Master in Applied Econometrics and Forecasting

Threshold Effects in the Wage Phillips Curve

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Abstract

The main purpose of this work is to evaluate the ability of the New Keynesian wage Phillips curve (NKWPC), proposed by Galí (2011), to describe U.S. wage inflation dynamics over the 1965-2018 period. To study this relationship, a threshold regression model that allows assessing the existence of regime-switching nonlinearity is employed.

Our results suggest that wage inflation dynamics are well described by a 3-regime threshold model where the best threshold variable is the current unemployment rate. The estimated thresholds split the NKWPC into regimes consistent with periods of deep recessions, moderate business cycle fluctuations and prolonged expansions. We find evidence that the negative relationship between wage inflation and unemployment implied by the NKWPC holds when unemployment is between the thresholds 5.69% and 7.63%; when unemployment is outside this band the relationship seems to break down.

To assess the robustness of our estimates, we account for possible endogeneity of the regressors and the threshold variable by using the structural threshold model proposed by Kourtellos et al. (2016). In this setting, we conclude that our baseline results are not very sensitive to endogeneity affecting the regressors. In contrast, the threshold estimates obtained when the threshold variable is considered as endogenous yield a substantial reduction in the number of observations in the second regime.

Keywords: New Keynesian wage Phillips curve; threshold regression model; nonlinearity; endogeneity.

JEL classification: C22, E24, E31, E52.

Resumo

Neste trabalho, avaliamos a capacidade da curva de Phillips salarial Neo-Keynesiana (CPSNK) proposta por Galí (2011) para descrever a inflação dos salários nos EUA durante o período 1965-2018. De forma a estudar esta relação, empregamos um modelo de regressão de limiar que nos permite examinar a existência de não-linearidades.

Os nossos resultados sugerem que a taxa de inflação salarial é bem descrita por um modelo de limiar com 3 regimes em que a variável de limiar é a taxa de desemprego. As estimativas para os parâmetros de limiar dividem a CPSNK em regimes consistentes com períodos de recessão profunda, de flutuações moderadas do ciclo económico e de crescimento prolongado. Encontramos evidência empírica consistente com a relação negativa entre a inflação salarial e a taxa de desemprego prevista pela CPSNK quando a taxa de desemprego está entre os limites de 5.69% e 7.63%. Quando a taxa de desemprego está fora deste intervalo, esta relação parece desaparecer.

Para avaliar a robustez das nossas estimativas, incorporamos a possível endogeneidade dos regressores e da variável de limiar ao estimar o modelo de regressão limiar estrutural proposto por Kourtellos et al. (2016). Neste contexto, concluímos que os nossos resultados não são muito diferentes quando permitimos que os regressores sejam endógenos. Por outro lado, as estimativas dos coeficientes de limiar obtidas quando a variável de limiar é considerada como endógena implicam uma redução significativa do número de observações no segundo regime.

Palavras-chave: curva de Phillips salarial Neo-Keynesiana; modelo de regressão de limiar; não linearidade; endogeneidade.

Classificação JEL: C22, E24, E31, E52.

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All remaining errors are, of course, my own.

Contents

1 Introduction

Ever since A. W. Phillips (1958) first documented the statistical evidence of a negative relationship between wage inflation and the rate of unemployment in the United Kingdom, the 'Phillips Curve' has become one of the most active fields of research in the economics profession.

The history associated with the Phillips curve is remarkably rich and full of fascinating debates; see Gordon (2011) for a comprehensive survey. In the last decades, we have witnessed the development of a new branch of this vast literature as a result of the emergence of the so-called New Keynesian (NK) macroeconomics. By combining the rigorous theoretical foundations of Real Business Cycle models with distinctive Keynesian ingredients (such as nominal rigidities and monopolistic competition), NK models have become an increasingly dominant framework for monetary policy analysis. In this setting, inflation dynamics are described by the NK Phillips curve. The appealing theoretical microfoundations and early empirical success led the NK Phillips curve to become a popular formulation to study the relationship between inflation and real economic activity.

More developed NK models often extend the specification of the wage-setting block by modelling wage inflation in a similar fashion as price inflation. Under appropriate assumptions, the NK wage Phillips curve (NKWPC) states wage inflation as a function of future expected wage inflation and the current deviation of wage markups from their optimal value, which reflects the degree of labour market slack. Despite the empirical success of NK models, there has been some criticism in relation to the lack of reference to unemployment, a central variable in policy debate. An important reformulation of the NKWPC that tackles this shortcoming was presented by Galí (2011). By adopting the formalism of Erceg et al. (2000), Galí derives a NKWPC that explicitly introduces the rate of unemployment in the wage inflation equation.

As a consequence of the negligible wage growth recorded in the aftermath of the 'Great Recession', the empirical assessment of Galí's version of the NKWPC has received increasing attention. In fact, the relationship between wage growth and unemployment may provide important guidance for monetary policy in the slow recovery environment experienced after the 2007-2008 recession. A strong negative estimate of this relationship would favour the strand of the literature that argues the need for policy normalization to prevent inflationary pressures (Yellen, 2014), given the low level of unemployment. On the other hand, a different body of research argues that the economy is still not near full employment (Blanchflower and Levin, 2015). In this case, evidence of a weak relationship provides an additional argument in support of more accommodative policies since unemployment can be reduced further without bearing the cost of higher inflation.

Although the relationship between wage inflation and unemployment is commonly assumed as linear, the role of nonlinearities is often emphasized in the Phillips curve literature. As highlighted by G. Akerlof et al. (1996), one of the main reasons that might lead us to expect the existence of nonlinearities is downward wage rigidity. This feature implies that wages might be more responsive to variations in the unemployment rate in periods of low labour market slack in comparison with periods of high labour market slack. While firms tend to increase wages more rapidly during periods of low unemployment, downward wage rigidity reduces the responsiveness of wages as the unemployment rate increases. Therefore, this obstacle to wage adjustment implies that the NKWPC should exhibit an asymmetric behavior, as the slope of the curve varies according to labour market conditions.

Our approach to capture the possible nonlinearity of the NKWPC is to estimate a threshold regression model, which splits the observations according to different regimes defined by a threshold variable. The key advantages of using this framework is that the thresholds are endogenously estimated and the study of nonlinearity can be performed through a formal and well developed econometric framework. Thus, the objective of the present work is to estimate and test a threshold regression model for the U.S. NKWPC and to provide an empirical assessment of the model's fit to the wage inflation dynamics over the 1965-2018 period.

In many ways this dissertation closely follows the empirical work of Donayre and Panovska (2016) who also study the formulation of the NKWPC presented by Galí (2011) using a threshold model. However, we complement our analysis by examining the robustness of the threshold model estimates to possible endogeneity in the NKWPC. In fact, it might be reasonable to consider the possibility that wage inflation and the adopted measure of labour market slack are jointly determined. Since our regime-switching regression is motivated by the idea that the NKWPC varies according to regimes defined by the level of labour market slack, we allow for endogeneity in both the regressors and in the threshold variable. To account for endogeneity, we resort to the structural threshold regression model of Kourtellos et al. (2016) which relies on the use of instrumental variables.

Our results provide evidence that the NKWPC is well described by a 3-regime threshold model. The relationship implied by the NKWPC changes when unemployment crosses the estimated thresholds of 5.69% and 7.63%, which split the observations into 3 regimes that correspond to deep recessions, moderate business cycle fluctuations and prolonged expansions. We only find evidence consistent with the negative relationship between wage inflation and unemployment predicted by the NKWPC in the intermediate unemployment regime. In both the low and high unemployment regimes the NKWPC relationship seems to break down. We show that these results are robust to endogeneity of the regressors. On the other hand, we find some sensitivity of the NKWPC to endogeneity of the threshold variable, considering that, in this setting, we obtain a higher estimate for the threshold separating the low and intermediate unemployment regimes. This causes the number of observations in the second regime to be reduced in almost half and yields a smaller estimate for the slope NKWPC in this regime.

The present work is organized as follows: section (2) presents the NKWPC; we briefly describe some theoretical foundations for nonlinearity in section (3); the threshold regression model is presented in section (4); we discuss the empirical evidence in section (5); in section (6) we analyze the robustness of our results to endogeneity in the NKWPC; finally, section (7) concludes.

2 The New Keynesian Wage Phillips Curve

In this section, we present the theoretical foundations of the NKWPC proposed by Galí (2011). This formulation for wage inflation incorporates the staggered wage setting specification presented by Erceg et al. (2000) in which workers are unable to re-optimize their nominal wage with probability θ_w. Because this probability is independent from the time elapsed since the last wage revision, θ_w can be interpreted as an index of nominal wage rigidities. The structural model for wage inflation can be expressed as

$$\pi_t^w = \beta_w E_t \pi_{t+1}^w - \lambda_w(\mu_t^w - \mu^w) \ . \tag{1}$$

Wage inflation is defined as π_t^w and μ^w denotes the desired level of the average wage markup μ_t^w. It can be shown that λ_w can be written as

$$\lambda_w = \frac{(1-\theta_w)(1-\beta_w\theta_w)}{\theta_w(1+\epsilon_w\varphi)} \ ,$$

where φ corresponds to the inverse of the Frisch labour supply elasticity[1], ϵ_w is the wage elasticity of demand and β_w is the standard utility discount factor.

Galí shows that a simple relationship between the wage markup and the unemployment rate may be obtained by making the additional assumption that members of the representative household decide to participate in the labour market only when the prevailing real wage rate is superior to the disutility from work. Under this setting, the real wage rate depends positively on the labour supply. Considering that the unemployment rate is defined as the difference between the labour supply and the employment rate, the wage markup can be written as a linear function of the unemployment rate

$$\mu_t^w = \varphi u_t \ . \tag{2}$$

Defining u_t^n as the natural rate of unemployment, this is the unemployment rate prevailing if there were no nominal wage rigidities, equation (2) implies

$$u^n = \frac{\mu^w}{\varphi} \ . \tag{3}$$

[1]The Frisch labour supply elasticity measures the response of labour supply given a variation in the real wage rate.

Plugging equations (2) and (3) into (1) yields the following expression for the NKWPC:

$$\pi_t^w = \beta_w E_t \pi_{t+1}^w - \lambda_w \varphi (u_t - u^n) \ . \tag{4}$$

In this formulation, wage inflation and unemployment are inversely related. It is important to emphasize that this is a microfounded equilibrium relationship. Note that the steepness of the curve is decreasing with respect to φ and to θ_w (through the parameter λ_w) so that in the absence of nominal rigidities the NKWPC becomes vertical. Also, the forward looking nature of wage inflation is embedded in equation (4). Indeed, if we iteratively substitute the expectation term, the NKWPC can be expressed as

$$\pi_t^w = -\lambda_w \varphi \sum_{i=0}^{\infty} \beta^i E_t\{(u_{t+i} - u^n)\} \ ,$$

implying that π_t^w is determined in a purely forward-looking manner, since it is a function of the discounted sums of the present and future unemployment rate.

In order to capture wage inflation persistence, the model can be extended to allow for automatic wage indexation to price inflation. In this setting, nominal wages are revised even when the representative household is not given the chance to re-optimize. Considering the wage indexation rule used by Galí (2011), the NKWPC can be written as

$$\pi_t^w = \alpha + \rho \overline{\pi}_{t-1}^p + \beta_w E_t\{\pi_{t+1}^w - \overline{\pi}_t^p\} - \lambda_w \varphi (u_t - u^n) \ ,$$

where $\overline{\pi}_t^p$ denotes the price inflation measure to which the wages are indexed.

To derive a simple reduced form representation of the NKWPC we follow Galí (2011) and assume that cyclical unemployment, defined as $\hat{u}_t = u_t - u^n$, follows a stationary AR(2) process with parameters ϕ_1 and ϕ_2. Then, the reduced form representation for the $(\pi_t^w, \hat{u}_t)$ relationship is

$$\pi_t^w = \alpha + \rho \pi_{t-1}^p + \psi_0 \hat{u}_t + \psi_1 \hat{u}_{t-1} \ , \tag{5}$$

where

$$\psi_0 = -\frac{\lambda_w \varphi}{1 - \beta_w(\phi_1 + \beta_w \phi_2)} \qquad \psi_1 = -\frac{\lambda_w \varphi \beta_w \phi_2}{1 - \beta_w(\phi_1 + \beta_w \phi_2)} \ . \tag{6}$$

Galí (2011) documented the ability of equation (5) to describe U.S. data. Although the model can provide a reasonable fit, it fails to account for much of the wage inflation dynamics, especially during the Great Recession period. Galí (2011) suggests that this poor fit results from the fact that the model presented above ignores the role played by downward wage rigidities. If this is the case, a nonlinear model would yield a better characterization of the data. In the next section we present some theoretical foundations for a nonlinear Phillips curve and review some previous empirical results.

