

MASTER
MONETARY AND FINANCIAL ECONOMICS

MASTER'S FINAL WORK
DISSERTATION

RELATIONSHIP BETWEEN GOVERNMENT DEBT AND ECONOMIC
GROWTH IN BRAZIL

ÁGATHA LECHNER DA SILVA

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ÁGATHA LECHNER DA SILVA

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GLOSSARY

AIC – Akaike Information Criteria.

ADF – Augmented Dickey Fuller.

ADF^{GLS} – Modified Dickey Fuller.

ARDL – Autoregressive Distributed Lag.

ECM – Error Correction Model.

EMU – European Monetary Union.

FPE - Final Prediction Error.

GDP – Gross Domestic Product.

GLS – Generalized Least Squares.

GMM – Generalized Method of Moments.

IPCA– Broad Consumer Price Index.

MAIC – Modified Akaike Information Criteria.

MZ_{α}^{GLS} – Modified Phillips Perron.

OLS – Ordinary Least Squares.

OCDE – Organization for Economic Co-operation and Development.

PEC – Constitutional Amendment Bill.

PP – Phillips Perron.

SIC – Schwarz Information Criteria.

SL – Saikkonen & Lütkepohl.

VAR – Vector Autoregressive.

VEC – Vector Error Correction.

Selic – Special System for Settlement and Custody.

SL – Vogelsang & Perron.

ABSTRACT

This dissertation provides new insights on the relationship between public debt and economic growth in Brazil. We applied Granger causality tests, in multivariate and bivariate analyses using respectively VEC and ARDL methodologies. We used monthly data over the period of January 1998 until November 2019. The interaction between other variables such as interest rate, inflation rate, exchange rate, Emerging Market Bond Index Plus (Embi+) and primary surplus was considered as well.

The most relevant findings of this dissertation are summarized as follows: Debt-to-GDP and GDP growth rate have a bi-directional Granger causality relationship. Debt can improve growth in the short run and become harmful in the long run. Also, GDP growth rate always reduces debt, both in the short and long run. The dynamic between debt and growth in the long run is influenced by inflation rate, exchange rate and Embi+, these variables are positively Granger caused by changes in debt and negatively Granger cause GDP growth rate; while GDP growth rate negatively Granger causes Embi+, that in turn positively Granger causes debt. Therefore, the negative impact of debt on growth is also indirect by changes on inflation rate, exchange rate and Embi+, while the reduction on Debt ratio provoked by GDP growth rate is also indirectly, by the reduction on Embi+.

KEYWORDS: Granger causality; VEC; ARDL; government debt; economic growth.

JEL CODES: C32; C22; O40; H63; H69.

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1. INTRODUCTION

Increasing government indebtedness worldwide has become a problem since the Global Financial Crisis (GFC) of 2008-2009, raising concerns related to the vulnerability of countries. In order to address this problem many economists argue that governments should enforce fiscal consolidations, to decrease public debt, explaining that this would result in economic growth. On the other hand, there are economists who advocate that fiscal consolidations could result in increasing debt-to-GDP ratios, moreover, this reduction in government size would affect the growth rate of the GDP.

Brazil, as many other countries, faces an increasing government debt burden. Between 1998 and 2020 Gross General Government Debt increased from 40% of GDP to 89%. In addition, the economic recession worsened the problem, in 2019, real GDP grew only 1.1% and in 2020 its expected to have a significant decrease. During the period under analysis (January 1998 until November 2019) the Brazilian government had five different presidents and alternated between moments of fiscal expansions and consolidations. Furthermore, Brazil faced hyperinflation in the past, which also makes the Brazilian Central Bank strongly conservative in relation to the interest rate, indirectly implying higher costs to government debt, by increasing its debt service.

Since 1998, after the *Plano Real*¹, Brazil has followed an inflation targeting regime that includes floating exchange rate and primary surplus targets in addition to inflation targets. Therefore, macroeconomic variables such as interest rate, inflation rate, exchange rate and primary surplus may be correlated with the pattern followed by the GDP growth rate and the debt-to-GDP ratio.

In December 2016, the Brazilian Congress approved a Constitution Amendment that created a ceiling for public spending² and the government has tried to implement some austerity measures, even during recession. However, the government has not succeeded in reducing government debt, moreover it was not able to overcome the recession. The

¹ The Plano Real was a set of economic reforms implemented in Brazil, with the main objective of combating the hyperinflation.

² Constitutional Amendment n. 95.

lack of consensus about the implications of public debt makes it hard for the government to make the best policy decisions.

The actual situation raises the question: “What is the relationship between public debt and economic growth in Brazil?” This dissertation aims to empirically investigate the dynamic between economic growth and public debt in Brazil in the period after the *Plano Real*. It will also include other variables that are related to both public debt and economic growth. The assessment is conducted through the analysis of growth equations, followed by a Vector Autoregressive (VAR) model and an Autoregressive Distributed Lag (ARDL) model, applying Granger causality tests. The data frequency is monthly, over the period of January 1998 and November 2019.

Since there are just a few empirical studies applied to Brazil, this dissertation contributes to the literature providing empirical results using Brazilian data. Moreover, we are not aware of any other study that has analysed the interaction between debt and growth considering the interrelationships with the other variables used in this composition.

The most relevant findings of this dissertation are summarized as follows: Debt-to-GDP ratio and GDP growth rate have a bi-directional Granger causality relationship. Debt can improve growth in the short run and become harmful in the long run. Also, GDP growth rate always reduces debt ratio, both in the short and long run. The dynamic between debt and growth in the long run is influenced by inflation rate, exchange rate and Embi+, these variables are positively Granger caused by changes in debt-to-GDP ratio and negatively Granger cause GDP growth rate, while GDP growth rate negatively Granger causes Embi+, that in turn positively Granger causes debt. Therefore, the negative impact of debt on growth is also indirect by changes on inflation rate, exchange rate and Embi+, while the reduction on debt ratio provoked by GDP growth rate is also indirectly, by the reduction on Embi+.

This remaining part of this dissertation is structured as follows: section two reviews the literature related to the topic, section three presents the methodology and section four presents the data and the empirical estimation results. Section five is the conclusion.

2. LITERATURE REVIEW

The literature associated with the relation between public debt and economic growth is well developed, despite the lack of empirical studies applied to Brazil, for the best of our knowledge. In addition, one should consider that empirical results have mixed conclusions, divided between those which concluded that debt improves growth, therefore generates a reduction on debt-to-GDP ratio, or those which advocate that debt hurts growth. Another strand of literature argues that government debt is useful until some threshold, after that becoming harmful. These different conclusions strengthen the idea that results are country and time specific.

Next, the empirical literature review will be divided into three different perspectives. Firstly, international studies that use Granger causality. Secondly, international studies that employed different methodologies. Lastly, studies applied to Brazil.

2.1. International Empirical literature using Granger Causality Tests

Afonso & Jalles (2014) studied the two-way causality between government spending, revenues and growth. They constructed different models applying OLS and GMM estimators and Granger causality test for one hundred fifty-five developed and developing countries, for the period of 1970 to 2010. They found weak evidence of causality from per capita GDP to expenditures. However, they have found stronger evidence supporting the reverse causality, in the short and long run. Moreover, they applied the same methodology only for OECD sub-sample countries and found stronger evidence for Granger causality from government spending to GDP in the short-run, although no significant long-run effect. The reverse relationship still holds for OECD sub-sample, in the short and long-run.³

Additionally, Gómez-Puig & Sosvilla-Rivero (2015) also used Granger causality for eleven EMU countries over 1980 and 2013, analysing the bi-directional relationship between debt and growth. Their study considered cross-country heterogeneity by including central and peripheral countries. Before allowing for endogenous breaks they found evidence of a positive effect of the change in debt on growth and vice versa. After

³ See Afonso & Alves(2016) for a complementary analyses of the possibility of Wagner's law.

allowing breaks, they found a “diabolic loop” between low growth and high debt for Spain, Belgium, Greece, Italy and the Netherlands. However, they found a positive relationship from debt to growth for Austria, Finland and France. Their results somehow explain why empirical studies are not always clear and can show ambiguous conclusions, depending on the period analysed and the country considered. According to the authors, causality should be examined from a dynamic and country specific point of view.

Lai *et al* (2015) explored the casual relationship between government debt, GDP and inflation in France, using annual data between 1980 and 2010. After performing unit root tests, they concluded that there is no long run co-integration between these variables. Therefore, they implemented VAR models and Granger causality test to check if there is short run relationship. They found a unidirectional causality from debt to GDP and from inflation to GDP, either a bidirectional relation between inflation and government debt.

Butts (2009) studied the relationship between economic growth and short-term external debt of twenty-seven Latin American and Caribbean countries, using data from 1970 to 2003. He concluded by the existence of Granger causality from economic growth to short term external debt in thirteen countries.

2.2. International Empirical literature using different methodologies

Afonso & Jalles (2013) investigated the effect of government debt ratio on economic growth using a panel of one hundred fifty-five countries, over the period of 1970 and 2008. They concluded that government debt has a negative effect on growth. Moreover, they concluded that the longer the average maturity of government debt the higher the growth rate for OECD countries in the group. They also found a threshold of 59% of GDP to European countries and 79% for emerging countries.

Additionally, Afonso & Alves (2015) also used panel data techniques to analyse the effect of government debt on real per capita GDP for fourteen European countries, during the period 1970-2012. They found that debt has a negative effect on growth both in the short and in long run. Furthermore, debt service had a much more negative effect than debt on economic performance, they also found a debt threshold around 75%.

Cherif & Hasanov (2018) using a VAR model with debt feedback, analysed the impact of macroeconomic shocks on US public debt dynamics, with data from 1947 to 2015. They concluded that austerity shocks could make debt decline at a cost of lower growth, moreover, debt converged to its pre-shock path, suggesting that austerity is self-defeating. On the other hand, growth shocks could substantially reduce debt, with none of the pain associated with austerity.

Focusing on the inverted U-shape relationship between debt and growth, Reinhart & Rogoff (2010), used a data set of two hundred years for forty-four countries and found a threshold for public debt of 90%, this value is the same for advanced and for emerging markets. They also found that inflation is higher when public debt is higher, when they used data only from emerging markets.

However, Égert (2015) analysed a variant of Reinhart & Rogoff (2010) dataset, he employed nonlinear threshold models and concluded that this negative nonlinear relationship is not ensured, moreover, it changes across samples and different model specifications.

2.3. Studies Applied to Brazil

Gadelha (2011) investigated the relationship between GDP, public expenditure, public revenues, and government debt. He applied Granger causality in a bivariate and multivariate framework, using data over January 1997 and June 2009. Results indicated a bidirectional causality between government revenues and expenditures, concluding that there is fiscal synchronization in Brazil.

