566	-0.002726	0.291586	-0.270061	11.485179	10.347461
Lithuania	0.026793	-0.023638	0.036527	-0.072303	3.530776	2.682399
Luxembourg	0.018728	-0.019993	-0.308232	-0.070849	0.009939	1.783962
Netherlands	0.014816	-0.002200	0.220139	-0.289188	1.927579	1.744006
Portugal	0.046880	-0.386878	1.638792	-1.236006	0.489257	0.612912
Slovakia	0.026371	-0.060242	1.051857	-1.042339	0.917185	1.031749
Slovenia	0.027932	0.621663	-0.393937	-1.359934	1.232878	0.330309
Spain	0.035156	-0.035880	-0.251233	-0.094009	0.089443	1.937823
Sweden	0.029016	-0.013191	-1.088106	-0.077766	0.023536	2.750295
Switzerland	0.005696	-0.010164	-0.032022	0.117770	1.498992	0.003039

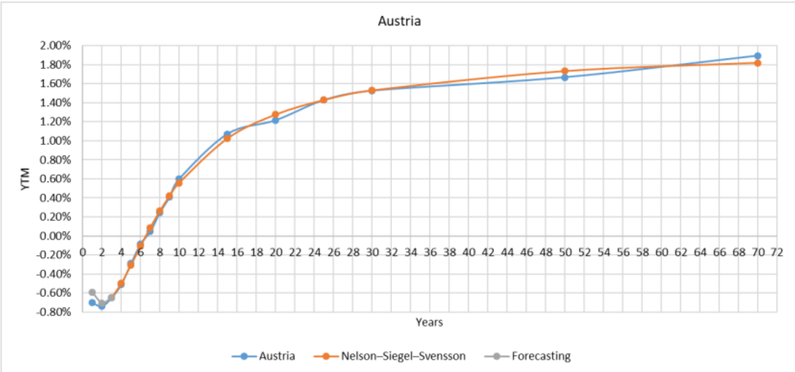

Figure 30 - Austria market and NSS yield curve (March 15, 2017) - STF

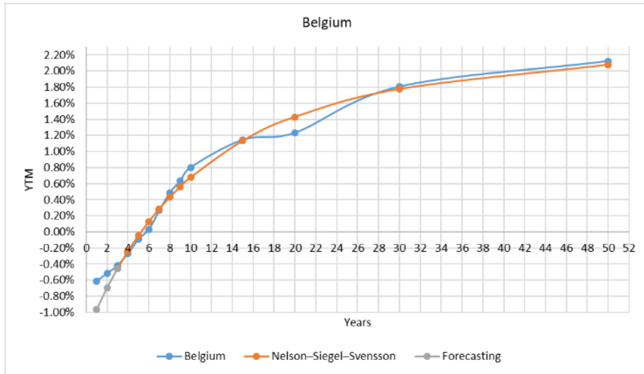

Figure 31 - Belgium market and NSS yield curve (May 5, 2017) - STF

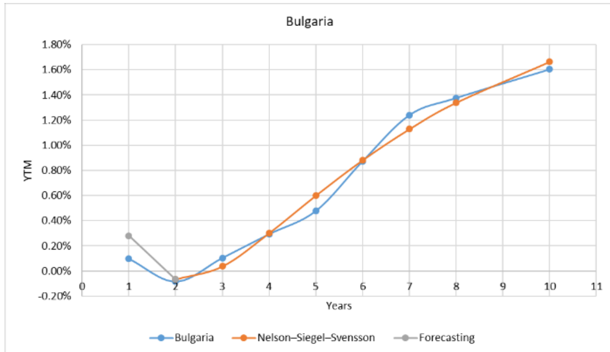

Figure 32 - Bulgaria market and NSS yield curve (May 5, 2017) - STF

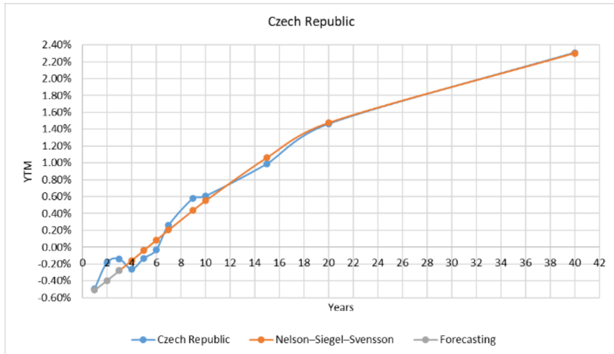

Figure 33 - Czech Republic market and NSS yield curve (May 5, 2017) - STF

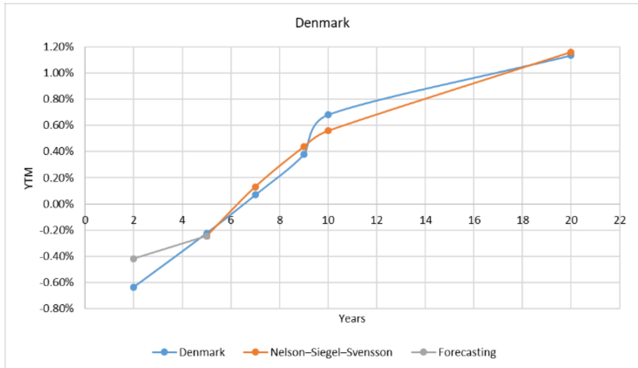

Figure 34 - Denmark market and NSS yield curve (March 15, 2017) - STF

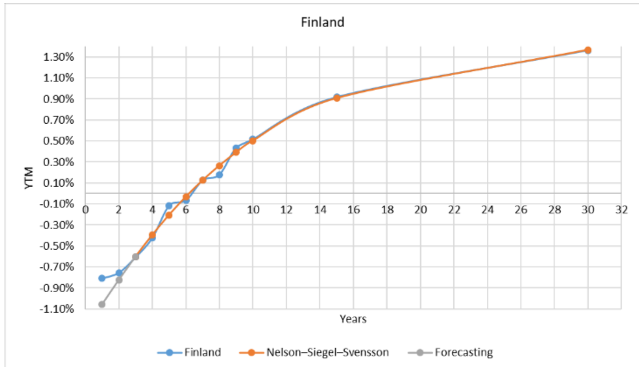

Figure 35 - Finland market and NSS yield curve (March 15, 2017) - STF

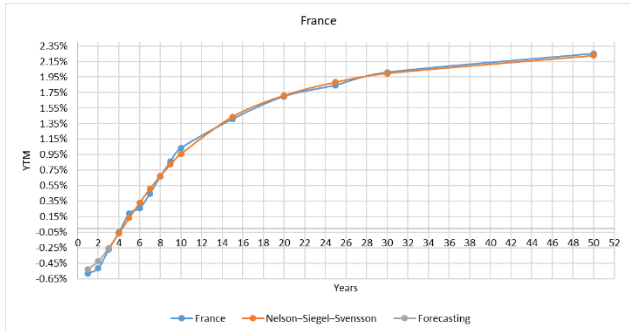

Figure 36 - France market and NSS yield curve (March 15, 2017) - STF

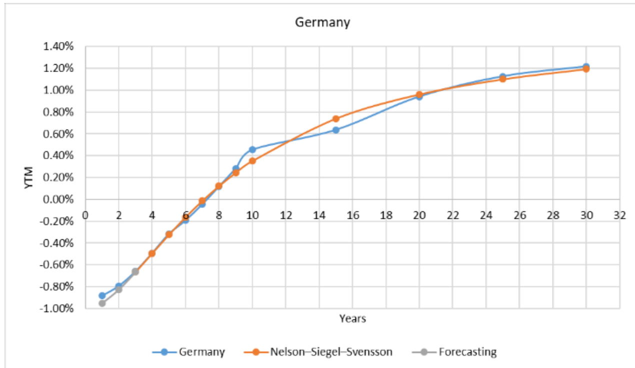