3 Nonlinearity in the Phillips Curve

The shape of the short-run Phillips curve is often regarded as nonlinear and dependent on labour markets conditions. A potential explanation for a nonlinear Phillips curve is downward wage rigidity[2]. An important branch of this literature is based in the efficiency wage theory, according to which wages are linked to effort so that firms resistance to cut wages during periods of high unemployment is consistent with a profit maximizing behavior. There are several theoretical models that can generate this feature. In Shapiro and Stiglitz (1984) firms pay higher wages to prevent shirking, while in the gift-exchange model (G. A. Akerlof, 1982) and the fair wage hypothesis (G. A. Akerlof and Yellen, 1990) lower wages have a negative effect on workers loyalty, thus reducing productivity.

A different strand of the literature is inspired by the insider-outsider theory of Lindbeck and Snower (1988). This theory states that incumbent workers have their position protected by labour turnover costs. Thus, firms may avoid to cut wages to prevent the materialization of these costs. Stiglitz (1974) shows how wage reductions increase the propensity of employees to quit and how increases in labour turnover have a negative effect on profits. The insider-outsider theory also provides a rationale for how institutions, such as labour unions and social norms, can contribute to increases in labour turnover costs and, therefore, to the degree of downward wage

[2]See Dupasquier and Ricketts (1998) for other explanations for nonlinearity in the Phillips curve and Babecky et al. (2009) for a discussion of theories that imply downward wage rigidity.

rigidities.

Motivated by these theoretical foundations, empirical studies of the U.S. wage Phillips curve have employed a wide variety of strategies to model nonlinearities. For brevity, we only review studies which use a threshold approach, since these are more closely related to the present work. Kumar and Orrenius (2016) use state-level panel data and introduce a linear spline term which allows for a different slope whenever the unemployment rate crosses the sample average of 6.1%. In addition, a three knot cubic spline model is also estimated. The chosen knots correspond to the quartiles of the unemployment rate empirical distribution. Taking into account that the different knots were chosen in a data-dependent fashion the authors perform a number of robustness checks, always obtaining results consistent with a strongly convex wage Phillips curve. Nalewaik (2016) models the U.S. wage Phillips curve using a 2-regime Markov-switching model. The strength of nonlinearity is measured as the coefficient of a quadratic function of cyclical unemployment. The author also studies nonlinearity through a standard regression with a break in the coefficient of the unemployment rate. The results imply that the U.S. wage Phillips curve is considerably steeper when unemployment decreases below 5%. Donayre and Panovska (2016) use a 3-regime threshold model to study the NKWPC of Galí (2011) using U.S. data. Their analysis suggests that the curve has a negative slope during mild recessions and consequent recovery periods. In contrast, the evidence of a negative relationship is weaker in the low unemployment regime while in the deep recessions regime this relationship breaks down. Also, the authors conclude that the results are consistent with the concept of downward wage rigidity. Donayre and Panovska (2018) use a threshold vector autoregressive model and find favourable evidence of a nonlinear wage Phillips curve. The authors allow for two different threshold variables and conclude that wage inflation dynamics change according to the unemployment rate and price inflation. Other related works that use threshold models to study the U.S. price Phillips curve include Doser et al. (2017) and Olivei and Barnes (2004) who find favourable evidence for nonlinearity using a threshold model.

4 The Threshold Regression Model

4.1 Representation

An intuitive and parsimonious strategy to model economic relationships using non-linear models seems to be to allow for the possibility that the data generating process changes according to *different states of the world* or *regimes*. A certain time series or economic relationship is said to be state-dependent or regime-switching if its underlying properties differ across regimes. An important class of state-dependent models assumes that the transition between regimes over time can be characterized by an observable stochastic variable. In this context, the regimes are delimited by certain observable thresholds. Given an estimate of such threshold it is possible to assign present and past observations to a particular regime.

The idea of approximating a general nonlinear relationship through a model with several regimes was initially proposed by Quandt (1958) and was further developed by Tong (1978) and Tong and Lim (1980). The latter authors formalized the class of threshold autoregressive (TAR) models. The essential characteristic of TAR models is the piecewise linearization through the introduction of an indicator variable that acts as the mathematical mechanism that enables a regime switch. Therefore, each regime follows a distinct AR(p) model over the state space defined by the thresholds. Tong (2011) referred to this idea as the *threshold principle*.

In this section we present the main features associated with the threshold regression model, a specific switching regression model that allows each regime to be described by a different multivariate linear model. Considering the time series $(y_t, x_t, q_t)_{t=1}^{T}$, a 2-regime threshold model is given by

$$y_t = \begin{cases} \boldsymbol{x_t'}\boldsymbol{\beta_1} + e_t, & q_t \leq \gamma \ , \\ \boldsymbol{x_t'}\boldsymbol{\beta_2} + e_t, & q_t > \gamma \ , \end{cases} \tag{7}$$

where $\boldsymbol{x}_t = (x_{t1}, x_{t2}, ..., x_{tk})'$ is a $k \times 1$ vector of exogenous explanatory variables, $\boldsymbol{\beta} = (\boldsymbol{\beta_1}, \boldsymbol{\beta_2})$ denotes the associated parameters and q_t is an observable threshold variable that splits the observations into two different regimes. Note that the explanatory variables are the same in both regimes, but the coefficients are allowed to change. The model determines which set of coefficients to use by evaluating the value of q_t in relation to the threshold γ. If $q_t \leq \gamma$ the slope coefficients are given by $\boldsymbol{\beta_1}$ and when $q_t > \gamma$ the regime coefficients correspond to $\boldsymbol{\beta_2}$. The regression error e_t is assumed to be a martingale difference sequence so that $E(e_t|\mathcal{F}_{t-1}) = 0$, where the conditioning set $\mathcal{F}_{t-1}$ denotes past information, and $\sigma^2 = E(e_t^2)$. From this representation the popular TAR model with regimes described by an AR(p) model arises if we set $\boldsymbol{x_t} = (y_{t-1}, y_{t-2}, ..., y_{t-p})'$. Additionally, if we consider the threshold variable to be the lagged dependent variable, $q_t = y_{t-d}$, where d is a positive integer delay parameter, the resulting model is the *self-exciting* threshold model (SETAR).

The threshold regression model of equation (7) may be written in an alternative way by considering the variable $I(q_t \leq \gamma)$ where $I(.)$ is the indicator function. Thus, the model takes the following representation

$$y_t = \boldsymbol{x_t'\beta_1} I(q_t \leq \gamma) + \boldsymbol{x_t'\beta_2} I(q_t > \gamma) + e_t \ . \tag{8}$$

The indicator function implies that the transition between regimes occurs instantaneously whenever the value of q_t crosses the threshold γ, which might not be a reasonable assumption for some applications. If this is the case, the indicator variable can be replaced by a continuous function $G(q_t;\, s, \gamma)$, defined between 0 and 1, that allows the parameters of the models to change slowly between regimes. The speed of the adjustment is controlled by the *smoothness* parameter s. The resulting model is the smooth transition model[3].

In some instances it might be reasonable to assume that the model is described by more than two regimes. Consider the 3-regime representation of the threshold model

$$y_t = \boldsymbol{x_t'\beta_1} I(q_t \leq \gamma_1) + \boldsymbol{x_t'\beta_2} I(\gamma_1 < q_t \leq \gamma_2) + \boldsymbol{x_t'\beta_3} I(q_t > \gamma_2) + e_t \ . \tag{9}$$

Intuitively, the relationship between the two variables can be seen to change whenever the value of the threshold variable lies outside the band defined by two thresh-

[3]A comprehensive review of this class of models is provided by Lütkepohl and Krätzig (2004).

olds γ_1 and γ_2.

A general m-regime threshold model can be written by considering the variable $I(\gamma_{j-1} < q_t \leq \gamma_j)$ and setting $\boldsymbol{x_t}(\gamma) = \boldsymbol{x_t}\, I(\gamma_{j-1} < q_t \leq \gamma_j)$. Thus,

$$y_t = \sum_{j=1}^{m} \boldsymbol{x}_t(\gamma)' \boldsymbol{\beta_j} + e_t \ ,$$

where $j = 1, ..., m$ and $-\infty = \gamma_0 < \gamma_1 < ... < \gamma_m = \infty$. Finally, it will be useful to write the threshold model in matrix form. Defining $\boldsymbol{y}$ and $\boldsymbol{e}$ as $T \times 1$ vectors by stacking y_t and e_t and $\boldsymbol{X}_{(\gamma)}$ as a $T \times K$ matrix by stacking $\boldsymbol{x_t}(\gamma)$ we can express the model as

$$\boldsymbol{y} = \sum_{j=1}^{m} \boldsymbol{X_{(\gamma)}} \boldsymbol{\beta_j} + \boldsymbol{e} \ . \tag{10}$$

4.2 Estimation

Despite being nonlinear in the parameters, the model depicted in equation (10) is a regression equation and therefore a natural estimation method is least squares (LS). This method corresponds to maximum likelihood estimation under the auxiliary assumption that e_t is iid $N(0, \sigma^2)$. If the threshold parameters were known, we could simply split the observations into the different regimes and estimate each linear segment of the model using LS. However, in most instances the values of the thresholds are unknown and must be estimated. Fortunately, Chan (1993) showed that consistent estimates of the model parameters can be obtained through sequential conditional least squares (CLS), which we describe next.

Let the model parameters be collected as $\boldsymbol{\theta} = (\boldsymbol{\beta}_j, \boldsymbol{\gamma})$, where $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, ..., \gamma_{m-1})$ denote the threshold parameters. Define $\hat{\boldsymbol{e}}_m$ as the LS residuals of the m-regime threshold regression and write the sum of squared residual as $S_m = \hat{\boldsymbol{e}}_m' \hat{\boldsymbol{e}}_m$. The LS estimates $\hat{\boldsymbol{\theta}}$ solves the following problem

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{argmin}\ S_m \ .$$

Since S_m is discontinuous with respect to $\boldsymbol{\gamma}$, the estimates $\hat{\boldsymbol{\theta}}$ must be obtained through a grid-search over the possible values of $\boldsymbol{\gamma}$. A standard approach is to restrict the search of $\boldsymbol{\gamma}$ to the observed values of q_t. Additionally, it is useful to assume that $\boldsymbol{\gamma}$ is bounded by the set $\Gamma = [\underline{\gamma}, \bar{\gamma}]$, where $\underline{\gamma}$ and $\bar{\gamma}$ are the τth and $(1-\tau)$th percentiles of q_t, respectively, so that each regime has at least $T\tau$ observations. This assumption not only ensures that an adequate number of observations is used in the estimation of each regime parameters, but also prevents abnormal realizations of test statistics. The estimation procedure can be implemented by sorting the threshold variable q_t and evaluating the sum of squared residuals function sequentially for each set of different values for $\boldsymbol{\gamma} \in \Gamma$. Conditionally on $\boldsymbol{\gamma}$, we can estimate equation (10) in order to obtain the estimates $\hat{\boldsymbol{\beta}}_j(\boldsymbol{\gamma})$ and the corresponding sum of squared residuals $S_m^*(\boldsymbol{\gamma}) \equiv S_m|_{\boldsymbol{\beta}_j=\hat{\boldsymbol{\beta}}_j(\boldsymbol{\gamma})}$. Since $S_m^*(\boldsymbol{\gamma})$ can only assume a finite number of distinct values, $\hat{\boldsymbol{\gamma}}$ can be obtained as

$$\hat{\boldsymbol{\gamma}} = \underset{\boldsymbol{\gamma}}{argmin}\ S_m^*(\boldsymbol{\gamma}) \ .$$

Plugging in the threshold estimate into $\hat{\boldsymbol{\beta}}_j(\boldsymbol{\gamma})$, the slope parameters can be computed as $\hat{\boldsymbol{\beta}}_j = \hat{\boldsymbol{\beta}}_j(\hat{\boldsymbol{\gamma}})$.