Rodrigues & Teixeira (2013) analysed the relationship between public spending and debt using Granger causality, over the period of 1950 and 2000. They concluded that public spending did not cause GDP growth, it is mostly a consequence of economic growth, supporting Wagner's Law.

Gadelha & Divino (2008) investigated whether there is monetary dominance or fiscal dominance⁴ in Brazil. They applied models of multivariate and bivariate Granger

⁴ In a monetary dominance the fiscal authority generates primary surplus that is enough to keep the debt-to-GDP ratio stable, therefore the monetary authority can exercise their role. On the other hand, when there is a fiscal dominance, the monetary authority needs to allow prices adjust to ensure that the current

causality, with monthly data over January 1995 and December 2005. The variables analysed were interest rate, debt to GDP ratio, primary surplus to GDP ratio, real exchange rate and risk premium. They concluded that Brazil was under monetary dominance and both interest rate and primary surplus unidirectionally Granger caused the debt-to-GDP ratio.

The studies applied to Brazil did not focused on the relationship between public debt and growth, most of them emphasised the relation between government expenditures and revenues or government expenditures and growth⁵. Therefore, this study differentiates from empirical literature applied to Brazil by analysing the relation between government debt and GDP. Moreover, aiming to find a more complete relationship we included the variables used by Gadelha & Divino (2008), that are interest rate, inflation rate, exchange rate, primary surplus and Embi+⁶, since these variables are also important for the changes in public debt and economic growth.

3. METHODOLOGY

3.1. Data Description and Unit Root Test

The dataset was collected from several sources: the Gross Domestic General Government Debt (ratio of GDP), which is called debt for simplicity throughout the text, is represented by D ; Y represents the GDP (growth rate), called growth throughout the text; Over Selic⁷ interest rate is noted by I ; Nominal Exchange rate direct quotation (R\$/US\$), is noted by E , all the variables listed above were sourced from the Brazilian Central Bank. R represents the inflation rate⁸ (percentage change), which has Brazilian Institute of Geography and Statistics as source. S represents the primary budget (ratio of GDP), which is sourced from Brazilian National Treasure. Embi+⁹ is sourced from JP

value of the outstanding government debt is equal to the actual real value of the future primary surplus. See, for instance, Afonso (2008).

⁵ Reinhart & Rogoff (2010) included Brazil in his sample, however it did not focused specifically on Brazilian data, but on a set of emerging and advanced countries.

⁶ Emerging Markets Bond Index is a benchmark index for measuring the total return performance of some emerging countries bonds compared to a similar American bond.

⁷ Interest rate set by Central Bank, used as a monetary policy instrument.

⁸ Consumer Index Price (IPCA).

⁹ Measures the spread between the Brazilian bond and US bond, used as a proxy for risk.

Morgan. The study uses monthly time series data, starting in January 1998 and ending in November 2019.

The data processing was as follows: First, we treated the outlier presented in the series of primary surplus in September 2010, by excluding values that represented atypical revenues and expenditures¹⁰. Then series of GDP and debt, which showed some seasonal component, were seasonally adjusted by the methodology Census X-13. The seasonal adjusted series and the one of primary surplus were converted to real terms, deflated by consumer index price, which considered January 1998 as the base value. Thereafter, the series were converted to annual values, by adding up the twelve rolling window values, thus they could be analysed in the same bases as public debt, which is a stock variable. Lastly, the values were converted in ratios of GDP.

The Augmented Dickey Fuller (*ADF*) and Phillips-Perron (*PP*) unit root tests are the most widely applied, however they can present problems related to power and size in finite samples. Moreover, Maddala & Kim (2004) explain that structural change does affect inference on unit roots and on cointegration, being important to allow for possible breaks at the estimation stage. Therefore, the study of stationarity is going to be conducted by a new generation of tests that address these related problems. Firstly, it is applied the modified Dickey-Fuller (ADF^{GLS}) test, suggested by Elliot *et al* (1996), then the Phillips-Perron (MZ_{α}^{GLS}) suggested by Ng and Perron (2001).

Elliot *at al* (1996) proposed the use of generalized least squares (GLS) estimators instead of ordinary least squares (OLS), to purge deterministic terms presented in the regression, since OLS estimators are inefficient in the presence of heteroscedasticity. Moreover, Ng & Perron (2001) explained that distortions of size in the presence of negative moving averages, related to outliers, implicate an incorrect selection of lags by the Akaike (AIC) and Schwarz (SIC) criteria. They also proposed the use of GLS estimators in place of OLS, for the traditional PP test. Therefore, this study applies both tests, making use of the modified Akaike Criteria (MAIC) for lag selection.

However, considering economic changes during the period, we may account for structural breaks. Furthermore, the modified ADF^{GLS} and MZ_{α}^{GLS} tests still have low

¹⁰ In this month primary surplus was affected by capitalization and onerous operations with the Brazilian oil company, Petrobras, that was atypical. The treatment followed (Gadelha & Divino, 2013).

power in the presence of breaks. Therefore, we applied two tests with endogenous breaks. The first one is the test proposed by Saikkonen & Lütkepohl (2002), hereinafter referred to as SL. The SL test considers that the change can occur over some period, and using a level change function ($f(\theta) \gamma$) it is possible to have a smooth transition function, which is added to the deterministic term. The general model is expressed in the following equation:

$$y_t = \mu_0 + \mu_1 t + f(\theta) \gamma + \nu_t \quad (1)$$

where y_t is the data series, μ_0 is the intercept, μ_1 is the deterministic trend coefficient; θ and γ are unknown parameters, ν_t are residuals generated by an autoregressive process, which may have a unit root. There are three possible changing functions for $f(\theta) \gamma$: shift dummy, exponential shift and rational shift. In this study it is going to be applied the last one, rational shift, which represents a rational function in the lag operator applied to a shift dummy. In this test, deterministic trends are estimated by GLS, then they are subtracted from the original series, generating a new series. Then, an ADF test is applied for the adjusted series. Critical values were tabulated by Lanne *et al.* (2002).

The second test implemented is the one proposed by Vogelsang & Perron (1998), hereinafter referred to as VP, that also allows for endogenous breaks by innovation outlier, VP similarly to SL assumes the breaks to occur gradually. Two models are used to check the stationarity hypothesis: intercept break and, trend and intercept break, both in level and in first difference. The general model is expressed in the following equation:

$$y_t = \mu_0 + \mu_1 y_{t-1} + \mu_2 t + \beta_1 D_l + \beta_2 D_p + \beta_3 D_t + \sum_{i=1}^j \rho_i \Delta y_{t-i} + \varepsilon_t \quad (2)$$

where y_t is the data series, μ_0 is the intercept, μ_2 is the deterministic trend coefficient; β_1 , β_2 and β_3 are breaking parameters to be estimated; D_l , D_p and D_t are dummy variables for the intercept break, one time break, and trend break, respectively; ρ_i and μ_1 are unknown parameters, Δ is the first lag operator, j is the optimum lag length to be selected by the Akaike criterion; and ε_t are i.i.d. innovations.

3.2. Growth Equations

The first specification we used to understand the interaction between variables is an estimation of the linear relationship between D and Y , which follow Afonso & Alves (2015) and Afonso & Jalles(2013), using different variables from them in the vector X_t^j , as follows:

$$Y_t = \alpha_t + \beta_1 X_t^j + \beta_2 D_t + \varepsilon_t, t = 1, \dots, T \quad (3)$$

where Y_t represents the growth rate of GDP; D_t is the debt-to-GDP ratio, and ε_t is the error term. α , β_1 and β_2 are unknown parameters to be estimated. The vector X_t^j includes variables that may impact on the relation between public debt and economic growth that were described in Section 3.1.

Next, with the inclusion of D_t^2 in the equation (3), one can check if there is some non-linear relationship. Thus, in equation (4), if β_2 is positive and β_3 is negative, we have support for the inverted U-shape relationship, meaning that we can check if debt has a positive effect on growth until some threshold:

$$Y_t = \alpha_t + \beta_1 X_t + \beta_2 D_t + \beta_3 D_t^2 + \varepsilon_t, t = 1, \dots, T. \quad (4)$$

where; D_t^2 is the debt-to-GDP ratio squared, β_3 is an unknown parameter to be estimated.

3.3. Multivariate Causality

The investigation of the causality among the variables begins by estimating a Vector Autorregressive (VAR) model, following Gadelha (2011) and Gadelha & Divino (2008). The VAR model considers all variables as endogenous, which is a common characteristic in economic series, in the sense that each variable can influence and be influenced by the behaviour of other variables. The VAR in its reduced form is represented as:

$$X_t = A_0 + A_1 X_{t-1} + A_2 X_{t-2} + \dots + A_p X_{t-p} + \xi_t \quad (5)$$

where X_t is a vector of stationary variables, p is the number of lags, A_0 is a vector of intercepts, A_i is a matrix of coefficients, and ξ_t is a vector of residuals not autocorrelated and homoscedastic. The lag selection is made by the usual lag length criteria tests, selecting the one that is considered the best for most of the test criteria results.

If the series are not stationary it is necessary to perform cointegration tests to examine if there is a long run equilibrium relation among the series. This study will perform co-integration tests following the procedures suggested by Johansen & Juselius (1990), Johansen (2002) and Johansen *at al* (2000). The test equation is defined as follow:

$$\Delta X_t = \mu + \pi X_{t-1} + \sum_{i=1}^{p-1} \pi_i \Delta X_{t-i} + \varepsilon_t \quad (6)$$

where X_t is a column vector, μ is a vector of constants, π and π_i represent a matrix of coefficients, p is the lag order, and ε_t is the residual not autocorrelated and homoscedastic. The matrix π is the co-integrating matrix, which represents the long run information about the relationship among the variables. The number of values of π that are statistically different from zero, provides the number of co-integration equations. Johansen proposed the use of two statistics to test for co-integration:

$$\lambda_{trace}(r) = -T \sum_{i=r+1}^n (1 - \widehat{\lambda}_i) \quad (7)$$

$$\lambda_{trace}(r, r+1) = -T \ln(1 - \widehat{\lambda}_{r+1}) \quad (8)$$

where $\widehat{\lambda}$ are the values estimated for the matrix π , and T is the number of observations. The test follows a recursive procedure, where the null hypothesis is that there are, at least, r cointegrated vectors.

