LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

MASTER IN
FINANCE

MASTER'S FINAL WORK
DISSERTATION

SECURITY SELECTION IN POST-MODERN PORTFOLIO THEORY: AN APPLICATION TO THE EUROPEAN STOCK MARKET

EMÍLIA MARÍLIA DE LIMA ROCHA

OCTOBER – 2016

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SUPERVISION:

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Abstract

In this work, we compare tangent portfolios and minimum risk portfolios derived from the modern portfolio theory (MPT) and the post-modern portfolio theory (PMPT) to analyse the differences in stock selection. We base our study on a set of 16 stocks included in the EURO STOXX 50 index and estimate inputs from historical data since 1997 until 2015. To measure risk in PMPT, we use semivariance in relation to three target returns - 0, the risk-free rate and the European stock market return. To attest the results' robustness, we replicate the analysis estimating inputs from equilibrium models. We find that PMPT's portfolios select stocks that display return distributions with positive skewness and/or leptokurtosis. Additionally, these portfolios' composition favors stocks with low semivariance, characterized by low downside frequency and/or average downside deviation.

Keywords: modern portfolio theory; post-modern portfolio theory; stock selection; efficient frontier; semivariance.

JEL Classification: G10, G11, G12, G15

Resumo

Neste trabalho, comparamos as carteiras tangentes e carteiras de risco mínimo obtidas com a teoria moderna da carteira (MPT) e a teoria pós-moderna da carteira (PMPT) com o propósito de analisar as diferenças na seleção de ações. Baseamos o nosso estudo num conjunto de 16 ações do índice EURO STOXX 50 e estimamos os *inputs* com dados históricos entre 1997 e 2015. Para medir o risco na PMPT, usamos a semivariância em relação a três retornos alvo - 0, a taxa de juro sem risco e a taxa de retorno do mercado bolsista Europeu. Para atestar a robustez dos resultados, replicamos a análise estimando os *inputs* a partir de modelos de equilíbrio. Observamos que as carteiras da PMPT escolhem ações que exibem uma distribuição de retorno com assimetria positiva e/ou leptocúrtica. Adicionalmente, a composição destas carteiras privilegia ações com baixa semivariância, caracterizada por baixa frequência de retornos inferiores ao retorno alvo e/ou baixo desvio médio.

Keywords: teoria moderna da carteira; teoria pós-moderna da carteira; seleção de ações; fronteira eficiente; semivariância.

Classificação JEL: G10, G11, G12, G15

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List of Abbreviations

ADD - Average Downside Deviation

CAPM - Capital Asset Pricing Model

DF - Downside Frequency

DM - Downside Magnitude

DR - Downside Risk

ECB - European Central Bank

EF - Efficient Frontier(s)

HP - Homogeneous Portfolio(s)

LPM - Lower partial moment(s)

MAR - Minimum Acceptable Return(s)

MPT - Modern Portfolio Theory

MRP - Minimum Risk Portfolio

PMPT - Post-Modern Portfolio Theory

PMPT_0 - PMPT with semivariance below 0

PMPT_{R_f} - PMPT with semivariance below the risk-free rate

PMPT_{R_m} - PMPT with semivariance below the market return

TP - Tangent Portfolio(s)

List of Notation

β_i - Asset's beta

β_i^D - Asset's downside beta

β_p - Portfolio's beta

$E(R_i)$ - Asset's expected return

$E(R_m)$ - Market expected return

$E(R_p)$ - Portfolio expected return

ϵ_i^2 - Asset's error term

$\bar{R}_i$ - Asset's mean return

R_i - Asset's realized return

R_f - Risk-free rate

X_i - Asset's weight in the portfolio

σ_i^2 - Asset's variance

σ_i - Asset's standard deviation

$\sigma_{\epsilon_i}^2$ - Asset's error term variance

$\Sigma_{e_i}^2$ - Asset's error term semivariance

σ_{ij} - Covariance between asset i and j

σ_{im} - Covariance between asset i and the market

ρ_{ij} - Correlation coefficient between asset i and j

$\rho_{ij_{MAR}}$ - Downside correlation coefficient between asset i and j

σ_p^2 - Portfolio variance

σ_p - Portfolio standard deviation

σ_m^2 - Market variance

$\Sigma_{i_{MAR}}^2$ - Asset's semivariance

$\Sigma_{i_{MAR}}$ - Asset's semideviation

$\Sigma_{p_{MAR}}^2$ - Portfolio semivariance

$\Sigma_{p_{MAR}}$ - Portfolio semideviation

$\Sigma_{m_{MAR}}^2$ - Market semivariance

$\Sigma_{ij_{MAR}}$ - Semicovariance between asset i and j

$\Sigma_{im_{MAR}}$ - Semicovariance between asset i and the market

1 Introduction

Since its inception, modern portfolio theory (MPT) (Markowitz, 1952, 1959) has been the predominant framework in portfolio selection. Post-modern portfolio theory (PMPT) (Rom and Ferguson, 1994) appears as an alternative approach for asset allocation, whose main goal is to use a risk measure that best captures an investor's risk perception. Such measure is the downside risk (DR), which focuses on return deviations below a desired target rate, the so-called minimum acceptable return (MAR). This perspective diverges from the classical one, in which risk is associated with volatility around the mean return.

In this work, we compare tangent portfolios (TP) and minimum risk portfolios (MRP) derived from MPT and PMPT to analyse the differences in stock selection. We apply both theories to a set of 16 European stocks, estimating inputs from historical data since 1997 until 2015. To measure PMPT's downside risk, we use the semivariance - the average squared deviation below the MAR, which we define as 0, the risk-free rate or the European stock market return. We perform a robustness analysis to the results, replicating the process with inputs estimated from equilibrium models.

This work adds a contribute to the literature that compares MPT and PMPT, such as Harlow (1991), Rom and Ferguson (1994), Grootveld and Hallerbach (1999), Swisher and Kasten (2005), Cumova and Nawrocki (2011), and Vasant et al. (2014). The major findings are that, relatively to MPT, PMPT's portfolios focus on stocks with positive skewness and lower significantly the downside risk, while maintaining or improving expected returns.

We find that PMPT's portfolios favor in their composition stocks whose return distribution displays positive skewness and/or leptokurtosis, as well as stocks with

low semivariance, characterized by low downside frequency and/or average downside deviation.

The remainder of the text is organized as follows. Chapter 2 embodies the literature review on PMPT, focusing on its foundations and the findings related to the comparison of this theory with MPT. Chapter 3 describes the methodological process to perform the analysis. Chapter 4 presents the results and lastly, Chapter 5 summarizes the main conclusions and discusses further research.

2 Literature Review

This Chapter presents a literature review on PMPT, addressing its scope and focusing on the major findings associated with this theory. Section 2.1 introduces PMPT and what triggers its origin. Section 2.2 explores the DR concept. Section 2.3 looks at the controversy surrounding return distribution, namely the criticism that MPT assumes normally distributed returns. Finally, Section 2.4 enunciates the major findings of the literature that compares portfolio selection in MPT and PMPT.

2.1 Introducing PMPT

The term PMPT first appears in the literature with Rom and Ferguson (1994), in which the authors present a new approach theory for asset allocation, adding a contribute to the risk/return paradigm. The authors consider that MPT has two major limitations in its formulation: (i) the variance of returns is an appropriated measure of investment risk and (ii) assets' return can be adequately represented by the normal distribution.[1]

[1] Markowitz (2014) recalls that Gaussian (normal) return distributions or quadratic utility functions are just sufficient but not necessary conditions for the use of mean-variance analysis. Section 2.3 presents more details on this matter.

2.2 Downside Risk

The variance measures volatility or dispersion of returns, given by the average squared deviation from their mean. It is a symmetric risk measure, penalizing the uncertainty on the upside in the same way that it does on the downside. This issue is precisely what Rom and Ferguson (1994) criticize. They argue that risk is not symmetrical since most investors are concerned with facing losses (downside). Swisher and Kasten (2005) share this view and claim that standard deviation is a "poor proxy for how humans experience risk". The authors state that risk is an "emotional condition", such as fear of loss or underperformance.

Harlow (1991) defines DR as an asymmetric measure that quantifies return deviations below a specified target rate. Rom and Ferguson (1994) highlight that DR measures enable each investor to consider a specific target return and only any outcome below that goal constitutes risk. In PMPT's framework, the target rate of return is called MAR and represents the rate of return that an investor must earn to assure his financial objective. Thus, DR is considered a most plausible risk measure (Harlow, 1991), (Markowitz et al., 1993), (Rom and Ferguson, 1994), (Swisher and Kasten, 2005), (Estrada, 2006, 2007).

DR provides investors with extra statistics. Sortino and Satchell (2001) refer to these elements as (i) downside frequency (DF), which measures the likelihood of falling below the MAR; (ii) average downside deviation (ADD), which quantifies the average shortfall below the MAR and (iii) downside magnitude (DM), that represents the worst-case scenario, i.e., the return below the MAR at the 99th percentile. All these statistics combined result in the DR statistic.

The lower partial moment (LPM) is one of the DR measures since it considers only the left-hand tail of the return distribution (Harlow, 1991). Using Grootveld and

Hallerbach (1999) notation, the LPM of order α around τ is given by:

$$\mathrm{LPM}_\alpha(\tau; R) \equiv \int_{-\infty}^{\tau} (\tau - R)^\alpha \mathrm{d}F(R) = E\{(\max[0, \tau - R])^\alpha\}, \quad (1)$$

where F(R) represents the cumulative distribution function of the investment return R. τ is the target rate, i.e, the MAR using PMPT's nomenclature. The parameter α is related to the type of the investor's utility function, u, consistent with the risk measure. Harlow (1991) notes that LPM_0 suites all investors who prefer more wealth than less ($u' > 0$). LPM_1 is appropriated for risk-averse investors ($u' > 0$ and $u'' < 0$). LPM_2 is indicated for all risk-averse investors that display skewness preference ($u' > 0$, $u'' < 0$ and $u''' > 0$).

2.3 Return Distribution

At the heart of MPT's foundations lies the discussion around return distributions. Markowitz (1959) observes that if a utility function can be approximated to a quadratic one for a sufficiently wide range of returns, then expected utility is approximately equal to a function of expected return and variance. Levy and Markowitz (1979) find that mean-variance approximations to expected utility are usually accurate. Rom and Ferguson (1994) deduce that MPT assumes Gaussian assets' return distributions. The authors claim that apart from mean and variance, skewness and kurtosis[2] play a determinant role in portfolio selection. Rom and Ferguson (1994) analyse the degree of asymmetry in several asset classes during 10, 20 and 30 years prior to 31/12/1992 and observe that the majority display positive skewness. They believe the results proof that MPT's assumption is "inappropriate" and potentially

[2]Skewness is a measure of asymmetry. Positive skewness indicates a distribution tilted to the right compared with a symmetric one, while negative skewness reveals a distribution tilted to the left. Kurtosis is a measure of the frequency of outliers, the fatter the tails of the distribution the higher its kurtosis. A normal distribution is characterized by a skewness coefficient of 0 and kurtosis of 3.

induces "incorrect" results.

Kaplan and Siegel (1994) defend that mean-variance optimization does not depend on the return distribution. On the other hand, Hlawitschka (1994) declare that mean-variance analysis is valid even when securities have asymmetric distributions. The author examines the efficacy of mean-variance approximation to expected utility for portfolios of calls and concludes that it succeeds.

Markowitz (2014) deepens this topic asserting that sufficient and necessary conditions for MPT's application are often confused. The author explains that normal distribution of returns or quadratic utility functions are only sufficient, but not necessary conditions for MPT application. He further highlights that formulas relating expected return and variance of portfolios to the expected returns, variances and covariances of securities do not depend on the form of the probability distribution.

2.4 Comparing MPT and PMPT

Markowitz (1959) refers that when a return distribution is not symmetric or display different degrees of skewness, the efficient portfolios produced by mean-variance approach may differ from the ones produced using mean-semivariance approach. The author explains that for a given expected return and variance, the mean-semivariance analysis chooses portfolios with greater skewness to the right or lesser skewness to the left in their distribution. Markowitz (1959) admits that semivariance tends to produce better portfolios than those produced with variance, but highlights that the latter cannot be considered "bad or undesirable". For him, variance's greatest handicap is sacrificing expected returns since it considers both upside and downside volatility. However, Markowitz (1959) affirms that variance is superior to semivariance on cost, convenience and familiarity and that deriving the mean-semivariance EF is time-consuming. The author further notes that the classical approach in-

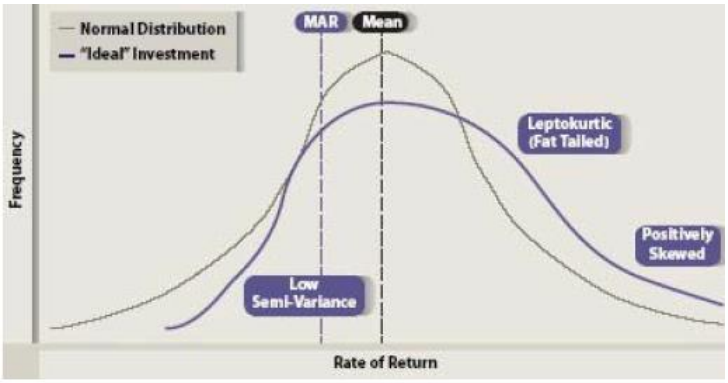
puts include only means, variances, and covariances, while semivariance requires the entire joint distribution of returns.

Rom and Ferguson (1994) compare MPT and PMPT and observe that in two portfolios with an equivalent risk level, the DR portfolio allocates a higher proportion to large-capitalization stocks and a lower one to foreign stocks and bonds than the mean-variance portfolio. The authors justify these differences by assets' skewness. They assert that with DR, the positive skewness of large-capitalization stocks makes them more attractive than in the mean-variance case, in which the skewness is ignored. Under the same reasoning, the negative skewness explains foreign stocks and bonds' underweighting.

Cumova and Nawrocki (2011) also find that mean-semivariance portfolios have higher skewness than portfolios derived from the mean-variance analysis. Additionally, they observe that mean-semivariance portfolios are less diversified than mean-variance ones, once skewness enables diversification with fewer stocks.[3]

On this matter, Swisher and Kasten (2005) characterize the "perfect" investment as (i) positively skewed - Negative outcomes are less frequent and scenarios with extreme losses are not as likely; (ii) leptokurtic - distribution with fatter tails, meaning larger chances of extreme outcomes compared with the normal distribution; (iii) low downside semivariance - when falling below the mean or any other target, do not fall too far below. Figure 1 illustrates these features in contrast with the normal distribution features.

[3]Simkowitz and Beedles (1978) conclude that 92% of the diversifiable skewness in a portfolio is diversified away with at least 5 stocks in the portfolio.



Source: Swisher and Kasten (2005), p.5.

Figure 1 – Normal distribution and the "perfect" investment distribution.

Other findings are from Harlow (1991), who highlights that a DR approach can lower risk, while maintaining or improving the level of expected return offered by mean-variance approach. Comparing the EF produced with both theories, he concludes that PMPT provides a higher allocation to bonds than MPT. Grootveld and Hallerbach (1999) verify that DR approaches tend to favor stocks in MRP, while bonds in TP. Swisher and Kasten (2005) consider that DR optimization is more intuitive on finding the optimum portfolio allocation than mean-variance optimization. Vasant et al. (2014) conclude that in pure equity cases, mean-semivariance portfolios have lower absolute returns but offer a significant benefit in terms of risk-adjusted returns. On this last issue, Estrada (2008) highlights that risk-adjusted returns from both theories should not be compared since the risk measure is not the same. The author stresses that doing it is "non informative". When comparing MPT and PMPT's EF in a mean-variance graph, the former outperforms the latter, while the opposite happens when plotted on a mean-semivariance graph. The author further notes that it all comes down to the investor decision on the measure that best captures his risk perception.

In this work, our focus is to compare stock selection in MPT and PMPT's TP and MRP. We contribute to the literature revised extending the analysis to the European

stock market. In addition, we analyse PMPT's portfolios subject to different MAR, measuring semivariance below 0, the risk-free rate and the European stock market return.

3 Data and Methodology

This Chapter focuses on the methodological process followed to apply MPT and PMPT. It is structured as follows. In section 3.1, we present the data used, as well as the processing that it is subject to. Section 3.2 explores the methodology of both theories. Subsection 3.2.1 reminds the classical approach methodology, while Subsection 3.2.2 details the PMPT methodology, giving particular emphasis to the risk measure adopted, the semivariance and the generation of the semicovariance matrix.

3.1 Data

We base our analysis on a set of 16 stocks included in the EURO STOXX 50 index. The index is organized in 16 supersectors and each of the stocks represents a supersector leader. We filter the data considering the stocks that belong to the index for at least 15 years and present the highest weight per supersector as of 04/03/2016[4]. Table I reports the supersector and representative firms.

[4]The index is weighted according to free-float market capitalization.

Table I
Stocks description

Number	Supersector	Name	Country
1	Automobiles & Parts	Daimler	Germany
2	Banks	Banco Santander	Spain
3	Chemicals	Bayer	Germany
4	Construction & Materials	VINCI	France
5	Food & Beverage	Danone	Belgium
6	Healthcare	Sanofi	France
7	Industrial Goods & Services	Siemens	Germany
8	Insurance	Allianz	Germany
9	Media	Vivendi	France
10	Oil & Gas	Total	France
11	Personal & Household Goods	Unilever NV	Netherlands
12	Real Estate	Unibail-Rodamco	France
13	Retail	Carrefour	France
14	Technology	SAP	Germany
15	Telecommunications	Deutsche Telekom	Germany
16	Utilities	Iberdrola	Spain

We collect data from Thompson Reuters DataStream from 01/01/1997 to 31/12/2015[5]. This yields a total of 4755 observations.

Figure 2 shows the evolution of the cumulative stocks' return during the sample period. Table II presents the data descriptive statistics.

[5]There is no available data prior to November of 1996 for any of the 16 stocks.

