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DISSERTATION

METHODOLOGIES FOR THE CALCULATION OF NON-LIFE PREMIUM PROVISIONS IN SOLVENCY II ENVIRONMENT

SÍLVIA MENDES BARATA PINTO DO NASCIMENTO

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ABSTRACT

Under the Solvency II regime, a new concept in the valuation of technical provisions is established, namely for the premium provisions. These provisions relate to claims events occurring after the valuation date and during the remaining in-force coverage period of policies. The cash flow projection should comprise all future claims payments and claims management expenses arising from those events, cash flows arising from ongoing administration of the in-force policies and expected future premiums stemming from those contracts. The valuation of such provisions should take account of the time value of money, the best estimate (B.E.) should not include margins and the calculation should be done summing the present value of all future costs subtracted by the present value of all expected future premiums.

In this context, this dissertation aims to present different methodologies to calculate Non-Life premium provisions and to analyse the impact of the contract boundaries of the policies and the number of instalments of the premiums on its calculation, for two lines of business (LoBs), Motor Vehicle Liability Insurance and Other Motor Insurance. As this is a recent investigation area, three different methodologies are proposed and the results, for the LoBs considered, are analysed and compared with the Solvency I equivalent provisions, in terms of gains/losses on the level of own funds.

KEYWORDS: Solvency II, Premium Provisions, LoB, CBNI Claims, B.E., Contract Boundaries, Instalments of the Premiums.

RESUMO

No regime de Solvência II, é estabelecido um novo conceito na avaliação das provisões técnicas, nomeadamente para as provisões para prémios. Estas provisões estão relacionadas com sinistros que ocorrem depois da data de fecho do exercício, decorrentes de apólices em vigor, e durante o restante período de cobertura das mesmas. Para tal, é necessário projetar os *cash flows* de todos os futuros montantes pagos de sinistros e despesas de gestão dos mesmos, *cash flows* de despesas de administração das apólices em vigor e *cash flows* de prémios futuros expetáveis dessas mesmas apólices. A avaliação destas provisões deve ter em conta o valor temporal do dinheiro, a melhor estimativa não deve incluir margens de prudência e deve ser calculada como a soma do valor atual dos custos futuros subtraída do valor atual dos prémios futuros esperados.

Neste contexto, a presente dissertação tem como principal objetivo apresentar diferentes metodologias para o cálculo destas provisões nos ramos Não Vida e analisar o impacto das fronteiras dos contratos e do fracionamento dos prémios no cálculo das mesmas, para duas linhas de negócio, Automóvel Responsabilidade Civil e Automóvel Outras Coberturas. Sendo uma recente área de investigação, três diferentes metodologias são propostas e os resultados obtidos, para as linhas de negócio consideradas, são analisados e comparados com as provisões equivalentes existentes em Solvência I, em termos de ganhos/perdas no nível de fundos próprios.

PALAVRAS-CHAVE: Solvência II, Provisão para Prémios, Sinistros CBNI, Melhor Estimativa, Linhas de Negócio, Fronteiras de Contratos, Fracionamento dos Prémios.

LIST OF ACRONYMS AND ABBREVIATIONS USED

B.E. – Best Estimate

CBNI Claims – Covered But Not Incurred Claims

CEIOPS – Committee of European Insurance and Occupational Pensions Supervisors

CPNP – Chargeable Premiums Not Processed

EIOPA – European Insurance and Occupational Pensions Authority

LoB – Line of Business

LR – Loss Ratio

PUP – Provision for Unearned Premiums

PUR – Provision for Unexpired Risks

PVFP – Present Value of Future Premiums

TP.i.j – Chapter Technical Provisions, sub-chapter i, paragraph j of the Technical Specifications for the Preparatory Phase (EIOPA-14/209, 30th April 2014)

TVM – Total Volume Measure

UP – Unearned Premiums

WP – Written Premium

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1. INTRODUCTION

The new solvency regime, Solvency II (to be fully applied starting on 1st January 2016), which is going to replace the current Solvency I regime, is the result of a deep and comprehensive review of the regulatory and supervisory framework of the European (re)insurance sector.

The Level 1 framework Directive, of the European Parliament and of the Council (Directive 138/2009/EC), dated of 25th November 2009, establishes the general principles and the main objectives of this new regime, which are, basically, the protection of policyholders and beneficiaries, the promotion of a risk-based culture within all the functions of the undertaking, the increase of the sensitiveness of the capital measures to the risks to which the company is effectively exposed, the convergence of practices between supervisors and undertakings, the enhancement of the transparency and the market discipline.

With this new regime, a global and integrated view of risks will be promoted (3 pillars): quantitative requirements, governance and disclosure of information.

Within the quantitative requirements, a high level of importance is given to the technical provisions. Chapter VI of the Directive, Section 2 (Articles 76 to 83), defines some important general principles on the calculation of such item. Article 76, paragraph 2, states that the value of technical provisions should correspond to the current amount that an insurance (or reinsurance) undertaking would have to pay if they want to transfer, immediately, their insurance (or reinsurance) obligations to another insurance (or reinsurance) undertaking. Then, Article 77, paragraph 1, emphasizes that the value of technical provisions should be equal to the sum of a best estimate and a risk margin. The best estimate, as defined on paragraph 2 of the same Article, shall correspond to the probability weighted average of future cash flows (taking into account the time value of money and using the relevant risk-free interest rate term structure provided by EIOPA[1], for all major currencies), the cash flow projection should consider all the cash in and out-flows required to settle all the obligations until the run-off and should be calculated gross, without deduction, of the amounts recoverable from reinsurance and special purpose vehicles. The risk margin, referred on paragraph 3, shall be calculated in order to ensure that the value of technical provisions is actually equivalent to the amount that insurance and reinsurance undertakings would be expected to require in order to take over and assume the insurance and reinsurance obligations.

According to the technical specifications for the preparatory phase (EIOPA-14/209)[2], published by EIOPA on 30th April 2014, insurance and reinsurance obligations should be segmented, as a minimum, by lines of business (LoBs) in

[1] European Insurance and Occupational Pensions Authority (EIOPA), established as a consequence of the reforms to the structure of supervision of the financial sector in the European Union, is part of a European System of Financial Supervisors that comprises three European Supervisory Authorities, one for the banking sector, one for the securities sector and one for the insurance and occupational pensions sector, as well as the European Systemic Risk Board.

[2] These technical specifications have been drafted to reflect the content of the Directive 138/2009/EC and the amendments agreed afterwards in the Omnibus II Directive, the content of the working documents of the (Level 2) Delegated Acts available at the time this document was drafted and, also, the relevant working documents of the (Level 3) Guidelines.

order to calculate technical provisions (TP.1.6). This segmentation into LoBs distinguishes between Life and Non-Life insurance obligations and should be based on the nature of the underlying risk (TP.1.10).

About Non-Life insurance obligations, technical provisions include the best estimate for claims provisions[3], the best estimate for premium provisions and the risk margin. According to TP.2.65, the valuation of the best estimate for claims provisions and for premium provisions should be carried out separately.

With respect to premium provisions, the best estimate should be calculated as the difference between the discounted expected future claims payments and expenses, arising from policies in force at the valuation date, falling within the contract boundary and considering the full run-off of all the corresponding liabilities, and the discounted expected future premiums stemming from those policies. So, this best estimate is the amount that the undertaking needs to have as a provision in order to cover the gap between future costs and future income, also taking into account the future policyholder behaviour such as likelihood of policy lapses during the remaining period (TP.2.69).

As technical provisions are one of the main financial guarantees of an insurer, it is of the major importance that premium provisions are calculated in an accurate and

[3] The best estimate of claims provisions relates to claims events having occurred before or at the valuation date, whether the claims arising from those events have been reported or not, as well as claims administration expenses arising from those events (TP.2.70). In summary, it is the amount that the undertaking needs to set aside in order to cover all the payments that still need to be done relating to the events that have already occurred before the valuation date.

reliable manner, with sound actuarial and statistical methods, in order to guarantee the solvency position of the undertaking at all times.

This dissertation starts with a brief description of Non-Life technical provisions under Solvency I regime and how they are going to change under Solvency II principles.

The concept of the best estimate for premium provisions is, then, explored with detail and three different methodologies are proposed for its calculation. After, a sensitivity analysis of the premium provisions to the contract boundaries and to the number of instalments of the premiums is carried out, covering two of the factors that significantly affect this best estimate.

The methodologies are then applied to the data of two LoBs, Motor Vehicle Liability and Other Motor Insurance, of three anonymized undertakings and the results are discussed, analysed and compared to the equivalent provisions that exist under Solvency I.

2. OVERVIEW ON NON-LIFE TECHNICAL PROVISIONS

Technical Provisions are, by definition, the amount that an insurer needs to set aside to fulfil its insurance obligations and to settle all commitments to policyholders and other beneficiaries arising over the lifetime of the portfolio, including the expenses of administering the policies, reinsurance and they are also used in order to compute the capital required to cover the remaining risks.

2.1. NON-LIFE TECHNICAL PROVISIONS UNDER SOLVENCY I

Under the current regime[4], technical provisions are one of the main financial guarantees[5]. They are calculated based on a prudent valuation but without a quantifiable measure of the embedded level of prudence. For instance, in Non-Life there is no allowance for discounting the cash flows and in Life, discounting is possible but using prudent rates (expected return on investments less a prudential margin).

Supported by the Decree-Law in force[6], the relevant technical provisions required to be established and maintained for Non-Life business are:

1- Provision for unearned premiums (PUP)[7] – It is the amount representing the part of gross written premiums, relating to each one of the policies in force,

[4] The Decree-Law number 94-B/98, dated April 17th, regulates the conditions, for access and exercise, of insurance and reinsurance activities, within the European Community and within the institutional framework of the free zones.

[5] Under Solvency I, the main financial guarantees are the technical provisions, the required solvency margin (minimum capital requirement aimed to protect the undertaking from adverse claims experience. The available solvency margin must be sufficient to cover this item) and the guarantee fund (Decree-Law number 94-B/98, dated April 17th, tittle III, chapter I, section I, article 68, paragraph 1).

[6] Decree-Law number 94-B/98, dated April 17th, tittle III, section II, subsection I, article 70, paragraph 1.

which is to be allocated to the following financial years and it is calculated on a *pro-rata temporis* basis (policy by policy) as the unearned premiums minus the deferred acquisition costs;

2- Provision for unexpired risks (PUR)[8] – It is an add-on to the provision for unearned premiums to cater for the potential of future claims and expenses to exceed the unearned premiums and any additional premiums receivable on contracts in force (it signals a potential insufficiency of the premiums charged). It is triggered by a combined ratio (which is defined as the sum of the claims, expenses and ceding ratios) greater than one and it is computed as the product of the sum of the unearned premiums with the chargeable premiums not yet processed of policies in force by the combined ratio minus one;

3- Provision for claims outstanding (PCO)[9] - Corresponds to the estimated remaining costs of settling all claims occurred before or at the reference date, whether reported or not (includes IBNR claims);

4- Equalisation Provision[10] – The aim of this provision is to create a buffer to offset exceptionally high claims experience for certain volatile insurance classes (e.g. credit and suretyship insurance, seismic risk).