Engle, & Granger (1987) explain that if there is co-integration among the series, there must exist a long run relationship between them. Co-integration implies that deviations from equilibrium are stationary, with finite variance. If that is the case, one should estimate a Vector Error Correction Model (VEC) using the linear combination of the series corrected by their co-integrating vector. The VEC model is represented as follow:

$$\Delta X_t = \mu + \Gamma_1 \Delta X_{t-1} + \dots + \Gamma_{p-1} \Delta X_{t-p+1} + \Pi X_{t-1} + \varepsilon_t \quad (9)$$

where, p is the number of lags already selected in the VAR model. $\Pi = \alpha\beta'$, where β is a matrix ($p \times r$), whose columns contain the cointegration vectors, α is a matrix ($p \times r$) with the adjustment coefficients. The linear combinations of $\beta'X_{t-1}$ represents the r number of cointegration equations.

3.4. Bivariate Causality

The bivariate analysis is conducted by ARDL models, following Gadelha (2011) and Gadelha & Divino (2008). In this model both the dependent variable and the independent variables are related contemporaneously and in its lagged values. The advantages of the ARDL technique is that it accepts different lags between the variables, which allows to capture the dynamic of the system without omission of important lag lengths. However, ARDL models in a bivariate system can be affected by the omission of important variables, this problem is overcome in this study by the multivariate causality.

The Error Correction Model (ECM) in a bivariate relationship can be derived as follows:

$$Y_t = \mu + \beta_1 X_t + e_t \quad (10)$$

where Y_t and X_t are vectors respectively of the dependent variable and the independent, e_t is the error term.

Solving for e_t we find the cointegration equation for X_t and Y_t . The ECMs for both variables are respectively:

$$\Delta X_t = \mu_x + \alpha_x e_{x,t-1} + \sum_{i=1}^p \alpha_{11} \Delta X_{t-i} + \sum_{i=1}^q \alpha_{12} \Delta Y_{t-i} + \varepsilon_{xt} \quad (11)$$

$$\Delta Y_t = \mu_y + \alpha_y e_{y,t-1} + \sum_{i=1}^l \alpha_{21} \Delta Y_{t-i} + \sum_{i=1}^m \alpha_{22} \Delta X_{t-i} + \varepsilon_{yt} \quad (12)$$

where ε_{xt} and ε_{yt} are uncorrelated residuals, $e_{x,t-1}$ and $e_{y,t-1}$ are estimated parameters for the lagged residual, that came from the solution of equation (10), the parameters α_x and α_y from equations (11) and (12) measures the speed of adjustment of X_t and Y_t respectively in direction to the long-run equilibrium. p , q , l and m are the optimal lags. The parameters α_{11} , α_{21} , α_{12} and α_{22} represents the short-run relationship.

In equations (11) and (12), the null hypothesis $H_0: \alpha_{12} = 0$ and $\alpha_x = 0$ means that ΔY_t does not Granger cause ΔX_t , on the other hand, the alternative hypothesis

$H_1: \alpha_{12} \neq 0$ and $\alpha_x \neq 0$ means that ΔY_t Granger cause ΔX_t . Similarly, $H_0: \alpha_{22} = 0$ and $\alpha_y = 0$ means that ΔX_t does not Granger cause ΔY_t , on the other hand, the alternative hypothesis $H_1: \alpha_{22} \neq 0$ and $\alpha_y \neq 0$ means that ΔX_t Granger cause ΔY_t .

4. EMPIRICAL ANALYSIS

4.1. Unit Root Tests and Data Analysis

Table I, in the appendix, reports the results of ADF^{GLS} and MZ_{α}^{GLS} unit root tests applied to the series in level and in first differences. Results show that the primary surplus, the debt, the exchange rate and the Embi+ are stationary in first difference; the inflation rate is stationary in level; the GDP growth rate and the interest rate are not stationary in neither of these tests. These results were expected because of the presence of structural changes, which are represented by breaks in the series, that are also graphically noticeable in Figure 1.

Therefore, the analysis was improved by the use of unit root tests with structural endogenous breaks, presented in Table II, in the appendix. Both SL and VP tests reached the same conclusions, in which series of the GDP growth rate, the interest rate, the Embi+ and the inflation rate are stationary in level. However, the debt, the primary surplus and the exchange rate are stationary in first difference.

Most of the selected break dates occurred between September 1998 and April 1999. During this period multiple changes occurred in the economic policy, the most relevant one was the switch from foreign exchange anchor to inflation target policies, which came out with sharp exchange rate devaluation and strong control of interest rates to achieve the inflation target.

Another important break selected by the tests were from October and November 2002. This period reflects a crisis of external confidence related to the election of president Lula, from Worker's Party, which is called Lula's effect. The third important break is related to the economic recession that hit Brazil in the second quarterly of 2014, which may explain the breaks that appeared between April 2014 and December 2015.

Therefore, we used three dummies variables, considering the breaking dates that appeared in the unit root tests and the information presented in Pastore *et al* (2020)¹¹, which reinforces the breaks pointed in the unit root tests. These dummies take the values “1” for the specific period when some event occurred, and “0” otherwise. The dummies used were: *dexchangerate*, *dlula* and *dcrisis*. The first one selected the period of January 1998 until March 1999, and it is related to changes in economic policy due to exchange rate depreciation; the second one selected the period between June 2002 and April 2003, which is related to Lula’s effect; the last is referred to the period between April 2014 and December 2016, a period of a strong economic recession.

4.2. Growth Equation

We estimated five different static models: where model 1, model 2, model 3 and model 4 are applications of equation (3) and model 5 checks the possibility of non-linear relationship, as presented in the equation (4). We also used the dummies *exchangerate*, *dlula*, and *dcrisis* which appeared to be statically significant at 10% level in all the models. Results are presented in Table III.

In model 1 we found significantly positive coefficients for debt and primary surplus, significantly negative coefficient for interest rate, exchange rate and Emib+. Results shows that debt has a positive impact on the GDP growth rate.

In model 2 we checked if the Brazilian Constitution amendment, which imposed a ceiling for government expenditure, representing a fiscal consolidation, had some impact on the debt and growth relationship. For this analysis we used a dummy represented as *dconsolidation*, which receives the value “1” if in the period considered there was a government consolidation, and “0” otherwise. However, *dconsolidation* showed not to be significant at the 10% level. Moreover, the results for the coefficients are very similar to those found on Model 1.

In model 3 and 4 we analysed if the relationship between debt and growth changes when the debt ratio is “high” or “low”. In model 3, we used a dummy which received the

¹¹ Report presented by the Business Cycle Dating Committee (CODACE), which presents the most relevant changes in economic cycles of Brazilian economy.

value “1” if in the period government debt was placed in between 30% and 60% of GDP, and “0” otherwise, represented by $d3060$. In model 4 we used a dummy which received the value “1” if in the period government debt was placed in between 60% and 90%, and “0” otherwise, represented by $d6090$. These values followed those applied by Reinhart and Rogoff (2010). The dummies used presented significant coefficients. Although the results of the debt coefficient were almost the same in both models, our results also suggest that growth intensifies when debt is in between 60% and 90%, since $d6090$ has a positive coefficient and $d3060$ has a negative coefficient. These results are in line with the findings of Reinhart and Rogoff (2010), they showed that growth rates in Brazil are larger when debt ratio is between 60% and 90%, and smaller when it is above 90% or in between 30% and 60%. They also found a threshold of 90% for debt-to-GDP ratio for advanced and emerging countries.

Previous results lead us to estimate Model 5, where we followed equation (4), to check for the possibility of an inverted U-shape relationship between debt and growth. Although, β_2 and β_3 presented signals as expected, indicating the U-shape relationship, β_3 was not statistically significant at 10% level. Therefore, we could not confirm the threshold pointed by Reinhart and Rogoff (2010) for emerging countries.

In every model the results of the coefficients were very similar, all of them showed debt presenting positive impacts on growth; interest rate, exchange rate and Embi+ presented negative impact; and primary surplus presented positive coefficients. In none of the models inflation presented significant coefficients. The results of this section do not consider the possibility of lagged effects, as well as the possible interaction between the dependent and independent variables, which are going to be analysed in sections 4.3 and 4.4.

4.3. Multivariate Causality

Since half of the series became stationary only after the first difference, the next step was testing for cointegration. However, before testing for the cointegration one needs to select the correct lag length to be used. We applied the lag length criteria to the VAR of the series and the decision was to select the lag pointed by most of the criteria's results,

which was seven. This value was selected by Final Prediction Error (FPE) and Akaike Information Criteria (AIC).

Firstly, we applied the Johansen Trace and Max-Eigenvalues tests, and results are reported in Table IV, in the appendix. They suggest a long run relationship between the variables, as we do not reject the null hypothesis for the presence of co-integration vector after the fourth rank. Since it is known that the series have breaks, we selected the breaks that appeared the most on the unit root test, then applied Johansen cointegration tests using these breaks. Three pairs of dates were selected for the application of the test with breaks, they are: January 1999 and November 2002, results are presented in Table V, in the appendix; January 1999 and December 2015; and November 2002 and December 2015¹². All of them came to the same conclusion: the existence of a long run relationship between the variables with five co-integrating vectors. The presence of co-integration denotes that the multivariate analysis should be conducted using a VEC. Thus, we estimated the VEC with five co-integration vectors, we used only *exchangerate*, *dlula* and *dcrisis*, we did not use *dconsolidation*, *d3060* and *d6090* since they were not significant for most of the vectors.

In addition, we also tested the significance of the coefficients of the co-integration equations in the VEC by employing a χ^2 Wald Test. If the null hypothesis is rejected, we can validate the results of Granger causality, moreover we can follow the strategy of analysing all the variables as endogenous in the system. We rejected the null hypothesis for all the coefficients 1% of significance, results are presented in Table VI, in the appendix.

The VEC model satisfies the stability condition, since none of the roots of the model lies outside the unit circle. Results of the Roots of AR Characteristic Polynomial are presented in Figure 2, in the appendix. The autocorrelation LM test was applied to check for the presence of serial correlation in the error terms. Results conclude for no autocorrelation after lag seven, since we cannot reject the null hypothesis of no serial correlation at a significance level of 5%. Results of the LM test are presented in Table VII, in the appendix.

¹² Results of the last two tests are not presented for reasons of parsimony.

The results of the Granger causality are presented in Table VIII, in the appendix, tests conclude that the GDP growth rate has a bidirectional relation with the debt ratio. Also, it is Granger caused by the exchange rate and by the primary surplus. Moreover, the debt ratio has a bidirectional relation with the interest rate, inflation rate and Embi+. Therefore, it is possible to say that the interest rate, inflation rate and Embi+ can influence the relation between debt and growth, since they affect the behaviour of the debt ratio. Moreover, these results are in accordance with the finds of Gadelha & Divino (2008), that also concluded that interest rate and Embi + Granger causes debt.