Figure 37 - Germany market and NSS yield curve (March 16, 2017) - STF

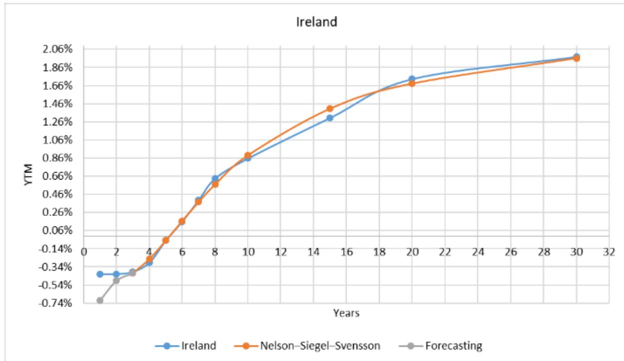

Figure 38 - Ireland market and NSS yield curve (May 5, 2017) - STF

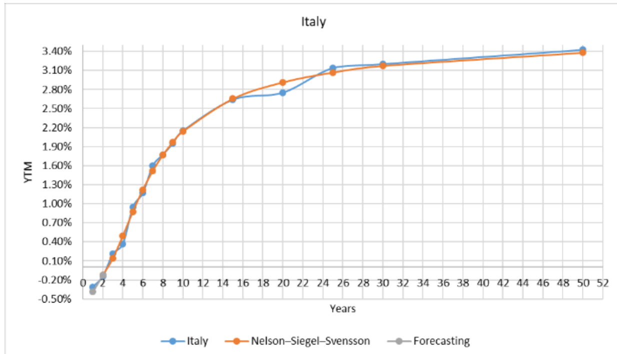

Figure 39 - Italy market and NSS yield curve (May 5, 2017) - STF

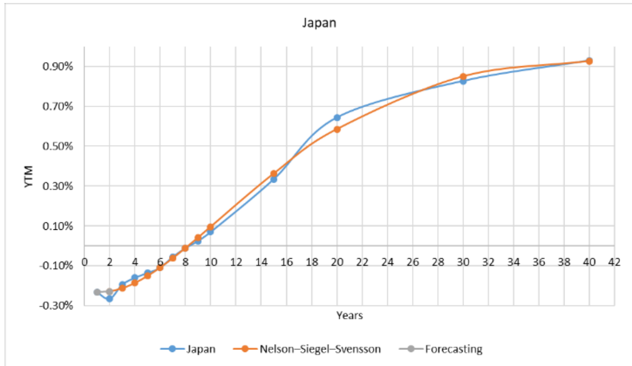

Figure 40 - Japan market and NSS yield curve (March 16, 2017) - STF

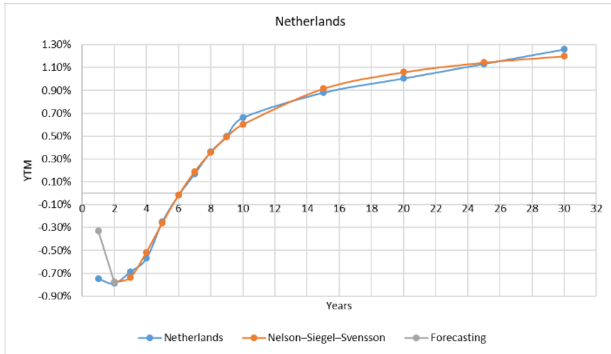

Figure 43 - Netherlands market and NSS yield curve (March 15, 2017) - STF

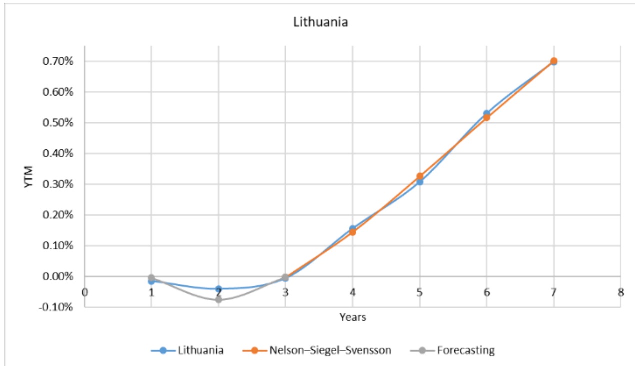

Figure 41 - Lithuania market and NSS yield curve (May 5, 2017) - STF

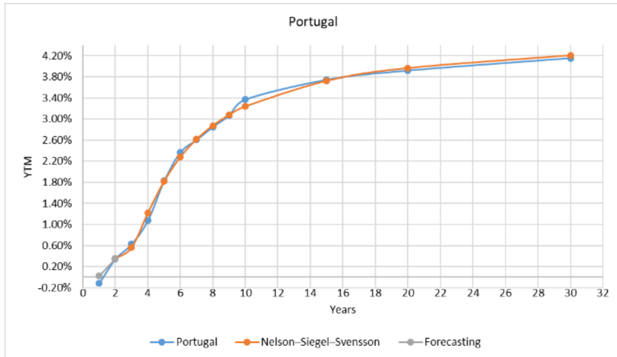

Figure 44 - Portugal market and NSS yield curve (May 5, 2017) - STF

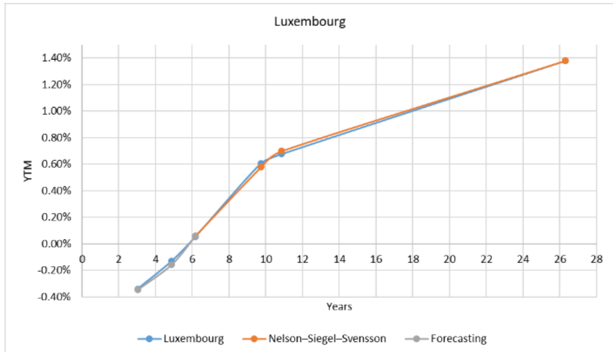

Figure 42 - Luxembourg market and NSS yield curve (May 5, 2017) - STF

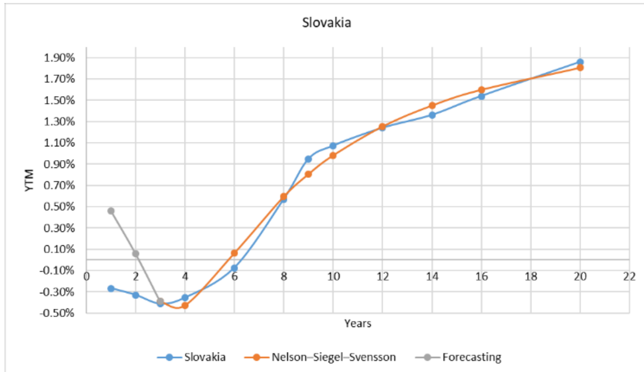

Figure 45 - Slovakia market and NSS yield curve (May 5, 2017) - STF

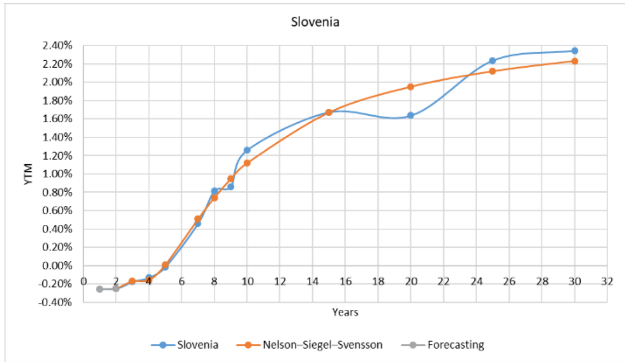

Figure 46 - Slovenia market and NSS yield curve (May 5, 2017) - STF

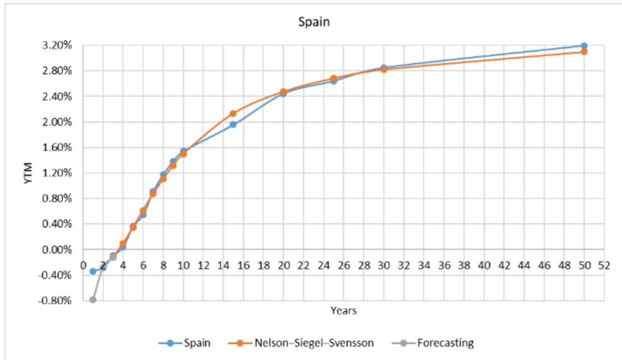

Figure 47 - Spain market and NSS yield curve (May 5, 2017) - STF

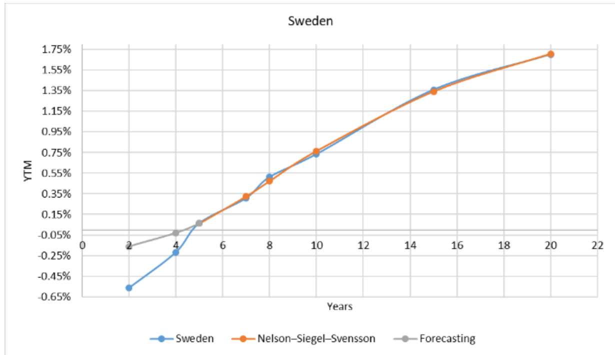

Figure 48 - Sweden market and NSS yield curve (March 15, 2017) - STF

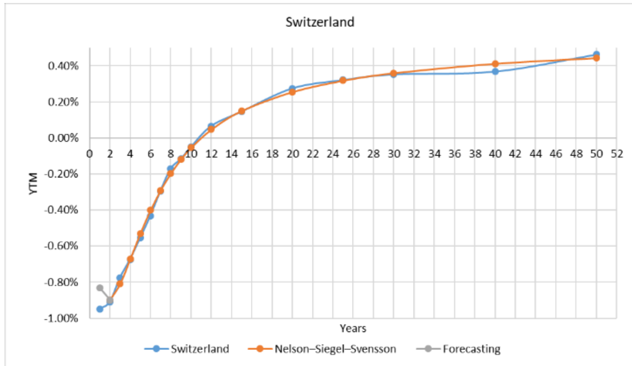

Figure 49 - Switzerland market and NSS yield curve (March 15, 2017) - STF

APPENDIX IV – COUNTRIES MARKET AND NSS MODEL YIELD CURVES (INTERMEDIATE TERM FORECAST)

