Lisbon School of Economics & Management
Universidade de Lisboa

# MASTER OF SCIENCE IN
FINANCE

# MASTER´S FINAL WORK
DISSERTATION

Is the Hierarchical Risk Parity (HRP) Portfolio the Optimal Choice for Passive Investors? A Comparative Analysis of 60/40, HRP, and All-Weather Portfolio

DIEGO NICOLA

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**SUPERVISION:**
LUÌS FILIPE ÁVILA DA SILVEIRA DOS SANTOS


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*Anything’s possible, you gotta dream like you never seen obstacles.*

*J.Cole - "The Autograph"*

# GLOSSARY

- HRP – Hierarchical Risk Parity
- AIC – Akaike Information Criterion
- BIC – Bayesian Information Criterion
- CAPM – Capital Asset Pricing Model
- ARCH – Autoregressive Conditional Heteroskedasticity
- GARCH – Generalized Autoregressive Conditional Heteroskedasticity
- TGARCH – Threshold GARCH
- EGARCH – Exponential GARCH
- APARCH – Asymmetric Power ARCH
- VaR – Value at Risk
- CVaR – Credit Value at Risk

ABSTRACT

In a world shaped by economic instability, geopolitical conflicts, and volatile interest rates, many investors are gravitating towards more passive investment strategies in search of stability. This thesis explores a compelling question: Can innovative portfolio methods such as the Hierarchical Risk Parity (HRP) beat conventional models in this time of unpredictability while providing a secure refuge for passive investors?

Through a detailed analysis of three distinct portfolios, covering the period from January 1, 2014, to January 1, 2024 — a decade marked by both economic crisis and recoveries — this research investigates whether the HRP model, with its automated, risk-parity-driven structure, provides a superior solution compared to more conventional approaches like the All-Weather portfolio and the 60/40 one. Advanced libraries such as arch, pandas, numpy, scikit-learn and Yfinance were used along with Python to achieve a meticulous and meticulous numerical analysis.

Key performance metrics including returns and maximum drawdown were used to evaluate the portfolios at first. Further, more complex statistical analyses were performed, including VaR and CVaR testing, stress testing, quantile regression, the Fama-French five-factor model, CAPM, and ARCH-GARCH models. The findings revealed that the HRP portfolio not only exceeded expectations but also demonstrated superior resilience and risk management, outperforming the other two portfolios across various market conditions.

These results lead to a fascinating insight: Innovating portfolio strategies with an emphasis on risk parity might present a new path for passive investors seeking better outcomes.

JEL Codes: G01;G11;G15;G17;C15;C21;C22;C38;C61

KEYWORDS: Portfolio Analysis, HRP, 60/40, All-Weather, Arch & Garch

RESUMO

Em um mundo moldado pela instabilidade econômica, conflitos geopolíticos e taxas de juros voláteis, muitos investidores estão se voltando para estratégias de investimento mais passivas em busca de estabilidade. Esta tese explora uma questão instigante: Métodos de portfólio inovadores, como o Hierarchical Risk Parity (HRP), podem superar os modelos convencionais neste momento de imprevisibilidade, oferecendo um refúgio seguro para investidores passivos?

Através de uma análise detalhada de três portfólios distintos, cobrindo o período de 1º de janeiro de 2014 a 1º de janeiro de 2024 — uma década marcada tanto por crises econômicas quanto por recuperações — esta pesquisa investiga se o modelo HRP, com sua estrutura automatizada e orientada pela paridade de risco, oferece uma solução superior em comparação com abordagens mais convencionais. Bibliotecas avançadas como arch, pandas, numpy, scikit-learn e yfinance foram utilizadas juntamente com Python para alcançar uma análise meticulosa e numérica.

Métricas-chave de desempenho, incluindo retornos e drawdown máximo, foram inicialmente usadas para avaliar os portfólios. Além disso, análises estatísticas mais complexas foram realizadas, incluindo testes de VaR e CVaR, testes de estresse, regressão quantílica, o modelo de cinco fatores de Fama-French, CAPM e modelos Arch-Garch. Os resultados revelaram que o portfólio HRP não apenas superou as expectativas, mas também demonstrou uma resiliência e gestão de risco superiores, superando os outros dois portfólios em diversas condições de mercado.

Esses resultados levam a um insight fascinante: Inovar nas estratégias de portfólio, com ênfase na paridade de risco, pode representar um novo caminho para investidores passivos que buscam melhores resultados.

PALAVRAS-CHAVE: Portfolio Analysis, HRP, 60/40, All-Weather, Arch & Garch

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# Acknowledgments

*To my mother Luisella and my grandmother Lucia, who have always been and will always be a source of inspiration for me, continuously believing in me. To my entire family, who have stood by me, even in the most challenging moments, and who have always taught me the true value of life and how to face every situation with a smile.*

*To all my friends in Italy, who have been part of my life since the beginning, and to those I met during these university years, with whom I've shared both joyful and difficult moments—without them, this journey would not have been the same.*

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*All of this will always be a part of me, and I will forever be grateful for it.*

# 1. INTRODUCTION

In the world of passive investing, finding techniques that offer the best possible balance between risk and return has grown in significance. The 60/40 portfolio, which consists of 60% equities and 40% bonds, has been the standard for decades among investors who want to balance risk mitigation with capital gain. However, the ongoing viability of this conventional strategy has come under scrutiny due to extraordinary shifts in financial markets, such as the aftermath of the global financial crisis and the COVID-19 epidemic. López de Prado's (2016) Hierarchical Risk Parity (HRP) portfolio and Ray Dalio's All-Weather portfolio are two of the most noteworthy innovations, offering alternative strategies designed to address the limitations of traditional approaches and enhance diversification and risk management.

This thesis investigates three distinct portfolio strategies 60/40, HRP, and All-Weather and evaluates their performance over the last decade, from 2014 to 2024. Based on a passive investing framework, the research aims to determine which of these approaches provides the optimum balance between risk and return. This study was motivated by the identification of a substantial gap in the literature: although individual studies have looked at these portfolios in isolation, a comprehensive and comparative examination utilizing sophisticated econometric models and stress-testing techniques is still lacking. This paper uses a comprehensive method that combines contemporary statistical tools with classic financial measures in an effort to close that gap.

