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INTERNSHIP REPORT

USING ONE-YEAR CLAIM DEVELOPMENT TO CHOOSE A LARGE CLAIM RESERVING TECHNIQUE

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SUPERVISION:

WALTHER NEUHAUS
ISABEL RIBEIRO (LIBERTY SEGUROS)

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LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Abstract

This internship report is written with the purpose of suggesting a best practice for dealing with large motor bodily injury claims at Liberty Seguros Portugal. To do this, claim splitting techniques already used by Liberty Seguros and other high performing countries in Liberty International are considered. One can then apply these techniques to the situation in Portugal to see if the current way of working with large claims is the optimal one.

Within each technique, several well-known methods of reserving will be analyzed. These methods include the Chain Ladder Method, Cape Cod Method, and Benktander Method.

Through model validation using one-year claim developments it is possible to create a statistic, which the author calls the one-year sensitivity measure, to measure the effectiveness of a technique to absorb large claims. A specific combination of technique and methodology will be suggested to most accurately predict ultimate reserves based on anonymized data from Liberty Seguros. Because the data has been altered to protect Liberty Seguros' sensitive data, the read should focus on the process of comparing techniques and not the actual value of the results.

This report relies heavily on the software used at Liberty International, ResQ by Towers Watson, but the results will be summarized in Microsoft Excel.

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Contents

1. Introduction

There is a common question in insurance companies revolving around large claims. Specifically, should or should not a company split large claims from normal claims when conducting reserve projections. Furthermore, additional questions arise if a company does decide to split large claims: what should be considered a large claim, how should they treat claim severities that fall below the large claim threshold and what methodology should be used to project reserves resulting from large claims.

Liberty International gives freedom to managers in each country to handle large claims as they feel appropriate for their country's realities. Liberty International's Portuguese branch, Liberty Seguros, separates large claims at a 100,000 threshold and projects large claim ultimates using a combination of incurred cost for reported claims and a frequency-driven model for unreported claims. The purpose of this internship report is to compare Liberty Seguros' current large claim splitting practices against other countries' practices (as well as not splitting) to recommend the best way to deal with large bodily injury (BI) motor insurance claims when estimating reserve projections.

This report will arrive at a conclusion by explaining to the reader the basic reasoning behind splitting large claims as well as the most common methods for BI reserving. It is assumed that the reader has fundamental understanding of the insurance industry, motor insurance and BI, and the reserving process. It is necessary to explain the practices used by Portugal and other Liberty International countries to form an opinion of those practices by applying them to Portugal's claim information. Furthermore, the question of whether to split large claims or not will be thoroughly evaluated. Finally, there will be an aggregate suggestion as to the best splitting practice and reserving methodology specific for Liberty Seguros Portugal.

To do this, several shocked scenarios will be simulated. Additional large claims will be introduced to the total incurred claims triangle and large claim count triangles. The one-year claim development (OCD) will then be compared using different reserving methodologies, the Chain Ladder Method, Cape Cod Method, and Benktander Method. The one-year claim development is measured by the change in the aggregate reserve ultimate between 2014 and 2015 (excluding the 2015 cohort for which no aggregate reserve ultimate was available in 2014).

The standard deviation of each method's one-year uncertainty will be calculated by computing the OCD of each method under three shocked scenarios. The technique that yields the lowest average of standard deviations, called the one- year sensitivity measure by the author, will be selected as the best approach for handling large claims.

2. What is a Large Claim?

There isn't one universally accepted quantitative definition as to what constitutes a large claim. Large in the sense of magnitude is relative and has to be looked at from the perspective of the country, company and line of business and can rarely be compared equally across these classifications. Where a motor claim of 1,000,000 may not be considered exceptionally large in Ireland, it could be relatively significant in Portugal. It is possible to mathematically set a standard for large claims, i.e. 1.64 standard deviations from the mean claim cost, but that wouldn't tell you much about the claim other than it is uncommon. In any case, standard deviation is not a good measure for heavily skewed distributions. Percentiles could be better, if available.

Qualitatively classifying large claims can be much more beneficial for reserving. The importance of classifying a claim as large is to be able to treat them in a special manner. This could be done on an individual basis or by looking at all claims as a whole. Large claims are ones that occur less frequently, occur in irregular patterns, and have a higher magnitude than average. Normally, they make up the lion's share of aggregate claim cost. This leads to the common characteristic of all large claims; they are hard to predict.

It is up to management to decide why they are splitting large claims and then to decide the criteria that constitutes a large claim.

2.1 The Optimal Threshold

There is a lot of ambiguity surrounding the topic of setting an appropriate threshold for splitting large and non-large claims. One may also see the term non-large claims referred to as normal claims or attritional losses. This topic is surprisingly underrepresented in scholarly publications but for good reason. Choosing a threshold is, at best, a situation by situation, company to company decision and is difficult to generalize.

The end goal of choosing an appropriate threshold should be two fold. One outcome should be to ensure that there is enough non-large or low volatility claims to produce a statistically sound and consistent estimate of normal claims. Secondly, the large claims should similar enough to be able to estimate them together, even if the bond that ties them is that they are completely random.

An article from the 1998 General Insurance Convention & ASTIN Colloquium describes some approaches for finding the split threshold.

- Plot the claim size distribution and read off the value above which a fixed percentage of the claims lie. e.g. 95% of claims are below £50,000 therefore cap all claims at £50,000.
- Select an arbitrary round number.
- Select a point equal to the reinsurance retention limit. This can work if the reinsurance retention limit is particularly low, however in practice this is unlikely to be low enough to remove the distortion caused by larger claims.

The truncation point will generally be lower for assessing relativities for pricing purposes than for reserving as the need for more stable results is greater. (Czernuszewicz, et al., 1998)

2.2 The Threshold at Liberty Seguros

At Liberty Seguros, the threshold is set at 100,000. This may seem to be low but on average 98.24% of the total claim count is below this value. If the threshold would be raised, there would be even fewer large claims, which could limit the variety of techniques that could be used to project large claim ultimates (i.e. there wouldn't be a large enough volume of claims to use a stochastic method.)

Choosing the optimal large claim threshold is beyond the scope of the internship project and will not be covered in this report. The optimal threshold will be assumed to be the current practice of splitting large and normal claims at 100,000. This assumption is justified because:

- experienced actuaries have set it at this value.
- management at Liberty Seguros has consistent results when projecting the normal incurred claim ultimates.
- only 1.76% of total claims are above the large claim threshold; raising it would result in practically eliminating the split and evaluating all claims together.
- the average claim severity is 6,322. From this point of view, 100,000 is relatively high and claims above this threshold could be considered exceptionally large.

3. Handling Large Claims

There are many ways to deal with large claims and no shortage of literature, notably *A Bifurcation Approach for Attritional and Large Losses in Chain Ladder Calculations*. (Riegel, 2014) Beyond the concept of splitting, there is also a need to decide how to handle large claims with the two main categories being removing or not removing large claims.

One can remove large claims by taking out any claims that distort the statistics of the projections. However, the article *Reserving and Pricing for Large Claims* rejects the process. The authors reason, "If the truncation point is £50,000, why should a claim of £49,999 be kept in the record and one of £50,001 be discarded?" Although the article continues to justify some removal by stating "When assessing an individual risk, however, there may be large claims where the circumstances which led to the claim simply cannot recur and this can be justification for removing the total claim." (Czernuszewicz, et al., 1998)

It seems to make more sense to leave in large claims and then decide how to split them. The report will only focus on this option. For the purpose of this report three main splitting techniques will be explained and analyzed. These include Total, In and Out (Count – Portugal and Excess – USA/Spain), and Leave In (Count – Ireland).

Here we see a simple example of how large claims are accounted for in each technique. In all of the scenarios in the example, the threshold is set at 100,000 and 1 individual claim is examined. The first technique, Total, can be used as the baseline reference for the next 3 approaches since Total also represents the incurred claims reality.