Table X - NSS model $\beta_{1,2,3,4}$ and $\lambda_{1,2}$ factors (intermediate term maturities forecast)

	β_1	β_2	β_3	β_4	λ_1	λ_2
Austria	0.020162	-0.019143	-0.116890	-0.077807	0.086461	1.729619
Belgium	0.023496	-0.029780	-0.884799	-0.078639	0.017087	2.027378
Bulgaria	0.029918	-0.018996	0.006211	-0.077114	1.476223	1.531002
Czech Republic	0.030088	-0.027950	-0.058276	-0.087537	0.316866	3.134142
Denmark	0.016939	-0.002688	-0.000051	-0.079249	0.009988	1.345077
Finland	0.022371	-0.029255	-0.039402	-0.011852	2.291256	19.348220
France	0.025399	-0.030418	0.002341	-0.040293	2.503659	2.320827
Germany	0.016482	-0.026273	0.000792	-0.025125	2.714309	2.723765
Ireland	0.025917	-0.034485	-0.047909	-0.085281	0.179730	1.968295
Italy	0.036582	-0.036717	-0.059474	2.155394	1.633825	77820.987544
Japan	0.000658	-0.003019	0.291665	-0.269408	11.597784	10.501835
Lithuania	0.029082	-0.027602	0.046827	-0.065106	5.709136	3.564919
Luxembourg	0.018629	-0.019993	-0.308299	-0.069999	0.009942	1.799840
Netherlands	0.015123	-0.014851	-0.061748	-0.067745	0.082956	1.248862
Portugal	0.046554	-0.243628	1.245474	-1.042822	0.545787	0.662713
Slovakia	0.027724	-0.038131	0.742350	-0.789988	1.292437	1.393741
Slovenia	0.027570	-0.034833	-0.207497	-0.083209	0.057211	1.768952
Spain	0.035623	-0.035633	-0.245482	-0.103250	0.070087	1.878328
Sweden	0.027821	-0.030039	-1.216174	-0.074959	0.024818	2.475677
Switzerland	0.005543	-0.013453	-0.024206	-0.000303	1.620451	0.002583