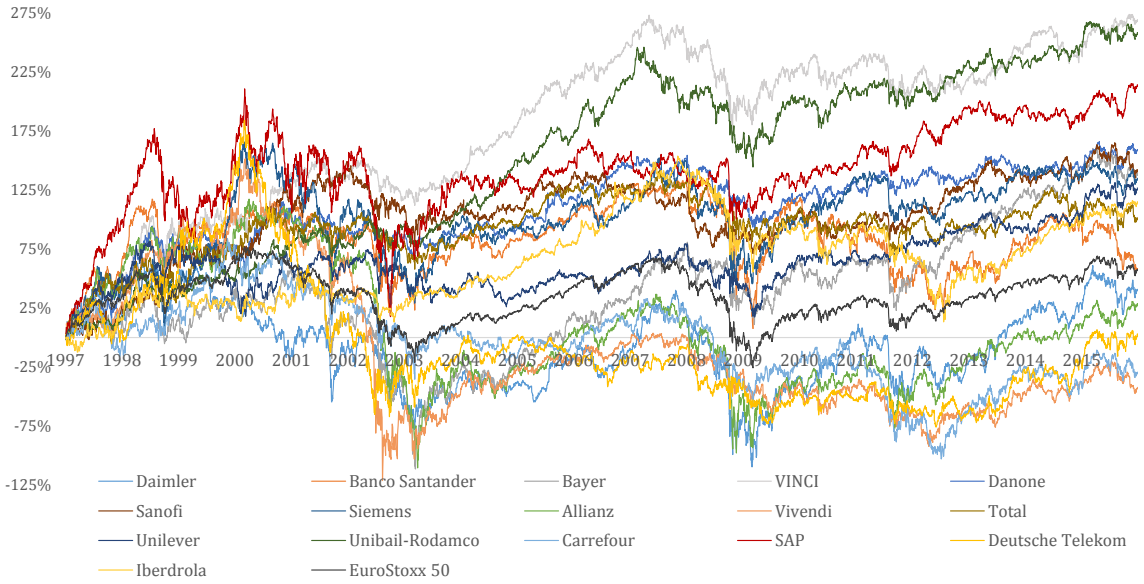


Figure 2 – Cumulative stocks' return from 1997 to 2015

Table II
Descriptive statistics

	Daimler	Banco Santander	Bayer	VINCI	Danone	Sanofi	Siemens	Allianz	Vivendi	Total	Unilever	Unibail-Rodamco	Carrefour	SAP	Deutsche Telekom	Iberdrola
Mean	0.01%	0.01%	0.03%	0.06%	0.03%	0.03%	0.03%	0.01%	-0.01%	0.02%	0.03%	0.05%	-0.01%	0.04%	0.00%	0.02%
Standard Error	0.03%	0.03%	0.03%	0.03%	0.02%	0.03%	0.03%	0.03%	0.03%	0.03%	0.02%	0.02%	0.03%	0.04%	0.03%	0.02%
Standard Deviation	2.26%	2.28%	2.07%	1.98%	1.60%	1.94%	2.26%	2.36%	2.31%	1.83%	1.59%	1.64%	2.02%	2.55%	2.26%	1.71%
Minimum	-15.7%	-16.2%	-19.4%	-13.3%	-11.1%	-14.0%	-18.7%	-15.2%	-29.5%	-13.2%	-10.7%	-8.7%	-11.7%	-19.4%	-16.4%	-13.4%
Maximum	19.4%	20.9%	33.0%	16.7%	9.7%	13.7%	21.6%	23.3%	20.3%	12.8%	10.4%	11.3%	11.1%	22.7%	14.5%	17.2%
Kurtosis	5.17*	6.41*	18.04*	5.04*	3.72*	3.52*	6.60*	7.65*	24.81*	4.07*	5.02*	2.85*	2.79*	9.24*	4.88*	10.07*
Skewness	0.11*	-0.01	0.54*	0.28*	-0.03	-0.01	0.01	0.25*	-1.36*	-0.01	-0.12*	0.06	-0.01	0.35*	0.06	0.33*

*Significant at p<0.05. Two-tailed tests on excess kurtosis and skewness ≠ 0. All the stocks in the sample period (1997-2015) exhibit significant leptokurtosis (kurtosis >3). Daimler, Bayer, VINCI, Allianz, Vivendi, Unilever, SAP and Iberdrola are significantly positively skewed. Vivendi and Unilever display significant negative skewness.

3.2 Methodology

In the core part of our work, we perform the analysis with inputs estimated from historical data. Then, we perform a robustness analysis obtaining those inputs from equilibrium models to check whether the initial results are corroborated.

Regarding our historical estimations, we compute the daily return for each stock, applying the Neperian logarithm between the observation in moment t and the previous one, in moment t-1. We consider five investment periods - 1, 5, 10, 15 and 30 years. We divide the 18 years of available data in consecutive periods of 1, 5,

10 and 15 years, respectively and take the average of daily returns for each period. Then, we annualize it multiplying by 250, which is the average number of trading days per year in the sample[6]. For the 30-year horizon, we consider the 18-year annualized average of daily returns.

Table III reports the annualized expected returns for each of the 16 stocks.

Table III
Expected returns

	Daimler	Banco Santander	Bayer	VINCI	Danone	Sanofi	Siemens	Allianz	Vivendi	Total	Unilever	Unibail-Rodamco	Carrefour	SAP	Deutsche Telekom	Iberdrola
1 Year	2.1%	3.3%	7.1%	14.4%	8.5%	7.7%	7.2%	1.6%	-2.0%	5.4%	6.8%	13.6%	-1.4%	11.4%	0.2%	6.1%
5 Years	-0.9%	0.0%	5.4%	10.9%	5.3%	3.7%	3.9%	-5.5%	-7.8%	3.0%	3.1%	13.2%	-5.1%	5.4%	-6.1%	4.1%
10 Years	-0.1%	0.8%	7.4%	11.7%	5.7%	2.9%	4.2%	-4.4%	-5.7%	3.3%	3.3%	13.3%	-4.6%	4.9%	-5.8%	5.5%
15 Years	-0.7%	0.9%	6.6%	11.6%	5.9%	5.9%	5.2%	-3.1%	-6.2%	3.8%	3.8%	13.3%	-3.9%	6.5%	-4.9%	3.9%
30 Years	2.1%	3.1%	7.2%	14.1%	8.4%	7.4%	7.0%	1.5%	-2.2%	5.3%	6.7%	13.5%	-1.6%	11.2%	0.1%	6.0%

3.2.1 MPT

In the classical approach, risk is measured by variance, the average squared deviation from the mean return. The variance of an asset is given by the equation below.

$$\sigma^2 = \frac{\sum_{t=1}^{T}(R_t - \bar{R})^2}{T-1} \tag{2}$$

Equivalently, the standard deviation of an asset is the square root of variance, as shown in equation 3.

$$\sigma = \sqrt{\frac{\sum_{t=1}^{T}(R_t - \bar{R})^2}{T-1}} \tag{3}$$

Table IV presents the historical standard deviations for each of the 16 stocks.

Another input is the covariance, a measure of how returns on assets move together (equation 4). However, a more intuitive measure is the correlation coefficient (equa-

[6]Taking the 5-year investment horizon as an example, we compute the average daily return for each stock during 1997 and 2002, then we move to the next period of 5 years - from 1998 to 2003, and so on until reaching the last 5-year period between 2010 and 2015. We compute the annualized average return based on all these periods and consider it the 5-year expect return.

Table IV
Historical volatilies

	Daimler	Banco Santander	Bayer	VINCI	Danone	Sanofi	Siemens	Allianz	Vivendi	Total	Unilever	Unibail-Rodamco	Carrefour	SAP	Deutsche Telekom	Iberdrola
1 Year	35.7%	36.1%	32.8%	31.4%	25.4%	30.9%	35.9%	37.4%	36.7%	29.0%	25.3%	25.9%	32.0%	40.7%	36.0%	27.0%
5 Years	36.8%	36.4%	33.8%	31.5%	25.1%	30.1%	37.0%	39.2%	39.2%	28.1%	24.8%	26.7%	31.8%	40.4%	36.0%	27.6%
10 Years	36.4%	35.3%	33.3%	31.4%	24.9%	29.4%	36.3%	38.6%	37.4%	27.7%	24.4%	26.9%	31.0%	38.6%	34.5%	27.6%
15 Years	36.3%	36.2%	33.3%	31.5%	25.4%	30.3%	36.7%	38.4%	38.3%	28.3%	24.9%	26.4%	32.1%	41.0%	36.4%	27.6%
30 Years	35.7%	36.1%	32.7%	31.4%	25.3%	30.7%	35.8%	37.4%	36.6%	28.9%	25.2%	25.9%	31.9%	40.3%	35.8%	27.0%

tion 5) which varies between a range of -1 to +1. Correlation plays a determinant role in diversification and risk mitigation, especially if it assumes low values (Markowitz, 1952).

$$\sigma_{ij} = \sum_{t=1}^{T} \frac{(R_{it} - \bar{R}_i)(R_{jt} - \bar{R}_j)}{T-1} \tag{4}$$

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \tag{5}$$

Table XIX reports the correlation matrices for each investment period.

At this stage we derive the efficient frontier (EF), a set of portfolios that offer the maximum possible expected return for a given level of risk (Markowitz, 1952). We start by obtaining the portfolio with the lowest standard deviation, the MRP. Equations 6 and 7 give portfolio expected return and variance, respectively.

$$E(R_p) = \sum_{i=1}^{n} E(X_i R_i) = X_i E(R_i) \tag{6}$$

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j \sigma_{ij} \tag{7}$$

In the equations above, n denotes the number of assets in the portfolio. We assume riskless lending and borrowing at the same rate. We use the spot rate taken from the Euro Area Yield Curve for AAA Bonds reported by the European Central Bank (ECB), considering the same maturity as the investment period. Table V portrays

the rates.[7] In addition, we derive the EF with and without short-selling restrictions. When allowed, we use the standard definition of short-selling.

Table V
Risk-free rates

1 Year	5 Years	10 Years	15 Years	30 Years
-0.64%	-0.52%	-0.12%	0.15%	0.47%

Source: ECB (2016).

Finally, we arrive at the TP, the portfolio that maximizes the EF slope given by the Sharpe Ratio (Sharpe, 1966), as follows.

$$\text{Sharpe Ratio} = \frac{E(R_p) - R_f}{\sigma_p} \tag{8}$$

3.2.2 PMPT

PMPT uses downside risk measures to quantify risk and Markowitz (1959) elects semivariance as the most robust measure. Semivariance is a particular case of the LPM when in equation 1, α equals 2. Unlike variance, which measures volatility around the mean distribution, semivariance determines the average squared deviations below the MAR, which can be distribution mean or any other pre-specified target. We use equation 9 based on Estrada (2006) to compute semivariance.

$$\Sigma^2_{MAR} = (1/T) \cdot \sum_{t=1}^{T} [\min(R_t - MAR, 0)]^2, \tag{9}$$

Where Σ^2_{MAR} denotes the semivariance in relation to any MAR, t indexes time and T represents the number of observations. Equivalently, the semideviation is given by the square root of semivariance, as follows.

[7] Yield curve spot rates are negative until the 10-year maturity, which may impact results since theoretical models do not assume negative risk-free rates.

$$\Sigma_{MAR} = \sqrt{(1/T) \cdot \sum_{t=1}^{T} [\min(R_t - MAR, 0)]^2} \tag{10}$$

Markowitz (1959) suggests estimating the portfolio semivariance with the following set of equations.

$$\Sigma^2_{p_{MAR}} = \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j S_{ij_{MAR}} \tag{11}$$

$$S_{ij_{MAR}} = (1/T) \cdot \sum_{t=1}^{K} (R_{it} - MAR)(R_{jt} - MAR), \tag{12}$$

Where $S_{ij_{MAR}}$ represents the semicovariance between asset i and j in relation to the MAR and periods 1 to K are those in which the portfolio underperforms the MAR. Estrada (2008) notes that equation 11 provides an exact estimation of the portfolio semivariance. However, it implies an endogenous semicovariance matrix since one needs to know whether the portfolio performs below the MAR. Stocks' weights determine whether that scenario happens and, consequently, change the semicovariance matrix. To overcome this issue, we follow the heuristic approach proposed by Estrada (2008)[8], in which the semicovariance between assets i and j is defined according to expression 13.

$$\Sigma_{ij_{MAR}} = (1/T) \cdot \sum_{t=1}^{T} [\min(R_{it} - MAR, 0) \cdot \min(R_{jt} - MAR, 0)] \tag{13}$$

The heuristic proposed by Estrada (2008) enables an exogenous semicovariance matrix required for estimating portfolio semivariance and also ensures its symmetry. Thus, the portfolio semivariance can be approximated with the next expression.

[8]For further proposals see for instance Hogan and Warren (1972), Hogan and Warren (1974), Ang (1975), Bawa and Lindenberg (1977), Nawrocki (1983), Markowitz et al. (1993), Nawrocki (1991), De Athayde (2001), Huang et al. (2001), Ballestero (2005), and Cumova and Nawrocki (2011).

$$\Sigma^2_{p_{MAR}} = \sum_{i=1}^{n}\sum_{j=1}^{n} X_i X_j \Sigma_{ij_{MAR}} \tag{14}$$

We note that this heuristic implies that only assets that share returns below the MAR at the same time are included in equation 13. However, Estrada (2008) finds evidence that for a wide range of portfolios, the heuristic yields portfolio semivariances highly correlated to the *ex-post* portfolio semivariances. He also stresses that it is particularly accurate when portfolio optimization is performed to allocate funds across asset classes rather than between individual stocks.

Based on this heuristic approach, we compute semivariance regarding three different MAR. The first reflects the investor's concern with any loss of capital (MAR - 0). Secondly, we explore the scenario in which the investor seeks to get at least the risk-free rate (MAR - R_f). We recall the reader that we collect the rates from the ECB Euro Area Yield Curve for AAA Bonds for the same maturities as the investment periods, from 1997 to 2015[9]. Finally, the third MAR chosen is the stock market return (MAR - R_m). We use the STOXX Europe 600 index as representative of the European stock market. We collect data for the period identified above and compute the logarithmic daily returns.

Table VI reports the historical semideviations for each of the 16 stocks and respective MAR.

We obtain the downside correlation coefficient between two assets, dividing their semicovariance (equation 13) by the product of its semideviations, as follows.

$$\rho_{ij_{MAR}} = \frac{\Sigma_{ij_{MAR}}}{\Sigma_{i_{MAR}} \cdot \Sigma_{j_{MAR}}} \tag{15}$$

[9]Rates are only available from the last quarter of 2004 on. As the 15-year rates are not provided, we compute a proxy through linear interpolation using the 10-year and 20-year rates (Martellini et al., 2003).

Table VI
Historical semideviations

		Daimler	Banco Santander	Bayer	VINCI	Danone	Sanofi	Siemens	Allianz	Vivendi	Total	Unilever	Unibail-Rodamco	Carrefour	SAP	Deutsche Telekom	Iberdrola
PMPT 0	1 Year	25.2%	25.7%	22.8%	21.3%	17.6%	21.6%	25.3%	26.2%	27.2%	20.4%	17.8%	17.9%	22.6%	27.9%	25.3%	18.8%
	5 Years	25.9%	25.6%	23.6%	21.4%	17.6%	21.2%	26.1%	27.7%	29.5%	19.9%	17.6%	18.5%	22.6%	27.7%	25.6%	19.2%
	10 Years	25.6%	24.9%	23.1%	21.4%	17.4%	20.7%	25.6%	27.2%	28.0%	19.6%	17.4%	18.7%	22.1%	26.6%	24.5%	19.2%
	15 Years	25.6%	25.6%	23.2%	21.5%	17.7%	21.2%	25.8%	27.1%	28.6%	20.0%	17.7%	18.3%	22.7%	28.2%	25.7%	19.3%
	30 Years	25.2%	25.6%	22.7%	21.2%	17.6%	21.5%	25.2%	26.2%	27.1%	20.4%	17.7%	17.9%	22.5%	27.7%	25.2%	18.8%
PMPT Rf	1 Year	24.4%	24.5%	19.6%	21.5%	16.5%	18.4%	21.7%	22.9%	18.3%	18.5%	15.1%	19.3%	20.8%	17.6%	18.3%	20.5%
	5 Years	26.6%	27.0%	20.6%	23.5%	17.3%	19.0%	23.5%	25.4%	19.9%	19.2%	15.8%	20.4%	22.5%	18.4%	19.1%	22.7%
	10 Years	24.5%	24.6%	19.6%	21.6%	16.6%	18.4%	21.7%	23.0%	18.3%	18.6%	15.1%	19.3%	20.9%	17.6%	18.3%	20.6%
	15 Years	24.5%	24.6%	19.7%	21.6%	16.6%	18.4%	21.7%	23.0%	18.4%	18.6%	15.2%	19.3%	20.9%	17.6%	18.3%	20.6%
	30 Years	24.5%	24.6%	19.7%	21.6%	16.6%	18.4%	21.7%	23.0%	18.4%	18.6%	15.2%	19.3%	20.9%	17.6%	18.3%	20.6%
PMPT Rm	1 Year	17.6%	17.6%	17.5%	17.3%	15.6%	18.5%	17.4%	18.5%	22.1%	14.8%	15.5%	16.3%	17.6%	23.0%	20.4%	15.4%
	5 Years	17.9%	17.0%	18.2%	16.7%	15.6%	17.8%	17.6%	19.4%	23.8%	13.6%	15.6%	16.5%	17.4%	22.9%	20.5%	15.4%
	10 Years	17.6%	16.3%	17.8%	16.3%	15.4%	17.4%	17.2%	18.9%	22.4%	13.2%	15.2%	16.4%	17.0%	21.8%	19.7%	15.0%
	15 Years	17.9%	17.3%	17.9%	17.0%	15.9%	18.0%	17.6%	19.0%	23.1%	14.1%	15.9%	16.4%	17.8%	23.4%	20.8%	15.6%
	30 Years	17.6%	17.6%	17.4%	17.2%	15.6%	18.4%	17.3%	18.4%	22.0%	14.7%	15.4%	16.2%	17.5%	22.8%	20.3%	15.3%

PMPT correlation matrices lie in the appendix from table XX to XXII. The process to derive the EF is equivalent to that applied when using MPT, but instead of minimizing the portfolio standard deviation, the target is to minimize semideviation. The TP is the portfolio that maximizes the EF slope. The problem is formulated as follows.

$$\text{Max}_{x_1,x_2,\cdots,x_n} \quad \frac{E(R_p) - R_f}{\Sigma_{p_{MAR}}} \tag{16}$$

When deriving the EF without short-selling, we impose the additional restriction of $\sum_{i=1}^{n} X_i \geq 0$.

In PMPT, the Sortino ratio measures the performance of risk-adjusted returns (Rom and Ferguson, 1994). Developed by Frank Sortino in 1980, this ratio is equivalent to the Sharpe ratio used with MPT but incorporates semideviation instead of standard deviation and measures excess return in relation to the MAR, as expressed below.

$$\text{Sortino Ratio} = \frac{E(R_p) - MAR}{\Sigma_{p_{MAR}}} \tag{17}$$

3.2.3 Robustness Analysis

The robustness analysis intends to check whether the results obtained with the previous methodology are corroborated given a change in inputs. Thus, we repeat the analysis estimating inputs from equilibrium models.

We start by addressing the methodology to estimate MPT inputs. Sharpe (1964), Lintner (1965) and Mossin (1966) developed independently the Capital Asset Pricing Model (CAPM), based on MPT. The model enables determining the expected return of an asset given its beta through a linear relationship.

In the classical framework, the variance measures the risk of an asset (equation 2). In a diversified portfolio, covariance measures the asset's risk in relation to the market portfolio (σ_{im}). We obtain an asset's beta dividing its covariance with the market by the market variance (σ_m^2), as follows.

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} \tag{18}$$

Beta measures the sensitivity of an asset return to the market as a whole. It captures the systematic risk, i.e, the risk that cannot be eliminated by diversification. A positive beta indicates that an asset's return follows the overall market trend, while a negative beta shows an opposite trend to that of the market.

Thus, according to CAPM, an asset's expected return is given by the following expression.

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f] \tag{19}$$

We compute in-sample betas for both approaches since PMPT's betas follow their own methodology. We recall the reader that we consider that the STOXX Europe

600 index represents the European stock market portfolio. We repeat the process of dividing the whole data period into consecutive periods matching the investment horizons (when applicable) and calculate the annualized average returns. Table VII reports the annualized market returns.