In order to understand the whole scenario of the interactions between the variables in the system, we complemented the analyses of the causality by impulse response functions and variance decomposition. By the impulse response function, we can see the response of one variable, over a future period of time, to a shock of another variable in the VEC. Therefore, we analysed the response of GDP growth rate and debt ratio to innovations of the rest of the endogenous variables in the system over the period of 18 months.

Figure 3, in the appendix, presents the response of the GDP growth rate to one standard innovation in the other endogenous variables of the VEC over time. Results suggest that a shock in debt ratio has negative effects on GDP growth rate, there are only a few positive effects during the third and fifth months. This behaviour is in accordance with the theory that government debt has a negative impact on growth. Moreover, the results of the first six months after a positive shock in primary surplus generates a positive impact on economic growth, validating the theory of expansionary fiscal consolidations¹³. Further, Matheson & Pereira(2016) concluded that fiscal multipliers related to spending and credit have dropped to near zero in Brazil between the global financial crisis and 2014, thus non-Keynesian effects are more likely to prevail.

The GDP growth rate shows a positive response to a shock in interest rates, and it is possible that an increase in short term interest rate¹⁴ could lead to an increase in savings, which may have a positive impact on the GDP growth rate. The response to a shock in the exchange rate is negative, meaning that a depreciation of the currency provokes a negative impact on the GDP, and this negative impact intensifies until the eighth month,

¹³ See Afonso & Martins (2016) for a better understanding of expansionary fiscal consolidations.

¹⁴ We used the Selic interest rate, that is the interbanking day interest rate.

then it starts to decrease and erodes after eighteen months, which may be related to capital imports. Inflation causes a negative impact on GDP growth rate during the first ten months, then it vanishes. Embi+ is negative for GDP growth rate during the first sixteen months, then it vanishes. Both, inflation and Embi+ negative effects on growth may be related to bad expectations.

We also tested the response of debt to a shock on the system variables. Results are presented in Figure 4, in the appendix, they show that the debt ratio decreases when the GDP growth rate increases. A shock on interest rate made debt increase over the first nine months. This behaviour is explained by part of the public debt being indexed to the interest rate. The exchange rate also makes debt increase over the first seven months, then it vanishes. This behaviour is explained by the external debt, that may increase when there is a currency depreciation. Innovations in the inflation rate provokes a decrease in the debt ratio, in this case, although the monetary authority sets the interest rate independently of the fiscal authority, the debt ratio is somehow benefited by the seigniorage. Embi+ increases debt ratio over all the period, meaning that when Embi+ increases, expectations of the country get worse; therefore, investors demand more risk premium, which increases the debt.

The result validated the theory of expansionary fiscal consolidations for Brazil, since a decreasing debt ratio has positive impacts on economic growth. Moreover, as it is presented in Figure 5, in the appendix, debt shocks generate increases on inflation rate most of the time, corroborating the fiscal theory of the price level (FTPL)¹⁵. Debt shocks also generate interest rate increases, meaning that the monetary authority may try to control inflation. Additionally, it improves profitability of government bonds, which may be required by investors in response to the increase in debt ratio; Embi+ increases, since investors demand more risk premium and currency depreciates. Inflation rate, exchange rate and Embi+ will provoke a negative impact on GDP growth rate. Therefore, debt may directly provoke a negative impact on the GDP growth rate and indirectly have negative impact by changes in inflation rate, exchange rate and Embi+.

Furthermore, a shock on GDP growth rate provokes an increase on interest rate, mixed effects on inflation, appreciation of currency, decreases Embi+ and increases primary

¹⁵ The FTPL posits that the increase in government debt increases demand and leads to price pressures.

surplus. The effect of the last three will result in a further debt reduction. Results are presented in Figure 6, in the appendix. Thus, the increase in the GDP growth rate may also indirectly decrease the debt ratio by the effect of exchange rate, Embi+ and primary surplus.

The variance decomposition quantifies the contribution of innovations in one variable to changes in another variable. Therefore, we aim to quantify the proportion of the variation in GDP growth rate and debt ratio that is related to each other and to the other endogenous variables in the VEC. Results are presented in Table IX and Table X, in the appendix, values are presented as percentage.

Table IX provides information about the variance decomposition of GDP growth rate. Most of the change is related to its past values, however one important part seems to be related to exchange rate, followed by primary surplus and debt ratio. These results also reinforces the conclusion of the Granger causality, since these three variables appeared Granger causing GDP growth rate.

Table X provides information about the variance decomposition of debt ratio. GDP growth rate, Embi + and inflation contributes the most to changes in debt ratio. That validates results for Granger causality, since these variables appeared Granger causing debt ratio.

4.4. Bivariate Causality

The ARDL model in a bivariate causality allows us to have an embracing analysis of the relationship between both variables included. Since it does not demand the same number of lags for both variables, we do not run the risk of omitting important lags. However, in a bivariate causality we run the risk of omission of important variables. Therefore, both methodologies, VEC and ARDL, are going to be used as complementary to each other.

The first step in the analysis was to perform the Engle-Granger cointegration test, in which we used the AIC and the SIC criteria for the lag selection. Results are presented in Table XI, in the appendix. All the pairs presented cointegration for at least one side of the

selection criteria used, which means we may find long run equilibrium for these pairs of variables.

Therefore, we performed the ARDL model for all the pair of variables. The model was carried out using restricted constant in the trend specification, we also included the same dummies used in the VEC model as fixed regressor. After the results, we tested if the dummies and the constant were significant and excluded those that did not show to be significant in at least 10% of significance level.

Hereafter, the Error Correction was included, when it presented to be statistically significant. Thus, we estimated the long run model of the error correction, otherwise we estimated the short run model, which is the ARDL with the differenced lagged variables without the error correction term. Results are presented in Table XII, in the appendix. The causality test was checked by the joint significance in the Wald test.

Results show that in most of the cases, when GDP growth rate is the dependent variable, variables exhibited long run relationship, meaning that in the long run there is a univariate relationship from the variables selected to GDP growth rate, except for debt, which exhibited only short run relation with GDP growth rate. On the other hand, GDP growth rate also Granger causes the pattern of most of the variables, although in the short run. The opposite relation occurred for the debt, which appeared to have a long run relationship with most of the variables, only when it was the independent variable, however, in the short run debt is Granger caused by the pattern of most of the variables.

Primary surplus is not Granger caused neither by GDP growth rate nor by debt. The same was found in the VEC Granger causality.

4.5.Results

In the first exercise we estimated the static relationship between the group of variables. In this part we did not allow for lagged variables impacting in the dependent variable, since the primordial objective was to understand if the relation would change after the Constitutional ceiling amendment and also after changing in debt ratios. Results showed the same signals and almost small changes in the coefficients. In all the models employed we find debt impacting positively in GDP growth rate.

Hereafter, we studied the causality between the variables, by the use of VEC in a multivariate analysis, and ARDL the bivariate one. The bivariate analysis allowed the impact of lagged variables and showed a similar result as the static model. In this methodology debt Granger causes GDP growth rate in a positive way in the short run. Moreover, GDP growth rate is Granger caused by the rest of the system of variables, but in a negative way. On the other hand, the multivariate analysis presented a different result, where debt also Granger causes GDP growth rate, but in a negative way and in the long run. Furthermore, the rest of the result of the bivariate analysis confirmed the results of the multivariate when debt or GDP growth rate are the dependent variable, except for the impact of primary surplus on the GDP growth rate.

Table XIII presents a comparison of both methodologies for the cases that we find Granger causality. In the VEC, the signal is a result of the cumulative values of the impulse response function. In the ARDL are the values of the summation of the coefficients of the lagged values of the dependent variables. Overwritten letters “S” and “L” represent that we find Granger causality respectively in short and long run models.

This result brings us the question: Why could debt impact positively on the GDP growth rate in the short run and negatively in the long run? The answer to this question takes in consideration the difference between ARDL and VEC methodologies. Since ARDL runs the risk of omission of important variables and VEC runs the risk of the omission of important lags, both analyses should be used in a complementary way. Results of the VEC consider the impact that debt may generate in other variables, that will also impact on GDP growth rate, such as inflation rate, exchange rate and Embi+, that are positively Granger caused by debt and negatively Granger causes GDP growth rate. That is, when debt increases, inflation rate, exchange rate and Embi+ also increases, however all of them will work to decrease growth. Moreover, the impulse response of the VEC showed positive impacts of debt on growth during the third and fifth periods, which is in accordance with ARDL results. Therefore, it is possible to say that the short run causality from debt to growth is positive, however this relationship changes in the long run, when the causality becomes negative, and part of it is related to the impact of inflation rate, exchange rate and Embi+. On the other hand, the causality from growth to debt is also negative, meaning that increasing in GDP growth rate causes reduction in debt-to-GDP ratio. Moreover, part of it is related to the impact of growth on Embi+, since growth

negatively Granger causes Embi+ in the long run, that in turns, positively Granger causes debt; that is, when the GDP growth rate increases, it decreases Embi+ in the long run, therefore the following reduction on debt is related to growth directly and indirectly.

Primary surplus showed to be completely exogenous in the VEC, in the sense that it was not Granger caused by none of the variables. In ARDL it appeared to be statistically significant when it was dependent only by its lagged value. This may be explained by the findings of Gadelha (2011) who concluded by a synchronization between government revenues and expenditures, that makes primary surplus much more dependent on both variables than in those used in this study. When primary surplus is the independent variable, it decreases debt in the short run; however, it presents opposite results for its relation with the GDP growth rate, which does not allow us to make a reliable statement about this relationship.

Interest rate presented negative long run Granger causality with GDP growth rate only in the ARDL and positive long run Granger causality with debt only in the VEC, the last proposition is in accordance with the findings of Gadelha & Divino (2008). When interest rate is the dependent variable, it showed to be positively Granger caused by GDP growth rate both in the short and long run. However, it is not possible to make any statement about its dependence of debt, since it has presented opposite signals in the VEC and in the ARDL.

Another approach that one could takes to explain the difference in the relationship from debt to growth in short and in long run is that fiscal multipliers are not long lasting, therefore they are more likely to prevail in short run than in long run, while in long run crowding-out effects are more likely to prevail. Results of Matheson & Pereira (2016) shows that fiscal multipliers in Brazil are short lived. Moreover, they came to the same conclusion of Gadelha (2011) about the fiscal synchronization, explaining that a surprising increases in government spending in a given quarterly I likely to generate a consolidation in the later on. Therefore, in long run crowding-out effects are more expected than Keynesian multipliers.

