LISBON SCHOOL OF ECONOMICS & MANAGEMENT
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MASTER
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MASTERS' FINAL WORK
DISSERTATION

COMMODITIES AND PORTFOLIO DIVERSIFICATION: MYTH OF FACT?

FÁBIO DOS SANTOS RUANO

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Resumo

Este estudo pretende analisar se as matérias-primas apresentam potencial de diversificação para portefólios de ações de investidores com aversão às perdas. A recente financialização do mercado das matérias-primas pode estar a afetar a vida de milhões de famílias a nível global, uma vez que determina o custo de vida. Alargamos a abordagem de Bessler & Wolff (2015) com o uso de indicadores de desempenho com o principal foco no risco de queda. A análise empírica considera a perspetiva das finanças comportamentais na avaliação dos benefícios de diversificação de 16 contratos futuros individuais e um índice de matérias-primas.

Este estudo confirma a elevada sensibilidade das matérias-primas às condições económicas do mercado. O sector energético de matérias-primas tem um melhor desempenho durante períodos de expansão económica. Os metais preciosos apresentam benefícios de diversificação tanto em períodos de expansão como de recessão, enquanto as matérias-primas do sector da pecuária apresentam um grande potencial de diversificação durante recessões. No geral concluímos que continuamos a observar benefícios de diversificação, mas estes dependem do período em análise, e têm vindo a decrescer ao longo do tempo.

Classificação JEL: C61, G10, G11, G41 **e** Q02

Palavras-chave: Aversão às perdas, matérias-primas, diversificação, modelos de alocação de ativos

Abstract

This study aims to investigate whether commodities yield diversification benefits to stock portfolios for loss-averse investors. The recent financialization of the commodity market increased correlations with stocks and thus may be hurting millions of households around the world, as it determines the cost of living. We extend the framework of Bessler & Wolff (2015) by using alternative performance measures mainly related to the downside risk. The empirical analysis accounts for a behavioral finance perspective in the assessment of diversification benefits from 16 individual future contracts and one index future on commodities.

Our study confirms the high sensitivity of commodities to market economic conditions. The energy sector performs better under economic expansion periods. Precious metals yield diversification benefits both in expansion and recession periods, while livestock commodities display a high potential to reduce risk especially during recessions. Overall, our findings yield that there is still a diversification benefit, but it is time-dependent and the benefits have been decreasing over time.

JEL Classification: C61, G10, G11, G41 and Q02

Keywords: Loss-averse, commodities, diversification, asset allocation models

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The way to get started is to quit talking and begin doing.

by Walt Disney

List of acronyms

Bbl. – Barrels
Bps – Basis points
BrO – Brent Oil
Bu. - Bushels
CBOT – Chicago Board of Trade
Coc – Cocoa
Cof – Coffee
Cop – Copper
Crn – Corn
CrO – Crude Oil
FeC – Feeder cattle
Gal. – Gallons
Gld – Gold
HeO – Heating Oil
ICE – International Exchange, Inc.
Inc. – Incorporated
IR – Information ratio
JB – Jarque Bera
Kurt. – Excess kurtosis
Lb. – Pound
LeH – Lean hogs
LIBOR – London Inter-Bank offered rate
LiC – Live cattle
MinVar – Minimum variance
MSCI – Morgan Stanley Capital International
MT – Metric tons
MV – Mean variance
NYB – New York Board of Trade
OR – Omega ratio
Pla – Platinum
p-value – Probability value
Ret. – Return
RP – Risk parity
RRA – Relative risk aversion or risk aversion coefficients
S&P 500 – Standard and Poor's 500 Index
Sil – Silver
Skew. – Skewness
Soy – Soybeans
SPGSCI – Standard and Poor's Goldman Sachs Commodity Index
SR – Sharpe ratio
St.w. – Strategic weight
Sug – Sugar
T oz. Troy ounce
TVaR – Tail value at risk
US – United States
USD – United States Dollars
VaR – Value at risk
Vol. – Volatility
Wht – Wheat

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1 Introduction

This study aims to investigate whether commodities yield diversification benefits to stock portfolios for loss-averse investors. Recent increases in the correlations with the stock market may have decreased the diversification potential of commodities. This financialization effect increased volatilities of commodity prices in the future market but also spilling over to the spot market. The increase in correlations is undoubtedly affecting the world's cost of living. Irrational trading in commodities affects not only market participants but all households around the world by turning prices more volatile and increasing their price. We extend the framework of Bessler & Wolff (2015) by using different performance measures mainly concern with the downside risk. We focus the entire research on loss-averse investors using individual commodities.

Commodity trading runs back to ancient civilizations. They use commodities as one of the first forms of money taking place even before written history. Worldwide institutional investors and high net worth investors are now shifting portfolio allocation into commodities. They seek risk diversification and opportunities to use active management skills provided by these alternative investments with risk and return characteristics that differ from traditional assets.

Tangible and homogeneous in nature, commodities are traded usually in contracts with standardized terms. Besides combining producers and direct users of the raw materials in the physical market serving as an economic need, the commodity market is also present in the financial markets with hedgers, speculators and arbitragers. This is possible due to the establishment of organized futures exchanges providing a more liquid market, where settlement is not necessarily physical and investors do not need to carry and store commodities. The first modern organized futures exchange was the Dojima Rice Exchange in Osaka, Japan, which was founded in 1710. In 1865 the CBOT developed the first standardized futures contracts on grain trading (Levine, Ooi, Richardson, & Sasseville, 2018). The availability of future commodity contracts grew rapidly in the 20th century, from less than 10 commodities in the 60s to more than 25 in the 90s.

Over the last 12 years, the commodity future market grew in popularity with a compound annual growth rate of 18.7%[1], trading 5.9 billion contracts in 2018 compare to only 0.6 billion in 2006. The commodity market is the most actively traded derivative product representing 18.8% of total derivatives volumes and is mainly traded in three exchanges: Shanghai Futures Exchange, Chicago Mercantile Exchange Group and Dalian Commodity Exchange.

Portfolio managers and investors constantly seek for new opportunities to improve their portfolio performances. Recent crisis force central banks to engage in monetary policies that resulted in a low interest rates environment in an attempt to stimulate investment and economic growth. These new rules are a game changer for market participants that now search for new investments and opportunities. Recent crisis also affected investors' perceptions about risk, particularly the fear of a downside risk. We see this by the more tight rules and restrictions that politicians and supervisors placed on financial markets. The alarm of a possible repetition of past mistakes regarding risk managing makes this study more valuable if we address the question from a behavioral finance perspective rather than a traditional one. Therefore, we focus and depict our investors as loss-averse and not purely rational and risk averse. Loss-aversion behavior is an emotional bias[2] from behavioral finance, it was firstly identified by Kahenman & Tversky (1979) when they develop the prospect theory. According to this bias, investors prefer the avoidance of a loss as opposing to achieving a gain.

Commodities have been found a useful alternative asset to manage risk and to diversify portfolios, this is shown in Daigler, Dupoyet, & You (2017) where one important benefits of adding commodities to a portfolio is the diversification potential that is possible due to the low correlations (or even a negative ones) with stocks and the heterogeneity of this asset class. Although recent studies reject the idea of these risk reduction benefits (Yan & Garcia, 2017; Zaremba, 2015), meaning that we may be overestimating these benefits. Basak and Pavlova (2016) addresses a new phenomenon referred as the financialization of commodities,

[1] For further information see the April 2019 WFE IOMA 2018 derivatives report from the World Federation of Exchanges available in *https://www.world-exchanges.org/storage/app/ media/statistics/WFE%202018%20IOMA%20Derivatives%20Report%20FINAL%2010.04.19.pdf.*
[2] A bias originate from impulse or intuition rather than conscious process, different from a cognitive bias, which is the result of a faulty judgment process.

triggered by an unprecedented inflow of institutional funds since 2004. The growth forced an increase in correlations between stocks and commodities. These effects begin by affecting future prices[3] but rapidly spill over the spot market. Commodity prices in this market are key determinants of the worldwide cost of living. An individual investor investing in commodities face two main sources of risk: supply and demand. When we include institutional investors, they face additional risks such as falling behind the benchmark index that increases volatility. The recent increases in energy and food prices intensifies the debate on whether these inflow of investors in commodities maybe hurting millions of households[4]. But not all is bad, the convenience yield, which is a compensation for bearing risk carried by inventories, is an important mechanism for producers who can transfer risk and supply liquidity to hedgers (Erb & Harvey, 2016).

Commodities differ from stocks as they do not produce income, so prices are not driven by traditional valuation techniques (Belousova & Dorfleitner, 2012). Thus, how do we value them? Levine et al. (2018) uses a 140 years dataset and shows that prices are related with business cycles depending on supply and demand. Bessembinder (1992) also highlight the importance of the future curve shape, as it affects production, and Georgiev (2001) claims that a downward sloping may provide positive roll yields[5].

Market participants invest in commodities through different instruments, each suits a specific set of investors with different characteristics and investment objectives. Jensen and Mercer (2011) details a wide range of exposures to commodities via exchange traded funds and notes, mutual funds, structured notes, swaps, commodity pool operators, commodity trading advisers, indirect claims on the stocks of commodity-based companies, cash market purchase of physical commodities or by derivative contracts futures and options. Our study will focus on futures contracts as they grew in popularity, have a large liquidity, are

[3] Institutional investors mainly trade future contracts.
[4] Yan and Garcia (2017) explores recent actions taken to reduce this effect when California Public Employees Retirement System, the California State Teachers' Retirement System and the Ohio Police & Fire Pension Fund reduced its commodity allocation.
[5] Bessembinder (1992) claims that production occurs when discounted future prices are below spot prices, in backwardation, indicating a temporary scarcity (or oversupply in contango). Bessembinder (2018) defends that roll yields are more informational for future gains and losses and not an actual gain or loss, gains and losses are only earn when we sell or buy assets.

standardized contracts with virtually no counterparty risk due to the role of the clearing houses. They also require low initial investment, no storage is needed, can be cash settled and are easily available to investors. We will use futures on individual commodities and an index for comparison reasons. Daigler et. all (2017) refers that studies that use only index futures may ignore potential benefits of the low correlations between individual commodities and creating a bias towards specific sectors of commodities.

We contribute to society by further assessing the real benefits of commodities given impact they have on the cost of living. Up until now, most studies focused on an in-sample analyze and narrow asset allocation strategies to mean variance using commodity indices. As Bessler & Wolff (2015) study we wide previous literature to different allocation strategies using an out-of-sample approach better suited to the real world investment decision making process. But we went further, we use alternative performance measures focus on tail risk and use individual commodities, avoiding sector bias selection with commodity indices. We also improve literature by using a behavioral finance perspective, by depicting investors as loss-averse given the recent post-crisis concerns about downside risk.

Our findings confirm the recent increases in correlations across sub-periods between stocks and commodities, supporting the idea of a financialization effect. Nevertheless, we also support that commodities still show potential for diversification and protection against downside risk. We also find that commodity returns are not normally distribute. Overall, the results show that risk reduction benefits are preserved, while for the majority of the augment portfolios both the volatility and the tail risk is reduced. Even by allocating a fixed constant weight to commodities can effectively reduce risk. Results change across different market environments. To summarize, our results shed light that there is still a diversification benefit, although it is time dependent and the benefits have been decreasing over time. Robustness check supports our conclusions.

The following sections are organized as follows. Section 2 reviews previous literature. Section 3 presents our data and the descriptive statistics. The applied methodology and the asset allocations models are detail in section 4. Our empirical results are present in section 5. Section 6 includes the robustness checks. Section 7 concludes.

2 Literature Review

When market participants face the decision of adding commodities to portfolios the majority of existence literature claims there is diversification potential, although this evidence is not consensual as show by Daskalaki & Skiadopoulos (2011).

For instance, Bodie & Rosansky (1980) and Fortenbery & Hauser (1990) found evidence that blending individual commodity futures with stock and equity index portfolios can effectively reduce risk without sacrificing returns, increasing Sharpe ratios without the need to an increase of returns. Ankrim & Hensel (1993) in a mean-variance framework using the S&P Goldman Sachs Commodity Index (SPGSCI) as a proxy for the commodity market, found diversification benefits. They improved optimal mean variance portfolios for different risk tolerance coefficients by mainly reducing risk. In similar observation periods, Jensen, Johnson, & Mercer (2000) points to the existence of diversification benefits only during restrictive phases of the monetary cycle, when typically inflation is high, on those periods the portfolios took larger weights to the SPGSCI. Georgiev (2001) detected a downside portfolio protection and improvements in the mean-variance space obtaining larger Sharpe ratios. Similar results where obtained by Gibson (2004). Evidences of a positive correlation between commodity futures and (unexpected) inflation was also found by Conover, Jensen, Johnson, & Mercer (2010); Gorton & Geert Rouwenhorst (2006) and Levine et al. (2018). Moreover, Conover et al. (2010) found larger Sharpe ratios during expansive monetary policy periods by the US Federal Reserve.

Commodity future returns historically exhibit significant volatility levels which explain the substantial differences found by Levine et al. (2018) when they use different historical returns measures (arithmetic and geometric). Nevertheless, Gorton & Geert Rouwenhorst (2006) findings reveal that by frequently rebalancing portfolios we can promote a less volatile environment. A number of previous studies also found a positive impact when portfolios are rebalanced, which generates roll returns. Erb & Harvey (2006) states that this rebalancing premium generates positive excess returns which are not a result of the individual commodity futures performance but rather the process of rebalancing the portfolio to its optimal weights. When assets appreciate (depreciate) in the relative value, they get larger (lower) weights than the target weight forcing the portfolio manager to sell (buying) those assets until the desired weight is achieved, this generates incremental returns as the assets fluctuate in value (Willenbrock, 2011).

You & Daigler (2010) went further when detected diversification potentials for the higher-moment risk of the portfolio when using the four-moment tail risk. They focus not only on standard deviation but also on skewness and kurtosis, which is especially useful for portfolio managers concern about downside losses. Later, You & Daigler (2013) detected diversification benefits of commodity futures, using mean-variance and Sharpe optimization models. In another study Daigler et al. (2017) continued to support the inclusion of future contracts both to reduce risk and to enhance returns. Concerning the tail risk, they found that for future portfolios, extreme losses are consistently smaller than for various equity index benchmarks.

After the mid-2000's, the literature focused on addressing a new phenomenon, previously named as the "financialization" of the commodity market (Basak & Pavlova, 2016; Büyüksahin, Haigh, & Robe, 2009; Cao, Jayasuriya, & Shambora, 2010; Daskalaki & Skiadopoulos, 2011; Main, Irwin, Sanders, & Smith, 2018; Tang & Xiong, 2012; Zaremba, 2015). In this new scenario, the diversification potential of commodity futures has been challenged by the large inflow from institutional investors (Basak & Pavlova, 2016). While evidence from Willenbrock (2011) suggests a low correlation between commodities and both bonds and stocks, the financialization of commodities introduces new sources of risks, that are common of traditional assets. The commodity market turned to be more volatile, more equity-like, increasing their correlations with stocks and reducing the inflation-hedging characteristics to a point where the diversification properties are insignificant or even nonexistent (Büyüksahin et al., 2009).

Cao et al. (2010) contrary to previous literature, found no increases of the efficient frontiers, under a mean-variance framework, when they add commodities to the portfolio. Daskalaki & Skiadopoulos (2011) conducted a study, using both a mean-variance and a non-mean variance framework, combining utility analysis and regression techniques. The authors found the diversification benefits when including commodity indices and five individual commodity futures in the investable space. The analysis was made in an out-of-sample framework, which is a more realist view for investors. In the real world, investors make choices of portfolio management based on uncertain parameters of future returns. In this out-of-sample framework, they found that regardless of the aversion level, efficient frontiers coincide denying the diversification properties and the value-adding of commodities to investors. Another interesting fact found by Daskalaki & Skiadopoulos (2011) was a decrease in portfolio turnover as more risk averse the investor is.

Belousova & Dorfleitner (2012) discusses the value of commodities for Euro denominated investors, as previously mentioned the large majority of commodities are priced and traded in USD, triggering an additional source of risk, the currency risk. The authors focus on 25 individual commodities, both future and physical contracts, as indices tend to overweight particular sectors, like SPGSCI that it is heavily weighted on the energy sector, see Erb & Harvey (2006). Belousova & Dorfleitner (2012) concluded that commodities remain valuable investments in the perspective of diversification. Agriculture, livestock and industrial metals commodities are attributable to the reduction of portfolio risk level, whether the energy and precious metals sectors yield the highest value growth to investors. These last two sectors improve portfolios both in bull and bear markets, enhancing the performance in return and risk reduction making them suitable for both conservative and aggressive investors.

Tang & Xiong (2012) continues the debate around the financialization of commodities. They also found increasing participation of pension and endowment funds leading to the increase in correlations and price volatility. As a result, they argue that commodity prices are no longer determined solely by supply and demand but also by the aggregate risk appetite for commodities and the investment behavior of diversified commodity index investors[6]. Similar to Basak & Pavlova (2016), Tang & Xiong (2012) found a more pronounced effect on commodities indices. Nevertheless, the authors highlighted one important benefit of financialization, which accounts for the risk-sharing mechanism between producers and the increasing number of institutional investors reducing premiums and increasing prices[7]. Therefore, the authors advise caution on constraints impositions by policymakers, which

[6] Basak & Pavlova (2016) discusses the benchmarking effect where managers' behavior face the risk of falling behind the index.
[7] On the other hand, for the small individual investors a reduction of risk premium is unattractive, forcing them to sell their positions to the large institutional investors (Basak & Pavlova, 2016).

might limit the potential risk-sharing benefit, and suggest that a simple exposure of the increase correlations and overestimation of diversification benefits might dissuade institutional investors from further investments.

In Zaremba (2015), that focus on passive commodity investment, results show that the expected roll yields decline. The authors argue that the inclusion of commodity futures in a traditional stock-bond portfolio is no longer reasonable, although Bessembinder (2018) disagrees and states that the roll yield should not be seen as an actual gain or loss.

Yan & Garcia (2017) use shrinkage estimator for the expected return, addressing the concern of You & Daigler (2013) about parameters estimation errors. Results show that with the reduction in estimation errors, optimal asset allocations are more balanced and stable. The results demonstrate that commodities can only reduce risk significantly in highly concentrated portfolios, as in more diversified portfolios this ability almost cease to exist. Only indices based on active strategies (momentum and term structure signals) could improve portfolio performance. Passive and mixed investment style indices fail to improve Shape ratios. Bessler & Wolff (2015) also highlights the impact of estimation errors. Their results show that in an out-of-sample study the benefits of commodities are not as significant as previously suggest by various in-sample analysis. However, they use historical returns, variances and covariance for the returns forecasts and the covariance matrix. The author suggest that a more realist and valuable approach is to first develop forecasting models and then use it in portfolio optimization.

Erb & Harvey (2016) decompose the commodity futures returns and argue that the previous positive correlation found in the literature between commodity returns and inflation is mainly driven by the interest rate-adjusted carry of future returns rather than spot returns. Therefore, commodity prices do not provide an inflation hedge.

Opposing to the general concept of financialization of commodities, Main et al. (2018) found that in non-energy sectors the majority of risk premiums increase, contrary to the concept of risk-sharing. This conclusion advocates a general decrease of risk premiums across all commodity futures that were brought by the financialization. The authors suggest that commodity future returns are mainly driven by idiosyncratic random fluctuations in supply and demand and that the average level of unconditional risk premium remains unaffected.