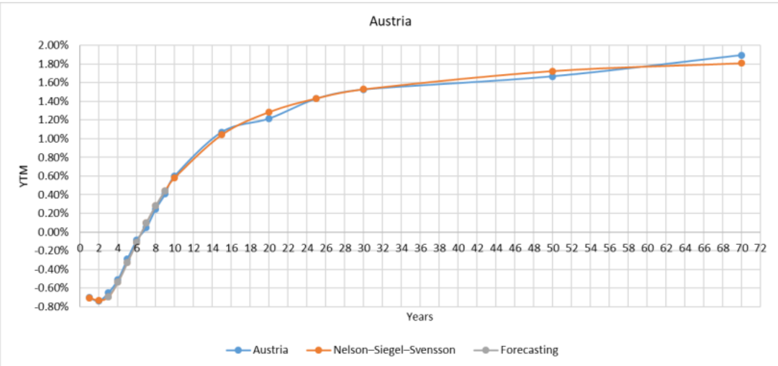

Figure 50 - Austria market and NSS yield curve (March 15, 2017) - ITF

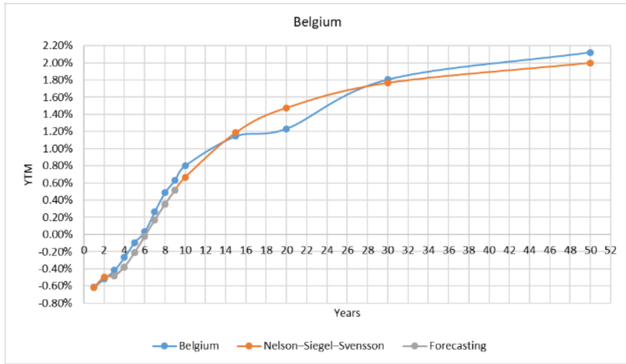

Figure 51 - Belgium market and NSS yield curve (May 5, 2017) - ITF

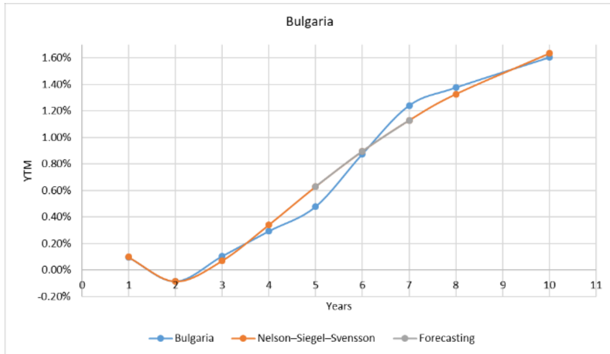

Figure 52 - Bulgaria market and NSS yield curve (May 5, 2017) - ITF

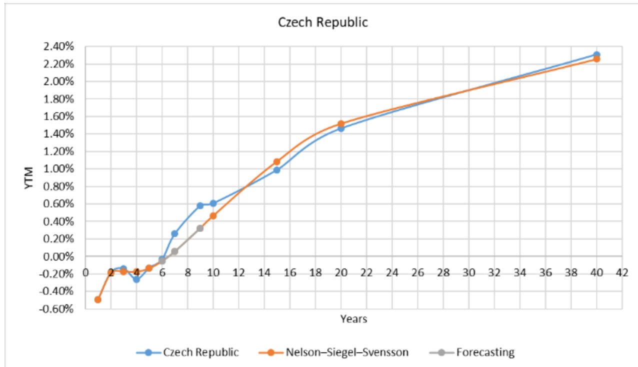

Figure 53 - Czech Republic market and NSS yield curve (May 5, 2017) - ITF

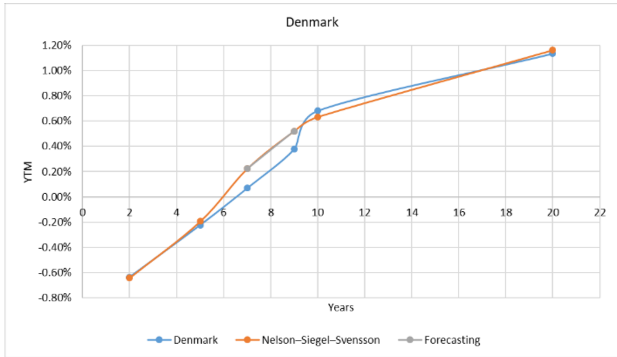

Figure 54 - Denmark market and NSS yield curve (March 15, 2017) - ITF

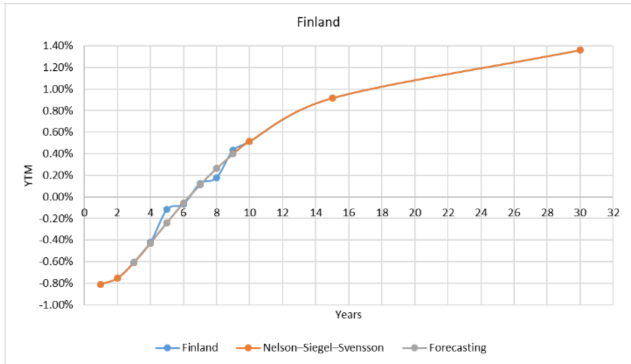

Figure 55 - Finland market and NSS yield curve (March 15, 2017) - ITF

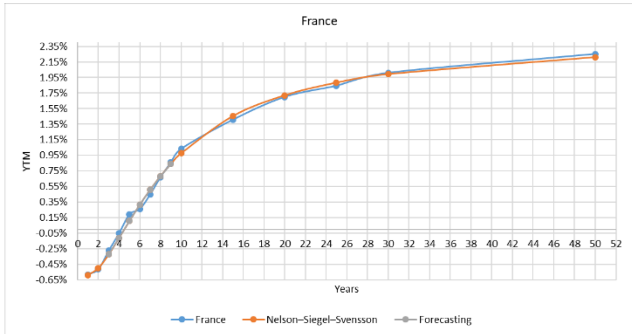

Figure 56 - France market and NSS yield curve (March 15, 2017) - ITF

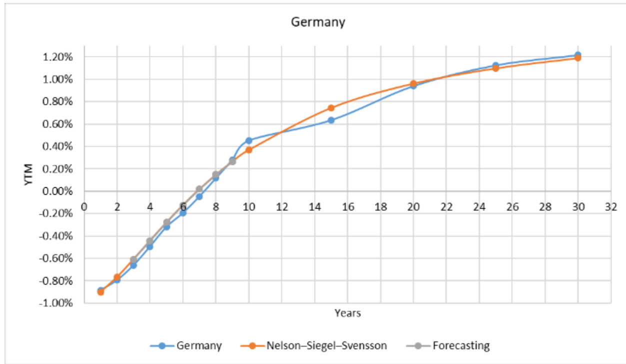

Figure 57 - Germany market and NSS yield curve (March 16, 2017) - ITF

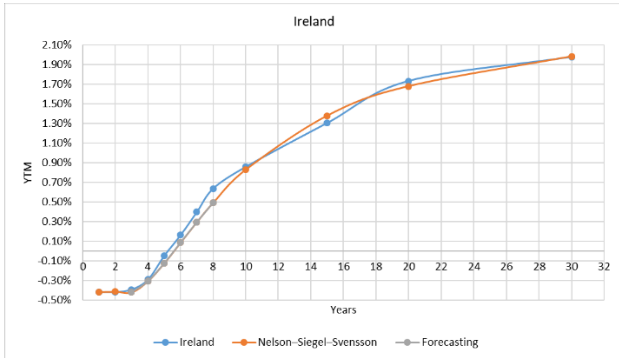

Figure 58 - Ireland market and NSS yield curve (May 5, 2017) - ITF

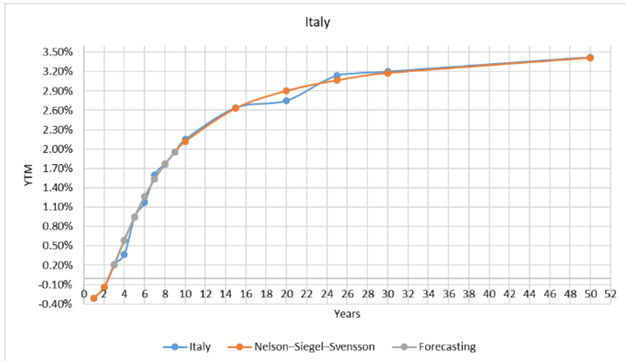

Figure 59 - Italy market and NSS yield curve (May 5, 2017) - ITF

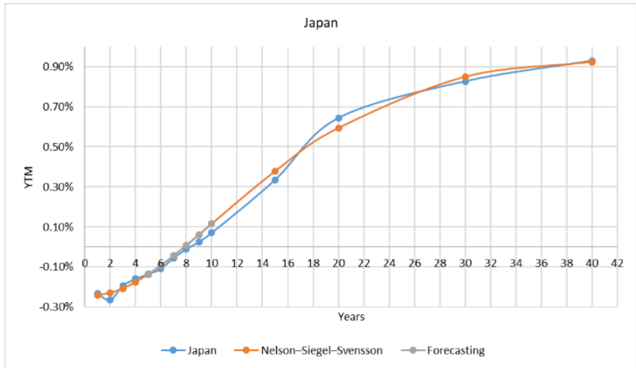

Figure 60 - Japan market and NSS yield curve (March 16, 2017) - ITF

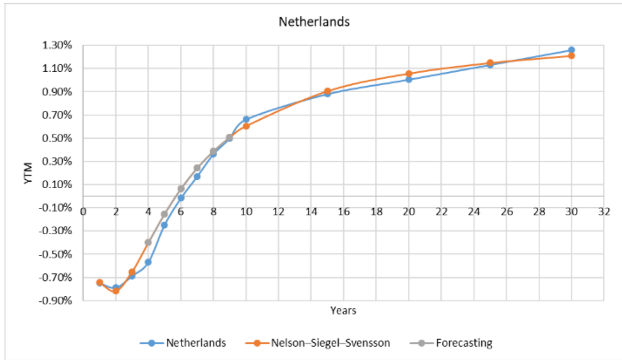