5. CONCLUSION AND POLICY IMPLICATIONS

The objective of this dissertation was to understand the relationship between public debt and economic growth in Brazil. Moreover, we aimed to understand the interaction between debt and growth with other variables, that are interest rate, inflation rate, exchange rate and Embi+. To achieve that objective we studied growth equations, and multivariate and bivariate Granger causalities, by applying VEC and ARDL methodologies.

We concluded that debt and growth have a bi-directional relationship, meaning that one causes and is caused by each other. Although the presence of causality in both directions, the behaviour is not the same in the short and long run. Debt may improve growth in the short run, however, it can be harmful to growth in the long run, not only by its direct relation to the GDP growth rate, but by its indirect impact over inflation rate, exchange rate and Embi+. On the other hand, growth reduces debt, both in the short and long run, there is also an indirect impact of growth in debt by the reduction that growth causes in Embi+.

The important policy implication of this result is that if we can better understand the relation between debt and inflation rate, exchange rate and Embi+, we may also be able to soften the negative impact of debt on growth, by the use of other policies that can impact on this variable.

Another issue that came up is why studies applied to other countries could find a positive long run relationship between debt and growth, as the findings of Gómez-Puig & Sosvilla-Rivero (2015). What is the difference between Brazil and other countries that found this positive relation? Maybe the answer is related to what this debt is used for, or else, the negative impact of other variables such as Embi+ is not that strong as it is in Brazil. These questions can be evaluated more deeply by other studies and should be taken into consideration by the policy makers.

In addition, the fact that debt may improve growth in the short run and harm it in the long run also emphasizes the trade-off faced by governments to correctly evaluate if it is time to promote aggregate demand or to implement austerity measures. The actual situation of high debts and economic recession also made the decision harder, since some austerity measures can deteriorate growth in the short run. Therefore, evaluating the

quality of public expenditures, namely its efficiency and effectiveness could be the better way to help the decision about in what policy or program to adopt austerity or expansionary measures.

Further, bills such as the PEC 187/2019 that propose the use of resources of public funds to pay down debts seems to be a more efficient way to use this feature, since part of it has not been used and at the same time government pays interest on its debt. Similarly, the use of part of Dollar reserves of the Central Bank to pay down public debt may be a good solution, since the current exchange rate depreciation increased a lot the Central Bank's profit on its reserves for much more than what is necessary to face possible future losses. Furthermore, measures such as the use of part of the amount saved by austerity measures in infrastructure projects, presented in the PEC 188/2019 may also be growth inducing and reduce public debt.

Finally, we have not included external public debt in our analysis because since October 2006 the Brazilian net external public debt is negative. This occurs because of the effort made by the government to increase dollars reserves, which decreases the vulnerability related to exchange rate depreciations. However, we think that further work could evaluate the interaction between external debt, dollar reserves, exchange rate and Embi+. Moreover, we think that the impact from Embi+ in external debt may be much stronger than in total debt.

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APPENDICES

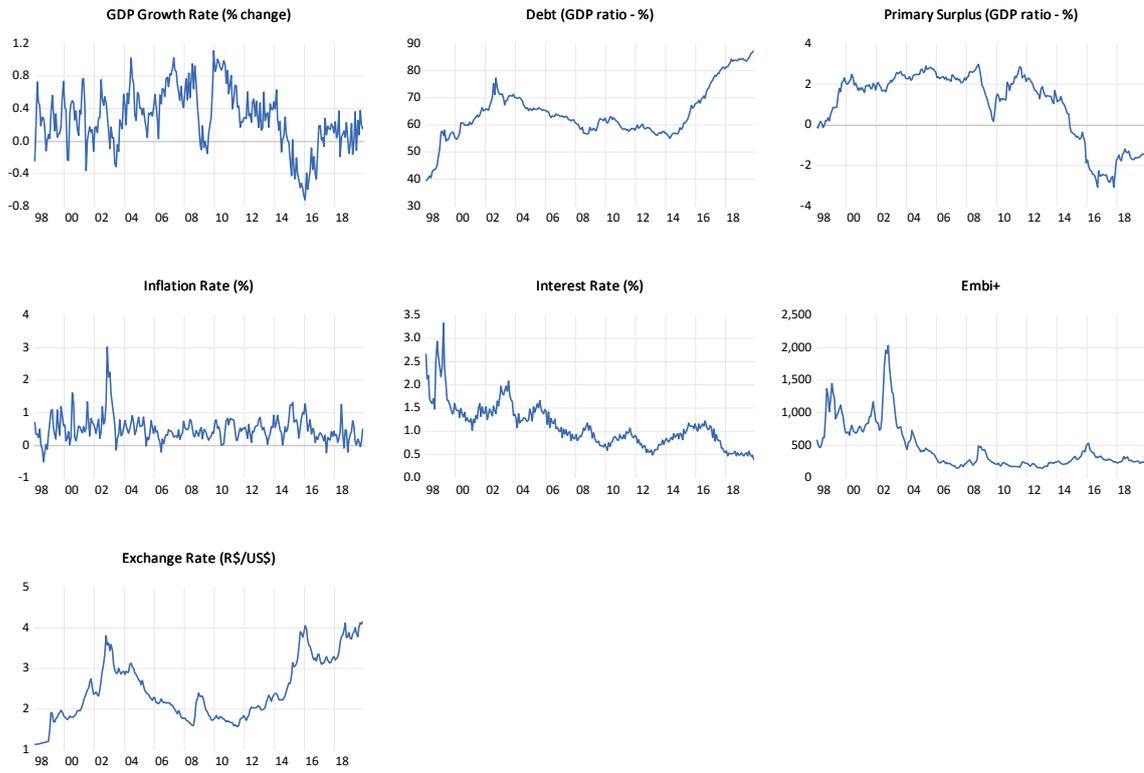


FIGURE 1 – Treated Series.

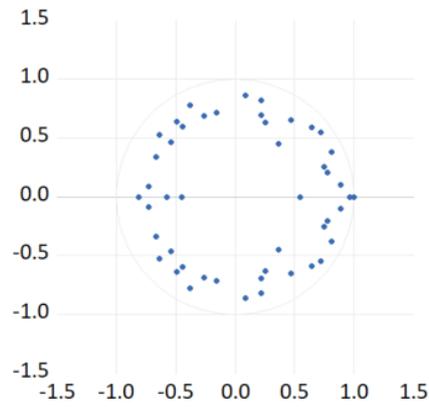


FIGURE 2 – ROOTS OF CHARACTERISTIC POLYNOMIAL.

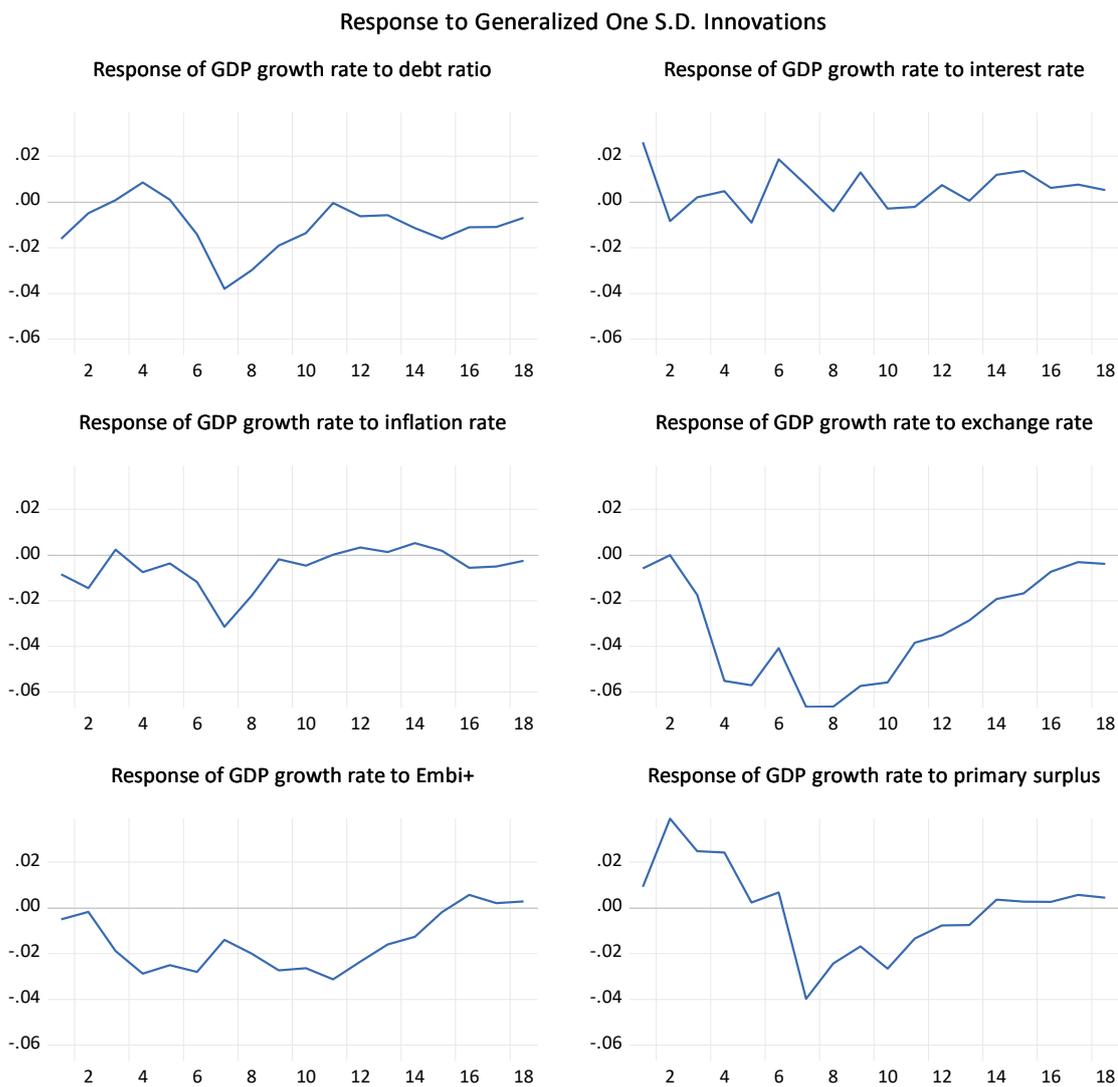


FIGURE 3 – Response of GDP growth rate to innovations on debt ratio, interest rate, exchange rate, Embi+ and primary surplus.