Levine et al. (2018) demonstrate that futures depend largely on the aggregate economic conditions in the long run, and that commodity futures index returns were positive and significant, especially during expansion and high inflation periods. Levine et al. (2018) corroborates Erb & Harvey (2016) findings that commodity futures market in backwardation outperform at contango. Further, the authors found that even in contango periods commodities can outperform stocks and bonds when inflation is up or when the economy is expanding. However, during severe recessions (the great-depression and the recent global financial crisis) commodities perform poorly, underperforming stocks and bonds.

A major innovation differentiating Gao & Nardari (2018) study from the existing literature is that they recursively construct one-period-ahead optimal portfolios by exploiting the predictability of all the first four moments of asset returns. They found that, by exploiting predictability, the inclusion of commodities into traditional asset portfolios does generate significant out-of-sample economic gains.

In summary, despite the growing popularity around commodities after the mid 2000's and the findings of potentials diversification benefits, results are mixed, complex and depend on several variables, some of which hard to grasp and to model. The conclusions from Tang & Xiong (2012) adverts for cautions when analyzing the issue, the recent change in the market perception of commodities might have trigger fashion decisions rather than rational ones, which might be a result of possible overestimations of the real benefits when we add commodities to a diversified portfolio. This study sets out to better understand the real value-add of this asset class, in a rigorous framework addressing previous concerns and red flags in literature, especially concerning parameters uncertainty, with a new and more up to date database.

3 Data

To test whether commodities yield diversification benefits regarding stock portfolios, we use daily and monthly prices of individual commodities futures contracts for the last 30 years, since 1st January 1989 to 31st December 2018. Bloomberg constructs a continuous time series of future prices rolling over contracts that fall within a certain range of days-to-maturity. We will use the Bloomberg generic first shortest futures time series as used by Daskalaki & Skiadopoulos (2011), which is the shortest maturity futures contract traded at any point in time. Chantziara & Skiadopoulos (2008) also explores Bloomberg generic contracts. The 16 futures contracts in the analysis represent the different commodity sectors. The main reason to use individual futures contracts arises from the fact that indices tend to overweight particular commodity sectors (Belousova & Dorfleitner, 2012). We then might neglect some of the benefits of a specific commodity sector if we only use broader indices. By narrowing to individual commodities we expect to better understand and account all the major properties and relationships between stocks and commodities. Similar approaches to that of ours were also used by Bessler & Wolff (2015) and Daigler et al. (2017).

Individual commodities were selected on the grounds that the underlying commodity reflects the characteristics of the commodity sector to which it belong. From the various sectors we select future contracts on Bloomberg available for the full period under analyze and with a high volume of trading, avoiding low liquid assets. Additionally to control these liquidity issues we have chosen the generics to roll to the next contract month five days prior to expiration to avoid unnecessary volatility due to increased trading volume.

Table I shows the final sample which includes: three energy futures (crude oil, brent oil and heating oil); three precious metals futures (gold, silver and platinum); one industrial metal future (copper); three softs futures (cocoa, coffee and sugar); three grains futures (corn, wheat and soybeans); and three livestock futures (feeder cattle, live cattle and lean hogs). We only use one future contract for the industrial metals sector due to the lack of generic first shortest futures series, during the full period under analysis. Daigler et al. (2017) combine all metals into one single group, but we believe precious metals and industrial metals behave differently. Past studies attribute to precious metals some hedging properties during periods of high stock market volatility (Hillier, Draper, & Faff, 2006), while industrial metals tend to be more correlated with business-cycles (Fama & French, 1988). Additionally, we also include a total return commodity index - S&P Goldman Sachs Commodity Index (SPGSCI) - to represent the commodity future market. In Belousova & Dorfleitner (2012) study we see that the index composition is highly concentrated on the energy sector, roughly 70%, and so we might expect SPGSCI to perform similarly as the energy commodities.

The current investment opportunity set for the diversified investor in the stock market is represent by the total return on the equity index S&P 500, which serves as our benchmark. In the robustness check tests, we also used the MSCI World and Russel 2000 to depict different types of investment opportunity sets and investors. For the risk-free we use LIBOR 1 month.

Our data display outliers and missing values, which required some adjustments. First, we remove days were data was not available for all assets. Second, for all the other missing values we use a simple interpolation method. For the outliers instead of removing we opt to use monthly data. Daily data exhibits a larger number of outlier and extreme leptokurtic distributions. The central limit theorem dictates that the distribution of more long periods returns becomes more normal than the distribution of daily returns, see Meucci (2010).

3.1 Descriptive Statistics

Table II exhibits the descriptive statistics of the monthly returns from January 1989 to December 2018, with 360 monthly observations. As we interpret the results key statistics vary between individual commodities indicating that commodities belong to a very heterogeneous asset class, also claimed by Belousova & Dorfleitner (2012) and Erb & Harvey (2006).

In the 30 years period under analysis, we see that commodities in a risk-return perspective underperformed compared to stocks, with negative or low Sharpe ratios, lower returns and higher volatility. When the sample is divided in sub-period, results show that the results for full period are largely affected by the period selection. From 1999 to 2008 all commodities had larger Sharpe ratios compared to stocks. Thus, we might also be in presence of some evidence of the financialization effect of commodities as already addressed by Basak & Pavlova (2016) and Cao et al. (2010). A positive fact for investors that is shown in the sub-period analysis is that stocks and commodities tend to perform on opposite directions. The most negative 10 year period for stocks was the best 10 year period for commodities.

Energy tend to have larger returns but also larger volatilities, precious metals have the lowest volatilities, and all other sectors seem to have low returns with the same volatility levels as the energy sector.

Regarding tail risk, both stocks and commodities exhibit leptokurtic distributions, which means a larger probability of occurring extreme returns. Despite commodities exhibit less negative skewness or even positive skewness which is a desirable feature for investors, that get less downside risk and high upward returns. The risk free, LIBOR 1 month, average annual return during the full period was 3.29%, ranging from 0.53% to 5.72% in the sub-period analysis.

Application of the Jarque–Bera test showed that at the 5% significance level all assets' monthly returns reject the null hypothesis of a normal distribution, which corroborates the findings of Bessler & Wolff (2015). Nevertheless, in the sub-period analysis (see table IV) we see some individual commodities in specific sub-periods where they do not reject the null hypothesis. These findings should alert investors when making investment decisions based on investment frameworks. Especially models or performance indicators that use the assumption of normally distributed returns. It also highlights the need and the importance of alternative asset allocation strategies and performance measures which is one of aims of this study.

The main benefit of adding commodities to a portfolio exhaustively discussed in literature is the diversification potential that is possible due to the low correlations and the heterogeneity of this asset class. Table III presents the Pearson correlations between and within the different commodity sectors, stocks and LIBOR. We calculated correlation between each group pair as the average correlations between each pair of individual assets within those two groups. The within group correlation is the average correlation between each pair of individual assets belonging to the same group, the same approach used in Daigler et al. (2017). We also exhibit the individual asset correlations in table V. In table III we also find the sub-period correlation matrix in intervals of 10 years[8].

[8] We need to alert that when data is nonnormally distributed, a test of the significance of Pearson's correlation may inflate Type I error rates and reduce its power. However, the relative performance of alternative methods has been unclear.

Observing the changes in correlations across the last 30 years, we see an increasing correlation between stocks and commodities, a strong evidence of the financialization effect. In the 90's the group with the largest correlation between these two assets class was in the range of 0.10 and 0.11 for industrial metals, grains and livestock. In this period, we even had negative correlations of -0.22 for the energy sector, which is a desired characteristic for investors seeking diversification. In the period were the literature points to the start of the financialization of commodities, results show that the energy sector rises from a correlation of -0.22 to 0.18. The industrial metals are more correlated to the business-cycle, and display the largest correlation pair with stocks. In the last sub-period correlations continue to grow and only livestock and softs remain at low levels. Livestock is the only sector that decreased their correlation with stocks in the sub-period comparison and are not significant at 5% level.

The high correlations between the commodity index and the energy sector, about 0.94 for the full period, are again evidence that there is a large energy component in the SPGSCI[9], which supports our decision to include individual commodity futures for a better understanding of the usefulness of this asset class.

In the full period analysis, we can see that correlations between stocks and commodities are still low, although not as low as before but low enough to evidence some diversification potential. Energy and industrial metals display the largest correlations from 0.41 to 0.53, all other sector range from 0.27 to 0.03. Livestock and softs have the lowest within sector correlations 0.67 and 0.60, respectively, making them the more heterogeneous sector. This might be an indication of a larger potential for diversification for these two sector as they have low correlation with stocks and within the sector.

Between industrial metals and precious metals we observe a correlation of 0.4, which supports our decision that we should separate industrial from precious metals has they have different characteristics, see Fama & French (1988) and Hillier et al. (2006).

In table V we test the statistical significance of correlations. At 5% significance level, we do not reject the null hypothesis for gold, sugar and all the livestock commodities. Gold

[9] This happens because the SPGSCI has a world-production weighting scheme, based on a five-year moving average, making the index largely biased towards the energy sector. A market-cap weighting scheme is just not possible for indices on future contracts. Futures contracts have a short position for every long position so the market capitalization is always zero.

and sugar have a much lower correlation with stocks than the sector they belong. Thus, within each sector we might have a superior diversification potential for specific commodities. Livestock commodities have also low correlations with stocks and within the sector.

In summary besides the increasing correlations across sub-periods between stocks and commodities, supporting the idea of a financialization effect, commodities still show potential for diversification and protection against downside risk.

4 Methodology

Investors are not purely rational as the tradition finance advocates. This study aims to concentrate the investigation on a specific set of investors. Investors that are loss-averse and for whom the basic utility function and the pure mean variance optimization model will not maximize their true utility. A common definition of utility is that it is the level of relative satisfaction received from the consumption of goods and services, subject to a budget constraint. Utility functions for these investors have different behaviors and levels of risk aversion when they are in the range of a relative gain or a loss. This loss-aversion behavior is an emotional bias[10] from behavioral finance, it was firstly identified by Kahenman & Tversky (1979) when they develop the prospect theory. This bias makes investors prefer the avoidance of a loss as opposing to achieving a gain. Although the goal of the study is not derive theirs true utility functions, we will rather focus on alternative investment strategies that better suit these investors.

These investors are not as worried about positive returns as they are with variance and extreme losses. As discuss in literature section, the main benefits of commodities are the diversification potential and protection against downside risk, which might be beneficial for these investors. Our results from the descriptive statistics confirm previous studies that neither stock nor commodity returns are normally distribute, which challenge investors to seek alternative investment strategies that rely less on this assumption.

Another important point from Bessler & Wolff (2015) and Yan & Garcia (2017) is the importance of the parameters' estimation errors. We will follow a similar approach of Bessler

[10] A bias originate from impulse or intuition rather than conscious process, different from a cognitive bias, which is the result of a faulty judgment process.

& Wolff (2015) whose allocation strategies require a lesser number of parameters. We then expect that with less required parameter we have less estimation errors. Yan & Garcia (2017) opt to use shrinkage estimators[11] that we will not use.

We begin with a strategy that requires zero input parameters, the strategic weights strategy. Then we choose two strategies that rely solely on one parameter the variance-covariance matrix, risk parity and minimum variance optimization. Risk parity requires only estimations on the volatilities (the diagonal line of the matrix), while the minimum variance optimization requires both variances and covariance. The two strategies rely on risk estimates that accordingly to Chopra & Ziemba (2013) present less estimation errors compared to returns estimations.

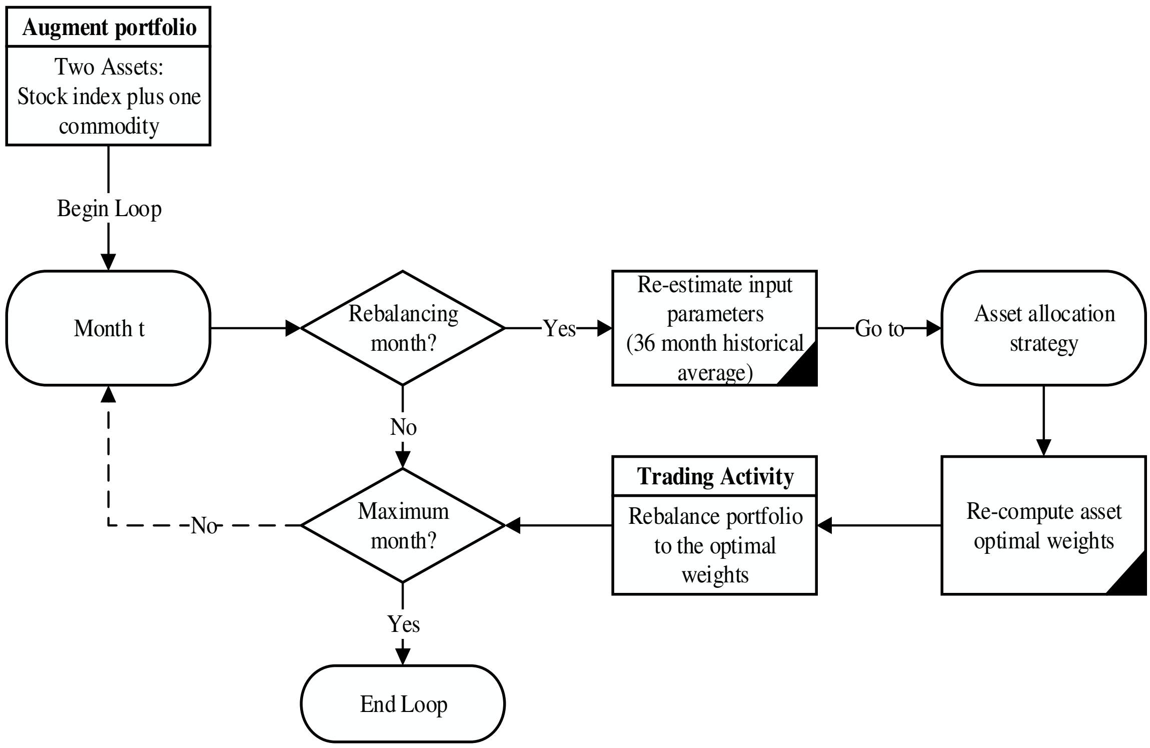
Given the popularity of the mean variance optimization in the literature, we will also use this strategy. Although mean variance requires the assumption of a normal distribution, we will use some constraints and different risk aversion coefficients that will better suit and model loss-averse investors that exhibit different risk aversion behaviors when they face a loss relative to a gain.

Each individual augment portfolio is construct by combining the stock index, our base portfolio, with each individual commodities under the different allocation strategies. Asset weights are rebalance every quarter and to exactly to the optimal weight. We recomputed the optimal weight at every rebalance period. The estimators for the parameters used in the allocation strategies were based on a 36 months historical average. Accordingly to Bessler & Wolff (2015) historical windows larger or equal to 48 fail to be point in time and a to low window period generates instability on the parameters. Parameters estimators based on forecast were applied by Gao & Nardari (2018). We decide to not follow such approach as we might drift to the field of economic forecasting models which is not the aim of this study. We also limit our strategies to long positions. Therefore, no short selling is allow. Transaction

[11] Shrinkage estimators are based on the idea that the shrinkage estimator is derive from the sample estimate, the prior and a shrinkage factor (a relative precision factor), as the shrinkage factor grows the shrinkage estimator converges to the sample estimate. This shrinkage factor is based on judgment, see Yan & Garcia (2017).

costs are not contemplate in our base scenario. We will account the impact of these costs in the robustness check. Figure 1 summarizes the investment process.

Figure 1 - Investment process for each augment stock-commodity portfolio.



Notes: Each augment portfolio is a combination between the stock index and one of the 17 commodity assets. We constructed 17 augment portfolios: 16 using individual commodities and 1 using the commodity index. The same methodology is applied to each asset allocation strategy. Each strategy has specific rules and constrains detail in the next sections. The input parameters required for each asset allocation strategy are estimated on a 36 month historical average. The input parameters are returns and the variance matrix that has variances and covariance. We rebalance portfolio every 3 months, no short selling is allowed and there are no transaction costs in our base scenario.

4.1 Strategic Weights (Naïve Strategy)

The strategic weights (st.w.) is the simplest allocation strategy, it sets a constant weight to the different asset classes. The selected weight to commodities is based on Bessler & Wolff (2015) study that used a 5% and a 15% weight on commodities, but those portfolio also included bonds. Our study focus solely on stocks and based on the relative weights of commodities to stocks in that study, we choose an allocation of 20% to commodities.

4.2 Risk Parity

Risk parity (RP) grown in popularity, due to their ability to control risk and to the simplicity of the strategy. The idea of risk parity strategy is that each asset contributes equally to the portfolio risk without considering the correlations such as:

(1) $$\omega_i = \frac{1/\hat{\sigma}_i^2}{\sum_{i=1}^{N}(1/\hat{\sigma}_i^2)}$$

In this strategy, assets with a lower (larger) volatility, $\hat{\sigma}_i^2$, will have a larger (lower) weight, ω_i, as the weights are anti-proportional to their volatilities. Due to the low number of require parameters it may address the issue of estimation errors, although it disregards correlations between assets, which has been a key potential benefit of commodities.

4.3 Minimum Variance Optimization

Minimum Variance Optimization (MinVar) requires the entire variance-covariance matrix as input, which means it considers the correlations between all assets, incorporating the diversification potential.

The MinVar is an optimization strategy that requires some advance software packages to solve the optimization problem, contrary to st.w. and RP strategies that simplify the problem by applying an heuristic[12]. The MinVar goal is to minimize the portfolio variance with the set of available assets as following:

(2) $$\min_{\omega} \omega' \sum \omega,$$

where ω' is the transpose vector of assets' weights, Σ the variance-covariance matrix and ω the vector of weights.

Using constrains in the optimization process it avoids getting extreme and unreasonable results for the real world, bounding results to the real world limitations. For our problem, we bound the weights of each asset, ω_i, to a minimum of 1%. It would be unreasonable to say that we combine commodities with stocks with a 0% weight. We do not allow short sales, which means we need to have positive weights that sum 1:

(3) $$\sum_{i=1}^{N} \omega_i = 1, \text{ with } \qquad \omega_i \geq 1\%, i = 1,2, \dots n$$

[12] Any approach that solves a specific problem by applying a simple and practical method not guaranteed to the be the best fit, but achieves satisfactory immediate goals.

In this strategy, the largest weights will be attribute for the asset with the lowest volatility and lest correlated with stocks.

4.4 Mean Variance Optimization (MV)

Markowitz (1952), introduces Mean Variance Theory that solves a tradeoff between risk and return, by maximizing the utility function, *U*, of an investor. We select a commonly used utility function commonly like in the study of Bessler, Opfer, & Wolff (2017)[13],as following:

(4) $$\max_{\omega} U = \omega'\mu - \frac{\delta}{2}\omega' \sum \omega$$

For a portfolio of n assets, the porfolio's return μ_p and volatilty σ_p^2 are calculated as:

(5) $$\mu_p = \sum_{i=1}^{n} \mu_i \omega_i \text{ and,}$$

(6) $$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{i,j} \omega_i \omega_j.$$

To avoid outliers from the optimization process and unreasonable results we constrain portfolio standard deviation to:

(7) $$\sqrt{\omega' \sum \omega} \leq \hat{\sigma}_c$$

We set the portfolio volatility in Equation (7) to a maximum value of 15%. In the period under analysis (see table II and IV), the S&P 500 volatility remain at or below the 15% threshold, from the perspective of an investor worried about losses it will be unlikely that he prefers an augment portfolio with a higher volatility relative to the base portfolio (the S&P 500). Additionally we set the same constrains from MinVar, see equations (3).