The study moves in a systematic yet interrelated manner. It starts with an overview of the fundamental theories and body of research, delving into the development of passive investing over time and the theoretical frameworks supporting each portfolio strategy. This lays the groundwork for a thorough explanation of the methodology, which involves applying the ARCH-GARCH, Fama-French Five-Factor, and Capital Asset Pricing Model (CAPM) models to assess how sensitive these portfolios are to different economic conditions. A key component of the investigation is the construction and analysis of the portfolios using Python-based simulations. The creation of the investment portfolios was inspired by the work of Luca Donghi (2022), whose ideas provided a foundation for their structure. This research, however, extends his approach by incorporating advanced statistical analyses to gain a deeper understanding of how these portfolios behave in various market conditions.

The findings section is at the heart of this thesis, where the portfolios are put through rigorous testing like as stress testing, volatility analysis, and quantile regression. These evaluations offer insightful information on how each portfolio performs both in times of market stability and in more extreme circumstances, such inflationary surges or recessions. The results show whether cutting-edge strategies like HRP, which rely on sophisticated clustering techniques and machine learning algorithms, can beat more conventional models like the All-Weather and 60/40 portfolios in terms of robustness and returns.

This thesis provides an analytical and useful narrative by combining historical study, economic modeling, and real-world applications. In addition to addressing the main research question, "Is the Hierarchical Risk Parity portfolio the best option for passive investors?" it aims to thoroughly analyze which investment strategy among the three portfolios offers the most effective balance between risk and return for passive investors in today's volatile financial landscape. The ultimate goal of this work is to make a significant contribution to the current discussion on portfolio optimization by providing a thorough analysis of risk-return strategies in an increasingly unstable investing environment. The structure of this thesis is as follows: Section 2 provides a literature review, outlining the fundamental principles of portfolio management. Section 3 describes the models used in this analysis and their relevance to passive investing. Section 4 explains the methodology and data employed, while Section 5 presents the results. Finally, Section 6 concludes the analysis.

## 2. Literature Review

### *2.1. Markowitz Theory*

In 1952, Harry Markowitz changed the world of finance with his Portfolio Theory introducing the concept of risk into the calculation of expected returns. This innovation allowed investors to adjust portfolios to create optimal risk and return trade-offs. At its heart is that so called efficient frontier, which illustrates the portfolios that offer the greatest return for a given level of risk or the smallest risk for a given return (Elton & Gruber, 1997). Markowitz started from the assumption that investors act rationally, with full access to all market information, and do not have to worry about fees or transaction costs (Sharpe, 1964). In addition, the theory assumes that all investors operate on a single time horizon (Tobin, 1958). These assumptions may seem like oversimplifications, but it was these assumptions on which Markowitz theory founded modern portfolio management, incorporating a basic principle of diversification.

A portfolio's efficiency is determined by minimizing the variance of its returns, represented by:

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{J=1}^{n} w_i w_j \sigma_{ij} \tag{1}$$

Here, $w_j$ and $w_i$ represent the weights of assets $i$ and $j$ in the portfolio, respectively, and $\sigma_{ij}$ is the covariance between the returns of assets $i$ and $j$. The portfolio variance $\sigma_p^2$ is calculated by summing over all pairs of assets, incorporating both the individual asset variances and covariances.

*2.2 Mathematical foundations of MPT*

In Markowitz's Portfolio Theory, the first step is to calculate the expected return of a portfolio, denoted as $\mu_p$, which represents the weighted average of the expected returns of its individual securities. For a portfolio with multiple assets, this is expressed as:

$$\mu_p = \sum_{i=1} w_i \mu_i \tag{2}$$

where $w_i$ is the proportion of the $i$-th security in the portfolio, and $\mu_i$ is its expected return.

After calculating the expected return, the risk must be quantified, which can be done by measuring variance and standard deviation. This variance measures the dispersion of returns from their expected mean, i.e., potential risk. For a single asset, variance is calculated as:

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(R_i - \bar{R})^2 \tag{3}$$

where $R_i$ is the return in the $i$-th period, and $\bar{R}$ is the average return across $n$ periods.

Another measure of risk is the standard deviation, which is the square root of the variance. The higher the variance, the higher the risk associated with the asset. The formula is:

$$\sigma_p = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov(r_i, r_j)} \tag{4}$$

Where $\sigma_p$represents the portfolio's standard deviation (total risk), $w_i$and $w_j$are the weights of assets $i$ and $j$ in the portfolio, and $Cov(r_i, r_j)$is the covariance between the returns of assets $i$ and $j$.

This metric is incredibly useful because it makes it easy to compare the risk of different investments since it quantifies volatility in the same units as returns. (Gruber, 2003)

Strategic allocation of capital across different assets relies on risk measures such as variance and standard deviation by the investors. This enables them to understand the potential price fluctuations of each asset and, in turn, build portfolios that match their desired risk and reward profile in accordance with their investment goals and risk tolerance (Sharpe, 1966). A primary foundation for modern portfolio management are these risk metrics that help investors to optimize their decision making. A second measure of importance is covariance, which is vital for portfolio diversification to measure the extent to which returns move in opposite directions for two assets (Elton & Gruber, 1997). The covariance analysis allows investors to form portfolios that give a maximum expected return at a minimum amount of risk by constructing portfolios consisting of assets that have low correlations with each other (Fama & French, 1992). Covariance is a key factor in strategies like the 60/40 portfolio, Hierarchical Risk Parity, and Ray Dalio’s All-Weather model, which all seek to optimize risk and return through diversification, whether by balancing equities and bonds, clustering assets based on correlations, or diversifying across asset classes to perform in various economic environments. Investors can reduce risk during volatile periods by relying on the fact that stocks and bonds have low covariance

(Asness, 2000) in the traditional 60/40 portfolio. Lopez de Prado (2016) has developed the HRP strategy that creates clusters of assets on variance to maximize diversification to minimize risk. Likewise, Dalio's All-Weather model uses covariance to balance asset classes in such a way that enables them to perform relatively consistently across varying economic cycles and resist big draws from market instability (Dalio, 2017).

$$Cov_{ij} = E[R_i R_j] - E[R_i]E[R_j] \tag{5}$$

where $E[R_i R_j]$ is the expected value of cross product of the returns between assets $i$ and $j$ and $E[R_i]E[R_j]$ are the expected returns of assets $i$ and $j$.