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[7] Calculated as defined on Decree-Law number 94-B/98, dated April 17th, tittle III, chapter I, section II, subsection II, article 79 and on the Regulatory Norm number 19/94-R, dated December 6th, chapter I.
[8] Calculated as defined on Decree-Law number 94-B/98, dated April 17th, tittle III, chapter I, section II, subsection II, article 79-A and on the Regulatory Norm number 19/94-R, dated December 6th, chapter II.
[9] Calculated as defined on Decree-Law number 94-B/98, dated April 17th, tittle III, chapter I, section II, subsection II, article 80 and on the Regulatory Norm number 19/94-R, dated December 6th, chapter V, section I.
[10] Defined on Decree-Law number 94-B/98, dated April 17th, tittle III, chapter I, section II, subsection I, article 77 and on the Regulatory Norm number 19/94-R, dated December 6th, chapter VI.

Under Solvency I, these technical provisions are the most important liability on the balance sheet of Non-Life insurance companies. However, long-established actuarial practices of estimating these technical provisions on an undiscounted basis have received many criticisms from modern finance practitioners as not being market consistent and, in some markets, overly prudent.

On the other hand, the current regime, in place since 2002, has been criticized by a number of other reasons, such as, for instance, the lack of harmonization and transparency in the calculation of technical provisions, the valuation of liabilities not being based on economic principles, the insufficient scope of risks[11], the no recognition of the positive effect of risk diversification and risk mitigation instruments and the very limited coverage of qualitative requirements, as the system of governance. So in 2009, and in order to try to fix all these drawbacks, it was published the Framework Directive with the general principles (Directive 138/2009/EC - Level 1) of a new solvency regime, Solvency II.

2.2. NON-LIFE TECHNICAL PROVISIONS UNDER SOLVENCY II

Under Solvency II, the calculation of technical provisions will remain an essential component in the construction of the solvency balance sheet but the valuation basis to be used will change from nowadays.

[11] For instance, the formula for the calculation of the required solvency margin is not sensitive to several important risks and penalizes the undertakings which calculate technical provisions in a more prudent manner.

This new regime intends the balance sheet (Figure 1) to reflect the true economic nature of assets and liabilities, in order to be used as a tool for managing and assessing the risks and the solvency position. It will also provide regulators with an accurate position to assess the solvency of the insurer, going beyond the objectives underlying the accounting approach.

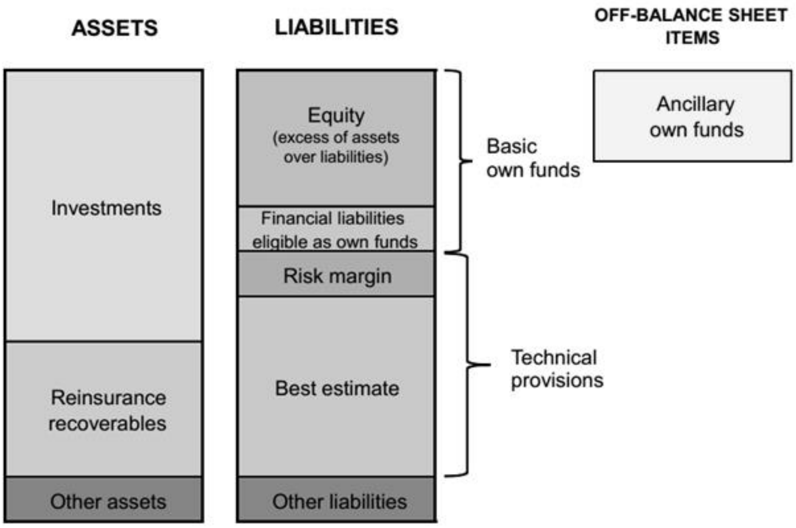

FIGURE 1: SOLVENCY II BALANCE SHEET

Under this new regime, the calculation of Non-Life insurance obligations changes significantly, using more sound economic principles. The allowance for discounting the cash flows (long-tailed lines of business will show the biggest differences from the current calculations), the absence of implicit margins of prudence, the removal of the equalisation provisions and the upfront recognition of profits on unearned premiums are the main reasons for the expected changes.

Segmented into 12 defined LoBs[12], the Non-Life technical provisions are made up of:

B.E. Claims Provisions + B.E. Premium Provisions + Risk Margin

The best estimate for claims provision, similar to the provision for claims outstanding in Solvency I, is the present value of all future cash flows (claim payments and allocated expenses) relating to claims events occurred prior or at the valuation date.

On the other hand, the best estimate for premium provision, whose calculation basis is one of the major differences between Solvency I and Solvency II and that can be conceptually seen as equivalent to the sum of the current provision for unearned premiums with the provision for unexpired risks, is given by the discounted value of all future cash out-flows (claim payments and expenses) subtracted by the present value of all future cash in-flows (future premiums due), relating to future exposure arising from policies in force at the valuation date (new future business cannot be taken into account, as stated on TP.2.12, but current business and renewals of such business, which comply with the contract boundaries rules, are included).

Compared to the current common accounting practice, the calculation of this best estimate reflects a fundamental change from the calculation of the current unearned premium provisions, using a *pro-rata temporis* method, and of the unexpired risks provisions. As a simplification, this new concept of calculating premium provisions means that companies will need to consider what would be their ultimate combined

[12] Medical expenses insurance, Income protection insurance, Worker´s compensation insurance, Motor vehicle liability insurance, Other motor insurance, Marine, aviation and transport insurance, Fire and other damage to property insurance, General liability insurance, Credit and suretyship insurance, Assistance insurance and Miscellaneous financial insurance (defined on TP.1.13 of the technical specifications for the preparatory phase).

ratio for the next future years (covering future claims costs, related expenses and other overhead costs) associated with their future exposure. It follows that where an a priori discounted combined ratio applied to the volume measure of the future exposure is lower than 100%, expected profits are immediately recognized, which is not the case under the current unearned premiums provision (PUP) methodology; and that an a priori discounted combined ratio greater that 100% would give rise to the recognition of a loss, due to the set up of an implicit unexpired risk provision (PUR), somewhat similar to the current approach except that the added PUR would be smaller (if there aren't any recognised insufficiencies on its calculation) due to the effect of discounting. Another important difference is that deferred acquisition costs are no longer allowed to be recognized as an asset and thus, the calculation of calendar-year earned premiums will just be equal to the written premiums less the variation in the premium provision, which does not consider the effect of the variation of the deferred acquisition costs.

At last, the risk margin, calculated net of reinsurance, is intended to be the amount that another (re)insurer taking on the liabilities at the valuation date would require over and above the best estimate and it is computed using the cost-of-capital approach.

3. NON-LIFE PREMIUM PROVISIONS

As stated above, the Solvency II framework directive considers the best estimate for Non-Life premium provision, calculated gross of reinsurance, as a replacement for the current provisions for unearned premiums and unexpired risks.

According to the technical specifications, this best estimate should be calculated as the expected present value (using compound interest) of future in and out-going cash flows, being a combination of:

a. Cash flows resulting from future claims events (CBNI claims) of the policies in force at the valuation date and cash flows arising from allocated claims management expenses (expenses relating and allocated to future claims);
b. Cash flows arising from unallocated claims management expenses, ongoing administration of the in-force policies and overhead expenses (the recurrent overheads attributable to the existing business at the calculation date of this best estimate, as referred on TP.2.55), allocated to premium provisions[13] and, as stated on TP.2.40, projected on the assumption that the undertaking will write new business in the future (called general management expenses);
c. Future premiums receivable, without taking into account future renewals that are not included within the current insurance contracts (contract boundaries) and new business.

[13] As stated on TP.2.71, undertakings should allocate the part of the expenses which is apportioned to existing business to premium provisions and to claims provisions.

From the three items referred above, c. may be considered to be known when the undertaking is computing its premium provision but items a. and b. need to be estimated.

At the valuation date, the undertaking should have knowledge how many policies has in-force at the valuation date, how many of them will give rise to future cash in-flows within the contract boundary of those policies, when those cash flows will be received and their amounts (called chargeable premiums not yet processed of type I, if the contract boundaries of the policies are just one year) and how many policies will be renewed in the next month, after the valuation date, and that the company does not have the legal right to reject them and also the premiums (called chargeable premiums not yet processed of type II). So, the companies should be able to derive the time distribution of the policies and of the future premiums, to which an accurate estimate of the expected lapse rates should be considered, calibrated based on their past experience.

Relatively to future general management expenses (overhead expenses[14], unallocated claims management expenses and cash flows arising from ongoing administration of the in-force policies), the (re)insurance company should be able to derive an accurate estimate, based on its own past experience, of these cash flows that will arise, on the next financial year and until run-off, and all companies will need to have a realistic method to allocate those expenses to claims provisions and to premium provisions.

[14] Overhead expenses include, for example, salaries of the general managers, auditing costs and regular day-to-day costs, like utility bills, rent for accommodations, IT costs and others.

3.1. METHODOLOGIES FOR THE CALCULATION OF THE BEST ESTIMATE FOR NON-LIFE PREMIUM PROVISIONS

Due to the difficulties that the undertakings may have for computing the best estimate of premium provisions, EIOPA allows, on the technical specifications for the preparatory phase (TP.6.80), the use of a simplification, described as:

$$BE_{Premium\ Provision} = CR \times (VM + PVFP) + (AER \times PVFP) - PVFP \qquad \textbf{[1]}$$

Where:

$\mathrm{CR} = \frac{\text{(claims + claims related expenses)}}{\text{(earned premiums gross of acquisition expenses)}}$ is the estimated combined ratio, gross of acquisition costs, for the LoB being considered;

VM is the volume measure for unearned premiums;

AER is the estimate of acquisition expenses ratio, for each LoB;

PVFP is the present value of future premiums, gross of commissions.

The first methodology proposed is an attempt to improve this simplification and the other two try to apply and model all the items on the definition of this best estimate.

However, the three proposed methodologies are described considering that the contract boundaries[15], of the policies in force at the valuation date, are one year (as it is expected) and then, it is explained how they can be modified if that is not the case.

[15] The definition of contract boundary is explained more carefully on sub-chapter 3.2.1.

Also, it is used, as an approximation of the real volume measure (total exposure or total amount at risk) for the future period, the sum of the unearned premiums (UP_n) with the chargeable premiums not yet processed of types I and II ($CPNP_n$).

On the other hand, and in order to estimate the pattern of the cash out-flows, it is used the run-off triangles of the cumulative paid amounts (which includes the cumulative paid claims amounts and the cumulative allocated management costs), defined by the random variables $A_{i,j}$ (representing cumulative paid amounts on accident year i and development year j – Figure 2).