Figure 63 - Netherlands market and NSS yield curve (March 15, 2017) - ITF

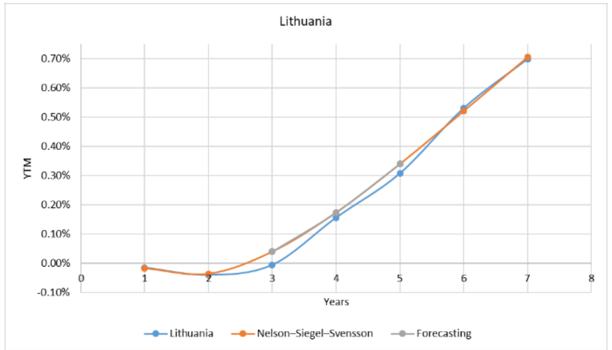

Figure 61 - Lithuania market and NSS yield curve (May 5, 2017) - ITF

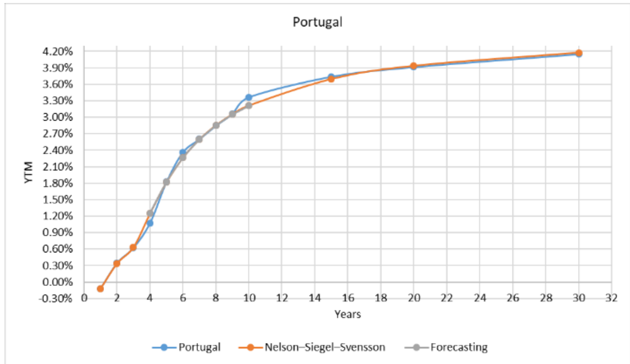

Figure 64 - Portugal market and NSS yield curve (May 5, 2017) - ITF

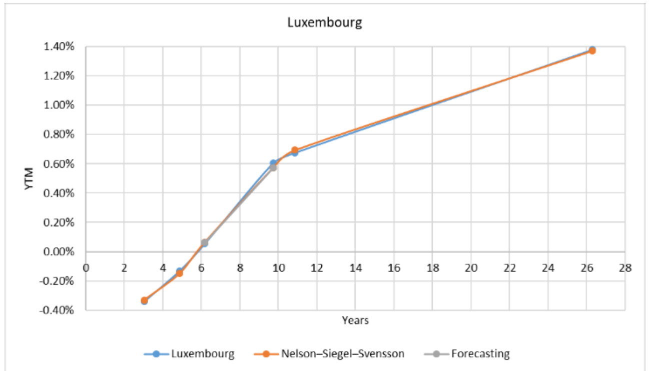

Figure 62 - Luxembourg market and NSS yield curve (May 5, 2017) - ITF

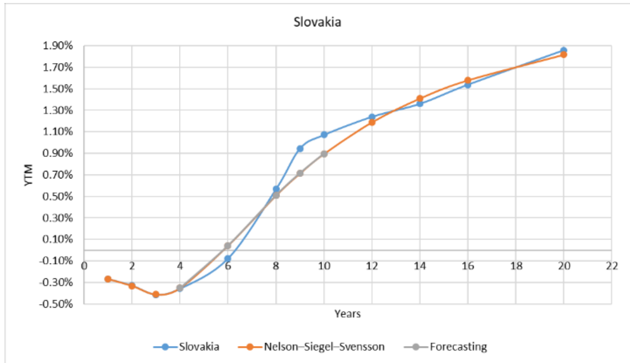

Figure 65 - Slovakia market and NSS yield curve (May 5, 2017) - ITF

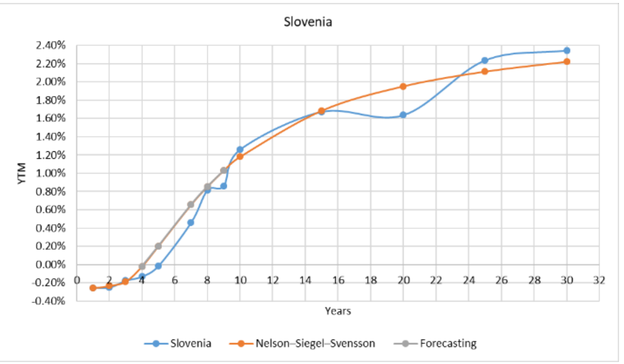

Figure 66 - Slovenia market and NSS yield curve (May 5, 2017) - ITF

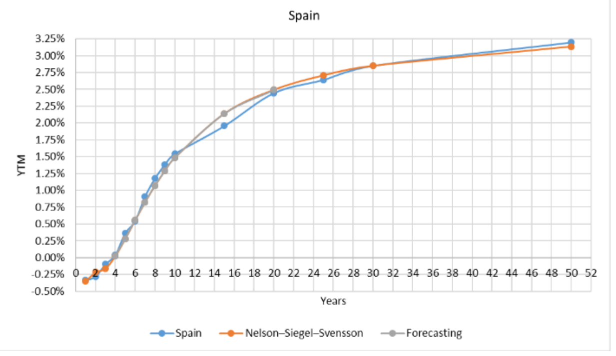

Figure 67 - Spain market and NSS yield curve (May 5, 2017) - ITF

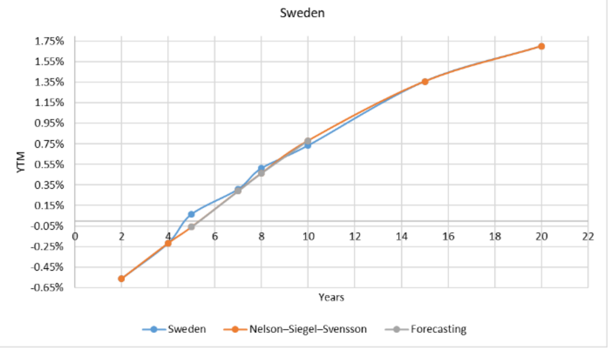

Figure 68 - Sweden market and NSS yield curve (March 15, 2017) - ITF

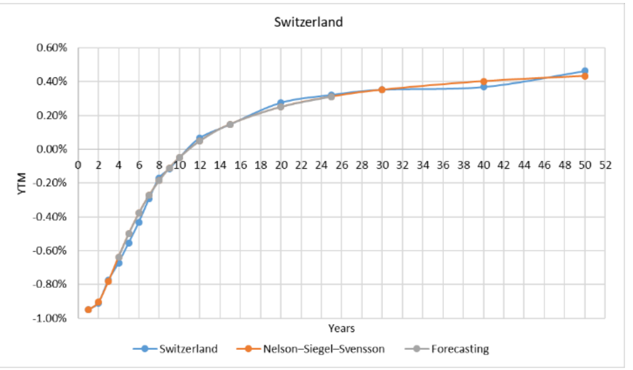

Figure 69 - Switzerland market and NSS yield curve (March 15, 2017) - ITF

APPENDIX V – COUNTRIES MARKET AND NSS MODEL YIELD CURVES (LONG TERM FORECAST)

Table XI - NSS model $\beta_{1,2,3,4}$ and $\lambda_{1,2}$ factors (long term maturities forecast)

	β_1	β_2	β_3	β_4	λ_1	λ_2
Austria	0.019247	-0.019139	-0.116741	-0.073730	0.086327	1.733630