Correlation is another critical measure of diversification and quantifies the strength and direction of the linear relationship between two assets. The correlation coefficient is given by:

$$\rho_{i,j} = \frac{Cov_{ij}}{\sigma_i \sigma_j} \tag{6}$$

where $\sigma_i$ and $\sigma_j$ are the standard deviations of assets $i$ and $j$, respectively correlation values range from -1, indicating perfect negative relations, to +1, indicating a perfect positive relation. Managing correlation is crucial in portfolio construction, as it helps reduce overall risk, especially in diversified strategies like HRP and Ray Dalio's All-Weather portfolio (Dalio, 2017).

### *2.3 Passive investing: beyond returns, an advanced analysis of volatility and efficiency*

In portfolio management, predicting and managing volatility is crucial for optimizing asset allocation and mitigating risk. Accurate models for forecasting volatility are critical, given the fundamental role volatility plays in determining portfolio risk. The full importance of this becomes evident with the ability of the ARCH (Autoregressive Conditional Heteroskedasticity) model, introduced by Engle (1982), to capture volatility clustering and provide more accurate risk estimates over time. Similarly, the Generalized ARCH (GARCH) model introduced by Bollerslev (1986) allows for both past residuals and past volatility to inform future volatility predictions, significantly improving the forecasting accuracy for both active and passive investors.

The time-varying nature of volatility, governed through these models, allows us to depict and predict periods of high and low market fluctuations. These models help portfolio managers anticipate periods of increased risk, offering better decision-making frameworks for portfolio rebalancing during volatile times (Poon & Granger, 2003).

The ARCH model was developed to account for the fact that volatility is often not constant over time, but instead exhibits clustering, periods of high volatility are likely to be followed by further high volatility, and similarly for low volatility (Engle, 1982). Engle's (1982) work demonstrated that incorporating conditional heteroskedasticity significantly improves the accuracy of risk estimates in portfolio management, particularly in active trading strategies, where frequent short-term volatility adjustments are most relevant.

The ARCH model can be expressed mathematically as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \tag{7}$$

In this equation, $\sigma_t^2$ represents the conditional variance at time $t$, which is the key quantity we are trying to predict. The parameter $\alpha_0$ is a constant that represents the long-term average level of volatility (often referred to as the unconditional variance). The terms $\alpha_i$ are coefficients that measure the influence of past squared residuals (errors) from previous periods on the current variance. These residuals, represented by $\varepsilon_{t-i}^2$, are squared values of the deviations from the expected return at time $t-i$, capturing how shocks to the system propagate through time.

The GARCH model builds on the ARCH model by incorporating both past squared residuals and past conditional variance into the forecast of future volatility:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{8}$$

In this formulation, $\sigma_t^2$ is the conditional variance at time $t$, like the ARCH model. The parameter $\alpha_0$ is again a constant representing the long-term average variance. The coefficient $\alpha_i$ captures the impact of past squared residuals (or shocks) on the current period's variance, with larger shocks increasing the variance for future periods. The term $\beta_j$ represents the persistence of volatility over time by measuring the contribution of the past conditional variance $\sigma_{t-j}^2$ to the current period's variance. The GARCH model thus extends ARCH by acknowledging that not only do past shocks affect current volatility, but past volatility itself also plays a significant role, capturing the tendency for volatility to persist over time (Bollerslev, 1986).

This innovation allowed GARCH models to better capture the persistence of volatility, a feature essential for portfolio managers aiming to adjust their strategies based on expected market fluctuations (Bollerslev, 1986). For instance, during periods of high volatility, portfolio managers may shift their allocations to more stable or inversely correlated assets to reduce overall portfolio risk (Michaud & Michaud, 2008). This is especially useful for risk-parity strategies, such as the Hierarchical Risk Parity (HRP) approach (López de Prado, 2016), where asset allocation is done by balancing risk.

GARCH models are also valuable in the context of passive investing, where long-term market exposure requires careful consideration of market volatility over extended periods. According to Swensen (2009), passive investing reduces costs and avoids the pitfalls of active management but still necessitates volatility management. GARCH models help passive investors anticipate or mitigate risks during periods of high volatility, especially since index-tracking funds are more exposed to market swings. Volatility forecasts allow for informed portfolio rebalancing to keep the risk-return ratio aligned with optimal levels, even during challenging market conditions.

Further, empirical studies (Linton & Perron, 2003; Andersen et al., 2006) indicate that GARCH models significantly improve the forecasting of time-varying risk, which is crucial for risk-adjusted performance analysis and constructing portfolios capable of withstanding volatile market conditions. These models also enable portfolio managers to assess potential drawdowns during periods when tail risks are higher than usual.

Additionally, ARCH and GARCH models are extensively used for stress-testing passive portfolios, particularly those that replicate broad market indices, such as Exchange Traded Funds (ETFs). These models simulate various economic scenarios, offering an improved view of how passive portfolios might behave under extreme market conditions (McAleer, 2005). Since passive funds cannot rely on active trading to mitigate downturns, understanding and preparing for periods of high volatility is key to protecting long-term returns (Bali & Engle, 2010).

Another powerful technique complementing volatility forecasting is quantile regression (Koenker & Bassett, 1978), which provides a granular understanding of how portfolios performs accross different quantiles of the return distribution. The quantile regression model can be represented as:

$$Q_y(\tau/\mathrm{X}) = X\beta_\tau \tag{9}$$

Where $Q_y(\tau/\mathrm{X})$ is the conditional quantile of the dependent variable $y$, given the matrix of explanatory variables $X$. In this expression, $X$ is an $n\ x\ k$ matrix of explanatory variables, and $\beta_\tau$ is a $k\ x\ 1$ vector of coefficients corresponding to the quantile $\tau$. This method is especially valuable for passive investors seeking to understand how their portfolios might behave during extreme market conditions, such as severe downturns or extraordinary gains. By combining GARCH models with quantile regression, investors can gain a more comprehensive understanding of the distribution of risk, which is vital for both risk management and portfolio optimization (Bali et al., 2008).