In our exposition so far, we have assumed that the threshold variable is known. Although economic theory might indicate the appropriate threshold variable, this might not always be the case. In such circumstances, it is possible to use an extension of the estimation procedure described above in order to determine the optimal threshold variable. Consider a $T \times \bar{d}$ matrix constituted by the set of candidate threshold variables. Let the column indexes of this matrix be denoted by $D = [1, \bar{d}\,]$ and define the optimal threshold variable index as $d \in D$. Then, the estimation of the optimal threshold variable amounts to the estimation of d, which we can think of as an additional parameter of the model. The CLS procedure can be extended to consider a grid-search over $(\boldsymbol{\gamma}, d) \in \Gamma \times D$. Therefore, the sum of squared residuals function is evaluated at all the admissible threshold values of all the considered threshold variables. The CLS estimates of $(\boldsymbol{\gamma}, d)$ minimize the sum of squared residuals

$$(\hat{\boldsymbol{\gamma}}, \hat{d}\,) = \underset{(\boldsymbol{\gamma},d)}{argmin}\ S_m^*(\boldsymbol{\gamma}, d) \ . \tag{11}$$

The possible threshold variables considered in the grid-search can be exogenous variables included as regressors in the model or variables outside the model. In the case of a SETAR model, where $q_t = y_{t-d}$, the search for the optimal threshold variable corresponds to the estimation of the optimal delay lag of the dependent variable.

4.3 Testing for the Number of Regimes

An important step when proposing a nonlinear model is to assess whether it obtains a better fit to the data than a linear model. Formally, the hypothesis of a linear model against a 2-regime threshold model can be stated as $H_0 : \boldsymbol{\beta}_1 = \boldsymbol{\beta}_2$. Because the linear model is nested in the 2-regime threshold model, an intuitive approach is to reject the null hypothesis for large values of the standard F statistic

$$F_{12}(\hat{\gamma}) = T\left(\frac{S_1 - S_2}{S_2}\right) , \tag{12}$$

which corresponds to the likelihood ratio statistic under the auxiliary assumption that e_t is iid $N(0, \sigma^2)$. If γ is known, F_{12} could be approximated by a $\chi^2(k)$ distribution in large samples. However, standard asymptotic theory cannot be used to derive the limiting distribution of $F_{12}(\hat{\gamma})$ because the linearity test suffers from the unidentified nuisance parameter problem[4]. To understand this problem note that γ is not present in the linear model. Thus, in this context, the threshold parameter is an unidentified parameter since any value of γ is consistent with the null hypothesis. Naturally, the value of S_1 does not depend on γ because this parameter is not present in the linear model. In contrast, γ must be estimated in order to estimate the 2-regime model and obtain S_2. Then, even if $\beta_1 = \beta_2$ there is no reason to expect that $S_1 = S_2$ since S_2 is a function of γ, which is estimated in order to yield the best possible fit of the 2-regime model. As a consequence, the limiting distribution of $F_{12}(\gamma)$ is non-standard and depends on the nuisance parameter γ.

To see that $F_{12}(\hat{\gamma})$ does not have a $\chi^2(k)$ asymptotic distribution note that equation (12) is a monotonic function of S_2, since F_{12} always increases when S_2 decreases. Because the LS estimates minimize S_2 over the sets Γ and D we can

[4]This problem was first presented and discussed by Davies (1977).

write

$$F_{12} = \underset{\boldsymbol{\gamma}}{max}\ F_{12}(\hat{\boldsymbol{\gamma}})\ , \tag{13}$$

Thus, the test statistic F_{12} is actually the maximum of a set of distinct $\chi^2(k)$ random variables and, therefore, the distribution of F_{12} is considerably greater than the $\chi^2(k)$. To derive the asymptotic distribution of F_{12} we need to think of this test statistic as the random maximum of the random function $F_{12}(\gamma)$. Following a general investigation of inference in the presence of nuisance parameters, B. E. Hansen (1996) shows that the asymptotic distribution of F_{12} depends on the moments of the regressors and of the threshold variable q_t (which are application specific), thereby making the tabulation of critical values impossible.

A practical method to compute a bootstrap approximation to the sampling distribution of F_{12} is provided by B. E. Hansen (1999). Before describing the procedure let us define a more general version of the test statistic F_{12} that can be used to test models with a different number of regimes against each other. Specifically, for $i < l$, the test statistic (13) can be generalized as

$$F_{il} = \underset{\boldsymbol{\gamma}}{max}\ F_{il}(\hat{\boldsymbol{\gamma}}) = T\left(\frac{S_i - S_l}{S_l}\right)\ . \tag{14}$$

The key feature of the bootstrap approximation of B. E. Hansen (1999) is to generate multiple simulated times series that satisfy the null hypothesis that the data follows a i-regime threshold model. Under the assumption that e_t is conditionally homoskedastic, $E(e_t^2|\mathcal{F}_{t-1}) = \sigma^2$, the procedure can be described as follows: (i) Generate a sample $\hat{\boldsymbol{e}}^{\boldsymbol{*}}$ by making random draws (with replacement) from the i-regime threshold model residuals; (ii) Set the simulated observations as $\boldsymbol{y}^* = \sum_{j=1}^{i} \boldsymbol{X}_{(\hat{\gamma})}\hat{\boldsymbol{\beta}}_{\boldsymbol{j}} + \hat{\boldsymbol{e}}^*$, where the estimates $\hat{\boldsymbol{\theta}} = (\hat{\boldsymbol{\beta}}_{\boldsymbol{j}}, \hat{\gamma})$ are taken from the estimated i-regime threshold model; (iii) Use the estimation method described in subsection (4.2) and the data $(\boldsymbol{y}^*, \boldsymbol{X})$ to estimate the i and l-regime threshold models to obtain S_i^* and S_l^*, respectively, and compute F_{il}^* in a similar way as in equation (14); (iv) Repeat the previous steps a large number of times to obtain successive draws from F_{il}^*; (v) Compute the bootstrap p-value as the percentage of simulated samples for which F_{il}^* is greater than the observed value for F_{il}. Although this procedure is easy to implement and achieves a reasonable rate of convergence to the asymptotic distribution, it requires the assumption that the errors e_t are independent from the past information $\mathcal{F}_{t-1}$, which is considerably stronger than the martingale difference sequence assumption.

If e_t is conditionally heteroskedastic the bootstrap procedure just described has to be modified in order to obtain simulated times series with heteroskedastic errors. Because the only difference in relation to the homoskedastic bootstrap lies in how the errors are generated we relegate the details of this procedure to the appendix (B).

4.4 Inference

We now turn to the topic of inference on the threshold model estimates. Chan (1993) finds that the LS estimator of γ is T-consistent but has a non-standard limiting distribution which depends on a host of nuisance parameters[5]. Specifically, he showed that the threshold estimator follows a compound Poisson process which is a function of the marginal distribution and coefficients of the regressors. Thus, the asymptotic theory developed by Chan (1993) does not yield a practical procedure for inference on $\boldsymbol{\gamma}$ or for the computation of confidence intervals. Hansen proposed solution to overcome this difficulty is to let the threshold effect tend to zero asymptotically, in a similar fashion as in the change point literature[6]. Under this assumption the threshold model simplifies to a linear model as the sample size increases. B. E. Hansen (1997) and B. E. Hansen (2000) show that using this approach it is possible to derive an asymptotic distribution for γ which is free from nuisance parameters other than a scale parameter.

Consider a 2-regime threshold model. Under the auxiliary assumption that e_t is iid $N(0, \sigma^2)$, we can test the hypothesis $H_0 : \gamma = \gamma_0$ using the likelihood ratio statistic. Define this statistic as

$$LR(\gamma) = T\,\frac{S(\gamma) - S(\hat{\gamma})}{S(\hat{\gamma})}\;. \tag{15}$$

[5] A generalization of this asymptotic theory to the m-regime model was provided by Li and Ling (2012).

[6] See Picard (1985) and Bai (1997).

We would reject H_0 for large values of $LR(\gamma)$. Note that $S(\hat{\gamma})$ is evaluated at all admissible values of γ during the estimation of the model. Therefore, the test statistic $LR(\gamma)$ is a by-product of estimation. B. E. Hansen (2000) shows that the asymptotic distribution of the $LR(\gamma)$ is given by

$$LR(\gamma) \xrightarrow{d} \eta^2 \xi \; , \tag{16}$$

where ξ has a distribution function given by $P(\xi \leq x) = (1 - e^{-x/2})^2$ and η^2 is a nuisance parameter which depends upon the error variance and on moments of the regressors. If the homoskedasticity assumption holds then $\eta^2 = 1$ and the limiting distribution of $LR(\gamma)$ is non-standard and free from the nuisance parameter η^2. If the errors are conditionally heteroskedastic the scale parameter η^2 has to be estimated. Details on the estimation of η^2 are relegated to the appendix (C).

This theory enables the computation of asymptotically valid confidence intervals for γ. For this purpose, B. E. Hansen (2000) suggests inverting the $LR(\gamma)$ statistic. Let $c_\xi(C)$ be the critical value for predefined confidence level C, then an asymptotically valid confidence interval can be obtained as

$$\hat{\Gamma} = \{\gamma : LR(\gamma) \leq c_\xi(C)\} \; . \tag{17}$$

A practical method to visualize the confidence intervals is to plot the $LR(\gamma)$ statistic as a function of γ along with a line at the relevant critical value $c_\xi(C)$. All values of γ below $c_\xi(C)$ are included in $\hat{\Gamma}$. If the errors are believed to be heteroskedastic we can construct valid confidence intervals by substituting the $LR(\gamma)$ statistic in (17) by a scaled likelihood statistic defined as $LR(\gamma)^* = LR(\gamma)/\hat{\eta}^2$.

Finally, we discuss inference on the slope parameters. Note that the estimator $\hat{\boldsymbol{\beta}}(\gamma)$ depends on the threshold parameter. When γ is known it is possible to show that

$$\sqrt{T}\,(\hat{\boldsymbol{\beta}}(\gamma) - \boldsymbol{\beta}) \xrightarrow{d} N(0, \boldsymbol{V}_{\boldsymbol{\beta}}) \; ,$$

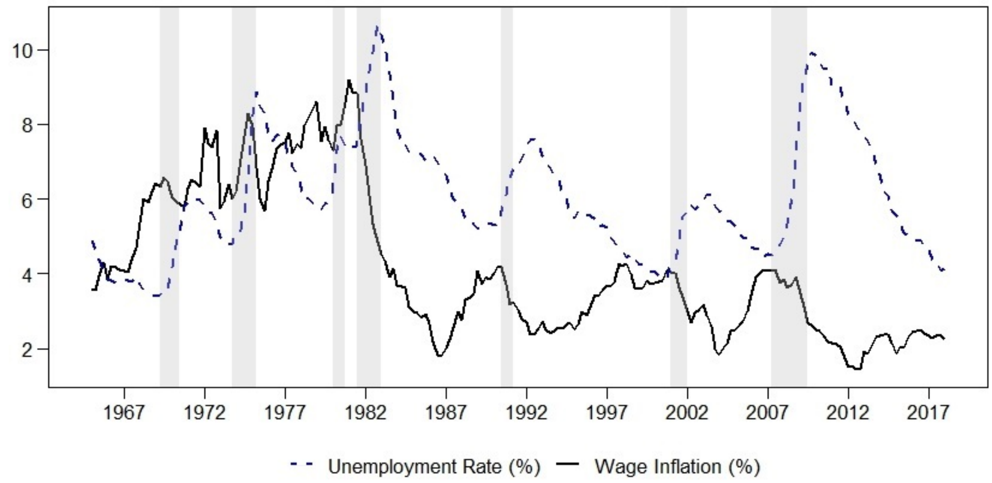
where $\boldsymbol{V}_{\boldsymbol{\beta}} = (\boldsymbol{X}'_{(\gamma)}\boldsymbol{X}_{(\gamma)})^{-1}\boldsymbol{X}'_{(\gamma)}E(\boldsymbol{e}\boldsymbol{e}')\boldsymbol{X}_{(\gamma)}(\boldsymbol{X}'_{(\gamma)}\boldsymbol{X}_{(\gamma)})^{-1}$ is the asymptotic covariance matrix under heteroskedasticity. Because the estimator $\hat{\gamma}$ is super consistent, B. E. Hansen (2000) argues that the parameter γ is not of first order asymptotic importance to the distribution of $\hat{\boldsymbol{\beta}}(\gamma)$. Therefore, valid confidence intervals may be constructed using the normal distribution as an approximation, even when the threshold parameter is estimated.