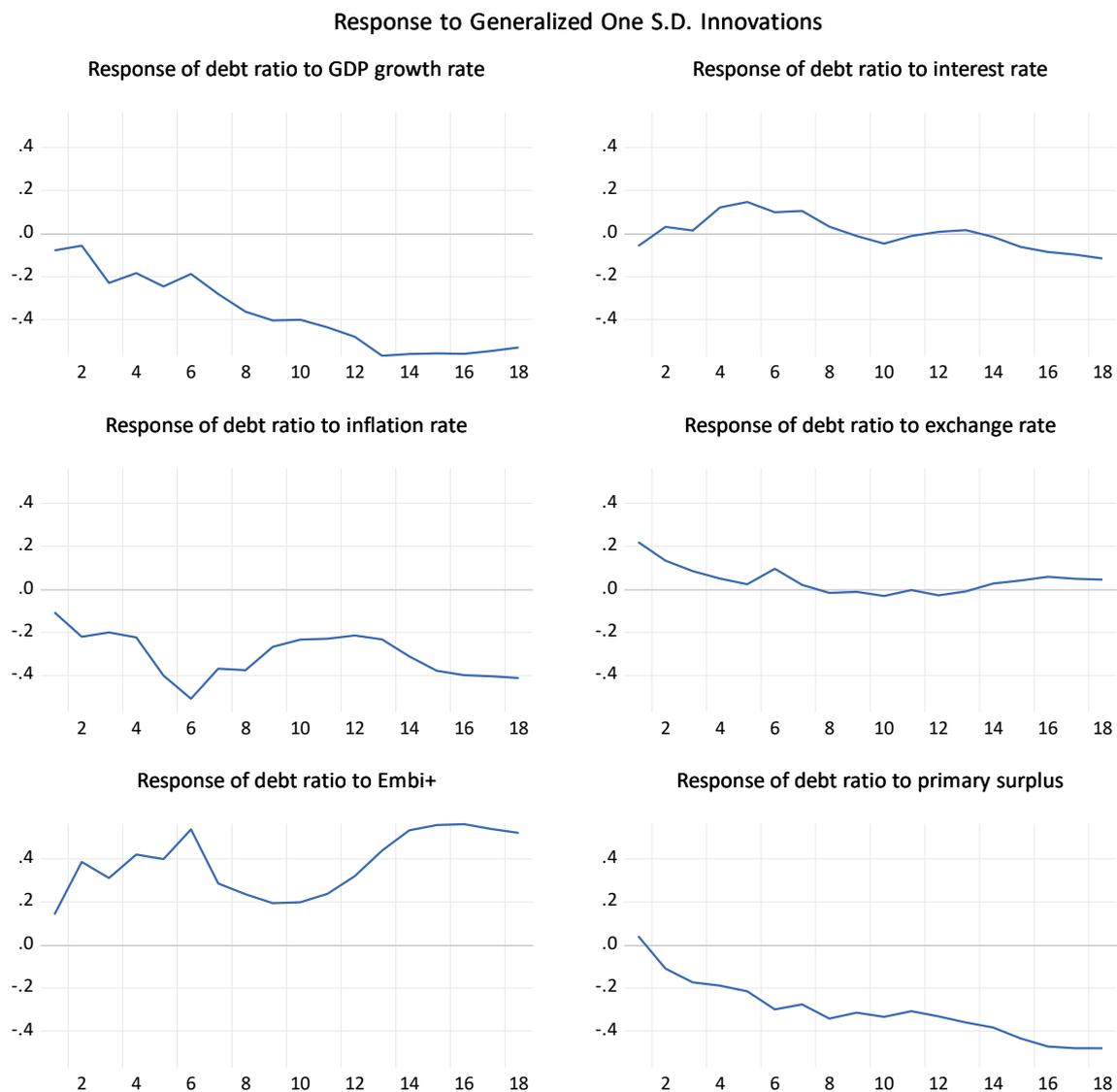


FIGURE 4 – Response of debt ratio to innovations on GDP growth rate, interest rate, inflation rate, exchange rate, Embi+ and primary surplus.

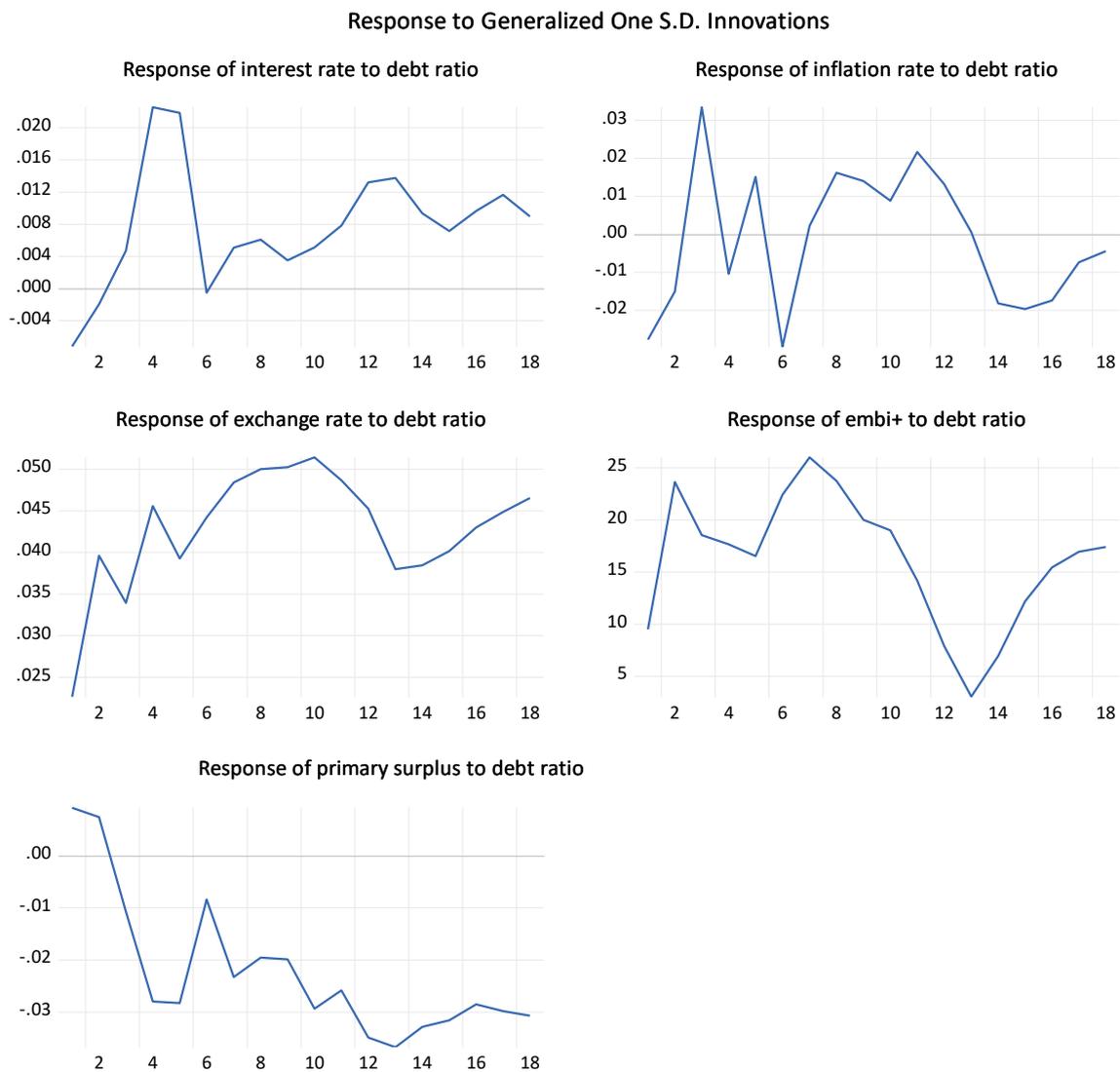


FIGURE 5 – Response of interest rate, inflation rate, exchange rate, Embi+ and primary surplus to innovations on Debt.

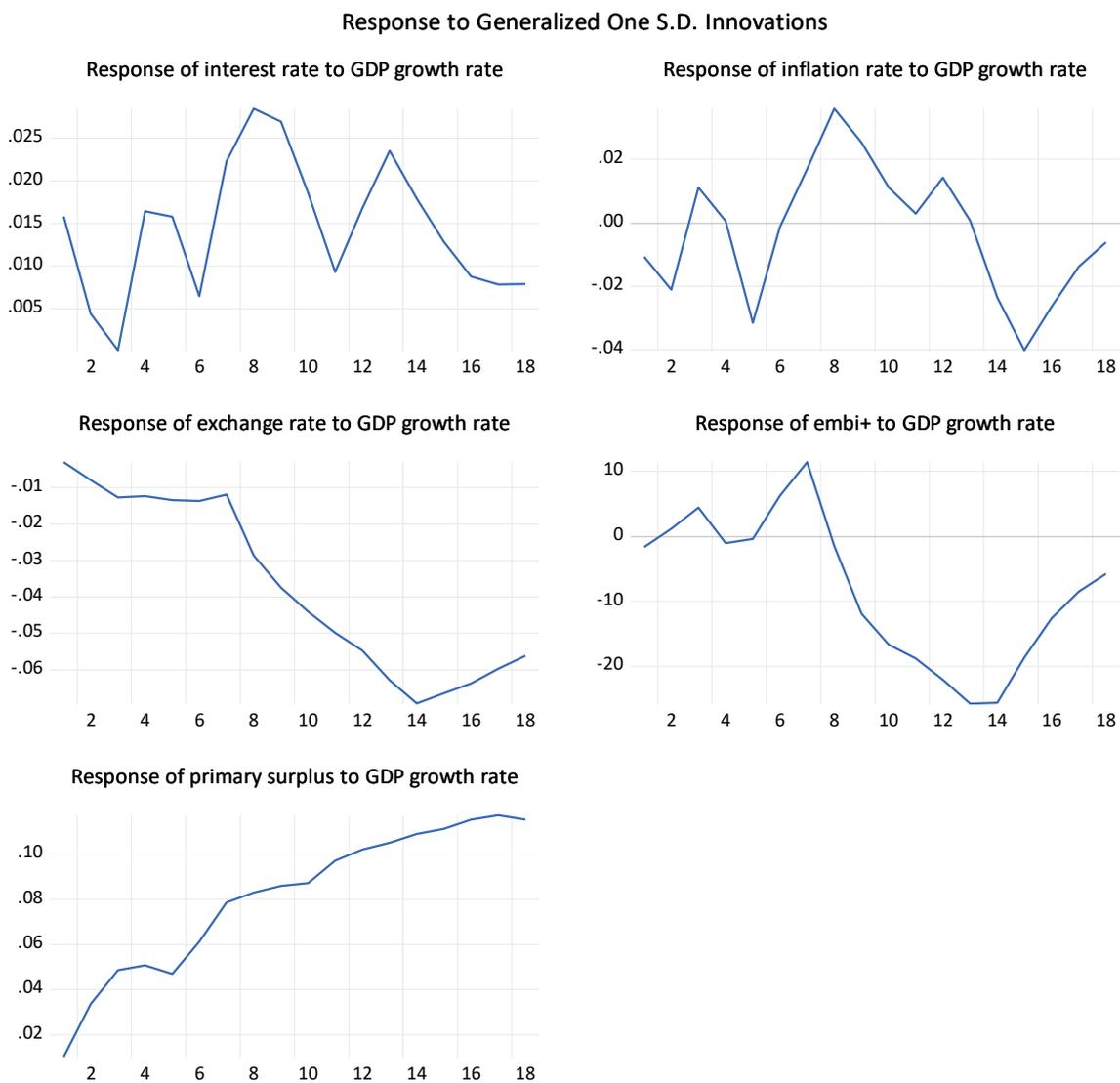


FIGURE 6 – Response of interest rate, inflation rate, exchange rate, Embi+ and primary surplus to innovations on GDP growth rate.

