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Capital Dispersion and Productivity in Portugal During  
the Troika Period

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## ABSTRACT

Portugal has struggled economically since the beginning of the twenty-first century. One reason for this is the low productivity of the portuguese manufacturing sector, which is particularly punishing to a country in the context of the eurozone. Using data for manufacturing firms in Portugal between 2011 and 2019, I document a countercyclical movement of productivity losses from capital misallocation, the increased dispersion of labor since 2011, even with a low and steady dispersion of the marginal product of labor, and the very high dispersion of capital and its marginal product. Using the same model as in Gopinath et al. (2017), even under a similar setting of falling real interest rate, I find evidence pointing the problems of productivity in Portugal away from being purely related with financial frictions, and provide markups and competition as a potential avenue to complement an explanation to this phenomenon.

**Keywords:** Financial Frictions, Heterogeneous Firms, Macroeconomics

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# CAPITAL DISPERSION AND PRODUCTIVITY IN PORTUGAL DURING THE TROIKA PERIOD

By Rui Vieira Santos

## 1 INTRODUCTION

Productivity differs across firms and explaining these differences implies understanding how factor allocation varies amongst them. In order to do so, one needs to analyze the empirical facts on the eventual resource misallocation across firms, understand what factors are most relevant for this resource misallocation, and how big of an issue is the productivity differential of amongst firms arising from factor misallocation.

This dissertation will be going over the evolution of these facts in Portugal from 2011 to 2019. This is a period of particular turmoil in portuguese history. Still trying to recover from the 2008 financial crisis, and a decade of virtually no economic growth, Portugal was forced to seek international help to avoid full fledged state default, and, shortly after this, there were a series of problems regarding the banking sector, namely the bankruptcy of its largest private bank, and the subsequent shock led most banks to turn to the state for emergency liquidity.

For at least two decades now, the portuguese manufacturing sector has found itself in a strange conundrum - portuguese manufacturing firms have tended to shrink and become less and less efficient. There are many reasons for this, such as restrictive labor laws, but one of specific interest is low investment levels. It is reasonable to believe that Portugal would be a prime candidate to be looked at when analyzing the evolution of capital allocation, particularly in the twenty-first century, when the overall evolution of the firm distribution has been so peculiar.

I extend the work of Gopinath et al. (2017), closely following their methodology, and apply it to Portugal - which is mentioned in the paper but it is not the focus - in the years comprised between 2011 and 2019. The main focus of analysis of their paper is on Spain, and their analysis period is comprised between 1994 and 2010. The point of this paper was to look at how capital and productivity evolved in a period where the real interest rate had been falling and there was a financial crisis, with ramifications into financial frictions faced by firms. Under these circumstances they look at how this interest rate evolution and overall economic conditions affected capital allocation, and through this, productivity, both of capital and total factor productivity. The novelty of this paper was to introduce a size dependent factor to the borrowing constraint firms face - frictions faced by the heterogeneous firms become heterogeneous themselves - which reflects the empirical fact of larger firms being more prone and able to take loans.

What I am trying to expand on, in comparison to Gopinath et al. (2017), is the appli-

cation of their methods to a new crisis period that has some common features, and focus this analysis only on Portugal to study the evolution of productivity and misallocation.

I document the evolution of the portuguese manufacturing sector in terms of its productivity and factor dispersion on a time of very hard economic conditions, to find that although most firms are small scale operations, the largest part of output and the wage bill come from the larger firms in the sample. Also, I show that the portuguese manufacturing sector has had an increase of aggregate productivity in the years where the country was undergoing the adjustment process, 2011 to 2015, more so than in the years of better economic growth of 2016 to 2019. Furthermore, using simple measures of markups, a positive correlation between larger market power and larger differential between observed and efficient productivity - the more market power, the more inefficient firms seem to be.

I succeeded in replicating the empirical facts of portuguese factor and productivity dispersion with the model used. Though not perfect, the simulations produce moments that mostly match those found in the data. The setback here is that the baseline model with heterogeneous financial frictions did not significantly outperform versions of the model with homogeneous financial frictions, and even the version of the model with no frictions at all.

The following sections of this dissertation are organized as follows. Section 2 offers an overview of the relevant literature. Section 3 describes the data used, as well as a quick glance into the weight of the sample in the overall economy. Section 4 provides a brief rundown of the theoretical model used. Section 5 presents the bulk of the results, goes over the empirical facts associated with factor productivity dispersion in the sample of portuguese manufacturing firms, a comparison of said facts to the models simulated results. Section 6 concludes.

## 2 LITERATURE REVIEW

Portugal has been "diagnosed" with many economic inefficiencies across the years, and has persistently been unable to cure its ailments. One such diagnosis is that of Blanchard (2007) which goes back a few years, but unfortunately remains relevant, and has been proven to be an accurate assessment. One of the main points Blanchard makes, is that Portugal needed to balance its trade and budget deficits, but also, that it has featured low levels of productivity as far back as the mid 1990s, and especially from the time it joined the euro area, which makes these persistent macroeconomic imbalances particularly punishing. The low productivity of Portuguese firms cannot be overlooked in the context of the free market, as Portugal has been falling off the "middle of the pack" in the European Union, as more productive countries - like those of Eastern Europe - that a little time ago, as the early 2000s were behind Portugal in measures like GDP per capita,

and have now caught up to Portugal and even surpass it in some cases. Productivity has been the object of many reform attempts, such as the ones of the Troika period included in this research, but it would seem the effects of such policies have not been immediately apparent.

Following the last two decades of the 20th century, when Portugal experienced a period of convergence with other European economies, many structural problems began to take their toll on the economy. Reis (2013) provides a good survey of this topic and he starts by pointing to the "classic" issues of the Portuguese economy: i) historically low levels of education; ii) low aggregate productivity; iii) trouble controlling public finances (leading to IMF interventions in 1977-1978 and 1983-85); iv) rigid labor market; v) slow judicial system; and vi) the inability to compete on low-value-added products after the integration of Eastern Europe and China in the world markets. Though these facts are all relevant to understand the portuguese economy, Reis makes the argument that they do not explain the very sluggish growth of the early 2000's, instead, he points to a misallocation of capital flows generating inefficiency and TFP losses. Since these problems all still exist to this day, and since Portugal, after the Sovereign Debt Crisis, has yet to prove that it can sustainably grow at a good pace I aim to demonstrate that this is still problem. Even after being extensively diagnosed, not enough has been done, either from the side of policy makers to create more robust systems to encourage more productive firms to invest, neither from the private sector to recognize worthwhile allocations of capital.

Restuccia and Rogerson (2008) use a model of heterogeneous plants to generate differences in capital accumulation, relative prices, and measured TFP. They make the point that policies that do not change aggregate relative prices, but do generate idiosyncratic changes in relative prices, can produce decreases in both output and measured TFP. These changes in idiosyncratic prices can be generated by policies, regulations, or institutions, such as government contracting (funded by all firms and households and that only benefit one or a few firms), non-competitive banking systems may result in banking institutions favoring some firms over others for non-economic factors, imperfect financial markets generate misallocation, public companies receiving public subsidies, corruption can have similar idiosyncratic effects, imposition and enforcement of trade restrictions.

An important precursor to the specific model used in this dissertation - featuring financial frictions and their impact in factor and productivity dispersion - is that of Hsieh and Klenow (2009). They measure resource misallocation - more resources are funneled into firms which are on the descending region of their marginal products - and its impact on TFP across China and India, and compare these countries with the United States where resource misallocation is not as dire. They find that China and India have more misallocation across their respective industries than the US and that there would be large TFP

gains (more than 50% in both countries) for these countries if their inputs were only as misallocated as in the US.

The introduction of adjustment costs to the model in Gopinath et al. (2017), was made relevant, by the empirical work in Asker et al. (2014), among others. Analyzing data from 40 countries, the authors find that industries where time-series volatility of productivity is higher, also feature high cross-section volatility in the marginal revenue product of capital. Furthermore, they use an investment model with adjustment costs to demonstrate that the volatility of productivity can explain up to 90% of cross-industry and cross-country differences in the dispersion of the marginal revenue product of capital.

Braguinsky et al. (2011) argues that overly restrictive labor protections (among other factors, such as tearing down monopolies) are hindering firms, especially more productive ones, from attaining their optimal sizes and that would result in slower growth. The curious result of this paper is that they make note of a trend in Portuguese manufacturing firms: the size distribution is not only skewed to the left (smaller firms) but that there is a trend for the distribution to become more skewed. This is very relevant when discussing the subject of productivity - size can be a bottleneck to increase productivity, and if firms can't size up, they be entirely unable to increase productivity.

In the same vein, Duarte and Restuccia (2010), and more specifically Duarte and Restuccia (2007), analyze TFP catchup between countries, and point to the role of labor reallocation across sectors as an explanation for this fact. Duarte and Restuccia (2007) document a convergence of TFP levels between Portugal and the US, where Portugal roughly halved its TFP gap relative to the US between 1956 and 1995, from 23% to 56% of the US level of TFP. The authors argue that most of this convergence can be explained by the rise in relative importance of the manufacturing sector, as productivity in both agriculture and services remains roughly constant throughout the period. If this hypothesis is correct, the sector that benefits most with an increase in labor supply would be manufacture, and the services sector would benefit least from increased labor allocated to it. However, since services has the highest number of workers, aggregate TFP would benefit more if productivity per worker increased in this sector, than if it did in agriculture or manufacture.

One other important factor in productivity differences is the markup. The markup essentially translates a firms market power, and it would be logical to assume that a firm with more market power is a larger firm (at least when compared to firms on the same sector). If this is true, the markup is very closely linked to productivity as a larger firm has a better chance of attracting funding than its competitors. But when considering the aggregate level of productivity, the firm where such funding would be better applied could be the smaller firm if it is in the ascending region of its marginal product, supposing the

larger firm is in the descending region.