In conclusion, the integration of ARCH and GARCH models into portfolio management has proven essential for forecasting volatility and managing risk, particularly in passive investment strategies. These models enable investors to adjust their portfolios in response to expected market variations, safeguarding long-term returns against short-term fluctuations. As financial markets continue to evolve, these advanced volatility models will remain a cornerstone of both active and passive portfolio management (Engle, 2001).

# 3. MODELS DESCRIPTION

## *3.1 CAPM and Fama and French five factor model*

Capital Asset Pricing Model (Sharpe, 1964) suggests that the expected return on a security or on a portfolio is a function of its exposure to market-wide, systematic risk, and not total risk which consists of both systematic and unsystematic components.

The formula is given by:

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f) \quad (10)$$

Where $E(R_i)$ is the expected return on the security, $R_f$ is the risk-free rate, $\beta_i$ is the asset's beta, and $E(R_m) - R_f$ represents the market risk premium.

The market beta (β) quantifies how much an asset's returns vary when compared with the market. The beta value can be defined as a measure of systematic risk inherent in the asset that is unable to diversify away and it is denoting the relative variability of a security's or the value of a portfolio of security, as compared to a benchmark, or the market as a whole, wherein a beta of 1 indicates that a 1% change in the market generates proportional change in the asset value.

This model, an extension of Markowitz's portfolio theory, simplifies decision making concerning betting by considering only systematic risk measured by beta. The assumptions include homogeneous expectations (investors expect the same returns and risks) and the ability to borrow or lend an unlimited amount at the risk-free rate. The CAPM is considered a foundational tool and although it has been extensively analysed and critiqued due to its simplistic assumptions and failure to relate to real market behaviour. Since then, the CAPM has been further developed, as with the Fama and French (1993) multi-factor models that have added additional risk factors to explain asset returns.

The Fama-French regression model used for this analysis is expressed as:

$$R_{p,t} - R_{f,t} = \alpha + \beta_1\left(Mkt_t - R_{f,t}\right) + \beta_2(SMB_t) + \beta_3(HML_t) + \beta_4(RMW_t) + \beta_5(CMA_t) + \varepsilon_t \quad (11)$$

Here, $\alpha$ represents the regression intercept, or the portfolio's alpha, while $\beta_1$ reflects the portfolio's sensitivity to the market risk premium, $\left(Mkt - R_{f,t}\right)$. The factor $SMB_t$ (Small Minus Big) represents the return spread between small-cap and large-cap stocks, capturing size risk. The $HML_t$ (High Minus Low) factor measures the return spread between value stocks (with high book-to-market ratios) and growth stocks (with low book-to-market ratios), thus capturing value risk. The $RMW_t$ (Robust Minus Weak) factor represents the return spread between firms with high profitability and those with low profitability, capturing profitability risk. Finally, the $CMA_t$ (Conservative Minus

Aggressive) factor captures the return spread between firms that follow conservative investment policies and those with aggressive investment patterns. The term $\varepsilon$ represents the error term in the regression model.

### *3.2 60/40 Strategy*

The 60/40 investment strategy comes out of Harry Markowitz's Modern Portfolio Theory (MPT) of what are known as Markowitz portfolios and allocates 60% of a portfolio to equities for growth and 40% to fixed income (bonds) as a hedge against volatility. This allocation aims to strike the right balance between risk and return, with the bond component keeping portfolio value steady during market slumps, especially when interest rates are cut by central banks to give the economy a much-needed boost (Robinson & Langley, 2017). The 60/40 approach soon gained popularity as a sensible application of MPT, especially during periods of economic expansion and low interest rates. This strategy became popular with investors during the late 20th century because bonds provided stability in equity market declines (Bogle, 1999).

However, market changes in recent times have brought into question whether the 60/40 allocation remains appropriate. With low bond yields remaining in effect post-pandemic, bond performance worsens on weaker equity days. Negative returns from 10-year U.S. Treasury bonds during market drops have led some to question the traditional performance of the 60/40 portfolio. Besides, the evolution of behavioural finance, which illustrates psychological biases in investment decisions, has further complicated this strategy compared to its original simplicity (Louraoui, 2020).

The diversification of the 60/40 portfolio is one of its main strengths. The strategy reduces risk by dividing investments into both stocks and bonds and aligns with newer portfolio theories that show how diversifying across different types of assets outperforms concentration in a single sector. This structure shields portfolios against systemic risks that cannot be mitigated through diversification. Equities typically perform poorly when bonds perform well, as bonds tend to do better during economic downturns (Asness, Frazzini, & Pedersen, 2012).

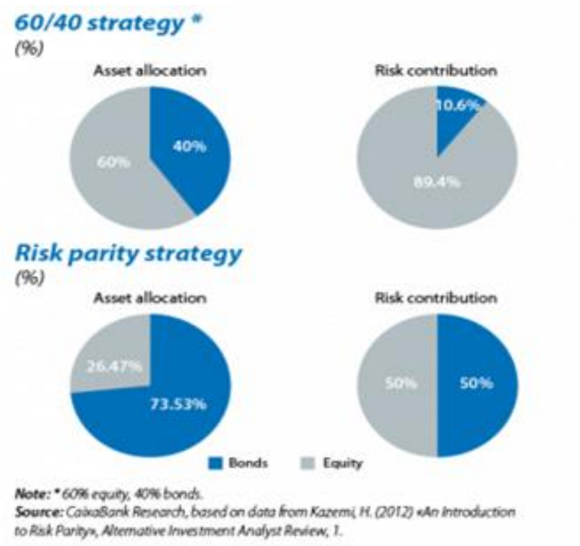
Historically, equities have outperformed, which has led to a preference for a 60/40 split over other allocations like 50/50 or 40/60. The greater potential returns make rational investors allocate more to equities, as these returns are considered sufficient compensation for the greater associated risk. When pricing stocks relative to safer assets like bonds, equities have generally provided superior long-term returns (Siegel, 2008). However, like equities, bonds are also subject to market volatility, though such periods are typically less severe and shorter-lived. Thus, a 60/40 allocation strikes a balance between higher returns and moderate risk, representing the core tenets of diversification and risk management that underpin Modern Portfolio Theory. (Liew & Ajakh, 2020)