Incurred Severity (Total)

AY/DY	0	1	2	3	4	5	…
2010	50k	90k	120k	120k	90k Settled	90k	90k

Portugal

Large Claim Count (In and Out)

AY/DY	0	1	2	3	4	5	…
2010	0	0	1	1	0	0	0

USA/Spain

Incurred Severity Excess (In and Out)

AY/DY	0	1	2	3	4	5	…
2010	0	0	20k	20k	0	0	0

Ireland

Large Claim Count (In Only)

AY/DY	0	1	2	3	4	5	…
2010	0	0	1	1	1	1	1

For Portugal (In and Out), a large claim is counted in lag 2 when the claim severity exceeds 100,000 and is removed from the triangle in the 4th lag when the severity drops below 100,000. In the USA/Spain example (In and Out), the amount of the claim severity that exceeds 100,000, or 20,000, is recognized in the large claim triangle in the 2nd lag. The excess becomes 0 in the 4th lag when the claim severity drops below 100,000. Finally, in the Ireland example (In Only or also called Leave In), a large claim is counted in lag 2 when the claim severity exceeds 100,000 but is not removed from the triangle in the 4th lag. It instead remains in the large triangle forever.

Each technique will now be evaluated in greater detail.

3.1 Total Approach

An option for Liberty Seguros is to not split the claims or the Total technique. Basically, the incurred claims triangle is projected to obtain a final ultimate estimate with no special attention to large claims.

This strategy of dealing with large claims can be used at Liberty Seguros for multiple reasons.

- The large claims are not that large compared to other countries like the United States and Ireland. Since 2009, the average large claim size has only been 183,846.

- There will be very few, if any, large claims occurring in the later development years (DYs). About 90% of the ultimate incurred cost of large claims is recognized in DY 0 and about 97.8% is recognized by the end of DY 1. This indicates that incurred costs are set conservatively by the company. Since the most harmful risk associated with large claims arise in the tail, including large claims in the total incurred triangle is not overly risky.

However, the Total technique could not be ideal based on the large claim count. Referring again to the 1998 General Insurance Convention & ASTIN Colloquium article,

> The decision over whether to include or exclude large claims in the reserving triangulations will depend on the class involved and the incidence of large claims. If there have been a high number in the past and the link ratios from year to year are not distorted then it may be acceptable to leave the triangles unadjusted. (Czernuszewicz, et al., 1998)

There may not be enough large claims to justify a Total technique.

3.1.1 Advantages

- There is no need to create separate large triangles for reserving purposes.
- There isn't a need to estimate average claim sizes when dealing with incurred units. The results are already represented in the same units as the ultimate reserves, which is not the case with the following two techniques.

3.1.2 Disadvantages

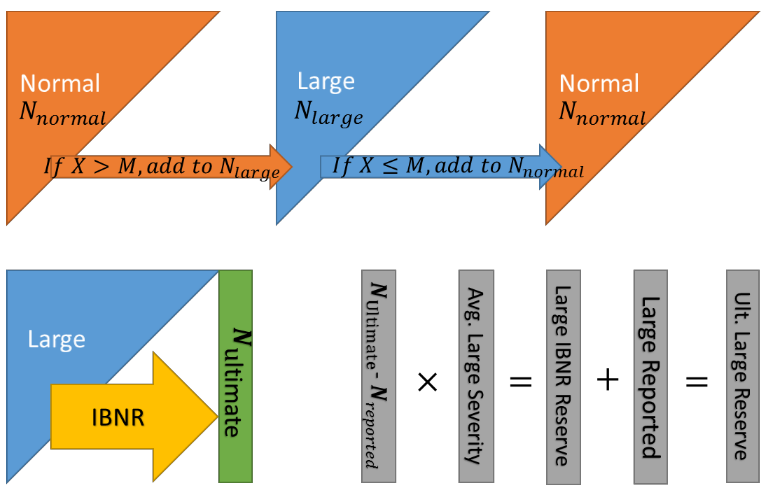
- There is more volatility in the total triangle since the large claims are not separated. In theory, this volatility can almost be eliminated with a split, leading to at least one highly stable, normal claims triangle.
- Separating large claims can give valuable information on the reality of the claims situation in Portugal.

3.2 In and Out Technique

The In and Out technique is used by Portugal and USA/Spain differently. In Portugal, the number of large claims are counted. In USA/Spain, the excess above the threshold is aggregated.

3.2.1 In and Out - Claim Count

At Liberty Seguros Portugal, the In and Out technique is used to project large claim counts. In this technique, a large claim threshold is determined. When a claim surpasses this threshold in magnitude, it is recognized in the large claims count triangle.



N_{normal} is the number of normal claims.
N_{large} is the number of large claims.
X is the severity of a claim. x_i represents the severity of claim i.
M is the large claim threshold.

If $x_i \leq M, n_{normal_i} = 1$ and $n_{large_i} = 0$. If $x_i > M, n_{normal_i} = 0$ and $n_{large_i} = 1$. A claim can enter the large claim count triangle if it is initially estimated to be above the threshold or if it was once estimated to be below the threshold and then becomes large because of some new information.

The large claim number, N_{large}, is used to project the ultimate number of large claims. Once the large IBNR claims count is estimated, it is multiplied by the average large claim severity to yield the IBNR reserve for large claims. To obtain the ultimate large claim reserve, the incurred cost of reported large claims is added to the IBNR reserve for large claims.

The normal claims ultimate can be projected using paid claims, incurred claims, claim count, or other measures. This report deals only with the claims above the large claim threshold and the normal triangle and normal claim ultimates will not be analyzed.

The following is an example of the large claim ultimate estimate process:

AY/DY	0	1	2	3	…	10	11	12	13	14	Ultimate Count	IBNR Count	Severity	IBNR Reserve for Large
2001	11	41	41	47		37	36	36	36	36	36	0	228,605	0
2002	29	32	30	27		24	25	26	26		26	0	228,605	0
2003	21	25	23	22		20	20	20			20	0	228,605	0
2004	15	17	24	25		21	21				21	0	228,605	45,721
2005	20	35	44	41		37					38	1	228,605	218,281
2006	18	34	37	40							40	0	228,605	8,581
2007	20	21	20	23							23	0	228,605	0
2008	25	30	35	43							46	0	228,605	0
2009	20	25	25	31							34	0	228,605	0
2010	10	22	25	29							32	-4	228,605	-914,420
2011	23	36	42	47							46	-2	228,605	-457,210
2012	11	22	31	32							33	1	228,605	228,605
2013	15	26	31								33	2	228,605	457,210
2014	10	16									32	16	228,605	3,700,262
2015	7										35	28	228,605	6,511,530
Total	255	382	408	407	…	139	102	82	62	36				**10,712,979**

Note that the negative values for IBNR in 2010 and 2011 indicate that claims will be removed from the large triangle. It is possible to see examples of this in 2001-2005. Specifically, looking at the 2nd lag in 2001, one sees that there are 30 large claims. In the 3rd lag, the count drops to 27. When the count drops from one DY to the subsequent DY, large claims have been removed. A decrease in large claims at later development stages is a common phenomenon when incurred costs are set conservatively by the company. Also, the large claim severity is constant for this example. It could have been changed in each AY to more accurately reflect reality.

3.2.1.1 Advantages

- This is an intuitive way to represent the reality. At the end of each evaluation period, the number of large claims is accurately depicted.
- Non-daunting administratively.
- Non-actuaries can easily understand the clean division of large and normal claims. If a claim severity is currently 90,000, it is normal. If it is 130,000, it is large.