Table VII
Annualized market return

1 Year	5 Years	10 Years	15 Years	30 Years
4.08%	0.64%	1.00%	1.56%	4.06%

Additionally, we calculate the STOXX Europe 600 index variance and its covariance with each of the 16 stocks. In this model, total risk is expressed as follows.

$$\sigma_i = \sqrt{\beta_i^2 \sigma_m^2 + \sigma_{e_i}^2} \tag{20}$$

e_i is the error term, the difference between expected returns and realized returns given non-market changes, as follows.

$$R_i - R_f = \beta_i (R_m - R_f) + e_i \tag{21}$$

Table XIII (appendix) portrays the betas, expected returns and standard deviations.[10]

The methodology to compute betas in PMPT is adjusted to the downside risk framework. We follow the proposal of Estrada (2006, 2007), in which an asset's downside beta is obtained dividing its semicovariance with the market portfolio ($\Sigma_{im_{MAR}}$) by

[10]The in-sample betas match the market betas regarding the STOXX Europe 600 index in all investment horizons.

the market's semivariance of returns ($\Sigma^2_{m_{MAR}}$), as expressed below.

$$\beta_i^D = \frac{\Sigma_{im_{MAR}}}{\Sigma^2_{m_{MAR}}} = \frac{\sum_{t=1}^{T}[\min(R_i - MAR, 0) \cdot \min(R_{m_t} - MAR_m, 0)]}{\sum_{t=1}^{T}[\min(R_{m_t} - MAR_m, 0)]^2} \tag{22}$$

According to Estrada (2007), downside betas can be integrated into an adjusted CAPM based on downside risk, which originates the following equation to compute expected returns.

$$E(R_i) = R_f + \beta_i^D[E(R_m) - R_f)] \tag{23}$$

This model merely replaces the classical beta by the downside beta, the measure of systematic risk in the downside risk framework.

We use the same MAR mentioned in Subsection 3.2.2. For the market MAR (MAR_m) in equation 22, we use the average daily market return for each time horizon. Table XIV (appendix) displays the downside betas, expected returns and semideviations for each time horizon and MAR. We then derive the MRP, minimizing the variance (equation 7) and semivariance (equation 14) for MPT and PMPT, respectively. To find the TP, we maximize equation 8 for MPT and equation 16 for PMPT. Recalling that,

$$\beta_p = \sum_{i=1}^{n} X_i \beta_i \tag{24}$$

$$E(R_p) = R_f + \beta_p[E(R_m) - R_f] \tag{25}$$

and

$$\sigma_p^2 = \beta_p^2 \sigma_m^2 + \sum_{i=1}^{n} x_i^2 \sigma_{e_i}^2 \tag{26}$$

We adjust equation 26 replacing σ_m^2 by Σ_m^2 and $\sigma_{e_i}^2$ by $\Sigma_{e_i}^2$ to compute $\Sigma^2_{p_{MAR}}$.

4 Results

This Chapter presents and discusses the results. Section 4.1 details the composition of MRP derived from MPT and PMPT, with and without short-selling restrictions. Section 4.2 replicates the same content for TP. Section 4.3 displays the MPT and PMPT's EF in all investment periods. Lastly, Section 4.4 presents the robustness analysis results.

4.1 Minimum Risk Portfolios

Table VIII details the composition of MPT and PMPT's MRP.

Table VIII
Minimum risk portfolios without short-selling

		Daimler	Banco Santander	Bayer	VINCI	Danone	Sanofi	Siemens	Allianz	Vivendi	Total	Unilever	Unibail-Rodamco	Carrefour	SAP	Deutsche Telekom	Iberdrola	$E(R_p)$	σ_p/Σ_p
1 Year	MPT	0%	0%	0%	5%	19%	7%	0%	0%	2%	3%	19%	26%	0%	2%	4%	14%	8.8%	18.3%
	PMPT 0	0%	0%	0%	0%	23%*	6%	0%	0%	0%	1%*	24%*	30%*	0%	0%*	0%*	16%*	9.2%*	14.3%
	PMPT Rf	0%	0%	0%	0%	14%*	5%	0%	0%	10%	0%*	38%*	2%*	0%	18%*	14%*	0%*	6.3%*	13.7%
	PMPT Rm	5%	8%*	2%	3%	8%*	1%	10%*	3%	3%	18%*	13%*	10%*	4%	0%*	2%*	11%*	6.2%*	10.0%
5 Years	MPT	0%	0%	0%	2%	20%	8%	0%	0%	0%	3%	22%	25%	0%	3%	5%	12%	6.0%	18.5%
	PMPT 0	0%	0%	0%	0%	24%*	8%	0%	0%	0%	2%*	26%*	27%*	0%	0%*	0%*	13%*	6.5%*	14.5%
	PMPT Rf	0%	0%	0%	0%	12%	7%	0%	0%	5%	0%*	40%*	1%*	0%	19%*	16%*	0%*	1.9%*	14.3%
	PMPT Rm	4%	10%*	3%	4%	6%*	4%	9%*	0%	2%	24%*	11%*	9%*	4%	0%*	1%*	9%*	3.6%*	9.8%
10 Years	MPT	0%	0%	0%	1%	20%	9%	0%	0%	0%	3%	23%	24%	0%	4%	6%	11%	6.0%	18.5%
	PMPT 0	0%	0%	0%	0%	24%*	9%	0%	0%	0%	3%*	26%*	25%*	0%	0%*	1%*	12%*	6.5%*	14.5%
	PMPT Rf	0%	0%	0%	0%	14%*	5%	0%	0%	9%	0%*	38%*	2%*	0%	18%*	14%*	0%*	2.0%*	13.7%
	PMPT Rm	3%	11%*	2%	4%	6%*	4%	9%*	0%	2%	24%*	12%*	8%*	4%	0%*	2%	9%*	3.9%*	9.5%
15 Years	MPT	0%	0%	0%	3%	19%	8%	0%	0%	0%	4%	22%	25%	0%	3%	5%	12%	6.6%	18.4%
	PMPT 0	0%	0%	0%	0%	23%*	7%	0%	0%	0%	3%*	26%*	28%*	0%	0%*	0%*	13%*	7.1%*	14.4%
	PMPT Rf	0%	0%	0%	0%	14%*	5%	0%	0%	9%	0%*	38%*	2%*	0%	18%*	14%*	0%*	2.7%*	13.7%
	PMPT Rm	4%	9%*	3%	4%	6%*	4%	10%*	1%	2%	22%*	11%*	10%*	4%	1%*	1%*	9%*	4.3%*	10.0%
30 Years	MPT	0%	0%	0%	4%	19%	7%	0%	0%	2%	3%	20%	26%	0%	2%	4%	14%	8.7%	18.3%
	PMPT 0	0%	0%	0%	0%	19%*	7%	0%	0%	0%	2%*	26%*	31%*	0%	0%*	0%*	17%*	9.0%*	14.3%
	PMPT Rf	0%	0%	0%	0%	14%*	5%	0%	0%	9%	0%*	38%*	2%*	0%	18%*	14%*	0%*	6.1%*	13.7%
	PMPT Rm	5%	8%*	4%	3%	8%*	3%	5%*	3%	3%	18%*	13%*	10%*	3%	0%*	2%*	10%*	6.1%*	10.0%

*Significant at p<0.05 - Paired t-tests of mean deviation from MPT's $E(R_p)$ and stocks' weights, realized for Banco Santander, Danone, Siemens, Total, Unilever, Unibail-Rodamco, SAP, Deutsche Telekom and Iberdrola.

In all investment periods, MPT and $PMPT_0$ produce similar MRP, with Unibail-Rodamco, Unilever, Danone and Iberdrola accounting on average for 78% of MPT's portfolios and 90% of $PMPT_0$. These stocks are the least risky, either measuring with variance or semivariance in relation to 0 (tables IV and VI, respectively). The four stocks display leptokurtosis and Iberdrola also has positive skewness (table II).

In PMPT$_{R_f}$'s MRP, Unilever presents an outstanding weight and it is followed by SAP, Danone, and Deutsche Telekom. SAP and Deutsche Telekom exhibit leptokurtosis and the latter has also positive skewness (table II). As pointed out by Markowitz (1959), Rom and Ferguson (1994), Swisher and Kasten (2005), and Cumova and Nawrocki (2011), semivariance gives rise to the investor's preference for positive skewness. Both stocks have low ADD in relation to the risk-free rate, with SAP displaying the lowest DF (table XV). Such features justify lesser semideviation regarding this MAR (table VI). Additionally, their lower correlation coefficients with the remaining stocks and especially between each other (table XXI) promote risk mitigation in the portfolio.

PMPT$_{R_m}$'s MRP are the most diversified once correlations between stocks reach the lowest values when semideviation is measured below the market return (table XXII). Total presents the largest weight in this portfolio as a result of its lowest ADD and semideviation regarding the market return (tables XV and VI, respectively).

Figure 3 compares the MRP's expected returns in all investment periods.

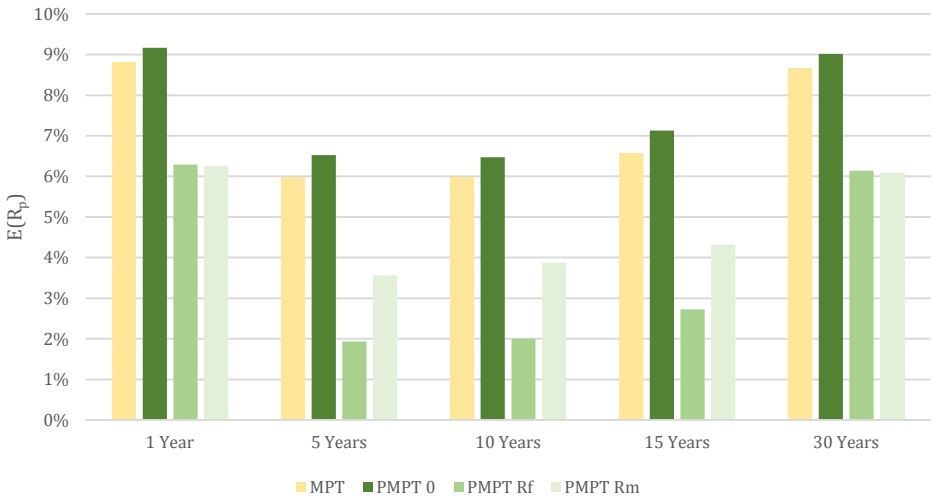


Figure 3 – Minimum risk portfolios' expected returns

Figure 3 reveals that PMPT$_0$'s MRP display in all investment periods the greatest expected returns, followed closely by MPT's MRP. Regarding risk among PMPT's

MRP, PMPT_{R_m} is the least risky given the inferior semideviations in relation to market return and the lowest correlation coefficients among stocks (table XXII), which favor risk mitigation.

The table below specifies the MRP's composition in all investment periods when short-selling is permitted.

Table IX
Minimum risk portfolios with short-selling

		Daimler	Banco Santander	Bayer	VINCI	Danone	Sanofi	Siemens	Allianz	Vivendi	Total	Unilever	Unibail-Rodamco	Carrefour	SAP	Deutsche Telekom	Iberdrola	$E(R_p)$	σ_p/Σ_p
1 Year	MPT	-6%	-11%	3%	7%	18%	7%	-1%	-7%	5%	7%	19%	27%	2%	5%	7%	18%	9.5%	17.7%
	PMPT 0	-10%	-13%	5%	3%	24%	8%	-3%	-10%	3%	9%	25%	32%	-1%	1%	5%	24%	10.4%*	13.7%
	PMPT Rf	-15%	-12%	-1%	-7%	16%	8%	-3%	-8%	19%	11%	37%	13%	-3%	26%	19%	0%	7.2%*	12.8%
	PMPT Rm	4%	8%	4%	3%	7%	3%	10%	3%	3%	17%	13%	10%	4%	-1%	1%	10%	6.3%*	10.0%
5 Years	MPT	-6%	-11%	4%	6%	18%	9%	-4%	-9%	2%	8%	21%	25%	3%	7%	9%	16%	7.0%	17.8%
	PMPT 0	-10%	-13%	5%	2%	24%	10%	-5%	-12%	1%	13%	26%	30%	0%	2%	7%	21%	8.3%*	13.7%
	PMPT Rf	-13%	-11%	1%	-14%	13%	9%	-3%	-10%	16%	15%	37%	15%	-3%	27%	21%	0%	2.7%*	13.1%
	PMPT Rm	4%	10%	2%	4%	7%	4%	10%	0%	2%	24%	11%	9%	4%	-1%	1%	9%	3.5%*	9.8%
10 Years	MPT	-6%	-10%	4%	6%	18%	9%	-5%	-9%	3%	9%	22%	25%	3%	7%	10%	15%	7.0%	17.7%
	PMPT 0	-10%	-12%	4%	1%	24%	11%	-7%	-12%	1%	13%	26%	28%	0%	4%	8%	20%	8.1%*	13.7%
	PMPT Rf	-15%	-12%	-2%	-8%	15%	8%	-2%	-8%	19%	11%	37%	13%	-3%	26%	19%	1%	2.8%*	12.9%
	PMPT Rm	3%	11%	2%	4%	6%	4%	9%	0%	2%	24%	12%	8%	4%	0%	2%	9%	3.9%*	9.5%
15 Years	MPT	-6%	-11%	4%	6%	18%	8%	-3%	-8%	3%	8%	21%	26%	3%	6%	8%	17%	7.4%	17.7%
	PMPT 0	-10%	-13%	5%	2%	23%	9%	-4%	-11%	1%	13%	26%	31%	0%	1%	6%	21%	8.7%*	13.7%
	PMPT Rf	-14%	-15%	-2%	-10%	15%	8%	-2%	-8%	19%	10%	37%	13%	-4%	26%	18%	10%	3.8%*	12.8%
	PMPT Rm	4%	9%	3%	4%	6%	4%	10%	1%	2%	22%	11%	10%	4%	-1%	1%	9%	4.2%*	10.0%
30 Years	MPT	-6%	-11%	3%	7%	18%	7%	-1%	-7%	5%	7%	20%	26%	2%	5%	7%	18%	9.4%	17.7%
	PMPT 0	-10%	-9%	5%	8%	27%	9%	-4%	-9%	2%	11%	28%	36%	0%	0%	6%	0%	11.0%*	14.0%
	PMPT Rf	-14%	-15%	-2%	-9%	15%	8%	-3%	-8%	19%	10%	37%	13%	-4%	26%	18%	10%	7.2%*	12.8%
	PMPT Rm	4%	8%	4%	3%	7%	3%	10%	3%	3%	18%	13%	10%	3%	-1%	1%	10%	6.2%*	9.9%

*Significant at $p<0.05$ - Paired t-test of mean deviation from MPT's $E(R_p)$.

Long positions follow an identical trend to that when MRP do not include short-selling, whereby we focus our analysis on the short positions. MPT, PMPT_0, and PMPT_{R_f} have in common short positions in Daimler, Banco Santander, Siemens and Allianz in all periods. PMPT_{R_f}'s MRP comprise additional short positions in Bayer, VINCI and, Carrefour. VINCI and Carrefour are among the stocks that have higher ADD with respect to the risk-free rate. The latter is also the stock with the largest DF below all MAR considered. PMPT_{R_m}'s MRP include only a short position in SAP which has the largest ADD concerning this MAR (table XV). Again, PMPT_0's MRP are the first in the ranking of expected returns, followed by MPT's MRP and PMPT_{R_m}'s MRP have the lowest risk among all PMPT's MRP.

4.2 Tangent Portfolios

Figure 4 illustrates stock selection in TP derived from MPT and PMPT with short-selling restrictions. Table X details the TP's composition.

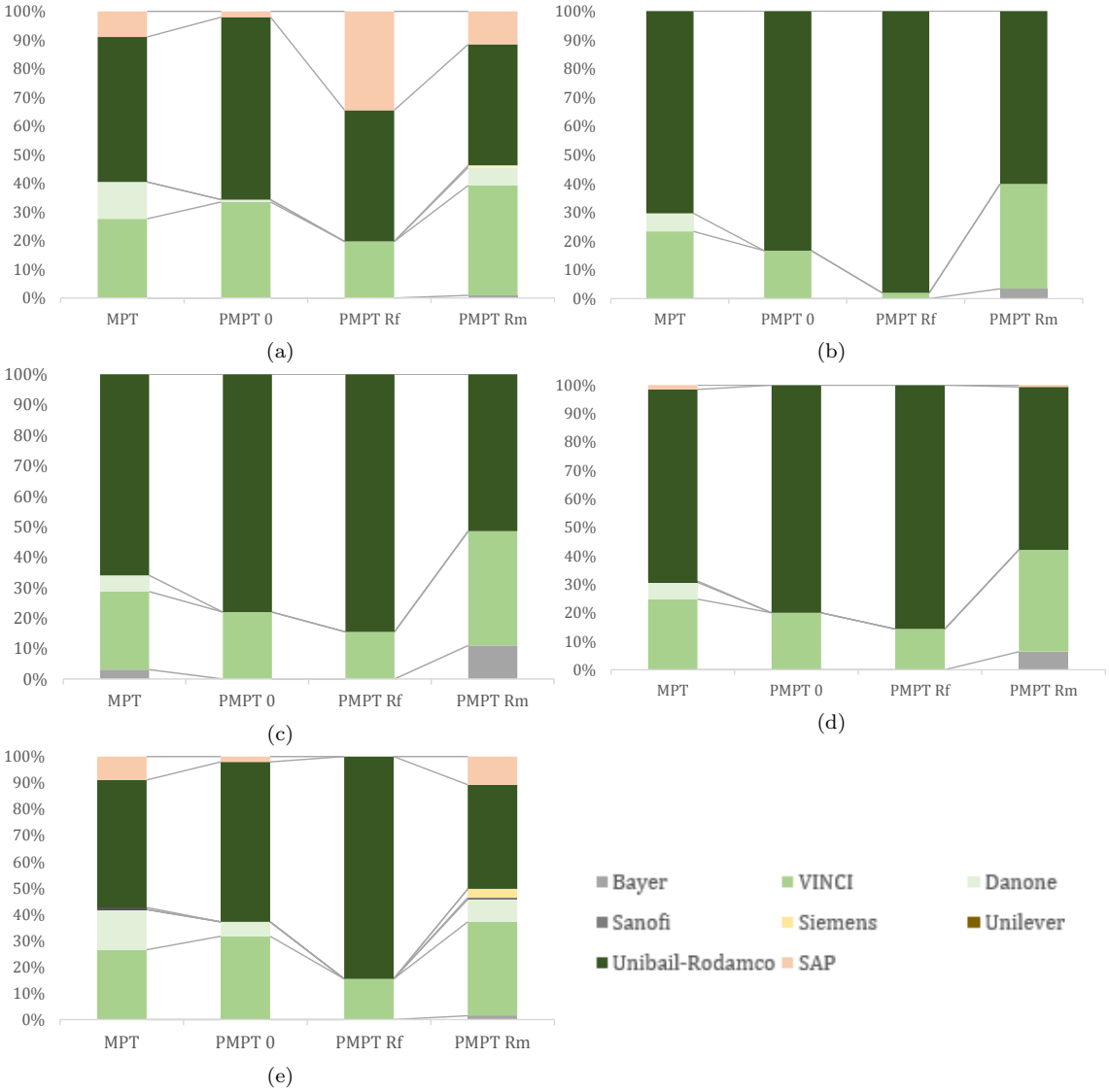


Figure 4 – Stock selection in tangent portfolios, 1 Year, 5 Years, 10 Years, 15 Years and 30 Years.