5 Empirical Evidence

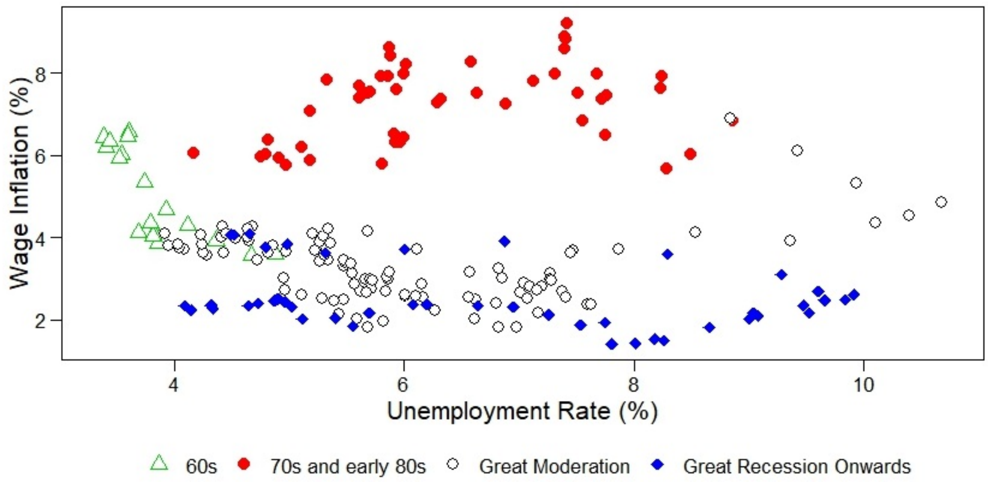
5.1 Data Description

To analyze the U.S. NKWPC we employ quarterly time series from the Federal Reserve Data (FRED) over the period 1964:Q1-2018:Q1. Wage inflation is based on earnings data for production and non-supervisory workers while price inflation is measured as the consumer price index, both variables are computed as four quarter growth rates. The unemployment rate is calculated as a hundred times the ratio between the number of unemployed and the civilian labour force. Finally, the natural unemployment rate corresponds to the Congressional Budget Office estimate. We use the same variables (and the same data transformations) as in Donayre and Panovska (2016) but we work with an extended sample which includes more observations on the recovery period following the 'Great Recession'. Standard Dickey-Fuller tests suggest that the considered time series are stationary. Wage inflation also seems to be stationary if we allow for a level shift in the early 80s. In appendix (A) we provide additional information regarding the variables employed in this work.

The NKWPC implies an inverse relationship between wage inflation and unemployment. Thus, as a preliminary step to our analysis, we proceed to briefly discuss if the data reflects this feature. Panel (a) of figure (1) plots wage inflation and the unemployment rate over our sample period. Additionally, we display a scatter plot in panel (b) of figure (1), where the observations are grouped according to different periods of the American economy. We consider the following time spans: 1965:Q1-1969:Q4 (60s), 1970:Q1-1981:Q4 (70s and early 80s), 1982:Q1-2006:Q4 ('Great Moderation') and 2007:Q1-2018:Q1 ('Great Recession' onwards). From this graphical analysis it is not easy to identify the negative relationship predicted by the NKWPC. In fact, the correlation coefficient between the two variables over the entire sample is practically zero. Also, as noted by Galí (2011), the relation-



(a)



(b)

Figure 1: The (π_t^w, u_t) relationship in the U.S. (1965:Q1-2018:Q1)

ship between wage inflation and the unemployment rate seems to be rather unstable over time[7]. However, a closer look reveals that the data seems to be consistent with the NKWPC during specific periods. The negative relationship is very pronounced during the 1960s and, to a lesser extent, during the 'Great Moderation'. It is worth noting that a common feature between these two periods is the absence of very severe recessions; in fact both periods are associated with solid economic expansions, despite the mild recessions in 1991 and 2001. Conversely, the relationship between the two variables is considerably different during the 1970s and early 1980s and in

[7]Evidence of instability in the context of a price Phillips curve is discussed in, for instance, King et al. (1995) and Musso et al. (2007)

the period following the 'Great Recession'. In the former period, the U.S economy was marked by the event of stagflation which translates into simultaneous high levels of wage inflation and unemployment. In fact, during this period of macroeconomic instability, the correlation between the two series (0.36) is positive, which contradicts the predicted sign of the NKWPC introduced in section (2). Similarly, there is also a mismatch between the data and the model predictions regarding the 'Great Recession' episode and the subsequent recovery period. The discrepancy lies in the fact that, taking into account the sharp increase in unemployment after 2007, inflation did not decrease as much as it would be expected if the data followed a linear wage Phillips curve. In contrast to other severe recessions (most notably the 'Great Depression') the economy did not enter into a deflationary period, leading the recent phenomenon to be labelled as the missing disinflation (see Doser et al., 2017). Additionally, the recovery period following the 'Great Recession' also seems to be at odds with the model predictions, given that the decrease of unemployment towards pre-crisis levels did not generate on inflationary pressure on wages. This observation suggests that the wage Phillips curve relationship became weaker during this period. The assessment of the existence and strength of a NKWPC type relationship has important implications to understand inflation dynamics and to the conduct of monetary policy.

This preliminary graphical analysis provides additional support for the case of nonlinearity, as it displays how the relationship between wage growth and unemployment changed over different periods of the American economy. Indeed, the wage Phillips curve appears to be rather unstable. Therefore, a model that aims to provide a good characterization of the data must be able to reproduce the changes in the relationship between wage inflation and the unemployment rate over different macroeconomic environments.

5.2 Specification Issues

We now turn to the estimation of the U.S. NKWPC. As discussed in section (2), the linear version of the reduced form NKWPC represented in equation (5) depends on the assumption that cyclical unemployment follows a stationary AR(2) process. An univariate analysis of this variable suggests that this assumption is reasonable since an AR(2) model is sufficient to eliminate symptoms of residual autocorrelation. Additionally, the estimates for the autoregressive parameters ϕ_1 and ϕ_2 are positive and negative, respectively. This implies that the NKWPC predicts a negative sign on current cyclical unemployment and a positive sign on its lag, since, as we can see from equation (6), the signs of the NKWPC reduced form parameters ψ_1 and ψ_2 only depend on the signs of ϕ_1 and ϕ_2.[8]

Given this preliminary note, and taking into account the evidence of nonlinearities in the (π_t^w, u_t) relationship discussed in the last subsection, we now consider the ability of a 2 and 3-regime threshold regression model to describe the U.S. NKWPC. Using the 2-regime model representation in equation (8), and the 3-regime representation in equation (9), we can write the NKWPC by setting $y_t = \pi_t^w$ $\boldsymbol{x}_t = (1\ \pi_{t-1}^p \hat{u}_t\ \hat{u}_{t-1})'$ and $\boldsymbol{\beta}_j = (\alpha_j\ \rho_j\ \psi_{0,j}\ \psi_{1,j})$. At this point, it is important to discuss some options adopted in our estimation procedure. To simplify the exposition, we use the 2-regime model results to present our choices. We begin by discussing the selection of the threshold variable, q_t. It is important to emphasize that our regime switching regression approach is motivated by evidence that the NKWPC varies across the different phases of the business cycle. Therefore, we restrict the set of admissible threshold variables, q_t, to the current and lagged series of unemployment (u_{t-d}), cyclical unemployment $(\hat{u}_{t-d})$ and variations in unemployment $(\Delta u_{t-d} = u_{t-d} - u_{t-d-1})$, which are often regarded as indicators of the state of the business cycle. Also, we set $\hat{d} = 3$ to define the maximum value considered for the delay parameter d. The results from the CLS grid-search indicate that the sum of squared residuals function is minimized when u_t is considered as the threshold variable and therefore we set $q_t = u_t$ in our empirical application.

This result seems to indicate that current labour market conditions have an immediate effect on the prevailing NKWPC dynamics, as the regime switches are

[8]This implication of the NKWPC is consistent with the idea that wage inflation depends negatively on the level and change of cyclical unemployment. See A. W. Phillips (1958), Blanchard and Galí (2007) and Blanchard and Galí (2010).

Table 1: Threshold estimates for the 2-regime NKWPC

Threshold Variable	τ	$\hat{\gamma}$	95% Confidence Interval	SSR 2-Regimes
u_t	0.15	7.63	[7.41; 7.75]	221.83
u_t	0.1	8.18	[7.63; 8.28]	219.88

determined by a contemporaneous variable. If this is the case, it seems reasonable to consider the possibility that π_t^w and u_t are jointly determined and take into account the endogeneity of q_t when estimating the NKWPC. In contrast to our result, Donayre and Panovska (2016) use a similar, but shorter, sample and conclude that u_{t-1} is the optimal threshold variable. In any case, the selection of u_t as the threshold variable is not a novelty in the literature (see Kumar and Orrenius, 2016 and Donayre and Panovska, 2018), even though these studies devote little attention to the robustness of their estimates to the possibility of an endogenous threshold variable. Additionally, we also want to assess the sensibility of our results to the potential endogeneity of the regressor $\hat{u}_t$, due to the fact that this variable is also an indicator of the current level of labour market slack. For now, we proceed our study assuming that both u_t and $\hat{u}_t$ are pre-determined and analyze the robustness of our results when these variables are taken as endogenous in the next section. Table (1) displays the CLS results for the 2-regime model. We report the estimated threshold along with the respective 95% confidence intervals for different values of the trimming parameter, τ, which determines the minimum number of observations in each regime. The table reveals that the estimated threshold depends on the value of τ. When $\tau = 0.15$ we obtain $\hat{\gamma} = 7.63$ with a 95% confidence interval given by $\hat{\Gamma} = [7.41; 7.75]$. However, it is possible to obtain a new global minimum for the sum of squared residuals function by considering $\tau = 0.1$. In this case, the threshold estimate is $\hat{\gamma} = 8.18$ with $\hat{\Gamma} = [7.63; 8.23]$. Thus, the change in regime appears to occur at high values of u_t. In fact, the 90th percentile of u_t is 8.29 which implies that the estimate $\hat{\gamma} = 8.18$ almost reaches the maximum admissible value. Because our sample size is moderate, we take a conservative approach and set $\tau = 0.15$ in our application to ensure a larger number of observations in the high unemployment

Table 2: Tests for linearity and for the number of regimes

	Test Statistic	Homo. Bootstrap p-value	Het. Bootstrap p-value
F_{12}	42.68	0.00	0.00
F_{13}	67.86	0.00	0.00
F_{23}	20.97	0.01	0.01

regime.

An important conclusion from our discussion of the 2-regime model is that the estimated threshold corresponds to a value close to the right end of the empirical distribution of u_t. This, in turn, implies that u_t seldom crosses the estimated threshold, which can be interpreted as favourable evidence for a linear specification. However, our threshold estimate is very accurate as reflected by the narrow confidence interval for $\hat{\gamma}$. The high and precise threshold estimate seems to indicate that there is a significant change in wage inflation dynamics when the economy enters in a period of severe recession. Therefore, the nonlinearities in the NKWPC appear to justify the distinction of, at least, two different regimes.

In order to define the appropriate number of regimes, we report in table (2) the F-statistics and respective bootstrapped p-values associated with the test procedure described in subsection (4.3). The estimated sampling distributions of the F-statistics are reported in appendix (D). The statistics F_{12} and F_{13} refer to the test of the linear model against the 2 and 3-regime model, respectively. To discriminate between the two nonlinear models we use the statistic F_{23} to test for remaining nonlinearity in the 2-regime model. The linearity tests provide strong evidence for the rejection of the linear model as the p-values for F_{12} and F_{13} are approximately zero. Also, the p-values obtained for F_{23} clearly favor a 3-regime model. These results are identical for both the homoskedastic and heteroskedastic versions of the bootstrap. We arrive at the same conclusion as in Donayre and Panovska (2016) and, in fact, our results for the F-statistics and respective p-values are very similar to those reported in this study. Given these results, we present and discuss the results for the 3-regime NKWPC in the next section.