Accident Year i \ Development Year j	0	1	2	3	…	d-1	Ultimate
1	$A_{1,0}$	$A_{1,1}$	$A_{1,2}$	$A_{1,3}$	…	$A_{1,d-1}$	$A_{1,Ult}$
2	$A_{2,0}$	$A_{2,1}$	$A_{2,2}$	$A_{2,3}$	…		
3	$A_{3,0}$	$A_{3,1}$	$A_{3,2}$	…			
4	$A_{4,0}$	$A_{4,1}$	…				
…	…						
d	$A_{d,0}$						

FIGURE 2: RUN-OFF TRIANGLE IN TERMS OF RANDOM VARIABLES

3.1.1. METHODOLOGY I

This first methodology aims to improve the simplification proposed by EIOPA to calculate the best estimate for premium provisions, allowing for a more precise consideration of the future general management expenses, allocated to this provision, and for discounting the cash out-flows. It is proposed that:

$$BE_{Premium\ Provision} = (TVM \times LR) \times (1 - f_1) + \left(EC \times (1 - f_2)\right) - PVFP \qquad \textbf{[2]}$$

Where:

$TVM = UP_n + CPNP_n$, is the total volume measure, or total risk exposure, for the future accident year (d+1) and n is the year under evaluation;

$LR = \frac{1}{k} \times \sum_{i=n-k+1}^{n} LR_i$, is the ultimate loss ratio, $LR_i = \frac{E[A_{i,Ult}]}{e_i}$ is the estimated ultimate loss ratio of accident year i[16] and e_i is the exposure of that accident year (considered to be the earned premiums);

$EC = \frac{1}{k} \times \sum_{i=n-k+1}^{n} EC_i$, are the estimated general management expenses allocated to premium provisions for the next year (and over the lifetime thereof) and $EC_i = \frac{Unallocated\ Expenses_{LoB,i} \times TVM_{i+1}}{Written\ Premiums_i}$ are the estimated general management expenses of accident year i[17];

$(1 - f_1) = (1 + r)^{-d_1}$, where d_1 is the average duration of insurance contracts in a given LoB and r is the risk-free interest rate corresponding to duration d_1, taken from the risk-free interest rate curve provided by EIOPA[18] (also $(1 - f_2) = (1 + r)^{-d_2}$, where d_2 is the average duration of the general management expenses);

$PVFP$ is the present value of future premiums (the discounted value of the $CPNP_n$).

[16] In this methodology, to estimate $E[A_{i,Ult}]$, for each accident year i, it was used the Mack's Model, proposed by Mack (1993), using the R package 'ChainLadder', the tail factor was estimated via linear extrapolation of the logarithms of the Chain Ladder development factors minus one, the standard error of the tail factor and the individual tail variability were estimated via log-linear regression.

[17] For each previous accident year i, the general management expenses of a given LoB, EC_i, are obtained proportionally to the operating costs of that year, deducted by the part of those expenses already allocated to the run-off triangle, and the proportionality constant is the weight of the volume measure for the period ahead ($TVM_{i+1} = UP_i + CPNP_i$) over the written premiums of that accident year i.

[18] It is a discounting proxy published on the Report on Proxies (CEIOPS-DOC-27/08, July 2008).

This basic approach to compute the best estimate for premium provisions[19] adds all the future costs relating to covered but not incurred claims (CBNI claims, estimated as $TVM \times LR$) with the future general management expenses (estimated by an average of the more recent historical years), discount these cash out-flows (using a discounting proxy), and subtracts the present value of the future premiums.

3.1.2. METHODOLOGY II

This second methodology tries to fix one of the major problems of the first one, which is the fact that the pattern of cash out-flows, relating to CBNI claims and to allocated management costs, is not modeled (Figure 3).

Development Year j / Accident Year i	0	1	2	3	...	d-1	Ultimate
1	$A_{1,0}$	$A_{1,1}$	$A_{1,2}$	$A_{1,3}$	...	$A_{1,d-1}$	$A_{1,Ult}$
2	$A_{2,0}$	$A_{2,1}$	$A_{2,2}$	$A_{2,3}$	...		
3	$A_{3,0}$	$A_{3,1}$	$A_{3,2}$	...			
4	$A_{4,0}$	$A_{4,1}$	...				
...	...						
d	$A_{d,0}$						
d+1	$A_{d+1,0}$	$A_{d+1,1}$	...		...		$A_{d+1,Ult}$
...	...						

FIGURE 3: PROJECTING THE RUN-OFF TRIANGLE FOR FUTURE ACCIDENT YEARS

So, this proposed methodology tries to predict those cash out-flows and is presented as:

$$BE_{Premium\ Provision} = \sum_{i \geq d+1} \left[\sum_{j \geq 0} \left(\frac{E[A_{i,j} - A_{i,j-1}]}{\left(1 + r_{i-(d+1)+j+1}\right)^{i-(d+1)+j+\frac{1}{2}}} \right) + \frac{EC_i}{\left(1 + r_{i-(d+1)+1}\right)^{i-(d+1)+\frac{1}{2}}} \right] - PVFP \qquad [3]$$

[19] This methodology can be adapted for contract boundaries greater than one year out, assuming, for each one of the future accident years that needs to be projected, the same estimated ultimate loss rate and that the future general management expenses will decrease proportionally with an estimate of the lapse rate. Also, the volume measure (as it will be explained later) needs to be recalculated and the discount factors have to be adjusted.

In order to estimate the cash out-flows (grey part of Figure 3), it is used an adaptation of the Hayne's Model. Hayne (1985) proposed a stochastic model to project the run-off triangles under the assumption that the development factors have a lognormal distribution and that they are independent from each other's. Wacek (2007) adapted the Hayne's Model in order to construct a model to estimate the future ultimate loss ratios because he claimed that any estimate of the ultimate loss ratio, for a particular accident year, is quickly made obsolete by subsequent actual loss emergence as that loss development affects the new estimate in two ways: first, actual accident year loss emergence replaces the expected emergence in the loss ratio projection (*accident year development effect*) and second, loss emergence with respect to older accident years might cause a revision in the prospective age to ultimate factor (*tail factor revision*).

After having these two effects modelled, it is possible to combine them in order to determine the distribution of the revised ultimate loss ratio estimate that will be determined in one year's time, extrapolating with the main purpose of finding an estimate of the ultimate loss ratio for the next accident year and, then, model all the payment pattern of the cash out-flows.

First, to present the method used to estimate the ultimate loss ratio, for the next accident year (d+1)[20], some mathematical assumptions and definitions need to be considered:

[20] If it is needed to project more than one accident year ahead (for policies with contract boundaries higher than one year), it is assumed that the estimated ultimate loss ratio for the accident year (d+1) will remain the same for the future accident years and, also, the estimated development factors ($\hat{f}_j$) will remain unchanged. Relatively to the risk exposure for the next accident years, it will need to be recalculated and it will need to reflect the effect of the

I. Let $A_{i,j}$ be a random variable that represents the accumulated paid claims amounts (which also include the accumulated allocated management costs) on accident year i ($i \in \{1, 2, \dots, d\}$) and development year j ($j \in \{0, 1, \dots, d-1\}$), where d is the number of accident years, e_i is the exposure on accident year i (considered to be the earned premiums) and $f_j = \frac{A_{i,j+1}}{A_{i,j}}$, with $j \geq 0$, is a random variable, assumed to be lognormal distributed[21] ($f_j \sim lognormal(\mu_j, \sigma_j)$), which represents the development factor between year j and (j+1);

II. To estimate the parameters of each f_j with unbiased estimators, it is used $\widehat{\mu_j} = \frac{1}{d-(j+1)} \times \sum_{k=1}^{d-(j+1)} \ln(\frac{a_{k,j+1}}{a_{k,j}})$ and $\widehat{\sigma_j} = [\frac{1}{d-(j+1)-1} \times \sum_{k=1}^{d-(j+1)} (\ln(\frac{a_{k,j+1}}{a_{k,j}}) - \widehat{\mu_j})^2]^{\frac{1}{2}}$ and each f_j is estimated by its expected value;

III. Let $lr_{i,j} = \frac{a_{i,j}}{e_i}$, for $j \leq (d-i)$, be the observed loss ratio on accident year i and development year j;

IV. Let $LR_{i,d-1} = \frac{a_{i,d-i}}{e_i} \times \prod_{k=d-i}^{d-2} f_k$, be the loss ratio of accident year i and development year (d-1);

V. Let $LR^*_{i,d-1} = \frac{A_{i,d-i+1}}{e_i} \times \prod_{k=d-i+1}^{d-2} f^*_k$ be the expected loss ratio of accident year i and development year (d-1), assuming that there is one more year of information, and f^*_j is the modified (because the empirical mean and variance changed after the appearance of new information) development factor;

estimated lapse rate. Also, the future general management expenses, allocated to this best estimate, are assumed to decrease proportionally with that estimated lapse rate.

[21] According to Zehnwirth (1994), one of the principal reasons for taking logarithms of the data (and to assume a lognormal distribution for the development factors) is because the difference of two logarithms is equivalent to analyse trends and, approximately, to analyse percentage changes. Also, Butsic (1992) stated that loss estimates change in their march through time, recognizing that they, like stock prices, are governed by a diffusion process (a type of continuous stochastic process with time-dependent probability structure).

VI. Rearranging the formula on IV., $LR_{i,d-1} = \frac{a_{i,d-i}}{e_i} \times f_{d-i} \times \prod_{k=d-i+1}^{d-2} f_k$, and comparing with V., it is possible to observe that $\frac{a_{i,d-i}}{e_i} \times f_{d-i}$ changes to $\frac{A_{i,d-i+1}}{e_i}$ (first source of variation, *accident year development*, because this effect is captured by the lognormal random variable of this development year) and $\prod_{k=d-i+1}^{d-2} f_k$ becomes $\prod_{k=d-i+1}^{d-2} f_k^*$ (second source of variation, *tail factor revision*);

In order to model the *accident year development*, and knowing that f_j is a lognormal distributed random variable, it is possible to conclude that $\frac{a_{i,d-i}}{e_i} \times f_{d-i} \sim lognormal\left(\mu_{j_1}, \sigma_{j_1}\right)$, with $\widehat{\mu_{J_1}} = \ln\left(lr_{i,d-i}\right) + \widehat{\mu_{d-\iota}}$ and $\widehat{\sigma_{J_1}} = \widehat{\sigma_{d-\iota}}$.

On the other hand, to model the *tail factor revision*, it is necessary to recalculate the mean and the variance of each one of the new f_j (which are the f_j^*) as the empirical μ_j and σ_j are now obtained with (d-j) factors instead of (d-(j+1)). So, the new estimates of the standard deviations ($\widehat{\sigma_{J_2}}$) are obtained weighting the previous ones by the respective weights ($\frac{1}{d-j}$). The estimates of the revised means ($\widehat{\mu_{J_2}}$) are calculated in order to guarantee that the expected values of the f_j^*, which are lognormal random variables with parameters μ_{j_2} and σ_{j_2}, are the same of the expected values of f_j and so, for each j, $\widehat{\mu_{J_2}} = \ln\left(\widehat{f_J}\right) - 0{,}5 \times \hat{\sigma}_{j_2}^2$.

VII. It follows that $LR_{i,d-1}^* \sim lognormal(\mu_i^*, \sigma_i^*)$, where $\mu_i^* = \mu_{j_1} + \sum_{k=d-i+1}^{d-2} \mu_{k_2}$ and $\sigma_i^* = \left(\sigma_{j_1}^2 + \sum_{k=d-i+1}^{d-2} \sigma_{k_2}^2\right)^{\frac{1}{2}}$, and the estimate of each $LR_{i,d-1}^*$ is obtained by its expected value.