Loss-averse investors, that are the focus of this study, exhibit different risk aversion levels for gains and losses. To model this type of behavior on the utility function we use two different values for the risk aversion coefficients, δ, in equation (4). Following Bessler & Wolff (2015) and Daskalaki & Skiadopoulos (2011) studies we use the same risk aversion coefficients of 10 for a more conservative behavior and 2 for a more aggressive behavior. The constrain on volatility, equation (7), will prevent the portfolio from getting to much volatile for our investor.

The Mean Variance strategy requires return as an additionally input parameter, as previous mentioned return estimators have larger estimation errors than risk estimates. By

[13] To see additional popular utility functions see Daskalaki & Skiadopoulos (2011).

comparing the in-sample results that use the true parameters and the out-of-sample results that use estimated parameters, we will see the impact of estimation errors.

4.5 Performance Measures

To evaluate the results of our strategies, we have to select suitable performance indicators to our type of investor and to the specific characteristics of our sample. We will start by computing the portfolio turnover as in DeMiguel, Garlappi, & Uppal (2007), define as the average sum of absolute value of the trades across the N available assets:

(8) $$PT = \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{N}(|\omega_{j,t+1} - \omega_{j,t+}|),$$

where $\omega_{j,t+1}$ is the new weight of asset j and at time t+1 after rebalance, and $\omega_{j,t+}$ is the weight just before rebalance at time t+1. T is the number of rebalancing intervals. In the robustness check we expect to see that the strategies with the higher portfolio turnovers, will have a larger negative impact when we introduce transactions costs.

We select two risk-adjusted performance appraisal measures, Sharpe ratio and information ratio, they focus not only on return but return adjusted to volatility. Although they rely on the assumption of normal distribution of returns.

(9) $$SR_p = \frac{R_p - r_f}{\sigma_p}, \text{ and}$$

(10) $$IR_p = \frac{\frac{1}{T}\sum R_p - R_B}{\sigma_{p-B}}.$$

Sharpe ratio (SR), equation (9), is the most commonly used performance indicator in finance also known as reward-to-variability, it compares the excess return over the risk free, $R_p - r_f$, to the total risk of the portfolio, σ_p, see Sharpe (1966). Information ratio (IR), equation (10), measures the reward earned by managers per the incremental risk created by deviating from the benchmark. Our benchmark will be the base portfolio without commodities. This indicator allow us to compute the active return, the T monthly excess returns over the benchmark, $(R_p - R_B)$, divided by the active risk which is the standard deviation of the active returns also kwon as the tracking error, σ_{p-B}.

Omega ratio (OR) develop in Keating & Shadwick (2002) it is a simple and elegant universal performance indicator. The indicator is useful for any type of sample, as it does not require any assumption on the distribution of returns. It has been applied in Bessler & Wolff (2015) study. The ratio measures the proportion of averages gains by the average losses. A gain (loss) is define as the returns above (below) the minimum acceptance return for the investor, in this case, we use the risk free.

(11) $$OR_p = \frac{\int_r^b (1-F(x))dx}{\int_a^r (F(x))dx}$$ where,

F(x) is the cumulative probability distribution that the return is less than the minimum acceptance return, *r*, *a* and *b* are the investment intervals. In practice, it is equal to the probability of weighted gains divided by the probability of the weighted losses.

This study focus on loss-averse investors that have a large concern about downside risk, or tail risk, so the last performance indicator that we will use is the 5% historical monthly value at risk (VaR) used to account for tail risk as in Daigler et al. (2017) and Hammoudeh, Araújo Santos, & Al-Hassan (2013). Using the historical value at risk suits better our sample by avoiding any assumption about the distribution of returns, as opposing to the Gaussian value at risk that requires the assumption of a normal distribution. To better account the tail risk we will also use an extension of value at risk, which is the tail value at risk (TVaR). VaR measures the portfolio loss of the 5% worst historical case, TVaR measures the average loss of the 5% worst historical cases. Remember that value at risk does not measure the maximum loss, the maximum loss at any given point is 100% of the portfolio. Value at risk indicators are presented in money units, but for simplicity we exhibit in percentage terms of the total portfolio value.

5 Results

5.1 In-sample performance

We begin the analysis by examining the in-sample benefits of adding individual commodities with stocks. In-sample means that the investors have perfect forecasts for the expected returns for all assets. When computing the weights for month *t* we have all data until and for month *t*. This analysis allows us to remove the estimation errors and compare to the out-of-sample results that have those errors. As the st.w. strategy requires no parameters we will leave results for the out-of-sample analysis.

Table VI presents the in-sample results for our augment stock portfolios with commodities. In all strategies we find a reduction in the VaR and TVaR values compared to the stock-only portfolio, although when we look for risk-return performance only in MV strategies we find benefits. The results from the MinVar and RP strategies show us a reduction of volatility for all commodities and for the case of silver we can also see return improvements. The energy sector in the MinVar strategy reduce volatility without sacrificing the returns as we see by the higher Sharpe ratios.

In both MV strategies, gold, live cattle and feeder cattle achieve the highest benefits for all performance indicators with the exception for the IR where crude oil, gold and soybeans perform better. The high active risk of the livestock sector's excess returns reduced the IR. Nevertheless, feeder cattle show the highest benefits regarding the tail risk.

Across individual commodities and sectors, we have different results showing the importance of this individual analysis. The index performs similarly to the energy sector, due to the largest weight to this sector as already mentioned.

5.2 Out-of-sample performance

Table VII shows the results for the out-of-sample analysis. Comparing to the in-sample results we note a large reduction in the returns but no large change in volatility. This indicates that we have larger estimation errors for returns than for the variance matrix, also stated by Chopra & Ziemba (2013). The out-of-sample results are time dependent and we may have some time selection bias. Nevertheless, commodities performance are very sensitive to the different market environments that might explain the different results obtain under different sub-periods. We analyze these impacts in section 5.3 and in the robustness check we also apply different estimation window periods for the parameters estimators.

In both MV strategies the energy and metal sectors improve returns but were unable to reduce risk, the same is true for some grains. VaR indicator might be a misleading indicator, where in both strategies all commodities reduce VaR, but not all improve TVaR meaning that the extreme losses get even worse. Figure 3 displays the rolling TVaR and the MV strategies have the worst performance compared to all other strategies. The softs sector did not show benefits for the less risk averse investor and for the more risk averse it marginally reduce risk. The livestock sector show a large risk reduction for both investors. Overall, the gold augment portfolio exhibit by far the highest benefits for both MV strategies.

MinVar and RP strategies have similar results compared to the in-sample performance. Results show that MinVar, that uses the entire variance matrix of assets, achieve better results than RP. In MinVar silver is the only commodity achieving a positive IR, exhibiting positive excess returns relative to the base portfolio. All sectors reduce the risk measures with livestock and gold showing the highest benefits. The energy sector not only reduce risk but marginally improve Sharpe ratios. The st.w. strategy depict the same risk reduction benefits of the MinVar and RP strategies, but at lower ranges. These results show that even a simple allocation of commodities to a stock portfolio can reduce risk, especially the extreme losses as we see by figure 3.

Figure 2 depicts the out-of-sample performance for all strategies across the full period under analysis regarding the total portfolio's value growth. Keep in mind that MV focus on risk and returns, while MinVar and RP uses only variances to solve the asset allocation problem. When we look to the total value growth across the entire period the MV strategies achieve the highest total growth. MinVar and RP marginally increase total portfolio value, showing the inability to generate positive excess returns display by the negative IR in table VII. Gold show the highest benefits for both MV strategies, while surprisingly in the MinVar and RP it was the augment portfolio with the lower growth. Silver achieve the highest growth in these two asset allocation strategies. St.w. strategies did not improve the portfolio growth, both with a 20% and 30% allocation to commodities.

5.3 Performance under different market environments

Levine et al. (2018) show that commodities perform differently under different market environments. We divide our full period into sub-periods of economic expansions and recessions. We use Bessler & Wolff (2015) division and methodology that uses monetary policy and stock market signals to characterized the economic environment. Tables VIII, IX, X, XI and XII resume the results for the different sub-periods under the five asset allocation strategies. For all strategies, the benefits of commodities are time dependent. The strongest periods for commodities are 2001 to 2004 and 2004 to 2008, during the tech bubble and right before the subprime financial crisis.

In the most recent period, we mainly see risk reduction benefits across all strategies. The subprime financial crisis period is characterized by a strong performance for gold and sugar allocations, enhancing returns and reducing risk. In the MinVar, RP and st.w. strategies, allocations to the livestock sector benefit the stock portfolio during this period. From 2004 to 2008 commodities record the highest benefits by boosting returns especially for the energy and metal sectors, under a more aggressive strategy (MV RRA 2) grains also exhibit benefits. This is also the only period where RP perform better than MinVar, and a period where risk reduction benefit are scarce. During another recession, the Tech Bubble, gold and grains (especially soybeans) generate benefits for all performance indicators. In the case of MinVar, RP and st.w. the entire metal sector and feeder cattle register also the same benefits. The period from 1994-2001 we mainly achieve benefits regarding risk reduction. For MV strategies heating oil and platinum generate some additional benefits. Softs were unable to benefit the stock portfolio in all strategies. The last period from 1991 to 1994 is the least propitious for commodities. Although we see little volatility decreases, there are no major benefits for the tail risk. Nevertheless in the MinVar, RP and st.w. strategies soybeans improve all performance indicators and the same is valid for silver under the st.w. strategies with the exception for the tail risk indicators.

5.4 Analysis of commodity portfolio weights

Up until now, we show the performance of the augment portfolios under the different asset allocation strategies and across different market environments. To obtain further evidence on the different results obtain from the different strategies and periods, we analyze the portfolio's weights. Under the st.w. strategy the weights did not drift significantly from the 20% desire weight and so we remove this strategy from this analyze.

Figure 4 represents the portfolio weights along the entire analyze period. Table XIII exhibits the portfolio turnover for all strategies and for all individual commodities and the commodity index. MV strategies exhibit corner solutions, see Best & Grauer (1991), where they allocate large portions on commodities from 1999 to 2009, while in the remaining periods despites some sporadic spikes overall they allocate a value close to 1%. During periods of economic recession all strategies allocate a large weight to the livestock sector. Feeder and live cattle receiving higher average allocations in the range of 28% to 47% across strategies. The soft sector receives the lower weights in all strategies with average values under the 20% threshold. Strategies focus solely on variances give lower weights to the energy sector. Soybeans dominates the grain sector and gold dominates the metal sector with average value ranging from 32% to 45%.

Regarding portfolio turnovers, they are higher for the livestock and the metal sector, while the energy sector exhibits lower ones. Due to the low correlation between stocks with the metal (especially gold) and livestock sectors it originate larger diversification opportunities across the full period. Allocations to commodities where very large when stock perform badly and vice versa generating these large portfolio turnovers compared to most correlated sector with stocks, see individual correlations in table V. In the two MV strategies the less (more) risk averse has higher (lower) turnover values. We also see that for the MinVar, which is an optimization technique, displays higher turnovers compared to the RP. The turnover values in the st.w. strategy are just a result of the normal drift of assets weights between rebalancing periods.

6 Robustness checks

6.1 Alternative benchmarks (base portfolios)

Up until now we use as our benchmark for the diversified stock portfolio the S&P 500 Index. Although we want test if commodities suit a slightly different investors. For a more global investor we use the MSCI World Stock Index. To check the effect on a different US investor we use the Russel 2000 Index that tracks only small cap stocks. Commodities exhibit similar benefits but at lower levels with the MSCI World Index. Crude oil and brent oil cease to have meaningful benefits for the stock portfolio. Using the Russel 2000 Index, commodities provide even higher benefits regarding risk reduction[14], though gold and silver are the only commodities improving returns. Overall, results do not change significantly.

6.2 Alternative estimation windows

The alternative estimation windows affect the parameters' estimation errors which are an important concern in (You & Daigler, 2013). Long estimation windows react to slow to

[14] Russel 2000 displays different higher risk than the S&P 500's. Volatility, VaR and TVaR are 19%, 8% and 13% respectively for Russel 2000 and 14%, 7% and 10% for the S&P 500, returns are similar for both. This might indicating a higher potential for commodities regarding risk reduction benefits.

structural changes while to short windows inflate turnovers and transaction costs creating instable portfolios with lower net performances. We vary the estimation windows for 12 months and 48 months. Using 12 month window the asset weights exhibit large turnovers, this instability affect negatively all performance indicators. On the other hand, the 48 month window create a stability on weights that decrease the turnovers without any meaningful change on the performance indicators. To sum up the theoretical benefits of a smaller estimation window were not achieve.

6.3 Alternative rebalancing frequencies

Smaller (higher) rebalancing frequencies may intensify (reduce) portfolio turnovers but also avoid (create) large undesired drifts from the optimal ones that might increase (decrease) performance. To investigate these effects and the sensitivity of our results we change the rebalancing frequency to 1 month and 6 months. Monthly periods reduce the weights drifts generating much smaller portfolio turnovers, but the performance increases are so small that in the presence of transaction costs, at 50 basis points, they cease to exist. Semiannually periods allow larger drifts increasing portfolio turnovers without significantly affecting results. To summarize our results were robust under alternative rebalancing periods.

6.4 Transaction costs

Transaction costs are a major concern for investors with large impacts on performance. The presence of real world transactions costs may invalidate strategies that perform well under no transactions costs. To test the sensitivity of our results under the assumption of no transaction costs we use a 50 and 200 basis points (bps) scenarios. Results were robust with transactions costs at 50 bps and only at 200 bps we see decreases in returns higher than 0.5%, with higher sensitivity for the optimization strategies. Changes in risk indicators were not significant.

7 Conclusions

This study aims to investigate whether commodities yield diversification benefits to stock portfolios for loss-averse investors. We use Bessler & Wolff (2015) methodology improving the existing literature that mainly focus on in-sample analyze and narrow asset allocation to mean variance strategies. We employ four different asset allocation strategies: strategic weight, risk-parity, minimum variance and mean variance. In the mean variance we use 2 different relative risk aversion levels to model the two risk behaviors exhibit by the loss-averse investor when he is the range of losses and gains. We widen research using six different performance appraisals: two risk-adjust measures, two drawdown measures, portfolio turnovers and one that measures relative gains to losses.

The empirical analysis accounts for a behavioral finance perspective in the assessment of diversification benefits from 16 individual future contracts and one index future on commodities. We were able to achieve far more rich conclusions extracting specific individual commodities benefits that are not accounted when we just use indices. Given the post-crisis concerns about downside risk and a repetition of past mistakes regarding risk managing, we depict investors as loss-averse.

We confirm the increasing correlations across sub-periods between stocks and commodities, supporting the idea of a financialization effect. Empirical results reveal a lower performance in the out-of-sample analyze compare to the in-sample results, achieving much lower returns. This might mean that previous benefits from in-sample analysis might be overstated to the real world conditions were we have uncertain return forecasts. Nevertheless, risk benefits are preserved. The majority of the augment portfolios reduce volatility and tail risk. Improvements in the risk-adjust returns are only attainable in mean variance allocation strategies, but these strategies have lower benefits regarding the tail risk. We conclude that a simple allocation to commodities can effectively reduce risk as we see by the results from the 20% strategic weight strategy.

Gold exhibits the highest performance with stocks reducing risk and enhancing returns when we use allocation strategies that require return and risk estimators. Using strategies that require only risk estimators silver performs better. Soybeans and sugar had abnormal performance during specific economic recessions, creating unique diversification benefits.

Overall, energy and grain commodities show the potential to improve returns and risk-adjust performances especially during expansion periods. However they fail on average to substantial reduce risk. Livestock commodities display a high potential to reduce risk especially during recessions. The precious metal sector yields diversification benefits during both recessions and expansions environments. Thus, we confirm Belousova & Dorfleitner (2012) findings that precious metals can be recommended, when investors face the choice concerning the commodity sector exposure. Our study confirms the high sensitivity of commodities to market economic conditions. In a more recent period, commodities did not yield as much benefits as before.

To summarize, our results shed light that there is still a diversification benefit, although it is time dependent and the benefits have been decreasing over time.

The stress tests shows that results can be affected by shorter parameters estimation windows, creating such an instability on weights that negatively affect performance. More frequent rebalancing periods enhance performance but in the presence of transaction costs these benefits cease to exist. Transactions costs only show a negative significant impact on returns at the level of 200 bps. Results were robust under larger rebalancing periods and estimation windows, with transaction costs at 50 bps and with different benchmarks for the stock portfolio.

This study also has some caveats, the most relevant are the parameter estimators that are based on historical estimations. Historical estimators lack the forward-looking information, we did not use forecast models[15], as it was not the purpose of this study.

Future research should focus on developing alternative utility functions for the target investors, expand research to different types of investors that exhibit other emotional and cognitive biases. Further studies should also include the bond market along with stocks and commodities.

[15] Not only we should build forecast models for returns and volatility but also for other higher-order moments of return distribution like skewness and kurtosis as they as they are key determinants of tail risk.

8 Bibliography

Ankrim, E. M., & Hensel, C. R. (1993). Commodities in Asset Allocation: A Real-Asset Alternative to Real Estate? *Financial Analysts Journal*, *49*(3), 20–29.

Basak, S., & Pavlova, A. (2016). A Model of Financialization of Commodities. *The Journal of Finance*, *71*(4), 1511–1556.

Belousova, J., & Dorfleitner, G. (2012). On the diversification benefits of commodities from the perspective of euro investors. *Journal of Banking and Finance*, *36*(9), 2455–2472.

Bessembinder, H. (1992). Systematic Risk, Hedging Pressure, and Risk Premiums in Futures Markets. *Review of Financial Studies*, *5*(4), 637–667.

Bessembinder, H. (2018). The "Roll Yield" Myth. *Financial Analysts Journal*, *74*(2), 41–53.

Bessler, W., Opfer, H., & Wolff, D. (2017). Multi-asset portfolio optimization and out-of-sample performance: an evaluation of Black–Litterman, mean-variance, and naïve diversification approaches. *European Journal of Finance*, *23*(1), 1–30.

Bessler, W., & Wolff, D. (2015). Do commodities add value in multi-asset portfolios? An out-of-sample analysis for different investment strategies. *Journal of Banking & Finance*, *60*, 1–20.

Best, M. J., & Grauer, R. R. (1991). On the sensitivity of mean-variance-efficient portfolios to changes in asset means: some analytical and computational results. *The Review of Financial Studies*, *4*(2), 315–342.

Bodie, Z., & Rosansky, V. I. (1980). Risk and Return in Commodity Futures. *Financial Analysts Journal*, *36*(3), 27–39.

Büyüksahin, B., Haigh, M. S., & Robe, M. A. (2009). Commodities and Equities: Ever a "Market of One"? *The Journal of Alternative Investments*, *12*(3), 76–95.

Cao, B., Jayasuriya, S., & Shambora, W. (2010). Holding a commodity futures index fund in a globally diversified portfolio: A placebo effect. *Economics Bulletin*, *30*(3), 1842–1851.

Chantziara, T., & Skiadopoulos, G. (2008). Can the dynamics of the term structure of petroleum futures be forecasted? Evidence from major markets. *Energy Economics*, *30*(3), 962–985.