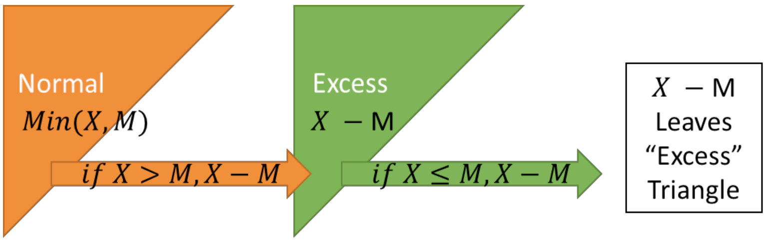
3.2.1.2 Disadvantage

Splitting claims that are close to the threshold does not tell much about the reality of the claims. For example, if the threshold is 100,000, statistically, there isn't much difference between a claim that is 99,000 and one that is 101,000. However, with this method, there is a major impact to the statistics of the large claim count triangle, affecting frequency, volatility, and other crucial parameters for reserving estimation.

3.2.2 In and Out - Excess

Unlike the In and Out technique used in Portugal where claims numbers are projected, the In and Out "excess" technique for splitting large claims places the excess severity above M into a large incurred claims triangle.

When the severity, X, of a claim surpasses the large claim threshold, M, the excess, $X - M$, is put into a triangle of its own. The claims in the normal triangle remain in that triangle but are censored at M. The excess is then adjusted accordingly as more information is gained. If a claim drops below the threshold, the excess becomes 0 and it is essentially removed from the triangle and the entire claim severity is accurately represented in the normal incurred claims triangle.



3.2.2.1 Advantages

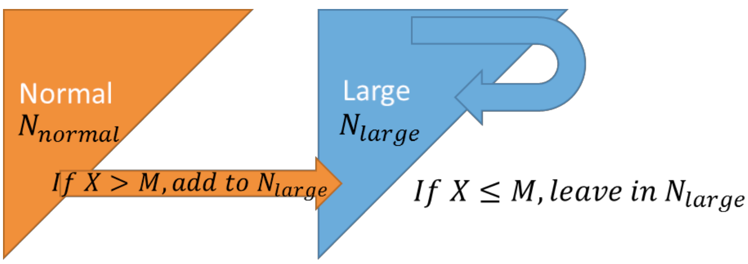
- The main advantage is that history is frozen so that Chain Ladder methods result in link factors that are appropriate for application to the immature data. At the same time, the excess portion, which tends to be more variable from one DY to the next is removed so it does not distort the normal triangle.
- This technique is similar to the concepts used in excess loss reinsurance, which most actuaries are familiar with.

3.2.2.2 Disadvantages

- Sparse data in the excess triangle makes estimations of the excess difficult.
- The claim severities in the normal triangle are right censored which will affect its statistics.

3.3 Leave In Technique

The leave in technique is similar to the current way Ireland is managing large claims. When a claim surpasses the large claim threshold in magnitude, it is recognized in large claims count triangle. However, when a claim is large and then drops below the threshold in severity, it is not removed from the large claim count triangle but is, instead, left in.



This method is described in detail in the article *A Method For Projecting Individual Large Claims* in the Causality Actuarial Society *Forum.*

> In dealing with the known large claims, we allow for the possibility that a currently large claim will ultimately settle below the large threshold. In our large number projection, we need a definition of large claim numbers that can cope with these outcomes. We deal with this by projecting a triangle of claim numbers, where a claim is counted once in the development year it became large. Claims which subsequently fall below the threshold are included in this triangle. (Murphy & McLennan, 2006)

3.3.1 Advantages

- The large claim count ultimate should have less volatility since the triangle never decreases.
- The normal claim count ultimate should have less volatility since the triangle never decreases.

3.3.2 Disadvantages

- It could be harder to account for claims that have been once large but now are not. Because the database cannot be simply quarried for claim size, but also claim size history, the database must have been set up to capture such information to build the large claim triangle.
- The large claim count triangle does not accurately represent the real number of large claims (ie claims that are greater than 100,000) and will eventually need to be adjusted.
- There is volatility added when deciding how to adjust the large claim numbers to represent the aggregate severity of the large claims. Since not all large claims are still large, one must now also estimate the portion of the ultimate that will actually be large on top of the average claim severity.

4. Data

To understand the processes and results in the analysis sections to come, it is important to understand the data available to analyze. Liberty Seguros has an extensive amount of data that has been made available for this report. However, this data had to be transformed in a way to remove all sensitive information that Liberty Seguros does not want published and yet maintain similar statistical characteristics of the original data.[1] The following is a description of the data that was used in the internship.

4.1 Motor Bodily Injury

Liberty Seguros has robust data dating back to 1993 for motor bodily injury claims. However, there are some limitations to this data due to a lack of accounting or changes to the way data was recorded in the data base.

For the Total technique, data is available from 1993 for paid claims and claim counts. However, there is no incurred claims data available in the top left of the triangle.

Accident Year	1	2	3	4	5	6	7	8	9	10
1993								3,383,553	3,392,399	3,229,012
1994							5,758,668	5,829,559	5,966,237	6,016,770
1995						5,797,827	6,166,037	6,677,142	7,113,612	7,351,916
1996					7,655,982	8,431,612	9,765,715	10,427,903	10,721,454	10,866,148
1997				8,606,473	9,442,589	10,633,230	10,736,142	10,757,488	10,547,553	10,590,614
1998			8,934,463	9,393,989	10,377,841	10,976,689	11,720,157	11,619,978	11,529,836	11,273,147
1999		10,180,974	11,008,740	14,980,153	16,516,353	16,847,633	17,007,086	16,458,746	16,240,840	16,356,172
2000	10,817,560	11,356,357	13,578,085	14,666,933	14,115,256	13,943,601	13,655,921	13,307,087	13,404,554	13,451,677
2001	11,918,751	14,670,948	14,775,927	16,024,684	14,904,376	14,942,851	14,765,690	14,654,728	14,742,691	14,618,023

[1] All numbers are altered from the original figures.

For the In and Out technique, there is large claim count information from 2000 but the top left of the triangle is missing for large incurred. This is one of the major reasons why claim count ultimates will be projected rather than incurred claims. Consequently, the Excess technique used in USA/Spain will not be considered.

Accident Year	1	2	3	4	5	6	7	8	9	10
2000							5,253,747	5,070,930	4,804,854	4,924,439
2001						6,905,817	6,887,261	6,442,220	6,683,940	6,388,552
2002					5,398,476	5,551,766	5,283,943	5,250,328	5,119,959	4,884,690
2003				3,361,375	3,767,584	3,801,680	4,105,803	3,791,534	3,985,491	3,962,225
2004			4,420,059	5,074,591	4,928,720	4,694,632	5,236,279	5,275,451	5,275,284	5,273,464
2005		6,753,387	7,877,875	8,800,831	8,436,338	8,695,504	8,657,730	8,835,813	8,828,406	8,648,142
2006	2,844,447	5,538,072	7,157,452	8,292,955	8,637,209	9,062,558	8,614,479	8,620,144	8,354,185	8,400,791
2007	2,794,207	3,267,732	3,156,137	3,463,015	3,700,673	3,816,001	3,586,936	3,562,658	3,750,945	
2008	3,720,496	4,651,696	5,497,542	6,473,460	6,539,097	6,892,237	7,522,958	7,546,091		

For In Only, the top left corner is missing for claim count and incurred claims.

Accident Year	1	2	3	4	5	6	7	8	9	10
2000					29	31	31	32	33	34
2001				47	48	50	51	52	53	53
2002			30	32	34	35	35	35	35	35
2003		24	24	25	26	27	27	28	28	28
2004	15	22	25	26	26	26	30	30	30	30
2005	22	37	40	48	48	52	53	54	54	54

Accident Year	1	2	3	4	5	6	7	8	9	10
2000					5,166,525	5,425,904	5,430,426	5,245,659	5,337,299	5,521,922
2001				7,816,683	7,379,414	7,679,568	7,660,444	7,627,859	7,857,123	7,765,248
2002			5,552,639	6,091,129	5,880,929	6,024,408	6,055,408	6,029,715	5,935,722	5,758,893
2003		3,777,504	3,727,557	3,795,790	4,094,250	4,486,222	4,793,064	4,575,996	4,685,621	4,660,535
2004	2,680,141	3,664,634	4,636,072	5,300,184	5,406,435	5,162,599	5,792,111	5,831,913	5,837,747	5,827,917
2005	4,309,788	7,665,274	8,124,456	9,609,427	9,355,954	9,720,203	9,858,324	10,066,046	10,058,639	9,878,375
2006	3,509,013	6,552,854	7,593,977	8,902,463	9,261,286	9,918,128	9,649,856	9,644,010	9,375,800	9,420,607
2007	2,173,541	3,655,632	3,594,367	3,852,978	4,205,046	4,362,723	4,130,941	4,107,462	4,295,393	

4.2 Baremo

It is important to explain what the Baremo is when discussing the availability of Liberty Seguros' data and usability and appropriateness of that data. Baremo is a Spanish word that translates to "scale" in English.