Given these assumptions, the algorithm used to calculate the costs of future claims (CBNI claims, including the allocated management costs) is:

i. Calculate an estimate of $LR^{*}_{d+1,d-1}$ using its expected value ($\widehat{LR^{*}_{d+1,d-1}} = e^{\widehat{\mu^{*}_{d+1}}+\frac{1}{2}*\widehat{\sigma^{*}_{d+1}}^{2}}$), with $\widehat{\mu^{*}_{d+1}} = \frac{1}{d} \times \sum_{k=1}^{d} \ln(\frac{a_{k,0}}{e_k}) + \sum_{k=0}^{d-2} \mu_{k_2}$ and $\widehat{\sigma^{*}_{d+1}} = [(\frac{1}{d-1} \times \sum_{k=1}^{d} \left[\ln\left(\frac{a_{k,0}}{e_k}\right) - \frac{1}{d} \times \sum_{i=1}^{d} \ln\left(\frac{a_{i,0}}{e_i}\right)\right]^2) + \sum_{k=0}^{d-2} \sigma_{k_2}^2]^{\frac{1}{2}}$;

ii. Obtain the total claim costs, in the development year (d-1) and for the future accident year, as $\widehat{LR^{*}_{d+1,d-1}} \times (UP_n + CPNP_n)$;

iii. Using the development factors estimated from the run-off triangle ($\widehat{f_j}$), it is possible to calculate the accumulated claims costs, $\widehat{A_{d+1,j}}$, with $j \in [0, \mathrm{d}-1]$;

iv. Analyzing the data from previous years not included in the run-off triangle, it is possible to have an idea of how many years should be projected beyond the development year (d-1) and then estimate the development factors for each one of those years, until all claims are expected to be completely settled.

To project the development factors f_j for those years ($j \geq d-1$), it is used the linear least squares regression applied to $\ln(\widehat{f_j}-1)$, for $j < d-1$. So, the main goal is to minimize $\sum_{j=0}^{d-2}\left(\ln(\widehat{f_j}-1) - \beta_0 - \beta_1 \times j\right)^2$ in order to obtain the best estimates for β_0 and β_1. As the approximation used, for $j \geq d-1$, is $\ln(\widehat{f_j}-1) = \widehat{\beta_0} + \widehat{\beta_1} \times j$, it is possible to find the expected values of the development factors beyond the development year (d-1), assuming that $\widehat{\sigma_j} = \widehat{\sigma^{*}_{d+1}}$ and estimating $\widehat{\mu_j} = \ln\Big(1 +$

$e^{\widehat{\beta_0}+\widehat{\beta_1}\times j}\Big) - \frac{1}{2}\times\widehat{\sigma_{d+1}^{*}}^{2}$. So, for each one of those development years, it is possible to find the estimates $\widehat{A_{d+1,J}}$.

v. Then, using the properties of the lognormal random variables, it is possible to find the confidence intervals[22] for all the estimates $\widehat{A_{d+1,J}}$ (with a confidence level α).

Having the estimates of the incremental future claims costs (CBNI claims and the allocated management costs), it is possible to calculate the present value of those cash out-flows and, in order to obtain the discounted value of all future costs, it is necessary to add the present value of the future general management expenses allocated to premium provisions, estimated as the average of the last k financial years ($EC = \frac{1}{k}\times \sum_{i=n-k+1}^{n} EC_i$). Then, to compute the best estimate for premium provisions, it is subtracted the discounted value of the future premiums of the policies in force at the valuation date (the discounted value of the $CPNP_n$).

3.1.3. METHODOLOGY III

This third methodology is an alternative to the one presented before (methodology II), to model the best estimate for premium provisions (defined on equation [3]), using a different stochastic approach.

[22] Assuming that all the estimates found (for μ's and σ's) correspond to their real values. Otherwise, it would be necessary to calculate the confidence intervals based on a log Student's T-distribution.

In order to estimate the future claims costs (and also the future allocated management costs), it was adapted (introducing variability) the deterministic Bornhuetter-Ferguson method, firstly described in the paper by Bornhuetter and Ferguson (1972).

The mathematical assumptions and definitions behind this methodology are:

I. It is assumed the same delay-specific claim frequencies, for each development year and independently of the accident years;

II. It is considered that the ultimate of the run-off triangle (i=1 and j>d-1) is the outstanding claims provision allocated to the first accident year and this is the amount that the insurance company is expecting to pay in the future (after development year d-1) from claims that happened on the first accident year that appears in the run-off triangle;

III. Let $I_{i,j} = e_i \times \theta_j$ be a random variable, representing the incremental paid claims amounts (which also include the incremental allocated management costs) on accident year i ($i \in \{1, 2, \dots, d\}$, where d is the number of accident years) and development year j ($j \in \{0, 1, \dots, d\}$ and it is assumed that when j=d is actually when j>d-1), where e_i is the observed exposure on the accident year i and θ_j is a random variable representing the delay-specific claim frequencies for development year j;

IV. It is assumed that the observed exposure, e_i, is the mean number of policies in force on accident year i and that θ_j, for each development year j, has a lognormal distribution with parameters μ_j and σ_j, estimated by the unbiased estimators $\widehat{\mu_j} = \frac{1}{d-j} \times \sum_{k=1}^{d-j} \ln\left(\frac{I_{k,j}}{e_k}\right)$ and $\widehat{\sigma_j} = (\frac{1}{d-j-1} \times \sum_{k=1}^{d-j} \left(\ln\left(\frac{I_{k,j}}{e_k}\right) - \mu_j\right)^2)^{\frac{1}{2}}$.

Given these assumptions, the algorithm used to calculate the costs of future claims (CBNI claims, including the allocated management costs) is:

i. Calculate the average premium per policy (p_i , with $i = 1,2, \dots, d$), for each accident year i, using the earned premiums and the mean number of policies in force in each accident year;

ii. Assuming that the average premium per policy for the year ahead (the random variable P_{d+1}) follows a lognormal distribution with parameters μ_p and σ_p, estimate those parameters as $\widehat{\mu_p} = \frac{1}{k} \times \sum_{i=n-k+1}^{n} \ln(p_i)$ and $\widehat{\sigma_p} = (\frac{1}{k-1} \times \sum_{i=n-k+1}^{n} \left(\ln(p_i) - \widehat{\mu_p}\right)^2)^{\frac{1}{2}}$, using the data from the last k years;

iii. Estimate the average premium per policy for the year ahead as the expected value of the lognormal random variable with the estimated parameters found in ii.;

iv. Assuming that the risk exposure for the future period is the sum of the unearned premiums of the previous year (UP_n) with the chargeable premiums not yet processed ($CPNP_n$) and, in order to calculate the estimate of the mean number of policies at risk for the period ahead ($E_{d+1} = \frac{UP+CPNP}{P_{d+1}}$), the parameters of this lognormal random variable need to be computed ($E_{d+1} \sim lognormal(\mu_e; \sigma_e)$) as $\widehat{\mu_e} = \ln(UP_n + CPNP_n) - \widehat{\mu_p}$ and $\widehat{\sigma_e} = \widehat{\sigma_p}$;

v. For each development year j, calculate the estimates of the parameters of the random variable θ_j ($\widehat{\mu_J}$ and $\widehat{\sigma_J}$);

vi. Knowing that $\widehat{I_{d+1,J}} = \widehat{\theta_J} \times \widehat{E_{d+1}} \sim lognormal\left(\mu_{d+1,j}; \sigma_{d+1,j}\right)$, find the estimates for $\mu_{d+1,j}$ and for $\sigma_{d+1,j}$, for each development year j, as $\widehat{\mu_{d+1,J}} = \widehat{\mu_e} + \widehat{\mu_J}$ and $\widehat{\sigma_{d+1,J}} = \left(\widehat{\sigma_e}^2 + \widehat{\sigma_J}^2\right)^{\frac{1}{2}}$;

vii. Calculate, for each development year j, the best estimate of the incremental costs of future claims in the period ahead by its expected value;

viii. The confidence intervals (with a confidence level α), for the estimates found in vii., for each development year j, can be obtained as $\left[e^{\widehat{\mu_{d+1,J}} - \Phi^{-1}\left(1-\frac{\alpha}{2}\right) \times \widehat{\sigma_{d+1,J}}} ; e^{\widehat{\mu_{d+1,J}} + \Phi^{-1}\left(1-\frac{\alpha}{2}\right) \times \widehat{\sigma_{d+1,J}}}\right]$.

Having the estimates of the expected values of the incremental costs of future claims in the period ahead[23] (which also include the allocated management costs), it is possible to calculate the present value of each of those estimates (using the risk-free interest rate term structure given by EIOPA), add the discounted value of the estimate of the future general management expenses (estimated as in methodology II) and subtract the present value of the future premiums (the discounted value of the $CPNP_n$), in order to find the best estimate for premium provisions.

[23] If it is needed to project more than one accident year ahead, it is assumed that the estimates of the delay-specific claim frequencies ($\widehat{\theta_J}$), for each development year j, will remain the same, the average premium per policy will also remain unchanged (and equal to $E[P_{d+1}]$) and that the risk exposure will have to be recalculated in order to take into account the estimated lapse rate. Also, the general management expenses, allocated to this best estimate, are assumed to decrease proportionally with that estimated lapse rate.

3.2. SENSITIVITY ANALYSIS

As it is possible to observe in equation [3], the best estimate for premium provisions depends strongly on the number of accident years ahead that needs to be projected (which depends on the contract boundaries of the policies in force at the valuation date), in order to compute the future claims costs, and, also, on the future premiums expected to be received by the undertaking (as stated on TP.2.34, the calculation of this best estimate should take into account the future premium cash-flows which fall due after the valuation date). So, the temporal distribution of the premiums of the insurance portfolio (number of instalments of the premiums) is, also, strongly correlated with the calculation of this best estimate. These presented reasons explain why it is of the major importance to study the impact of the contract boundaries and of the number of instalments of the premiums in the best estimate for premium provisions.

3.2.1. CONTRACT BOUNDARIES

The calculation of the best estimate for premium provisions should only include future cash-flows associated with recognized obligations within the boundary of the contracts in force at the valuation date and thus, should not include the future business of the (re)insurance company (TP.2.12). So, it is important to clarify that the boundary of an insurance policy ends whenever the following facts happen first: the undertaking has the unilateral right to terminate the contract; the undertaking has the unilateral right to reject the premiums payable under the contract or the undertaking has the unilateral right to amend the premiums or the benefits payable under the contract in such a way that the premiums fully reflect the risks (TP.2.18).