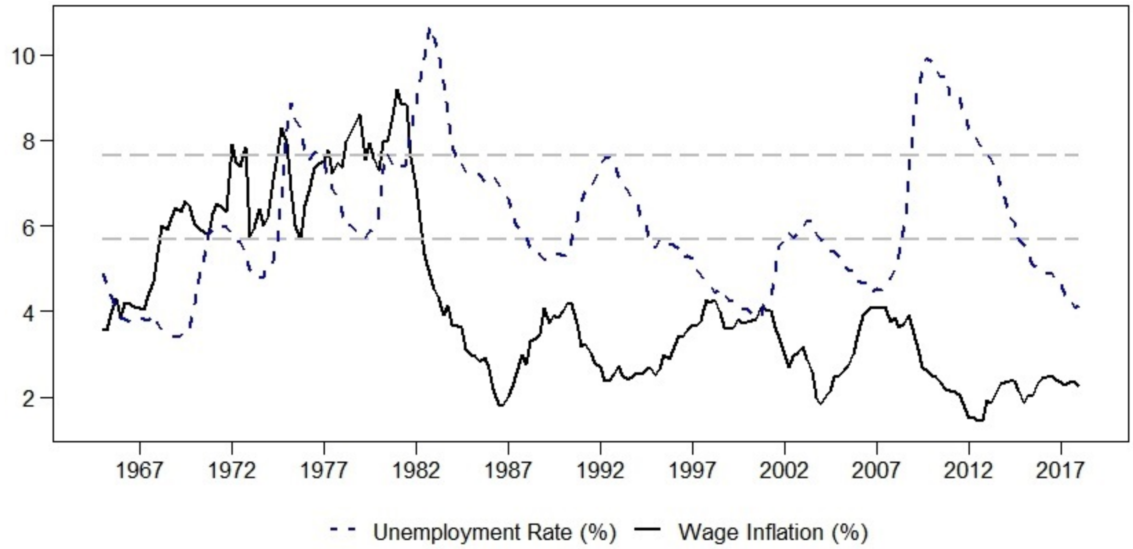


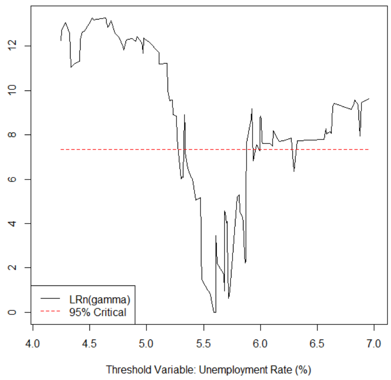
Figure 2: Estimated thresholds for the U.S. NKWPC (1965Q1:2018Q1)

5.3 Results

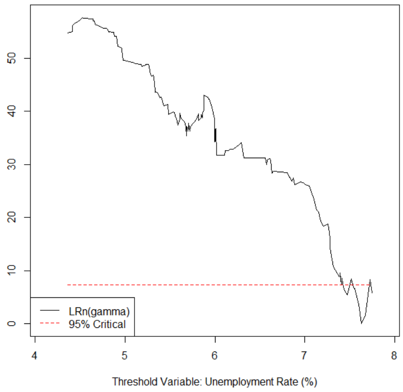
The estimated thresholds of the 3-regime NKWPC correspond to $\hat{\gamma}_1 = 5.69$ and $\hat{\gamma}_2 = 7.63$. Figure (2) displays how the model splits the data into the different regimes. Based on the threshold estimates, we can observe that wage inflation dynamics are described by the first regime, which is active when $u_t \leq 5.69$, during the most part of the 1965-1975 and 1994-2008 periods. We can think of this regime as corresponding to prolonged economic expansions associated with reduced levels of cyclical unemployment. It is also possible to conclude that this regime includes most of the low and stable wage inflation periods, which justifies the inclusion of the 1969-70 recession in the first regime, as the wage inflation was not yet at the very high levels observed during most of the 1970s and early 1980s.

When unemployment lies in the boundary defined by $u_t \in (5.69; 7.63]$ the NKWPC is described by the second regime. This regime can be associated with moderate business cycle fluctuations. Lastly, deep recessions are included in the third regime. In particular, this regime captures the most severe years of the early 1980 and 2008 recessions.

Confidence intervals for the estimated threshold are represented in figure (3), where we plot the $LR(\gamma)$ statistic profile along with the 95% critical value. As dis-



(a) 1st Threshold $\hat{\gamma}_1$



(b) 2nd Threshold $\hat{\gamma}_2$

Figure 3: Likelihood ratio sequence in γ for the two thresholds

cussed in the previous subsection, we obtain a very precise estimate of the threshold $\hat{\gamma}_2 = 7.63$ as reflected by the narrow confidence interval given by $\hat{\Gamma}_{\hat{\gamma}_2} = [7.41; 7.75]$. On the other hand, the confidence interval for $\hat{\gamma}_1$ is $\hat{\Gamma}_{\hat{\gamma}_1} = [5.28; 6.29]$ which implies a higher level of uncertainty regarding this estimate. In any case, this interval is still not very wide and, since there is a significant distance between the upper limit for $\hat{\gamma}_1$ and the lower limit for $\hat{\gamma}_2$, we interpret these estimates as consistent with the conclusion that a 3-regime threshold model seems to be appropriate to model the NKWPC.

Table (3) reports the estimation results for the 3-regime NKWPC. The diagnosis of the model residuals using the Breusch-Pagan test provides significant evidence for the rejection of homoskedastic errors. Additionally, standard autocorrelation tests based on the Ljung-Box and LM statistics reveal that the 3-regime model residuals show symptoms of autocorrelation. Therefore, we report HAC standard errors in addition to heteroskedastic robust standard errors[9]. The intercept estimate is similar across the first and second regimes, while in the third it rises to 3.60. The coefficients associated with lagged price inflation π_{t-1}^p suggest that price indexation is economically relevant, especially in the second regime. It is also worth noting that the standard errors of the coefficients associated with π_{t-1}^p are remarkably small and

[9]HAC standard errors are estimated using a Bartlett Kernel and the Newey-West fixed bandwidth method option in Eviews 10.

Table 3: Estimation results for the 3-regime threshold model

	Threshold Variable	u_t		1st Threshold	5.69		SSR		202.3
	Trimming Parameter	0.15		2nd Threshold	7.63		Residual Variance		0.95

	Regime 1 ($u_t \leq 5.69$)			Regime 2 ($5.69 < u_t \leq 7.63$)			Regime 3 ($u_t > 7.63$)		
		Standard Errors			Standard Errors			Standard Errors	
Variable	Estimate	HAC	White	Estimate	HAC	White	Estimate	HAC	White
Constant	2.216	0.190	0.115	2.161	0.678	0.392	3.604	1.858	1.245
π^p_{t-1}	0.373	0.067	0.040	0.612	0.075	0.047	0.377	0.094	0.064
$\hat{u}_t$	-0.709	0.647	0.483	-2.177	0.725	0.482	0.318	0.420	0.284
$\hat{u}_{t-1}$	-0.145	0.675	0.499	1.351	0.602	0.420	-0.710	0.327	0.249
Observations (% of total)	95	(44.6%)		83	(39%)		35	(16.4%)	
Regime Variance	0.478			1.424			1.104		

that this variable is statistically significant across all regimes.

In our analysis, we are most interested in the coefficients associated with cyclical unemployment as they allow us to evaluate if the data supports the relationship predicted by the NKWPC. Overall, the NKWPC predictions that cyclically unemployment should exhibit a negative signal on its current value and a positive signal on its lag does not hold in all regimes. Additionally, and in contrast to Donayre and Panovska (2016), our results seem to indicate that the NKWPC is convex since the sum of the coefficients associated with cyclical unemployment is decreasing as we move from the first regime to the third.

In the first regime, associated with prolonged economic expansions, the coefficients associated with cyclical unemployment are both negative. Furthermore, none of them is statistically significant for standard significance levels. In contrast, Donayre and Panovska (2016) find a marginally significant coefficient on $\hat{u}_t$ at the 10% significance level. We can justify this difference by the fact that we estimate the first regime parameters including the observations from the 2014-2018 period which, in turn, seem to contribute to the conclusion of a weak relationship between wage inflation and cyclical unemployment. This description of the first regime seems consistent with the recent experience in the U.S. of weak wage growth and a simultaneous low level of unemployment.

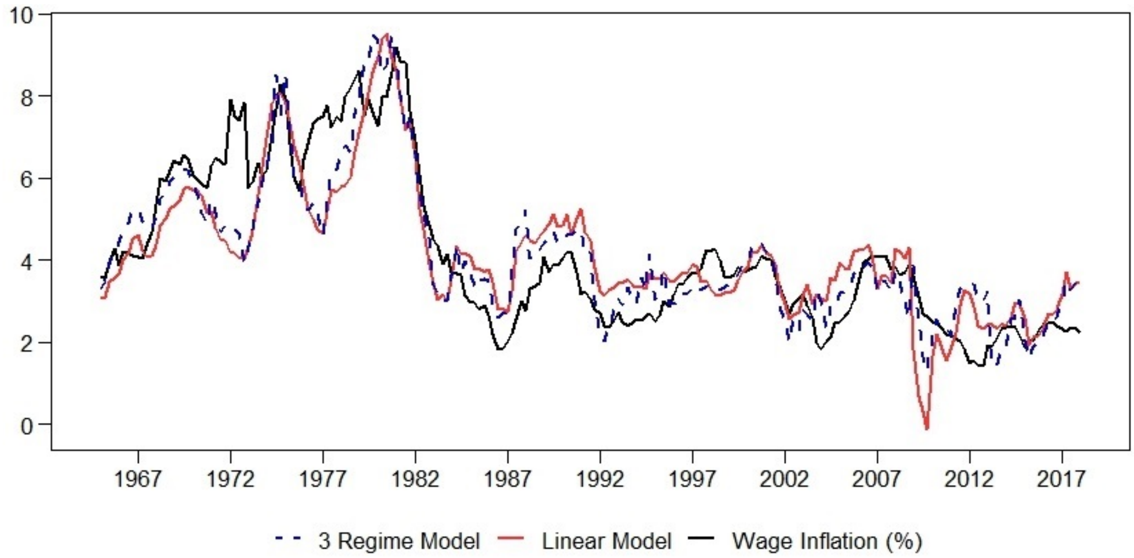


Figure 4: Fitted values for the U.S. wage inflation (1965Q1:2018:Q1)

The second regime results are consistent with the predictions of the NKWPC as the estimates for the cyclical unemployment coefficients have the correct signals and are statistically significant. Furthermore, cyclical unemployment appears to have an important effect on wage growth as a percentage point increase in cyclical unemployment results in a 2.18 percentage point decrease in wage inflation on impact and 1.35 percentage point increase after one quarter. Our results suggest that when unemployment is in an intermediate level there is a traditional Phillips curve relationship between wage inflation and cyclical unemployment as the two variables appear to be negatively related.

In the deep recessions regime, the NKWPC provides a poor description of the data. In particular, the two coefficients associated with cyclical unemployment have the incorrect sign. Additionally, only lagged cyclical unemployment is statistically significant. Therefore, the relationship embodied in the NKWPC does not seem to yield a practical monetary policy guide during severe economic depressions.

To further evaluate the ability of the 3-regime NKWPC to describe U.S. wage inflation, figure (4) plots the model fitted values along with the observed wage inflation. Also, the fitted values from the linear model are also displayed for comparison. In general, the 3-regime NKWPC seems to provide a reasonable fit to the data as it captures most of the general movement in wage inflation. On the other hand, the model estimates misses much of the high-frequency variation and fails to account for the 1971-1972, 1976-1977 and 'Great Recession' periods. When compared to the linear model, the fit of the 3-regime model is substantially better. In fact, the root mean squared error of the 3-regime model is 13% inferior to the linear model, while there is an improvement of 6% in the correlation between the actual and fitted series of the 3-regime model in relation to the linear model.