> The Baremo legal system for the assessment of personal damage caused by road accidents was introduced by Law 30 in 1995 [in Spain]. The assessment system is a legal and rating system that seeks to value all types of damages, both pecuniary and non-pecuniary.

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> The actual level of damages is controlled by the 'Baremo' personal injury system (which uses actuarially derived tables to calculate the payout a claimant would receive). The severity of an injury is measured on a range of 1 to 100; an injury level of 100 would typically be associated with cases such as total quadriplegia, whereas whiplash related injuries are typically limited to 3. This results in a typical payout of around 3,000, with lawyers fees taken as a percentage of this, usually around 10 per cent (or approximately 300) as a conditional fee (no win - no fee) arrangement. (Axa, 2013)

This system has been adopted as a new custom in the legal environment in Portugal in cases involving motor injury claims. Since 2008, Portuguese judges have been referring to the Baremo tables to establish a ruling on the payout of motor claims. Although they are not obligated by law to enforce the Baremo tables, many have been.

The result is shorter settlement times and a decrease in severity and legal costs of large claims, both good for insurers. This makes the most recent claim severity data more relevant but the claim number data and some development patterns from past years should still remain valuable.

4.3 Exposure

It is important to use the appropriate exposure measure depending on what units you are estimating. In this report, we are projecting incurred claims ultimates and claim number ultimates. Therefore, it is reasonable to assume that there is a need for two different exposures; they are earned premiums and total number of reported bodily injury claims.

4.3.1 Earned Ultimate Premium

The importance of knowing and having a good handle on a line of business's loss ratio has become very apparent in the work environment. For this reason, earned ultimate premiums will be used as the exposure measure for the Total technique. Earned premium is the amount of the premium that corresponds to portion of the accident year that the policy is in force. For example, if a policy holder pays 150 for a policy signed on September 1st in AY 1993, the earned premium for AY 1993 is 50.

Unfortunately, Liberty Seguros only has ultimate premiums data from 2008 to the present. This would severely limit the amount of data that could be used for exposure based methods such as Cape Cod and Benktander. Rather than excluding the data from 1993 to 2008, the premiums were estimated for those years. Regression analysis was considered but in the end, a good enough fit could not be found. Therefore, paid claims were used as a proxy to exposure. Another alternative could have been number of claims reported.

Accident Year	8	9	10	11	12	13	14	15	16
1993	2,582,220	2,807,233	3,024,433	3,076,899	3,086,930	3,117,544	3,119,530	3,282,582	3,282,582
1994	4,679,435	4,914,626	5,255,094	5,384,470	5,443,953	5,462,043	5,490,517	5,486,005	5,486,005
1995	5,218,212	5,884,420	6,125,247	6,211,675	6,693,280	6,789,123	6,970,941	7,125,876	7,139,935
1996	8,002,267	8,601,045	9,351,391	9,590,818	9,979,401	10,052,142	10,304,062	10,306,087	10,357,902
1997	9,607,474	10,135,809	10,290,628	10,367,229	10,383,697	10,633,753	10,652,088	10,664,709	10,772,926
1998	9,910,775	10,401,671	10,587,951	10,835,988	10,970,040	10,992,847	11,197,710	11,196,574	11,197,606
1999	14,262,597	14,449,006	14,731,711	14,847,571	15,520,457	15,976,242	15,980,164	16,149,841	16,150,502
2000	11,007,417	12,013,832	12,376,216	12,657,431	12,902,132	12,946,945	13,076,920	13,078,704	13,091,281
2001	12,566,603	12,760,876	12,952,149	13,507,509	13,595,456	13,604,106	14,098,252	14,277,794	
2002	10,594,973	10,772,837	10,988,984	11,081,999	11,422,364	11,452,594	11,453,595		

One can see from the above table that paid claims mostly stabilizes at the 8th DY. This corresponds with the missing premium data and enables the use of the paid claims from that time as an estimate.

For the years 1992 to 2007, the total paid for each year was divided by the total paid in 2008, Δ_i. The ultimate premium in 2008 was multiplied by Δ_i to produce an estimate for the missing premium data.

DY	Ultimate Premium	Total Paid	% of x_{2008}
i	$\bar{P} = P_{2008} \cdot \Delta_i, i \leq 2007$	x_i	$\Delta_i = x_i / x_{2008}$
1993	17,015,212	3,282,528	25.51%
1994	28,475,846	5,493,482	42.69%
1995	38,385,684	7,405,261	57.54%
1996	55,572,903	10,720,972	83.30%
1997	56,265,364	10,854,560	84.34%
1998	58,073,477	11,203,376	87.05%
1999	83,694,803	16,146,172	125.46%
2000	67,859,563	13,091,281	101.72%
2001	74,009,937	14,277,794	110.94%
2002	59,370,502	11,453,595	89.00%
2003	44,759,881	8,634,954	67.10%
2004	56,769,582	10,951,832	85.10%
2005	73,211,547	14,123,771	109.74%
2006	72,238,855	13,936,122	108.29%
2007	52,615,396	10,150,418	78.87%
2008	66,711,042	12,869,712	
2009	61,059,073	13,179,194	
2010	60,848,729	10,935,029	
2011	68,566,443	14,333,215	
2012	76,572,203	11,907,055	
2013	77,874,951	9,543,023	
2014	75,696,763	6,310,037	
2015	72,291,737	2,483,083	

Having this data allows us to utilize all available incurred claims and claim count information. In the article *Using Best Practices to Determine a Best Reserve Estimate:*

> Whenever an appropriate exposure base has been identified, the actuary should rely on a loss reserving method that mixes the loss development, or chain ladder, method with exposure-based expected loss methods. The most common of these blended methods in use are the Bornhuetter-Ferguson (BF) and Cape Cod (CC) methods (Struzzieri & Hussian, 1998).

Accurate estimates using the Cape Cod method and Benktander method would not be possible without these earned premium estimations.

4.3.2 Total Bodily Injury Claims

Total claims is used as an exposure measure for estimating claim number ultimates for the In and Out and Leave In techniques. This measure was chosen because:

- The exposure measure, IBNR claims, and ultimate are in the same units.
- Other measures like ultimate earned premium and paid claims are too big when dealing with large claim numbers, which are relatively quite small.
- The relationship between the number of large claims and the total number of claims is intuitive. If there are more total claims, it is expected that there are more large claims.

5. Methodologies

The methods that are considered include the Chain Ladder, Cape Cod, and Benktander methods. There was extensive research into other possible methods including the Munich Chain Ladder, stochastic Chain Ladder and other stochastic models, and some distribution based reserving methods. In the end, there wasn't enough improvement to the results to justify complicating the system. Liberty Seguros' software, ResQ, includes a version of the weighted Chain Ladder method, Cape Cod, and Bornhuetter Ferguson Methods. If a different method is chosen, it will have to been programmed into the software in order to consider long term use of the results of this report. Because one of the goals of this report is to suggest the best practice for Liberty Seguros to use as the new standard for handling large claims, it will be more practical to suggest a method from the ResQ library, however, a different method could be suggested if it yields a convincingly low one-year claim development measure.

This report will not focus on the derivation of the methods because the three methods used are quite common and a there is no lack of literature on the subject. However, there is a need to establish some notation.