One other important factor in productivity differences is the markup. The markup essentially translates a firm's market power, and it would be logical to assume that a firm with more market power is a larger firm (at least when compared to firms on the same sector). If this is true, the markup is very closely linked to productivity as a larger firm has a better chance of attracting funding than its competitors. But when considering the aggregate level of productivity, this could very likely be a faulty allocation of capital, as larger firms with more market power, are not necessarily the ones where capital translates to more productivity gains. De Loecker and Eeckhout (2018) document a considerable increase in the average markup between 1980 and 2016 using a firm-level dataset, with 70000 firms from 134 countries. Though this is the case, Portugal actually has a small decrease in its average markup over time, but as documented ahead, though the markup may not be increasing, there are reasons to believe it is not innocuous, and is a potential driver of misallocation.

Khan and Thomas (2013) present a full fledged DSGE model with financial frictions, adjustment costs, investment decisions and productivity differences, featuring, in this case, market entry and exit. They find that under the setting they analyze - specifically due to a level of irreversible investment - a subset of firms in the economy will end up accumulating a disproportionate level of capital compared to their respective productivity levels, lowering aggregate TFP considerably. In a related work, Khan, Senga, et al. (2014), the authors focus on crisis events, namely, those caused by an aggregate TFP shock, and those caused by a shock to economy-wide credit conditions. They find that shocks to credit conditions are qualitatively different and better explain financial crisis, such as the Great Recession in the US.

Baqae and Farhi (2020) explore a different avenue within the DSGE approach to productivity and misallocation. The model used, as it allows for arbitrary elasticities of substitution, factor endowments, different number of factors, as well as input-output network linkages, and by accounting for all these different aspects, the authors arrive to the conclusion that TFP losses arising from the very existence of markups, may be larger than previously estimated. On top of this already very relevant contribution, the authors also combine the literature on misallocation to that of growth accounting, developing a framework to analyze the social costs of markups, and in doing so, find that though there are large gains in TFP to be had from reallocation and markup reduction, and this has happened specifically in the period between 1997-2015 in the US, but the distance to the optimal level has increased, as the Pareto-efficient frontier has grown at a faster pace.

Still in the DSGE framework, Peters (2020) proposes a tractable model of Schumpeterian growth, in the tradition of Aghion and Howitt (1992), with endogenous market power,

where the markup distribution arises from equilibrium conditions. In their model, firms start at an initial equilibrium where they charge a limit price due to Bertrand competition. These firms make a risky investment decision which increases their productivity, allowing them to charge a higher markup, but they are stochastically replaced by competitors. The higher the churning rate of firms, the lower the overall level of markups is - older firms have higher market power and higher markups. This mechanism works through two opposite signal channels, i) the own-innovation channel, where markups are higher on old firms, and a market with more older firms will necessarily have higher markups; and ii) the creative destruction channel where markups are lower due to the entrance of newer firms into markets. The churning rate is a key variable in the model as it affects the markup distribution, misallocation, TFP, and the labor share, and the higher it is, the better the economic outcome will be.

Augusto, Mateus, et al. (2021) analyze Portuguese firms' vulnerability to debt, taking into account the COVID-19 shock. They find that the manufacturing sector is one of the sectors where it is most likely for vulnerable firms to increase their debt held and excess debt. This seems to hold especially for small- to middle-sized firms, who tend to exhibit more excess borrowing than large firms. The increase in the number of vulnerable firms in the 2020-2022 horizon, seems to be more related to a drop of operating results than to an increase of debt. Coupling this result to the observations regarding firm size distributions made in Braguinsky et al. (2011), it is reasonable to expect that Portuguese firms may be more vulnerable than before to this shock.

### 3 DESCRIPTION OF THE DATA

All micro data for this work has been taken from the Orbis Europe database. It is a firm-level dataset that collects administrative data from European firms. The sample is composed of 99.9% private manufacturing firms, and industries are defined by the four-digit NACE Rev. 2 industry classification.

I extract data from balance sheet and profit/loss accounts of manufacturing firms, that are part of their financial reporting. Since Portugal is part of the EU, such reporting is regulatory, which contributes positively to the sample, as it results in higher presence of small firms than if the data was, for instance, for the United States. I take data from these firms for the years between 2011 and 2019.

Specific data not present in the firm-level database, such as national accounts, and price indices, was taken from the Eurostat database. Data regarding the real interest rate, namely, expected inflation and corporate nominal interest rate, was taken from, the European Central Bank, and the Bank of Portugal databases.

Table I shows how representative the sample is when compared to aggregate data from Portugal. In this table are the percentages of the sum of employment, wage bill, and value added, of firms in the dataset, relative to the total national aggregates in Portugal, from 2011 to 2019. Firms in the sample represent from 6% to 8% of employment across all of the Portuguese economy in the years considered. This tendency is also reflected in value added, wage bill, and the adjusted wage bill.<sup>1</sup> that points to some underrepresentation of the wage bill in national accounting. When taking this measure into account the sample is less representative then when compared to the simple aggregate wage.

TABLE I: REPRESENTATIVENESS OF THE SAMPLE IN PORTUGUESE NATIONAL ACCOUNTS

	Employment (%)	Wage bill (%)	Value added (%)	Adj. wage bill (%)
2011	6.75	7.11	5.87	5.93
2012	6.93	7.31	5.77	6.06
2013	7.04	7.20	5.77	6.00
2014	7.44	7.97	6.10	6.69
2015	7.73	8.52	7.43	7.22
2016	7.95	8.90	7.25	7.59
2017	7.92	8.48	7.24	7.26
2018	7.72	8.03	6.46	6.89
2019	8.54	8.69	7.02	7.45

<sup>1</sup>The adjusted wage bill is a measure computed by multiplying the adjusted wage share by GDP. The adjusted wage share is the percentage of GDP that comes from wages, but includes wage payments from the self employed to the aggregate wage.

Therefore, the sample can be considered as moderately representative. Of course, restricting the analysis to manufacturing firms, which is not the largest sector of the portuguese economy. With this in mind, the number of firms is large and representative enough for the manufacturing sector, which itself is about 20% of the portuguese economy in the variables considered here.

Table II informs us on the relative importance of each class of firm size (by employment) inside the sample, for the variables of employment, wage bill, and value added. It is interesting to note that the smallest firms (under 20 employees) take on the lion's share of the wage bill in the sample, while not exhibiting the highest share of employment and featuring the lowest share of value added in the sample. In fact, the larger firms (over 250 employees) represent nearly half of value added and medium firms (20-250 employees) employ more than half of the total number of workers, but make up for the smallest percentage of the wage bill.

TABLE II: WEIGHT OF EACH SIZE CLASS IN THE SAMPLE

	Employment (%)	Wage Bill (%)	Value Added (%)
1-19 employees	13.61	9.80	5.90
20-250 employees	56.45	51.63	40.90
Over 250 employees	29.94	38.57	53.19

Notably, firms with over 250 employees represent more than 50% of total value added, and the second most employment and wage bill, while medium-sized firms generate about 40% of value added, and small-sized firms are almost vestigial in this measure.

Table III characterizes the sample by firm size, as measured by average number of employees over time. The vast majority of firms have less than 20 employees and they account for just under 70% of the total sample, with firms employing more than 250 employees representing a very small percentage of the overall number of firms in the sample.

TABLE III: Within Sample Firm Size

	Number of firms	Percentage
1-19 Employees	9539	69.60
20-250 Employees	3978	29.02
Over 250 Employees	189	1.38
Total	13706	100.00

Here there is a clear inversion of what happens in the importance of firms in these size ranges in Table II. Though smaller firms are more numerous, they do not account for the

lion's share of neither employment, value added, or the wage bill, and the largest firms in the sample (250+) account for more than half of value added - with a very small overall representation within the sample. This would point to an overarching issue, previously discussed by Braguinsky et al. (2011), of optimal firm size and of the largely left-skewed firm size distribution of portuguese firms.

For a visual representation of this employee distribution, Figure 2 presents the partial distributions by firm size, compared to the overall distribution of all employees. Here, it is clear that the left skewness is present on all sub samples, be it the larger or the smaller firms, though the tendency seems to be aggravated as firm dimension increases. It is also visible that most of the firms are small and in fact most firms are between 0 and 100 employees, so most firms are in the small- to middle-size range.

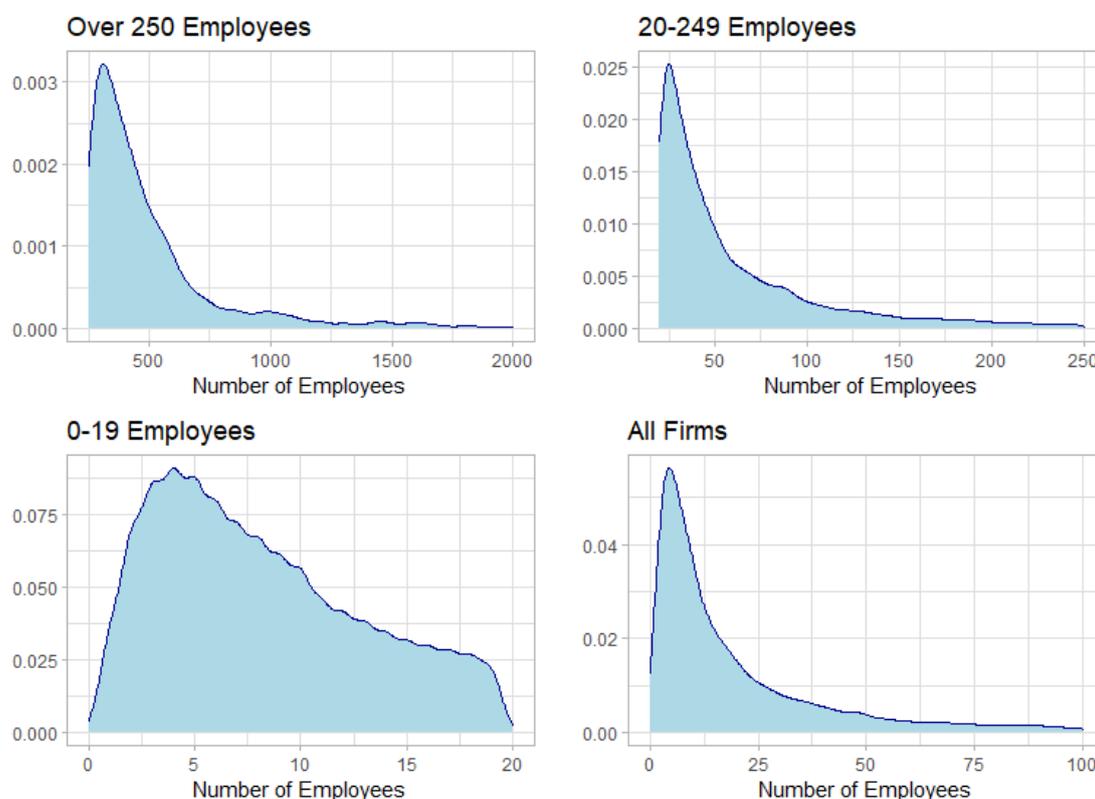


FIGURE 1: Firm Distributions by Number of Employees

#### 4 MODEL

In this section I will describe the model used to replicate the stylized facts associated with capital dispersion and factor productivity, in a world with financial frictions namely, borrowing constraints. Exposition in this chapter follows very closely that of Gopinath

et al. (2017).