Belgium	0.022702	-0.029756	-0.864602	-0.074378	0.016690	1.885299
Bulgaria	0.042973	-0.036530	0.006278	-0.102254	1.309432	2.074813
Czech Republic	0.030279	-0.228187	0.009047	-0.090860	0.099065	2.629441
Denmark	0.017726	-0.002688	-0.000051	-0.082415	0.009988	1.456279
Finland	0.022365	-0.029326	-0.037821	-0.015600	2.100928	12.573914
France	0.025001	-0.030761	0.002349	-0.038693	2.402382	2.414755
Germany	0.013610	-0.029384	0.004170	-0.054150	0.495952	1.652236
Ireland	0.024499	-0.033156	-0.045477	-0.081230	0.165971	1.796850
Italy	0.037199	-0.035035	-0.067227	-0.003852	1.576561	194.113267
Japan	0.000680	-0.002844	0.291656	-0.270609	11.352144	10.239932
Lithuania	0.029534	-0.027475	0.046842	-0.066333	5.816622	3.420271
Luxembourg	0.019445	-0.019994	-0.308483	-0.072912	0.009948	1.800200
Netherlands	0.016811	-0.015129	-0.069832	-0.071701	0.096109	1.416197
Portugal	0.048890	-0.247341	1.246504	-1.044196	0.565397	0.693006
Slovakia	0.030081	-0.051019	0.746092	-0.781254	1.150972	1.281889
Slovenia	0.030972	-0.035226	-0.223980	-0.094916	0.063076	1.996553
Spain	0.035312	-0.035484	-0.239406	-0.102905	0.068051	1.781474
Sweden	0.027667	-0.030075	-1.273167	-0.070311	0.026011	2.613614
Switzerland	0.006376	-0.014368	-0.024799	-0.000303	1.797835	0.002583