The notation and formulas in this chapter come from an article in the International Journal of Advanced Research by Werner Hürlimann. (Hürlimann, 2015)[2]

5.1 Notation

Where there is n years of data, a $n \times n$ triangle can be constructed.

	Development Year (DY)					
Accident Year (AY)	1	2	…	…	n-1	n
1	$S_{1,1}$	$S_{1,2}$	…	…	$S_{1,n-1}$	$S_{1,n}$
2	$S_{2,1}$	$S_{2,2}$	…	…	$S_{2,n-1}$	
…	…	…	…	…		
…	…	…	…			
n-1	$S_{n-1,1}$	$S_{n-1,2}$				
n	$S_{n,1}$					

	Development Year (DY)					
Accident Year (AY)	1	2	…	…	n-1	n
1	$S_{1,1}$	$C_{1,2}$	…	…	$C_{1,n-1}$	$RC_{1,n}$
2	$S_{2,1}$	$C_{2,2}$	…	…	$RC_{2,n-1}$	
…	…	…	…	…		
…	…	…	…			
n-1	$S_{n-1,1}$	$RC_{n-1,2}$				
n	$RC_{n,1}$					

Incremental Incurred Claims
$S_{i,k}, 1 \leq i, k \leq n$

[2] This article is used solely for its concise descriptions and notion of the methods used in this report.

Cumulative Incurred Claims

$C_{i,k} = \sum_{j=1}^{k} S_{i,j},\ i \in \{1, \dots, n\},\ \ k \in \{1, 2, \dots, n-i+1\}$

Most Recent Cumulative Incurred Claims ("the diagonal")

$RC_i = C_{i,n-1+1}$

Chain Ladder Factors

$f_k^{CL} = \sum_{i=1}^{n-k} C_{i,k+1} / \sum_{i=1}^{n-k} C_{i,k},\ k = 1, 2, \dots, n-1$

Loss Development Factors to Ultimate

$F_k = \prod_{j=k}^{n-1} f_j^{CL},\ \ k = 1, \dots, n-1,\ \ F_n = 1$

Chain Ladder Lag Factors

$p_i^{CL} = \frac{1}{F_{n-i+1}},\ \ i = 1, \dots, n$

Chain Ladder IBNR Factors

$q_i^{CL} = 1 - p_i^{CL},\ \ i = 1, \dots, n$

5.2 Standard IBNR Methods

5.2.1 Chain Ladder (CL)

The chain ladder is the most commonly used standard IBNR method. It is attractive to actuaries because of its ease of computation and that it uses all past data. This can be especially appealing for companies like Liberty Seguros that have a robust database of past data.

For the project, the standard weighted average Chain Ladder was used.

Ultimate and IBNR

$U_i^{CL} = \frac{RC_i}{p_i^{CL}},\ \ IBNR_i^{CL} = q_i^{CL} \cdot U_i^{CL},\ \ i = 1, \dots, n$

5.2.2 The Cape Cod Method (CC)

The Cape Cod method was chosen over the Bornhuetter Ferguson method because the constant loss ratio (LR) is derived solely from the data rather than a value selected by an actuary (select value). For the purposed of this report, the comparison of methods was to rely on as little actuarial judgement as possible. This is to show results that one would obtain without the expertise of an experienced actuary which often relies on a "gut" feeling about the legal and economical environment, marketing and sales concerns, and other factors rather than pure mathematical theory.

Loss Ratio

$$LR = \sum_{i=1}^{n} RC_i \,/ \sum_{i=1}^{n} p_i^{CL} \cdot P_i$$

Ultimate and IBNR

$U_i^{CC} = RC_i + IBNR_i^{CC} = p_i^{CL} \cdot U_i^{CL} + \left(1 - p_i^{CL}\right) \cdot LR \cdot P_i,\ i = 1, \dots, n,\ IBNR_i^{CC} = q_i^{CL} \cdot LR \cdot P_i$

5.2.3 Bornhuetter-Ferguson (BF)

The BF method will not be used to analyze the data when making a suggestion on the claim splitting technique. The reason for this is that rather than using an average loss ratio like with CC, the loss ratio is selected by the actuary. It can vary by DY or can be set to a constant. It is shown here in order to recognize how it differs from the CC method.

Ultimate and IBNR

$$U_i^{BF} = RC_i + IBNR_i^{CC} = p_i^{CL} \cdot U_i^{CL} + \left(1 - p_i^{CL}\right) \cdot LR_i \cdot P_i, \ \ i = 1, \dots, n, \ IBNR_i^{BF} = \ q_i^{CL} \cdot LR_i \cdot P_i$$

5.3 IBNR Loss Ratio (LR) Methods

The loss ratio is the amount of claims over premiums. This method gives a nice contrast to the link ratio methods above (CL, CC, and BF) and instead is based on "the incremental amount of reported claims per unit of premium in each development period." (Hürlimann, 2015)

Incremental Loss Ratios

$$m_k = \sum_{i=1}^{n-k+1} S_{i,k} \, / \sum_{i=1}^{n-k+1} P_i, k \in \{1, \dots, n\}$$

Lag Ratio Factors

$$p_i^{LR} = \sum_{k=1}^{n-i+1} m_k \, / \sum_{k=1}^{n} m_k, \ \ i = 1, \dots, n$$

$$q_i^{LR} = 1 - p_i^{LR}, \ \ i = 1, \dots, n$$

5.3.1 Individual LR Method

"As in the chain-ladder method, the [ultimate] of each origin period depends on the current *individual* claims experience at analysis date" (Hürlimann, 2015):

$$U_i^{ind} = \frac{RC_i}{p_i^{LR}}, \ \ IBNR_i^{ind} = q_i^{LR} \cdot \ U_i^{ind}, \ \ i = 1, \dots, n$$

5.3.2 Collective LR Method

"The [ultimate aggregate paid claims] of each origin period depends on the overall *collective* claims experience and the premium assigned to the origin period" (Hürlimann, 2015):

$$U_i^{coll} = \ P_i \cdot \sum_{k=1}^{n} m_k \ \ IBNR_i^{coll} = q_i^{LR} \cdot \ U_i^{coll}, i = 1, \dots, n$$

5.3.3 Credibility LR Method

$$U_i^{cred} = Z_i \cdot U_i^{ind} + (1 - Z_i) \cdot U_i^{coll},$$
$$IBNR_i^{cred} = Z_i \cdot IBNR_i^{ind} + (1 - Z_i) \cdot IBNR_i^{coll}, \ i = 1, \dots, n$$

Benktander Credibility LR Method (BM)

$$Z_i^{BC} = p_i^{LR}, \ i = 1, \dots, n$$

6. Determining the Technique

The primary purpose of internship project is to decide which of the three techniques is best for handling large claims under the current circumstances at Liberty Seguros as well as in "worst case" scenarios. This can be done by simulating various realities in which large claims can be introduced into the triangles. The techniques will be stressed, or shocked, with randomly and intentionally placed large claims to see how well they absorb the shocks. If a technique has consistent results under the various stressed circumstances, it can be considered to be a good technique for dealing with large claims. The technique that best absorbs the large claims will be selected as the ideal one for Liberty Seguros. Then a method will be selected and optimized to produce the best possible reserve estimate.

Although it would have added another dimension to the report, the large claim threshold will not be tested and kept constant at 100,000.

6.1 Stressing the Large Claim Triangles

In all of these scenarios, large claims were added in 2014 to the original data and (except in scenario 3) retained in 2015 to simulate extreme real world possibilities.

6.1.1 Scenario 1: 10 Random Large Claims

The first scenario is adding 10 large claims randomly in to the incurred claims triangle.

Rather than simulating the claim size of the 10 large claims, the average large claim severity, 228,605, was used. This figure was calculated by the actuaries at Liberty Seguros and assumed to be accurate for the current data. The average is used to allow the process to be more easily replicated for each of the large claim splitting techniques. In the case of the Total technique, 228,605 was added directly on top of the total amount of incurred claims in that cell for each random large claim. For the In and Out and Leave In techniques, the number of large claims was increased by one and then the total ultimate count was multiplied by 228,605 to obtain the ultimate reserve.