As it was referred when the proposed methodologies where described, these methodologies can be adapted (with the assumptions that were mentioned) to contract boundaries greater than one year. The key important points that need to be referred are that, for each one of the future accident years that have to be considered, the considered risk exposure for the periods ahead (the total volume measure) needs to be recalculated and it is also necessary to estimate the lapse rates. So, for each one of those future accident years, it is considered:

1. $TVM_i = \left(1 - \widehat{LpR}\right) \times [WP_{i-1} + UP_{i-2} - UP_{i-1}]$, for each i>d, is the total volume measure, or the total risk exposure, calculated for each group of policies with the same contract boundaries, and where WP_i $(= [1 - \widehat{LpR}] \times WP_{i-1})$ are the written premiums of the accident year i and $UP_{i-1}(= [1 - \widehat{LpR}] \times UP_{i-2})$ are the unearned premiums of the previous accident year (it is also necessary to subtract UP_i because the unearned premiums of year i are considered to be exposure for the year i+1);

2. The present value of the future cash in-flows (PVFP) is, now, the sum of the discounted values of the WP_i, for each accident year i>d;

3. $\widehat{LpR} = \frac{1}{k} \times \sum_{i=d-k+1}^{d} LpR_i$, for i=d+1, is the estimate of the lapse rate, estimated as the average of the observed lapse rates of the last k years and assumed to be constant after i>d+1.

3.2.2. NUMBER OF INSTALMENTS OF THE PREMIUMS

In order to study the impact, on the value of the best estimate for premium provisions, of the number of instalments of the premiums, it was tested three different scenarios: 1) All the policies in force at the valuation date pay the premiums annually; 2) All the policies pay the premiums semiannually and 3) All the policies pay the premiums monthly.

Then, for each of those scenarios, it was assumed four different temporal distributions of the beginning date of the insurance policies: a) Uniformly starting distribution throughout the year (from 31st of January ahead); b) All starting at 1st of July; c) All starting at 1st of December and d) All starting at 31st of January. So, for each of these scenarios and sub-scenarios, what has to be recalculated, for each of the proposed methodologies, is the total volume measure (it is necessary to compute the unearned premiums, UP_n, on a *pro-rata temporis* basis, and the chargeable premiums not yet processed, $CPNP_n$, calculated from the total volume of business (VB) of the year under analysis – Table I and the general formulas used to derive these ones are presented on Annex 4) and the present value of the future premiums (PVFP), obtained as the discounted value of the $CPNP_n$.

Scenario	Unearned Premiums (**UP$_n$**)	Chargeable Premiums (**CPNP$_n$**)	Scenario	Unearned Premiums (**UP$_n$**)	Chargeable Premiums (**CPNP$_n$**)
1.a)	$\frac{VB}{12} \times \sum_{k=1}^{12} \frac{i}{12}$	0	**2.c)**	$\frac{VB}{2} \times \frac{5}{6}$	$\frac{VB}{2}$
1.b)	$\frac{VB}{12} \times 6$	0	**2.d)**	$\frac{VB}{2} \times \frac{1}{6}$	0
1.c)	$\frac{VB}{12} \times 11$	0	**3.a)**	$\frac{VB}{12}$	$\frac{VB}{12} \times \sum_{k=1}^{11} \frac{i}{12}$
1.d)	$\frac{VB}{12}$	0	**3.b)**	0	$\frac{VB}{12} \times 6$
2.a)	$\frac{VB}{12} \times \sum_{k=1}^{6} \frac{i}{6}$	$\frac{VB}{12} \times \frac{6}{2}$	**3.c)**	0	$\frac{VB}{12} \times 11$
2.b)	0	$\frac{VB}{2}$	**3.d)**	$\frac{VB}{12}$	0

TABLE I: FORMULAS TO CALCULATE UP$_N$ AND CPNP$_N$

4. APPLICATION TO MOTOR VEHICLE LIABILITY AND OTHER MOTOR INSURANCE

The different proposed methodologies and the sensitivity analysis presented on the previous chapters were applied to the data of three anonymised undertakings (Insurance Company A, B and C), for these lines of business.

It was used the information on claims payments and case reserves, both including allocated management costs, by origin and development year, for accidents occurred from 2004 to 2013. It was also used information about the general management expenses, the written premiums, the unearned premiums and the chargeable premiums not processed.

All the estimated cash in and out-flows were discounted using the risk-free interest rate term structure, provided by EIOPA[24], with reference date 31st December 2013 and without making use of any transitional measure. Also, the present values were obtained using compound interest and assuming that the future cash flows occur at the middle of the year.

This application of the theoretic models was performed using *Microsoft Excel* and additional calculations were done using the statistical software *R* for *Windows GUI* front-end.

[24] The risk-free interest rate term structure, provided by EIOPA for the Stress Test 2014, is available from: https://eiopa.europa.eu/activities/financial-stability/insurance-stress-test-2014/stress-test-specifications/index.html

In order to preserve the confidentiality of the data, this chapter will only present some final results in terms of the best estimate for premium provisions.

4.1. RESULTS: ONE YEAR CONTRACT BOUNDARIES

Assuming that all the policies in force at the valuation date (31st December 2013), of the three undertakings, have contract boundaries of one year, the overall results of the best estimates for premium provisions are presented in the following tables (more detailed information can be found in Annex 5).

	Insurance Company A				Insurance Company B				Insurance Company C			
	Motor Vehicle Liability	Other Motor Insurance	**Total**	$\triangle_{SII/SI}$	Motor Vehicle Liability	Other Motor Insurance	**Total**	$\triangle_{SII/SI}$	Motor Vehicle Liability	Other Motor Insurance	**Total**	$\triangle_{SII/SI}$
Methodology I $BE_{Premium\ Provisions}$	33.390,4	18.874,9	**52.265,3**	**11,5%**	33.806,2	19.222,0	**53.028,2**	**-4,2%**	83.609,0	33.837,8	**117.446,8**	**-8,5%**
Methodology II $BE_{Premium\ Provisions}$	34.185,1	18.368,3	**52.553,4**	**12,1%**	34.522,8	17.284,3	**51.807,1**	**-6,4%**	81.146,3	33.459,1	**114.605,4**	**-10,7%**
Methodology III $BE_{Premium\ Provisions}$	34.088,2	15.815,8	**49.904,0**	**6,5%**	36.297,4	15.700,5	**51.997,9**	**-6,0%**	85.368,6	34.917,2	**120.285,8**	**-6,3%**

(in thousands of Euros)

TABLE II: BEST ESTIMATES FOR PREMIUM PROVISIONS

As it is possible to observe, the three methodologies give almost the same estimates. The differences that exist are explained, mainly, by the ultimate loss rates embedded in the calculations (Table III), which sometimes vary between the presented methodologies due to some irregularities of the run-off triangles (these irregularities are more evident in the LoB Other Motor Insurance, where the run-off triangles used are less stable).

It is important to be referred is that these analyses should be done with run-off triangles well behaved (taking out the outliers and separating material damages from bodily injuries). This was not possible to do because of the lack of such granular information and, as stated, this fact justifies that, sometimes, the ultimate loss rates vary between the three methodologies.

	Insurance Company A		Insurance Company B		Insurance Company C	
One Year Out	Motor Vehicle Liability	Other Motor Insurance	Motor Vehicle Liability	Other Motor Insurance	Motor Vehicle Liability	Other Motor Insurance
Methodology I $LR_{Ultimate}$	90,3%	77,7%	76,6%	89,0%	78,6%	75,2%
Methodology II $LR_{Ultimate}$	87,5%	70,5%	74,0%	73,9%	72,8%	69,6%
Methodology III $LR_{Ultimate}$	90,4%	64,2%	81,6%	69,9%	78,2%	74,3%

TABLE III: ESTIMATED ULTIMATE LOSS RATES

Comparing to the relevant Solvency I provisions, for undertaking A, it is expected the best estimate for premium provisions to be higher (resulting in a negative contribution to the level of own funds), while for companies B and C, the best estimates allow for the recognition of a positive contribution to the level of own funds.

4.2. RESULTS: FIVE YEARS CONTRACT BOUNDARIES

Assuming that all the policies held by the undertaking at the valuation date have contract boundaries of five years, the overall results are presented in Table IV (and more specific data can be found in Annex 6).

	Insurance Company A				Insurance Company B				Insurance Company C			
	Motor Vehicle Liability	Other Motor Insurance	**Total**	$\Delta_{5years/1year}$	Motor Vehicle Liability	Other Motor Insurance	**Total**	$\Delta_{5years/1year}$	Motor Vehicle Liability	Other Motor Insurance	**Total**	$\Delta_{5years/1year}$
Methodology I $BE_{Premium\ Provisions}$	31.018,4	4.638,9	**35.657,3**	**-31,8%**	9.146,1	13.987,8	**23.133,9**	**-56,4%**	16.894,0	3.522,3	**20.416,3**	**-82,6%**
Methodology II $BE_{Premium\ Provisions}$	37.561,4	3.679,0	**41.240,5**	**-21,5%**	14.365,4	10.010,1	**24.375,5**	**-52,9%**	11.786,3	4.863,6	**16.649,9**	**-85,5%**
Methodology III $BE_{Premium\ Provisions}$	38.973,0	-7.674,6	**31.298,4**	**-37,3%**	18.694,4	3.156,8	**21.851,2**	**-58,0%**	13.165,6	2.129,8	**15.295,4**	**-87,3%**

(in thousands of Euros)

TABLE IV: BEST ESTIMATES FOR PREMIUM PROVISIONS (CONTRACT BOUNDARIES: 5 YEARS)

Increasing the contract boundaries from one year to five, the best estimates for premium provisions, as expected, decrease because expected profits, for a longer period, are recognised (and they are higher the lower is the ultimate loss rate embedded in the calculations).

As it is possible to observe on Insurance Company A, LoB Other Motor Insurance and in Methodology III, this best estimate is negative and this means that the present value of the cash in-flows exceeds the present value of the cash out-flows, being these kind of situations acceptable under the Solvency II economic valuation principles and, as stated on TP.2.68, the companies are not required to set these values to zero.

However, as in Non-Life business it is not likely that the contract boundaries of the insurance policies are higher than one year, this sensitivity analysis was just made in order to prove that when the contract boundaries are extended, the values of these best estimates are strongly affected and expected to decrease.

4.3. RESULTS: NUMBER OF INSTALMENTS OF THE PREMIUMS

For each scenario and sub-scenario, defined previously, the three proposed methodologies (with contract boundaries of one year) were applied in order to compute the best estimate for premium provisions at 31st December 2013 (considering $VB = WP_{2013} + CPNP_{2013}$, which represents the total volume of business made in the year 2013, although $CPNP_{2013}$ are cash flows that are going to be received in 2014), and the overall results are presented in the following tables (Table V, VI and VII).