#### 4.1 Firms' Problem

Firm  $i$ 's output at the time  $t$  is given by a Cobb-Douglas production function,  $y_{it} = Z_{it}k_{it}^\alpha\ell_{it}^{1-\alpha}$ , where  $Z_{it}$  represents firm TFP,  $k_{it}$  stands for capital, and  $\ell_{it}$  labor. Labor is hired on a competitive market at an exogenous wage rate  $w_t$ .

Firm productivity is defined as  $Z_{it} = Z_t^A z_i^P \exp(z_{it}^T)$ , the product of an aggregate component ( $Z_t^A$ ), a permanent firm-specific component ( $z_i^P$ ), and a transitory component ( $z_{it}^T$ ) that follows an AR(1) process given by:

$$z_{it}^T = -\frac{\sigma^2}{2(1+\rho)} + \rho z_{it-1}^T + \sigma u_{it}^z, \quad \text{with } u_{it}^z \sim \mathcal{N}(0, 1) \quad (1)$$

where  $\rho$  governs persistence of the transitory effect, and  $\sigma$  denotes standard deviation of the productivity shocks  $u_{it}^z$ .

Firms sell differentiated varieties of a product to a monopolistically competitive global market. Demand faced by firm  $i$  at time  $t$  is a downward sloping function of price,  $p_{it}$ ,  $y_{it} = p_{it}^{-\varepsilon}$ , where  $\varepsilon$  is the elasticity of demand. The markup is constant and comes as  $\mu = \varepsilon/(\varepsilon - 1)$ .

This economy is populated by family-firm agents, that chooses consumption  $c$ , debt  $b$ , investment  $x$ , and labor  $\ell$ , to maximize the discounted sum of utility flows,

$$\max_{\{c_{it}, b_{it+1}, x_{it}, \ell_{it}, p_{it}\}} E_0 \sum_{t=0}^{\infty} \beta^t \mathcal{U}(c_{it}), \quad (2)$$

subject to a budget constraint:

$$c_{it} + x_{it} + (1 + r_t)b_{it} + \frac{\psi(k_{it+1} - k_{it})^2}{2k_{it}} = p_{it}y_{it} - w_t\ell_{it} + b_{it+1}, \quad (3)$$

where  $k$  stands for the capital stock,  $r$  represents the return on capital,  $\psi$  is a non-negative parameter that governs the magnitude of the quadratic adjustment cost represented in the budget constraint. Capital accumulation comes as:

$$k_{it+1} = (1 - \delta)k_{it} + x_{it}. \quad (4)$$

where  $\delta$  denotes capital depreciation.

Utility is a standard isoelastic function of consumption of goods:

$$\mathcal{U}(c_{it}) = \frac{c_{it}^{1-\gamma} - 1}{1-\gamma}, \quad (5)$$

where  $\gamma$  denotes the reciprocal of the elasticity of intertemporal substitution.

Unlike other related papers in the literature, such as Hsieh and Klenow (2009), this model exhibits a firm size-dependent borrowing constraint:

$$b_{it+1} \leq \theta_0 k_{it+1} + \theta_1 \Psi(k_{it+1}) \quad (6)$$

The importance of size in this borrowing constraint is measured by the value of  $\theta_1$ . Should this parameter be set to  $\theta_1 = 0$  the borrowing constraint would be rendered the same as the literature standard, where the ratio of borrowing to capital must be lower than a constant exogenous level of  $\theta_0$ .

where  $\Psi(k) = e^k - 1$  is an increasing and convex function of capital.

## 4.2 Recursive Form

Rewriting the problem in the recursive form, first define firm net worth as  $a_{it} = k_{it} - b_{it}$  and denote by  $\mathbf{X}$  the vector of exogenous shocks. To simplify notation, henceforth subscripts are dropped, and next period variables are denoted with an apostrophe (e.g.  $k_{it+1} = k'$ ). Thus the maximization problem can be written as:

$$V(a, k, z^P, z^T, \mathbf{X}) = \max_{a', k', \ell, p} \left\{ \mathcal{U}(c) + \beta EV(a', k', z^P, (z^T)', \mathbf{X}') \right\}, \quad (7)$$

subject to the budget constraint:

$$c + a' + \frac{\psi(k' - k)^2}{2k} = p(y)y - w\ell - (r + \delta)k + (1 + r)a, \quad (8)$$

the production function  $y = Zk^\alpha \ell^{1-\alpha}$ , the demand function  $y = p^{-\varepsilon}$ . The borrowing constraint that becomes:

$$k' \leq \lambda_0 a' + \lambda_1 \Psi(k') = [\lambda_0 + \lambda_1 \Psi(k')/a']a', \quad (9)$$

in which  $\lambda_0 = 1/(1 - \theta_0)$  and  $\lambda_1 = \theta_1/(1 - \theta_0)$  with  $\lambda_0 + \lambda_1 \geq 1$ .

## 4.3 Dispersion in Factor Returns

To start, take the first order-condition for labor, which gives labor demand:

$$\ell = \mu^{\frac{-\varepsilon}{1+\alpha(\varepsilon-1)}} \left( \frac{w}{1-\alpha} \right)^{\frac{-\varepsilon}{1+\alpha(\varepsilon-1)}} Z^{\frac{\varepsilon-1}{1+\alpha(\varepsilon-1)}} k^{\frac{\alpha(\varepsilon-1)}{1+\alpha(\varepsilon-1)}}. \quad (10)$$

One can notice that labor demand depends positively in capital and productivity, and is a decreasing function of wages, and productivity. By construction, the allocation of labor is undistorted across firms, so that the marginal revenue product of labor equals the wage:

$$\text{MRPL} = \frac{1 - \alpha}{\mu} \frac{py}{\ell} = w. \quad (11)$$

Now, the optimal solution for next period capital is given by:

$$E_0 \left[ \beta \frac{\mathcal{U}'(c')}{\mathcal{U}'(c)} \right] \left[ \text{MRPK}' - (r' + \delta) - \frac{\partial AC'}{\partial k'} \right] = \chi \frac{(1 - \lambda_1 \Psi'(k'))}{\mathcal{U}'(c)} + \frac{\partial AC}{\partial k'}, \quad (12)$$

where  $AC = (\psi/2)/(k' - k)^2/k$  represents the adjustment cost, and  $\chi$  is the Lagrange multiplier associated with the borrowing constraint, and  $\text{MRPK}'$  is defined as:

$$\text{MRPK}' = \frac{\varepsilon - 1}{\varepsilon} (Z' k'^{\alpha} \ell'^{1-\alpha})^{\frac{-1}{\varepsilon}} \alpha Z' k'^{1-\alpha} \ell'^{1-\alpha}. \quad (13)$$

In Equation (12) it is clear that in a world without adjustment costs, risk in capital accumulation, and borrowing constraints, the marginal productivity of capital would be given by the sum of the interest rate, and the capital depreciation rate:  $r + \delta$ . Due to these existing frictions,  $\text{MRPK}$  is distorted across firms.

Similarly to Asker et al. (2014), there is an underlying assumption that even in an undistorted economy, TFP gains from increasing capital would be uncertain.

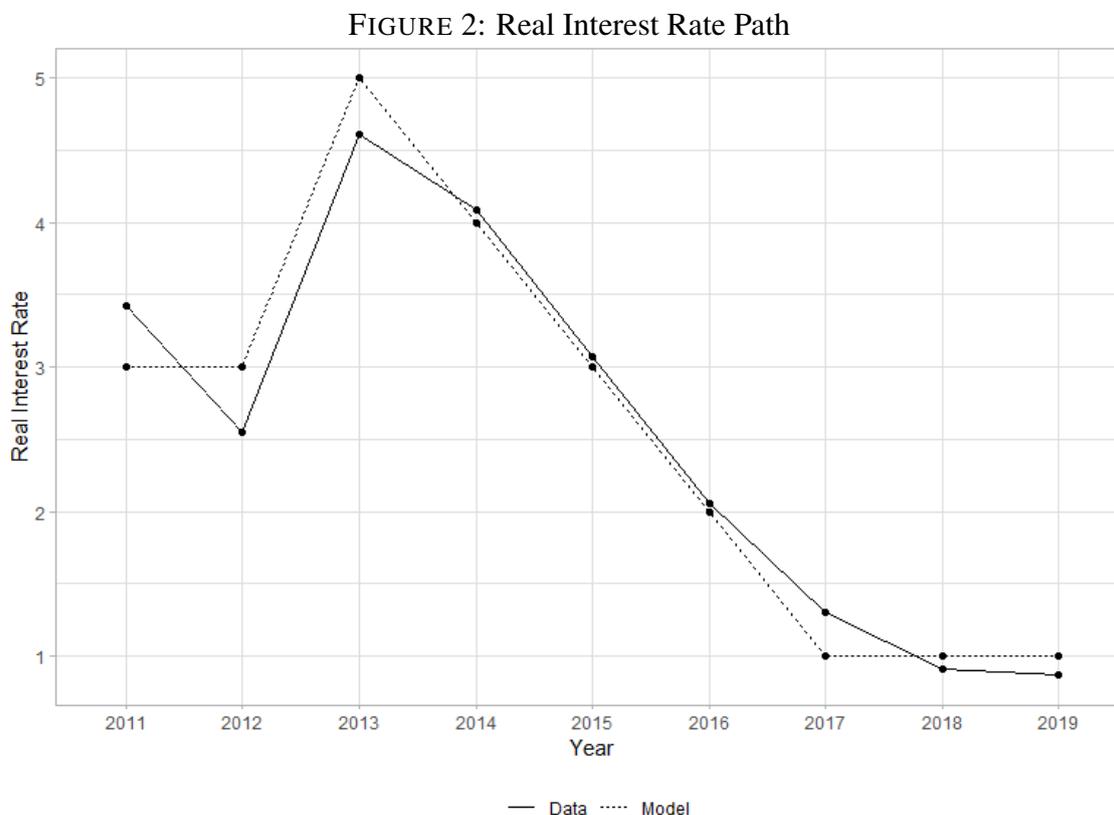
#### 4.4 The Real Interest Rate Path

In this version of the model built upon the baseline version of Gopinath et al. (2017), I assume that firms have perfect foresight and are hit by shocks to  $r_t$ , which are perceived as unexpected and permanent.