From our analysis so far we can conclude that the NKWPC is significantly non-linear and that a 3-regime threshold model can provide a reasonable characterization for the behavior of wage inflation in the U.S. from 1965 to 2018. The model estimates show that the relationship between wage inflation and cyclical unemployment is consistent with the NKWPC in the second regime, which is active when the unemployment rate is in an intermediate level. Therefore, the traditional negative relationship between wage inflation and unemployment seems to hold only during moderate business cycle fluctuations. On the other hand, the NKWPC seems to break down in the first and third regimes, associated with deep recessions and prolonged expansions periods, respectively. As a result, and according to our model, further reductions in the unemployment rate will not contribute to the recovery of the wages from the sluggish growth observed since the 'Great Recession'.

6 Endogeneity

6.1 Theoretical Motivation

In the previous section, we conducted our study assuming that both current unemployment and cyclical unemployment were orthogonal to the error term. However, contemporaneous variables that aim to capture the degree of labour market slack are often regarded as endogenous in the literature[10].

The case for endogeneity is motivated by the existence of a feedback effect between wage inflation and the level of economic activity. NK models derive the

[10] See chapter 7 of Bårdsen et al. (2005), Malikane and Mokoka (2014), Piazza (2018), Ho and Njindan Iyke (2018) and Albuquerque and Baumann (2017)

interdependence between these two variables and typically treat them as a system[11]. McLeay and Tenreyro (2018) provide further rationale for endogeneity by analyzing a simple model where the monetary authorities take this relationship into account when setting the optimal policy rule. By assuming that the inflation rate is determined by a Phillips curve and that the policymaker can determine the level of cyclical unemployment, the authors show that it is not possible to identify the Phillips curve from the data, precisely because the central bank can offsets this relationship. Intuitively, this lack of identification is the result of a simultaneity bias, as the observed inflation rate is the equilibrium outcome between the central bank policy rule and the Phillips curve.

Based on the conclusions of McLeay and Tenreyro (2018), we could argue that the absence of evidence for NKWPC in the low and high unemployment regimes reflects the increase in central bank efforts to counter inflationary or deflationary pressures in the economy, which offsets the Phillips curve relationship. Thus, in order to assess the robustness of our results, we now relax the orthogonality assumption regarding the unemployment rate, which plays the role of threshold variable, and the current cyclical unemployment rate. To accommodate this new set of assumptions in the threshold regression model we need a different estimation and inference theory from the one presented in section (5). In the next subsection we describe this new framework.

6.2 The Structural Threshold Model

Endogeneity is an increasingly popular topic in the threshold regression literature. An extension of the asymptotic framework of B. E. Hansen (2000) to the case of endogenous regressors and an exogenous threshold variable is presented by Caner and B. E. Hansen (2004). Kourtellos et al. (2016) developed a consistent estimator and a distribution theory that allows for endogeneity in both the regressors and the threshold variable. To address the endogeneity problem, these studies rely on the utilization of instrumental variables, leading to the emergence of two stage least

[11]See, for instance, the basic 3 equation NK model presented in Galí (2015).

squares (2SLS) and general method of moments (GMM) estimators in the threshold model literature. A nonparametric approach that does not require the use of instrumental variables was introduced by Yu and P. C. Phillips (2018), which also allows for endogeneity in both the regressors and threshold variable. However, the application of this nonparametric estimator is restricted to iid data, in contrast with Kourtellos et al. (2016) which allows for stationary and ergodic data. Additional related literature includes Kapetanios (2010), who provides a procedure to test the exogeneity of the regressors based on the bootstrap of a Hausman-type statistic, and Dentler et al. (2014), who analyze the conditions under which the endogeneity of an explanatory variable does not affect nonlinearity tests.

In our empirical application, we are interested in an estimator that addresses the case of endogeneity in both the threshold variable and the regressors. Therefore, we now provide a description of the Kourtellos et al. (2016) structural threshold model. Intuitively, consistent estimation of the threshold is achieved by adding parametric assumptions regarding the structural model. In fact, Kourtellos et al. (2016) relate the problem of having an endogenous threshold variable to the problem of having an endogenous sample selection variable, as in the limited dependent variable literature; see Heckman (1979). The difference between the two frameworks, however, is that in the sample selection model the assignment of observations into the different regimes is observed (this is, the threshold is known) but the sample selection (or threshold) variable is taken as latent, where in threshold models it is not possible to known with certainty which observations belong to each regime (that is, the threshold is unknown) and the threshold variable is observable. Once again, consider the 2-regime threshold model studied by B. E. Hansen (2000) augmented by a reduced form equation for q_t and for $\boldsymbol{x}_t$

$$y_t = \boldsymbol{x_t'\beta_1} I(q_t \leq \gamma) + \boldsymbol{x_t'\beta_2} I(q_t > \gamma) + e_t \ ,$$

$$q_t = \boldsymbol{z}_t'\boldsymbol{\delta}_q + v_{qt} \ ,$$

$$\boldsymbol{x}_t = \boldsymbol{z}_t'\boldsymbol{\delta}_x + v_{xt} \ ,$$

where $\boldsymbol{z}_t = (z_{t1}, z_{t2}, ..., z_{tp})'$ is a $1 \times p$ vector of exogenous explanatory variables and $\boldsymbol{x}_t = (x_{t1}, x_{t2}, ..., x_{tk})'$, such that $p \geq k$. The reduced form errors v_{qt} and v_{xt} are martingale difference sequences and therefore $E(v_{qt}|\mathcal{F}_{t-1}) = 0$ and $E(v_{xt}|\mathcal{F}_{t-1}) = 0$. We denote the conditional expectation for $\boldsymbol{x}_t$ by $\boldsymbol{g}_t = E(\boldsymbol{x}_t|\mathcal{F}_{t-1}) = \boldsymbol{z}_t'\boldsymbol{\delta}_x$. We are interested in the case where q_t is endogenous and therefore we assume that $E(e_t|\mathcal{F}_{t-1}, v_{qt}) \neq 0$. As shown by Yu (2013), the CLS estimation method of B. E. Hansen (2000) is inconsistent in this setting. Additionally, Yu (2013) demonstrates that a 'naïve' 2SLS procedure, where q_t is substituted by the adjusted values $\hat{q}_t = \boldsymbol{z}_t'\hat{\boldsymbol{\delta}}_q$ in the CLS grid-search, also fails to produce consistent estimates. Intuitively, the inconsistency of these two estimators is caused by the fact that the endogeneity bias is not taken into account in the CLS objective function. In order to achieve consistency, Kourtellos et al. (2016) adds a set of parametric assumptions regarding the (e_t, v_{qt}) relationship to derive the bias correction terms. The assumptions are the following

$$\boldsymbol{A.1.}\;\; E(e_t|\mathcal{F}_{t-1}, v_{qt}) = E(e_t|v_{qt}); \quad \boldsymbol{A.2.}\;\; E(e_t|v_{qt}) = \kappa v_{qt}; \quad \boldsymbol{A.3.}\;\; v_{qt} \sim N(0,1)\ .$$

The first assumption establishes conditional mean independence of e_t from the information set $\mathcal{F}_{t-1}$. Assumption $\boldsymbol{A.2.}$ implies a linear relationship between $E(e_t|v_{qt})$ and v_{qt}. Lastly, $\boldsymbol{A.3.}$ assumes that the reduced form error v_{qt} follows a normal distribution. Using these assumptions it can be shown that

$$E(e_t|\mathcal{F}_{t-1}, v_{qt} \leq \gamma - \boldsymbol{z}_t'\boldsymbol{\delta}_q) = \kappa\lambda_{1t}(\gamma - \boldsymbol{z}_t'\boldsymbol{\delta}_q)\ , \tag{18}$$

$$E(e_t|\mathcal{F}_{t-1}, v_{qt} > \gamma - \boldsymbol{z}_t'\boldsymbol{\delta}_q) = \kappa\lambda_{2t}(\gamma - \boldsymbol{z}_t'\boldsymbol{\delta}_q)\ , \tag{19}$$

where $\lambda_{1t}(\gamma - \boldsymbol{z}_t'\boldsymbol{\delta}_q) = -\frac{\phi(\gamma - \boldsymbol{z}_t'\boldsymbol{\delta}_q)}{\Phi(\gamma - \boldsymbol{z}_t'\boldsymbol{\delta}_q)}$ and $\lambda_{2t}(\gamma - \boldsymbol{z}_t'\boldsymbol{\delta}_q) = \frac{\phi(\gamma - \boldsymbol{z}_t'\boldsymbol{\delta}_q)}{1-\Phi(\gamma - \boldsymbol{z}_t'\boldsymbol{\delta}_q)}$ denote the inverse Mills ratios terms. The parameter κ corresponds to the covariance between e_t and v_{qt} and the functions $\phi(.)$ and $\Phi(.)$ are the normal pdf and cdf, respectively. We can write the inverse Mills ratio terms in a single equation by using the following definition

$$\Lambda_t(\gamma) = \lambda_{1t}(\gamma - \boldsymbol{z}_t'\boldsymbol{\delta}_q)\, I(q_t \leq \gamma) + \lambda_{2t}(\gamma - \boldsymbol{z}_t'\boldsymbol{\delta}_q)\, I(q_t > \gamma)\ , \tag{20}$$

which allows us to write the structural threshold regression model, with endogenous threshold and slope variables, as

$$y_t = \boldsymbol{g_t'\beta_1} I(q_t \leq \gamma) + \boldsymbol{g_t'\beta_2} I(q_t > \gamma) + \kappa\, \Lambda_t(\gamma) + \varepsilon_t\ , \tag{21}$$

where $\varepsilon_t = e_t - \kappa \Lambda_t(\gamma)$. It is possible to see how this model nests different variations of the threshold regression model. If $\kappa = 0$, we get the threshold model studied by Caner and B. E. Hansen (2004). If $\boldsymbol{x}_t = \boldsymbol{z}_t$ the model corresponds to the one studied in Seo and Linton (2007), in which q_t is a linear function of observed variables. Additionally, if the slope variables are exogenous and q_t corresponds to a single variable we have the threshold model of B. E. Hansen (2000).

Three steps are required to estimate the model. First, estimate the reduced form regression for $\boldsymbol{x}_t$ and q_t by LS to obtain $\hat{\boldsymbol{\delta}}_x$ and $\hat{\boldsymbol{\delta}}_q$, in order to set $\hat{\boldsymbol{g}}_t = \boldsymbol{z}_t'\hat{\boldsymbol{\delta}}_x$ and $\hat{\Lambda}_t(\gamma) = \lambda_{1t}(\gamma - \boldsymbol{z}_t'\hat{\boldsymbol{\delta}}_q)\, I(q_t \leq \gamma) + \lambda_{2t}(\gamma - \boldsymbol{z}_t'\hat{\boldsymbol{\delta}}_q)\, I(q_t > \gamma)$. Second, perform a CLS grid-search to obtain the conditional estimates for the slope parameters $\hat{\boldsymbol{\beta}}_{\mathbf{1}}(\gamma)$, $\hat{\boldsymbol{\beta}}_{\mathbf{2}}(\gamma)$ and $\hat{\kappa}(\gamma)$. Then, we can estimate γ by minimizing the SSR function as

$$\hat{\gamma} = \underset{\gamma}{argmin} \sum_{t=1}^{T} \left(y_t - \hat{\boldsymbol{g}}_t'\hat{\boldsymbol{\beta}}_1(\gamma) I(q_t \leq \gamma) - \hat{\boldsymbol{g}}_t'\hat{\boldsymbol{\beta}}_2(\gamma) I(q_t > \gamma) - \hat{\kappa}(\gamma)\hat{\Lambda}_t(\gamma)\right)^2 . \quad (22)$$

Finally, split the observations into different regimes according to $\hat{\gamma}$ and obtain the estimates for the slope parameters $\boldsymbol{\beta}$ and κ by LS or GMM. Assuming that both the threshold effect, $\boldsymbol{\beta}_1 - \boldsymbol{\beta}_{\mathbf{2}}$, and the endogeneity bias, κ, tend to zero, Kourtellos et al. (2016) shows that inference and construction of confidence intervals is similar to the case of an exogenous threshold variable.