Chopra, V. K., & Ziemba, W. T. (2013). The effect of errors in means, variances, and covariances on optimal portfolio choice. In *Handbook of the Fundamentals of Financial Decision Making: Part I* (pp. 365–373). World Scientific.

Conover, C. M., Jensen, G. R., Johnson, R. R., & Mercer, J. M. (2010). Is Now the Time to Add Commodities to Your Portfolio? *The Journal of Investing*, *19*(3), 10–19.

Daigler, R. T., Dupoyet, B., & You, L. (2017). Spicing Up a Portfolio with Commodity Futures: Still a Good Recipe? *The Journal of Alternative Investments*, *19*(4), 8–23.

Daskalaki, C., & Skiadopoulos, G. (2011). Should investors include commodities in their portfolios after all? New evidence. *Journal of Banking and Finance*, *35*(10), 2606–2626.

DeMiguel, V., Garlappi, L., & Uppal, R. (2007). Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? *The Review of Financial Studies*, *22*(5), 1915–1953.

Erb, C. B., & Harvey, C. R. (2006). The Strategic and Tactical Value of Commodity Futures. *Financial Analysts Journal*, *62*(2), 69–97.

Erb, C. B., & Harvey, C. R. (2016). Conquering Misperceptions about Commodity Futures Investing. *Financial Analysts Journal*, *72*(5), 5–6.

Fama, E. F., & French, K. R. (1988). Business cycles and the behavior of metals prices. *The Journal of Finance*, *43*(5), 1075–1093.

Fortenbery, T. R., & Hauser, R. J. (1990). Investment Potential of Agricultural Futures Contracts. *American Journal of Agricultural Economics*, *72*(3), 721.

Gao, X., & Nardari, F. (2018). Do Commodities Add Economic Value in Asset Allocation? New Evidence from Time-Varying Moments. *Journal of Financial and Quantitative Analysis*, *53*(1), 365–393.

Georgiev, G. (2001). Benefits of Commodity Investment. *The Journal of Alternative Investments*, *4*(1), 40–48.

Gibson, R. C. (2004). The Rewards of Multiple-Asset-Class Investing. *Journal of Financial Planning*, *17*(7), 58–69.

Gorton, G., & Geert Rouwenhorst, K. (2006). Facts and fantasies about commodity futures. *Financial Analysts Journal*, *62*(2), 47–68.

Hammoudeh, S., Araújo Santos, P., & Al-Hassan, A. (2013). Downside risk management and VaR-based optimal portfolios for precious metals, oil and stocks. *North American Journal of Economics and Finance*, *25*, 318–334.

Hillier, D., Draper, P., & Faff, R. (2006). Do precious metals shine? An investment perspective. *Financial Analysts Journal*, *62*(2), 98–106.

Jensen, G. R., Johnson, R. R., & Mercer, J. M. (2000). Efficient use of commodity futures in diversified portfolios. *Journal of Futures Markets*, *20*(5), 489–506.

Kahenman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, *47*(2), 363–391.

Keating, C., & Shadwick, W. F. (2002). A universal performance measure. *Journal of Performance Measurement*, *6*(3), 59–84.

Levine, A., Ooi, Y. H., Richardson, M., & Sasseville, C. (2018). Commodities for the Long Run. *Financial Analysts Journal*, *74*(2), 55–68.

Main, S., Irwin, S. H., Sanders, D. R., & Smith, A. (2018). Financialization and the returns to commodity investments. *Journal of Commodity Markets*, *10*(June), 22–28.

Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, *7*(1), 77–91.

Meucci, A. (2010). Quant nugget 4: annualization and general projection of skewness, kurtosis and all summary statistics. *GARP Risk Professional-" The Quant Classroom*, 59–63.

Jensen, G. & Mercer J. (2011). Commodities as an Investment. *Research Foundation Literature Reviews*, 1–33.

Sharpe, W. F. (1966). Mutual fund performance. *The Journal of Business*, *39*(1), 119–138.

Tang, K., & Xiong, W. (2012). Index investment and the financialization of commodities. *Financial Analysts Journal*, *68*(6), 54–74.

Willenbrock, S. (2011). Diversification return, portfolio rebalancing, and the commodity return puzzle. *Financial Analysts Journal*, *67*(4), 42–49.

World Federation of Exchanges (2019). *WFE IOMA 2018 Derivatives Report*, April 2019. Retrieve from: https://www.worldexchanges.org/storage/app/media/statistics/WFE%20 2018%20IOMA%20Derivatives%20Report%20FINAL%2010.04.19.pdf

Yan, L., & Garcia, P. (2017). Portfolio investment: Are commodities useful? *Journal of Commodity Markets*, *8*(May 2016), 43–55.

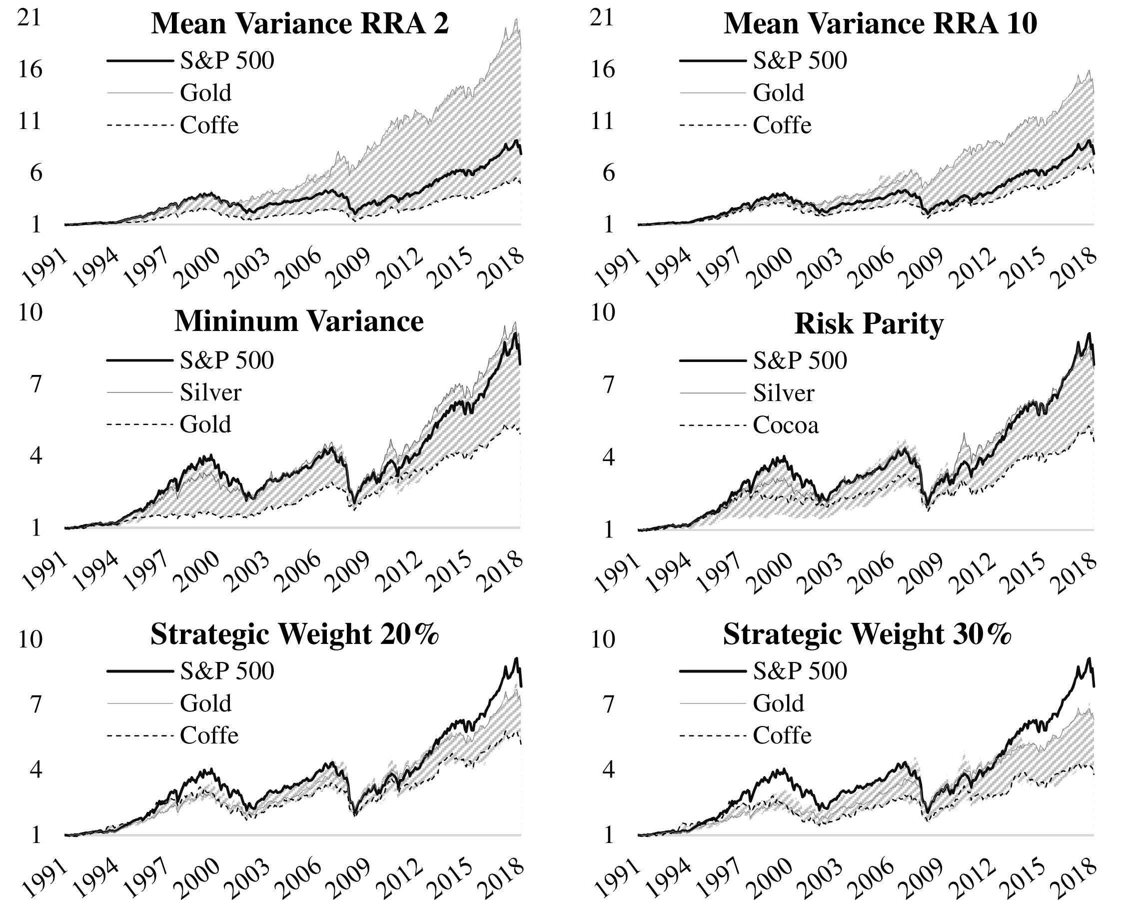
You, L., & Daigler, R. T. (2010). Using Four-Moment Tail Risk to Examine Financial and Commodity Instrument Diversification. *Financial Review*, *45*(4), 1101–1123.

You, L., & Daigler, R. T. (2013). AM arkowitz Optimization of Commodity Futures Portfolios. *Journal of Futures Markets*, *33*(4), 343–368.

Zaremba, A. (2015). Is Financialization Killing Commodity Investments? *The Journal of Alternative Investments*, *18*(1), 66–91.

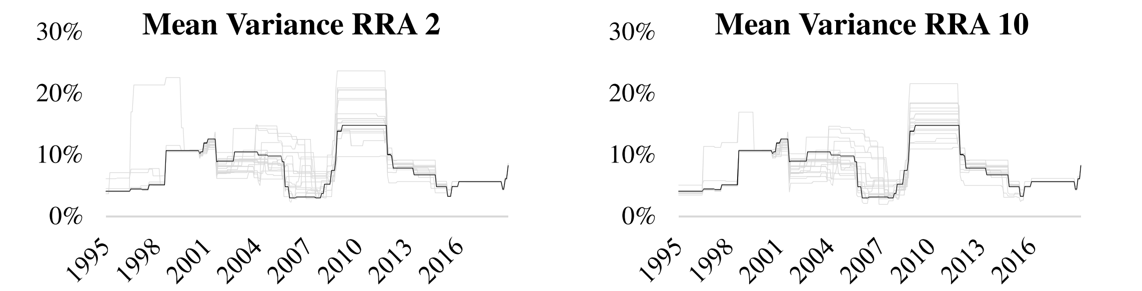
9 Appendix

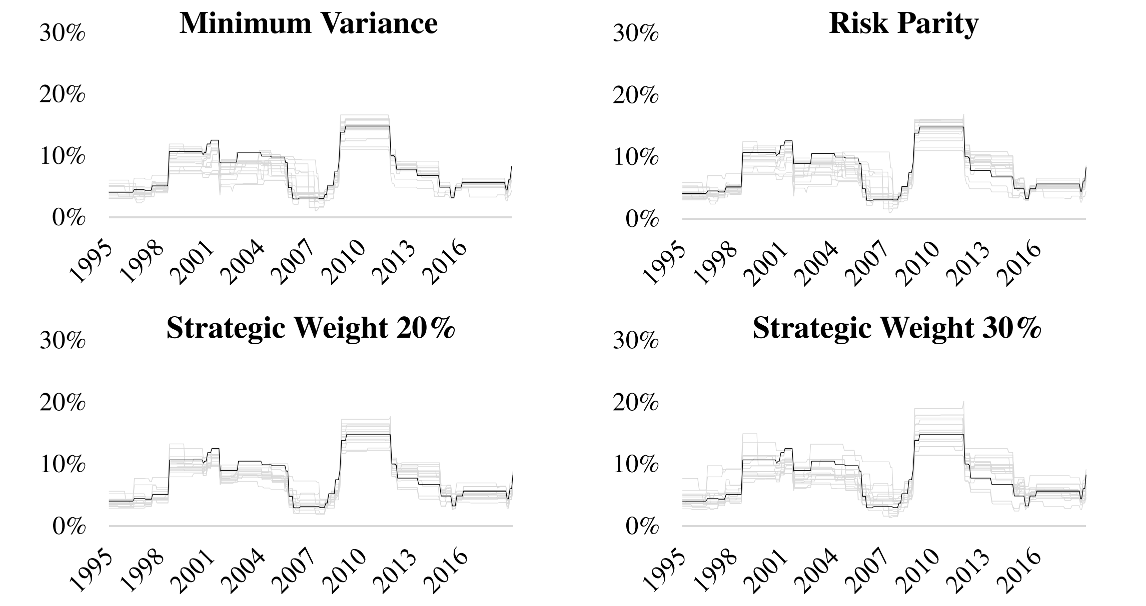
Figure 2 - Out-of-sample portfolio total growth for each investment strategy.



Notes: The graphs represent the portfolio value in U.S. Dollars across time with an initial investment in each portfolio of 1 U.S. Dollar. The grey area represents the range of possible outcomes for the individual commodities in combination with stocks. The stock portfolio without commodities is represented by the S&P 500 index, as mentioned in the methodology. In each graph we also highlight the commodity with the highest and the lowest final portfolio value represented by a solid grey line and a dash line respectively.

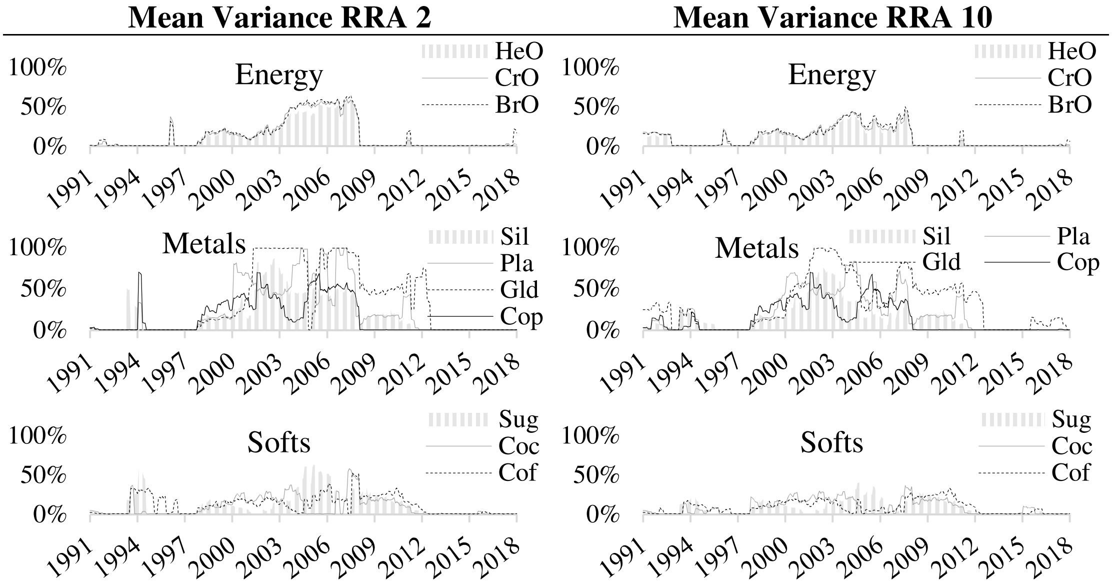
Figure 3 - Out-of-sample portfolios 36 month rolling monthly TVaR.

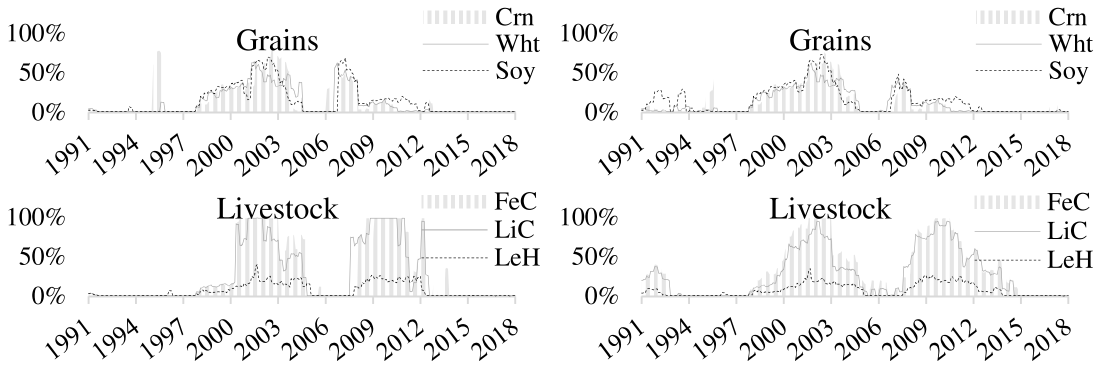




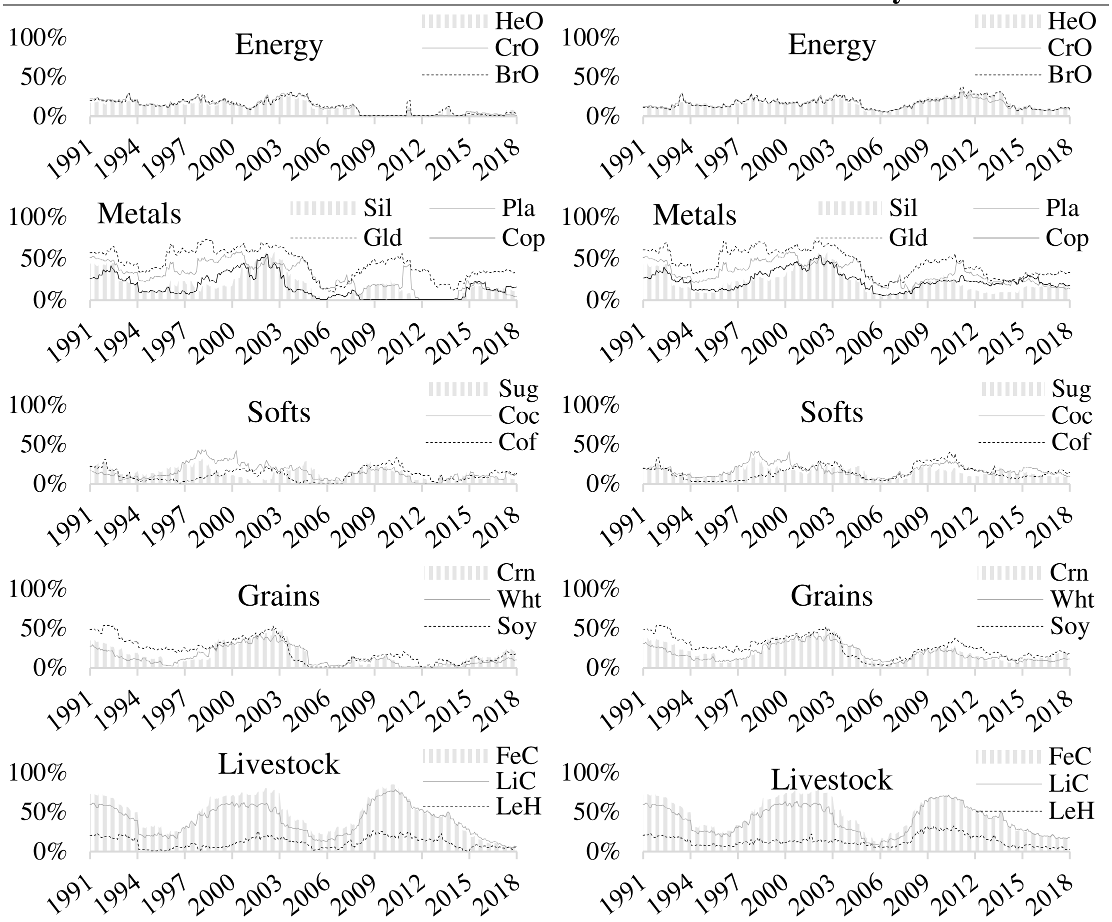
Notes: The graphs represent the out-of-sample rolling 36 month 5% historical tail value at risk from January 1989 to December 2018. TVaR are expressed in percentage with positive values for all the allocation strategies and roll over for the full period over 36-month periods. The grey lines represent the augment portfolios, the highlight line is the stock only portfolio. Lines below the base portfolio indicate benefits by reducing the tail risk. Lines above the base portfolio do not decrease the portfolio tail risk.

Figure 4 - Augment portfolios weights for MV, MinVar and RP strategies by commodity sector.