Table X
Tangent portfolios without short-selling

		Daimler	Banco Santander	Bayer	VINCI	Danone	Sanofi	Siemens	Allianz	Vivendi	Total	Unilever	Unibail-Rodamco	Carrefour	SAP	Deutsche Telekom	Iberdrola	$E(R_p)$	σ_p/Σ_p	Sharpe/ Sortino Ratio
1 Year	MPT	0%	0%	0%	26%	15%	0%	0%	0%	0%	0%	1%	49%	0%	9%	0%	0%	12.8%	20.8%	0.6444
	PMPT 0	0%	0%	0%	32%	6%*	0%	0%	0%	0%	0%	0%	61%*	0%	2%	0%	0%	13.5%*	16.5%	0.8203
	PMPT Rf	0%	0%	0%	19%*	0%*	0%	0%	0%	0%	0%	0%	43%	0%	38%	0%	0%	12.9%*	16.5%	0.8213
	PMPT Rm	0%	0%	1%*	36%*	9%*	1%	3%	0%	0%	0%	0%	40%*	0%	11%	0%	0%	12.9%	12.6%	0.6958
5 Years	MPT	0%	0%	0%	23%	6%	0%	0%	0%	0%	0%	0%	70%	0%	0%	0%	0%	12.2%	23.5%	0.5410
	PMPT 0	0%	0%	0%	17%	0%*	0%	0%	0%	0%	0%	0%	83%*	0%	0%	0%	0%	12.9%*	17.7%	0.7242
	PMPT Rf	0%	0%	0%	2%*	0%*	0%	0%	0%	0%	0%	0%	98%	0%	0%	0%	0%	13.2%*	20.3%	0.6750
	PMPT Rm	0%	0%	3%*	36%*	0%*	0%	0%	0%	0%	0%	0%	60%*	0%	0%	0%	0%	12.1%	13.9%	0.8290
10 Years	MPT	0%	0%	3%	26%	5%	0%	0%	0%	0%	0%	0%	66%	0%	0%	0%	0%	12.3%	23.4%	0.5320
	PMPT 0	0%	0%	0%	22%	0%*	0%	0%	0%	0%	0%	0%	78%*	0%	0%	0%	0%	13.0%*	17.8%	0.7302
	PMPT Rf	0%	0%	0%	15%*	0%*	0%	0%	0%	0%	0%	0%	85%	0%	0%	0%	0%	13.1%*	18.8%	0.7014
	PMPT Rm	0%	0%	11%*	38%*	0%*	0%	0%	0%	0%	0%	0%	52%*	0%	0%	0%	0%	12.1%	13.0%	0.8483
15 Years	MPT	0%	0%	0%	25%	6%	0%	0%	0%	0%	0%	0%	67%	0%	2%	0%	0%	12.3%	22.9%	0.5307
	PMPT 0	0%	0%	0%	20%	0%*	0%	0%	0%	0%	0%	0%	80%*	0%	0%	0%	0%	13.0%*	17.4%	0.7456
	PMPT Rf	0%	0%	0%	14%*	0%*	0%	0%	0%	0%	0%	0%	86%	0%	0%	0%	0%	13.1%*	18.8%	0.6864
	PMPT Rm	0%	0%	6%*	36%*	0%*	0%	0%	0%	0%	0%	0%	57%*	0%	1%	0%	0%	12.3%	13.6%	0.7852
30 Years	MPT	0%	0%	0%	28%	13%	0%	0%	0%	0%	0%	0%	51%	0%	9%	0%	0%	12.8%	21.1%	0.5838
	PMPT 0	0%	0%	0%	33%	1%*	0%	0%	0%	0%	0%	0%	64%*	0%	2%	0%	0%	13.6%*	16.8%	0.8117
	PMPT Rf	0%	0%	0%	20%*	0%*	0%	0%	0%	0%	0%	0%	46%	0%	34%	0%	0%	12.8%*	16.7%	0.7415
	PMPT Rm	0%	0%	1%*	38%*	6%*	0%	1%	0%	0%	0%	0%	42%*	0%	12%	0%	0%	13.1%	12.9%	0.6979

*Significant at p<0.05 - Paired t-tests of mean deviation from MPT's $E(R_p)$ and stocks' weights, realized for Bayer, VINCI, Danone, Unibail-Rodamco and SAP.

Unibail-Rodamco and VINCI dominate the portfolio allocation in both theories and regardless the MAR. Unibail-Rodamco has the strongest position in MPT, $PMPT_0$, and $PMPT_{R_f}$'s TP, while in $PMPT_{R_m}$'s TP, VINCI approximates Unibail-Rodamco's weight. VINCI's crescent allocation is explained by its positive skewness and leptokurtosis (table II). Danone presents the third highest weight in MPT's TP while in PMPT's TP, SAP or Bayer occupy that position (except in $PMPT_0$'s TP). As we have seen with MRP, SAP displays positive skewness and leptokurtosis, as well as low DF in relation to all MAR considered (table XV). Bayer also exhibits positive skewness and leptokurtosis (table II).

Figure 5 compares the TP's expected returns in all investment periods.

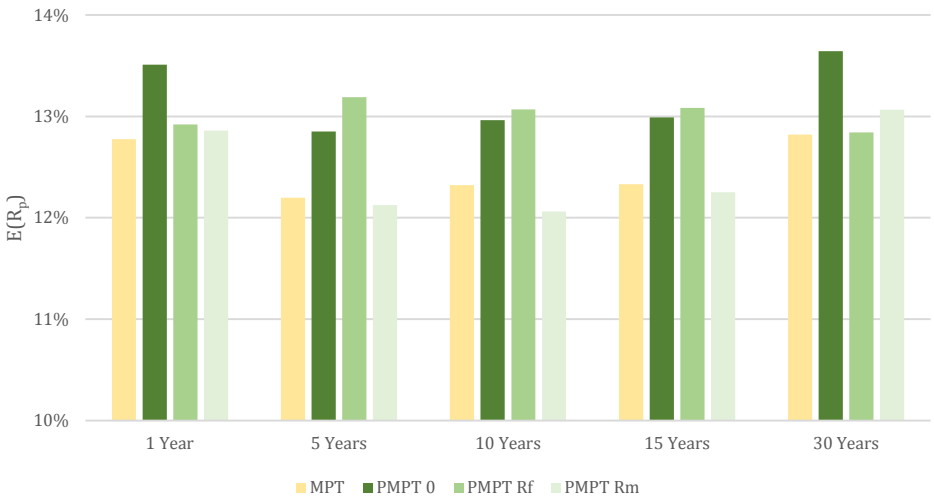


Figure 5 – Tangent portfolios' expected returns

Figure 5 shows that PMPT$_{R_f}$'s TP achieve in most investment periods (5, 10 and 15 years) the greatest expected return. In the remaining years, PMPT$_0$'s TP occupy that position. Similarly to MRP, PMPT$_{R_m}$'s TP offer the lowest semideviations among all PMPT's TP.

As the performance of risk-adjusted returns between MPT and PMPT's portfolios should not be compared, we analyse only the Sortino ratios among PMPT's TP. For the 1-year investment period, PMPT$_{R_f}$'s TP achieves the best performance and in the 30-year investment period is the PMPT$_0$'s TP. In the remaining periods, PMPT$_{R_m}$'s the TP performs above the others. The explanatory factors lie in the semideviation and respective MAR. In the 1-year investment period, SAP constitutes approximately 40% of the PMPT$_{R_f}$'s TP and presents low semideviation in relation to the risk-free rate (table VI). On the other hand, the 1-year risk-free has the lowest value, which favors the ratio numerator. In the 30-year period, the PMPT$_0$'s TP has the highest expected return but both the risk-free rate and the market return are considerably superior to zero, which provides an advantage over the other TP. In the remaining periods, Unibail-Rodamco and VINCI have marked weights in the

portfolios and these stocks' semideviation regarding market return is lower than for any other MAR (table VI), which triggers PMPT_{R_m}'s TP to perform above the other PMPT's TP.

Table XI contains the TP's composition including short-selling.

Table XI
Tangent portfolios with short-selling

		Daimler	Banco Santander	Bayer	VINCI	Danone	Sanofi	Siemens	Allianz	Vivendi	Total	Unilever	Unibail-Rodamco	Carrefour	SAP	Deutsche Telekom	Iberdrola	$E(R_p)$	σ_p/Σ_p	Sharpe/ Sortino Ratio
1 Year	MPT	-26%	-13%	12%	44%	28%	18%	14%	-20%	-18%	1%	16%	59%	-37%	25%	-10%	9%	24.0%	27.6%	0.8944
	PMPT 0	-41%	-17%	27%	65%	35%	19%	13%	-26%	-23%	-1%	20%	80%	-59%	31%	-23%	2%	31.8%*	23.4%	1.3589
	PMPT Rf	-77%	-17%	30%	161%	53%	41%	-4%	-60%	-97%	-10%	34%	102%	-105%	98%	-48%	0%	59.7%*	35.5%	1.7008
	PMPT Rm	-22%	5%	21%	57%	21%	12%	24%	-17%	-22%	9%	10%	56%	-50%	24%	-25%	-5%	25.9%*	19.6%	1.1166
5 Years	MPT	-26%	-9%	30%	52%	27%	22%	26%	-50%	-30%	7%	9%	86%	-54%	26%	-26%	8%	33.8%	38.0%	0.9015
	PMPT 0	-41%	-13%	55%	70%	31%	24%	25%	-61%	-29%	8%	10%	114%	-76%	31%	-44%	-3%	44.0%*	30.8%	1.4260
	PMPT Rf	-140%	-13%	152%	389%	103%	99%	-9%	-218%	-317%	36%	2%	345%	-222%	101%	-208%	0%	173.8%*	95.9%	1.8180
	PMPT Rm	-31%	10%	56%	89%	21%	15%	42%	-67%	-40%	19%	-3%	115%	-87%	29%	-62%	-8%	48.3%*	34.3%	1.3916
10 Years	MPT	-28%	-10%	44%	57%	31%	14%	24%	-52%	-26%	2%	7%	85%	-59%	26%	-32%	18%	35.2%	39.6%	0.8916
	PMPT 0	-50%	-23%	72%	73%	18%	4%	21%	-68%	-33%	-7%	1%	111%	0%	29%	-53%	5%	40.0%*	30.2%	1.3271
	PMPT Rf	-154%	2%	252%	443%	139%	46%	2%	-272%	-300%	-2%	8%	372%	-284%	102%	-256%	0%	191.9%*	104.2%	1.8422
	PMPT Rm	-32%	10%	71%	92%	24%	2%	36%	-63%	-33%	13%	-5%	104%	-86%	25%	-66%	8%	47.4%*	33.0%	1.4070
15 Years	MPT	-34%	-10%	31%	53%	25%	30%	29%	-41%	-31%	6%	7%	86%	-51%	25%	-27%	2%	33.3%	37.8%	0.8759
	PMPT 0	-53%	-13%	56%	74%	30%	34%	29%	-51%	-33%	7%	7%	115%	-74%	29%	-46%	-11%	44.0%*	31.1%	1.4147
	PMPT Rf	-155%	-15%	143%	337%	92%	120%	35%	-184%	-278%	10%	2%	290%	-220%	112%	-199%	11%	152.3%*	83.2%	1.8282
	PMPT Rm	-54%	1%	52%	87%	21%	27%	38%	0%	-44%	14%	-7%	109%	-84%	24%	-64%	-20%	43.6%*	32.3%	1.3020
30 Years	MPT	-28%	-14%	14%	49%	29%	19%	15%	-23%	-22%	-1%	16%	64%	-43%	27%	-12%	8%	25.9%	29.8%	0.8518
	PMPT 0	-45%	-18%	32%	73%	36%	20%	14%	-29%	-27%	-3%	19%	86%	-67%	34%	-26%	0%	34.4%*	25.4%	1.3558
	PMPT Rf	-86%	-25%	39%	186%	57%	46%	-3%	-70%	-119%	-16%	33%	119%	-124%	109%	-62%	17%	68.6%*	41.0%	1.6610
	PMPT Rm	-26%	3%	27%	68%	24%	13%	27%	-21%	-27%	7%	9%	66%	-60%	29%	-31%	-7%	29.9%*	22.6%	1.1435

*Significant at p<0.05 - Paired t-test of mean deviation from MPT's $E(R_p)$.

Concerning long positions, stock selection is in line with the results in the TP when short-selling is not permitted. The major difference is Siemens' outstanding position in PMPT_{R_m}'s TP. This stock exhibits leptokurtosis (table II). Both theories go short in roughly the same stocks: Daimler, Allianz, Vivendi, Carrefour, and Deutsche Telekom. PMPT_{R_m}'s TP do not incorporate a short position in Banco Santander but short Iberdrola (1 year) and Unilever (5, 10 and 15 years). Unilever exhibits negative skewness (table II).

Concerning the TP's features, MPT's TP present the lowest expected returns and PMPT_{R_f} the highest among all TP. PMPT_{R_m}'s TP presents the lowest semideviation in the 1-year and 30-year investment horizons while in the other periods is PMPT_0's TP. As for PMPT's TP performance, PMPT_{R_f}'s TP achieve the best result in all investment periods since their expected returns are much superior than the remaining.

4.3 Efficient Frontiers

Figure 6 plots the EF derived from MPT in all investment periods.

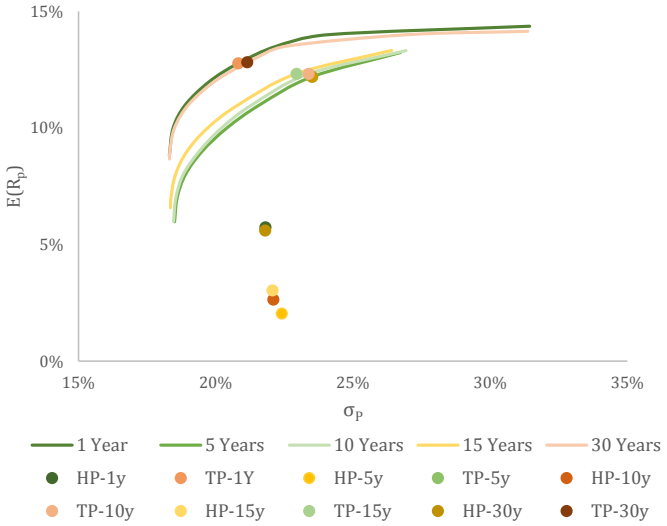


Figure 6 – MPT's efficient frontiers

This figure elucidates that the 1-year and 30-year EF have the most favorable risk-return combination of portfolios.

Figure 7 exhibits PMPT's EF considering all MAR and investment periods.

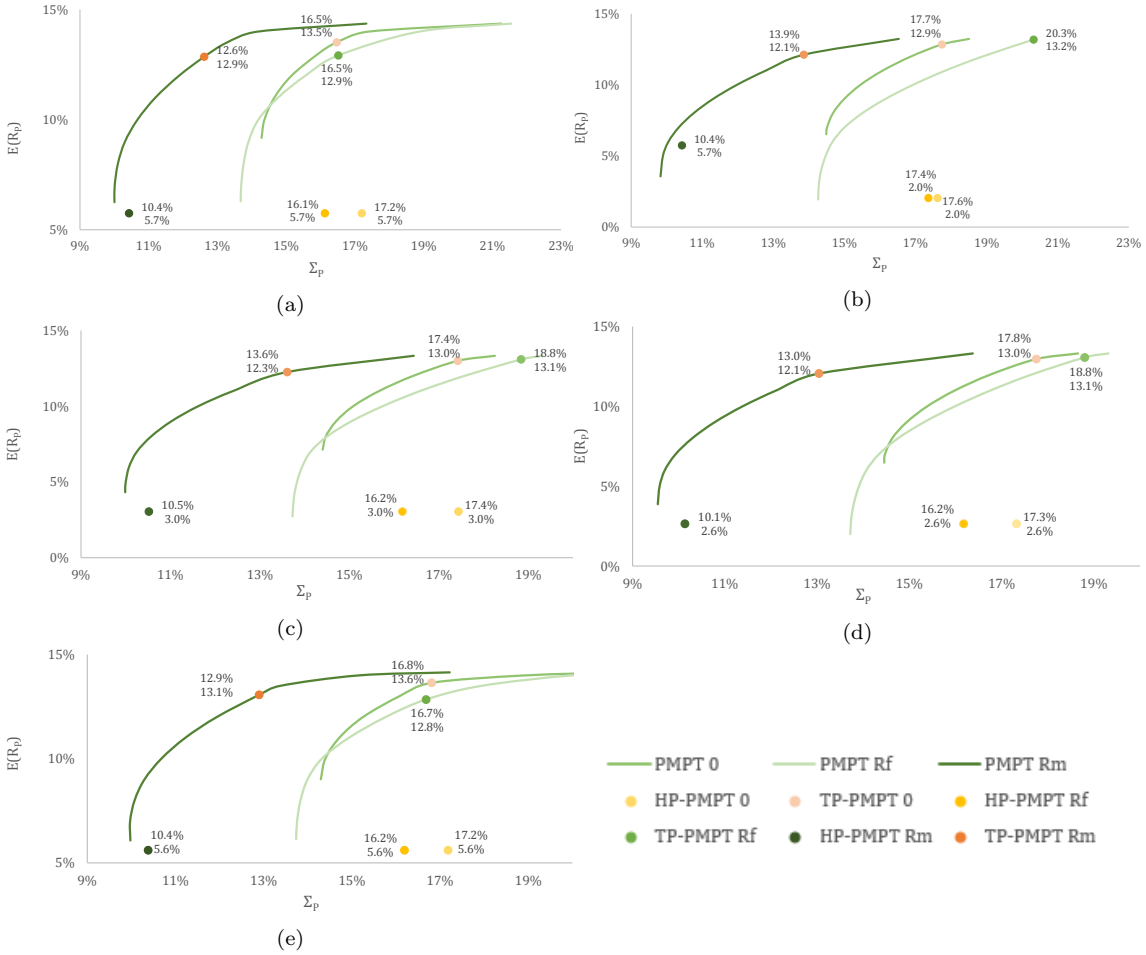


Figure 7 – PMPT's efficient frontiers, 1 Year, 5 Years, 10 Years, 15 Years and 30 Years.

As we have seen in Section 4.1 and 4.2, this figure illustrates that PMPT$_{R_m}$ present the least risky efficient portfolios since all the stocks have lower semideviation in relation to market return, except Vivendi, Unilever, SAP and Deutsche Telekom (table VI).

4.4 Robustness Analysis Results

Table XVI (appendix) exposes the MRP's composition when inputs are estimated from equilibrium models - CAPM for MPT and a CAPM-adjusted model to downside risk for PMPT. Results are in line with the analysis based on historical inputs, except that PMPT_{R_m}'s MRP have a similar composition to MPT and PMPT_0's MRP. The stocks that stand out are Unilever, Danone, Unibail-Rodamco, and Iberdrola. Regarding MPT's MRP with historical inputs, Unilever become the stock with the highest weight, followed by Danone. These stocks have the lowest semideviation (table XIV). PMPT's MRP note similarities, except PMPT_{R_f}'s MRP that give preference to SAP and Deutsche Telekom over Unibail-Rodamco and Iberdrola. Table XIV shows that SAP and Deutsche Telekom have low downside betas and are among the stocks with lower semideviation regarding the risk-free rate. Additionally, as we have noted in Section 4.1, both stocks display leptokurtosis and SAP is positively skewed (table II).