6.1.2 Scenario 2: 5 Large Claims in One Year

Liberty Seguros has a reinsurance contract with an excess of loss for claims above 1 million. For any claim above that value, Liberty Seguros' liability is capped at 1 million. However, if several large claims occur to different policies the cost to the insurance company could be up to 1 million times the number of policies involved. To simulate the effect if 5 large claims occur in one year, potentially a massive road accident involving 5 different policy holders, a random cell is selected like in scenario 1 and 5 claims are added.

6.1.3 Scenario 3: 1 Exceptionally Large Claim Settled for 0

In the third scenario, an exceptionally large claim is added to a recent AY and early DY and then settled for 0 in the subsequent year. This is to test how the methods react within each technique to a large, sudden increase and decrease in a short time frame. It is reasonable to have a claim estimated as large in the first or second lag and then settle for a lower value within a year in the Portuguese legal system, but unlikely to occur in later lags. Therefore, the second lag in AY 2014 was intentionally chosen rather than chosen at random.

6.2 One-year Claim Development (OCD)

The one-year claim development (OCD) is measured by the change in the aggregate reserve ultimate between 2014 and 2015 (excluding the 2015 cohort for which no aggregate reserve ultimate was available in 2014).

The table below shows the In and Out technique for Scenario 1 with the Chain Ladder method applied to find the large claim number ultimates. The OCD is found by subtracting the 2015 Cohort Ultimate and the 2014 Aggregate Ultimate from the 2015 Aggregate Ultimate.

	CL Ultimate	
AY	**2015**	**2014**
2000	25	25
2001	38	38
2002	26	26
2003	20	21
2004	23	24
2011	46	45
2012	31	32
2013	33	31
2014	19	19
2015	13	**477**
	495	

	495	2015 Aggregate Ultimate
–	13	2015 Cohort Ultimate
–	**477**	2014 Aggregate Ultimate
=	**5**	OCD Measure

In this case, the OCD is 5. This means that the ultimate estimate with another diagonal of data is predicting 5 more claims than it previously did the year before, indicating an under-estimation in 2014. Had this number been negative, the sensitive measure would be indicating that the 2014 ultimate was over-estimated. The ideal result would be an OCD of 0.

6.3 One-year Sensitivity Measure (OSM)

The OCD is a way to understand how reliable a method's ultimate projection is; it is a measure of the precision of the method. However, for the purpose of this internship report, a measure of the robustness of a technique must be defined. The one-year sensitivity measure (OSM) is a technique's ability to absorb large claims.

Each technique (Total, In and Out, and Leave In) will yield a different OSM based on the consistency of each method's OCDs in each shocked scenario. Basically, it is answering the question, is the CL method producing a similar OCD in the original scenario and shocked scenarios 1, 2 and 3? One can take the standard deviation of the OCDs for a method to understand how much on average it is varying over the different scenarios. If a method is estimating with a similar OCD in each scenario, the standard deviation will be low, indicating that the method (under the specific splitting technique) is not overly affected by the introduction of new large claims. This can be better understood with an example.

The following table includes the OCDs for the CL method under the Leave In technique.

	Original	Scenario 1	Scenario 2	Scenario 3
CL Ultimate	-2	1	16	-3

With the original data, the 2014 ultimate is overestimated by 2. It is underestimated it by 1 in Scenario 1, 16 in Scenario 2, and overestimated by 3 in Scenario 3. To measure a method's ability to handle shocked scenarios, the standard deviation of the OCDs can be used. In this case, it is 7.64. This number can be averaged with the standard deviations of the other methods in the Leave In technique to for a statistic that can be compared across all techniques. The average of the standard deviations is the OSM.

The following tables contains the OCDs, standard deviations, and OSM for each technique. Remember that the OSM is the average of each method's standard deviation of OCDs. In the first table, 647,324 is the standard deviation of the CL's OSMs from the original data and stressed scenarios. 727,578 is the OSM or average of the standard deviations 647,324, 707,965 and 827,445.[3]

6.3.1 Total[4]

	Original	Scenario 1	Scenario 2	Scenario 3	Standard Deviation
CL Ultimate	1,621,171	285,936	1,695,665	454,792	647,324
CC Ultimate	1,735,676	-4,228	1,386,798	396,285	707,965
Benktander	1,104,283	-1,014,828	344,707	-615,415	827,445
				OSM	**727,578**

6.3.2 In and Out

	Original	Scenario 1	Scenario 2	Scenario 3	Standard Deviation
CL Ultimate	9	5	6	7	1.51
CC Ultimate	1	-3	-2	0	1.44
Benktander	-5	-10	-9	-6	2.00
				OSM	**1.65**

6.3.3 Leave In

	Original	Scenario 1	Scenario 2	Scenario 3	Standard Deviation
CL Ultimate	-2	1	16	-3	7.64
CC Ultimate	-10	-8	5	-10	6.30
Benktander	-10	-15	12	-8	10.05
				OSM	**8.00**

The In and Out and Leave In techniques are both represented in claim count units so their standard deviations can be compared directly. The results show that of these two methods, the In and Out method is the better choice. However, the standard deviation of the Total is estimated by using incurred claims. To compare the In and Out and Total techniques, additional analysis had to be performed to make a final decision.

6.4 Confirming the Results

To compare the Total technique results to the In and Out technique results, one can multiply the OCDs of the In and Out technique by the average large claim severity.[5]

The following table is the Total technique's OCDs, the standard deviations of each method's OCDs, and the average of the standard deviations.

[3] The standard deviations for all methods and OSMs for all techniques are consistent with the real data. However, the OCDs in the following tables are intentionally not consistent with the real data.
[4] In this case, the shocked results are actually improving the OCDs of the Total technique and in some of the methods in the other techniques. This was not anticipated and a coincidental result; likely a consequence of the adjustment of the original data.
[5] One only has to multiply the claim number OCD by the average large claims severity to compare with the sensitivity of Total because the OCD of normal claims triangle for the In and Out technique, in theory, should be close to zero.

6.4.1 Total

	Original	Scenario 1	Scenario 2	Scenario 3	Std	Average Std
CL Ultimate	1,621,171	285,936	1,695,665	454,792	647,324	727,578
CC Ultimate	1,735,676	-4,228	1,386,798	396,285	707,965	
Benktander	1,104,283	-1,014,828	344,707	-615,415	827,445	

The following table is the In and Out technique's OCDs, the OCDs times the average large claim severity, the standard deviations of each method's OCDs large claim severity, and the average of the standard deviations (OSM).

6.4.2 In and Out

	Original		Scenario 1		Scenario 2		Scenario 3			
	Ultim.	• Avg Sever.	Ultim.	• Avg Sever.	Ultim.	• Avg Sever.	Ultim.	• Avg Sever.	Std	Average Std
CL Ultimate	9	2,057,315	5	1,140,316	6	1,340,454	7	1,651,404	346,184	377,138
CC Ultimate	1	182,837	-3	-646,862	-2	-468,276	0	-51,042	328,636	Avg Severity
Benktander	-5	-1,229,791	-10	-2,331,601	-9	-2,131,767	-6	-1,458,396	456,594	228,605

The results show that the In and Out technique is superior to the Total technique because the OSM is smaller.

One may ask, how sensitive is the OSM to the estimated average large claim severity? To answer that, a goal seek operation in Microsoft Excel can be used to find the value of the average large claim severity that would make the OSMs of the Total and In and Out techniques equal. The difference between the original average large claim severity and the result of the goal seek will be the sensitivity measure.