### *3.3 Risk Parity Approach*

The Hierarchical Risk Parity (HRP) was introduced by López de Prado in 2016 (López de Prado, 2016). HRP utilizes asset correlations to maintain minimum risk in a portfolio by using concepts of minimum variance, risk diversification, and risk hierarchy. Unlike traditional methods, HRP aims at minimizing return variability in contrast to future return prediction, portfolio management in such conditions becomes more stable. By employing hierarchical clustering, assets are grouped based on their correlations, creating a more diversified portfolio and reducing overall risk. Risk parity is applied within clusters to allocate capital proportionally to each asset's risk contribution, minimizing volatility (López de Prado, 2018). HRP distributes risk more evenly than does the 60/40 strategy, thereby limiting concentration risk and mitigates estimation errors common to Modern Portfolio Theory (MPT) and adapts well to market changes. However, its reliance on historical data may limit adaptability to sudden market shifts, like geopolitical or climate crises, and its complexity requires continuous monitoring, which may be challenging for some investors. (Palit & Prybutok, 2023) Despite this, HRP offers better risk management, especially for tail risks, by balancing risk across assets and reducing volatility during market crises.

HRP provides an improvement over traditional risk parity techniques in risk management as machine learning technologies help HRP better represent the relationships between the assets and subsequently avoid the adverse effects of concentrating risk in a few assets, particularly during times of turbulence (López de Prado, 2018). For instance, HRP outperforms standard methods such as conventional variance in treating tail risks and more generally accounts for dispersed interdependencies between assets during market crises (Palit & Prybutok, 2023). Fourthly, HRP clustering leads to a more balanced risk contribution than typical strategies, which fail to consider such hierarchical relationships, thus magnifying portfolio vulnerability in periods of stress.

Figure I HRP vs 60/40

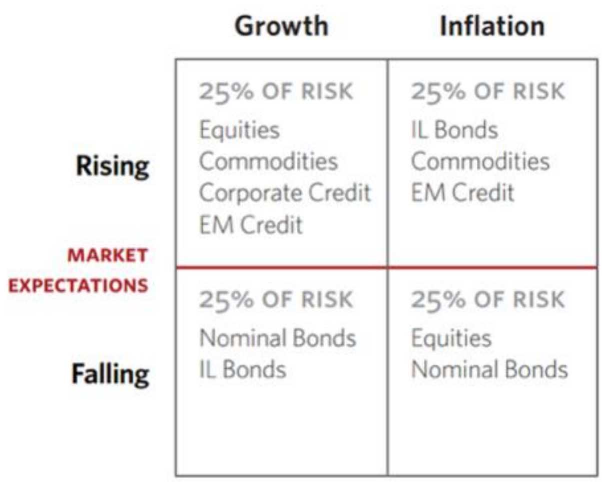



The image above illustrates asset allocation as well as risk contribution for the traditional 60/40 strategy versus a risk parity strategy. The 60/40 portfolio has 60% capital allocated to equities and 40% to bonds, but more than 89.4% of the risk comes from equities only (as depicted). The difference is that the risk parity strategy allocates the risk equally into both equities and bonds, although the actual asset allocation is 73.53% bonds to 26.47% equities. The employment of this balanced approach reduces the concentration risk (Kazemi 2012).

### *3.4 All-Weather Strategy*

In 1996, Ray Dalio developed an investment strategy called the All-Weather Portfolio, a strategy that seeks to produce relatively stable returns with lower risks under a variety of market scenarios. The portfolio is based on the idea that the economy follows four main cycles: growth or inflation values in the process of rising or falling. The idea behind this strategy is to share risk between many different asset classes to protect against economic changes by taking advantage of long-term and medium-term bonds, gold, equities and commodities.

Figure II All-Weather idea




One crucial aspect of the All-Weather Portfolio is that it can be treated as passive investment strategy, since little intervention is needed from the investor's side once the right asset allocation is determined (Dalio, 2011). In contrast to active investment strategies such as those that attempt to outperform the market by trading very frequently and trying to predict the future, All-Weather makes a strong effort to hold to a balanced portfolio that self corrects to different economic settings (Dalio, 2017). This passive strategy limits transaction costs and minimizes market timing risk for investors by concentrating on steady, long run investment (Anadu, Kruttli, McCabe, & Osambela, 2020). Keeping investors emotionally stable during market uncertainties is one key of this strategy. But this approach also diminishes the likelihood of impulsive decisions prompted by short term events and induces more disciplined investment behavior (Quantified Trading, 2024). However, the portfolio has some limitations in extreme scenarios where real interest rates tend to zero or negative, or inflation is high, as the assets may not perform well then. In the face of global financial crisis of 2007 or the COVID 19 pandemic of 2020, the portfolio has proved resilient, retaining stability and enabling the investors to sail through the market during the rough times.

*3.5 Arch-Garch models*

In portfolio management, predicting and managing volatility is crucial for optimizing asset allocation and mitigating risk. The ARCH model, introduced by Engle (1982), and the GARCH model, proposed by Bollerslev (1986), are widely used to address the issue of conditional heteroskedasticity, capturing time-varying volatility in financial markets (equations 7 and 8). These models allow portfolio managers to anticipate periods of heightened market risk and make informed decisions in asset allocation and risk management. Beyond the standard GARCH framework, more advanced models such as GJR-GARCH, EGARCH, and APARCH have been developed to account for market asymmetries and the leverage effect, where negative shocks have a larger impact on volatility than positive shocks. These models improve the accuracy of volatility forecasts, making them indispensable tools for managing portfolio risk during volatile market conditions (Glosten et al., 1993; Nelson, 1991; Ding et al., 1993). The GJR-GARCH model, introduced by Glosten, Jagannathan, and Runkle (1993), incorporates an asymmetry term that captures the leverage effect, offering a more accurate volatility estimate, especially during periods of negative market performance. The GJR-GARCH (p, q) model is formulated as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{q} \gamma_i \varepsilon_{t-i}^2 I_{\{\varepsilon_{t-i<0}\}} \tag{12}$$

In this model, $\gamma_i$ captures the asymmetric response of volatility to negative shocks, with the indicator function $I_{\{\varepsilon_{t-i<0}\}}$ taking the value of 1 when the shock is negative, reflecting the leverage effect.