When we allow short-selling in MRP, long positions are similar to that when portfolios do not include short positions (table XVII - appendix). Both theories go short in Daimler, Banco Santander, Siemens, and Allianz. Only PMPT_{R_f}'s MRP include an additional short position in VINCI. We observe in table XIV that VINCI exhibits one of the highest downside betas and semideviation in relation to the risk-free rate.

Table XVIII (appendix) shows the TP's composition when inputs are estimated from equilibrium models. Unlike the analysis with historical inputs, all TP's are very diversified with no stocks exhibiting outstanding weights. In the former case, Unibail-Rodamco and VINCI account in both theories and with all MAR more than 60% of the portfolios in all periods. Both stocks combined register no more than 15% in this scenario. Given the balance among all stocks' weights, we do not note

substantial differences in the TP generated by both theories. When short-selling restrictions are imposed, we obtain the exactly same TP.

Concerning the portfolios' features, we highlight the significant lowest expected returns of MPT's MRP as lies in table XII. MPT's TP present the lowest expected returns as well, except in the 10-year and 15-year investment periods, in which all TP have the same expected returns. Regarding semideviation, in most years, PMPT_{R_f}'s MRP and TP have the lowest figures. PMPT_{R_f}'s TP achieves the best performance in the 1, 5 and 10-year investment periods while PMPT_0's TP performs above the others in the 15 and 30-year horizons.

Table XII
Paired t-tests of mean deviation from MPT's expected returns

	E(Rp) - MRP				**E(Rp) - TP**			
	MPT	PMPT_0	PMPT_{Rf}	PMPT_{Rm}	MPT	PMPT_0	PMPT_{Rf}	PMPT_{Rm}
1 year	3.0%	3.6%*	3.7%*	3.6%*	4.4%	4.8%*	4.7%	4.8%*
5 years	0.3%	0.5%*	0.4%*	0.5%*	0.7%	0.8%*	0.8%*	0.8%*
10 years	0.7%	0.8%*	0.8%*	0.8%*	1.1%	1.1%*	1.1%*	1.1%*
15 years	1.1%	1.3%*	1.4%*	1.3%*	1.7%	1.7%*	1.7%*	1.7%*
30 years	3.0%	3.6%*	3.6%*	3.6%*	4.3%	4.5%*	4.4%*	4.5%*

*Significant at p<0.05. The expected returns are from MRP and TP without short-selling.

The robustness analysis results confirm that differences in stock selection in MPT and PMPT's MRP are identical either estimating inputs from historical data or through equilibrium models (except for PMPT_{R_m}'s MRP). In the TP, we do not observe the same pattern. There are no substantial differences in the TP from both theories in all investment periods, which might indicate that results are sensitive to a change in inputs.

5 Conclusion

MPT is used worldwide by academics and practitioners in portfolio selection. PMPT appears as an alternative approach that measures downside risk. Under this framework, risk is perceived as failure to accomplish a pre-determined goal. Our main objective is to analyse the differences in portfolios produced by both theories. We find that although displaying similar stock selection trends, PMPT's portfolios favor in their composition stocks that display positive skewness and/or leptokurtosis, as well as stocks with low semideviation, mainly due to low DF and/or ADD. Additionally, in most cases, we observe that MPT portfolios achieve lower expected returns than PMPT portfolios. The robustness analysis results are coherent with the previous findings, particularly for MRP.

For further research, it would be of interest to perform a similar analysis with asset classes given that it is more common than allocating funds across individual stocks. Another suggestion would be to perform an *ex-post* analysis to PMPT's portfolios, namely to compare the realized returns with those from MPT.

References

Ang, J. S. (1975). A note on the E, SL portfolio selection model. *Journal of Financial and Quantitative Analysis 10*(05), 849–857.

Ballestero, E. (2005). Mean-semivariance efficient frontier: A downside risk model for portfolio selection. *Applied Mathematical Finance 12*(1), 1–15.

Bawa, V. S. and E. B. Lindenberg (1977). Capital market equilibrium in a mean-lower partial moment framework. *Journal of Financial Economics 5*(2), 189–200.

Cumova, D. and D. Nawrocki (2011). A symmetric LPM model for heuristic mean–semivariance analysis. *Journal of Economics and Business 63*(3), 217–236.

De Athayde, G. M. (2001). Building a mean-downside risk portfolio frontier. *Managing downside risk in financial markets: Theory, practice and implementation 1*, 194–211.

ECB (2016). Euro area yield curve. `https://www.ecb.europa.eu/stats/money/yc/html/index.en.html`. [Accessed on: 2016/8/2].

Estrada, J. (2006). Downside risk in practice. *Journal of Applied Corporate Finance 18*(1), 117–125.

Estrada, J. (2007). Mean-semivariance behavior: Downside risk and capital asset pricing. *International Review of Economics & Finance 16*(2), 169–185.

Estrada, J. (2008). Mean-semivariance optimization: A heuristic approach. *Journal of Applied Finance (Formerly Financial Practice and Education) 18*(1), 57–72.

Grootveld, H. and W. Hallerbach (1999). Variance vs downside risk: Is there really that much difference? *European Journal of operational research 114*(2), 304–319.

Harlow, W. V. (1991). Asset allocation in a downside-risk framework. *Financial Analysts Journal 47*(5), 28–40.

Hlawitschka, W. (1994). The empirical nature of taylor-series approximations to expected utility. *American Economic Review 84*(3), 713–719.

Hogan, W. W. and J. M. Warren (1972). Computation of the efficient boundary in the E-S portfolio selection model. *Journal of Financial and Quantitative Analysis 7*(04), 1881–1896.

Hogan, W. W. and J. M. Warren (1974). Toward the development of an equilibrium capital-market model based on semivariance. *Journal of Financial and Quantitative Analysis 9*(1), 1–11.

Huang, T., V. Srivastava, and S. Raatz (2001). Portfolio optimisation with options in the foreign exchange market. *Journal of Derivatives & Hedge Funds 7*(1), 55.

Kaplan, P. D. and L. B. Siegel (1994). Portfolio theory is alive and well. *The Journal of Investing 3*(3), 18–23.

Levy, H. and H. M. Markowitz (1979). Approximating expected utility by a function of mean and variance. *The American Economic Review 69*(3), 308–317.

Lintner, J. (1965). Security prices, risk, and maximal gains from diversification. *The Journal of Finance 20*(4), 587–615.

Markowitz, H., P. Todd, G. Xu, and Y. Yamane (1993). Computation of mean-semivariance efficient sets by the critical line algorithm. *Annals of Operations Research 45*(1), 307–317.

Markowitz, H. M. (1952). Portfolio selection. *The Journal of Finance 7*(1), 77–91.

Markowitz, H. M. (1959). Portfolio selection: efficient diversification of investments. *Cowies Foundation Monograph* (16).

Markowitz, H. M. (2014). Mean–variance approximations to expected utility. *European Journal of Operational Research 234*(2), 346–355.

Martellini, L., P. Priaulet, and S. Priaulet (2003). *Fixed-income securities: Valuation, Risk Management and Portfolio Strategies*. Chichester: John Wiley & Sons.

Mossin, J. (1966). Equilibrium in a capital asset market. *Econometrica: Journal of the Econometric Society*, 768–783.

Nawrocki, D. (1983). Comparison of risk measures when used in a simple portfolio selection heuristic. *Journal of Business Finance & Accounting 10*(2), 183–194.

Nawrocki, D. N. (1991). Optimal algorithms and lower partial moment: ex post results. *Applied Economics 23*(3), 465–470.

Rom, B. M. and K. W. Ferguson (1994). Post-modern portfolio theory comes of age. *The Journal of Investing 3*(3), 11–17.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The journal of finance 19*(3), 425–442.

Sharpe, W. F. (1966). Mutual fund performance. *The Journal of Business 39*(1), 119–138.

Simkowitz, M. A. and W. L. Beedles (1978). Diversification in a three-moment world. *Journal of Financial and Quantitative Analysis 13*(05), 927–941.

Sortino, F. A. and S. Satchell (2001). *Managing downside risk in financial markets*. Oxford: Butterworth-Heinemann.

Swisher, P. and G. W. Kasten (2005). Post-modern portfolio theory. *Journal of Financial Planning-Denver 18*(9), 74.

Vasant, J., L. Irgolic, R. Kruger, and K. Rajaratnam (2014). A comparison of mean-variance and mean-semivariance optimisation on the JSE. *Journal of Applied Business Research 30*(6), 1587.

Appendix

Table XIII
Betas, expected returns and volatilities - MPT

	Daimler	Banco Santander	Bayer	VINCI	Danone	Sanofi	Siemens	Allianz	Vivendi	Total	Unilever	Unibail-Rodamco	Carrefour	SAP	Deutsche Telekom	Iberdrola
β_{1y}	1.29	1.39	1.12	0.91	0.72	0.91	1.32	1.36	1.10	1.03	0.75	0.73	1.05	1.16	1.09	0.82
β_{5y}	1.33	1.37	1.04	0.91	0.64	0.83	1.36	1.42	1.19	0.99	0.66	0.64	0.99	1.10	1.06	0.78
β_{10y}	1.33	1.36	1.02	0.97	0.65	0.82	1.35	1.42	1.12	1.00	0.67	0.68	0.97	1.06	1.01	0.84
β_{15y}	1.31	1.37	1.02	0.92	0.64	0.82	1.36	1.40	1.15	0.98	0.65	0.64	0.98	1.12	1.07	0.80
β_{30y}	1.31	1.38	1.04	0.90	0.67	0.85	1.33	1.38	1.11	1.00	0.70	0.64	1.00	1.14	1.08	0.79
$E(R_i)_{1y}$	5.5%	5.9%	4.6%	3.7%	2.8%	3.7%	5.6%	5.8%	4.6%	4.2%	2.9%	2.8%	4.3%	4.9%	4.5%	3.2%
$E(R_i)_{5y}$	1.0%	1.1%	0.7%	0.5%	0.2%	0.4%	1.1%	1.1%	0.9%	0.6%	0.2%	0.2%	0.6%	0.8%	0.7%	0.4%
$E(R_i)_{10y}$	1.4%	1.4%	1.0%	1.0%	0.6%	0.8%	1.4%	1.5%	1.1%	1.0%	0.6%	0.6%	1.0%	1.1%	1.0%	0.8%
$E(R_i)_{15y}$	2.0%	2.1%	1.6%	1.5%	1.1%	1.3%	2.1%	2.1%	1.8%	1.5%	1.1%	1.1%	1.5%	1.7%	1.7%	1.3%
$E(R_i)_{30y}$	5.2%	5.4%	4.2%	3.7%	2.9%	3.5%	5.2%	5.4%	4.4%	4.1%	3.0%	2.8%	4.1%	4.6%	4.4%	3.3%
σ_{i1y}	34.2%	35.2%	30.4%	26.3%	23.6%	26.8%	32.4%	33.7%	28.9%	26.0%	22.3%	25.3%	30.7%	29.8%	29.3%	25.9%
σ_{i5y}	36.1%	36.6%	30.2%	26.1%	23.1%	26.6%	34.1%	36.4%	31.4%	25.5%	22.3%	24.9%	31.4%	29.8%	30.2%	26.8%
σ_{i10y}	34.8%	34.8%	29.0%	27.0%	22.8%	25.7%	33.1%	35.2%	29.5%	25.5%	21.5%	24.6%	29.6%	28.5%	28.7%	26.4%
σ_{i15y}	34.4%	34.9%	29.0%	26.9%	22.6%	25.7%	33.2%	34.7%	30.2%	25.2%	21.2%	24.6%	29.8%	29.8%	29.8%	26.4%
σ_{i30y}	34.1%	34.6%	29.0%	26.7%	22.8%	25.9%	32.3%	33.9%	29.1%	25.2%	21.7%	24.7%	29.8%	30.0%	29.7%	26.2%

Table XIV
Downside betas, expected returns and semideviations - PMPT

		Daimler	Banco Santander	Bayer	VINCI	Danone	Sanofi	Siemens	Allianz	Vivendi	Total	Unilever	Unibail-Rodamco	Carrefour	SAP	Deutsche Telekom	Iberdrola
PMPT 0	β_{1y}	1.39	1.41	1.19	1.05	0.82	0.99	1.34	1.40	1.19	1.10	0.84	0.83	1.16	1.25	1.14	0.86
	β_{5y}	1.39	1.38	1.14	1.02	0.75	0.92	1.39	1.48	1.28	1.05	0.76	0.78	1.08	1.17	1.13	0.86
	β_{10y}	1.39	1.37	1.11	1.07	0.76	0.89	1.38	1.47	1.21	1.05	0.76	0.81	1.06	1.13	1.06	0.91
	β_{15y}	1.38	1.39	1.12	1.04	0.76	0.91	1.39	1.46	1.24	1.05	0.75	0.78	1.08	1.20	1.13	0.88
	β_{30y}	1.38	1.41	1.13	1.03	0.78	0.93	1.38	1.43	1.19	1.07	0.79	0.77	1.09	1.22	1.13	0.87
	$E(R_i)_{1y}$	5.9%	6.0%	5.0%	4.3%	3.2%	4.1%	5.7%	6.0%	5.0%	4.5%	3.3%	3.3%	4.8%	5.2%	4.7%	3.4%
	$E(R_i)_{5y}$	1.1%	1.1%	0.8%	0.7%	0.3%	0.5%	1.1%	1.2%	1.0%	0.7%	0.4%	0.4%	0.7%	0.8%	0.8%	0.5%
	$E(R_i)_{10y}$	1.4%	1.4%	1.1%	1.1%	0.7%	0.9%	1.4%	1.5%	1.2%	1.1%	0.7%	0.8%	1.1%	1.1%	1.1%	0.9%
	$E(R_i)_{15y}$	2.1%	2.1%	1.7%	1.6%	1.2%	1.4%	2.1%	2.2%	1.9%	1.6%	1.2%	1.2%	1.7%	1.8%	1.7%	1.4%
	$E(R_i)_{30y}$	5.4%	5.5%	4.5%	4.2%	3.3%	3.8%	5.4%	5.6%	4.8%	4.3%	3.3%	3.2%	4.4%	4.9%	4.5%	3.6%
	Σ_{i1y}	29.5%	29.0%	26.2%	23.6%	22.0%	23.9%	26.7%	28.0%	25.1%	22.0%	20.3%	23.3%	27.4%	25.7%	25.5%	23.3%
	Σ_{i5y}	31.3%	31.0%	27.2%	23.6%	22.0%	24.4%	28.4%	30.8%	27.4%	21.8%	20.9%	24.0%	28.7%	26.1%	26.7%	24.9%
	Σ_{i10y}	29.8%	29.2%	26.1%	24.2%	21.6%	23.6%	27.5%	29.3%	25.7%	21.8%	20.0%	23.6%	27.1%	25.0%	25.3%	24.1%
	Σ_{i15y}	29.7%	29.4%	26.2%	24.4%	21.6%	23.8%	27.8%	29.3%	26.9%	21.8%	20.0%	23.4%	27.3%	26.5%	26.5%	24.2%
	Σ_{i30y}	29.5%	29.3%	26.1%	24.2%	21.6%	23.8%	27.3%	28.7%	26.0%	21.8%	20.2%	23.4%	27.3%	26.9%	26.4%	24.1%
PMPT Rf	β_{1y}	1.41	1.42	1.15	1.28	0.85	0.98	1.22	1.29	1.03	1.14	0.81	1.09	1.15	0.92	0.90	1.05
	β_{5y}	1.44	1.45	1.04	1.34	0.79	0.86	1.26	1.36	0.96	1.08	0.72	1.01	1.09	0.82	0.81	1.17
	β_{10y}	1.42	1.40	1.06	1.30	0.81	0.88	1.26	1.33	0.93	1.10	0.77	1.01	1.07	0.86	0.84	1.12
	β_{15y}	1.42	1.40	1.06	1.30	0.81	0.88	1.26	1.33	0.93	1.10	0.77	1.01	1.07	0.86	0.84	1.12
	β_{30y}	1.42	1.40	1.06	1.30	0.81	0.88	1.26	1.33	0.93	1.10	0.77	1.01	1.07	0.86	0.84	1.12
	$E(R_i)_{1y}$	6.0%	6.1%	4.8%	5.4%	3.4%	4.0%	5.1%	5.5%	4.2%	4.8%	3.2%	4.5%	4.8%	3.7%	3.6%	4.3%
	$E(R_i)_{5y}$	1.1%	1.2%	0.7%	1.0%	0.4%	0.5%	0.9%	1.1%	0.6%	0.7%	0.3%	0.6%	0.7%	0.4%	0.4%	0.8%
	$E(R_i)_{10y}$	1.5%	1.4%	1.1%	1.3%	0.8%	0.9%	1.3%	1.4%	0.9%	1.1%	0.7%	1.0%	1.1%	0.8%	0.8%	1.1%
	$E(R_i)_{15y}$	2.2%	2.1%	1.6%	2.0%	1.3%	1.4%	1.9%	2.0%	1.5%	1.7%	1.2%	1.6%	1.7%	1.4%	1.3%	1.7%
	$E(R_i)_{30y}$	5.6%	5.5%	4.3%	5.1%	3.4%	3.6%	5.0%	5.3%	3.8%	4.4%	3.2%	4.1%	4.3%	3.5%	3.5%	4.5%
	Σ_{i1y}	29.5%	28.9%	25.7%	25.6%	22.1%	23.6%	25.2%	26.7%	23.4%	22.3%	19.9%	25.4%	27.1%	22.6%	23.3%	24.6%
	Σ_{i5y}	32.1%	31.9%	26.4%	27.2%	22.5%	24.1%	27.3%	29.8%	24.4%	22.4%	20.8%	25.9%	28.9%	23.1%	24.2%	27.7%
	Σ_{i10y}	29.7%	29.1%	25.2%	26.1%	21.8%	23.3%	25.8%	27.5%	22.8%	22.0%	19.9%	24.9%	26.9%	22.3%	23.2%	25.7%
	Σ_{i15y}	29.7%	29.1%	25.3%	26.1%	21.8%	23.3%	25.8%	27.5%	22.8%	22.0%	19.9%	24.9%	26.9%	22.3%	23.2%	25.7%
	Σ_{i30y}	29.7%	29.1%	25.3%	26.1%	21.8%	23.3%	25.8%	27.5%	22.8%	22.0%	19.9%	24.9%	26.9%	22.3%	23.2%	25.7%
PMPT Rm	β_{1y}	1.38	1.40	1.19	1.04	0.82	1.00	1.34	1.39	1.19	1.10	0.84	0.83	1.15	1.24	1.14	0.87
	β_{5y}	1.39	1.38	1.14	1.02	0.75	0.92	1.39	1.48	1.29	1.05	0.76	0.78	1.08	1.18	1.13	0.86
	β_{10y}	1.39	1.37	1.11	1.07	0.76	0.89	1.38	1.47	1.21	1.05	0.76	0.81	1.06	1.13	1.06	0.91
	β_{15y}	1.38	1.39	1.12	1.04	0.76	0.91	1.39	1.46	1.24	1.05	0.76	0.78	1.08	1.20	1.13	0.88
	β_{30y}	1.38	1.41	1.13	1.03	0.78	0.93	1.37	1.43	1.19	1.07	0.79	0.77	1.09	1.22	1.13	0.87
	$E(R_i)_{1y}$	5.9%	6.0%	5.0%	4.3%	3.2%	4.1%	5.7%	5.9%	5.0%	4.5%	3.3%	3.3%	4.8%	5.2%	4.8%	3.5%
	$E(R_i)_{5y}$	1.1%	1.1%	0.8%	0.7%	0.3%	0.5%	1.1%	1.2%	1.0%	0.7%	0.4%	0.4%	0.7%	0.8%	0.8%	0.5%
	$E(R_i)_{10y}$	1.4%	1.4%	1.1%	1.1%	0.7%	0.9%	1.4%	1.5%	1.2%	1.1%	0.7%	0.8%	1.1%	1.1%	1.1%	0.9%
	$E(R_i)_{15y}$	2.1%	2.1%	1.7%	1.6%	1.2%	1.4%	2.1%	2.2%	1.9%	1.6%	1.2%	1.2%	1.7%	1.8%	1.7%	1.4%
	$E(R_i)_{30y}$	5.4%	5.5%	4.5%	4.2%	3.3%	3.8%	5.4%	5.6%	4.8%	4.3%	3.3%	3.2%	4.4%	4.9%	4.5%	3.6%
	Σ_{i1y}	30.9%	30.2%	27.9%	24.7%	22.9%	25.2%	27.6%	28.6%	25.7%	23.6%	21.6%	24.4%	28.5%	26.5%	27.0%	23.9%
	Σ_{i5y}	31.3%	31.0%	27.2%	23.6%	22.0%	24.4%	28.4%	30.8%	27.4%	21.8%	20.9%	24.0%	28.7%	26.1%	26.7%	24.9%
	Σ_{i10y}	29.8%	29.2%	26.1%	24.2%	21.6%	23.6%	27.5%	29.3%	25.7%	21.8%	20.1%	23.6%	27.1%	25.0%	25.3%	24.1%
	Σ_{i15y}	29.8%	29.4%	26.2%	24.4%	21.6%	23.8%	27.8%	29.3%	27.0%	21.8%	20.0%	23.4%	27.3%	26.6%	26.5%	24.2%
	Σ_{i30y}	29.6%	29.4%	26.2%	24.2%	21.7%	23.8%	27.3%	28.8%	26.1%	21.8%	20.3%	23.4%	27.3%	27.0%	26.5%	24.1%