The analysis begins in a point where the economy is in a stochastic steady state and induce a change to the interest rate path. As such, I assume the economy starts in such a stochastic steady state, where the interest rate is 3 per cent per year, and then induce a new path of the interest rate, shown in the dotted line in Figure 2. A stochastic steady state in this model, is one where all aggregate shocks are constant, firms are hit with idiosyncratic shocks to productivity and change their production, savings, and investment decisions over time.

Figure 2 shows the interest rate path, both from the data and the one induced to the model. Contrary to Gopinath et al. (2017), there is an initial increase in the real interest rate, which suggests different qualitative results from this paper, as the setting is different, featuring both a longer period of heavy crisis of the country under analysis, as well as a difference in the real interest rate path. The measure used for the real interest rate as the mean of monthly nominal corporate lending rate to non-financial firms weighted by

the monthly volume of credit loaned to non-financial firms, minus the yearly average expected Harmonized Index of Consumer Prices forecast as presented in the ECB survey of professional forecasters.<sup>2</sup>



#### 4.5 Parametrization

Table IV details the parameters used for calibration in the model.

TABLE IV: BASELINE PARAMETERS

$\beta$	$\gamma$	$\delta$	$\varepsilon$	$\alpha$	$\rho$	$\sigma$	$\pi$	$z_L$	$z_H$	$\psi$	$\lambda_0$	$\lambda_1$
0.87	2.00	0.06	3.00	0.35	0.10	0.49	0.80	0.57	2.72	1.25	1	0.55

The first five parameters ( $\beta, \gamma, \delta, \varepsilon, \alpha$ ) are chosen to take conventional values and match those used in Gopinath et al. (2017). Productivity parameters ( $\rho, \sigma, \pi, z_L, z_H$ ) are estimated to match TFP dispersion as seen in firm-level data. Finally, ( $\psi, \lambda_0, \lambda_1$ ) are calibrated as to match three moments in the data:

<sup>2</sup>This measure follows from Gopinath et al. 2017.

**Moment 1.** Within-firm regression coefficient of capital growth  $(k' - k)/k$  on productivity  $\log Z$ , which exhibits a (statistically significant) point estimate equal to 0.01.

**Moment 2.** Fraction of firms that borrow equal to 0.91.

**Moment 3.** Cross-sectional regression coefficient of firm leverage  $b/k$  on capital  $\log k$ , which exhibits a (statistically significant) point estimate equal to 0.06.

## 5 RESULTS

This section documents empirical results and the simulated results of the model described above.

### 5.1 Empirical Results

Measures for the Marginal Revenue Productivity of Labor (MRPL), and the Marginal Revenue Productivity of Capital (MRPK), respectively, are taken as:

$$\text{MRPL}_{ist} = \left( \frac{1 - \alpha}{\mu} \right) \left( \frac{p_{ist} y_{ist}}{\ell_{ist}} \right) = \left( \frac{1}{1 - \tau_{ist}^y} \right) w_{ist}, \quad (14)$$

$$\text{MRPK}_{ist} = \left( \frac{\alpha}{\mu} \right) \left( \frac{p_{ist} y_{ist}}{k_{ist}} \right) = \left( \frac{1 + \tau_{ist}^k}{1 - \tau_{ist}^y} \right) (r_t + \delta_{st}), \quad (15)$$

$$\text{TFPR}_{ist} = p_{ist} A_{ist} = \frac{p_{ist} y_{ist}}{k_{ist}^\alpha \ell_{ist}^{1-\alpha}} = \mu \left( \frac{\text{MRPK}_{ist}}{\alpha} \right)^\alpha \left( \frac{\text{MRPL}_{ist}}{1 - \alpha} \right)^{1-\alpha}, \quad (16)$$

and, additionally, the measure for misallocation:

$$\Lambda_{ist} = \frac{1}{\varepsilon - 1} \left[ \log \left( E_i Z_{ist}^{\varepsilon-1} \left( \frac{\overline{\text{TFPR}}_{st}}{\text{TFPR}_{ist}} \right)^{\varepsilon-1} + \text{Cov}_i \left( Z_{ist}^{\varepsilon-1}, \left( \frac{\overline{\text{TFPR}}}{\text{TFPR}_{ist}} \right)^{\varepsilon-1} \right) \right) \right] - \frac{1}{\varepsilon - 1} \log(E_i Z_{ist}^{\varepsilon-1}) \quad (17)$$

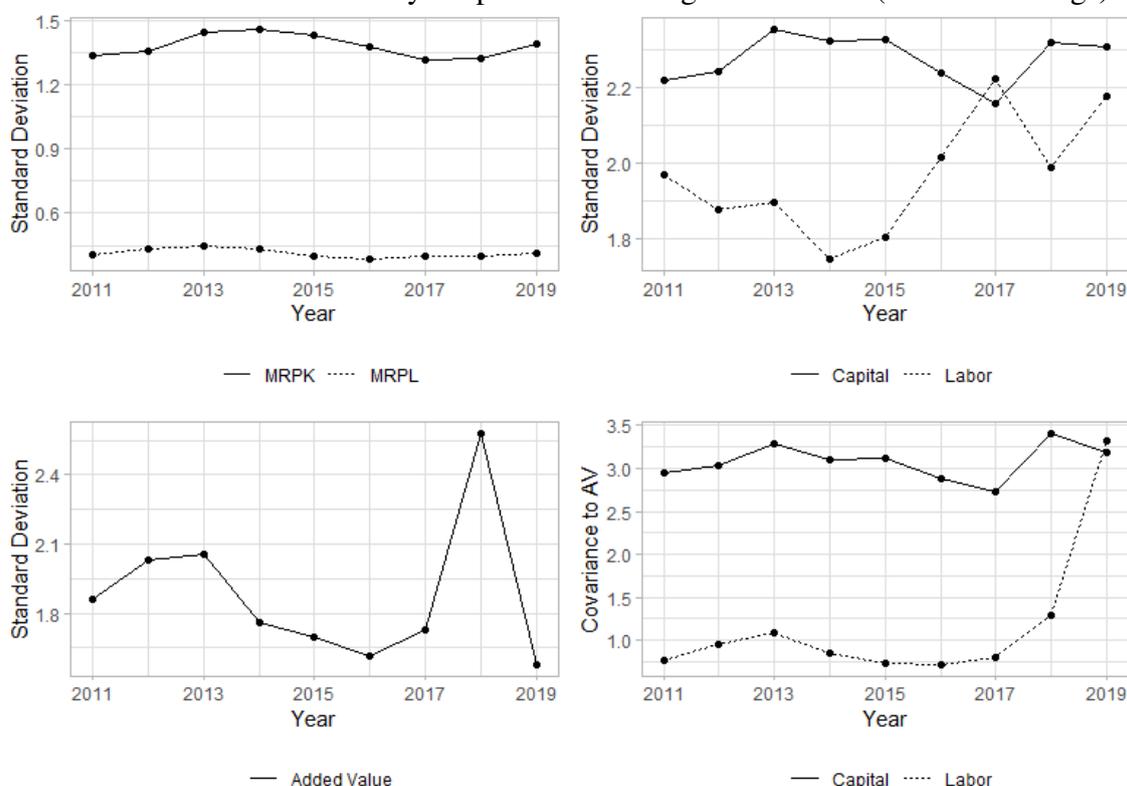
where  $\overline{\text{TFPR}}$  is the cross-firm mean of TFPR.

These measures replicate precisely those of Gopinath et al. (2017) (see Section 3 of the paper for the foundations of these measures). Notation follows from the Model section.  $\tau^y$  is a firm-specific output wedge, and  $\tau^k$  is a firm-specific wedge of capital relative to labor.

Note that, since labor is undistorted in this model, the only wedge affecting MRPL is that of output. Capital, on the other hand, is affected by an output wedge, as well as a wedge to capital itself.

To analyze dispersion in factor productivities, the first place to look is their standard deviations, and analyze how such dispersions could be achieved. Take Figure 3, for instance. The first panel shows the standard deviations of the marginal revenue product of capital, and that of labor. The other panels examine the dispersion of Value Added<sup>3</sup> (measured as operating revenue minus material costs), the dispersion of capital, and labor individually, and the covariance between capital and the measure of added value, and between labor and added value.

FIGURE 3: Factor Productivity Dispersion in Portugal 2011-2019 (Variables in logs)



The first thing to note, is that the standard deviation of MRPK is much higher than that of MRPL, but both are quite stable, though there are some small variations in line with the results for the dispersion of TFPR analyzed further on. Note that the standard deviation

<sup>3</sup>it isn't immediately clear whether the ORBIS database variable "Material Costs" takes into account intermediate costs such as energy, and comparing the difference between Turnover and Material Costs to the ORBIS variable "Added Value" actually shows there's a difference between the two, and so this isn't Value Added per se, but an intermediate variable somewhere between sales and actual Value Added. This measure is used to keep results comparable to Gopinath et al. 2017.

of MRPK varies more than that of MRPL, and this is much smaller in level compared to MRPK. In the standard deviation of factors, there is more happening, and it is clear that both labor and capital are more dispersed than their respective marginal productivities, and labor is on a much more comparable level to capital, than MRPL compared to MRPK.