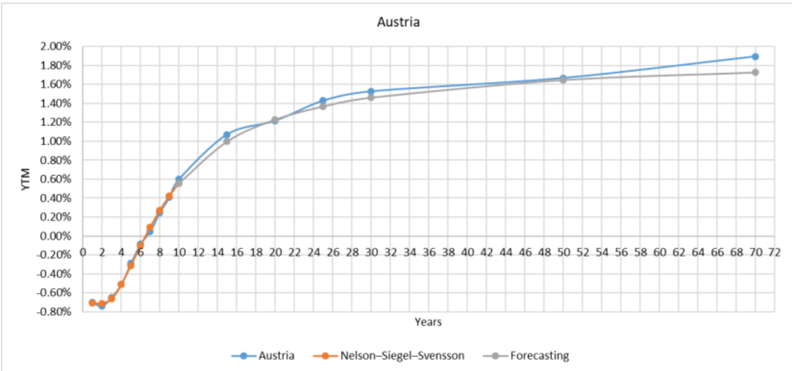

Figure 70 - Austria market and NSS yield curve (March 15, 2017) - LTF

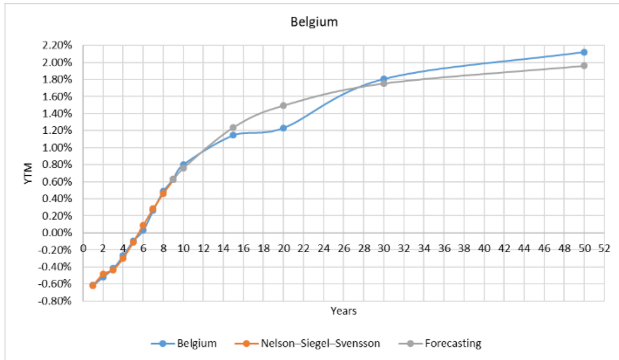

Figure 71 - Belgium market and NSS yield curve (May 5, 2017) - LTF

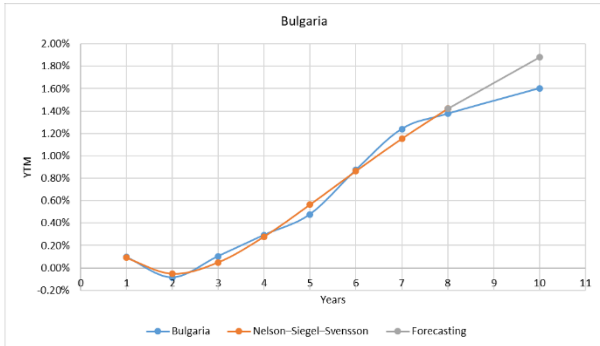

Figure 72 - Bulgaria market and NSS yield curve (May 5, 2017) - LTF

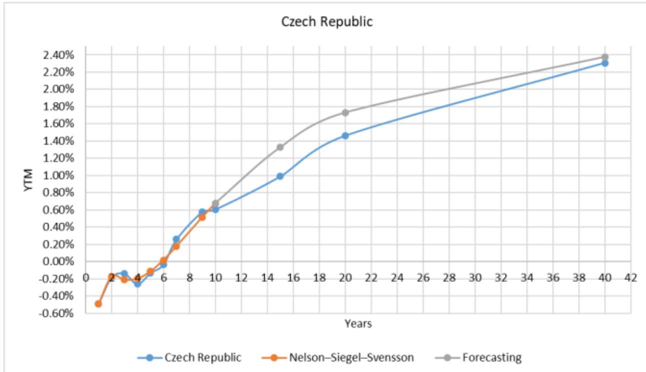

Figure 73 - Czech Republic market and NSS yield curve (May 5, 2017) - LTF

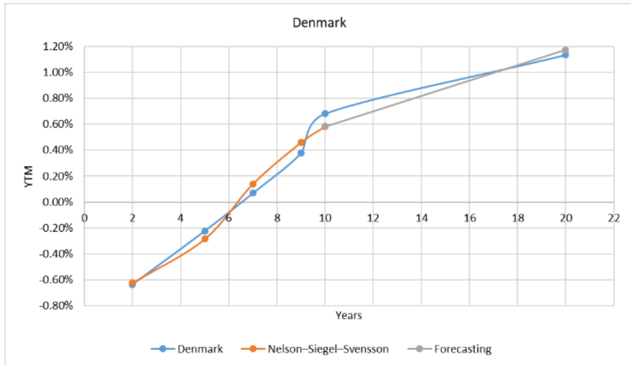

Figure 74 - Denmark market and NSS yield curve (March 15, 2017) - LTF

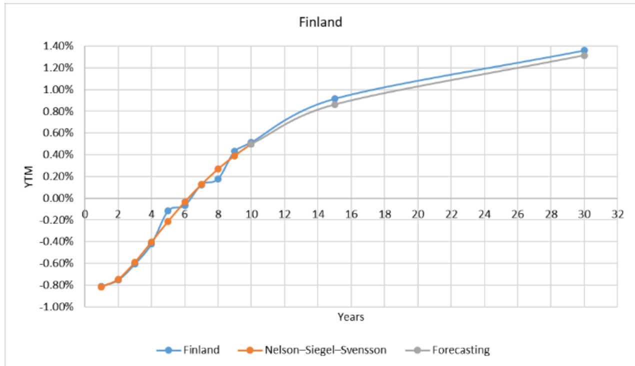

Figure 75 - Finland market and NSS yield curve (March 15, 2017) - LTF

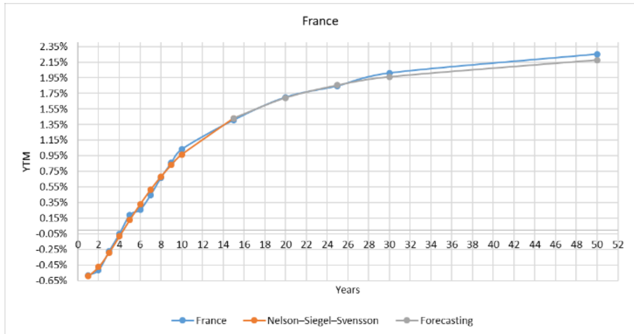

Figure 76 - France market and NSS yield curve (March 15, 2017) - LTF

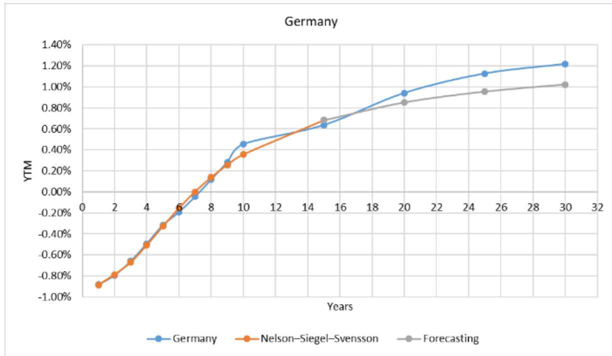

Figure 77 - Germany market and NSS yield curve (March 16, 2017) - LTF

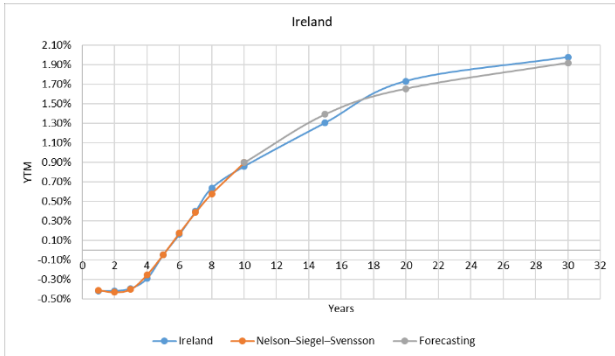

Figure 78 - Ireland market and NSS yield curve (May 5, 2017) - LTF

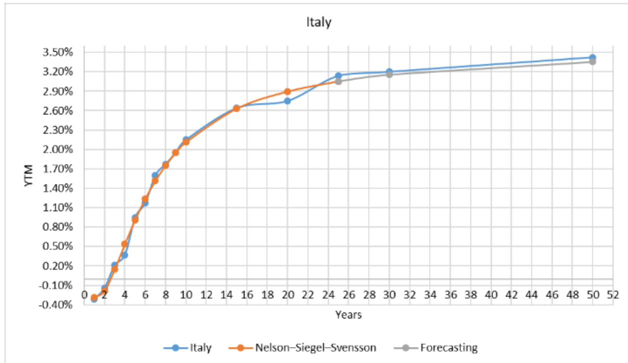

Figure 79 - Italy market and NSS yield curve (May 5, 2017) - LTF