The EGARCH model, introduced by Nelson (1991), models volatility asymmetry in logarithmic form, ensuring that volatility predictions remain non-negative without imposing non-negativity constraints on the model parameters. The EGARCH (p, q) model is expressed as:

$$\log(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{p} \beta_j \log(\sigma_{t-j}^2) + \sum_{i=1}^{q} \gamma_i \left( \frac{|\varepsilon_{t-1}|}{\sigma_{t-i}} - E\frac{|\varepsilon_{t-1}|}{\sigma_{t-i}} \right) \tag{13}$$

This model captures asymmetry through $\gamma_i$, which represents the differential impact of negative and positive shocks on future volatility.

The APARCH model, developed by Ding, Granger, and Engle (1993), generalizes GARCH by introducing a power parameter $\delta$ that governs the form of the volatility process, adjusting for the magnitude and direction of market movements. The APARCH (p, q) model is formulated as:

$$\sigma_t^\delta = \alpha_o + \sum_{i=1}^{q} \alpha_i ( |\varepsilon_{t-1}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^{p} \beta_j \log \sigma_{t-j}^\delta \tag{14}$$

Here, $\delta$ adjusts the power of the conditional standard deviation, while $\gamma_i$captures the asymmetric effect of shocks.

The use of these advanced GARCH models enhances the ability of portfolio managers to capture and forecast market volatility, improving decision-making in risk management (Abdul Aziz, Vrontos, & Hasim, 2018). Studies have shown that asymmetric GARCH models, such as GJR-GARCH and EGARCH, provide more accurate volatility estimates compare to the standard models such as GARCH, proving to be essential tools for financial analysts in managing portfolio risk (Ugurlu et al., 2014; Predescu & Stancu, 2011). Another study on Turkish financial markets (Ugurlu et al., 2014) demonstrated that EGARCH and GJR-GARCH provided better volatility forecasts during periods of financial instability, offering valuable tools for risk mitigation.

In summary, ARCH, GARCH, and their advanced variants, such as GJR-GARCH, EGARCH, and APARCH, play a pivotal role in modern portfolio management by offering enhanced tools for volatility forecasting. These models not only provide insights into current market conditions but also help portfolio managers mitigate risk in the face of uncertain and volatile market environments.

# 4. DATA AND METHODOLOGY

The All-Weather Portfolio is famous for its balanced allocation and is expected to perform well during economy dry periods or storms. Especially in the light of recent market volatility, this study assesses its behaviour in various economic environments. Inspired by an article on Medium by Luca Donghi (2022), the idea and methodology to compare the All-Weather Portfolio with the Hierarchical Risk Parity (HRP) portfolio and the 60/40 model were based on that article. Nevertheless, this analysis adjusts the asset allocations and extends the study by including other performance and risk metrics.

In the time horizon from January 1st, 2014 to January 1st, 2024, relevant Exchange-Traded Funds (ETFs) were available, and key events like inflation, interest rate increases and COVID, happened during this time which makes this time horizon especially interesting and relevant. Such a timeframe forms a sound basis for assessing portfolio performance in different economic environment.

The All-Weather Portfolio includes an allocation of 30% to stocks, 5% to corporate bonds, 5% to inflation-linked bonds, 30% to long-term government bonds, 15% to short-term bonds, and 15% to gold. ETFs were chosen to represent each asset class within the portfolio, ensuring sufficient historical data for modelling purposes. The ETFs selected for the portfolio were IEF, representing U.S. Treasury bonds with maturities between 7 and 10 years; GLD, tracking the price of gold; DBC, offering exposure to a broad range of commodities; TLT, providing exposure to long-term U.S. Treasury bonds with maturities of 20 years or more; VTI, tracking the performance of the overall U.S. stock market; TIP, offering exposure to U.S. Treasury inflation-protected securities (TIPS); and LQD, representing investment-grade corporate bonds.

One of the key features of the portfolio design is the low correlation between the ETFs, which enhances diversification and reduces overall portfolio risk, particularly valuable for a young, passive investor. A risk-free rate of 2.93%, represented by the yield on 10-year U.S. Treasury bonds, was used for portfolio evaluation. This rate aligns with the typical long-term investment horizon and is critical for calculating risk-adjusted return metrics such as the Sharpe Ratio, which measures portfolio performance relative to a risk-free asset.

All data processing and analysis for the three portfolios were conducted using Python, employing various libraries such as Yahoo Finance for data retrieval and packages like arch, matplotlib, numpy, and pandas to ensure accurate data handling, modelling, and visualization. Historical market data were downloaded directly using Python code, which streamlined the evaluation process and enabled a comprehensive comparison of the portfolios.

The number of daily observations obtained through the code is more than 2,500, as shown in the image below:

Figure III Dataset Statistics

| Ticker | DBC | GLD | IEF | LQD | TIP | TLT | VTI |
|---|---|---|---|---|---|---|---|
| count | 2516.000000 | 2516.000000 | 2516.000000 | 2516.000000 | 2516.000000 | 2516.000000 | 2516.000000 |
| mean | 17.201041 | 140.692468 | 95.374992 | 99.175008 | 97.008697 | 109.574691 | 141.694055 |
| std | 4.355425 | 26.262385 | 7.998556 | 11.488801 | 8.736963 | 17.468522 | 48.070050 |
| min | 9.948129 | 100.500000 | 80.740623 | 80.113274 | 84.572998 | 78.212257 | 75.444008 |
| 25% | 13.939955 | 118.930000 | 89.839977 | 90.767349 | 89.921894 | 98.310001 | 94.052483 |
| 50% | 15.580241 | 126.670002 | 92.918613 | 96.133373 | 92.349659 | 104.656883 | 130.798073 |
| 75% | 21.686802 | 167.775002 | 101.382942 | 107.847708 | 104.096407 | 122.738985 | 191.237244 |
| max | 28.925365 | 193.889999 | 113.309097 | 121.998787 | 116.286835 | 154.570251 | 236.576630 |