The estimation of the 3-regime model can be done using the following procedure[12]. We begin by estimating a 2-regime model using the complete sample in order to obtain the initial threshold estimate, $\hat{\gamma}_1$, that allows us to split the data into two subsamples. Then, by imposing that one element of $\boldsymbol{\gamma} = (\gamma_1, \gamma_2)$ equals $\hat{\gamma}$, say $\gamma_1 = \hat{\gamma}_1$, we proceed to estimate a 2-regime model for the subsample where the null of linearity is rejected, which yields the second-stage threshold estimate, $\hat{\gamma}_2$. It can be shown, that this method yields consistent estimates for $\boldsymbol{\gamma} = (\gamma_1, \gamma_2)$. By iterating this method at least once, this is, by imposing that one element of $\boldsymbol{\gamma} = (\gamma_1, \gamma_2)$ equals $\hat{\gamma}_2$ in order to obtain a refined estimate $\hat{\gamma}_1$, the threshold estimates can be made asymptotically efficient, in the sense that they have the same

[12]This method was developed by Bai (1997) and Bai and Perron (1998) in the context of change-point models. For an application of this procedure to the case of a SETAR model see B. E. Hansen (1999).

asymptotic distribution as those obtained by joint minimization of the CLS criteria with respect to the pair (γ_1, γ_2). Given the final estimate $\hat{\boldsymbol{\gamma}} = (\hat{\gamma}_1, \hat{\gamma}_2)$, slope variable estimates can be obtained by LS or GMM on each subsample.

6.3 Robustness of the NKWPC Estimates to Endogeneity

In this subsection, we examine the sensibility of the NKWPC estimates when potential endogeneity of u_t and $\hat{u}_t$, is taken into account. We consider three different cases: i) endogenous threshold variable only; ii) endogenous regressor only; iii) endogenous threshold variable and regressor. In order to accommodate these assumptions, we use the estimator developed by Kourtellos et al. (2016), which relies on the use of instrumental variables.

To select the appropriate instruments, we resort to the strategies typically used in studies of the Phillips curve. The first strategy we explore is the use of lags of endogenous variables as instruments. As noted by Albuquerque and Baumann (2017), this seems to be a common strategy when accounting for endogeneity since it is both intuitive and simple to implement. In fact, McLeay and Tenreyro (2018) argue that in order to commit to a certain optimal policy rule, monetary authorities have to consider the future development of labour market slack, thus generating a correlation between unemployment and its lags. This information can, therefore, be accounted in the estimation of the model.

We also consider as instruments the set of variables used in the linear NK Phillips curve literature[13]. A common practice in this literature is to directly estimate the structural equation of the NK Phillips curve using GMM to account for endogeneity in both the slack variable and in the expected inflation term. The basis of this approach relies on the assumption of rational expectations, which implies that the error term should be orthogonal to variables dated $t-1$ and earlier. The set of instrumental variables in these studies typically includes lags of the labour share, output gap, long-short interest rate spread and commodities price inflation.

[13] Some very influential articles in this field are for instance Galí and Gertler (1999) and Galí, Gertler, and Lopez-Salido (2001); for a critical review of these studies see Rudd and Whelan (2007).

To evaluate the validity of the instruments, we resort to the procedure presented by L. Hansen (1982) and test subsets of the orthogonality conditions. Given that there is no distribution theory to conduct these tests in the context of threshold models we assess the validity of the instruments in the linear setting. This simplifying approach allows us to obtain a certain degree of confidence in the instruments used for the regressor $\hat{u}_t$. However, by performing the tests in the linear setting, we can not directly assess the validity of the instruments employed for the threshold variable u_t. Notwithstanding, if a subset of instruments seems to be valid for $\hat{u}_t$ we consider it to be equally valid to instrument u_t. In appendix (E) we present the lists of instruments employed in estimation and report the results from these tests, which seem to indicate that all the considered instrumental variables are appropriate.

In table (4) we report the estimation results when we allow for endogeneity in the NKWPC. The slope parameters are estimated by GMM for all the presented exercises and the standard errors estimates are corrected for autocorrelation. Next to the threshold estimates we report the 95% confidence interval in brackets. The first main remark is that the estimated thresholds are very similar to those obtained in the previous section. In fact, we obtain an estimate of $\hat{\gamma}_2 = 7.63$ in all but one of the reported exercises. The estimate $\hat{\gamma}_1$ is more sensible to the set of adopted hypothesis and instruments, which is expectable given the higher variability associated with the estimation of this threshold as reflected by panel (a) of figure (3). It is worth noting that when we consider an endogenous threshold variable, the estimate for $\hat{\gamma}_1$ is slightly larger. However, this small increase leads to a substantial reduction in the number of observations in the second regime, when we compare with the results of section (5). In general, the confidence intervals are similar to those of the last section. The exception is when both the threshold variable u_t and the regressor $\hat{u}_t$ are taken as endogenous, in which case the confidence interval for γ_1 is considerably wider. Even though the confidence intervals for the thresholds overlap, the distance between the point estimates seems sufficient to support a 3-regime model.

We now turn to the evaluation of the NKWPC predictions for the coefficients related to cyclical unemployment. The qualitative implications of the estimates ob-

Table 4: Structural threshold regression estimates of the NKWPC

	Endogenous threshold variable only: u_t					
	Instrument set 1			Instrument set 2		
	1st Threshold, $\hat{\gamma}_1$= 6.1 [5.54; 6.32]			1st Threshold, $\hat{\gamma}_1$= 6.1 [5.55; 6.2]		
	2nd Threshold, $\hat{\gamma}_2$= 7.63 [7.47; 7.75]			2nd Threshold, $\hat{\gamma}_2$= 7.63 [7.42; 7.75]		
Variable	Regime 1	Regime 2	Regime 3	Regime 1	Regime 2	Regime 3
	($u_t \leq 6.1$)	($6.1 < u_t \leq 7.63$)	($u_t > 7.63$)	($u_t \leq 6, 1$)	($6.1 < u_t \leq 7.63$)	($u_t > 7, 63$)
Constant	1.483	2.108	5.098	1.632	1.805	5.431
	(0.186)	(0.625)	(1.200)	(0.165)	(0.520)	(0.836)
π^p_{t-1}	0.445	0.700	0.342	0.404	0.683	0.346
	(0.040)	(0.046)	(0.057)	(0.035)	(0.040)	(0.035)
$\hat{u}_t$	-0.907	-2.881	-0.048	-1.122	-2.652	-0.116
	(0.338)	(0.659)	(0.249)	(0.292)	(0.527)	(0.235)
$\hat{u}_{t-1}$	-0.155	2.290	-0.612	0.005	2.153	-0.645
	(0.311)	(0.532)	(0.2)	(0.28)	(0.422)	(0.165)
Observations	130	48	35	130	48	35

	Endogenous regressor only: $\hat{u}_t$					
	Instrument set 1			Instrument set 2		
	1st Threshold, $\hat{\gamma}_1$= 5.61 [5.48; 5.99]			1st Threshold, $\hat{\gamma}_1$= 5.72 [5.48; 6.95]		
	2nd Threshold, $\hat{\gamma}_2$= 7.75 [7.42; 7.75]			2nd Threshold, $\hat{\gamma}_2$= 7.63 [7.39; 7.75]		
Variable	Regime 1	Regime 2	Regime 3	Regime 1	Regime 2	Regime 3
	($u_t \leq 5.61$)	($5.61 < u_t \leq 7.75$)	($u_t > 7.75$)	($u_t \leq 5.72$)	($5.72 < u_t \leq 7.63$)	($u_t > 7.63$)
Constant	2.271	2.060	3.157	2.301	2.054	3.327
	(0.123)	(0.362)	(1.283)	(0.115)	(0.342)	(0.845)
π^p_{t-1}	0.362	0.650	0.393	0.367	0.594	0.399
	(0.043)	(0.043)	(0.06)	(0.038)	(0.041)	(0.041)
$\hat{u}_t$	0.070	-2.82	0.921	-0.006	-1.465	0.262
	(0.551)	(0.773)	(0.408)	(0.468)	(0.531)	(0.324)
$\hat{u}_{t-1}$	-0.909	1.995	-1.202	-0.868	0.823	-0.587
	(0.556)	(0.666)	(0.339)	(0.468)	(0.469)	(0.213)
Observations	96	85	32	107	71	35

	Endogenous threshold variable and regressor: u_t and $\hat{u}_t$					
	Instrument set 1			Instrument set 2		
	1st Threshold, $\hat{\gamma}_1$= 5.61 [5.48; 6.2]			1st Threshold, $\hat{\gamma}_1$= 6.15 [5.56; 6.32]		
	2nd Threshold, $\hat{\gamma}_2$= 7.63 [6.02; 7.75]			2nd Threshold, $\hat{\gamma}_2$= 7.63 [6.02; 7.75]		
Variable	Regime 1	Regime 2	Regime 3	Regime 1	Regime 2	Regime 3
	($u_t \leq 5.61$)	($5.61 < u_t \leq 7.63$)	($u_t > 7.63$)	($u_t \leq 6.15$)	($6.15 < u_t \leq 7.63$)	($u_t > 7.63$)
Constant	2.252	2.065	6.154	1.77	1.704	5.981
	(0.206)	(0.565)	(1.341)	(0.157)	(0.556)	(1.030)
π^p_{t-1}	0.314	0.649	0.293	0.360	0.691	0.302
	(0.054)	(0.050)	(0.064)	(0.041)	(0.040)	(0.042)
$\hat{u}_t$	0.196	-2.461	-0.028	-0.366	-2.573	0.236
	(0.480)	(0.710)	(0.300)	(0.414)	(0.508)	(0.246)
$\hat{u}_{t-1}$	-1.145	1.564	-0.867	-0.742	2.153	-1.090
	(0.532)	(0.637)	(0.263)	(0.422)	(0.487)	(0.186)
Observations	96	82	35	132	46	35

tained for regime 1 and 3 are similar to those described in section (5). In particular, the coefficients associated with cyclical unemployment have the wrong sign or are not significant. In contrast, examination of the second regime results shows that the coefficients of $\hat{u}_t$ and $\hat{u}_{t-1}$ are both significant and have the sign predicted by the NKWPC, negative on the contemporaneous variable and positive on the lag. In this regime, the instrument set 1 yields a larger estimate for the slope of the NKWPC, in comparison to the instrument set 2.

When we consider endogeneity in $\hat{u}_t$ only, our results indicate that a percentage point increase in cyclical unemployment has a total effect in wage inflation similar to that obtained in section (5), even though the individual estimates for the cyclical unemployment coefficients are substantially lower when we employ the instrument set 2 in estimation. On the other hand, endogeneity in the threshold variable seems to have an important impact given that, in this setting, the total effect of cyclical unemployment in wage inflation is considerably smaller. When both $\hat{u}_t$ and u_t are endogenous the estimate for the slope of the NKWPC is more volatile to the set of instruments used. However, we tend to favor the results obtained with the instrument set 2, given that it incorporates more (seemingly valid) orthogonality conditions in estimation, which also suggests a higher estimate for γ_1 and a smaller effect of cyclical unemployment on wage inflation.

Overall, the uncertainty regarding the slope of the NKWPC in the second regime seems to be closely related to the estimate of γ_1, considering that small variations in this threshold generate large differences in the number of observations in the second regime. Endogeneity of the regressor $\hat{u}_t$ does not seem to change the results of the last section. However, when we consider endogeneity in the threshold variable u_t the slope of the NKWPC seems to be smaller. Additionally, the substantial reduction in the number of observations in the second regime suggests that a 2-regime model might be sufficient to account for the variability in the U.S. wage inflation. Therefore, testing for the number of regimes in the context of the framework developed by Kourtellos et al. (2016) would certainly be an useful additional step in this study.

7 Conclusions

In this work we analyzed the NKWPC for the U.S. wage inflation over the 1965-2018 period. We provide evidence that this relationship is nonlinear and well described by a 3-regime threshold model, where the optimal threshold variable is given by the contemporaneous unemployment rate. The relationship implied by the NKWPC varies whenever unemployment crosses the estimated thresholds of 5.69% and 7.63%, which splits the observations into 3 regimes: deep recessions, moderate business cycle fluctuations and prolonged expansions. Our analysis shows that the negative relationship between wage inflation and unemployment predicted by the NKWPC is only observable in the second regime, while in the first and third regimes the NKWPC seems to break down. These results seem to indicate that further reductions in the U.S. unemployment rate by itself will not generate an acceleration in wage growth.