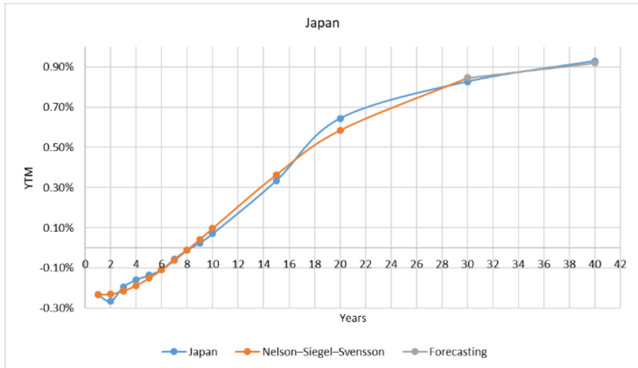

Figure 80 - Japan market and NSS yield curve (March 16, 2017) - LTF

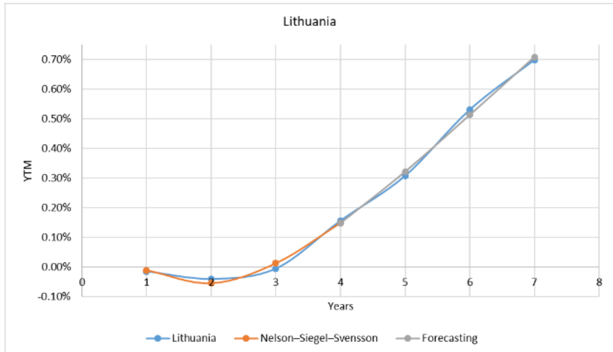

Figure 81 - Lithuania market and NSS yield curve (May 5, 2017) - LTF

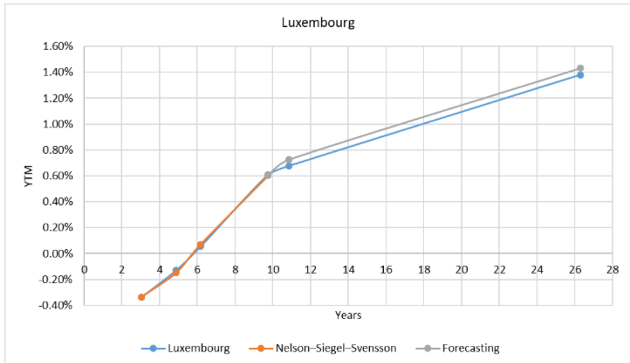

Figure 82 - Luxembourg market and NSS yield curve (May 5, 2017) - LTF

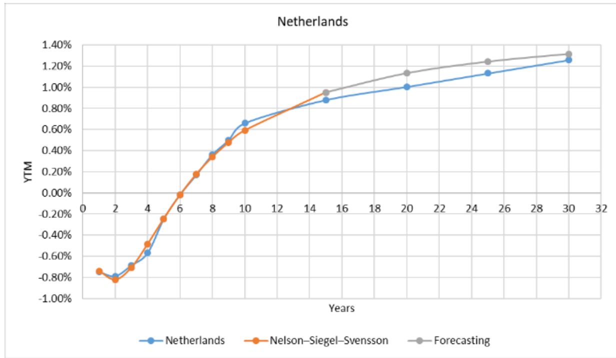

Figure 83 - Netherlands market and NSS yield curve (March 15, 2017) - LTF

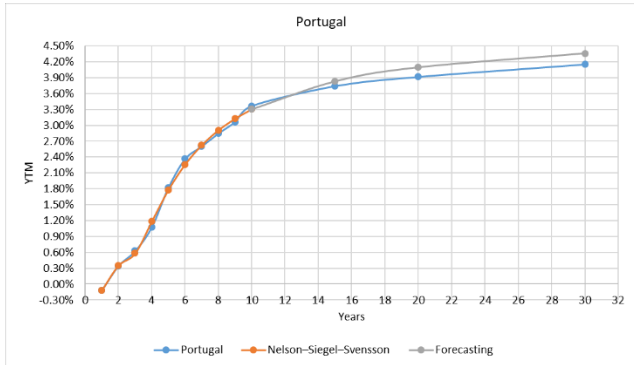

Figure 84 - Portugal market and NSS yield curve (May 5, 2017) - LTF

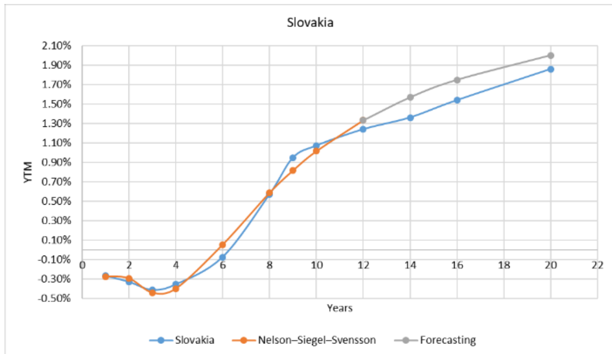

Figure 85 - Slovakia market and NSS yield curve (May 5, 2017) - LTF

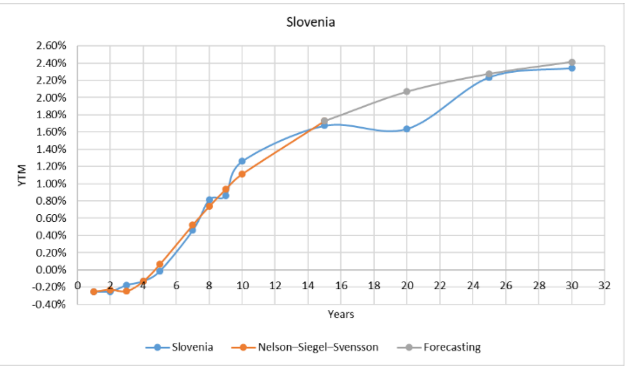

Figure 86 - Slovenia market and NSS yield curve (May 5, 2017) - LTF

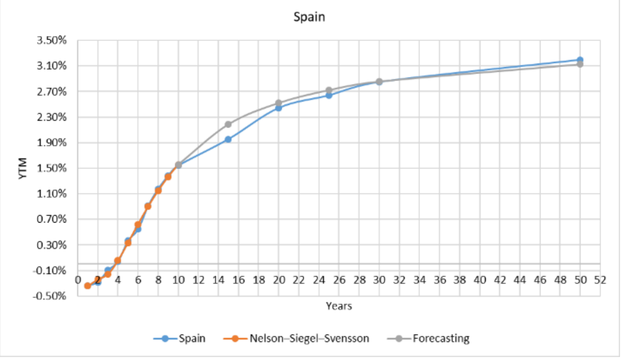

Figure 87 - Spain market and NSS yield curve (May 5, 2017) - LTF

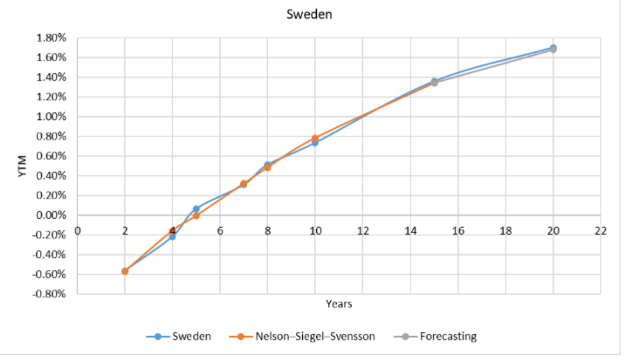

Figure 88 - Sweden market and NSS yield curve (March 15, 2017) - LTF

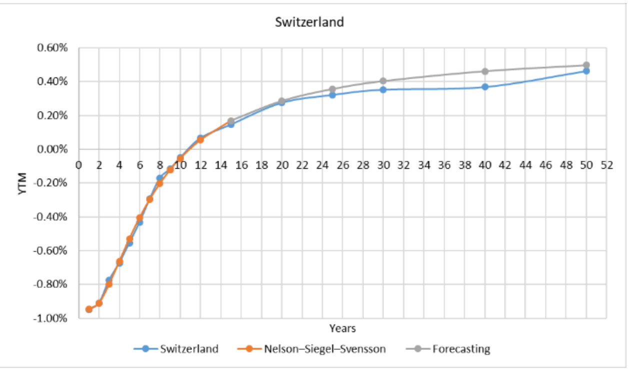

Figure 89 - Switzerland market and NSS yield curve (March 15, 2017) - LTF