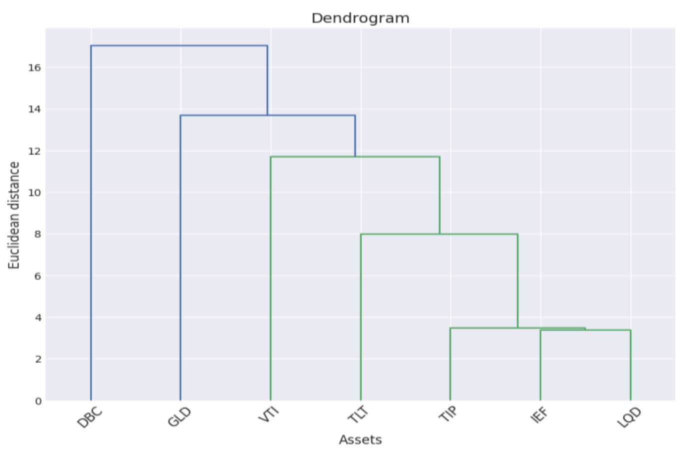
Daily returns for each asset were downloaded and cumulative returns were obtained, before constructing the investment portfolios. Then, the data was split into a training set and a test set, split at 40% and 60%. This split may seem strange initially, given that the test set usually makes up 20-30%, however this split was chosen to evaluate the model in operation over a longer test period. As discussed by James et al. (2013), a larger test set can help to reduce overfitting and improve the model's ability to generalize to new data, particularly in financial time series where diverse market conditions are common. This method will ensure that the model is performing in short run as well as robust and reliable in the long run.

The first portfolio analyzed is the 60/40 model, a traditional strategy that allocates 60% to stocks and 40% to bonds. For the equity portion, the VTI ETF was selected, representing a broad range of U.S. stocks across all market capitalizations. The bond component was represented by TLT, which focuses on long-term U.S. government bonds with maturities of 20 years or more. This allocation aligns with the use of the U.S. Treasury bond risk-free rate, ensuring consistency with the portfolio's economic environment.

The second portfolio, the All-Weather portfolio, follows risk parity and variance optimization principles. Inspired by Luca Donghi's study, this portfolio seeks to balance risk across different economic cycles using mean-variance optimization, ensuring each asset contributes proportionally to total risk. The portfolio is structured around four economic cycles: Rising Growth, Falling Growth, Rising Inflation, and Falling Inflation. Relevant assets are allocated to each cycle—commodities (DBC), U.S. stocks (VTI), gold (GLD), corporate bonds (LQD), and government bonds (TLT, IEF, TIP) to optimize performance under specific conditions. Using the riskfolio-lib library, expected return statistics and covariance matrices were calculated to estimate risk and return distribution, and equal weight (25%) was assigned to each economic scenario, following Bridgewater Associates' balanced approach.

The final portfolio, HRP, was constructed using hierarchical clustering, enriched by world GDP and inflation indicators. The annual correlations between assets and economic variables were calculated to identify diversification opportunities and risk reduction. Using agglomerative hierarchical clustering with Euclidean distance and Ward's linkage, assets were grouped into clusters based on their correlations. The process, starting from individual asset clusters and progressively merging the closest ones, ensures optimal risk diversification by minimizing variance within each cluster.

Figure IV HRP three



After combining the time series of returns of each asset within each cluster and calculating their standard deviation, the volatility of the clusters has been determined. In this step, asset weights were assigned inversely proportional to their volatility within each cluster, thus ensuring a balanced distribution of capital across clusters.

The process is recursive, iterating through top-level clusters, recalculating volatility and reassigning weights to balance risk and reduce dependency on potentially unstable covariance matrices.

### *4.1 Performance Metrics*

For all the investment portfolios daily returns were computed using adjusted closing prices daily. The daily returns for each portfolio were calculated as the weighted average of the individual assets returns $r_{p,t}$, where each return was weighted by the proportion of the total investment in the portfolio.

$$r_{p,t} = \sum_{i=1}^{n} w_i r_{i,t} \tag{15}$$

In this context, $w_i$ stands for the weight of the i-th asset in the portfolio, while $r_{i,t}$ corresponds to the daily return of the i-th asset at time $t$.

The annualized return, also known as the Compound Annual Growth Rate (CAGR), was calculated to standardize the portfolio returns over a 10-year time frame. The CAGR provides a geometric average return per year, offering a smooth rate of growth that accounts for compounding over the period.

$$CAGR = \left(\frac{V_T}{V_0}\right)^{\frac{252}{T}} - 1 \tag{16}$$

In terms of portfolio value, $V_T$ denotes the value at the end of the period, with $V_O$ representing the initial value. $T$ refers to the total number of trading days considered.

The annualized volatility represents the standard deviation of the daily returns, which is annualized using the square root of time rule to reflect the risk over the course of a year. This method adjusts the daily volatility to account for the longer time horizon, giving a clearer picture of the portfolio's risk on an annual basis.

$$\sigma_{ANNUAL} = \sqrt{252 * w^T \sum w} \tag{17}$$

Where $w$ represents the vector of asset weights in the portfolio, $\sum$ is the covariance matrix of asset returns, and 252 is the number of trading days in a year.

The Maximum Drawdown (MDD) measures the extent of the lowest point that the portfolio has reached from the initial investment and points to the lowest low. It emphasizes the greatest percentage decrease from the greatest value to the lowest level followed by a rise; it gives a clear picture of the portfolio's poor performance during a given period.

$$MDD = \min_t \left( \frac{V_t}{\max_{s \le t} V_s} - 1 \right) \tag{18}$$

As for the maximum portfolio value, $V_t$ indicates the portfolio's value at time $T$, while $\max_{s \le t} V_s$ identifies the maximum value reached by the portfolio before time $T$,.

Sharpe ratio is the measure of excess return of the portfolio over the risk-free rate of return divided by the portfolio's standard deviation. It measures the extra return that is earned per unit of risk assumed and offers a common method of ranking investment portfolios.

$$Sharpe\ Ratio = \frac{r_{p-}r_f}{\sigma_p} \tag{19}$$

Here, $r_p$ represents the average return of the portfolio, with $r_f$ being the risk-free rate. Additionally, $\sigma_p$ is used to denote the standard deviation of the portfolio's return.