Minimum Variance **Risk Parity**



Notes: The graphs represent the weights of commodities for each augment portfolio of individual commodities. The out-of-sample results are represent from January 1991 to December 2018. Results are divided by allocation strategy and commodity sector. Remember that each augment portfolio has two assets the commodity and the stock index. Thus, the stock index weight is equal to (1-$w_{commodity}$).

Table I - General overview of the futures commodities' sample

Commodity	Unit	Contract Size	Source
Energy			
Crude Oil	USD/bbl.	1,000 barrels	New York Mercantile Exchange
Brent Oil	USD/bbl.	1,000 barrels	ICE Futures Europe Commodities
Heating Oil	USD/gal.	42,000 US gallons	New York Mercantile Exchange
Precious metals			
Gold	USD/t oz.	100 troy oz.	Commodity Exchange, Inc.
Silver	USD/t oz.	5,000 troy oz.	Commodity Exchange, Inc.
Platinum	USD/t oz.	50 troy Oz.	New York Mercantile Exchange
Industrial metals			
Copper	USD/bu.	5,000 bushels	Chicago Board of Trade
Softs			
Cocoa	USD/MT	10 metric tons	NYB - ICE Futures US Softs
Coffee	USD/lb.	37,500 lbs.	NYB - ICE Futures US Softs
Sugar	USD/lb.	112,000 lbs.	NYB - ICE Futures US Softs
Grains			
Corn	USD/bu.	5,000 bushels	Chicago Board of Trade
Wheat	USD/bu.	5,000 bushels	Chicago Board of Trade
Soybeans	USD/bu.	5,000 bushels	Chicago Board of Trade
Livestock			
Feeder Cattle	USD/lb.	50,000 lbs.	Chicago Mercantile Exchange
Live Cattle	USD/lb.	40,000 lbs.	Chicago Mercantile Exchange
Lean Hogs	USD/lb.	40,000 lbs.	Chicago Mercantile Exchange

Notes: This table describes our sample of individual future commodity contracts.

Table II - Sample assets descriptive statistics for the full period (1989-2018)

Assets	Return (%)	Volatility (%)	Sharpe ratio	Skewness	Excess kurtosis	Jarque-Bera p-value
Indices						
S&P 500	9.29	14.27	0.42	-0.79	1.81	0.0000
S&P GSCI	2.42	20.64	-0.04	-0.51	2.64	0.0000
Energy						
Crude Oil	3.28	32.08	0.00	-0.21	1.76	0.0000
Brent Oil	4.07	31.74	0.02	-0.21	2.52	0.0000
Heating Oil	4.09	31.30	0.03	-0.15	1.55	0.0000
Precious metals						
Gold	3.96	15.25	0.04	-0.05	1.40	0.0000
Silver	3.27	27.64	0.00	-0.22	1.33	0.0000
Platinum	1.44	20.54	-0.09	-1.03	5.40	0.0000
Industrial metals						
Copper	1.97	25.64	-0.05	-0.46	4.70	0.0000
Softs						
Cocoa	1.72	30.19	-0.05	0.05	0.66	0.0365
Coffee	-0.88	35.51	-0.12	0.43	1.95	0.0000
Sugar	0.46	32.50	-0.09	-0.08	0.70	0.0217

Grains						
Corn	1.04	27.77	-0.08	-0.64	1.98	0.0000
Wheat	0.45	29.30	-0.10	0.16	0.99	0.0003
Soybeans	0.44	24.88	-0.11	-0.88	3.06	0.0000
Livestock						
Feeder Cattle	1.89	14.63	-0.10	-0.58	2.61	0.0000
Live Cattle	1.69	16.76	-0.10	-0.73	3.39	0.0000
Lean Hogs	1.14	36.52	-0.06	-0.37	1.74	0.0000

Notes: This table represents the descriptive statistics of the assets under analyze for the full period. We compute returns, volatilities, Sharpe ratios, skewness, excess kurtosis and the Jarque Bera p-value. We use the Libor 1 month as the risk free asset. We reject the null hypothesis that returns are normally distributed, but at 5% significance level.

Table III - Correlation matrix across asset sectors for the full period and across sub-periods.

Full Period 1989 - 2018	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
Stock Index (a)	1.00								
Commodity Index (b)	0.49	1.00							
Energy (c)	0.41	0.94	0.96						
Precious metals (d)	0.20	0.40	0.31	0.87					
Industrial metals (e)	0.53	0.55	0.43	0.40	1.00				
Softs (f)	0.15*	0.26	0.20	0.22	0.20	0.61			
Grains (g)	0.28	0.38	0.24	0.21	0.29	0.21	0.82		
Livestock (h)	0.04*	0.10*	0.09*	-0.04*	0.04*	-0.05*	-0.12*	0.68	
LIBOR (i)	-0.13*	-0.04*	-0.04*	-0.03*	-0.03*	-0.04*	0.02*	-0.02*	1.00
Sub-Period 1989 - 1998	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
Stock Index (a)	1.00								
Commodity Index (b)	-0.14*	1.00							
Energy (c)	-0.22	0.85	0.95						
Precious metals (d)	-0.01*	0.03*	0.01*	0.79					
Industrial metals (e)	0.11*	0.09*	0.02*	0.15*	1.00				
Softs (f)	-0.04*	0.03*	-0.01*	0.04*	0.01*	0.51			
Grains (g)	0.12*	0.14*	-0.07*	0.02*	0.06*	0.07*	0.72		
Livestock (h)	0.10*	0.12*	0.07*	0.06*	-0.03*	0.01*	0.01*	0.62	
LIBOR (i)	0.03*	0.06*	0.08*	-0.10*	-0.08*	-0.09*	-0.07*	0.02*	1.00
Sub-Period 1999 - 2008	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
Stock Index (a)	1.00								
Commodity Index (b)	0.22	1.00							
Energy (c)	0.18*	0.92	0.97						
Precious metals (d)	0.14*	0.35	0.26	0.78					
Industrial metals (e)	0.44	0.51	0.41	0.43	1.00				
Softs (f)	0.09*	0.12*	0.06*	0.22	0.21	0.61			
Grains (g)	0.19	0.23	0.10*	0.21	0.17*	0.16*	0.76		
Livestock (h)	0.02*	0.06*	0.02*	0.02*	0.06*	0.02*	0.02*	0.62	
LIBOR (i)	0.02*	0.07*	0.07*	-0.03*	0.02*	-0.05*	0.05*	0.01*	1.00

Sub-Period 2009 - 2018	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
Stock Index (a)	1.00								
Commodity Index (b)	0.49	1.00							
Energy (c)	0.41	0.94	0.96						
Precious metals (d)	0.17*	0.40	0.30	0.87					
Industrial metals (e)	0.51	0.54	0.43	0.41	1.00				
Softs (f)	0.13*	0.26	0.20	0.23	0.21	0.61			
Grains (g)	0.28	0.38	0.24	0.20	0.29	0.20	0.82		
Livestock (h)	0.05*	0.10*	0.09*	-0.05*	0.04*	-0.05*	-0.12*	0.68	
LIBOR (i)	-0.12*	-0.04*	-0.04*	-0.03*	-0.03*	-0.04*	0.02*	-0.02*	1.00

Notes: The table represents the calculated correlation between each group pair as the average correlations between each pair of individual assets within those two groups. The within group correlation represented by the diagonal line, is the average correlation between each pair of individual assets belonging to the same group. Due to the heterogeneity within each commodity sector the diagonal line is not 1. The more heterogeneous the group is the lower is the value of the diagonal correlation. The same approach is used in Daigler et al. (2017).

* statistical significant at 5% level.

Table IV - Sample assets descriptive statistics across sub-periods of 10 years, from 1989 to 2018.

	Period 1989 – 1998						Period 1999 – 2008						Period 2009 – 2018					
Assets	Ret. (%)	Vol. (%)	SR	Skew.	Kurt.	JB p-value	Ret. (%)	Vol. (%)	SR	Skew.	Kurt.	JB p-value	Ret. (%)	Vol. (%)	SR	Skew.	Kurt.	JB p-value
Benchmark Assets																		
S&P 500	17.01	13.30	0.85	-0.84	2.76	0.000	-1.82	15.40	-0.35	-0.84	1.79	0.000	12.33	13.60	0.87	-0.57	0.82	0.008
S&P GSCI	-3.14	15.73	-0.56	0.52	2.48	0.000	9.56	25.53	0.23	-0.95	2.58	0.000	0.70	19.56	0.01	-0.29	1.01	0.032
Energy																		
Crude Oil	-3.49	30.06	-0.31	0.45	3.74	0.000	12.63	36.11	0.25	-0.60	1.27	0.001	0.18	29.86	-0.01	-0.30	0.95	0.043
Brent Oil	-4.16	30.76	-0.32	0.48	4.77	0.000	14.02	36.04	0.29	-0.63	1.90	0.000	1.66	27.99	0.04	-0.38	1.08	0.013
Heating Oil	-3.68	30.98	-0.30	0.21	2.26	0.000	14.74	36.03	0.31	-0.39	1.23	0.005	1.52	26.31	0.04	-0.32	0.59	0.152
Precious metals																		
Gold	-3.06	11.16	-0.79	-0.08	0.71	0.266	11.37	17.03	0.46	-0.27	2.08	0.000	3.71	16.71	0.19	-0.04	0.19	0.903
Silver	-1.52	22.64	-0.32	0.04	0.93	0.117	7.75	28.25	0.15	-0.62	1.35	0.000	3.19	31.52	0.08	-0.06	1.10	0.046
Platinum	-3.53	14.98	-0.62	0.05	-0.39	0.667	10.01	24.69	0.26	-1.74	8.09	0.000	-1.62	20.74	-0.10	-0.43	0.37	0.112
Industrial metals																		
Copper	-7.83	22.90	-0.59	0.01	1.02	0.076	7.89	30.34	0.14	-0.79	6.20	0.000	6.24	23.02	0.25	-0.34	2.61	0.000
Softs																		
Cocoa	-0.48	25.35	-0.24	0.36	0.29	0.228	7.00	36.23	0.09	0.13	0.55	0.402	-0.98	28.27	-0.05	-0.48	-0.14	0.092
Coffee	-1.20	42.80	-0.16	0.25	1.80	0.000	0.76	33.59	-0.09	0.48	0.31	0.081	-0.95	29.03	-0.05	0.82	2.61	0.000
Sugar	-2.91	26.52	-0.33	0.20	1.69	0.001	5.12	36.05	0.04	-0.21	0.06	0.649	0.18	34.32	-0.01	-0.13	0.78	0.185
Grains																		
Corn	-2.54	24.61	-0.34	-1.65	6.24	0.000	6.46	28.70	0.10	-0.19	0.24	0.611	-0.82	29.99	-0.05	-0.49	1.27	0.002
Wheat	-4.71	25.66	-0.41	0.12	0.54	0.419	8.03	28.52	0.15	-0.04	-0.09	0.962	-1.94	33.40	-0.07	0.30	1.51	0.001
Soybeans	-3.65	17.74	-0.53	-0.69	0.77	0.002	6.57	30.66	0.10	-1.18	3.66	0.000	-0.97	24.70	-0.06	-0.46	0.08	0.117
Livestock																		
Feeder Cattle	-2.01	12.10	-0.64	0.22	1.30	0.009	2.61	14.96	-0.07	-1.07	5.33	0.000	4.59	16.51	0.25	-0.58	1.00	0.003
Live Cattle	-2.13	14.45	-0.54	-0.12	0.26	0.725	3.31	19.18	-0.02	-1.27	5.59	0.000	3.64	16.47	0.19	-0.32	0.81	0.068
Lean Hogs	-2.86	32.50	-0.26	-0.11	1.08	0.049	3.98	37.95	0.01	-0.68	4.03	0.000	0.02	38.50	-0.01	-0.30	0.06	0.406

Notes: This table represents the descriptive statistics of the assets under analyze divided in periods of 10 years across our full period. We compute returns, volatilities, Sharpe ratios, skewness, excess kurtosis and the Jarque Bera p-value. We use the Libor 1 month as the risk free asset. On average, we reject the null hypothesis that returns are normally distributed, but at 5% significance level we have some exceptions in some individual commodities.

Table V - Correlation analysis of individual assets for the full period 1989 to 2018

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)	(l)	(m)	(n)	(o)	(p)	(q)	(r)	(s)
S&P 500 (a)	1.00																		
S&PGSCI (b)	.49	1.00																	
Crude Oil (c)	.40	.94	1.00																
Brent Oil (d)	.41	.96	.92	1.00															
Heating Oil (e)	.43	.92	.87	.96	1.00														
Gold (f)	.03*	.30	.18	.23	.24	1.00													
Silver (g)	.22	.44	.32	.36	.34	.83	1.00												
Platinum (h)	.34	.46	.35	.37	.34	.69	.70	1.00											
Copper (i)	.53	.55	.42	.44	.43	.27	.41	.53	1.00										
Cocoa (j)	.21	.30	.23	.28	.32	.17	.24	.22	.15	1.00									
Coffee (k)	.15	.30	.21	.18	.19	.39	.39	.33	.23	.27	1.00								
Sugar (l)	.09*	.19	.14	.13	.11*	.06*	.06*	.16	.23	.11	.25	1.00							
Corn (m)	.23	.37	.20	.23	.26	.24	.30	.23	.27	.21	.35	.11	1.00						
Wheat (n)	.28	.32	.15	.17	.19	.19	.18	.23	.26	.16	.33	.12	.70	1.00					
Soybeans (o)	.33	.44	.28	.33	.33	.11*	.18	.22	.35	.11*	.29	.19	.68	.53	1.00				
Feeder Cattle (p)	.09*	.08*	.15	.12	.07*	-.15	-.08*	.02*	.03*	-.08*	-.04*	-.18	-.36	-.20	-.22	1.00			
Live Cattle (q)	-.02*	-.04*	-.02*	-.04*	-.07*	-.09*	-.02*	.04*	.01*	-.16	-.06*	-.08*	-.17	-.15	-.14	.67	1.00		
Lean Hogs (r)	.04*	.24	.23	.21	.16	-.07*	.01*	.02*	.08*	.07*	.06*	.05*	.02*	.12	.04*	.25	.12	1.00	
LIBOR (s)	-.13	-.04*	-.06*	-.03*	-.03*	-.01*	-.02*	-.06*	-.03*	.02*	-.07*	-.06*	.03*	.05*	-.03*	-.03*	-.01*	-.02*	1.00

Notes: We display the Pearson correlation across the assets under analyze. 16 individual commodity, the S&P 500 index, the S&P Goldman Sachs Index and the Libor 1 month. We test correlation significance. Although, we need to alert that when data is nonnormally distributed, a test of the significance of Pearson's correlation may inflate Type I error rates and reduce its power. However, the relative performance of alternative methods has been unclear. Sugar, livestock sector and LIBOR display the lower correlations, and for the majority they are not statistically significant at the 5% level.

* statistical significant at 5% level

Table VI - In-sample results for all augment-portfolios for all strategies

Strategy	**Performance Measure**	**Base Portfolio**	**Energy**			**Metals**				**Softs**			**Grains**			**Livestock**			Index
			CrO	BrO	HeO	Gld	Sil	Pla	Cop	Coc	Cof	Sug	Crn	Wht	Soy	FeC	LiC	LeH	
Mean Variance RRA 2	Return (%)	8.7	13.7	13.3	12.3	14.9	13.3	14.1	11.4	9.9	11.5	11.1	13.2	10.4	13.8	14.2	14.2	12.2	13.9
	Volatility (%)	14.1	14.9	15.3	14.6	13.8	15.3	14.8	15.2	13.9	16.1	14.7	14.2	14.8	14.5	11.8	12.4	15.3	14.5
	Sharpe	.41	.73	.69	.65	.88	.68	.76	.57	.51	.54	.57	.73	.51	.76	.96	.92	.62	.76
	IR	.00	.58	.50	.45	.51	.43	.45	.30	.11	.25	.26	.49	.18	.52	.46	.48	.39	.51
	Omega	1.4	1.7	1.7	1.6	1.9	1.7	1.8	1.5	1.5	1.5	1.5	1.7	1.5	1.8	2.0	2.0	1.6	1.8
	VaR 5%	-7.1	-6.3	-6.3	-6.3	-5.1	-6.2	-6.4	-6.6	-5.8	-6.6	-6.3	-5.9	-6.2	-6.2	-4.7	-5.9	-6.3	-6.3
	TVaR 5%	-9.7	-8.8	-9.3	-9.5	-8.0	-9.6	-8.8	-9.6	-8.4	-9.5	-9.5	-8.8	-9.3	-9.3	-6.4	-7.6	-9.6	-9.7
Mean Variance RRA 10	Return (%)	8.7	13.4	13.6	12.2	14.8	13.0	14.1	11.5	10.3	11.4	10.8	13.2	10.9	13.1	14.3	14.2	12.0	13.8
	Volatility (%)	14.1	14.9	15.1	14.5	13.6	15.2	14.5	15.1	13.7	16.2	14.5	14.0	14.7	14.3	11.6	12.2	15.2	14.3
	Sharpe	.41	.72	.71	.65	.89	.67	.78	.57	.55	.53	.55	.74	.55	.72	.99	.93	.60	.77
	IR	.00	.56	.55	.46	.50	.41	.47	.32	.17	.25	.23	.54	.25	.46	.48	.49	.37	.53
	Omega	1.4	1.7	1.7	1.6	1.9	1.6	1.8	1.5	1.5	1.5	1.5	1.7	1.5	1.7	2.0	2.0	1.6	1.8
	VaR 5%	-7.1	-6.3	-6.3	-6.3	-5.1	-6.2	-6.4	-6.6	-5.6	-6.5	-6.3	-5.9	-6.2	-6.2	-4.3	-5.9	-6.5	-6.2
	TVaR 5%	-9.7	-8.9	-9.3	-9.5	-8.0	-9.5	-8.3	-9.6	-8.2	-9.5	-9.5	-8.5	-9.3	-9.3	-6.3	-7.6	-9.6	-9.5
Minimum Variance	Return (%)	8.7	8.6	8.7	8.6	6.5	9.0	8.2	8.3	6.8	8.2	7.7	8.2	8.1	7.9	7.1	6.9	7.8	7.7
	Volatility (%)	14.1	13.4	13.5	13.6	10.5	13.5	12.7	13.7	12.8	13.8	13.5	13.3	13.4	13.3	10.7	11.0	13.7	12.7
	Sharpe	.41	.43	.44	.43	.35	.46	.42	.40	.31	.39	.37	.40	.40	.38	.40	.37	.36	.38
	IR	.00	-.02	-.00	-.01	-.22	.06	-.07	-.09	-.29	-.10	-.16	-.09	-.11	-.12	-.14	-.18	-.17	-.15
	Omega	1.4	1.4	1.4	1.4	1.3	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.3	1.3	1.4	1.3	1.3	1.3
	VaR 5%	-7.1	-6.3	-6.2	-6.3	-4.1	-6.0	-5.9	-6.4	-5.6	-6.5	-6.3	-5.9	-6.1	-6.6	-4.5	-4.8	-6.5	-5.7
	TVaR 5%	-9.7	-9.1	-9.0	-9.2	-6.6	-8.8	-8.8	-9.1	-8.0	-9.1	-8.9	-8.7	-8.9	-9.0	-7.3	-7.6	-9.5	-8.7
Risk Parity	Return (%)	8.7	7.9	7.9	7.6	6.5	8.7	7.5	8.0	6.6	8.2	7.5	7.9	7.6	7.5	7.0	6.6	7.7	6.6
	Volatility (%)	14.1	13.8	13.7	13.8	10.6	13.5	12.6	14.1	12.9	14.0	13.5	13.6	13.6	13.4	10.6	11.0	13.6	12.9
	Sharpe	.41	.37	.37	.35	.35	.44	.38	.36	.29	.38	.34	.37	.35	.35	.40	.35	.36	.29
	IR	.00	-.14	-.14	-.19	-.22	.01	-.15	-.13	-.31	-.08	-.20	-.12	-.18	-.17	-.16	-.20	-.16	-.28
	Omega	1.4	1.3	1.3	1.3	1.3	1.4	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.4	1.3	1.3	1.3
	VaR 5%	-7.1	-6.4	-6.2	-6.2	-4.3	-5.8	-5.6	-6.4	-5.9	-6.3	-6.5	-5.9	-6.1	-6.5	-4.5	-5.0	-6.8	-5.3
	TVaR 5%	-9.7	-9.5	-9.4	-9.6	-6.6	-8.8	-8.7	-9.3	-8.4	-9.1	-9.0	-9.0	-9.1	-9.2	-7.1	-7.5	-9.3	-9.0

Notes: This table reports the in-sample portfolio benefits of commodities. The evaluation period is from January 1991 to December 2018. In grey are the situations where no improvements were achieved compared to the performance indicator from the base portfolio. We divide the table by asset allocation strategy and represent the base portfolio (stock only) with the 17 augment portfolios. We compute returns, volatilities, Sharpe ratios, information ratios, omega ratios, the monthly 5% historical 36 months value at risk and the monthly 5% historical 36 months tail value at risk.