Insurance Company A

	Methodology I - $BE_{premium\ provisions}$				Methodology II - $BE_{premium\ provisions}$				Methodology III - $BE_{premium\ provisions}$			
Scenario	Motor Vehicle Liability	Other Motor Insurance	Total	$\Delta_{Scenario/SII}$	Motor Vehicle Liability	Other Motor Insurance	Total	$\Delta_{Scenario/SII}$	Motor Vehicle Liability	Other Motor Insurance	Total	$\Delta_{Scenario/SII}$
1.a)	54.108,7	32.122,2	**86.230,9**	**65,0%**	54.951,3	31.453,2	**86.404,5**	**64,4%**	55.001,5	28.605,0	**83.606,5**	**67,5%**
1.b)	50.852,7	30.255,4	**81.108,1**	**55,2%**	51.684,8	29.674,1	**81.358,9**	**54,8%**	51.640,6	26.984,6	**78.625,2**	**57,6%**
1.c)	83.412,6	48.923,3	**132.335,9**	**153,2%**	84.349,1	47.464,9	**131.814,1**	**150,8%**	85.249,8	43.188,7	**128.438,5**	**157,4%**
1.d)	18.292,8	11.587,5	**29.880,3**	**-42,8%**	19.020,5	11.883,3	**30.903,8**	**-41,2%**	18.031,4	10.780,5	**28.812,0**	**-42,3%**
2.a)	31.347,3	16.947,9	**48.295,3**	**-7,6%**	32.189,9	16.279,0	**48.468,9**	**-7,8%**	32.240,2	13.430,8	**45.671,0**	**-8,5%**
2.b)	5.330,0	-93,1	**5.236,9**	**-90,0%**	6.162,1	-674,4	**5.487,8**	**-89,6%**	6.117,9	-3.363,9	**2.754,1**	**-94,5%**
2.c)	37.889,9	18.574,8	**56.464,8**	**8,0%**	38.826,5	17.116,5	**55.942,9**	**6,4%**	39.727,1	12.840,2	**52.567,3**	**5,3%**
2.d)	18.292,8	11.587,5	**29.880,3**	**-42,8%**	19.020,5	11.883,3	**30.903,8**	**-41,2%**	18.031,4	10.780,5	**28.812,0**	**-42,3%**
3.a)	12.379,6	4.302,7	**16.682,3**	**-68,1%**	13.222,1	3.633,8	**16.855,9**	**-67,9%**	13.272,4	785,6	**14.058,0**	**-71,8%**
3.b)	5.330,0	-93,1	**5.236,9**	**-90,0%**	6.162,1	-674,4	**5.487,8**	**-89,6%**	6.117,9	-3.363,9	**2.754,1**	**-94,5%**
3.c)	-45,7	-6.715,6	**-6.761,2**	**-112,9%**	890,9	-8.173,9	**-7.283,0**	**-113,9%**	1.791,5	-12.450,2	**-10.658,6**	**-121,4%**
3.d)	18.292,8	11.587,5	**29.880,3**	**-42,8%**	19.020,5	11.883,3	**30.903,8**	**-41,2%**	18.031,4	10.780,5	**28.812,0**	**-42,3%**

(in thousands of Euros)

TABLE V: BEST ESTIMATES FOR PREMIUM PROVISIONS FOR THE DIFFERENT SCENARIOS (UNDERTAKING A)

Insurance Company B

	Methodology I - $BE_{premium\ provisions}$				Methodology II - $BE_{premium\ provisions}$				Methodology III - $BE_{premium\ provisions}$			
Scenario	Motor Vehicle Liability	Other Motor Insurance	Total	$\Delta_{Scenario/SII}$	Motor Vehicle Liability	Other Motor Insurance	Total	$\Delta_{Scenario/SII}$	Motor Vehicle Liability	Other Motor Insurance	Total	$\Delta_{Scenario/SII}$
1.a)	52.000,4	28.624,9	**80.625,4**	**52,0%**	52.692,7	26.281,8	**78.974,5**	**52,4%**	55.017,7	24.547,1	**79.564,8**	**53,0%**
1.b)	49.101,1	26.965,1	**76.066,2**	**43,4%**	49.806,1	24.834,7	**74.640,9**	**44,1%**	51.842,2	23.179,3	**75.021,5**	**44,3%**
1.c)	78.094,9	39.305,1	**117.400,0**	**121,4%**	78.672,1	39.305,1	**117.977,2**	**127,7%**	83.597,3	36.857,4	**120.454,7**	**131,7%**
1.d)	20.107,2	10.367,0	**30.474,2**	**-42,5%**	20.940,1	10.364,4	**31.304,5**	**-39,6%**	20.087,1	9.501,2	**29.588,3**	**-43,1%**
2.a)	28.087,5	16.846,9	**44.934,4**	**-15,3%**	28.779,8	14.503,8	**43.283,5**	**-16,5%**	31.104,8	12.769,1	**43.873,8**	**-15,6%**
2.b)	1.275,2	3.409,1	**4.684,3**	**-91,2%**	1.980,2	1.278,7	**3.258,9**	**-93,7%**	4.016,3	-376,8	**3.639,5**	**-93,0%**
2.c)	30.269,0	20.007,2	**50.276,2**	**-5,2%**	30.846,2	15.749,1	**46.595,3**	**-10,1%**	35.771,4	13.301,3	**49.072,8**	**-5,6%**
2.d)	20.107,2	10.367,0	**30.474,2**	**-42,5%**	20.940,1	10.364,4	**31.304,5**	**-39,6%**	20.087,1	9.501,2	**29.588,3**	**-43,1%**
3.a)	8.160,0	7.031,9	**15.191,9**	**-71,4%**	8.852,3	4.688,7	**13.541,1**	**-73,9%**	11.177,3	2.954,0	**14.131,4**	**-72,8%**
3.b)	1.275,2	3.409,1	**4.684,3**	**-91,2%**	1.980,2	1.278,7	**3.258,9**	**-93,7%**	4.016,3	-376,8	**3.639,5**	**-93,0%**
3.c)	-9.585,9	377,1	**-9.208,7**	**-117,4%**	-9.008,7	-3.880,9	**-12.889,6**	**-124,9%**	-4.083,5	-6.328,7	**-10.412,2**	**-120,0%**
3.d)	20.107,2	10.367,0	**30.474,2**	**-42,5%**	20.940,1	10.364,4	**31.304,5**	**-39,6%**	20.087,1	9.501,2	**29.588,3**	**-43,1%**

(in thousands of Euros)

TABLE VI: BEST ESTIMATES FOR PREMIUM PROVISIONS FOR THE DIFFERENT SCENARIOS (UNDERTAKING B)

Insurance Company C

	Methodology I - $BE_{premium\ provisions}$				Methodology II - $BE_{premium\ provisions}$				Methodology III - $BE_{premium\ provisions}$			
Scenario	Motor Vehicle Liability	Other Motor Insurance	Total	$\Delta_{Scenario/SII}$	Motor Vehicle Liability	Other Motor Insurance	Total	$\Delta_{Scenario/SII}$	Motor Vehicle Liability	Other Motor Insurance	Total	$\Delta_{Scenario/SII}$
1.a)	149.682,2	61.946,6	**211.628,8**	**80,2%**	146.760,2	61.441,8	**208.202,0**	**81,7%**	151.760,3	63.196,5	**214.956,9**	**78,7%**
1.b)	140.651,0	58.245,6	**198.896,6**	**69,4%**	138.102,8	57.843,4	**195.946,2**	**71,0%**	142.470,0	59.356,8	**201.826,8**	**67,8%**
1.c)	230.963,0	95.256,1	**326.219,1**	**177,8%**	224.677,3	93.827,3	**318.504,5**	**177,9%**	235.373,6	97.754,5	**333.128,1**	**176,9%**
1.d)	50.339,1	21.235,0	**71.574,1**	**-39,1%**	51.528,2	21.859,6	**73.387,8**	**-36,0%**	49.566,4	20.959,1	**70.525,5**	**-41,4%**
2.a)	77.154,1	30.863,1	**108.017,3**	**-8,0%**	74.232,1	30.358,3	**104.590,4**	**-8,7%**	79.232,2	32.113,1	**111.345,3**	**-7,4%**
2.b)	-4.405,2	-3.921,4	**-8.326,6**	**-107,1%**	-6.953,5	-4.323,5	**-11.277,0**	**-109,8%**	-2.586,2	-2.810,2	**-5.396,4**	**-104,5%**
2.c)	85.906,7	33.089,2	**118.995,9**	**1,3%**	79.621,1	31.660,3	**111.281,4**	**-2,9%**	90.317,4	35.587,5	**125.904,9**	**4,7%**
2.d)	50.339,1	21.235,0	**71.574,1**	**-39,1%**	51.528,2	21.859,6	**73.387,8**	**-36,0%**	49.566,4	20.959,1	**70.525,5**	**-41,4%**
3.a)	16.714,0	4.960,2	**21.674,3**	**-81,5%**	13.792,0	4.455,4	**18.247,4**	**-84,1%**	18.792,1	6.210,2	**25.002,3**	**-79,2%**
3.b)	-4.405,2	-3.921,4	**-8.326,6**	**-107,1%**	-6.953,5	-4.323,5	**-11.277,0**	**-109,8%**	-2.586,2	-2.810,2	**-5.396,4**	**-104,5%**
3.c)	-34.973,4	-18.716,6	**-53.690,1**	**-145,7%**	-41.259,1	-20.145,5	**-61.404,6**	**-153,6%**	-30.562,8	-16.218,3	**-46.781,1**	**-138,9%**
3.d)	50.339,1	21.235,0	**71.574,1**	**-39,1%**	51.528,2	21.859,6	**73.387,8**	**-36,0%**	49.566,4	20.959,1	**70.525,5**	**-41,4%**

(in thousands of Euros)

TABLE VII: BEST ESTIMATES FOR PREMIUM PROVISIONS FOR THE DIFFERENT SCENARIOS (UNDERTAKING C)

As it is possible to observe, and comparing with the best estimates found for the same three undertakings considering their specific structures of the number of instalments of the premiums (Table II), these best estimates are strongly affected by the way the premiums are paid, being scenario 2.c), when all the policies pay their premiums semiannually and all start paying them at 1st of December, the one which gives values for the best estimates for premium provisions more approximate to the general cases presented on Table II.

It is important to notice that if all the policies in force at the valuation date pay the premiums annually (scenario 1), there are no expected cash in-flows in the forward year ($PVFP = 0$), the total amount at risk for the future period is just the unearned premiums ($TVM_{2014} = UP_{2013}$) and the best estimate for premium provision assumes its highest value on sub-scenario c), because it is when the unearned premiums, calculated on a *pro-rata temporis* basis, are higher.

On the other hand, these best estimates, for the LoBs considered, assume their lowest values (even negative estimates, which means that the expected future cash in-flows exceed all the expected cash out-flows) on scenario 3.c), which is when all the policies in force at 31st December 2013 pay their premiums monthly and all start paying on the first day of that month (so $UP_{2013} = 0$ and $TVM_{2014} = CPNP_{2013}$, where $CPNP_{2013}$ assume their highest value as almost all the premiums are going to be received in the next year).

So, the payment structure of the premiums affects strongly the best estimate for premium provisions and undertakings, in order to compute these best estimates in an accurate and reliable manner, should study carefully the structure of their portfolios.

5. CONCLUSIONS AND FURTHER DEVELOPMENTS

The aim of this work was to understand the concept and the differences, from the current practices, of constructing provisions for future claims events under the Solvency II framework, which are expected to change significantly.

The Directive and all the documentation supporting the definition of the best estimate for premium provisions confirmed the complexity and extent of this project. However, the lack of technical literature on this specific subject created the need to use the literature on claims reserving in order to adapt the methods for this case study but, due to time limitations, it was necessary to focus on the construction of a restricted number of general methodologies.

The results obtained seem to support the idea that, comparing to the equivalent Solvency I provisions, the calculation of these best estimates not necessarily results on an increase on the level of own funds for the (re)insurance companies. Also, the obtained estimates depend strongly on several factors and the undertaking must select the methodology that believes to be more adequate to its own reality and data.