6.4.3 Total

	Original	Scenario 1	Scenario 2	Scenario 3	Std	Average Std
CL Ultimate	1,621,171	285,936	1,695,665	454,792	647,324	727,578
CC Ultimate	1,735,676	-4,228	1,386,798	396,285	707,965	
Benktander	1,104,283	-1,014,828	344,707	-615,415	827,445	

6.4.4 In and Out

	Original		Scenario 1		Scenario 2		Scenario 3			
	Ultim.	• Avg Sever.	Ultim.	• Avg Sever.	Ultim.	• Avg Sever.	Ultim.	• Avg Sever.	Std	Average Std
CL Ultimate	9	3,968,987	5	2,199,906	6	2,586,014	7	3,185,902	667,861	727,578
CC Ultimate	1	352,730	-3	-1,247,931	-2	-903,402	0	-98,470	634,007	Avg Severity
Benktander	-5	-2,372,522	-10	-4,498,142	-9	-4,112,621	-6	-2,813,548	880,866	441,026

In the tables above, one can see that the average severity must be 441,026 for the OSMs to be equal. This is a difference of 212,421 from the actual estimate, 228,605. There would have had to have been a severe miscalculation of the average severity for the Total technique to be less volatile than the In and Out technique when comparing their ability to absorb large claims. Therefore, the In and Out technique is confirmed to be the best splitting method.

6.5 Selecting the Method

Because the In and Out technique is selected, the best method for projecting ultimate reserves at Liberty Seguros can now be chosen. According to the article *Using Best Practices to Determine a Best Reserve Estimate*,

> When beginning a loss reserve study, the actuary has a wide array of tools and methods from which to choose. One school of thought says to use several methods, and average all of the methods to get to the

selected result. However, some of these methods may be more biased or more variable than others. A better practice would be to exclude these methods from the average. The selected result would then be less biased and/or have less variance. (Struzzieri & Hussian, 1998)

Adding on to this statement, the method chosen should be as simple as possible (ie, not blending or choosing a more complicated method when the results are marginal) while still achieving accurate results. To find the method with the least inconsistency overall, the absolute values of the OCDs are analyzed.

6.5.1 Absolute OCDs of In and Out Technique

	Original	Scenario 1	Scenario 2	Scenario 3	Sum
CL Ultimate	9	5	6	7	27
CC Ultimate	1	3	2	0	6
Benktander	5	10	9	6	30

It is important to know that the real values of the Benktander OCDs were closer to the CL as would theoretically be expected but the sum of the absolute OCDs are consistent with the real data. Therefore, since it has the highest summed value, the Loss Ratio Benktander is not considered.

The OCD of the CL with the original data is the highest and it will be rejected as a method candidate.

Further defense against the use of the Chain latter can be found by Struzzieri and Hussian and Murphy and McLennan. Struzzieri and Hussian say they following about the Chain Ladder method

> As many actuaries using the loss development method have discovered, early development is unstable, not a useful predictor of ultimate losses, and will understate ultimate losses when the current evaluation is less than average and overstate when the current evaluation is greater than average. Murphy and Patrik [11] make similar observations. (Struzzieri & Hussian, 1998)

Murphy and McLennan agree, stating:

> Due to the generally small number of claims which are reported as large in development years one and two, the projected number of large claims for the most recent origin periods may be artificially unstable.
>
> Further, we must ask ourselves if it is intuitive to suggest that if the most recent origin period has twice as many large claims per unit of exposure reported in development year one as the historical average, then it will have twice the number of large claims per unit of exposure ultimately. This does not seem to make sense in practice. (Murphy & McLennan, 2006)

The Chain Ladder and Benktander is out performed by the Cape Cod method which projects very well in the original scenario and also seems to be the suitable choice in shocked scenerios. It is for these reasons that the Cape Cod method will be chosen to as the best method for projecting large claims at Liberty Seguros.

Additional analysis will now be performed in ResQ. Extreme data will be eliminated and tail smoothing curves will be considered.

7. Optimizing the Cape Cod Method

Often, it is not enough to base a reserve estimate on the original data. An actuary will apply their intuition and may also want to remove extreme incremental differences and apply a tail smoothing curve. The following sections will explore these options.

7.1 Removing Extreme Link Ratios

There are several ways to remove extreme link rations. An actuary can hand pick the values that are too high, the average of the last x years can be used if more recent data is the most relevant, and the highest and lowest valued ratios can be removed. To avoid a method which requires actuarial expertise, hand picking will be excluded and the last 5 years and the highest and lowest approaches will be tested.

7.1.1 Last 5

Because of the Baremo, one can rationalize that the most recent data is the most relevant. Therefore, one can average the last link ratios to produce a more accurate series of Chain Ladder factors. This was tested with the In and Out technique with the Cape Cod method, however, the results did not improve. This could be because the Baremo shouldn't overly effect the claim count. Since the optimal technique is In and Out which uses the claim count, data from all of the accident years will be used evenly. The Last 5 approach will not be used for the ultimate reserve estimate.

7.1.2 Highest and Lowest

The highest and lowest development factors were removed in an effort to stabilize data and produce an average that represents well the reality of the claim development. This procedure can be seen in practice with several examples in an article *Unstable Loss Development Factors* from the Casualty Actuarial Society *E-Forum*. (Blumsohn & Laufer, 2009)

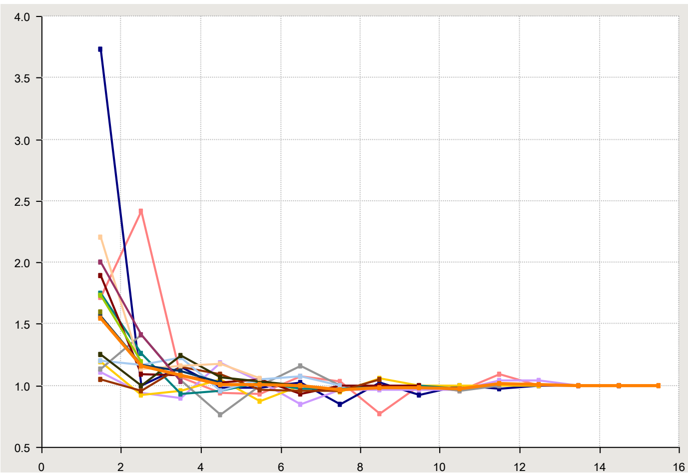
The follow tables will illustrate the effect on the averages of removing the extreme ratios:

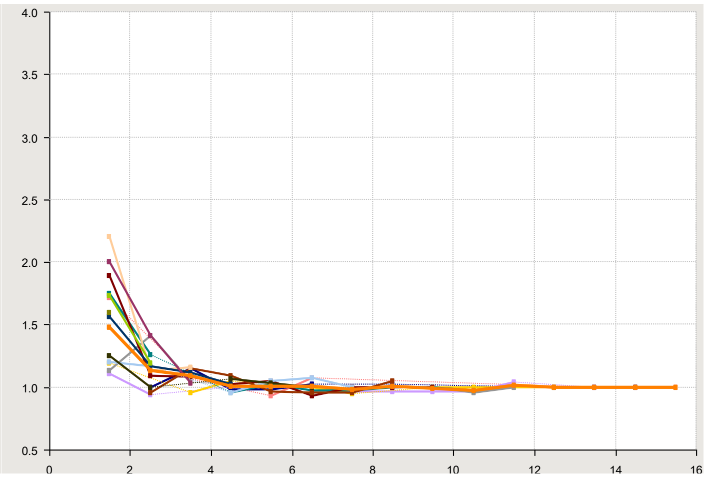
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2000	1.71429	2.41667	1.06897	0.93548	0.93103	1.07407	1.03448	0.76667	1.00000	0.95652	1.09091	1.00000	1.00000	1.00000	1.00000
2001	3.72727	1.00000	1.14634	0.97872	0.97826	1.02222	0.84783	1.02564	0.92500	1.00000	0.97297	1.00000	1.00000	1.00000	
2002	1.10345	0.93750	0.90000	1.18519	1.03125	0.84848	0.96429	0.96296	0.96154	0.96000	1.04167	1.04000	1.00000		
2003	1.19048	0.92000	0.95652	1.04545	0.86957	1.00000	0.95000	1.05263	1.00000	1.00000	1.00000	1.00000			
2004	1.13333	1.41176	1.04167	0.76000	1.00000	1.15789	1.00000	1.00000	1.00000	0.95455	1.00000				
2005	1.75000	1.25714	0.93182	0.95122	1.02564	0.97500	0.97436	1.00000	1.00000	0.97368					
2006	1.88889	1.08824	1.08108	1.02500	1.04878	0.93023	1.00000	1.00000	1.00000						
2007	1.05000	0.95238	1.15000	1.08696	0.96000	0.95833	0.95652	1.04545							
2008	1.20000	1.16667	1.22857	0.95349	1.04878	1.06977	1.00000								
2009	1.25000	1.00000	1.24000	1.06452	1.03030	1.00000									
2010	2.20000	1.13636	1.16000	1.17241	1.05882										
2011	1.56522	1.16667	1.11905	1.02128											
2012	2.00000	1.40909	1.03226												
2013	1.73333	1.19231													
2014	1.60000														
Averages	1.54510	1.15608	1.07882	1.00985	1.00552	0.99695	0.96587	0.97890	0.98086	0.97576	1.01613	1.00952	1.00000	1.00000	1.00000