Table VII - Out-of-sample results for all augment-portfolios for all strategies.

	Performance Measure	**Base Portfolio**	**Energy**			**Metals**				**Softs**			**Grains**			**Livestock**			Index
			CrO	BrO	HeO	Gld	Sil	Pla	Cop	Coc	Cof	Sug	Crn	Wht	Soy	FeC	LiC	LeH	
Mean Variance RRA 2	Return (%)	8.7	9.3	9.6	9.7	11.7	10.2	9.5	9.7	9.1	6.8	7.8	8.2	9.9	10.4	8.1	8.0	8.1	10.4
	Volatility (%)	14.1	15.3	15.4	15.0	13.6	15.0	15.9	15.5	13.9	14.2	14.5	15.7	14.3	14.5	13.2	13.1	14.0	15.0
	Sharpe	.41	.43	.44	.46	.65	.49	.42	.45	.46	.28	.34	.34	.50	.52	.40	.40	.38	.50
	IR	.00	.09	.11	.15	.25	.17	.08	.14	.07	-.33	-.12	-.05	.16	.21	-.05	-.06	-.10	.18
	Omega	1.4	1.4	1.4	1.4	1.6	1.5	1.4	1.4	1.4	1.2	1.3	1.3	1.5	1.5	1.4	1.4	1.3	1.5
	VaR 5%	-7.1	-6.4	-6.2	-6.8	-5.1	-6.2	-6.7	-6.3	-5.7	-6.9	-6.3	-6.5	-6.1	-6.7	-5.5	-6.2	-6.5	-6.4
	TVaR 5%	-9.7	-10.2	-10.3	-10.1	-8.2	-10.0	-11.2	-9.9	-8.8	-9.4	-9.5	-11.5	-9.3	-10.3	-9.0	-9.0	-10.0	-10.4
Mean Variance RRA 10	Return (%)	8.7	9.0	9.2	9.3	10.5	9.3	9.1	9.6	8.2	7.6	8.3	7.9	9.1	9.3	8.5	8.0	8.1	9.8
	Volatility (%)	14.1	14.3	14.4	14.2	12.6	14.3	14.6	14.9	13.5	13.9	14.0	14.5	13.7	14.1	12.4	12.7	13.8	14.0
	Sharpe	.41	.43	.44	.46	.61	.45	.43	.46	.40	.34	.39	.35	.46	.46	.46	.41	.38	.50
	IR	.00	.06	.09	.12	.18	.08	.05	.15	-.08	-.26	-.07	-.10	.06	.09	-.02	-.07	-.11	.15
	Omega	1.4	1.4	1.4	1.4	1.6	1.4	1.4	1.4	1.4	1.3	1.3	1.3	1.4	1.4	1.4	1.4	1.3	1.5
	VaR 5%	-7.1	-6.2	-6.2	-6.2	-5.0	-6.0	-6.4	-6.3	-5.7	-6.7	-6.3	-6.5	-5.7	-6.7	-4.6	-5.8	-6.5	-5.7
	TVaR 5%	-9.7	-9.7	-9.8	-9.6	-7.9	-9.6	-10.5	-9.6	-8.7	-9.3	-9.0	-10.3	-9.1	-9.8	-8.4	-9.1	-9.8	-9.8
Minimum Variance	Return (%)	8.7	8.5	8.5	8.5	6.5	8.9	8.0	8.3	6.9	8.1	7.7	8.0	8.2	7.7	7.1	6.8	7.9	7.5
	Volatility (%)	14.1	13.6	13.7	13.7	10.7	13.8	13.0	14.0	13.0	13.9	13.7	13.5	13.5	13.5	10.9	11.2	13.8	12.9
	Sharpe	.41	.42	.42	.42	.35	.44	.40	.40	.31	.38	.36	.38	.40	.36	.39	.36	.37	.36
	IR	.00	-.04	-.03	-.03	-.22	.03	-.10	-.07	-.27	-.12	-.17	-.12	-.10	-.15	-.15	-.19	-.14	-.17
	Omega	1.4	1.4	1.4	1.4	1.3	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.3	1.3	1.4	1.3	1.3	1.3
	VaR 5%	-7.1	-6.3	-6.2	-6.4	-4.3	-6.2	-6.4	-6.4	-5.7	-6.5	-6.4	-6.0	-6.1	-6.7	-4.5	-4.9	-6.5	-5.7
	TVaR 5%	-9.7	-9.3	-9.2	-9.4	-6.7	-9.1	-9.1	-9.2	-8.1	-9.1	-9.0	-8.9	-8.9	-9.3	-7.4	-7.8	-9.5	-8.9
Risk Parity	Return (%)	8.7	7.9	7.8	7.7	6.4	8.5	7.4	8.1	6.6	8.1	7.4	7.7	7.7	7.2	7.0	6.6	7.7	6.6
	Volatility (%)	14.1	13.8	13.8	13.8	10.7	13.7	12.8	14.2	13.1	14.1	13.7	13.7	13.6	13.6	10.7	11.1	13.6	12.9
	Sharpe	.41	.37	.36	.35	.34	.42	.36	.37	.29	.38	.34	.36	.36	.32	.39	.34	.36	.29
	IR	.00	-.14	-.15	-.18	-.22	-.02	-.17	-.10	-.30	-.09	-.20	-.14	-.16	-.20	-.17	-.21	-.17	-.28
	Omega	1.4	1.3	1.3	1.3	1.3	1.4	1.3	1.3	1.2	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
	VaR 5%	-7.1	-6.3	-6.2	-6.3	-4.3	-6.1	-5.7	-6.4	-5.9	-6.4	-6.8	-6.0	-6.0	-6.6	-4.6	-5.0	-6.9	-5.5
	TVaR 5%	-9.7	-9.5	-9.4	-9.6	-6.7	-9.1	-8.9	-9.4	-8.4	-9.1	-9.2	-9.2	-9.1	-9.5	-7.2	-7.6	-9.3	-9.0

Strategic Weights	Return (%)	8.7	7.8	8.0	7.9	7.9	7.9	7.6	7.8	7.3	7.1	7.1	7.4	7.1	7.4	7.4	7.3	7.1	7.6
	Volatility (%)	14.1	13.9	13.8	13.8	11.8	13.5	12.8	14.1	12.9	14.2	13.4	13.6	13.7	13.4	11.9	11.9	14.0	13.0
	Sharpe	.41	.36	.38	.37	.43	.37	.37	.35	.35	.30	.32	.34	.31	.34	.39	.38	.31	.37
	IR	.00	-.15	-.11	-.13	-.18	-.14	-.23	-.18	-.20	-.21	-.22	-.22	-.26	-.23	-.31	-.31	-.21	-.25
	Omega	1.4	1.3	1.3	1.3	1.4	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
	VaR 5%	-7.1	-6.0	-5.8	-5.9	-5.3	-6.0	-5.9	-6.4	-5.3	-6.7	-6.3	-6.1	-6.0	-6.5	-5.7	-5.6	-6.6	-5.0
	TVaR 5%	-9.7	-9.5	-9.4	-9.4	-7.9	-8.9	-8.8	-9.5	-8.2	-8.9	-8.9	-9.4	-8.9	-9.3	-8.1	-8.0	-9.2	-9.1

Notes: This table reports the out-of-sample portfolio benefits of commodities. The evaluation period is from January 1991 to December 2018. In grey are the situations where no improvements were achieved compared to the performance indicator from the base portfolio. We divide the table by asset allocation strategy and represent the base portfolio (stock only) and the 17 augment portfolios. We compute returns, volatilities, Sharpe ratios, information ratios, omega ratios, the monthly 5% historical 36 months value at risk and the monthly 5% historical 36 months tail value at risk.

Table VIII - Out-of-sample sub-period results for all augment-portfolios using Mean Variance with a RRA 10.

	Performance	**Base**	**Energy**			**Metals**				**Softs**			**Grains**			**Livestock**			Index
	Measure	**Portfolio**	CrO	BrO	HeO	Gld	Sil	Pla	Cop	Coc	Cof	Sug	Crn	Wht	Soy	FeC	LiC	LeH	
Up + (2013 to 2018)	Return (%)	11.5	11.2	10.9	11.2	7.8	11.2	11.2	11.3	11.0	11.0	11.0	11.0	11.3	11.3	10.1	9.2	10.8	11.3
	Volatility (%)	10.8	10.7	10.8	10.7	10.6	10.6	10.7	10.7	10.6	10.5	10.6	10.6	10.7	10.7	10.3	10.0	10.6	10.7
	Sharpe	1.00	.98	.95	.98	.67	.99	.99	.99	.97	.98	.97	.97	.99	.99	.91	.85	.95	.99
	IR	.00	-.68	-.63	-.65	-1.25	-.84	-1.10	-.97	-.53	-.55	-1.03	-.86	-.69	-.52	-.33	-.56	-.43	-1.09
	Omega	2.1	2.1	2.0	2.1	1.7	2.1	2.1	2.1	2.0	2.1	2.0	2.1	2.1	2.1	2.0	1.9	2.0	2.1
	VaR 5%	-5.1	-5.1	-5.1	-5.1	-5.2	-5.0	-5.1	-5.1	-6.2	-5.1	-5.2	-5.0	-5.0	-5.0	-5.1	-5.0	-4.9	-5.1
	TVaR 5%	-7.0	-7.0	-7.0	-6.9	-6.9	-6.9	-6.9	-6.9	-7.1	-6.9	-6.9	-6.9	-6.9	-6.9	-6.9	-6.9	-6.9	-6.9
Down – (04/2008 to 12/2012)	Return (%)	3.8	1.1	1.3	1.6	9.8	4.5	-4.2	0.5	4.1	4.0	5.9	4.2	2.6	3.9	3.7	2.2	3.0	1.0
	Volatility (%)	19.6	21.4	21.8	20.7	16.7	20.3	22.4	21.2	19.7	19.1	18.6	20.5	19.8	19.8	15.5	14.5	18.5	21.1
	Sharpe	.16	.02	.03	.05	.55	.19	-.22	-.01	.18	.18	.28	.17	.10	.17	.20	.11	.13	.02
	IR	.00	-.49	-.39	-.47	.38	.09	-.74	-.70	.04	.02	.24	.05	-.20	.02	-.01	-.10	-.13	-.45
	Omega	1.1	1.0	1.0	1.0	1.5	1.1	0.8	1.0	1.1	1.1	1.2	1.1	1.1	1.1	1.2	1.1	1.1	1.0
	VaR 5%	-9.3	-11.1	-11.1	-9.9	-7.6	-10.0	-10.2	-11.1	-10.0	-10.6	-8.0	-11.1	-10.7	-11.3	-8.4	-7.9	-8.9	-11.1
	TVaR 5%	-13.0	-15.4	-15.9	-14.4	-12.3	-13.3	-17.8	-16.0	-13.5	-13.2	-11.4	-13.8	-14.0	-14.3	-10.7	-10.1	-12.4	-15.1

Up + (07/2004 to 03/2008)	Return (%)	5.8	11.8	12.8	11.9	14.2	10.9	14.0	14.8	6.1	5.2	5.2	5.2	7.6	5.5	4.1	4.7	4.5	12.4
	Volatility (%)	8.7	10.0	9.6	10.2	9.1	10.0	10.8	13.8	8.9	8.7	10.5	9.9	9.9	12.0	7.1	8.1	8.8	10.4
	Sharpe	.19	.76	.90	.76	1.11	.67	.90	.77	.22	.12	.10	.10	.35	.11	-.00	.07	.04	.80
	IR	.00	.61	.69	.62	.85	.66	.62	.71	.05	-.23	-.09	-.12	.25	-.03	-.33	-.35	-.52	.61
	Omega	1.1	1.7	1.9	1.8	2.2	1.6	2.3	2.0	1.2	1.1	1.1	1.1	1.3	1.1	1.0	1.1	1.0	1.7
	VaR 5%	-3.4	-4.7	-3.8	-5.2	-3.1	-4.1	-2.9	-3.0	-3.5	-3.8	-5.3	-3.8	-4.2	-3.1	-3.1	-3.1	-3.2	-4.1
	TVaR 5%	-4.6	-5.3	-4.1	-5.7	-3.8	-4.3	-4.2	-4.2	-4.1	-4.4	-5.8	-5.9	-5.2	-8.8	-4.3	-4.5	-4.5	-4.8
Down – (02/2001 to 06/2004)	Return (%)	-3.7	-2.4	-2.6	-1.9	4.3	-0.0	1.8	1.3	-3.0	-3.3	-4.4	-1.6	1.4	7.1	0.9	-1.3	-3.7	2.8
	Volatility (%)	17.4	15.1	15.4	15.7	13.8	18.3	16.4	17.4	13.8	17.3	17.2	15.8	15.2	16.8	16.5	18.4	17.8	14.7
	Sharpe	-.32	-.29	-.29	-.25	.17	-.11	-.01	-.04	-.36	-.31	-.37	-.22	-.04	.30	-.07	-.18	-.32	.05
	IR	.00	.19	.18	.21	.42	.21	.43	.55	.06	.07	-.25	.14	.38	.79	.21	.11	-.00	.42
	Omega	0.8	0.8	0.8	0.8	1.1	0.9	1.0	1.0	0.8	0.8	0.8	0.9	1.0	1.3	0.9	0.9	0.8	1.0
	VaR 5%	-8.4	-8.4	-8.7	-9.2	-5.6	-6.3	-7.5	-6.4	-5.7	-7.9	-9.2	-6.7	-7.2	-7.3	-4.9	-8.9	-7.8	-7.0
	TVaR 5%	-9.8	-8.8	-8.8	-9.3	-6.9	-11.2	-9.3	-8.3	-6.1	-8.4	-10.0	-7.8	-8.6	-8.0	-11.1	-12.8	-11.2	-9.5
Up + (03/1994 to 01/2001)	Return (%)	17.4	17.3	17.6	17.6	16.0	15.4	17.9	17.0	15.6	14.0	15.7	14.2	16.7	14.9	17.0	17.7	17.1	17.6
	Volatility (%)	14.4	14.0	14.0	14.1	13.5	13.7	12.9	13.8	13.8	14.3	14.3	15.6	13.7	13.7	13.0	13.4	14.0	13.6
	Sharpe	.82	.84	.86	.85	.77	.71	.95	.82	.72	.59	.70	.55	.81	.68	.87	.90	.82	.88
	IR	.00	-.02	.05	.06	-.65	-.69	.14	-.11	-.52	-.81	-.35	-.41	-.17	-.53	-.10	.13	-.18	.08
	Omega	1.8	1.8	1.8	1.8	1.7	1.7	2.0	1.8	1.7	1.5	1.7	1.5	1.8	1.6	1.9	1.9	1.8	1.9
	VaR 5%	-4.8	-4.5	-5.2	-4.4	-4.5	-4.5	-4.2	-4.5	-4.4	-5.7	-5.7	-5.0	-4.7	-4.1	-4.4	-4.4	-4.5	-4.5
	TVaR 5%	-8.0	-7.6	-7.6	-7.4	-7.5	-7.6	-6.9	-7.6	-7.7	-8.3	-8.2	-10.0	-7.4	-7.0	-6.9	-7.2	-7.6	-7.0
Down – (1992 to 02/1994)	Return (%)	8.1	6.7	6.7	7.8	5.5	6.8	7.3	6.1	7.5	6.7	8.1	7.1	7.2	7.3	6.8	6.4	8.2	6.3
	Volatility (%)	7.0	6.5	6.4	6.4	6.3	6.6	6.9	6.8	6.9	7.1	6.9	6.4	6.8	6.1	5.9	6.2	7.0	5.7
	Sharpe	.66	.49	.51	.67	.32	.50	.56	.38	.58	.45	.68	.57	.55	.63	.56	.46	.67	.49
	IR	.00	-.69	-.68	-.16	-.64	-.39	-.95	-1.45	-1.01	-.77	.06	-.79	-1.10	-.36	-.45	-.61	.06	-.81
	Omega	1.6	1.4	1.4	1.6	1.2	1.4	1.5	1.3	1.5	1.4	1.6	1.5	1.5	1.5	1.5	1.4	1.6	1.4
	VaR 5%	-2.4	-2.0	-2.1	-2.1	-2.6	-2.4	-2.6	-2.7	-2.4	-2.3	-2.5	-2.4	-2.4	-2.0	-1.6	-2.7	-2.5	-2.2
	TVaR 5%	-2.6	-2.4	-2.5	-2.5	-2.7	-2.9	-2.6	-2.9	-2.6	-2.5	-2.5	-2.6	-2.8	-2.3	-2.2	-3.0	-2.6	-2.5

Notes: This table reports the out-of-sample portfolio benefits of commodities for the Mean Variance strategy with a risk aversion coefficient of 10 (a more risk averse behavior) across different market environments. Up+ periods represent economic expansions whether down- periods represent recessions. The evaluation period is from January 1991 to December 2018. In grey are the situations where no improvements were achieved compared to the performance indicator from the base portfolio. We divide the table by asset allocation strategy and represent the base portfolio (stock only) and the 17 augment portfolios. We compute returns, volatilities, Sharpe ratios, information ratios, omega ratios, the monthly 5% historical 36 months value at risk and the monthly 5% historical 36 months tail value at risk.

Table IX - Out-of-sample sub-period results for all augment-portfolios using Mean Variance with a RRA 2.