TABLE I
UNIT ROOT TESTS WITHOUT STRUCTURAL BREAK

Variable	Test Equation	ADF^{GLS}	Lags	MZ_{α}^{GLS}	Lags
GDP growth rate	Intercept	-1.290721	12	-1.34344	12
	Trend and Intercept	-1.753942	12	-1.85005	12
	D(Intercept)	-0.801839	11	-0.11943	11
	D(Trend and Intercept)	-2.115286	11	-0.84698	11
Debt	Intercept	1.328672	2	1.37089	2
	Trend and Intercept	-0.952966	2	-0.96881	2
	D(Intercept)	-3.598385***	12	-1.70343*	12
	D(Trend and Intercept)	-3.593600***	9	-2.32025	9
Primary Surplus	Intercept	-0.746254	12	-0.78408	12
	Trend and Intercept	-0.779822	12	-0.84424	12
	D(Intercept)	-3.265969***	10	-2.18834**	10
	D(Trend and Intercept)	-3.501313***	10	-2.42425	10
Exchange Rate	Intercept	0.267376	3	0.23421	3
	Trend and Intercept	-1.475775	2	-1.50945	2
	D(Intercept)	-6.005510***	4	-6.16886***	4
	D(Trend and Intercept)	-5.987699***	4	-6.11966***	4
Interest Rate	Intercept	0.786271	12	0.87691	12
	Trend and Intercept	-1.185095	12	-1.16717	12
	D(Intercept)	-0.329988	11	0.07366	11
	D(Trend and Intercept)	-1.688773	11	-0.23832	11
Embi+	Intercept	-1.766380*	7	-1.65462*	7
	Trend and Intercept	-2.393650*	7	-2.33533	7
	D(Intercept)	-2.698043***	11	-1.36413	11
	D(Trend and Intercept)	-4.847106***	11	-3.01152**	11
Inflation rate	Intercept	-3.256705***	8	-2.68997***	8
	Trend and Intercept	-4.319382***	8	-3.89828***	8
	D(Intercept)	-0.811405	11	0.53147	11
	D(Trend and Intercept)	-2.126006	12	-0.60669	12

Source: Authors Elaboration.

Note: *, ** and *** denotes significance at 10%, 5% and 1% levels; D() denotes tests in first difference.

TABLE II
UNIT ROOT TESTS WITH STRUCTURAL BREAK

Variable	Test Equation	SL - Rational Shift			VP- Innovational Outlier		
		Date	t- statistic	Lags	Date	t- statistic	lags
GDP growth rate	Intercept	2001 M6	-3.0334**	2	1998 M10	-5.672562***	0
	Trend and Intercept	2001 M6	-2.4919	2	1999 M01	-5.715973***	0
	D(Intercept)	2001 M6	-6.0331***	2	1999 M03	-21.14956***	0
	D(Trend and Intercept)	2001 M6	-4.7778***	2	1998 M06	-21.55431***	0
Debt	Intercept	1999 M1	-0.6209	2	2015 M06	-4.317432	5
	Trend and Intercept	1999 M1	-0.9221	2	2012 M07	-4.445778	4
	D(Intercept)	1999 M1	-7.3233***	2	1999 M01	-19.23603***	0
	D(Trend and Intercept)	1999 M1	-4.2197***	2	1999 M01	-19.51033***	0
Primary Surplus	Intercept	2015 M12	-0.6352	2	2014 M04	-4.450804	12
	Trend and Intercept	2015 M12	-1.3882	2	2015 M10	-3.372438	12
	D(Intercept)	2015 M12	-8.4324***	2	1998 M 06	-15.33422***	0
	D(Trend and Intercept)	2015 M12	-9.0591***	2	1998 M08	-15.30660***	0
Exchange Rate	Intercept	2002 M10	-0.8667	2	2005 M03	-3.381711	1
	Trend and Intercept	2002 M10	-1.1440	2	2008 M12	-3.554331	4
	D(Intercept)	2002 M10	-3.5849***	2	2002 M10	-13.11930***	0
	D(Trend and Intercept)	2002 M10	-4.8238***	2	2003 M04	-12.38337***	0
Interest Rate	Intercept	1998 M9	-3.1055**	2	1999 M03	-7.773424***	0
	Trend and Intercept	1998 M9	-0.8898**	2	1999 M03	-8.281520***	0
	D(Intercept)	1998 M9	-10.1937***	2	1999 M03	-23.45007***	0
	D(Trend and Intercept)	1998 M9	-5.1881***	2	1998 M11	-23.66599***	0
Embi+	Intercept	2002 M11	-3.0340**	2	2002 M10	-6.100274***	3
	Trend and Intercept	2002 M11	-2.2273	2	2002 M10	-6.285755***	3
	D(Intercept)	2002 M11	-3.8811***	2	1998 M09	-11.87812***	0
	D(Trend and Intercept)	2002 M11	-4.8686***	2	1998 M09	-12.23370***	0
Inflation rate	Intercept	2002 M11	-4.2899***	2	1998 M11	-8.149452***	0
	Trend and Intercept	2002 M11	-4.7562***	2	1998 M11	-8.074853***	0
	D(Intercept)	2002 M11	-4.9046***	2	1998 M08	-18.22486***	0
	D(Trend and Intercept)	2002 M11	-5.4818***	2	1999 M04	-18.34451***	0

Source: Authors Elaboration.

Note: *, ** and *** denotes significance at 10%, 5% and 1% levels; D() denotes tests in first difference.

TABLE III
GROWTH EQUATIONS

Dependent Variable: GDP growth rate	Model 1	Model 2	Model 3 [#]	Model 4 [#]	Model 5
D	0.024870*** (0.002983)	0.024935*** (0.003012)	0.015705*** (0.005455)	0.015705*** (0.005455)	0.026825*** (0.004447)
I	-0.132585** (0.0277)	-0.133311** (0.060150)	-0.186561*** (0.065033)	-0.186561*** (0.065033)	-0.147985** (0.065337)
R	0.042858 (0.047159)	0.041920 (0.047558)	0.044768 (0.047371)	0.044768 (0.047371)	0.036706 (0.048343)
E	-0.361561*** (0.046593)	-0.359584*** (0.048054)	-0.330204*** (0.056359)	-0.330204*** (0.056359)	-0.340081*** (0.059040)
Embi+	-0.000225*** (8.17E-05)	-0.000226*** (8.23E-05)	-0.000196** (8.14E-05)	-0.000196** (8.14E-05)	-0.000223*** (8.19E-05)
S	0.094739*** (0.017846)	0.091282*** (0.026777)	0.083513*** (0.018371)	0.083513*** (0.018371)	0.091210*** (0.018832)
D ²					-3.30E-05 (5.56E-05)
<i>dconsolidation</i>		-0.019213 (0.110776)			
<i>d3060</i>			-0.132068*** (0.045517)		
<i>d6090</i>				0.132068*** (0.045517)	
<i>dexchangerate</i>	-0.318553** (0.126371)	-0.317802** (0.126687)	-0.310161** (0.124783)	-0.310161** (0.124783)	-0.331970** (0.128535)
<i>dlula</i>	0.349224*** (0.111081)	0.348969*** (0.111303)	0.341118*** (0.110715)	0.341118*** (0.110715)	0.359455*** (0.112550)
<i>dcrisis</i>	-0.124405* (0.069517)	-0.129998* (0.076753)	-0.183423** (0.078511)	-0.183423** (0.078511)	-0.150904* (0.082689)
R-squared	0.559782	0.559834	0.574379	0.574379	0.560394

Source: Authors Elaboration.

Note: Models are estimated by OLS*, ** and *** denotes significance at 10%, 5% and 1% levels; std. error in between (), # represents models estimated with constant.

TABLE IV

JOHANSEN COINTEGRATION TESTE WITHOUT STRUCTURAL BREAK

Rank	λ^{trace}	P-value	$\lambda^{Eigenvalue}$	P-value
r = 0	250.11***	0.00	80.82***	0.00
r ≤ 1	169.29***	0.00	56.71***	0.00
r ≤ 2	112.58***	0.00	51.26***	0.00
r ≤ 3	61.316***	0.00	36.69***	0.00
r ≤ 4	24.62	0.18	14.38	0.33
r ≤ 5	10.24	0.26	10.18	0.20
r ≤ 6	0.05	0.82	0.05	0.82

Source: Authors Elaboration.

Note: *** denotes the rejection of the null hypothesis at 1% level of significance.

TABLE V

JOHANSEN COINTEGRATION TESTE WITH STRUCTURAL BREAK

Rank	LR	P-Value	90%	95%	99%
r = 0	325.12***	0.0000	155.31	160.86	171.61
r ≤ 1	218.79***	0.0000	121.52	126.47	136.11
r ≤ 2	135.43***	0.0000	91.65	96.00	104.53
r ≤ 3	86.33***	0.0009	65.79	69.55	76.97
r ≤ 4	55.08***	0.0064	43.98	47.15	53.48
r ≤ 5	24.68	0.1426	26.11	28.68	33.93
r ≤ 6	6.35	0.5306	12.24	14.25	18.57

Source: Authors Elaboration.

Note: *** denotes the rejection of the null hypothesis at 1% level of significance.

Breaks date used: January 1999 and December 2002.

TABLE VI

SIGNIFICANCE OF COINTEGRATION EQUATION

Dependent Varialbe	Chi-Square	P-Value
GDP growth rate	23.14593***	0.000317
Debt ratio	56.99717***	0
Interest Rate	69.72704***	0
Inflation Rate	55.38739***	0
Exchange Rate	16.03341***	0.006749
Embi+	23.499***	0.000271
Primary Surplus	26.70278***	0.000065

Source: Authors Elaboration.