In fact, Labor dispersion is significantly increasing over the period, but initially, between 2011 and 2014 it is actually falling, whereas that of Capital is increasing overall between 2011 and 2015. This countercyclical movement can be explained, to some extent, to the stage of the economic cycle of Portugal in these years, featuring an economic crisis, which was accompanied by policies aimed at liberalizing the labor market. Simultaneously this period featured very high unemployment, and after 2015 as the economy began to pick back up, employment increased, and at the same time the dispersion of labor increases.

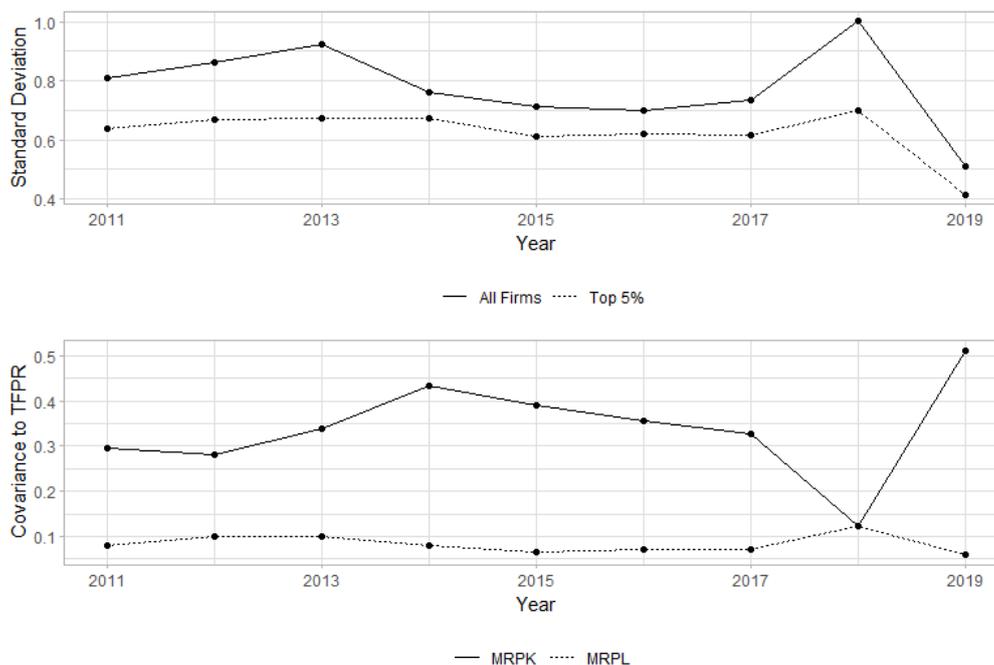
The dispersion of value added seems to be more in line with that of MRPK, with an initial increase up until 2015, followed by a small decrease and picking back up until 2018, with a stark fall in 2019, the reason for which is not fully clear, but the overall dispersion ends up much smaller than it started. Compared to the dispersion of labor, this indicates a countercyclical movement relative to the overall economic cycle of the period in Portugal. The similarity of the dispersion of Added Value to that of the Marginal Revenue Productivity of Capital, does not seem to be completely casual, as the fourth panel shows that Capital has a very high, positive, covariance to it. Again, compared to labor, the covariance of capital to value added is much higher, though the convergence seen in 2019 suggested by the standard deviation of factors and value added, seems to come from a sudden, stark fall in 2019 of the dispersion of value added, combined with the overall trend of Labor dispersion to go up all across the period.

Ignoring 2019 in the last panel, nothing too interesting seems to happen, with the covariance of both factors to value added being very stable, though capital comes at a much higher level when compared to that of labor. This is but the first strange result in 2019, which are not always plausible or easily explainable, and, more often than not, break away from the trends and cycles observed in the years leading up to it.

Figure 4 shows TFPR dispersion and its covariance with other factors' Marginal Revenue Productivities, as well as dispersion in the top 5% of firms with higher Capital. TFPR dispersion had again, a somewhat countercyclical behavior. In the first three years, 2011 to 2013 (peak crisis years in Portugal), there is a small increase in dispersion of TFPR, with a sustained fall leading up to 2017, an increase in 2018, and a dip in 2019. This reality is mirrored almost exactly when looking only at the top 5% of firms with the higher capital, but interestingly, this group of firms comes with a smaller overall level of dispersion. Having verified this, there is no reason to believe that the larger firms have

a dominating effect over that of smaller firms, as the dispersion is fairly close to that of the entire sample, and its actually smaller. Having said this, atomicity in the sample is probably a likelier cause of high TFPR dispersion, than a small group of large firms overpowering the results.

FIGURE 4: TFP Dispersion in Portugal 2011-2019 (Variables in logs)



Now looking at the second panel, a very small covariances of the marginal productivities with TFPR both are positive, though not very strongly. Covariance of TFPR and MRPK is almost an exact mirror image of the dispersion of TFPR, with a noticeable decrease in 2018, where it even matches the covariance of TFPR and MRPL, followed by a peak in 2019. Regarding the covariance of MRPL and TFPR there is little to mention, other than that it is very small, seemingly unresponsive, all across the period. Even in 2019 where the strangest movements happen in most variables, the movement in this covariance is not noticeable at all.

Concluding the analysis of the empirical results directly emerging from the data, Figure 5 plots the observed overall misallocation, measured as  $\Lambda_{ist}$ , and an efficient path of TFP<sup>4</sup> against the observed path of TFPR. Furthermore, in this figure, the full and permanent sample are shown. The permanent sample uses only firms with available data for the entire period. This is done to assess the possibility of results being skewed by new firms entering the sample.<sup>5</sup>

<sup>4</sup>Calculated as:  $\log(\text{TFPR} - \Lambda_{ist} = \log(\text{TFPR}^e))$

<sup>5</sup>When cleaning the dataset, firm-year observations are removed based on defined criteria. Because of

On the top panel, and on the same trend as Figure 4, there is a countercyclical movement in misallocation, with an initial ascending stage, from 2011 to 2015, and a descending stage between 2015 to 2018. Misallocation falls in years where the portuguese economy is weaker, and bounces back in years where the economy was picking up the pace and growing again. Coincidentally, there is a political shift in 2015, with a general election in this year, but there's no apparent reason to connect this to the shift in the path of productivity. A potential reasoning for this path is that in the recovery period, inefficient firms shut down, causing misallocation to fall. After the recovery, in the years following 2015, new, potentially less productive firms would enter the market and cause misallocation to increase. However, when taking into account only the permanent sample, this does not seem to hold up - this sample has only firms that are present in all years, and the same overarching trend is there.

It is worth noting how the permanent sample has lower misallocation all across the period though the gap is small and tends to be approximately constant, except for 2018. In both samples, 2019 features very low misallocation compared to previous years, and it would be tempting to claim this was due to economic performance. However, as previously discussed, 2015 through 2018 are good years for Portugal in terms of economic performance, and yet its misallocation is still increasing.

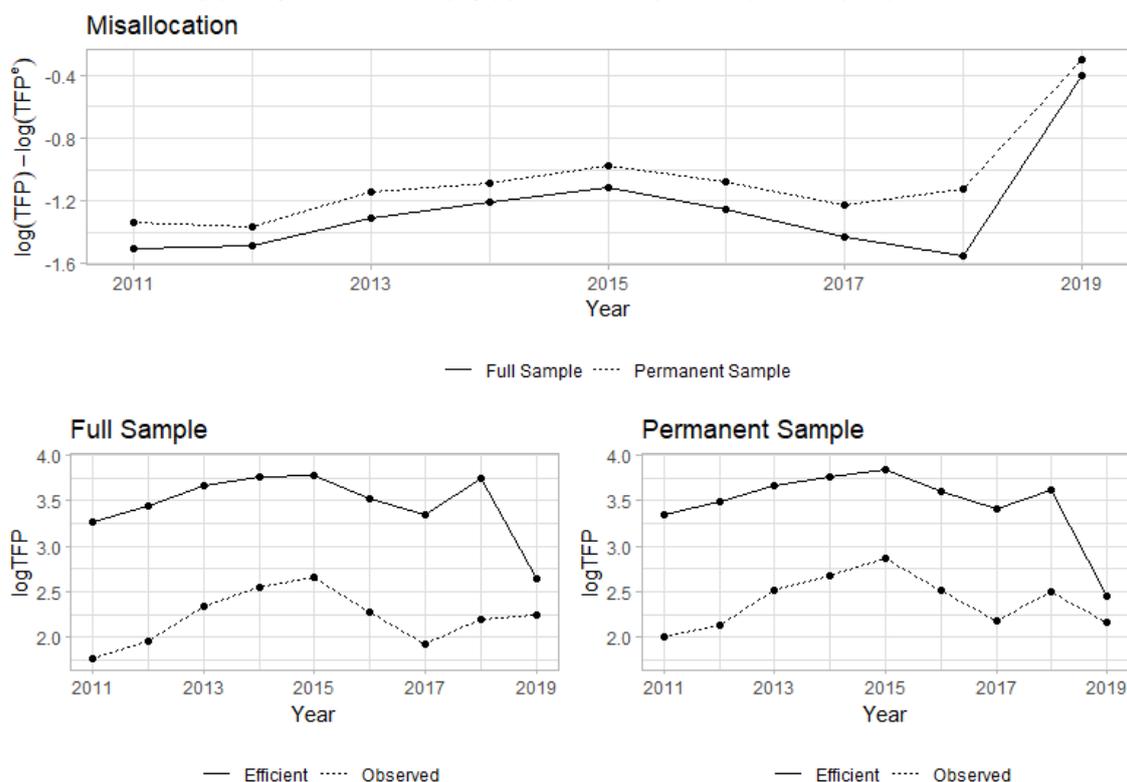
Now, let us analyze the bottom panels. They seem to show that misallocation fell so sharply in 2019. In both the full and permanent sample, the gap between efficient and observed TFP is apparent and large in all years except 2019. The same countercyclical behavior can be observed in both figures, where TFP increases in crisis years, and began to fall after 2015, but the novelty here is how the efficient level of TFP falls by so much in 2019, which explains the low level of misallocation in this year, as there is no large jump in productivity, but rather a large fall in the estimated efficient level in this year.