	Performance	**Base**	**Energy**			**Metals**				**Softs**			**Grains**			**Livestock**			Index
	Measure	**Portfolio**	CrO	BrO	HeO	Gld	Sil	Pla	Cop	Coc	Cof	Sug	Crn	Wht	Soy	FeC	LiC	LeH	
Up + (2013 to 2018)	Return (%)	11.5	11.2	10.3	11.3	7.8	11.2	11.2	11.3	11.3	11.2	11.2	10.1	11.2	11.4	9.1	9.0	11.2	11.3
	Volatility (%)	10.8	10.7	11.0	10.7	10.8	10.6	10.7	10.7	10.6	10.6	10.6	10.5	10.7	10.7	10.6	10.6	10.6	10.7
	Sharpe	1.00	.97	.87	.99	.65	.99	.99	.99	1.00	.99	.99	.90	.98	1.01	.79	.78	.98	.99
	IR	.00	-.66	-.53	-.80	-1.02	-.94	-1.10	-.93	-.47	-.67	-.82	-.55	-.75	-.09	-.56	-.65	-.57	-1.13
	Omega	2.1	2.1	1.9	2.1	1.6	2.1	2.1	2.1	2.1	2.1	2.1	2.0	2.1	2.1	1.8	1.8	2.1	2.1
	VaR 5%	-5.1	-5.1	-5.1	-5.1	-5.2	-5.0	-5.1	-5.1	-5.2	-5.1	-5.2	-5.0	-5.0	-5.0	-5.1	-5.0	-4.9	-5.1
	TVaR 5%	-7.0	-7.0	-7.1	-6.9	-6.9	-6.9	-6.9	-6.9	-6.9	-6.9	-6.9	-6.9	-6.9	-6.9	-6.9	-6.9	-6.9	-6.9
Down – (04/2008 to 12/2012)	Return (%)	3.8	0.5	1.1	0.9	10.7	4.6	-5.5	-1.2	7.0	4.1	7.1	4.8	2.5	5.2	3.1	3.4	2.9	0.4
	Volatility (%)	19.6	23.2	23.6	22.3	17.6	20.6	23.8	22.4	20.8	18.9	18.1	20.7	20.1	20.3	16.7	14.1	18.6	22.7
	Sharpe	.16	-.00	.02	.01	.57	.19	-.26	-.08	.31	.19	.36	.20	.09	.23	.15	.20	.12	-.01
	IR	.00	-.36	-.28	-.35	.39	.08	-.70	-.71	.29	.04	.33	.12	-.18	.16	-.04	-.03	-.14	-.35
	Omega	1.1	1.0	1.0	1.0	1.6	1.2	0.8	0.9	1.3	1.2	1.3	1.2	1.1	1.2	1.1	1.2	1.1	1.0
	VaR 5%	-9.3	-11.5	-12.3	-11.3	-8.8	-10.5	-12.2	-11.1	-10.3	-10.6	-8.0	-11.1	-11.2	-11.3	-8.8	-8.6	-9.2	-11.1
	TVaR 5%	-13.0	-17.5	-17.8	-16.4	-12.8	-14.1	-19.8	-17.6	-13.8	-12.9	-11.2	-13.9	-14.3	-14.6	-12.1	-9.4	-12.6	-16.5
Up + (07/2004 to 03/2008)	Return (%)	5.8	15.6	17.2	14.7	18.1	15.5	19.0	17.9	6.7	5.2	3.1	8.9	12.8	8.8	4.1	4.2	3.9	16.4
	Volatility (%)	8.7	14.1	13.8	13.2	13.5	14.8	17.3	15.3	9.5	9.3	14.7	10.6	13.5	14.5	8.3	8.7	9.1	14.8
	Sharpe	.19	.81	.95	.80	1.04	.77	.86	.90	.27	.12	-.07	.45	.64	.32	.00	.00	-.03	.83
	IR	.00	.63	.73	.63	.78	.69	.64	.80	.13	-.11	-.21	.36	.58	.22	-.24	-.37	-.53	.64
	Omega	1.1	1.7	1.8	1.8	2.1	1.7	2.3	2.1	1.2	1.1	0.9	1.4	1.7	1.3	1.0	1.0	1.0	1.8
	VaR 5%	-3.4	-5.2	-4.2	-5.9	-4.6	-5.6	-4.3	-3.8	-4.2	-3.8	-6.0	-3.3	-5.6	-3.1	-3.3	-3.9	-3.2	-5.7
	TVaR 5%	-4.6	-6.1	-5.1	-6.8	-5.1	-6.7	-6.9	-5.1	-4.4	-4.8	-8.4	-6.4	-6.7	-10.4	-4.5	-4.8	-4.5	-6.6
Down – (02/2001 to 06/2004)	Return (%)	-3.7	-2.4	-2.5	-1.8	4.9	2.0	2.3	1.3	-3.3	-3.3	-4.4	-0.7	2.4	8.0	-1.0	-1.8	-3.8	2.6
	Volatility (%)	17.4	15.1	15.4	15.7	15.1	18.5	16.7	17.4	13.8	17.3	17.2	16.7	15.4	16.8	17.3	19.7	18.6	14.8
	Sharpe	-.32	-.29	-.29	-.24	.20	.00	.02	-.04	-.38	-.31	-.37	-.16	.03	.36	-.17	-.19	-.31	.04
	IR	.00	.19	.18	.23	.45	.31	.45	.55	.03	.07	-.25	.19	.43	.83	.12	.08	-.01	.40
	Omega	0.8	0.8	0.8	0.8	1.1	1.0	1.0	1.0	0.8	0.8	0.8	0.9	1.0	1.3	0.9	0.9	0.8	1.0
	VaR 5%	-8.4	-8.4	-8.7	-9.2	-6.0	-5.9	-7.5	-6.4	-5.7	-7.9	-9.2	-6.2	-7.6	-7.3	-5.9	-9.4	-7.9	-7.1
	TVaR 5%	-9.8	-8.8	-8.8	-9.3	-7.8	-11.1	-9.4	-8.3	-6.1	-8.4	-10.0	-8.1	-8.8	-8.0	-11.9	-12.9	-12.2	-9.5

Period	Performance Measure	Base Portfolio	CrO	BrO	HeO	Gld	Sil	Pla	Cop	Coc	Cof	Sug	Crn	Wht	Soy	FeC	LiC	LeH	Index
Up + (03/1994 to 01/2001)	Return (%)	17.4	16.8	17.4	18.1	16.7	15.1	17.3	16.1	16.7	10.4	13.7	12.9	16.6	15.8	17.2	17.3	17.1	17.6
	Volatility (%)	14.4	14.1	14.1	14.1	13.7	14.0	13.4	13.9	14.3	15.4	15.0	19.2	13.7	13.6	13.6	13.6	14.0	13.6
	Sharpe	.82	.79	.84	.88	.81	.68	.87	.76	.78	.31	.54	.38	.80	.75	.85	.86	.82	.88
	IR	.00	-.11	.00	.19	-.39	-.65	-.02	-.28	-.16	-.90	-.52	-.34	-.22	-.36	-.13	-.04	-.19	.08
	Omega	1.8	1.8	1.8	1.9	1.8	1.6	1.9	1.7	1.8	1.2	1.5	1.4	1.8	1.7	1.8	1.9	1.8	1.9
	VaR 5%	-4.8	-5.5	-5.2	-4.4	-4.5	-4.7	-4.4	-4.5	-4.5	-5.7	-6.0	-5.2	-5.0	-4.4	-4.4	-4.4	-4.5	-4.5
	TVaR 5%	-8.0	-7.8	-7.6	-7.4	-7.5	-7.6	-7.4	-7.6	-7.7	-9.0	-9.0	-13.9	-7.5	-7.0	-7.5	-7.5	-7.6	-7.2
Down – (1992 to 02/1994)	Return (%)	8.1	7.5	7.2	8.0	8.0	8.2	8.2	8.1	7.5	7.8	8.2	8.1	7.8	8.2	8.1	8.1	8.3	8.0
	Volatility (%)	7.0	6.8	6.8	6.9	6.9	6.8	6.8	6.9	6.9	6.9	6.9	6.8	6.8	6.8	6.8	6.8	6.9	6.9
	Sharpe	.66	.58	.55	.66	.66	.69	.68	.66	.58	.63	.68	.68	.63	.69	.67	.68	.69	.66
	IR	.00	-1.46	-1.55	-.24	-.35	.31	.25	-.11	-1.01	-.62	.15	.04	-.59	.39	-.12	.06	.47	-.42
	Omega	1.6	1.5	1.5	1.6	1.6	1.6	1.6	1.6	1.5	1.5	1.6	1.6	1.5	1.6	1.6	1.6	1.6	1.6
	VaR 5%	-2.4	-2.4	-2.4	-2.4	-2.4	-2.3	-2.4	-2.6	-2.4	-2.3	-2.4	-2.4	-2.4	-2.4	-2.4	-2.5	-2.4	-2.4
	TVaR 5%	-2.6	-2.6	-2.6	-2.6	-2.5	-2.5	-2.5	-2.6	-2.6	-2.5	-2.5	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6

Notes: This table reports the out-of-sample portfolio benefits of commodities for the Mean Variance strategy with a risk aversion coefficient of 2 (a less risk averse behavior) across different market environments. Up+ periods represent economic expansions whether down- periods represent recessions. The evaluation period is from January 1991 to December 2018. In grey are the situations where no improvements were achieved compared to the performance indicator from the base portfolio. We divide the table by asset allocation strategy and represent the base portfolio (stock only) and the 17 augment portfolios. We compute returns, volatilities, Sharpe ratios, information ratios, omega ratios, the monthly 5% historical 36 months value at risk and the monthly 5% historical 36 months tail value at risk.

Table X - Out-of-sample sub-period results for all augment-portfolios using Minimum Variance

	Performance	**Base**	**Energy**			**Metals**				**Softs**			**Grains**			**Livestock**			Index
	Measure	**Portfolio**	CrO	BrO	HeO	Gld	Sil	Pla	Cop	Coc	Cof	Sug	Crn	Wht	Soy	FeC	LiC	LeH	
Up + (2013 to 2018)	Return (%)	11.5	11.0	10.3	10.7	6.7	10.6	9.8	10.6	9.8	9.8	10.4	10.2	10.9	10.1	8.8	7.1	10.7	9.7
	Volatility (%)	10.8	10.8	10.7	10.6	8.3	10.1	10.8	10.4	10.1	10.1	10.4	9.6	10.5	10.2	10.3	10.1	10.9	10.4
	Sharpe	1.00	.96	.90	.94	.72	.97	.85	.96	.90	.90	.93	1.00	.97	.92	.79	.63	.91	.86
	IR	.00	-.53	-1.14	-.62	-.83	-.33	-.56	-.32	-.57	-.59	-.48	-.41	-.34	-.53	-.50	-.72	-.21	-.84
	Omega	2.1	2.0	1.9	2.0	1.7	2.0	1.8	2.0	1.9	2.0	2.0	2.0	2.0	1.9	1.8	1.6	1.9	1.9
	VaR 5%	-5.1	-5.4	-5.0	-5.0	-4.1	-4.0	-4.6	-4.7	-5.4	-5.0	-5.7	-3.9	-4.7	-4.3	-5.3	-4.8	-4.4	-4.9
	TVaR 5%	-7.0	-7.0	-7.0	-6.9	-4.5	-5.9	-6.3	-6.6	-6.3	-6.2	-6.8	-5.5	-6.6	-6.2	-6.6	-6.5	-6.6	-6.7

Down – (04/2008 to 12/2012)	Return (%)	3.8	2.7	3.1	2.7	6.7	5.4	-0.1	2.7	3.0	4.0	5.6	4.2	3.2	3.7	5.4	4.1	3.7	2.4
	Volatility (%)	19.6	20.2	20.2	20.0	16.5	20.0	19.9	20.1	19.4	19.1	18.8	19.9	19.8	19.4	14.5	14.1	18.7	20.1
	Sharpe	.16	.10	.12	.10	.37	.24	-.03	.10	.12	.18	.26	.18	.13	.16	.33	.24	.16	.09
	IR	.00	-.54	-.31	-.58	.27	.27	-.74	-.74	-.16	.03	.21	.17	-.19	-.05	.10	.02	-.02	-.53
	Omega	1.1	1.1	1.1	1.1	1.3	1.2	1.0	1.1	1.1	1.1	1.2	1.1	1.1	1.1	1.3	1.2	1.1	1.1
	VaR 5%	-9.3	-9.9	-10.1	-9.4	-6.9	-10.0	-9.7	-10.2	-9.6	-10.6	-8.0	-9.9	-9.9	-11.3	-7.4	-7.2	-8.8	-10.0
	TVaR 5%	-13.0	-13.9	-14.1	-13.6	-11.9	-13.1	-14.3	-14.0	-13.2	-13.4	-11.5	-13.2	-13.4	-13.5	-9.8	-10.0	-12.4	-14.0
Up + (07/2004 to 03/2008)	Return (%)	5.8	10.1	10.7	10.1	10.1	7.9	10.8	7.5	6.1	5.9	8.5	3.4	5.9	3.4	3.7	4.8	4.5	10.3
	Volatility (%)	8.7	8.7	8.3	8.6	8.2	9.0	7.0	8.3	8.6	8.7	8.9	10.2	9.3	10.5	5.9	7.4	8.5	8.7
	Sharpe	.19	.69	.79	.69	.72	.42	.96	.40	.23	.21	.49	-.07	.19	-.07	-.07	.09	.05	.70
	IR	.00	.70	.78	.74	.86	.56	.87	1.11	.06	.06	.66	-.58	.02	-.54	-.38	-.26	-.51	.67
	Omega	1.1	1.7	1.8	1.6	1.6	1.3	2.0	1.3	1.2	1.2	1.4	1.0	1.1	0.9	0.9	1.1	1.0	1.7
	VaR 5%	-3.4	-4.4	-3.2	-3.8	-3.3	-3.2	-2.0	-2.7	-3.5	-3.8	-4.1	-4.2	-4.0	-4.2	-2.7	-2.7	-3.1	-4.2
	TVaR 5%	-4.6	-5.0	-4.3	-4.8	-3.6	-4.5	-2.6	-4.3	-4.1	-4.5	-4.4	-6.1	-4.9	-7.1	-3.4	-4.1	-4.2	-4.6
Down – (02/2001 to 06/2004)	Return (%)	-3.7	-2.4	-2.7	-2.1	5.6	0.6	0.5	1.0	-1.0	-3.3	-4.4	0.1	0.6	5.6	2.5	0.1	-3.5	0.1
	Volatility (%)	17.4	15.1	15.4	15.7	10.7	15.7	15.8	17.5	13.7	17.3	17.2	14.5	14.7	16.5	13.7	14.6	16.8	13.9
	Sharpe	-.32	-.29	-.30	-.26	.34	-.09	-.09	-.06	-.22	-.31	-.37	-.13	-.09	.22	.04	-.13	-.33	-.13
	IR	.00	.18	.16	.20	.62	.31	.39	.54	.23	.07	-.25	.33	.41	.80	.35	.22	.02	.34
	Omega	0.8	0.8	0.8	0.8	1.3	0.9	0.9	1.0	0.9	0.8	0.8	0.9	0.9	1.2	1.0	0.9	0.8	0.9
	VaR 5%	-8.4	-8.4	-8.7	-9.2	-5.3	-6.6	-6.9	-7.5	-5.7	-7.9	-9.2	-6.1	-7.1	-7.3	-4.5	-8.6	-7.4	-7.0
	TVaR 5%	-9.8	-8.8	-8.8	-9.3	-5.8	-9.1	-8.4	-8.3	-6.0	-8.4	-10.0	-6.8	-8.1	-7.5	-8.8	-9.9	-9.5	-8.4
Up + (03/1994 to 01/2001)	Return (%)	17.4	16.1	16.3	15.9	5.1	14.5	14.3	16.3	11.4	16.8	11.9	15.4	15.6	11.6	11.7	13.3	15.7	12.3
	Volatility (%)	14.4	13.3	13.4	13.4	9.7	13.6	10.7	13.3	13.0	14.0	13.7	13.9	13.2	12.8	10.2	10.8	14.3	11.7
	Sharpe	.82	.79	.80	.76	-.06	.66	.81	.80	.45	.80	.46	.70	.75	.47	.59	.71	.71	.58
	IR	.00	-.18	-.14	-.26	-1.11	-.67	-.36	-.22	-.68	-.14	-.79	-.38	-.41	-.91	-.61	-.50	-.54	-.54
	Omega	1.8	1.8	1.8	1.8	1.0	1.6	1.9	1.8	1.4	1.8	1.4	1.7	1.7	1.4	1.5	1.7	1.7	1.5
	VaR 5%	-4.8	-4.5	-5.1	-4.2	-3.8	-4.7	-3.9	-3.9	-4.9	-5.2	-6.0	-5.0	-4.7	-3.9	-3.1	-3.8	-4.8	-3.8
	TVaR 5%	-8.0	-7.1	-7.0	-7.0	-5.5	-7.7	-6.3	-7.2	-7.0	-8.0	-8.4	-7.9	-7.0	-7.2	-5.3	-5.4	-8.0	-5.9

Down – (1992 to 02/1994)	Return (%)	8.1	4.0	4.3	5.4	5.4	8.7	7.5	2.1	6.4	6.2	9.3	6.8	3.8	8.7	4.6	5.1	8.6	4.3
	Volatility (%)	7.0	7.0	6.8	7.3	7.7	9.5	7.8	8.9	6.9	9.6	8.7	6.3	6.9	6.3	6.5	6.8	7.7	6.3
	Sharpe	.66	.07	.12	.27	.25	.55	.51	-.16	.42	.28	.66	.52	.04	.83	.17	.24	.67	.13
	IR	.00	-1.18	-1.15	-.72	-.32	.06	-.08	-.91	-.56	-.25	.17	-.22	-.74	.08	-.46	-.40	.13	-.86
	Omega	1.6	1.1	1.1	1.2	1.2	1.6	1.4	0.9	1.4	1.2	1.7	1.5	1.0	1.8	1.1	1.2	1.6	1.1
	VaR 5%	-2.4	-2.6	-2.3	-2.2	-3.2	-3.5	-3.2	-4.9	-2.5	-3.8	-2.3	-2.5	-3.0	-2.3	-2.2	-3.5	-2.6	-2.2
	TVaR 5%	-2.6	-2.9	-2.9	-3.2	-3.3	-5.1	-3.7	-6.0	-2.8	-4.8	-3.0	-2.5	-3.2	-2.5	-2.9	-4.1	-2.8	-2.7

Notes: This table reports the out-of-sample portfolio benefits of commodities for the Minimum Variance strategy across different market environments. Up+ periods represent economic expansions whether down- periods represent recessions. The evaluation period is from January 1991 to December 2018. In grey are the situations where no improvements were achieved compared to the performance indicator from the base portfolio. We divide the table by asset allocation strategy and represent the base portfolio (stock only) and the 17 augment portfolios. We compute returns, volatilities, Sharpe ratios, information ratios, omega ratios, the monthly 5% historical 36 months value at risk and the monthly 5% historical 36 months tail value at risk.