Table XV
Downside risk components

MAR		Daimler	Banco Santander	Bayer	VINCI	Danone	Sanofi	Siemens	Allianz	Vivendi	Total	Unilever	Unibail-Rodamco	Carrefour	SAP	Deutsche Telekom	Iberdrola
0	DF	49.1%	47.1%	49.2%	47.8%	49.0%	48.2%	48.8%	49.0%	48.6%	47.7%	48.2%	46.7%	50.1%	47.9%	49.3%	47.0%
	ADD	-1.64%	-1.66%	-1.44%	-1.40%	-1.14%	-1.42%	-1.59%	-1.62%	-1.54%	-1.36%	-1.13%	-1.20%	-1.46%	-1.67%	-1.55%	-1.20%
	DM	-0.03%	-0.07%	-0.03%	-0.04%	-0.03%	-0.05%	-0.03%	-0.03%	-0.04%	-0.03%	-0.04%	-0.03%	-0.03%	-0.02%	-0.04%	-0.04%
Rf	DF_{1y}	48.7%	49.4%	48.9%	49.5%	50.0%	49.2%	49.4%	48.7%	49.2%	48.3%	49.7%	49.2%	50.9%	48.0%	50.6%	49.2%
	DF_{5y}	48.8%	49.5%	49.1%	49.7%	50.0%	49.2%	49.5%	48.9%	49.4%	48.5%	49.7%	49.2%	50.9%	48.2%	50.7%	49.4%
	DF_{10y}	48.8%	49.5%	49.1%	49.7%	50.1%	49.2%	49.6%	48.9%	49.5%	48.5%	49.7%	49.3%	50.9%	48.3%	50.7%	49.4%
	DF_{15y}	48.8%	49.5%	49.1%	49.7%	50.1%	49.2%	49.6%	48.9%	49.5%	48.5%	49.7%	49.3%	51.0%	48.3%	50.7%	49.4%
	DF_{30y}	48.8%	49.5%	49.1%	49.8%	50.1%	49.2%	49.6%	48.9%	49.5%	48.5%	49.7%	49.3%	51.0%	48.3%	50.7%	49.4%
	ADD_{1y}	-1.55%	-1.52%	-1.27%	-1.36%	-1.05%	-1.17%	-1.29%	-1.38%	-1.19%	-1.20%	-0.95%	-1.25%	-1.32%	-1.08%	-1.10%	-1.21%
	ADD_{5y}	-1.56%	-1.52%	-1.27%	-1.36%	-1.05%	-1.17%	-1.29%	-1.38%	-1.19%	-1.20%	-0.95%	-1.25%	-1.32%	-1.08%	-1.10%	-1.21%
	ADD_{10y}	-1.56%	-1.52%	-1.27%	-1.36%	-1.05%	-1.17%	-1.30%	-1.38%	-1.19%	-1.20%	-0.95%	-1.26%	-1.32%	-1.08%	-1.10%	-1.21%
	ADD_{15y}	-1.56%	-1.52%	-1.27%	-1.36%	-1.05%	-1.17%	-1.30%	-1.38%	-1.19%	-1.20%	-0.95%	-1.26%	-1.32%	-1.08%	-1.10%	-1.21%
	ADD_{30y}	-1.56%	-1.52%	-1.27%	-1.36%	-1.05%	-1.17%	-1.30%	-1.38%	-1.19%	-1.20%	-0.96%	-1.26%	-1.32%	-1.08%	-1.10%	-1.22%
	DM_{1y}	-0.02%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	0.00%
	DM_{5y}	-0.02%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.02%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%
	DM_{10y}	-0.02%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.02%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%
	DM_{15y}	-0.02%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.02%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%
	DM_{30y}	-0.02%	-0.01%	-0.01%	-0.01%	-0.01%	-0.02%	-0.02%	-0.01%	-0.01%	-0.02%	-0.01%	-0.01%	-0.02%	-0.01%	-0.01%	-0.01%
Rm	DF	50.8%	49.3%	50.2%	50.5%	50.4%	49.7%	50.3%	50.5%	51.3%	50.7%	49.1%	48.7%	53.0%	49.4%	51.5%	50.0%
	ADD	-1.12%	-1.09%	-1.06%	-1.10%	-0.97%	-1.11%	-1.03%	-1.09%	-1.11%	-0.91%	-0.93%	-1.09%	-1.06%	-1.29%	-1.18%	-0.97%
	DM	-0.01%	-0.02%	-0.02%	-0.01%	-0.01%	-0.02%	-0.01%	-0.02%	-0.01%	-0.01%	-0.01%	-0.02%	-0.01%	-0.01%	-0.01%	-0.01%

Table XVI
Minimum risk portfolios without short-selling (robustness analysis)

		Daimler	Banco Santander	Bayer	VINCI	Danone	Sanofi	Siemens	Allianz	Vivendi	Total	Unilever	Unibail-Rodamco	Carrefour	SAP	Deutsche Telekom	Iberdrola	$E(R_p)$	σ_p/Σ_p
1 Year	MPT	0%	0%	0%	8%	24%	7%	0%	0%	0%	0%	28%	19%	0%	0%	0%	14%	3.0%	17.5%
	PMPT 0	0%	0%	0%	7%	19%	9%	0%	0%	0%	6%	24%	16%	1%	0%	2%	15%	3.6%	14.8%
	PMPT Rf	0%	0%	1%	0%	17%	10%	0%	0%	8%	2%	25%	4%	1%	14%	13%	6%	3.7%	14.6%
	PMPT Rm	0%	0%	1%	8%	18%*	9%	0%*	0%	1%	7%*	21%*	15%*	2%	0%	3%	15%*	3.6%	14.9%
5 Years	MPT	0%	0%	0%	4%	26%	9%	0%	0%	0%	0%	28%	21%	0%	0%	0%	11%	0.3%	16.8%
	PMPT 0	0%	0%	0%	7%	20%	10%	0%	0%	0%	8%	23%	15%	2%	0%	1%	12%	0.5%	14.5%
	PMPT Rf	0%	0%	3%	0%	17%	12%	0%	0%	8%	3%	24%	5%	1%	15%	13%	0%	0.4%	14.5%
	PMPT Rm	0%	0%	0%	7%	20%*	10%	0%*	0%	0%	8%*	23%*	15%*	2%	0%	1%	12%*	0.5%	14.5%
10 Years	MPT	0%	0%	0%	0%	27%	11%	0%	0%	0%	0%	32%	20%	0%	0%	0%	10%	0.7%	16.8%
	PMPT 0	0%	0%	1%	4%	20%	12%	0%	0%	0%	7%	25%	14%	3%	1%	3%	10%	0.8%	14.5%
	PMPT Rf	0%	0%	3%	0%	16%	11%	0%	0%	10%	2%	23%	5%	2%	14%	13%	0%	0.8%	14.0%
	PMPT Rm	0%	0%	1%	4%	20%*	12%	0%*	0%	0%	7%*	25%*	15%*	3%	1%	3%	10%*	0.8%	14.5%
15 Years	MPT	0%	0%	0%	3%	26%	10%	0%	0%	0%	0%	31%	20%	0%	0%	0%	10%	1.1%	16.4%
	PMPT 0	0%	0%	1%	5%	20%	11%	0%	0%	0%	7%	25%	16%	2%	0%	1%	11%	1.3%	14.3%
	PMPT Rf	0%	0%	3%	0%	16%	11%	0%	0%	10%	2%	23%	5%	2%	14%	13%	0%	1.4%	14.0%
	PMPT Rm	0%	0%	1%	5%	20%*	11%	0%*	0%	0%	7%*	25%*	16%*	2%	0%	1%	11%*	1.3%	14.4%
30 Years	MPT	0%	0%	0%	5%	25%	9%	0%	0%	0%	0%	28%	21%	0%	0%	0%	12%	3.0%	16.6%
	PMPT 0	0%	0%	1%	6%	20%	11%	0%	0%	0%	7%	23%	16%	2%	0%	1%	12%	3.6%	14.3%
	PMPT Rf	0%	0%	3%	0%	17%	12%	0%	0%	10%	3%	23%	5%	2%	13%	12%	1%	3.6%	14.1%
	PMPT Rm	0%	0%	1%	6%	20%*	11%	0%*	0%	0%	7%*	23%*	16%*	2%	0%	1%	12%*	3.6%	14.4%

*Significant at p<0.05 - Paired t-test of mean deviation from $PMPT_{Rm}$ stocks' weights in MRP with historical inputs, realized for Danone, Siemens, Total, Unilever, Unibail-Rodamco and Iberdrola.

Table XVII
Minimum risk portfolios with short-selling (robustness analysis)

		Daimler	Banco Santander	Bayer	VINCI	Danone	Sanofi	Siemens	Allianz	Vivendi	Total	Unilever	Unibail-Rodamco	Carrefour	SAP	Deutsche Telekom	Iberdrola	$E(R_p)$	σ_p/Σ_p
1 Year	MPT	-7%	-12%	1%	14%	27%	13%	-12%	-12%	2%	9%	32%	22%	4%	-2%	3%	18%	2.1%	16.2%
	PMPT 0	-5%	-7%	2%	10%	21%	11%	-5%	-7%	3%	10%	25%	17%	3%	0%	4%	16%	3.2%*	14.4%
	PMPT Rf	-7%	-8%	3%	-3%	18%	11%	0%	-3%	10%	6%	27%	5%	3%	15%	14%	8%	3.3%*	14.4%
	PMPT Rm	-4%	-5%	3%	10%	19%	10%	-4%	-5%	3%	9%	23%	16%	3%	1%	4%	16%	3.4%*	14.7%
5 Years	MPT	-8%	-10%	4%	13%	27%	14%	-13%	-13%	-4%	11%	30%	22%	5%	1%	3%	15%	0.0%	15.2%
	PMPT 0	-4%	-4%	3%	10%	22%	12%	-7%	-8%	-2%	13%	25%	17%	4%	3%	4%	13%	0.3%*	14.1%
	PMPT Rf	-7%	-7%	6%	-6%	19%	14%	-2%	-5%	11%	9%	25%	7%	4%	17%	15%	1%	0.3%*	14.0%
	PMPT Rm	-4%	-4%	3%	10%	22%	12%	-6%	-8%	-2%	13%	25%	17%	4%	3%	4%	13%	0.3%*	14.1%
10 Years	MPT	-9%	-11%	5%	9%	27%	16%	-14%	-16%	0%	10%	33%	21%	6%	3%	5%	14%	0.4%	15.0%
	PMPT 0	-6%	-5%	4%	7%	21%	13%	-7%	-10%	1%	11%	26%	16%	5%	4%	6%	12%	0.7%*	14.0%
	PMPT Rf	-7%	-7%	6%	-4%	18%	13%	-2%	-5%	12%	7%	25%	7%	4%	15%	14%	3%	0.7%*	13.7%
	PMPT Rm	-6%	-5%	4%	7%	21%	13%	-7%	-10%	1%	11%	26%	16%	5%	4%	6%	12%	0.7%*	14.0%
15 Years	MPT	-8%	-11%	5%	10%	27%	15%	-13%	-14%	-1%	12%	33%	21%	6%	0%	2%	15%	0.8%	14.8%
	PMPT 0	-5%	-5%	4%	8%	22%	13%	-7%	-9%	0%	12%	27%	17%	5%	1%	4%	13%	1.2%*	13.9%
	PMPT Rf	-7%	-7%	6%	-4%	18%	13%	-2%	-5%	12%	7%	25%	7%	4%	15%	14%	3%	1.2%*	13.7%
	PMPT Rm	-5%	-5%	4%	8%	22%	13%	-7%	-9%	0%	12%	27%	17%	5%	1%	4%	13%	1.2%*	13.9%
30 Years	MPT	-8%	-11%	4%	12%	27%	15%	-12%	-13%	1%	11%	31%	22%	5%	-1%	2%	15%	2.3%	15.1%
	PMPT 0	-5%	-6%	4%	9%	21%	13%	-6%	-8%	2%	11%	25%	17%	5%	1%	4%	14%	3.2%*	14.0%
	PMPT Rf	-7%	-7%	6%	-4%	18%	13%	-2%	-5%	12%	7%	25%	7%	4%	15%	14%	3%	3.2%*	13.7%
	PMPT Rm	-5%	-6%	4%	9%	21%	13%	-6%	-8%	2%	11%	25%	18%	5%	1%	4%	14%	3.2%*	14.0%

*Significant at p<0.05 - Paired t-test of mean deviation from MPT's $E(R_p)$.

Table XVIII
Tangent portfolios (robustness analysis)

		Daimler	Banco Santander	Bayer	VINCI	Danone	Sanofi	Siemens	Allianz	Vivendi	Total	Unilever	Unibail-Rodamco	Carrefour	SAP	Deutsche Telekom	Iberdrola	$E(R_p)$	σ_p/Σ_p	Sharpe/ Sortino Ratio
1 Year	MPT	6%	7%	6%	6%*	5%	5%	8%	8%	7%	9%	6%	4%*	5%	8%	6%	5%	4.4%	22.0%	0.2318
	PMPT 0	6%	7%	6%	6%*	5%	5%	8%	7%	7%	9%	6%	4%*	5%	7%	6%	4%	4.8%	17.1%	0.2782
	PMPT Rf	6%	7%	6%	8%	5%	5%	7%	7%	6%	10%	6%	6%*	5%	5%	5%	6%	4.7%	16.6%	0.3198
	PMPT Rm	6%	6%	6%	6%*	5%	5%	8%	8%	8%	8%	6%	4%*	5%	8%	6%	5%	4.8%	17.1%	0.0394
5 Years	MPT	6%	6%	6%	7%*	4%	5%	9%	8%	8%	10%	5%	4%*	4%	7%	6%	4%	0.7%	22.4%	0.0550
	PMPT 0	6%	6%	6%	7%*	5%	5%	8%	7%	8%	10%	6%	4%*	4%	7%	6%	4%	0.8%	17.4%	0.0452
	PMPT Rf	6%	6%	5%	10%	5%	5%	8%	7%	6%	11%	5%	5%*	5%	5%	4%	6%	0.8%	17.4%	0.0738
	PMPT Rm	6%	6%	6%	7%*	5%	5%	8%	7%	8%	10%	6%	4%*	4%	7%	6%	4%	0.8%	17.4%	0.0085
10 Years	MPT	6%	7%	6%	6%*	4%	5%	9%	8%	7%	10%	6%	4%*	4%	7%	6%	5%	1.1%	22.1%	0.0536
	PMPT 0	6%	7%	6%	7%*	5%	5%	9%	8%	7%	9%	6%	4%*	5%	7%	6%	5%	1.1%	17.1%	0.0659
	PMPT Rf	6%	7%	6%	8%	5%	5%	8%	7%	6%	10%	6%	5%*	5%	5%	5%	6%	1.1%	16.4%	0.0752
	PMPT Rm	6%	7%	6%	7%*	5%	5%	9%	8%	7%	9%	6%	4%*	5%	7%	6%	5%	1.1%	17.2%	0.0079
15 Years	MPT	6%	7%	6%	6%*	4%	5%	9%	8%	7%	10%	6%	3%*	5%	7%	6%	4%	1.7%	22.1%	0.0680
	PMPT 0	6%	7%	6%	6%*	5%	5%	9%	8%	7%	10%	6%	4%*	5%	7%	6%	5%	1.7%	17.3%	0.1011
	PMPT Rf	6%	7%	6%	8%	5%	5%	8%	7%	6%	10%	6%	5%*	5%	5%	5%	6%	1.7%	16.4%	0.0949
	PMPT Rm	6%	7%	6%	6%*	5%	5%	9%	8%	7%	10%	6%	4%*	5%	7%	6%	5%	1.7%	17.3%	0.0106
30 Years	MPT	6%	7%	6%	5%*	5%	5%	9%	8%	7%	10%	6%	3%*	5%	7%	6%	4%	4.3%	21.7%	0.1759
	PMPT 0	6%	7%	6%	6%*	5%	5%	8%	8%	7%	10%	6%	4%*	5%	6%	6%	4%	4.5%	17.0%	0.2670
	PMPT Rf	6%	7%	6%	8%	5%	5%	8%	7%	6%	10%	6%	5%*	5%	5%	5%	6%	4.4%	16.4%	0.2413
	PMPT Rm	6%	7%	6%	6%*	5%	5%	8%	8%	7%	10%	6%	4%*	5%	6%	6%	4%	4.5%	17.1%	0.0275

*Significant at p<0.05 - Paired t-test of mean deviation from MRP stocks' weights with historical inputs, realized for VINCI and Unibail-Rodamco.