We also check for the robustness of our estimates to endogeneity in the context of the 3-regime threshold model by incorporating the parametric bias correcting term proposed by Kourtellos et al. (2016). We argue that, accounting for possible endogeneity in the regressors does not yield significant differences in our baseline results. However, when the threshold variable is treated as endogenous, we obtain a higher estimate for the threshold that separates the low and intermediate unemployment regimes. As a consequence, the number of observations in the second regime is significantly lower and the estimate for the slope of the NKWPC is smaller.

Useful topics for future research related to the study threshold effects in the NKWPC would be to test for the number of regimes when accounting for an endogenous threshold variable and to test for endogeneity of the regressors and the threshold variable. Additionally, it would be interesting to assess different, more dynamic, specifications of the NKWPC that would contribute to eliminate the residual autocorrelation. Finally, the assessment of the ability of different measures of the labour market slack to account for the variation in wage inflation in the context of a threshold model would also be an useful and current line of investigation.

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Appendix

A Variables Description

Table 5: Descriptive statistics

Variable	Average	Min	Max	St. Deviation	Description
Wage inflation	4,30	1,38	9,20	1,97	4-quarter growth rate of the earnings for production and non-supervisory workers.
Price inflation	4,17	-1,61	14,43	2,86	4-quarter growth rate of the consumer price index.
Unemployment rate	6,13	3,39	10,67	1,65	Ratio between the number of unemployed and the civilian labour force.
Natural rate of unemployment	5,63	5,00	6,27	0,45	Rate of unemployment consistent with the absence of cyclical fluctuations in aggregate demand.
Labour Share	61,20	55,97	65,32	2,27	Percentage of the economic product that reverts to workers.
Output gap	0,16	-8,51	6,63	3,17	Computed applying the procedure recommended in Hamilton (2017) to the series of real GDP.
Commodities price inflation	3,59	-13,27	22,40	5,22	4-year growth rate of the commodities price index.
Interest rate spread	1,07	-4,13	3,71	1,67	Computed as the difference between 10 years US government bond yields and the federal funds rate.

B Heteroskedastic Bootstrap

To approximate the sampling distribution of the test statistic (12) under the hypothesis of heteroskedasticity, bootstrap the data as described in section (4.3), but we now set $\hat{\boldsymbol{e}}^* = \hat{\boldsymbol{\sigma}}^* \odot \tilde{\boldsymbol{\varepsilon}}^*$ (where $\odot$ denotes the Hadamard product) in order to obtain simulated time series with heteroskedastic errors. In our application, we will follow B. E. Hansen (1999) and assume that the conditional variance $\boldsymbol{\sigma^2}$ is a linear function of the squared regressors, which we define as $\boldsymbol{Z} = \boldsymbol{X} \odot \boldsymbol{X}$. Thus, let $\boldsymbol{\sigma}^2 = \boldsymbol{Z}'\boldsymbol{\alpha}$

and $\boldsymbol{e}^2 = \boldsymbol{Z}'\boldsymbol{\alpha} + \boldsymbol{v}$ with $E(\boldsymbol{v}|\mathcal{F}_{t-1}) = \boldsymbol{0}$ so that $\hat{\boldsymbol{\alpha}}$ can be obtained by regressing the squared LS residuals on $\boldsymbol{Z}$. The fitted values $\hat{\boldsymbol{\sigma}}^2 = \boldsymbol{Z}'\hat{\boldsymbol{\alpha}}$ are then used to compute the rescaled residuals $\tilde{\boldsymbol{\varepsilon}} = \hat{\boldsymbol{e}}/\hat{\boldsymbol{\sigma}}$ from the i-regime threshold model. To generate $\tilde{\boldsymbol{\varepsilon}}^*$ we make independent draws (with replacement) from the empirical distribution of $\tilde{\boldsymbol{\varepsilon}}$, which in turn, enables the computation of $\tilde{\boldsymbol{e}}^*$. Naturally, the validity of the bootstrap depends upon the specification of the conditional variance. Consequently, the simplifying premise that the bootstrap results are not overly sensitive to the adopted model must inevitably complement this approach.

C Estimation of the Nuisance Parameter η^2

As discussed in subsection (4.4), the $LR(\gamma)$ statistic can be used to test hypothesis $H_0 : \gamma = \gamma_0$. B. E. Hansen (2000) shows that, when the errors are heteroskedastic, the asymptotic distribution of this statistic depends on the nuisance parameter η^2, which therefore must be estimated. Define $r_{1t} = [(\boldsymbol{\beta_1} - \boldsymbol{\beta_2})'\boldsymbol{x}_t]^2(e_t^2/\sigma^2)$ and $r_{2t} = [(\boldsymbol{\beta_1} - \boldsymbol{\beta_2})'\boldsymbol{x}_t]^2$, as well as the corresponding sample counterparts $\hat{r}_{1t} = [(\hat{\boldsymbol{\beta}}_{\boldsymbol{1}} - \hat{\boldsymbol{\beta}}_{\boldsymbol{1}})'\boldsymbol{x}_t]^2(\hat{e}_t^2/\hat{\sigma}^2)$ and $\hat{r}_{2t} = [(\hat{\boldsymbol{\beta}}_{\boldsymbol{1}} - \hat{\boldsymbol{\beta}}_{\boldsymbol{2}})'\boldsymbol{x}_t]^2$. Then, define the following ratio of conditional expectations

$$\eta^2 = \frac{E(r_{1t}\,|q_t = \gamma_0)}{E(r_{2t}\,|q_t = \gamma_0)} \ .$$

B. E. Hansen (2000) proposes a polynomial regression in q_t or a kernel regression to estimate η^2. For j=1 and 2, a quadratic polynomial regression can be obtained by estimating the following LS regressions

$$\hat{r}_{tj} = \hat{\mu}_{j0} + \hat{\mu}_{j1}q_t + \hat{\mu}_{j2}q_t^2 + \hat{\varrho}_{jt} \ .$$

Then the estimate $\hat{\eta}^2$ is obtained as

$$\hat{\eta}^2 = \frac{\hat{\mu}_{10} + \hat{\mu}_{11}\hat{\gamma} + \hat{\mu}_{12}\hat{\gamma}^2}{\hat{\mu}_{20} + \hat{\mu}_{21}\hat{\gamma} + \hat{\mu}_{22}\hat{\gamma}^2} \ .$$

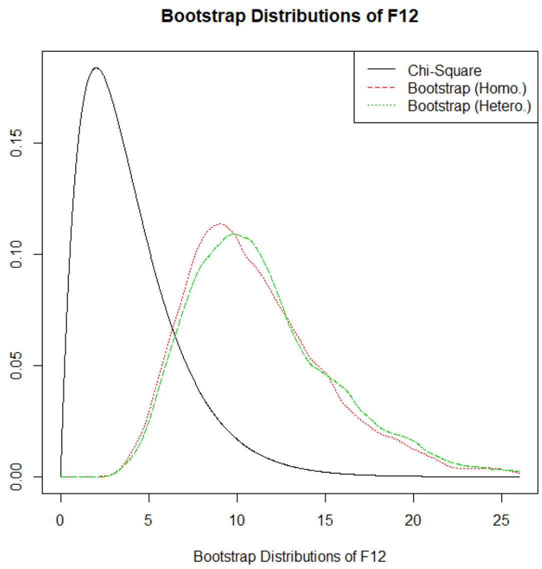
Alternatively, we can use the Nadaraya-Watson kernel estimator

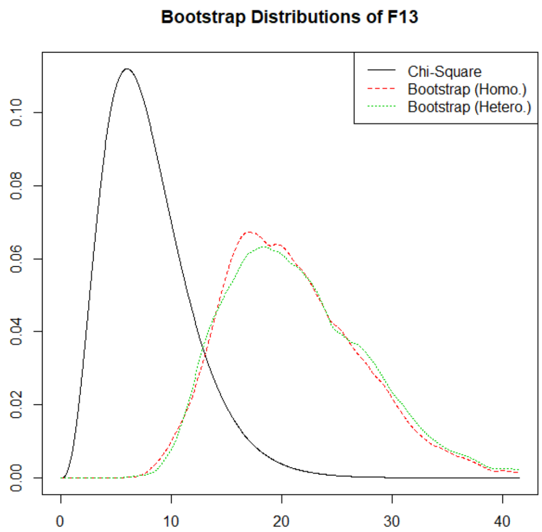
$$\hat{\eta}^2 = \frac{\sum_{t=1}^{T} K_h(\hat{\gamma} - q_t)\hat{r}_{1t}}{\sum_{t=1}^{T} K_h(\hat{\gamma} - q_t)\hat{r}_{2t}} \ .$$

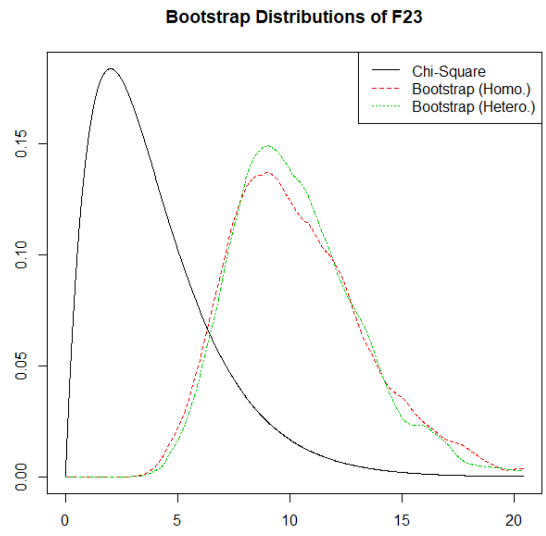
For a selected bandwidth h, we can compute $K_h(u) = h^{-1}K(u/h)$ where $K(u)$ is, for instance, the Epanechnikov kernel.

D Estimated Sampling Distributions for F_{12}, F_{13} and F_{23}

In the following figures we display the estimated densities of the bootstrap distribution for the test statistics employed in the present work. These estimates were computed using an Epanechnikov kernel.







E Testing for the Validity of the Instrumental Variables

To evaluate the validity of the set of instrumental variables employed in the empirical work, we estimate a linear version of the NKWPC by GMM where $\hat{u}_t$ is treated as an endogenous regressor, and test the validity of subsets of orthogonality conditions using the procedure described by L. Hansen (1982). We perform these tests using two different instruments sets:

- instrument set 1 includes a constant, 2 lags of price inflation, 3 lags of unemployment and 3 lags of cyclical unemployment;
- instrument set 2 includes a constant, 2 lags of price inflation, 3 lags of unemployment, 3 lags of cyclical unemployment, 2 lags of the output gap, 2 lags of the labour share, 2 lags of the interest rate spread, 2 lags of commodities price inflation

The underlying strategy of instrument set 1 is to employ lags of the endogenous variables as instruments. Given that we consider endogeneity of both cyclical unemployment and unemployment, we add lags of both these variables to instrument set 1 when we test for the validity of the orthogonality conditions in the linear setting.

However, it is important to note that the variables employed in the instrument set 1 to obtain the results reported in table (4) are different across the adopted set of assumptions regarding the NKWPC. When only the regressor $\hat{u}_t$ is endogenous we exclude the 3 lags of unemployment from the instrument set 1. On the other hand, when only u_t is considered as endogenous we exclude the 3 lags of cyclical unemployment. We only use 3 lags of unemployment and of cyclical unemployment simultaneously in the case where both the regressor $\hat{u}_t$ and the threshold variables u_t are treated as endogenous.

None of the results reported in table (6) provides evidence for the rejection of the null hypothesis and, therefore, all the variables are considered as valid instruments.

Table 6: Testing subsets of the orthogonality conditions

Instrument set 1

Valid instruments under the null hypothesis	Difference in J-stats	p-value
3 lags of unemployment	4.305	0.366
3 lags of cyclical unemployment	5.157	0.272
2 lag of price inflation	2.499	0.287

Instrument set 2

Valid instruments under the null hypothesis	Difference in J-stats	p-value
3 lags of unemployment	5.386	0.146
3 lags of cyclical unemployment	5.304	0.151
2 lags of the output gap	1.400	0.497
2 lags of the labour share	2.456	0.293
2 lags of the interest rate spread	1.098	0.578
2 lags of commodities price inflation	5.092	0.078
2 lag of price inflation	3.607	0.165