Figure 6 plots growth of efficient and observed TFP relative to 2011, as well as a counterfactual of 2% growth each year, for both the permanent and the full sample. In both samples, TFP grew well above this counterfactual rate in most years of the sample. Again, this figure reflects the strange evolution of TFP in 2019, as it is the only year where TFP actually falls, compared to the first year.

It is also apparent that these growth rates are lower in the permanent sample than the full sample, which may indicate that the permanent sample is a force driving TFP dispersion down.

Finally, Figure 7 depicts the misallocation for 2011 and 2015, the years where misallocation had a positive evolution. As shown here, misallocation in 2011 is high in general, but it has multiple peaks, in higher levels than observed in 2015, as well as a wider tail in this, some firms may be present in one year, while others may be present in all but one year.

FIGURE 5: Efficient vs Observed Paths and Total Misallocation



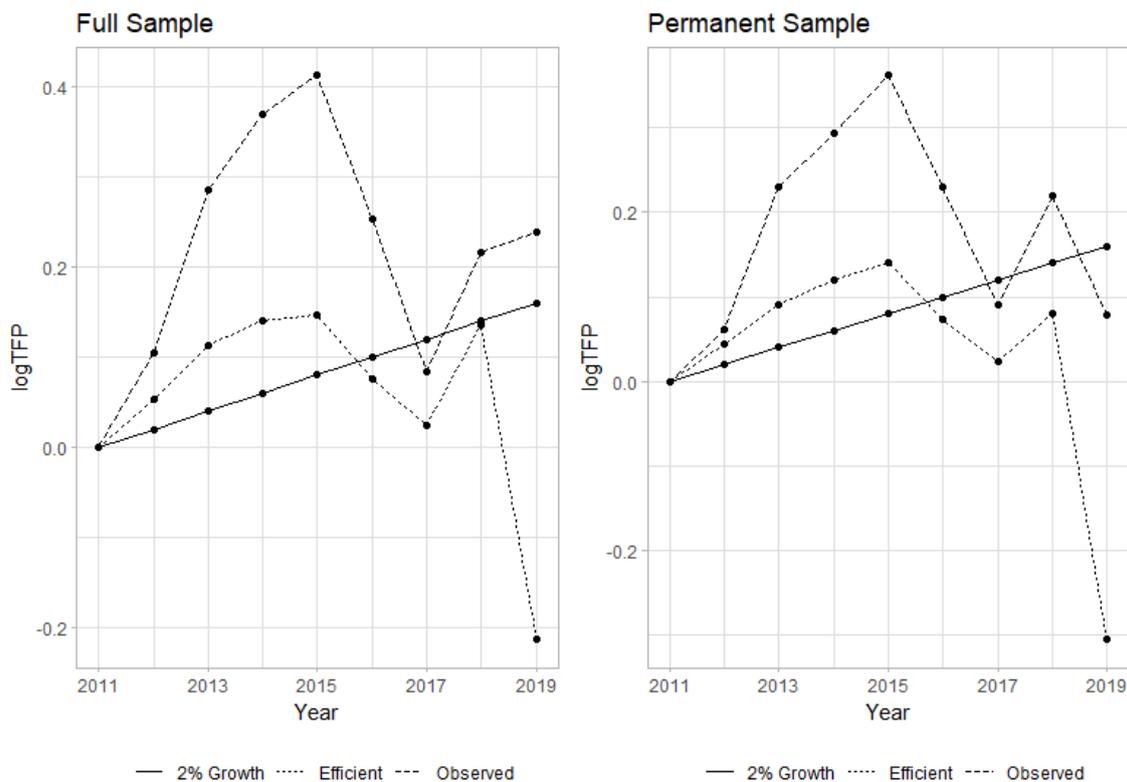
the left side of the distribution. In the year 2015 the distribution is more regular, exhibiting two peaks but very close to each other, and with a narrower tail on the left side. This denotes a clear trend of falling misallocation in the years where Portugal was under the Troika adjustment program.

Having said all this, the Troika adjustment period of 2011 to 2015 had a positive evolution of productivity in the manufacturing sector, which was not followed through in the following years up to 2019.

As a last note, 2019 has showed very strange results in the measures of productivity analyzed so far, especially with regards to the steep fall in efficient productivity. Looking at the permanent sample alone shows that the results are not dependent on new firms entering the sample in this specific year, but rather a strange evolution in the sector. I chose to keep this year in the sample due to this, but also because of the effect of removing 2019 would have in the size of the sample: in the end of all the data cleaning steps done, there are about 80.000 firm-year observations, and 2019 accounts for about 10.000 of those.<sup>6</sup> When considering that 2019 was not a recession year, these results are even more puzzling. The choice made was to keep this year in the sample, both due to the impact of

<sup>6</sup>Again, this doesn't mean there is a large portion of new firms in 2019, but rather that information for 2019 is more complete, and less firms are excluded in this year when cleaning the dataset.

FIGURE 6: 2% Simulated Rate vs Observed Growth in Efficient and Observed TFP



removing it in the sample size, and due to the small effect it has in the statistical moments shown below.

Table V depicts key statistical moments, which are separated into calibration moments (targeted in simulations) and the productivity parameters estimated directly from the data. When comparing the full and permanent samples the moments are very close to each other, and when looking at the moments obtained using the full sample except for the year 2019, moments are almost perfectly matched to the sample including this year. This gives credence to the notion that 2019 features some strange results, but it doesn't necessarily break away from the overarching trend observed.

### 5.2 Interjection: Markups and Misallocation, an Explanation?

A very high degree of misallocation is present in the data. do not have success in replicating this fact with the baseline model, or any of the variations used for comparison in the next section. An avenue to explain, or to complement the analysis is that of concentration, and whether or not there is a relation between the degree of concentration in the sample, and the misallocation verified. A rudimentary measure for markups is computed, by calculating the the materials share:

FIGURE 7: Distribution of Misallocation Across Time

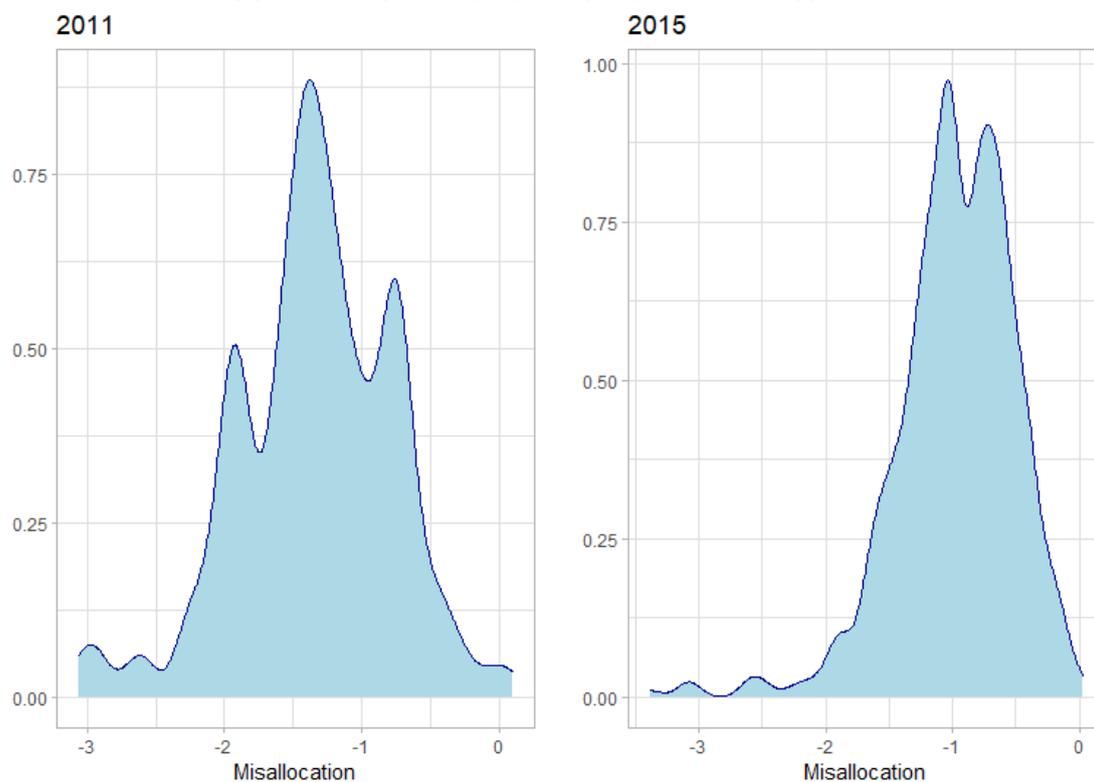


TABLE V: KEY MOMENTS: FULL SAMPLE, PERMANENT SAMPLE AND WITHOUT 2019

Moment	Full Sample	Permanent Sample	Without 2019
Number of firms	13706	2501	13387
<b>Calibration Moments</b>			
Coefficient of $(k' - k)/k$ on $\log Z$	0.01	0.01	0.01
Fraction Borrowing	0.91	0.98	0.90
Coefficient of $b/k$ on $\log k$	0.06	-0.0004	0.05
<b>Productivity Parameters</b>			
Standard Deviation of TFPR	0.86	0.69	0.86
$\sigma_{TFP}$	0.49	0.38	0.51
$\rho_{TFP}$	0.10	0.10	0.10

$$\text{Materials Share} = \frac{\text{Material Costs}}{\text{Sales}}, \quad (18)$$

and then, using the average elasticity of materials of portuguese firms, as calculated in Santos, Brito, et al. (2021) which is 0.606, to get the markup  $\mu$ , as,

$$\mu = \frac{0.606}{\text{Materials Share}}. \quad (19)$$

This is, of course, a simple calculation. but nonetheless gives an idea of whether or not there is any relation between firms being inefficient and having some degree of market power. I find that the average markup of the 10% of firms with the least misallocation is around 1.7, and the markup for the 10% of firms with the most misallocation is around 5.3. This strongly suggests that firms operating under more competitive conditions, having less market power, tend to be more efficient.