Table XIX
Correlation matrices - MPT

		Daimler	Banco Santander	Bayer	VINCI	Danone	Sanofi	Siemens	Allianz	Vivendi	Total	Unilever	Unibail-Rodamco	Carrefour	SAP	Deutsche Telekom	Iberdrola
1 Year	Daimler	1															
	Banco Santander	0.55*	1														
	Bayer	0.54*	0.50*	1													
	VINCI	0.42*	0.45*	0.40*	1												
	Danone	0.43*	0.43*	0.44*	0.36*	1											
	Sanofi	0.42*	0.44*	0.45*	0.35*	0.42*	1										
	Siemens	0.60*	0.58*	0.53*	0.43*	0.42*	0.42*	1									
	Allianz	0.60*	0.61*	0.54*	0.43*	0.44*	0.44*	0.61*	1								
	Vivendi	0.47*	0.52*	0.41*	0.39*	0.38*	0.39*	0.50*	0.50*	1							
	Total	0.52*	0.54*	0.49*	0.44*	0.46*	0.45*	0.53*	0.53*	0.47*	1						
	Unilever	0.43*	0.42*	0.44*	0.34*	0.54*	0.42*	0.41*	0.44*	0.37*	0.44*	1					
	Unibail-Rodamco	0.33*	0.38*	0.32*	0.36*	0.31*	0.30*	0.35*	0.38*	0.32*	0.37*	0.31*	1				
	Carrefour	0.48*	0.52*	0.43*	0.40*	0.46*	0.43*	0.48*	0.49*	0.45*	0.47*	0.43*	0.32*	1			
	SAP	0.48*	0.45*	0.44*	0.34*	0.37*	0.36*	0.55*	0.49*	0.41*	0.42*	0.36*	0.30*	0.40*	1		
	Deutsche Telekom	0.46*	0.49*	0.45*	0.33*	0.36*	0.38*	0.51*	0.51*	0.49*	0.42*	0.38*	0.27*	0.39*	0.42*	1	
	Iberdrola	0.43*	0.56*	0.40*	0.39*	0.37*	0.36*	0.43*	0.47*	0.41*	0.44*	0.40*	0.33*	0.41*	0.35*	0.40*	1
5 Years	Daimler	1															
	Banco Santander	0.60*	1														
	Bayer	0.55*	0.51*	1													
	VINCI	0.47*	0.48*	0.41*	1												
	Danone	0.41*	0.41*	0.42*	0.37*	1											
	Sanofi	0.44*	0.45*	0.45*	0.37*	0.43*	1										
	Siemens	0.66*	0.62*	0.55*	0.48*	0.4*	0.43*	1									
	Allianz	0.65*	0.65*	0.55*	0.48*	0.4*	0.45*	0.65*	1								
	Vivendi	0.49*	0.55*	0.43*	0.41*	0.35*	0.42*	0.52*	0.53*	1							
	Total	0.56*	0.58*	0.52*	0.48*	0.46*	0.48*	0.57*	0.56*	0.49*	1						
	Unilever	0.42*	0.42*	0.42*	0.36*	0.54*	0.45*	0.40*	0.41*	0.37*	0.47*	1					
	Unibail-Rodamco	0.35*	0.39*	0.30*	0.39*	0.30*	0.28*	0.36*	0.37*	0.33*	0.36*	0.30*	1				
	Carrefour	0.50*	0.54*	0.43*	0.42*	0.46*	0.46*	0.49*	0.50*	0.48*	0.51*	0.43*	0.31*	1			
	SAP	0.51*	0.45*	0.43*	0.36*	0.34*	0.37*	0.58*	0.51*	0.42*	0.43*	0.32*	0.28*	0.40*	1		
	Deutsche Telekom	0.48*	0.52*	0.47*	0.35*	0.35*	0.42*	0.55*	0.54*	0.50*	0.46*	0.36*	0.25*	0.41*	0.44*	1	
	Iberdrola	0.44*	0.55*	0.38*	0.43*	0.37*	0.37*	0.43*	0.47*	0.39*	0.47*	0.40*	0.34*	0.41*	0.35*	0.38*	1
10 Years	Daimler	1															
	Banco Santander	0.61*	1														
	Bayer	0.54*	0.50*	1													
	VINCI	0.51*	0.51*	0.41*	1												
	Danone	0.41*	0.41*	0.41*	0.39*	1											
	Sanofi	0.43*	0.44*	0.43*	0.37*	0.42*	1										
	Siemens	0.65*	0.61*	0.54*	0.50*	0.40*	0.42*	1									
	Allianz	0.65*	0.65*	0.54*	0.50*	0.39*	0.44*	0.65*	1								
	Vivendi	0.47*	0.53*	0.41*	0.38*	0.33*	0.40*	0.5*	0.51*	1							
	Total	0.57*	0.58*	0.51*	0.50*	0.46*	0.47*	0.57*	0.56*	0.46*	1						
	Unilever	0.43*	0.42*	0.41*	0.37*	0.54*	0.44*	0.40*	0.41*	0.36*	0.47*	1					
	Unibail-Rodamco	0.37*	0.41*	0.30*	0.43*	0.32*	0.28*	0.36*	0.38*	0.30*	0.38*	0.30*	1				
	Carrefour	0.50*	0.53*	0.42*	0.43*	0.46*	0.46*	0.48*	0.49*	0.46*	0.50*	0.43*	0.33*	1			
	SAP	0.48*	0.43*	0.41*	0.33*	0.32*	0.34*	0.57*	0.50*	0.40*	0.40*	0.30*	0.25*	0.38*	1		
	Deutsche Telekom	0.47*	0.50*	0.46*	0.33*	0.33*	0.41*	0.54*	0.53*	0.5*	0.44*	0.35*	0.24*	0.40*	0.44*	1	
	Iberdrola	0.47*	0.58*	0.39*	0.49*	0.38*	0.37*	0.45*	0.49*	0.37*	0.50*	0.41*	0.38*	0.42*	0.33*	0.37*	1
15 Years	Daimler	1															
	Banco Santander	0.58*	1														
	Bayer	0.54*	0.49*	1													
	VINCI	0.48*	0.48*	0.38*	1												
	Danone	0.39*	0.39*	0.39*	0.36*	1											
	Sanofi	0.42*	0.42*	0.42*	0.34*	0.40*	1										
	Siemens	0.62*	0.58*	0.52*	0.44*	0.36*	0.38*	1									
	Allianz	0.63*	0.63*	0.53*	0.46*	0.37*	0.42*	0.62*	1								
	Vivendi	0.44*	0.50*	0.39*	0.33*	0.28*	0.35*	0.48*	0.48*	1							
	Total	0.54*	0.55*	0.48*	0.45*	0.43*	0.44*	0.52*	0.53*	0.42*	1						
	Unilever	0.40*	0.39*	0.39*	0.33*	0.51*	0.40*	0.34*	0.39*	0.30*	0.43*	1					
	Unibail-Rodamco	0.37*	0.40*	0.29*	0.41*	0.30*	0.26*	0.33*	0.36*	0.27*	0.36*	0.28*	1				
	Carrefour	0.49*	0.52*	0.41*	0.41*	0.43*	0.43*	0.46*	0.48*	0.42*	0.47*	0.39*	0.31*	1			
	SAP	0.43*	0.39*	0.38*	0.26*	0.28*	0.30*	0.55*	0.44*	0.37*	0.34*	0.25*	0.21*	0.33*	1		
	Deutsche Telekom	0.44*	0.47*	0.43*	0.28*	0.28*	0.36*	0.54*	0.50*	0.48*	0.39*	0.29*	0.21*	0.36*	0.43*	1	
	Iberdrola	0.46*	0.58*	0.37*	0.46*	0.36*	0.35*	0.41*	0.46*	0.33*	0.46*	0.38*	0.38*	0.41*	0.27*	0.33*	1
30 Years	Daimler	1															
	Banco Santander	0.58*	1														
	Bayer	0.54*	0.50*	1													
	VINCI	0.46*	0.46*	0.37*	1												
	Danone	0.41*	0.40*	0.41*	0.35*	1											
	Sanofi	0.42*	0.41*	0.42*	0.33*	0.40*	1										
	Siemens	0.62*	0.57*	0.53*	0.42*	0.37*	0.37*	1									
	Allianz	0.63*	0.61*	0.54*	0.44*	0.39*	0.41*	0.61*	1								
	Vivendi	0.44*	0.49*	0.39*	0.33*	0.30*	0.35*	0.47*	0.47*	1							
	Total	0.53*	0.53*	0.48*	0.42*	0.43*	0.44*	0.51*	0.51*	0.41*	1						
	Unilever	0.42*	0.41*	0.41*	0.33*	0.51*	0.39*	0.36*	0.40*	0.31*	0.44*	1					
	Unibail-Rodamco	0.36*	0.39*	0.30*	0.39*	0.30*	0.27*	0.33*	0.36*	0.27*	0.35*	0.29*	1				
	Carrefour	0.49*	0.52*	0.42*	0.41*	0.44*	0.43*	0.46*	0.48*	0.42*	0.47*	0.41*	0.32*	1			
	SAP	0.44*	0.40*	0.39*	0.26*	0.29*	0.30*	0.54*	0.44*	0.36*	0.35*	0.28*	0.22*	0.34*	1		
	Deutsche Telekom	0.45*	0.47*	0.44*	0.28*	0.3*	0.36*	0.53*	0.51*	0.47*	0.39*	0.32*	0.22*	0.37*	0.43*	1	
	Iberdrola	0.45*	0.56*	0.38*	0.44*	0.36*	0.35*	0.41*	0.46*	0.33*	0.44*	0.39*	0.37*	0.41*	0.28*	0.34*	1

* Significant at $p<0.05$ - Two-tailed test of correlation ≠ 0.

Table XX
Correlation matrices - $PMPT_0$

		Daimler	Banco Santander	Bayer	VINCI	Danone	Sanofi	Siemens	Allianz	Vivendi	Total	Unilever	Unibail-Rodamco	Carrefour	SAP	Deutsche Telekom	Iberdrola
1 Year	Daimler	1															
	Banco Santander	0.65*	1														
	Bayer	0.65*	0.62*	1													
	VINCI	0.59*	0.59*	0.55*	1												
	Danone	0.59*	0.56*	0.57*	0.54*	1											
	Sanofi	0.55*	0.54*	0.57*	0.51*	0.55*	1										
	Siemens	0.70*	0.66*	0.66*	0.60*	0.57*	0.55*	1									
	Allianz	0.70*	0.68*	0.66*	0.59*	0.58*	0.56*	0.69*	1								
	Vivendi	0.58*	0.61*	0.56*	0.55*	0.54*	0.52*	0.60*	0.62*	1							
	Total	0.64*	0.64*	0.62*	0.59*	0.59*	0.56*	0.65*	0.64*	0.59*	1						
	Unilever	0.57*	0.55*	0.58*	0.51*	0.64*	0.53*	0.57*	0.58*	0.53*	0.57*	1					
	Unibail-Rodamco	0.50*	0.53*	0.49*	0.53*	0.48*	0.45*	0.51*	0.53*	0.49*	0.52*	0.47*	1				
	Carrefour	0.61*	0.63*	0.58*	0.56*	0.60*	0.55*	0.60*	0.62*	0.58*	0.60*	0.57*	0.49*	1			
	SAP	0.60*	0.56*	0.57*	0.50*	0.54*	0.50*	0.64*	0.60*	0.53*	0.55*	0.52*	0.46*	0.55*	1		
	Deutsche Telekom	0.57*	0.58*	0.58*	0.50*	0.50*	0.49*	0.60*	0.61*	0.57*	0.55*	0.51*	0.45*	0.52*	0.53*	1	
	Iberdrola	0.57*	0.66*	0.55*	0.57*	0.53*	0.50*	0.56*	0.59*	0.54*	0.57*	0.53*	0.50*	0.56*	0.49*	0.52*	1
5 Years	Daimler	1															
	Banco Santander	0.68*	1														
	Bayer	0.66*	0.62*	1													
	VINCI	0.62*	0.61*	0.56*	1												
	Danone	0.57*	0.54*	0.55*	0.54*	1											
	Sanofi	0.56*	0.55*	0.57*	0.52*	0.55*	1										
	Siemens	0.73*	0.68*	0.66*	0.62*	0.55*	0.55*	1									
	Allianz	0.74*	0.71*	0.67*	0.61*	0.54*	0.57*	0.72*	1								
	Vivendi	0.57*	0.62*	0.56*	0.53*	0.48*	0.52*	0.58*	0.62*	1							
	Total	0.67*	0.67*	0.64*	0.62*	0.59*	0.58*	0.67*	0.66*	0.58*	1						
	Unilever	0.56*	0.54*	0.56*	0.52*	0.64*	0.55*	0.55*	0.55*	0.50*	0.59*	1					
	Unibail-Rodamco	0.52*	0.54*	0.47*	0.55*	0.48*	0.44*	0.52*	0.53*	0.48*	0.52*	0.46*	1				
	Carrefour	0.62*	0.62*	0.58*	0.57*	0.59*	0.57*	0.60*	0.62*	0.57*	0.62*	0.56*	0.48*	1			
	SAP	0.60*	0.55*	0.54*	0.52*	0.52*	0.52*	0.65*	0.60*	0.51*	0.55*	0.49*	0.46*	0.54*	1		
	Deutsche Telekom	0.57*	0.60*	0.59*	0.50*	0.47*	0.53*	0.61*	0.62*	0.56*	0.56*	0.50*	0.45*	0.52*	0.53*	1	
	Iberdrola	0.58*	0.65*	0.53*	0.60*	0.53*	0.50*	0.56*	0.58*	0.49*	0.59*	0.53*	0.50*	0.55*	0.49*	0.51*	1
10 Years	Daimler	1															
	Banco Santander	0.69*	1														
	Bayer	0.65*	0.62*	1													
	VINCI	0.65*	0.63*	0.55*	1												
	Danone	0.57*	0.54*	0.54*	0.56*	1											
	Sanofi	0.55*	0.54*	0.56*	0.51*	0.54*	1										
	Siemens	0.73*	0.68*	0.65*	0.63*	0.55*	0.54*	1									
	Allianz	0.74*	0.71*	0.67*	0.61*	0.54*	0.57*	0.72*	1								
	Vivendi	0.54*	0.59*	0.54*	0.49*	0.46*	0.50*	0.56*	0.59*	1							
	Total	0.68*	0.66*	0.64*	0.64*	0.59*	0.57*	0.66*	0.66*	0.55*	1						
	Unilever	0.56*	0.54*	0.55*	0.53*	0.64*	0.54*	0.55*	0.55*	0.48*	0.59*	1					
	Unibail-Rodamco	0.54*	0.56*	0.47*	0.59*	0.49*	0.43*	0.53*	0.53*	0.44*	0.53*	0.46*	1				
	Carrefour	0.62*	0.62*	0.57*	0.58*	0.59*	0.56*	0.59*	0.61*	0.55*	0.62*	0.56*	0.49*	1			
	SAP	0.59*	0.54*	0.53*	0.50*	0.50*	0.50*	0.64*	0.59*	0.50*	0.53*	0.47*	0.43*	0.52*	1		
	Deutsche Telekom	0.56*	0.59*	0.59*	0.48*	0.46*	0.52*	0.61*	0.62*	0.56*	0.54*	0.49*	0.43*	0.51*	0.54*	1	
	Iberdrola	0.60*	0.67*	0.53*	0.65*	0.55*	0.49*	0.57*	0.59*	0.45*	0.61*	0.54*	0.54*	0.56*	0.46*	0.48*	1
15 Years	Daimler	1															
	Banco Santander	0.67*	1														
	Bayer	0.65*	0.60*	1													
	VINCI	0.63*	0.61*	0.54*	1												
	Danone	0.56*	0.53*	0.52*	0.54*	1											
	Sanofi	0.54*	0.53*	0.55*	0.50*	0.53*	1										
	Siemens	0.71*	0.66*	0.63*	0.60*	0.52*	0.52*	1									
	Allianz	0.72*	0.69*	0.66*	0.59*	0.53*	0.55*	0.70*	1								
	Vivendi	0.51*	0.56*	0.51*	0.45*	0.42*	0.47*	0.55*	0.56*	1							
	Total	0.65*	0.64*	0.61*	0.61*	0.56*	0.56*	0.63*	0.64*	0.51*	1						
	Unilever	0.55*	0.52*	0.53*	0.50*	0.61*	0.53*	0.52*	0.54*	0.44*	0.57*	1					
	Unibail-Rodamco	0.53*	0.54*	0.46*	0.57*	0.47*	0.42*	0.50*	0.52*	0.41*	0.51*	0.44*	1				
	Carrefour	0.60*	0.62*	0.56*	0.57*	0.57*	0.55*	0.58*	0.60*	0.52*	0.60*	0.53*	0.48*	1			
	SAP	0.55*	0.51*	0.49*	0.45*	0.47*	0.47*	0.62*	0.55*	0.48*	0.49*	0.44*	0.39*	0.49*	1		
	Deutsche Telekom	0.54*	0.56*	0.57*	0.45*	0.44*	0.49*	0.62*	0.60*	0.55*	0.52*	0.46*	0.40*	0.49*	0.55*	1	
	Iberdrola	0.59*	0.67*	0.51*	0.63*	0.53*	0.48*	0.53*	0.57*	0.41*	0.58*	0.51*	0.53*	0.55*	0.41*	0.45*	1
30 Years	Daimler	1															
	Banco Santander	0.67*	1														
	Bayer	0.65*	0.61*	1													
	VINCI	0.61*	0.60*	0.53*	1												
	Danone	0.57*	0.53*	0.54*	0.53*	1											
	Sanofi	0.54*	0.52*	0.55*	0.49*	0.53*	1										
	Siemens	0.71*	0.65*	0.64*	0.59*	0.53*	0.51*	1									
	Allianz	0.72*	0.68*	0.66*	0.58*	0.54*	0.54*	0.69*	1								
	Vivendi	0.51*	0.55*	0.51*	0.45*	0.43*	0.46*	0.54*	0.56*	1							
	Total	0.64*	0.63*	0.61*	0.58*	0.57*	0.55*	0.62*	0.63*	0.51*	1						
	Unilever	0.56*	0.53*	0.55*	0.50*	0.62*	0.52*	0.54*	0.54*	0.44*	0.57*	1					
	Unibail-Rodamco	0.53*	0.53*	0.47*	0.56*	0.48*	0.43*	0.50*	0.51*	0.41*	0.50*	0.45*	1				
	Carrefour	0.61*	0.62*	0.57*	0.57*	0.58*	0.54*	0.58*	0.60*	0.52*	0.59*	0.55*	0.48*	1			
	SAP	0.55*	0.52*	0.50*	0.45*	0.48*	0.47*	0.62*	0.56*	0.47*	0.50*	0.46*	0.39*	0.49*	1		
	Deutsche Telekom	0.55*	0.56*	0.58*	0.46*	0.46*	0.49*	0.62*	0.60*	0.54*	0.52*	0.47*	0.40*	0.50*	0.55*	1	
	Iberdrola	0.58*	0.66*	0.52*	0.61*	0.53*	0.48*	0.53*	0.57*	0.42*	0.57*	0.52*	0.52*	0.56*	0.42*	0.46*	1

* Significant at p<0.05 - Two-tailed test of correlation ≠ 0.