Note: *** denotes significance at 1% level

TABLE VII
VEC RESIDUAL SERIAL CORRELATION LM TESTS

Lag	F-statistic	P-Value
1	1.882022	0.0003
2	2.000301	0.0001
3	1.738961	0.0015
4	1.377469	0.046
5	1.807478	0.0007
6	1.813465	0.0007
7	1.334598	0.0646
8	1.245115	0.1239
9	0.942524	0.5873
10	0.636017	0.9761

Source: Authors Elaboration.

Note: Null hypothesis: no serial correlation at lag h.

TABLE VIII
GRANGER CAUSALITY TEST

Independent Variable	Dependent Variable							Direction of causality
	Y	D	I	R	E	Embi+	S	
Y	-	18.97380*** (0.0042)	14.53546** (0.0242)	10.96128* (0.0896)	9.820979 (0.1324)	16.47853** (0.0114)	3.821981 (0.7008)	Y → D; Y → I; Y → R; Y → Embi+
D	11.92609* (0.0636)	-	26.46606*** (0.0002)	21.87011*** (0.0013)	11.40607* (0.0766)	13.32261** (0.0382)	5.725819 (0.4546)	D → Y; D → I; D → R D → E; D → Embi+
I	6.819487 (0.3379)	27.62727*** (0.0001)	-	27.06110*** (0.0001)	8.311279 (0.2162)	23.70176*** (0.0006)	6.525338 (0.3670)	I → D; I → R; I → Embi+
R	6.532053 (0.3663)	30.84374*** (0.0000)	16.02811** (0.0136)	-	19.41075*** (0.0035)	14.47299** (0.0248)	6.533845 (0.3661)	R → D; R → I; R → E; R → Embi+
E	12.36035* (0.0544)	9.346753 (0.1550)	33.94641*** (0.0000)	10.94679* (0.0900)	-	12.50652* (0.0516)	8.922092 (0.1780)	E → Y; E → I; E → R; E → Embi+
Embi+	2.133336 (0.9070)	41.60621*** (0.0000)	43.68729*** (0.0000)	25.21548*** (0.0003)	15.43095** (0.0172)	-	3.891478 0.6914	Embi+ → D; Embi+ → I; Embi+ → R; Embi+ → E
S	15.53278** (0.0165)	3.176217 (0.7864)	6.516320 (0.3679)	7.688136 (0.2619)	2.334217 (0.8865)	4.296242 (0.6367)	-	S → Y

Source: Authors Elaboration.

Note: *, ** and *** denotes significance at 10%, 5% and 1% levels; variables between () are the p-values; all the other values are the Chi-square of the Granger Causality.

TABLE IX

VARIANCE DECOMPOSITION OF GDP GROWTH RATE

Variance Decomposition of GDP growth rate:								
Period	S.E.	GDP growth rate	Debt ratio	Interest Rate	Inflation Rate	Exchange Rate	Embi+	Primary Surplus
1	0.183151	100	0	0	0	0	0	0
2	0.209598	96.00433	0.023999	1.064935	0.098707	0.01221	0.001204	2.794612
3	0.222245	94.32781	0.111989	1.051877	0.202231	0.63681	0.272632	3.39665
4	0.239047	87.45872	0.426099	0.92354	0.188218	6.492936	0.317128	4.193356
5	0.247459	82.31036	0.410572	1.093467	0.17614	11.71551	0.29907	3.994875
6	0.253935	79.74094	0.591579	1.328337	0.480751	13.35328	0.530367	3.974745
7	0.269995	72.74237	2.157544	1.175015	2.05934	16.42001	0.78228	4.663444
8	0.279718	67.90714	3.08716	1.158759	2.448218	19.97127	0.826217	4.60124
9	0.286298	65.10216	3.333176	1.227076	2.368261	22.62983	0.802372	4.537124
10	0.292677	62.43653	3.376321	1.206154	2.295998	25.04519	0.77863	4.861186

Source: Authors Elaboration.

Note: Factorization by Cholesky Decomposition.

TABLE X

VARIANCE DECOMPOSITION OF DEBT RATIO

Variance Decomposition of debt ratio:								
Period	S.E.	GDP growth rate	Debt ratio	Interest Rate	Inflation Rate	Exchange Rate	Embi+	Primary Surplus
1	0.89585	0.767807	99.23219	0	0	0	0	0
2	1.153488	0.696747	88.87458	0.399082	1.941236	0.024476	7.448036	0.615844
3	1.395012	3.187934	83.56808	0.622883	2.398688	0.38676	8.258937	1.576718
4	1.602214	3.736209	76.55858	1.738067	3.248148	0.658492	12.06869	1.991818
5	1.771675	4.983667	67.26677	2.751705	7.880829	0.704996	14.23904	2.172995
6	1.975189	4.916541	58.76718	2.786985	12.69285	0.567856	17.41046	2.858131
7	2.103162	6.123217	55.67954	3.101031	14.20159	0.664024	16.73237	3.498234
8	2.248552	7.974579	53.11785	2.953769	14.97471	0.977567	15.47039	4.531138
9	2.401642	9.818936	52.86607	2.695913	14.03063	1.374447	13.99266	5.221348
10	2.550495	11.17809	52.80089	2.419296	12.97311	1.823604	12.87357	5.931436

Source: Authors Elaboration.

Note: Factorization by Cholesky Decomposition.

TABLE XI

ENGLE-GRANGER COINTEGRATION TEST

Dependent Variable	Independent Variable	AIC	Lags	SIC	Lags
y	d	-2.236107	15	-4.322247***	1
d	y	-1.562334	4	-2.010215	0
y	i	-2.101870	15	-4.096740***	1
i	y	-2.796474	15	-2.194599	12
y	r	-2.099894	15	-4.143831***	1
r	y	-4.007762***	8	-7.566273***	0
y	e	-2.350242	15	-5.110468***	1
e	y	-0.637093	15	-3.402884*	0
y	Embi+	-2.046844	15	-4.146842***	1
Embi+	y	-1.947773	7	-2.695038	1
y	s	-3.158550*	15	-4.938623***	1
s	y	-1.466781	15	-1.466781	15
d	i	-0.307425	14	-1.290000	1
i	d	-3.498478**	14	-2.235684	12
d	r	-0.122924***	14	-1.270687	2
r	d	-4.017607	8	-7.621434***	0
d	e	-2.924036	2	-4.500774***	2
e	d	-3.256299*	4	-4.535044***	2
d	Embi+	-2.924036	2	-2.946314	2
Embi+	d	-3.256299*	4	-2.472982	0
d	s	-1.369190	14	-3.487583	0
s	d	-2.070137	13	-3.032596**	0

Source: Authors Elaboration.

Note: *, ** and *** denotes significance at 10%, 5% and 1% levels.

TABLE XII
ENGLE-GRANGER CAUSALITY TEST

Null Hypothesis	Model	OBS	F-statistic	P-value	Causality	ADL
ΔD does not Granger causes ΔY	Short run	258	3.780155	0.0241	Yes	(4,2)
ΔY does not Granger causes ΔD	Short run	260	6.927241	0.009	Yes	(2,2)
ΔI does not Granger causes ΔY	Long run	250	15.86195	0.0000	Yes	(12,12)
ΔY does not Granger causes ΔI	Short run	250	12.58429	0	Yes	(12,12)
ΔR does not Granger causes ΔY	Long run	250	5.883427	0.0032	Yes	(12,6)
ΔY does not Granger causes ΔR	Long run	254	14.71783	0	Yes	(4,2)
ΔE does not Granger causes ΔY	Long run	250	15.90652	0.0000	Yes	(12,7)
ΔY does not Granger causes ΔE	No relationship	-	-	-	No	-
$\Delta Embi+$ does not Granger causes ΔY	Long run	250	5.195388	0.0005	Yes	(12,11)
ΔY does not Granger causes $\Delta Embi+$	Short run	256	2.857744	0.0593	Yes	(6,6)
ΔS does not Granger causes ΔY	Long run	250	17.01869	0.0000	Yes	(12,6)
ΔY does not Granger causes ΔS	No relationship	-	-	-	No	-
ΔI does not Granger causes ΔD	No relationship	-	-	-	No	-
ΔD does not Granger causes ΔI	Long run	250	7.955811	0	Yes	(12,10)
ΔR does not Granger causes ΔD	Short run	254	4.443769	0.0000	Yes	(12,7)
ΔD does not Granger causes ΔR	Long run	254	22.15795	0	Yes	(8,7)
ΔE does not Granger causes ΔD	Short run	260	12.82875	0.0004	Yes	(2,0)
ΔD does not Granger causes ΔE	Long run	261	15.00638	0	Yes	(1,1)
$\Delta Embi+$ does not Granger causes ΔD	Short run	253	12.14643	0.0000	Yes	(9,6)
ΔD does not Granger causes $\Delta Embi+$	Long run	254	3.863187	0.0022	Yes	(1,8)
ΔS does not Granger causes ΔD	Short run	261	3.441240	0.0647	Yes	(0,1)
ΔD does not Granger causes ΔS	No relationship	-	-	-	No	-

Source: Authors Elaboration.

Note: F-statistic is the result of the application of the Wald test for the joint coefficients of the dependent variable.

TABLE XIII
COMPARISON OF RESULTS OF GRANGER CAUSALITY: VEC AND ARDL

Causality	VEC	ARDL	Causality	VEC	ARDL
$D \rightarrow Y$	Negative ^L	Positive ^S	$Y \rightarrow D$	Negative ^L	Negative ^S
$I \rightarrow Y$		Negative ^L	$Y \rightarrow I$	Positive ^L	Positive ^S
$R \rightarrow Y$		Negative ^L	$Y \rightarrow R$	Negative ^L	Positive ^L
$E \rightarrow Y$	Negative ^L	Negative ^L	$Y \rightarrow E$		
$Embi+ \rightarrow Y$		Negative ^L	$Y \rightarrow Embi+$	Negative ^L	Positive ^S
$S \rightarrow Y$	Positive ^L	Negative ^L	$Y \rightarrow S$		
$I \rightarrow D$	Positive ^L		$D \rightarrow I$	Positive ^L	Negative ^L
$R \rightarrow D$	Negative ^L	Negative ^S	$D \rightarrow R$	Positive ^L	Positive ^L
$E \rightarrow D$		Positive ^S	$D \rightarrow E$	Positive ^L	Positive ^L
$Embi+ \rightarrow D$	Positive ^L	Positive ^S	$D \rightarrow Embi+$	Positive ^L	Positive ^L
$S \rightarrow D$		Negative ^S	$D \rightarrow S$		

Source: Authors Elaboration.

Note: S represents Granger causality only in the short run and L in the long run.