Furthermore, the average share of sales in the 10% of firms with the least level of misallocation is 0.13, but that of the 10% of firms with the largest distance from the efficient level of TFP, is 0.04. The relation here isn't clear cut, but this could indicate that there are firms operating under their optimal size.

Figure 8 shows the total misallocation calculated  $\log(\text{TFP}^e) - \log(\text{TFP})$  in the sample, to the log of markups. The correlation is negative and statistically significant, meaning, the larger markups seem to go hand-in-hand with the higher levels of misallocation. This follows nicely into the idea that there are competition effects involved in the behavior of firms, generating inefficiency.<sup>7</sup>

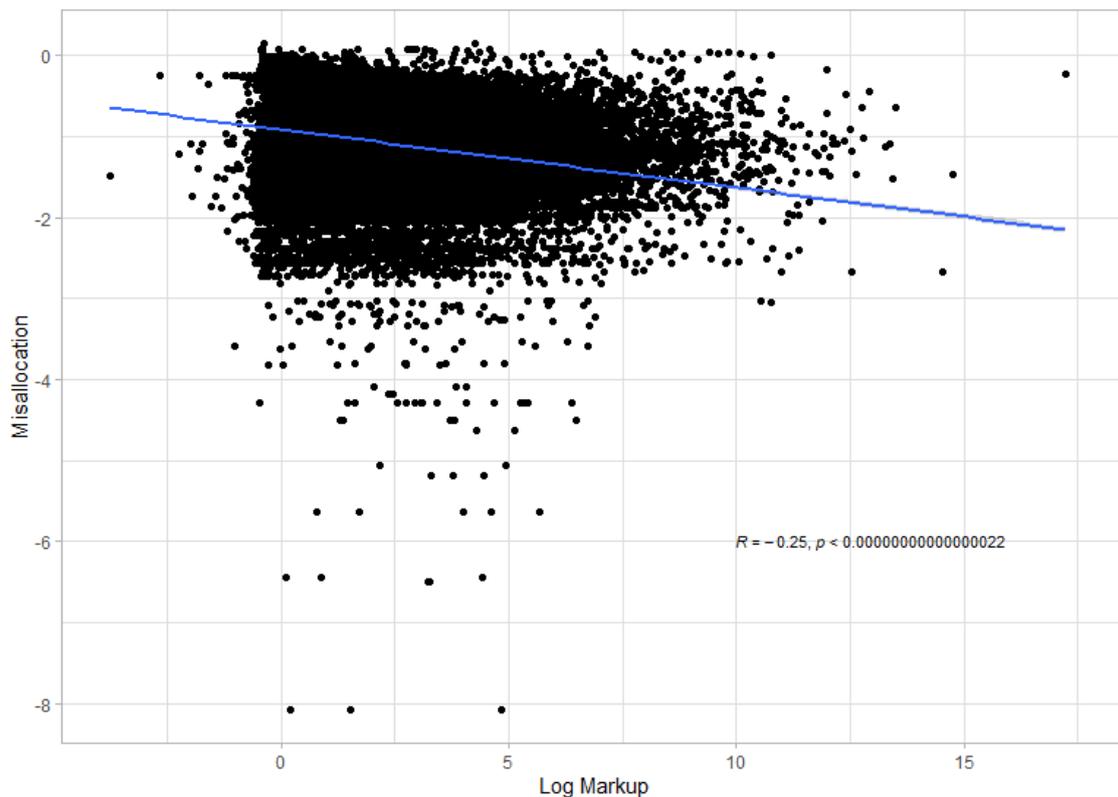
This is a worthwhile deviation as it is a possible reason to explain the behavior of firms operating below efficient levels of TFP, but this simple analysis should be taken as an indicator, and not a proven fact. As seen further on, heterogeneity in financial frictions faced by firms does not seem to be the larger reason why firms in the sample feature below-efficient TFP, as the model under this setting does not significantly overperform compared to the model featuring homogeneous frictions, so other explanations should be pursued, and this simple calculation points in the direction of markups.

### 5.3 Model Simulation Results

Table VI compares the moments in the data with those obtained by numerical simulations using the model in Section 4. For this purpose, data moments are compared to simulated results of three different models: the baseline model described in Section 4 featuring heterogeneous financial frictions (HEF), one with homogeneous financial frictions (HOF), and a model where firms face no financial frictions (NOF). Note that when calculating the moments in the data, the full sample of firms is considered.

<sup>7</sup>I realize that there are some very evident outliers in the graph, such as positive misallocation, negative markups, and very high levels of misallocation, but having removed all of these the correlation coefficient stays around the same value, and the p-value is not affected at all, so I chose to keep the figure as is.

FIGURE 8: Correlation of Markups and Misallocation



In the baseline HEF model takes the parameter vector  $(\psi, \lambda_0, \lambda_1) = (1.1, 1.25, 0.85)$  to match the three moments discussed in the end of Section 4, also shown in Table V. In the case of the HOF model the vector  $(\psi, \lambda_0, \lambda_1) = (1.25, 10, 0)$  was chosen to match the first two moments, and in the case of NOF the vector used  $(\psi, \lambda_0, \lambda_1) = (1.1, \infty, \infty)$  to match only the third moment.

**A. Distributional Moments.** Panel A in Table VI shows statistics regarding distributional aspects of firm size in the data and in the studied models, namely the standard deviations (SD) of TFP, labor, and capital, as well as the top 20% share of aggregate labor and capital. In the first row, all models have the same standard deviation of  $\log Z$  - this is by construction as this standard deviation is input directly in the model as a parameter in the productivity distribution. In row 2, all models overshoot regarding dispersion of labor, which in the data is milder than achieved with any of the three models. In row 3, all three models match around 60% of observed dispersion in capital.

Row 4 reports labor concentration in the top fifth of firms and results are somewhat similar to row 2, where all models are close to each other and surpass what is observed in the data. Row 5 depicts capital accumulation in the same percentile of firms, where, again, all models report very similar moments, matching about 90% of observed capital

concentration.

**B. Within-Firm Moments.** Panel B informs us on dynamics of capital, debt, and productivity at the firm-level. This panel encompasses results from two regressions, where rows 6 to 8 show point estimates from a regression of capital growth on productivity, net worth, and capital:

$$\frac{k_{ist+1} - k_{ist}}{k_{ist}} = d_i + d_{st} + \beta_z \log(Z_{ist}) + \beta_a \log(a_{ist}) + \beta_k \log(k_{ist}) + u_{ist}^k, \quad (20)$$

with  $d_i$  being a firm fixed effect and  $d_{st}$  a sector-year fixed effect, where sectors are identified at the four-digit level. Rows 9 through 11 present the estimated coefficients taken from a similar regression, where the left-hand side is now the debt to capital ratio  $(b_{ist+1} - b_{ist})/k_{ist}$ .

In row 6, you will notice all models have the same moment. This is by design. The data moment is 0.01, which is particularly hard to match, when trying to replicate as much productivity dispersion as in this case. What happens is that to reach such a low level of this specific coefficient, the parameter governing costs of adjustment to capital,  $\psi$ , has to be increased significantly, which when simulating with the high TFP dispersion target of about 0.8 (as in the data) puts a cap on fraction of borrowing reported in row 12. It is not linear, but in general, to match this moment, one would have to compromise on a much lower level of fraction of borrowing firms. For this reason, the choice was made to match row 6 at 0.1, which is roughly the point at which the other key moments can be effectively achieved.

In this panel, models are mostly close between each other, with a few rows where they differentiate between each other and show relations worth looking into. Row 7, for example, shows models very close to each other. There is a generalized overshoot of how much investment is affected by current period net worth in the models, compared to the data, though it should be noted that in the data, the relation is positive as theoretically expected, but it is weaker than intuition would lead one to believe.

TABLE VI: MODEL COMPARISON

Moment	Data	HEF	HOF	NOF
<b><u>A. Distributional Moments</u></b>				
1. SD( $\log Z$ )	0.86	0.80	0.80	0.80
2. SD( $\log \ell$ )	1.31	1.37	1.38	1.38
3. SD( $\log k$ )	2.01	1.26	1.27	1.26
4. Top 20% Share of Aggregate Labor	0.80	0.85	0.85	0.85
5. Top 20% Share of Aggregate Capital	0.93	0.83	0.84	0.83
<b><u>B. Within-Firm Moments</u></b>				
6. Coefficient of $(k' - k)/k$ on $\log Z$	0.01	0.10	0.10	0.10
7. Coefficient of $(k' - k)/k$ on $\log a$	0.06	0.12	0.13	0.13
8. Coefficient of $(k' - k)/k$ on $\log k$	-0.52	-0.40	-0.36	-0.40
9. Coefficient of $(b' - b)/k$ on $\log Z$	-0.03	-0.21	-0.22	-0.21
10. Coefficient of $(b' - b)/k$ on $\log a$	0.16	0.26	0.29	0.26
11. Coefficient of $(b' - b)/k$ on $\log k$	-0.24	-0.47	-0.45	-0.48
<b><u>C. Cross-Sectional Moments</u></b>				
12. Fraction Borrowing	0.91	0.95	0.94	0.95
13. Coefficient of $b/k$ on $\log k$	0.06	0.06	0.06	0.06
14. Corr( $\log Z, \log k$ )	-0.08	0.76	0.76	0.76
15. Corr( $\log Z, \log a$ )	0.16	0.66	0.68	0.66
16. Corr( $\log \text{MRPK}, \log Z$ )	0.78	0.61	0.61	0.62
17. Corr( $\log \text{MRPK}, \log k$ )	-0.51	-0.05	-0.06	0.04
18. Corr( $\log \text{MRPK}, \log a$ )	-0.15	-0.10	-0.10	-0.10
<b><u>D. Model Evaluation</u></b>				
19. Root Mean Squared Error		0.35	0.38	0.36
20. Mean Absolute Error		0.25	0.27	0.26

In rows 8 and 11, current capital stock has a negative effect in both investment (row 8) and debt (row 11) which is an interesting relation, but the effect is much smaller in debt than that of investment. This points to less investment and, by consequence, debt accumulation in firms with higher levels of capital. However, row 9 completes this picture:

debt is only slightly affected by productivity, though negatively. This seems to indicate that productive firms are not being rewarded, in a sense, for their good performance with a relaxation of their credit constraints, but rather this effect has little to no weight in the sample, instead most of debt reduction comes from pre existing capital stock.