Table XI - Out-of-sample sub-period results for all augment-portfolios using Risk Parity

	Performance	**Base**	**Energy**			**Metals**				**Softs**			**Grains**			**Livestock**			Index
	Measure	**Portfolio**	CrO	BrO	HeO	Gld	Sil	Pla	Cop	Coc	Cof	Sug	Crn	Wht	Soy	FeC	LiC	LeH	
Up + (2013 to 2018)	Return (%)	11.5	8.0	6.3	6.6	6.2	8.9	6.4	7.6	9.1	9.4	9.0	8.9	9.3	8.2	8.2	7.0	10.5	5.5
	Volatility (%)	10.8	10.9	11.0	10.8	8.3	9.7	10.4	10.3	9.9	10.2	10.2	9.4	10.5	9.9	10.2	9.8	10.8	10.4
	Sharpe	1.00	.67	.51	.54	.67	.84	.55	.67	.85	.85	.81	.87	.82	.75	.74	.65	.91	.46
	IR	.00	-.93	-1.16	-1.13	-.89	-.77	-1.10	-.90	-.55	-.51	-.67	-.72	-.71	-.78	-.56	-.70	-.26	-1.28
	Omega	2.1	1.7	1.5	1.5	1.6	1.9	1.5	1.6	1.9	2.0	1.8	1.9	1.8	1.8	1.7	1.6	1.9	1.4
	VaR 5%	-5.1	-5.5	-4.9	-5.0	-4.2	-4.0	-4.4	-5.2	-5.7	-4.8	-4.9	-4.1	-4.4	-4.1	-5.3	-5.0	-4.2	-4.7
	TVaR 5%	-7.0	-7.1	-6.8	-6.5	-4.6	-5.7	-5.7	-6.5	-6.6	-6.2	-6.6	-5.7	-6.4	-6.1	-6.2	-5.7	-6.7	-6.5
Down – (04/2008 to 12/2012)	Return (%)	3.8	2.7	3.9	3.5	6.6	5.6	0.5	3.4	0.9	3.3	4.1	4.0	2.2	3.1	5.0	3.8	3.3	2.5
	Volatility (%)	19.6	20.9	20.7	20.6	16.4	20.7	19.7	21.2	19.9	19.5	19.0	20.7	20.5	19.8	14.1	14.0	18.5	20.2
	Sharpe	.16	.10	.16	.14	.37	.24	-.00	.13	.02	.14	.19	.16	.08	.13	.31	.23	.15	.09
	IR	.00	-.23	.01	-.08	.26	.22	-.53	-.10	-.40	-.05	.04	.03	-.23	-.11	.09	-.00	-.06	-.27
	Omega	1.1	1.1	1.1	1.1	1.3	1.2	1.0	1.1	1.0	1.1	1.2	1.1	1.1	1.1	1.3	1.2	1.1	1.1
	VaR 5%	-9.3	-9.7	-10.0	-9.9	-6.4	-9.4	-11.5	-10.1	-10.1	-10.5	-8.3	-10.7	-10.5	-11.3	-7.4	-7.1	-8.7	-9.4
	TVaR 5%	-13.0	-13.7	-14.0	-14.0	-11.7	-14.0	-14.5	-14.6	-13.3	-13.9	-11.8	-13.8	-13.5	-13.8	-9.8	-10.1	-12.5	-13.5

Up + (07/2004 to 03/2008)	Return (%)	5.8	9.2	9.7	9.2	10.6	8.7	10.7	10.4	6.5	6.9	8.0	4.9	6.9	3.5	4.2	5.0	4.4	9.9
	Volatility (%)	8.7	8.3	8.0	8.4	8.1	8.8	7.0	8.5	8.1	9.2	8.5	10.3	9.4	11.1	6.2	7.5	8.4	8.5
	Sharpe	.19	.61	.69	.61	.80	.52	.94	.74	.30	.30	.45	.07	.30	-.05	.01	.11	.03	.68
	IR	.00	.70	.79	.75	.89	.68	.81	1.17	.19	.28	.58	-.20	.26	-.37	-.32	-.26	-.54	.67
	Omega	1.1	1.6	1.7	1.6	1.7	1.4	2.0	1.7	1.2	1.2	1.4	1.1	1.2	1.0	1.0	1.1	1.0	1.6
	VaR 5%	-3.4	-3.7	-2.9	-3.8	-2.8	-3.4	-2.1	-2.5	-3.3	-3.8	-3.7	-4.7	-3.6	-3.7	-2.9	-2.7	-3.1	-3.9
	TVaR 5%	-4.6	-4.7	-4.3	-4.6	-3.3	-4.1	-2.7	-4.1	-3.6	-4.5	-4.3	-6.0	-4.6	-7.3	-3.7	-4.2	-4.2	-4.5
Down – (02/2001 to 06/2004)	Return (%)	-3.7	-1.9	-2.1	-1.7	5.5	1.0	0.9	2.1	-0.3	-2.6	-4.0	0.4	1.1	5.9	3.1	0.1	-3.3	0.4
	Volatility (%)	17.4	14.9	15.2	15.5	10.9	15.6	15.8	18.0	13.3	17.7	16.5	14.4	14.7	16.7	13.2	14.6	16.4	13.9
	Sharpe	-.32	-.26	-.27	-.24	.32	-.06	-.07	.01	-.17	-.26	-.36	-.11	-.06	.24	.09	-.13	-.32	-.12
	IR	.00	.25	.23	.23	.60	.34	.41	.60	.33	.13	-.05	.35	.44	.80	.39	.22	.04	.36
	Omega	0.8	0.8	0.8	0.8	1.3	1.0	1.0	1.0	0.9	0.8	0.8	0.9	1.0	1.2	1.1	0.9	0.8	0.9
	VaR 5%	-8.4	-8.0	-8.3	-8.7	-5.3	-6.6	-6.7	-7.4	-5.7	-7.6	-8.8	-5.6	-7.1	-7.4	-4.5	-8.6	-7.3	-6.9
	TVaR 5%	-9.8	-8.8	-8.7	-9.2	-5.8	-9.1	-8.3	-8.3	-6.1	-7.8	-9.5	-6.6	-8.1	-7.8	-8.3	-10.0	-8.9	-8.4
Up + (03/1994 to 01/2001)	Return (%)	17.4	16.2	16.4	15.7	5.1	13.9	14.3	15.4	11.9	16.7	13.1	14.6	15.1	11.3	11.3	12.8	15.2	12.1
	Volatility (%)	14.4	13.4	13.4	13.5	9.6	13.4	10.6	13.0	13.1	13.9	14.1	13.8	13.1	12.8	10.1	10.8	14.1	11.6
	Sharpe	.82	.79	.81	.75	-.05	.62	.82	.75	.48	.80	.53	.65	.72	.44	.57	.67	.68	.56
	IR	.00	-.17	-.14	-.27	-1.11	-.68	-.36	-.35	-.65	-.16	-.57	-.44	-.44	-.86	-.64	-.55	-.46	-.56
	Omega	1.8	1.8	1.8	1.7	1.0	1.6	1.9	1.7	1.4	1.8	1.5	1.6	1.7	1.4	1.5	1.6	1.6	1.5
	VaR 5%	-4.8	-4.3	-5.0	-4.5	-3.8	-4.5	-4.2	-4.0	-4.9	-4.9	-6.3	-4.7	-4.8	-3.8	-3.1	-3.8	-4.7	-3.8
	TVaR 5%	-8.0	-7.1	-7.0	-7.0	-5.4	-7.6	-6.2	-7.1	-7.0	-7.9	-8.7	-8.2	-7.0	-7.3	-5.3	-5.4	-8.0	-5.9
Down – (1992 to 02/1994)	Return (%)	8.1	5.5	5.7	6.3	5.3	8.2	7.4	2.3	5.9	6.7	9.0	7.1	3.3	8.8	4.5	5.1	8.7	4.7
	Volatility (%)	7.0	6.9	6.8	7.0	8.0	8.8	7.8	8.7	7.1	9.9	8.6	6.6	7.3	6.3	6.5	6.8	7.6	6.3
	Sharpe	.66	.29	.33	.39	.23	.53	.50	-.14	.33	.32	.64	.54	-.02	.84	.15	.24	.68	.19
	IR	.00	-1.19	-1.17	-.77	-.32	.01	-.09	-.90	-.48	-.18	.12	-.15	-.71	.09	-.47	-.40	.14	-.84
	Omega	1.6	1.2	1.3	1.3	1.2	1.5	1.4	0.9	1.3	1.3	1.6	1.5	1.0	1.8	1.1	1.2	1.6	1.2
	VaR 5%	-2.4	-2.1	-2.1	-2.2	-3.3	-3.4	-3.1	-4.7	-2.6	-3.6	-2.3	-2.5	-3.1	-2.3	-2.2	-3.6	-2.6	-2.1
	TVaR 5%	-2.6	-2.6	-2.7	-3.0	-3.5	-4.8	-3.7	-5.9	-3.0	-5.0	-2.9	-2.5	-3.4	-2.5	-3.0	-4.1	-2.8	-2.6

Notes: This table reports the out-of-sample portfolio benefits of commodities for the Risk Parity strategy across different market environments. Up+ periods represent economic expansions whether down- periods represent recessions. The evaluation period is from January 1991 to December 2018. In grey are the situations where no improvements were achieved compared to the performance indicator from the base portfolio. We divide the table by asset allocation strategy and represent the base portfolio (stock only) and the 17 augment portfolios. We compute returns, volatilities, Sharpe ratios, information ratios, omega ratios, the monthly 5% historical 36 months value at risk and the monthly 5% historical 36 months tail value at risk.

Table XII - Out-of-sample sub-period results for all augment-portfolios using a 20% Strategic Weight

	Performance	Base	Energy			Metals				Softs			Grains			Livestock			Index
	Measure	**Portfolio**	CrO	BrO	HeO	Gld	Sil	Pla	Cop	Coc	Cof	Sug	Crn	Wht	Soy	FeC	LiC	LeH	
Up + (2013 to 2018)	Return (%)	11.5	7.1	7.1	7.3	8.4	7.1	6.9	8.0	9.5	8.0	7.6	7.2	7.6	7.9	9.1	8.9	8.8	7.5
	Volatility (%)	10.8	11.6	11.5	11.0	8.6	9.6	10.0	10.1	9.5	10.1	10.1	9.4	10.8	10.0	9.9	9.6	12.1	10.3
	Sharpe	1.00	.55	.55	.60	.89	.66	.61	.72	.93	.72	.68	.69	.64	.72	.84	.86	.67	.66
	IR	.00	-.76	-.81	-.80	-.79	-.85	-1.09	-.92	-.34	-.57	-.69	-.76	-.67	-.77	-.63	-.63	-.31	-1.13
	Omega	2.1	1.5	1.5	1.5	1.9	1.6	1.6	1.7	2.0	1.8	1.7	1.7	1.6	1.7	1.8	1.9	1.6	1.6
	VaR 5%	-5.1	-4.6	-4.6	-4.6	-3.3	-4.1	-4.5	-4.9	-4.1	-3.8	-4.0	-4.6	-5.6	-4.0	-5.3	-4.8	-6.1	-4.8
	TVaR 5%	-7.0	-7.1	-6.6	-6.3	-4.9	-5.3	-5.4	-6.4	-5.4	-5.8	-6.4	-5.6	-6.3	-6.0	-6.1	-5.4	-6.9	-6.5
Down – (04/2008 to 12/2012)	Return (%)	3.8	3.2	4.1	3.6	5.5	4.9	2.3	3.2	2.8	3.5	4.7	3.9	2.1	4.1	4.8	4.7	4.7	3.2
	Volatility (%)	19.6	21.0	20.7	20.5	17.0	20.2	19.1	21.5	19.6	18.5	18.3	20.6	20.3	19.4	16.6	16.2	18.4	19.9
	Sharpe	.16	.12	.17	.15	.29	.21	.09	.12	.11	.16	.23	.16	.07	.18	.25	.26	.22	.13
	IR	.00	-.11	.05	-.05	.31	.13	-.29	-.11	-.15	-.06	.10	.01	-.24	.05	.23	.20	.13	-.16
	Omega	1.1	1.1	1.1	1.1	1.2	1.2	1.1	1.1	1.1	1.1	1.2	1.1	1.1	1.1	1.2	1.2	1.2	1.1
	VaR 5%	-9.3	-9.7	-10.0	-9.6	-8.4	-9.5	-10.2	-10.8	-9.7	-10.6	-8.0	-10.5	-10.6	-11.3	-8.5	-8.0	-8.5	-9.1
	TVaR 5%	-13.0	-14.1	-14.4	-14.1	-11.8	-14.1	-14.2	-15.4	-13.3	-13.0	-11.5	-13.5	-13.7	-13.4	-11.0	-10.8	-12.1	-13.4
Up + (07/2004 to 03/2008)	Return (%)	5.8	10.1	10.6	10.5	9.2	10.3	9.8	11.0	7.2	7.3	6.8	9.0	10.1	6.3	4.2	4.7	2.7	9.3
	Volatility (%)	8.7	8.3	7.9	8.4	7.7	9.6	7.1	8.7	8.6	10.1	9.1	10.1	9.7	10.9	6.5	7.4	8.9	7.8
	Sharpe	.19	.72	.82	.75	.65	.64	.79	.79	.36	.31	.29	.48	.61	.20	.01	.08	-.17	.66
	IR	.00	.70	.80	.73	.95	.70	.81	.89	.22	.23	.14	.53	.67	.07	-.45	-.29	-.53	.71
	Omega	1.1	1.7	1.8	1.7	1.6	1.5	1.8	1.8	1.3	1.3	1.2	1.4	1.5	1.2	1.0	1.1	0.9	1.6
	VaR 5%	-3.4	-3.7	-3.1	-4.0	-2.9	-3.9	-2.4	-2.3	-3.5	-4.6	-3.9	-4.1	-3.9	-3.7	-2.9	-2.7	-3.1	-3.4
	TVaR 5%	-4.6	-4.8	-4.1	-4.8	-3.2	-4.0	-2.7	-3.6	-4.0	-5.3	-5.0	-4.9	-4.1	-7.0	-3.8	-4.1	-3.6	-4.2
Down – (02/2001 to 06/2004)	Return (%)	-3.7	-1.6	-1.7	-1.6	-0.6	-1.7	-1.4	-0.8	-1.3	-2.4	-4.1	-1.3	-1.2	0.8	-1.3	-2.4	-2.1	-1.7
	Volatility (%)	17.4	14.8	15.1	15.2	14.0	15.3	15.8	17.1	13.7	17.3	16.7	14.7	15.0	15.9	14.6	14.5	16.2	14.7
	Sharpe	-.32	-.25	-.24	-.24	-.18	-.24	-.21	-.16	-.24	-.25	-.36	-.23	-.21	-.07	-.22	-.30	-.25	-.25
	IR	.00	.28	.28	.26	.66	.33	.50	.63	.25	.19	-.06	.42	.42	.80	.50	.20	.16	.36
	Omega	0.8	0.8	0.8	0.8	0.9	0.8	0.9	0.9	0.8	0.8	0.8	0.9	0.9	0.9	0.9	0.8	0.8	0.8
	VaR 5%	-8.4	-7.8	-7.9	-7.5	-7.2	-7.5	-7.3	-8.5	-5.8	-7.6	-9.5	-6.9	-7.1	-7.9	-7.3	-7.0	-7.6	-7.7
	TVaR 5%	-9.8	-8.7	-8.6	-8.8	-7.7	-8.4	-7.9	-9.1	-6.2	-7.8	-9.8	-7.9	-7.8	-8.5	-7.9	-8.2	-9.1	-8.3

Up + (03/1994 to 01/2001)	Return (%)	17.4	15.9	16.1	15.5	12.9	13.6	15.1	14.0	13.3	13.5	13.5	13.4	13.2	12.9	14.0	14.0	13.8	14.8
	Volatility (%)	14.4	13.2	13.1	13.3	11.8	13.3	11.9	13.1	12.5	15.3	13.4	13.6	13.2	12.9	11.9	12.0	14.6	12.2
	Sharpe	.82	.78	.80	.74	.62	.60	.80	.64	.62	.52	.59	.57	.58	.57	.71	.70	.56	.75
	IR	.00	-.20	-.18	-.28	-1.23	-.81	-.56	-.69	-.65	-.36	-.60	-.72	-.70	-.97	-.88	-.82	-.45	-.57
	Omega	1.8	1.8	1.8	1.7	1.6	1.5	1.8	1.6	1.6	1.5	1.6	1.5	1.5	1.5	1.7	1.6	1.5	1.7
	VaR 5%	-4.8	-5.0	-5.1	-4.6	-4.3	-4.6	-4.6	-4.2	-3.9	-6.2	-4.9	-4.3	-4.3	-4.5	-3.9	-3.8	-4.8	-4.3
	TVaR 5%	-8.0	-7.1	-7.2	-6.8	-6.7	-7.5	-6.8	-7.2	-6.8	-8.4	-8.0	-8.4	-7.1	-7.3	-6.4	-6.3	-8.2	-6.5
Down – (1992 to 02/1994)	Return (%)	8.1	4.2	4.2	5.5	7.1	9.2	7.8	5.3	5.6	6.2	8.9	7.7	4.8	8.3	6.9	7.1	8.6	6.2
	Volatility (%)	7.0	6.9	6.8	7.2	6.0	6.9	6.1	7.3	7.0	9.0	7.8	5.8	6.8	5.6	5.9	5.9	7.6	6.2
	Sharpe	.66	.10	.10	.27	.60	.83	.71	.25	.30	.30	.69	.72	.19	.86	.58	.61	.67	.44
	IR	.00	-1.21	-1.18	-.69	-.32	.20	-.08	-.71	-.51	-.27	.12	-.10	-.64	.06	-.54	-.39	.11	-.90
	Omega	1.6	1.1	1.1	1.2	1.5	1.9	1.6	1.2	1.3	1.2	1.7	1.7	1.1	1.8	1.5	1.5	1.6	1.4
	VaR 5%	-2.4	-2.4	-2.3	-2.2	-2.2	-2.7	-1.9	-3.2	-2.6	-3.6	-2.5	-2.2	-2.7	-1.9	-1.9	-1.7	-2.5	-2.2
	TVaR 5%	-2.6	-2.8	-2.9	-3.2	-2.3	-3.4	-2.4	-4.0	-3.0	-4.3	-3.0	-2.3	-3.3	-2.2	-2.0	-2.5	-2.7	-2.6

Notes: This table reports the out-of-sample portfolio benefits of commodities for the Strategic Weight strategy with a 20% allocation to commodities across different market environments. Up+ periods represent economic expansions whether down- periods represent recessions. The evaluation period is from January 1991 to December 2018. In grey are the situations where no improvements were achieved compared to the performance indicator from the base portfolio. We divide the table by asset allocation strategy and represent the base portfolio (stock only) and the 17 augment portfolios. We compute returns, volatilities, Sharpe ratios, information ratios, omega ratios, the monthly 5% historical 36 months value at risk and the monthly 5% historical 36 months tail value at risk.

Table XIII – Out-of-sample portfolio turnover for the full period.

Portfolio Turnover	**Energy**			**Metals**				**Softs**			**Grains**			**Livestock**			Index
(%)	CrO	BrO	HeO	Gld	Sil	Pla	Cop	Coc	Cof	Sug	Crn	Wht	Soy	FeC	LiC	LeH	
Mean Variance RRA 2	5.4	6.2	5.4	11.7	10.3	14.5	9.1	7.5	9.1	7.9	11.1	7.9	8.0	16.0	15.1	4.6	6.5
Mean Variance RRA 10	5.0	5.6	5.5	12.6	7.8	10.9	8.3	5.9	5.9	5.6	7.4	5.9	8.8	11.9	10.8	3.8	7.6
Minimum Variance	4.2	5.0	4.7	8.4	6.0	8.1	6.4	5.7	5.3	5.9	5.3	4.9	5.9	7.7	7.2	4.8	7.0
Risk Parity	4.6	5.2	4.7	7.8	5.4	6.5	6.3	5.1	5.1	5.1	5.4	5.1	5.8	6.9	6.8	4.1	6.2
Strategic Weights	4.1	4.2	3.9	2.7	3.5	2.8	3.3	3.8	4.3	4.4	4.0	3.9	3.7	2.6	2.7	4.7	3.0

Notes: This table reports the out-of-sample augment portfolios turnovers across the different asset allocation strategies for the full period.