Across this work, some aspects had to be simplified, however they should be analysed more carefully in the future. Further developments to this work would be: 1) To test other methods to project the ultimate loss rates and to model the payments pattern, for the periods ahead; 2) To explore other methodologies to estimate and allocate the future general management expenses to the best estimate for premium provisions.

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ANNEXES

ANNEX 1. APPLICATION OF METHODOLOGY I (AN EXAMPLE)

1. Using the R package 'ChainLadder' to project the run-off triangle:

```
################### ChainLadder Package#######################
suppressPackageStartupMessages(library(ChainLadder))
#READING THE RUN-OFF TRIANGLE (PAID CLAIMS AMOUNTS WITH THE ALLOCATED MANAGEMENT COSTS)
#For confidentiality reasons, the run-off triangle presented below is taken from:
#Wacek, M. (2007). The Path of the Ultimate Loss Ratio Estimate. Winter 2007, pp.359
PaidClaims=matrix(c(17674,32062,38619,42035,43829,44723,45162,45375,45483,45540,
18315,32791,39271,42933,44950,45917,46392,46600,46753,NA,
18606,32942,39634,43411,45428,46357,46681,46921,NA,NA,
18816,33667,40575,44446,46746,47350,47809,NA,NA,NA,
20649,36515,43724,47684,49753,50716,NA,NA,NA,NA,
22327,39312,46848,51065,53242,NA,NA,NA,NA,NA,
23141,40527,48284,52661,NA,NA,NA,NA,NA,NA,
24301,42168,50356,NA,NA,NA,NA,NA,NA,NA,
24210,41640,NA,NA,NA,NA,NA,NA,NA,NA,
24468,NA,NA,NA,NA,NA,NA,NA,NA,NA),nrow=10,ncol=10,byrow=TRUE)
PaidClaims
#Application of Mack's Model:
Mack <- MackChainLadder(Triangle=PaidClaims, tail=TRUE, tail.se=NULL, tail.sigma=NULL)
Mack$FullTriangle #Forecasted Full Triangle
Mack$Mack.S.E #Total Variability in the Projection of Future Losses
Mack$f #Estimated Development Factors
```

2. Calculate $LR_{2014} = \frac{1}{4} \times \sum_{i=2010}^{2013} LR_i$ (the selection of the number of years to compute the average is made based on the moment that the stability of the company's business is reached), where $LR_i = \frac{E[A_{i,Ult}]}{e_i}$;

3. Calculate $TVM_{2014} = UP_{2013} + CPNP_{2013}$ and $EC_{2014} = \frac{1}{4} \times \sum_{i=2010}^{2013} EC_i$ (select the same number of years used to compute the ultimate loss ratio), where $EC_i = \frac{Unallocated\ Expenses_{LoB,i} \times TVM_{i+1}}{Written\ Premiums_i}$;

4. Calculate d_1 and d_2 as in the example shown below:

Time t	Proportion Paid (Incr.)	Risk-Free Interest Rates (EIOPA)	P(0,t)	E.P.V - Expected Present Value	t x E.P.V
1	1,9%	0,3%	0,997	0,018	0,018
2	6,7%	0,4%	0,991	0,066	0,132
3	15,3%	0,7%	0,981	0,150	0,451
4	21,5%	0,9%	0,964	0,207	0,828
5	20,4%	1,2%	0,943	0,192	0,962
6	14,8%	1,4%	0,919	0,136	0,816
7	9,1%	1,6%	0,893	0,081	0,567
8	5,0%	1,8%	0,866	0,044	0,349
9	2,7%	2,0%	0,838	0,022	0,201
10	2,7%	2,1%	0,809	0,022	0,222
				0,9	4,5
				Average Duration (d) =	***4,8***

Where $d = \frac{\sum_{t=1}^{10}(t \times E.P.V_t)}{\sum_{t=1}^{10}(E.P.V_t)}$ is the average duration;

5. Calculate $PVFP = \frac{CPNP_{2013}}{(1+r_1)^{\frac{1}{2}}}$ and applying Formula [2] it is possible to obtain the best estimate for premium provisions.

ANNEX 2. APPLICATION OF METHODOLOGY II (AN EXAMPLE)

1. Using the run-off triangle and the algorithm described, find the estimates for each development factor, $\widehat{f_j}$:

Development Factors Estimates

$\widehat{\mu_j}$	0,389	0,083	0,059	0,040	0,029	0,027	0,021	0,013	0,003
$\widehat{\sigma_j}$	0,040	0,018	0,027	0,019	0,011	0,012	0,007	0,014	0,014
$\widehat{f_j}$	**1,476**	**1,087**	**1,061**	**1,041**	**1,029**	**1,027**	**1,022**	**1,013**	**1,003**

2. Estimate $LR^*_{2014,9}$, following the algorithm described to estimate the ultimate loss ratio (an example is shown below):

Analysis of Estimated Loss Ratios - One Year Out

			First Effect		Second Effect					Lognormal - 95% C.I.	
			Accident Year Development		*Tail Factor Revision*		Est miu for	Est sigma for	Estimated	Est. LR 1	Est. LR 1
Accident	Development	Net Paid	Est miu for	Est sigma for	Est miu for	Est sigma for	Est Ult LR	Est Ult LR	$\widehat{LR^*_{i,d-1}}$	Year Out	Year Out
Year	Year	Loss Ratio	Actual 1-Yr LDF	Actual 1-Yr LDF	Revised Mean LDF	Revised Mean LDF	1 Year Out	1 Year Out	1 Year Out	Lower Bound	Upper Bound
2004	10	77,9%	0,000	0,000	0,000	0,000	-0,250	0,000	77,90%	77,90%	77,90%
2005	9	81,4%	0,003	0,014	0,000	0,000	-0,203	0,014	81,61%	79,36%	83,90%
2006	8	76,3%	0,013	0,014	0,003	0,007	-0,255	0,016	77,53%	75,15%	79,97%
2007	7	66,1%	0,021	0,007	0,016	0,009	-0,377	0,011	68,62%	67,16%	70,11%
2008	6	72,1%	0,027	0,012	0,038	0,009	-0,262	0,014	76,94%	74,78%	79,14%
2009	5	77,2%	0,029	0,011	0,065	0,009	-0,165	0,014	84,79%	82,46%	87,16%
2010	4	75,7%	0,040	0,019	0,094	0,009	-0,145	0,021	86,56%	82,99%	90,24%
2011	3	72,0%	0,059	0,027	0,134	0,010	-0,136	0,029	87,33%	82,47%	92,41%
2012	2	64,2%	0,083	0,018	0,193	0,010	-0,167	0,021	84,67%	81,25%	88,20%
2013	1	47,2%	0,389	0,040	0,276	0,010	-0,086	0,042	91,85%	84,55%	99,60%
2014	**0**	**0,00%**	**-0,811**	**0,081**	**0,666**	**0,011**	**-0,145**	**0,082**	**86,78%**	**73,71%**	**101,49%**

3. In order to project the development factors beyond development year (d-1), use the following R-Code to obtain the estimates for β_0 and β_1:

```
year=c(0,1,2,3,4,5,6,7,8)
rate=c(-0.741375909,-2.44527591,-2.796343308,-3.19305663,-3.523468075,-3.595811313,-3.834263084,-
4.329994194,-5.780311097) # an example of ln(f̂j − 1)
plot(year,rate,main="Regression of the Development Factors")
cor(year,rate)
fit <- lm(rate ~ year)
fit
abline(fit)
summary(fit)
```

4. Estimate $\widehat{A_{2014,9}} = \widehat{LR^*_{2014,9}} \times (UP_{2013} + CPNP_{2013})$ and using the estimated development factors, it is possible to obtain all the payment structure for the next accident year (2014) and the confidence intervals for each one of those estimates:

Development Year j	**Estimated $A_{2014,j}$**	**95th Confidence Intervals** Lower Bound	Upper Bound
0	18.871,7	16.969,3	20.927,4
1	27.863,3	24.376,0	31.706,5
2	30.279,1	26.358,3	34.615,3
3	32.127,1	27.673,0	37.090,5
4	33.445,8	28.665,2	38.791,9
5	34.432,4	29.464,7	39.993,8
6	35.377,1	30.220,7	41.157,1
7	36.141,9	30.855,1	42.070,5
8	36.617,8	31.180,9	42.725,9
9	36.730,9	31.197,8	42.958,2
10	36.851,6	29.196,6	45.898,4
11	36.927,3	27.714,0	48.230,2
12	36.974,6	26.497,0	50.239,0
13	37.004,1	25.450,8	52.040,3
14	37.022,6	24.527,1	53.695,1
15	37.034,1	23.697,6	55.240,3
16	37.041,3	22.943,1	56.700,1
17	37.045,8	22.250,6	58.090,9

(in thousands of Euros)

5. Calculate $EC_{2014} = \frac{1}{10} \times \sum_{i=2004}^{2013} EC_i$, using the average of the last ten years because when computing $\widehat{LR^*_{2014,9}}$ it is also considered the contribution of each of those years, and $PVFP = \frac{CPNP_{2013}}{(1+r_1)^{\frac{1}{2}}}$;

6. Having all these estimates, and applying equation [3], it is possible to get the best estimate for premium provisions:

Development Year j	Risk-Free Interest Rates (EIOPA)	Discounted Future Claims Costs (Incremental)
0	0,30%	18.843,2
1	0,44%	8.932,9
2	0,65%	2.376,9
3	0,91%	1.790,6
4	1,17%	1.251,3
5	1,40%	913,7
6	1,61%	851,5
7	1,80%	669,1
8	1,97%	403,2
9	2,12%	92,6
10	2,25%	95,6
11	2,36%	57,8
12	2,46%	34,9
13	2,54%	21,1
14	2,60%	12,7
15	2,65%	7,7
16	2,69%	4,6
17	2,71%	2,8
	Discounted Future Claims Costs =	**36.362,3**
	Discounted General Management Expenses =	**12.487,7**
	Discounted Future Premiums =	**14.664,9**
	***BE** Premium Provisions* **=**	**34.185,1**

(in thousands of Euros)

ANNEX 3. APPLICATION OF METHODOLOGY III (AN EXAMPLE)

1. Using the data about the average premium per policy, calculate $\widehat{\mu_p} = \frac{1}{3} \times \sum_{i=2011}^{2013} \ln(p_i)$ and $\widehat{\sigma_p} = (\frac{1}{2} \times \sum_{i=2011}^{2013} \left(\ln(p_i) - \widehat{\mu_p}\right)^2)^{\frac{1}{2}}$ (the consideration of just three years is justified by the significant decrease in premiums verified over the past ten years) and, then, estimate $\widehat{\mu_e} = \ln(UP_{2013} + CPNP_{2013}) - \widehat{\mu_p}$ and $\widehat{\sigma_e} = \widehat{\sigma_p}$;
2. Estimate the delay-specific claim frequencies, θ_j, as in the example bellow:

Logarithms of ($I_{i,j}/e_i$)