**C. Cross-Sectional Moments.** This panel shows correlations between firm size, productivity, leverage, net worth, and MRPK in the cross section of firms. Row 12 features the percentage of firms borrowing, and row 13 is the regression coefficient of the fraction of debt over capital on the capital stock itself. Row 12 is a calibration target for both the HEF and HOF models, and row 13 is a target when simulating the HEF model. This explains similarities in between these models in row 12, though the matching between models in row 13 is purely coincidental.

When looking at the results for correlations, it is clear that most miss their mark. In row 14 the sign is opposite to what is verified in the data, the effect is much larger. Row 15 even though the sign matches, the magnitude is way off in all three models. In row 14, the observed correlation is a strange: TFPR correlates negatively with the capital stock, which is not a theoretically or empirically sound result, when taking Gopinath et al. (2017) for reference. Nonetheless, it is the result in the data, which went through a thorough process of cleaning and error correcting. It could point to a larger problem than the scope of this dissertation, but this result is not to be replicable in other datasets of firm data. Being such a strange result, it follows that this model is not particularly suited to replicate it.

Row 17 comes in the same vein as row 15, the sign is the same but the size of the effect is not satisfactorily matched. For row 17 however, it is important to note that the correlation for the NOF model is negative while in the data, and the other models, it is positive.

In rows 18 and 16 correlations are replicated the best, in row 16 the models replicate almost 80% of the observed effect. In row 18, the three models replicate two thirds of the observed moment.

**D. Model Evaluation.** In this panel, the root mean squared error and the mean absolute error of the three models are presented. These measures are calculated using only the moments not targeted during calibration, so rows 6, 12, and 13 are excluded. All three models are very close to each other, so there is not a clear cut winner, as opposed to Gopinath et al. (2017) where the HEF model is a clear best, though even here it performs marginally better than the other two. Interestingly NOF has a small edge over HOF in both measures.

### 5.4 Aggregate Results

In this section, aggregate results stemming from model simulations are analyzed and compared with the data, as well as comparing the different models.

In Table VII the three models discussed so far are compared to the full dataset, and a subset that does not include 2019.<sup>8</sup> The reason this subset is used is due to the discussion of Section 5.1.

Table VII compares the percentage-point change in misallocation, between the first and final years. The same is done for the standard deviation of log MRPK, log sum of value added, and for the log of aggregate capital. Finally, the last row is the ratio between the changes in debt and capital in the period from 2011 to the final year.

TABLE VII: AGGREGATE RESPONSES

	Data	Data <sup>2018</sup>	HEF	HOF	NOF
1. $\Delta\Lambda$	1.12	-0.009	-0.0007	-0.003	0.0007
2. $\Delta(\text{Dispersion})$	-0.02	0.12	-0.007	-0.0005	-0.0051
3. $\Delta \log(\sum y)$	0.38	0.25	0.06	0.06	0.04
4. $\Delta \log K$	0.213	0.11	0.08	0.09	0.08
5. $\Delta B/\Delta K$	9.92	0.35	1.4	1.21	1.41

When comparing the models amongst themselves, it is clear that their performances are very similar. The biggest difference is that of row 5, where the HOF model exhibits a marginal ratio of debt to capital much smaller than that which is produced by the other models. This row also shows that in all three models, firms increased their leverage as a result of the decline in the real interest rate. Compared to the reality observed in the data, when including 2019, the increase is much steeper, but when that year is removed the dataset reports a much more moderate evolution, even more than what is depicted in the data.

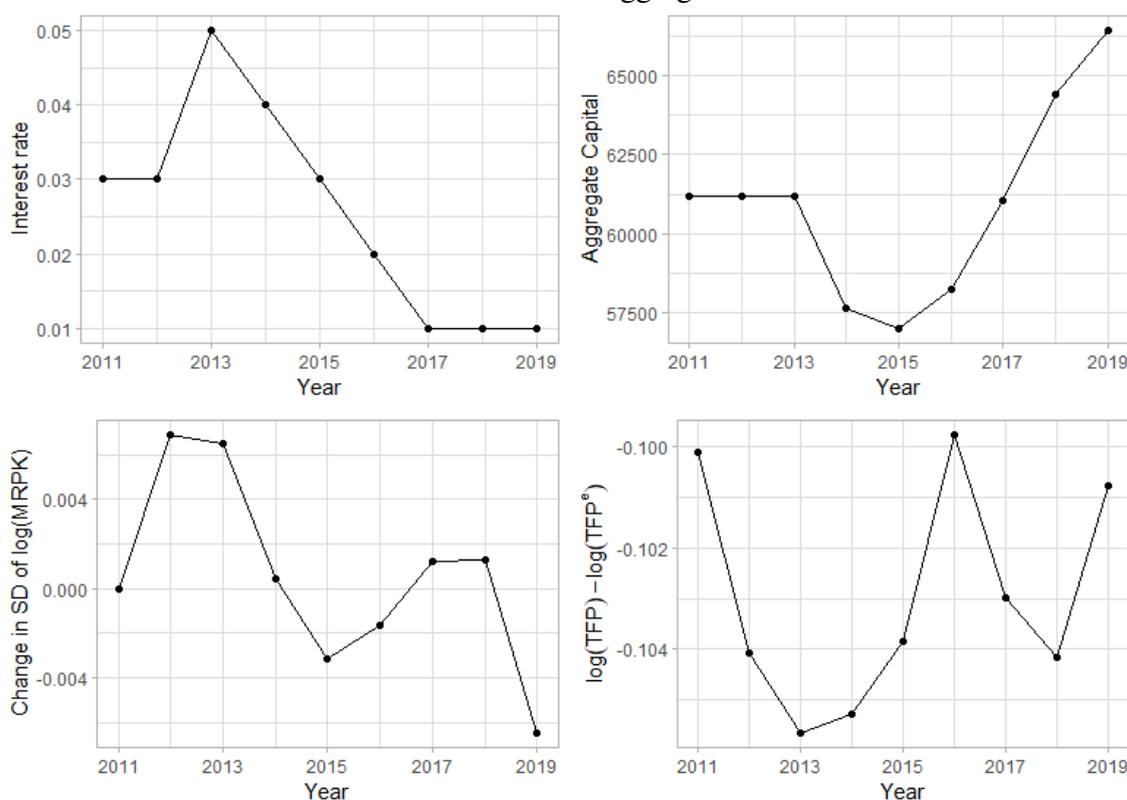
Misallocation evolves very modestly in all three models, which is an accurate picture of the data before 2019, but quite far off when including this year. Dispersion of MRPK in the models seems much more in line with what is in the data when including 2019 than in the years up until 2018, with a very modest decline. Value added grows more in the data than in any of the models studied and when it comes to aggregate capital, there is a

<sup>8</sup>Notation is "Data" for the full set, and "Data<sup>2018</sup>" for the subset without 2019.

decent match when comparing data until 2018 with the simulated results, but 2019 reports a steeper evolution of aggregate capital.

Finally, Figure 9 plots the evolution of aggregate variables in the HEF model. Aggregate capital seems to respond to the real interest rate at a lag. The smallest level is reached in 2015, which is coincidental with the year where the interest rate, after an initial rise, comes back to its initial level. After that, the trend is very clearly increasing, which comes in line with the period where the real interest rate is decreasing, and when it stabilizes at 1% in the last three years. Aggregate capital seems to respond to this in the last period, where the trend - though still steep - seems to be flattening out.

FIGURE 9: Evolution of Aggregate Variables



When it comes to the evolution of dispersion of MRPK the overall trend is for a generalized fall, though a very mild one. When the real interest rate is higher, however, the dispersion is actually increasing. This is the only aggregate variable where the evolution goes against that of Gopinath et al. (2017).

Misallocation has a strange evolution, as it is increasing over the period, and its evolution is a bit erratic, though it remains within a very narrow range, so even big jumps seen in the graph are very very small changes. This reflects the results of Table VII, where the change from the initial year to the last year in the HEF model is essentially zero.

## 6 CONCLUSION

The aim of this dissertation was to study capital and productivity dynamics in Portugal, in a period of dire financial and social conditions in the country, using quantitative methods. The shock of the European Sovereign Debt Crisis hit Portugal particularly hard, as the country had not yet recovered from the strong shock of the 2008 Financial Crisis and was still trying to catch up after the slow economic growth in the start of the twentieth century.

Empirically, I observed that the distributions of output and wages differ across different sizes of firms. Larger and medium-sized firms pay the largest share of the wage bill, as well as being the biggest contributors in output - even when they represent a very small percentage of firms in the dataset. This observation seems particularly worrying when considering the trend in Portuguese firms to shrink, as smaller firms do not contribute as much to output and wages.

In addition, I also observed a stark difference in the characteristics of production factors - capital and its marginal product are much more disperse than labor and its own marginal product.

Following this, I document the good recovery of productivity up until 2015 after an aggressive crisis. However, said recovery was followed by a change in trajectory - though it didn't altogether ruin the previously verified recovery of TFPR, the slowdown made it an overall mediocre, even if positive, evolution.

Simultaneously, the measures calculated for the difference between observed TFPR and what would be its efficient level, show a strong trend of improvement up until 2018, though the data seems to be faulty for 2019, preventing one to extrapolate much for this year. I use simple, though good enough to inform in a comparative sense, measures of markups to conclude that misallocation can have a very direct link to the competitive structures of markets (a very promising extension of the model, if possible, would be exactly this - dropping the fixed markup simplification in favor of a setting under some kind of non-perfect competition).

The three versions of the model simulated (i.e. baseline, homogeneous frictions and no frictions) performed reasonably well, but none stood out as the clear best. Even the model featuring the most complexity - with heterogeneous frictions - did not significantly outperform the version with no financial frictions. This seems to point away from financial frictions as a standalone explanation to the productivity issues of the Portuguese manufacturing sector.

It remains to be seen how the COVID-19 Pandemic Shock has altered this picture, and similar studies to what I have done, now applied to the current economic conditions of Portugal and the entire world, is an obvious path for future research.

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