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2000	1.71429	~~2.41667~~	1.06897	0.93548	0.93103	1.07407	~~1.03448~~	~~0.76667~~	~~1.00000~~	~~0.95652~~	~~1.09091~~	~~1.00000~~	1.00000	1.00000	1.00000
2001	~~3.72727~~	1.00000	1.14634	0.97872	0.97826	1.02222	~~0.84783~~	1.02564	~~0.92500~~	~~1.00000~~	~~0.97297~~	1.00000	1.00000	1.00000	
2002	1.10345	0.93750	~~0.90000~~	~~1.18519~~	1.03125	~~0.84848~~	0.96429	0.96296	0.96154	0.96000	1.04167	~~1.04000~~	1.00000		
2003	1.19048	~~0.92000~~	0.95652	1.04545	~~0.86957~~	1.00000	0.95000	~~1.05263~~	1.00000	1.00000	1.00000	1.00000			
2004	1.13333	1.41176	1.04167	~~0.76000~~	1.00000	~~1.15789~~	1.00000	1.00000	1.00000	0.95455	1.00000				
2005	1.75000	1.25714	0.93182	0.95122	1.02564	0.97500	0.97436	1.00000	1.00000	0.97368					
2006	1.88889	1.08824	1.08108	1.02500	1.04878	0.93023	1.00000	1.00000	1.00000						
2007	~~1.05000~~	0.95238	1.15000	1.08696	0.96000	0.95833	0.95652	1.04545							
2008	1.20000	1.16667	1.22857	0.95349	1.04878	1.06977	1.00000								
2009	1.25000	1.00000	~~1.24000~~	1.06452	1.03030	1.00000									
2010	2.20000	1.13636	1.16000	1.17241	~~1.05882~~										
2011	1.56522	1.16667	1.11905	1.02128											
2012	2.00000	1.40909	1.03226												
2013	1.73333	1.19231													
2014	1.60000														
Averages	1.48214	1.12903	1.08262	1.01412	1.00984	1.00362	0.98165	1.00532	0.99315	0.97143	1.01538	1.00000	1.00000	1.00000	1.00000

In the first two DYs, the average is decreased and there are mixed results in the following DYs.

The stabilization effect can be better visualized in the graphs below.





The graph on the left represents the original development factor data. The y-axis is the development factor magnitude and the x-axis is the development year. The graph on the right has had the extreme values removed.

After running the projections again, estimating with an average of all of the link ratio outperforms the removal of data in terms of OCD. Therefore the original Chain Ladder factors will be used.

7.2 Smoothing the Tail

In ResQ, several tail smoothing curves are available. They include exponential decay, inverse power, power, and Weibull. There is also an R-squared value for each of the curves to help the user decide which one fits best to the data.

DY	Initial Selection	Exponential Decay	Inverse Power	Power	Weibull	Selected Value
1	1.48214	1.10174	1.45573	1.09693	1.33297	**1.48214**
2	1.12903	1.07088	1.08116	1.06751	1.12863	**1.12903**
3	1.08824	1.04938	1.03638	1.04722	1.06317	**1.08824**
4	1.00680	1.03440	1.02145	1.03312	1.03451	**1.00680**
5	1.00984	1.02397	1.01445	1.02327	1.02007	**1.00984**
6	1.00362	1.01670	1.01055	1.01638	1.01218	**1.00362**
7	*0.98165*	1.01163	1.00811	1.01154	1.00763	**0.98165**
8	1.00532	1.00810	1.00648	1.00813	1.00490	**1.00532**
9	*0.99315*	1.00565	1.00532	1.00574	1.00321	**0.99315**
10	*0.97143*	1.00393	1.00447	1.00405	1.00214	**0.97143**
11	1.01538	1.00274	1.00382	1.00286	1.00144	**1.01538**
12	*1.00000*	1.00191	1.00331	1.00202	1.00099	**1.00000**
13	*1.00000*	1.00133	1.00290	1.00142	1.00068	**1.00000**
14	*1.00000*	1.00093	1.00257	1.00101	1.00048	**1.00000**
15	*1.00000*	1.00065	1.00230	1.00071	1.00034	**1.00000**
16	*1.00000*	1.00045	1.00207	1.00050	1.00024	**1.00000**
17	*1.00000*	1.00031	1.00188	1.00035	1.00017	**1.00000**
18	*1.00000*	1.00022	1.00171	1.00025	1.00012	**1.00000**
19	*1.00000*	1.00015	1.00157	1.00018	1.00009	**1.00000**
20	*1.00000*	1.00011	1.00144	1.00012	1.00007	**1.00000**
21	*1.00000*	1.00007	1.00134	1.00009	1.00005	**1.00000**
22	*1.00000*	1.00005	1.00124	1.00006	1.00004	**1.00000**
23 - Ult	1.00000	1.00012	1.01632	1.00015	1.00011	**1.00000**

	Expo Decay	Inverse Power	Power	Weibull
A	0.14604	0.15343	1.13997	1.38711
B	-0.36143	-1.57060	0.70618	0.64683
C		-0.50000		
R-squared %	45.22%	74.53%	44.89%	78.19%

There wasn't a curve that fit well enough in the early development years, therefore, no smoothing curve was applied. One could argue that the inverse power curve fits reasonably well in terms of deviance from the initial selected averages and R-squared. However, even a few percentage point difference in the early years can have a drastic impact on the final ultimate reserve estimate.

Also, none of the curves properly matched the reality of the tail of the large claims development. The large claim count is almost entirely known within the first few development years. It is very uncommon to see the addition of a large claim in a later DY. It is for this reason, that no tail smoothing was selected.

8. Conclusion

The result of the analysis is that the ideal technique is the In and Out technique which is already being used at Liberty Seguros. This is confirmed through the analysis of the OCDs of the Chain Ladder, Cape Cod, and Benktander methods and comparing the OSM with those of the other techniques.

Through further analysis of the methods with the In and Out data, it was found that the Cape Cod is the optimal method for estimating large claim reserves.

The following is the result of the 2014 and 2015 ultimate projections with the above assumptions.

	CC Ultimate	
AY	**2015**	**2014**
2000	24	24
2001	36	36
2002	26	26
2003	20	20
2004	21	21
2005	37	38
2006	40	40
2007	23	21
2008	45	44
2009	32	32
2010	34	32
2011	46	44
2012	30	31
2013	32	31
2014	21	26
2015	21	**466**
	488	**0.80**

One can see that the OCD is 0.80 proving that the precision of this method is close to the ideal result of 0. This statistic is found by subtracting the 2015 large claim number ultimate, 488.01, minus the 2015 cohort ultimate, 21.48, minus the 2014 large claim number ultimate, 466.73.

The get to a reserve ultimate, the IBNR large claim numbers can be easily calculated to produce a final estimate for the IBNR reserve and ultimate incurred reserve estimate for large claims.

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