Table XXI
Correlation matrices - $PMPT_{R_f}$

		Daimler	Banco Santander	Bayer	VINCI	Danone	Sanofi	Siemens	Allianz	Vivendi	Total	Unilever	Unibail-Rodamco	Carrefour	SAP	Deutsche Telekom	Iberdrola
1 Year	Daimler	1															
	Banco Santander	0.68*	1														
	Bayer	0.67*	0.64*	1													
	VINCI	0.70*	0.70*	0.65*	1												
	Danone	0.59*	0.59*	0.62*	0.61*	1											
	Sanofi	0.57*	0.58*	0.63*	0.59*	0.58*	1										
	Siemens	0.71*	0.69*	0.69*	0.71*	0.61*	0.59*	1									
	Allianz	0.71*	0.73*	0.69*	0.70*	0.60*	0.60*	0.72*	1								
	Vivendi	0.61*	0.65*	0.60*	0.64*	0.57*	0.56*	0.61*	0.66*	1							
	Total	0.69*	0.70*	0.68*	0.72*	0.63*	0.60*	0.71*	0.69*	0.64*	1						
	Unilever	0.60*	0.58*	0.63*	0.59*	0.69*	0.59*	0.60*	0.60*	0.58*	0.63*	1					
	Unibail-Rodamco	0.61*	0.63*	0.61*	0.67*	0.57*	0.54*	0.61*	0.64*	0.58*	0.63*	0.58*	1				
	Carrefour	0.62*	0.64*	0.62*	0.65*	0.63*	0.57*	0.62*	0.64*	0.61*	0.64*	0.60*	0.59*	1			
	SAP	0.61*	0.57*	0.60*	0.58*	0.56*	0.54*	0.63*	0.61*	0.54*	0.59*	0.56*	0.55*	0.57*	1		
	Deutsche Telekom	0.55*	0.60*	0.58*	0.56*	0.52*	0.52*	0.58*	0.60*	0.57*	0.57*	0.54*	0.53*	0.52*	0.49*	1	
	Iberdrola	0.63*	0.76*	0.61*	0.70*	0.57*	0.55*	0.63*	0.66*	0.61*	0.67*	0.56*	0.61*	0.61*	0.53*	0.58*	1
5 Years	Daimler	1															
	Banco Santander	0.70*	1														
	Bayer	0.69*	0.64*	1													
	VINCI	0.77*	0.76*	0.69*	1												
	Danone	0.60*	0.58*	0.63*	0.65*	1											
	Sanofi	0.58*	0.57*	0.63*	0.61*	0.61*	1										
	Siemens	0.76*	0.70*	0.69*	0.77*	0.61*	0.57*	1									
	Allianz	0.76*	0.75*	0.69*	0.77*	0.58*	0.59*	0.75*	1								
	Vivendi	0.63*	0.66*	0.61*	0.70*	0.58*	0.58*	0.62*	0.68*	1							
	Total	0.73*	0.72*	0.71*	0.78*	0.66*	0.63*	0.74*	0.73*	0.68*	1						
	Unilever	0.60*	0.57*	0.63*	0.62*	0.73*	0.61*	0.62*	0.58*	0.56*	0.66*	1					
	Unibail-Rodamco	0.65*	0.65*	0.60*	0.71*	0.59*	0.54*	0.63*	0.66*	0.61*	0.64*	0.58*	1				
	Carrefour	0.64*	0.64*	0.61*	0.70*	0.64*	0.59*	0.61*	0.64*	0.64*	0.65*	0.60*	0.60*	1			
	SAP	0.63*	0.55*	0.59*	0.64*	0.57*	0.55*	0.64*	0.62*	0.54*	0.61*	0.56*	0.56*	0.57*	1		
	Deutsche Telekom	0.55*	0.61*	0.60*	0.59*	0.53*	0.55*	0.57*	0.62*	0.56*	0.60*	0.55*	0.54*	0.52*	0.48*	1	
	Iberdrola	0.67*	0.80*	0.64*	0.76*	0.61*	0.57*	0.68*	0.71*	0.64*	0.72*	0.59*	0.63*	0.65*	0.55*	0.61*	1
10 Years	Daimler	1															
	Banco Santander	0.70*	1														
	Bayer	0.69*	0.64*	1													
	VINCI	0.76*	0.73*	0.68*	1												
	Danone	0.60*	0.58*	0.63*	0.63*	1											
	Sanofi	0.57*	0.58*	0.63*	0.59*	0.60*	1										
	Siemens	0.75*	0.69*	0.69*	0.75*	0.61*	0.57*	1									
	Allianz	0.76*	0.74*	0.69*	0.75*	0.58*	0.59*	0.75*	1								
	Vivendi	0.63*	0.65*	0.61*	0.68*	0.58*	0.58*	0.61*	0.67*	1							
	Total	0.72*	0.71*	0.71*	0.75*	0.65*	0.62*	0.73*	0.71*	0.66*	1						
	Unilever	0.61*	0.58*	0.65*	0.62*	0.72*	0.62*	0.62*	0.59*	0.57*	0.66*	1					
	Unibail-Rodamco	0.64*	0.64*	0.60*	0.69*	0.58*	0.54*	0.62*	0.65*	0.59*	0.62*	0.58*	1				
	Carrefour	0.64*	0.64*	0.62*	0.69*	0.64*	0.59*	0.61*	0.63*	0.63*	0.65*	0.61*	0.59*	1			
	SAP	0.63*	0.56*	0.60*	0.63*	0.57*	0.55*	0.64*	0.63*	0.55*	0.61*	0.57*	0.55*	0.57*	1		
	Deutsche Telekom	0.56*	0.61*	0.61*	0.58*	0.53*	0.56*	0.57*	0.61*	0.57*	0.60*	0.57*	0.54*	0.53*	0.50*	1	
	Iberdrola	0.67*	0.78*	0.64*	0.74*	0.60*	0.56*	0.66*	0.70*	0.62*	0.70*	0.59*	0.61*	0.64*	0.56*	0.60*	1
15 Years	Daimler	1															
	Banco Santander	0.70*	1														
	Bayer	0.69*	0.64*	1													
	VINCI	0.76*	0.73*	0.68*	1												
	Danone	0.60*	0.58*	0.63*	0.63*	1											
	Sanofi	0.57*	0.58*	0.63*	0.59*	0.60*	1										
	Siemens	0.75*	0.69*	0.69*	0.75*	0.61*	0.57*	1									
	Allianz	0.76*	0.74*	0.69*	0.75*	0.58*	0.59*	0.75*	1								
	Vivendi	0.63*	0.65*	0.61*	0.68*	0.58*	0.58*	0.61*	0.67*	1							
	Total	0.72*	0.71*	0.71*	0.75*	0.65*	0.62*	0.73*	0.71*	0.66*	1						
	Unilever	0.61*	0.58*	0.65*	0.62*	0.72*	0.62*	0.62*	0.59*	0.57*	0.66*	1					
	Unibail-Rodamco	0.64*	0.64*	0.60*	0.69*	0.58*	0.54*	0.62*	0.65*	0.60*	0.62*	0.58*	1				
	Carrefour	0.64*	0.65*	0.62*	0.69*	0.64*	0.59*	0.61*	0.63*	0.63*	0.65*	0.61*	0.59*	1			
	SAP	0.63*	0.56*	0.60*	0.63*	0.57*	0.55*	0.64*	0.63*	0.55*	0.61*	0.57*	0.55*	0.57*	1		
	Deutsche Telekom	0.57*	0.62*	0.61*	0.58*	0.53*	0.56*	0.57*	0.61*	0.57*	0.60*	0.57*	0.54*	0.53*	0.50*	1	
	Iberdrola	0.67*	0.78*	0.64*	0.74*	0.60*	0.56*	0.66*	0.70*	0.63*	0.70*	0.59*	0.61*	0.64*	0.56*	0.60*	1
30 Years	Daimler	1															
	Banco Santander	0.70*	1														
	Bayer	0.69*	0.64*	1													
	VINCI	0.76*	0.73*	0.68*	1												
	Danone	0.60*	0.58*	0.63*	0.63*	1											
	Sanofi	0.57*	0.58*	0.63*	0.59*	0.60*	1										
	Siemens	0.75*	0.69*	0.69*	0.75*	0.61*	0.57*	1									
	Allianz	0.76*	0.74*	0.69*	0.75*	0.58*	0.59*	0.75*	1								
	Vivendi	0.63*	0.65*	0.61*	0.68*	0.58*	0.58*	0.61*	0.67*	1							
	Total	0.72*	0.71*	0.71*	0.75*	0.65*	0.62*	0.73*	0.71*	0.66*	1						
	Unilever	0.61*	0.58*	0.65*	0.62*	0.72*	0.62*	0.62*	0.59*	0.57*	0.66*	1					
	Unibail-Rodamco	0.64*	0.64*	0.60*	0.69*	0.58*	0.54*	0.62*	0.65*	0.60*	0.62*	0.59*	1				
	Carrefour	0.64*	0.65*	0.62*	0.69*	0.64*	0.59*	0.61*	0.63*	0.63*	0.65*	0.61*	0.59*	1			
	SAP	0.63*	0.56*	0.60*	0.63*	0.57*	0.55*	0.64*	0.63*	0.55*	0.61*	0.57*	0.55*	0.57*	1		
	Deutsche Telekom	0.57*	0.62*	0.61*	0.58*	0.53*	0.56*	0.57*	0.62*	0.57*	0.60*	0.57*	0.54*	0.53*	0.50*	1	
	Iberdrola	0.67*	0.78*	0.64*	0.74*	0.60*	0.56*	0.66*	0.70*	0.63*	0.70*	0.59*	0.61*	0.64*	0.56*	0.60*	1

* Significant at $p<0.05$ - Two-tailed test of correlation $\neq 0$.

Table XXII
Correlation matrices - $PMPT_{R_m}$

		Daimler	Banco Santander	Bayer	VINCI	Danone	Sanofi	Siemens	Allianz	Vivendi	Total	Unilever	Unibail-Rodamco	Carrefour	SAP	Deutsche Telekom	Iberdrola
1 Year	Daimler	1															
	Banco Santander	0.35*	1														
	Bayer	0.39*	0.32*	1													
	VINCI	0.33*	0.33*	0.30*	1												
	Danone	0.28*	0.25*	0.34*	0.34*	1											
	Sanofi	0.27*	0.25*	0.35*	0.29*	0.37*	1										
	Siemens	0.41*	0.34*	0.36*	0.31*	0.27*	0.25*	1									
	Allianz	0.41*	0.40*	0.37*	0.31*	0.27*	0.28*	0.38*	1								
	Vivendi	0.28*	0.32*	0.30*	0.30*	0.30*	0.29*	0.30*	0.33*	1							
	Total	0.31*	0.30*	0.34*	0.33*	0.35*	0.32*	0.30*	0.32*	0.31*	1						
	Unilever	0.25*	0.22*	0.33*	0.28*	0.51*	0.35*	0.24*	0.24*	0.27*	0.30*	1					
	Unibail-Rodamco	0.26*	0.27*	0.30*	0.40*	0.36*	0.30*	0.25*	0.28*	0.30*	0.31*	0.33*	1				
	Carrefour	0.32*	0.35*	0.33*	0.34*	0.39*	0.33*	0.29*	0.32*	0.33*	0.34*	0.34*	0.29*	1			
	SAP	0.32*	0.26*	0.33*	0.25*	0.31*	0.27*	0.38*	0.32*	0.28*	0.26*	0.28*	0.26*	0.30*	1		
	Deutsche Telekom	0.30*	0.30*	0.38*	0.26*	0.30*	0.29*	0.33*	0.34*	0.35*	0.29*	0.30*	0.25*	0.29*	0.33*	1	
	Iberdrola	0.29*	0.40*	0.31*	0.42*	0.36*	0.29*	0.25*	0.32*	0.31*	0.33*	0.34*	0.38*	0.32*	0.23*	0.29*	1
5 Years	Daimler	1															
	Banco Santander	0.36*	1														
	Bayer	0.39*	0.28*	1													
	VINCI	0.35*	0.32*	0.28*	1												
	Danone	0.25*	0.21*	0.34*	0.33*	1											
	Sanofi	0.25*	0.22*	0.34*	0.28*	0.40*	1										
	Siemens	0.44*	0.33*	0.35*	0.32*	0.23*	0.23*	1									
	Allianz	0.46*	0.42*	0.37*	0.33*	0.21*	0.25*	0.43*	1								
	Vivendi	0.25*	0.30*	0.28*	0.23*	0.24*	0.27*	0.25*	0.32*	1							
	Total	0.30*	0.27*	0.34*	0.33*	0.34*	0.32*	0.27*	0.29*	0.27*	1						
	Unilever	0.24*	0.19*	0.32*	0.28*	0.55*	0.39*	0.20*	0.21*	0.25*	0.31*	1					
	Unibail-Rodamco	0.25*	0.25*	0.27*	0.40*	0.37*	0.28*	0.24*	0.26*	0.26*	0.29*	0.32*	1				
	Carrefour	0.30*	0.31*	0.30*	0.33*	0.39*	0.35*	0.25*	0.29*	0.30*	0.32*	0.35*	0.28*	1			
	SAP	0.34*	0.24*	0.32*	0.26*	0.30*	0.28*	0.39*	0.33*	0.26*	0.25*	0.27*	0.25*	0.29*	1		
	Deutsche Telekom	0.28*	0.30*	0.39*	0.24*	0.29*	0.32*	0.33*	0.35*	0.34*	0.27*	0.28*	0.24*	0.27*	0.34*	1	
	Iberdrola	0.29*	0.40*	0.30*	0.44*	0.37*	0.30*	0.24*	0.29*	0.25*	0.33*	0.35*	0.38*	0.31*	0.25*	0.29*	1
10 Years	Daimler	1															
	Banco Santander	0.35*	1														
	Bayer	0.38*	0.28*	1													
	VINCI	0.34*	0.31*	0.28*	1												
	Danone	0.25*	0.21*	0.34*	0.33*	1											
	Sanofi	0.25*	0.21*	0.33*	0.29*	0.40*	1										
	Siemens	0.44*	0.32*	0.35*	0.31*	0.22*	0.23*	1									
	Allianz	0.46*	0.41*	0.38*	0.31*	0.21*	0.26*	0.43*	1								
	Vivendi	0.24*	0.29*	0.27*	0.21*	0.22*	0.25*	0.25*	0.31*	1							
	Total	0.30*	0.26*	0.34*	0.32*	0.34*	0.32*	0.27*	0.29*	0.25*	1						
	Unilever	0.24*	0.19*	0.31*	0.29*	0.55*	0.39*	0.19*	0.21*	0.23*	0.31*	1					
	Unibail-Rodamco	0.25*	0.24*	0.27*	0.40*	0.37*	0.28*	0.24*	0.25*	0.24*	0.29*	0.32*	1				
	Carrefour	0.30*	0.31*	0.30*	0.33*	0.39*	0.35*	0.24*	0.29*	0.28*	0.32*	0.35*	0.28*	1			
	SAP	0.33*	0.23*	0.32*	0.25*	0.29*	0.27*	0.40*	0.34*	0.25*	0.25*	0.26*	0.24*	0.29*	1		
	Deutsche Telekom	0.28*	0.30*	0.38*	0.24*	0.28*	0.31*	0.34*	0.35*	0.34*	0.27*	0.27*	0.23*	0.27*	0.35*	1	
	Iberdrola	0.29*	0.40*	0.30*	0.43*	0.37*	0.30*	0.23*	0.29*	0.23*	0.32*	0.35*	0.37*	0.31*	0.24*	0.28*	1
15 Years	Daimler	1															
	Banco Santander	0.35*	1														
	Bayer	0.39*	0.28*	1													
	VINCI	0.33*	0.30*	0.29*	1												
	Danone	0.27*	0.23*	0.33*	0.36*	1											
	Sanofi	0.26*	0.22*	0.32*	0.32*	0.40*	1										
	Siemens	0.43*	0.32*	0.35*	0.29*	0.22*	0.23*	1									
	Allianz	0.45*	0.39*	0.38*	0.30*	0.23*	0.26*	0.41*	1								
	Vivendi	0.23*	0.28*	0.25*	0.20*	0.19*	0.22*	0.27*	0.29*	1							
	Total	0.30*	0.27*	0.33*	0.32*	0.35*	0.33*	0.26*	0.29*	0.23*	1						
	Unilever	0.25*	0.20*	0.31*	0.32*	0.54*	0.39*	0.19*	0.23*	0.21*	0.33*	1					
	Unibail-Rodamco	0.25*	0.25*	0.27*	0.41*	0.38*	0.30*	0.24*	0.26*	0.23*	0.31*	0.33*	1				
	Carrefour	0.31*	0.32*	0.30*	0.35*	0.38*	0.34*	0.25*	0.30*	0.26*	0.32*	0.34*	0.29*	1			
	SAP	0.32*	0.24*	0.30*	0.25*	0.28*	0.26*	0.42*	0.33*	0.24*	0.24*	0.24*	0.23*	0.27*	1		
	Deutsche Telekom	0.28*	0.29*	0.38*	0.25*	0.26*	0.29*	0.37*	0.35*	0.33*	0.26*	0.25*	0.23*	0.26*	0.35*	1	
	Iberdrola	0.30*	0.41*	0.31*	0.44*	0.38*	0.31*	0.22*	0.29*	0.20*	0.32*	0.37*	0.39*	0.33*	0.23*	0.26*	1
30 Years	Daimler	1															
	Banco Santander	0.35*	1														
	Bayer	0.39*	0.29*	1													
	VINCI	0.32*	0.29*	0.29*	1												
	Danone	0.28*	0.23*	0.33*	0.36*	1											
	Sanofi	0.27*	0.22*	0.32*	0.31*	0.39*	1										
	Siemens	0.43*	0.32*	0.35*	0.28*	0.23*	0.23*	1									
	Allianz	0.44*	0.38*	0.38*	0.29*	0.24*	0.26*	0.41*	1								
	Vivendi	0.23*	0.27*	0.25*	0.22*	0.21*	0.23*	0.26*	0.29*	1							
	Total	0.30*	0.27*	0.32*	0.32*	0.35*	0.33*	0.27*	0.29*	0.24*	1						
	Unilever	0.26*	0.20*	0.31*	0.32*	0.52*	0.37*	0.20*	0.23*	0.21*	0.32*	1					
	Unibail-Rodamco	0.25*	0.25*	0.28*	0.41*	0.37*	0.30*	0.24*	0.26*	0.24*	0.30*	0.33*	1				
	Carrefour	0.32*	0.34*	0.31*	0.35*	0.39*	0.34*	0.26*	0.30*	0.27*	0.33*	0.35*	0.29*	1			
	SAP	0.32*	0.25*	0.30*	0.25*	0.28*	0.25*	0.42*	0.33*	0.24*	0.24*	0.25*	0.23*	0.27*	1		
	Deutsche Telekom	0.29*	0.29*	0.38*	0.25*	0.27*	0.28*	0.37*	0.36*	0.33*	0.27*	0.26*	0.23*	0.26*	0.35*	1	
	Iberdrola	0.29*	0.39*	0.31*	0.43*	0.38*	0.31*	0.23*	0.29*	0.22*	0.31*	0.37*	0.39*	0.33*	0.22*	0.27*	1

* Significant at $p<0.05$ - Two-tailed test of correlation $\neq 0$.