	4,6	4,0	2,7	1,8	2,3	2,1	2,1	1,6	-0,5	-0,5	-0,6
	4,5	4,0	2,8	2,9	2,3	1,9	1,2	1,0	1,5		
	4,6	4,0	2,7	2,6	2,4	1,7	1,6	1,7			
	4,7	3,9	2,2	2,0	1,2	1,0	1,4				
	4,7	3,9	2,7	2,6	1,3	1,4					
	4,8	3,9	2,7	2,1	1,8						
	4,8	3,9	2,3	1,8							
	4,7	3,9	2,7								
	4,7	3,8									
	4,6										
Mean =	4,7	3,9	2,6	2,2	1,9	1,6	1,6	1,4	0,5	-0,5	-0,6
Standard Deviation =	0,1	0,1	0,2	0,4	0,5	0,4	0,4	0,4	1,4	1,4	1,4
$\widehat{\theta_j}$ =	**106,3**	**50,3**	**13,6**	**10,4**	**7,6**	**5,6**	**5,3**	**4,4**	**4,8**	**1,8**	**1,5**

3. As $\widehat{I_{d+1,J}} = \widehat{\theta_J} \times \widehat{E_{d+1}} \sim lognormal\left(\mu_{d+1,j}; \sigma_{d+1,j}\right)$, it is possible to estimate, for each development year j, $\widehat{\mu_{d+1,J}} = \widehat{\mu_e} + \widehat{\mu_J}$ and $\widehat{\sigma_{d+1,J}} = \left(\widehat{\sigma_e}^2 + \widehat{\sigma_J}^2\right)^{\frac{1}{2}}$ and, then, obtain the estimates of the future claims costs for each development year, calculate the present value of those estimates and the confidence intervals:

Development Year j	**Estimated $A_{2014,j}$**	**95th Confidence Intervals** Lower Bound	Upper Bound	Discounted Future Claims Costs (Incremental)
0	19.206,0	15.802,6	23.123,4	19.177,0
1	9.095,2	7.827,8	10.508,4	9.035,8
2	2.467,0	1.525,4	3.781,5	2.427,2
3	1.881,0	725,6	4.027,7	1.822,6
4	1.373,5	410,5	3.428,0	1.303,3
5	1.015,1	411,8	2.104,4	940,2
6	963,0	388,7	2.002,9	867,9
7	801,9	356,4	1.565,0	701,6
8	866,9	17,8	5.190,3	734,5
9	316,8	6,5	1.896,7	259,6
>	269,1	5,5	1.611,3	173,7

(in thousands of Euros)

4. Calculate $EC_{2014} = \frac{1}{3} \times \sum_{i=2011}^{2013} EC_i$, using the average of the last three years as it was done when estimating the parameters of P_{2014}, $PVFP = \frac{CPNP_{2013}}{(1+r_1)^{\frac{1}{2}}}$ and, using equation [3], it is possible to obtain the best estimate for premium provisions.

ANNEX 4. GENERAL FORMULAS TO CALCULATE UP_N AND $CPNP_N$

The formulas presented on Table I are special cases of the more general ones:

$$CPNP_i = \sum_{k=1}^{12} \frac{VB_{i,k}}{12} \times \left[z \times INT\left(\frac{k-1}{z}\right)\right];$$

$UP_i = \sum_{k=1}^{12} \frac{VB_{i,k}}{12} \times \left[z \times INT\left(\frac{12-k}{z}\right) + z - (12-k)\right]$, when the premiums payments start at the end of a particular month;

$UP_i = \sum_{k=1}^{12} \frac{VB_{i,k}}{12} \times \left[z \times INT\left(\frac{12-k}{z}\right) + z - \big(12 - (k-1)\big)\right]$, when the premiums payments start at the beginning of a particular month.

Where:

$VB_i = \sum_{k=1}^{12} VB_{i,k}$ is the total volume of business on accident year i;

$INT\left(\frac{a}{b}\right)$ or $\left\lfloor\frac{a}{b}\right\rfloor$ is the greater integer x such that: $(x \times b) \leq a$;

$z = \begin{cases} 12, under\ scenario\ 1 \\ 6, under\ scenario\ 2 \\ 1, under\ scenario\ 3 \end{cases}$, being the time interval (in months) between two premium payments.

ANNEX 5. *ONE YEAR CONTRACT BOUNDARIES – GENERAL RESULTS*

Contract Boundary 1 Year

	Insurance Company A			**Insurance Company B**			**Insurance Company C**		
	Motor Vehicle Liability	Other Motor Insurance	Total	Motor Vehicle Liability	Other Motor Insurance	Total	Motor Vehicle Liability	Other Motor Insurance	Total
Estimated Future Claims Costs	38.228,2	21.917,8	60.146,0	33.900,3	19.406,8	53.307,1	112.032,4	45.911,8	157.944,3
Estimated Future General Management Expenses	11.798,6	7.865,8	19.664,4	14.330,1	7.058,1	21.388,2	32.325,6	13.853,8	46.179,4
Present Value of Future Costs	48.055,3	28.651,5	76.706,8	46.476,2	25.462,5	71.938,6	138.583,6	57.398,3	195.981,9
Present Value of Future Premiums	14.664,9	9.776,6	24.441,5	12.670,0	6.240,4	18.910,4	54.974,6	23.560,5	78.535,1
Methodology I $BE_{Premium\ Provisions}$	**33.390,4**	**18.874,9**	**52.265,3**	**33.806,2**	**19.222,0**	**53.028,2**	**83.609,0**	**33.837,8**	**117.446,8**
$\triangle_{SII/SI}$	-	-	**11,5%**	-	-	**-4,2%**	-	-	**-8,5%**
Estimated Future Claims Costs	37.045,8	19.881,6	56.927,4	32.791,5	16.115,0	48.906,4	103.733,2	42.518,0	146.251,2
Estimated Future General Management Expenses	12.506,6	8.337,7	20.844,3	15.189,9	7.481,6	22.671,5	34.265,1	14.685,0	48.950,1
Present Value of Future Costs	48.850,0	28.144,9	76.994,9	47.192,8	23.524,7	70.717,5	136.120,9	57.019,7	193.140,6
Present Value of Future Premiums	14.664,9	9.776,6	24.441,5	12.670,0	6.240,4	18.910,4	54.974,6	23.560,5	78.535,1
Methodology II $BE_{Premium\ Provisions}$	**34.185,1**	**18.368,3**	**52.553,4**	**34.522,8**	**17.284,3**	**51.807,1**	**81.146,3**	**33.459,1**	**114.605,4**
$\triangle_{SII/SI}$	-	-	**12,1%**	-	-	**-6,4%**	-	-	**-10,7%**
Estimated Future Claims Costs	38.255,6	18.111,8	56.367,3	36.115,9	15.235,7	51.351,6	111.372,2	45.373,0	156.745,2
Estimated Future General Management Expenses	11.326,7	7.551,1	18.877,8	13.756,9	6.775,8	20.532,7	31.032,5	13.299,7	44.332,2
Present Value of Future Costs	48.753,0	25.592,4	74.345,5	48.967,3	21.940,9	70.908,3	140.343,2	58.477,8	198.821,0
Present Value of Future Premiums	14.664,9	9.776,6	24.441,5	12.670,0	6.240,4	18.910,4	54.974,6	23.560,5	78.535,1
Methodology III $BE_{Premium\ Provisions}$	**34.088,2**	**15.815,8**	**49.904,0**	**36.297,4**	**15.700,5**	**51.997,9**	**85.368,6**	**34.917,2**	**120.285,8**
$\triangle_{SII/SI}$	-	-	**6,5%**	-	-	**-6,0%**	-	-	**-6,3%**

(in thousands of Euros)

ANNEX 6. *FIVE YEARS CONTRACT BOUNDARIES – GENERAL RESULTS*

Contract Boundary 5 Years

	Insurance Company A			**Insurance Company B**			**Insurance Company C**		
	Motor Vehicle Liability	Other Motor Insurance	Total	Motor Vehicle Liability	Other Motor Insurance	Total	Motor Vehicle Liability	Other Motor Insurance	Total
Estimated Future Claims Costs	207.562,6	110.340,1	317.902,7	191.156,0	101.558,6	292.714,6	508.258,3	187.958,7	696.217,0
Estimated Future General Management Expenses	41.916,7	26.323,0	68.239,7	50.910,1	18.841,8	69.751,9	106.151,3	42.021,6	148.172,9
Present Value of Future Costs	230.587,4	126.735,6	357.323,0	224.504,5	111.371,5	335.876,0	568.948,8	213.561,9	782.510,7
Present Value of Future Premiums	199.569,0	122.096,8	321.665,7	215.358,4	97.383,7	312.742,1	552.054,8	210.039,6	762.094,4
Methodology I $BE_{Premium\ Provisions}$	**31.018,4**	**4.638,9**	**35.657,3**	**9.146,1**	**13.987,8**	**23.133,9**	**16.894,0**	**3.522,3**	**20.416,3**
$\triangle_{5years/1year}$	**-7,1%**	**-75,4%**	**-31,8%**	**-72,9%**	**-27,2%**	**-56,4%**	**-79,8%**	**-89,6%**	**-82,6%**
Estimated Future Claims Costs	201.348,1	100.093,9	301.442,0	184.903,8	84.331,9	269.235,7	470.607,1	174.067,4	644.674,5
Estimated Future General Management Expenses	44.431,7	27.902,4	72.334,1	53.964,7	25.037,4	79.002,1	112.520,4	44.542,9	157.063,3
Present Value of Future Costs	237.130,4	125.775,8	362.906,2	229.723,8	107.393,8	337.117,6	563.841,1	214.903,2	778.744,3
Present Value of Future Premiums	199.569,0	122.096,8	321.665,7	215.358,4	97.383,7	312.742,1	552.054,8	210.039,6	762.094,4
Methodology II $BE_{Premium\ Provisions}$	**37.561,4**	**3.679,0**	**41.240,5**	**14.365,4**	**10.010,1**	**24.375,5**	**11.786,3**	**4.863,6**	**16.649,9**
$\triangle_{5years/1year}$	**9,9%**	**-80,0%**	**-21,5%**	**-58,4%**	**-42,1%**	**-52,9%**	**-85,5%**	**-85,5%**	**-85,5%**
Estimated Future Claims Costs	207.711,4	91.179,6	298.890,9	203.649,4	79.730,7	283.380,1	505.262,9	185.752,7	691.015,6
Estimated Future General Management Expenses	40.240,0	22.675,4	62.915,4	40.477,2	22.675,4	63.152,5	80.301,3	30.134,5	110.435,9
Present Value of Future Costs	238.542,0	114.422,1	352.964,1	234.052,8	100.540,5	334.593,3	565.220,5	212.169,4	777.389,8
Present Value of Future Premiums	199.569,0	122.096,8	321.665,7	215.358,4	97.383,7	312.742,1	552.054,8	210.039,6	762.094,4
Methodology III $BE_{Premium\ Provisions}$	**38.973,0**	**-7.674,6**	**31.298,4**	**18.694,4**	**3.156,8**	**21.851,2**	**13.165,6**	**2.129,8**	**15.295,4**
$\triangle_{5years/1year}$	**14,3%**	**-148,5%**	**-37,3%**	**-48,5%**	**-79,9%**	**-58,0%**	**-84,6%**	**-93,9%**	**-87,3%**

(in thousands of Euros)