The Value at Risk (VaR) was calculated at a 95% confidence level using the quantile method. The formula used is as follows:

$$VaR_{95\%} = -Quantile(1 - \alpha) * 100 \tag{20}$$

Where α refers to the confidence level, and $-Quantile(1 - \alpha)$ reflects the quantile at the corresponding confidence interval, typically the 5th percentile. The negative sign indicates a focus on potential losses, thus quantifying the downside risk.

## *4.2 Statistical Analysis*

### *4.2.1 CAPM*

After evaluating the performance metrics and comparing the various portfolios, a more quantitative and statistical analysis was conducted. The first analysis involved the calculation of the Capital Asset Pricing Model (CAPM), which estimates the expected return for each portfolio based on the associated risk. The code for implementing the CAPM was adapted from an open-source implementation available on GitHub, provided by Harsh Parikh. In this process, after downloading the returns for each portfolio, the S&P 500 was used as a benchmark. Once the returns were adjusted to a monthly basis, the percentage variance on monthly prices was calculated to obtain both the portfolio and market returns. The beta of each portfolio was then calculated using the appropriate formula as you can see below:

$$\beta = \frac{Cov(r_m, r_p)}{Var(r_m)} \tag{21}$$

Where $Cov(r_m, r_p)$ is the covariance between the portfolio returns and market return and $Var(r_m)$ is the variance of markets return.

### *4.2.2 Monte Carlo Analysis*

The primary objective of this analysis was to simulate future scenarios based on historical asset returns and the covariance matrix of their daily returns. This approach provided a clearer view of potential portfolio behaviour in future periods. The code utilized for this analysis was inspired by Abdalla A. Mahgoub’s (2020) article on Medium, "Measuring Portfolio Risk Using Monte Carlo Simulation in Python (Part 1)" and was adapted to analyse three different portfolios. A total of 100 Monte Carlo simulations were run over a 100-day time horizon. Each simulation generated paths of correlated daily returns for the assets considered, using the Cholesky decomposition of the covariance matrix. This method ensured that the daily returns respected the historical correlations between assets, thereby producing more realistic results. For the simulation, the initial amount invested in the portfolio was set at $10,000, reflecting the intention to analyze the portfolio from the perspective of a passive management approach for a young investor. Additionally, two key risk metrics, Conditional Value at Risk (CVaR) and Value at Risk (VaR) were calculated based on the results of the Monte Carlo simulations. These metrics were derived from the distributions of the final values of the simulated portfolios and provided insights into the potential downside risks.

### *4.2.3 Fama and French 5 factors model*

One of the most widely used methodologies in portfolio analysis is the Fama-French Five-Factor Model. This model extends the traditional Capital Asset Pricing Model (CAPM) by incorporating additional factors to better explain portfolio returns. In this analysis, logarithmic returns were employed, as logarithmic returns are additive and are commonly used in statistical and econometric models. This approach helps to normalize the data more effectively than simple returns. The Fama-French factor dataset utilized for this study was obtained from the Ken French Data Library at the Tuck School of Business, Dartmouth College. Following this, an Ordinary Least Squares (OLS) regression was performed to determine the influence of the Fama-French factors on portfolio returns. To calculate excess returns, the risk-free rate was subtracted from the portfolio returns, allowing for the evaluation of the additional variance explained by the Fama-French factors (equation 11) beyond what is captured by CAPM (equation 10). It is important to note that the original Fama-French dataset uses a short-term risk-free rate based on the one-month Treasury bill. However, for this analysis, the risk-free rate was replaced with the 10-year U.S. Treasury yield. This adjustment was made to maintain consistency with the long-term nature of the study, which spans from 2014 to 2024 and focuses on passive investment portfolios. The 10-year yield was deemed more relevant as it better reflects the opportunity costs associated with long-term investments.

### *4.2.4 VaR & CVaR*

To evaluate the risk of the analyzed portfolios, a model was developed to calculate both Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR). VaR was computed using the portfolio's daily returns, derived from the historical price data of the assets. The VaR was then calculated as the percentile corresponding to the confidence level, specifically the 5th percentile, providing a clear measure of potential downside risk. CVaR, which measures the average loss occurring beyond the VaR threshold, was determined by averaging the returns that exceeded the VaR, capturing the worst-case returns and offering a more comprehensive view of potential portfolio losses (Rockafellar & Uryasev, 2000). To ensure the reliability of the VaR model, a backtesting process was implemented, comparing the actual number of VaR breaches with the expected number. A breach occurs when the portfolio's actual return falls below the predicted VaR, and the Violation Ratio was used to gauge the model's accuracy in predicting risk. To further enhance the model's precision, a GARCH (1,1) model was incorporated, accounting for volatility clustering and improving the forecasting of conditional VaR, making the risk estimates more responsive to fluctuating market conditions.

### *4.2.5 Stress Test Scenarios*

The analysis utilized a methodology based on synthetic data for macroeconomic variables covering the period from 2014 to 2024. The use of synthetic data was chosen to ensure complete control over the variables, maintaining time consistency and eliminating the possibility of missing data or anomalies that could affect the results (Breuer et al., 2009). We defined several macroeconomic stress scenarios such as high inflation, deflation and financial crises, linked to specific percentage shocks on key variables to simulate market stresses. Standard performance and risk metrics including Sharpe Ratio, VaR and CVaR were then applied to analyze the impact of these shocks to the HRP, All-Weather and 60/40 portfolios. The application of synthetic data enabled accurate modeling of hypothetical scenarios, isolating the direct effects of macroeconomic variables on the portfolios and allowing for an in-depth theoretical analysis. This approach also facilitated the exploration of the sensitivity of portfolios to different levels of stress, providing a clearer understanding of how macroeconomic shocks could influence financial performance (Glasserman et al., 2015).

Table I Stress Test

| | Earthquake | High Inflation | Deflation | Financial Crisis | Global Recession |
|---|---|---|---|---|---|
| GDP | (0.01) | (0.05) | (0.02) | (0.04) | (0.03) |
| Interest Rates | 0 | 0.02 | (0.015) | (0.02) | (0.01) |
| CPI | 0.005 | 0.06 | (0.01) | (0.05) | (0.005) |
| Un. Rate | 0.005 | 0.01 | 0.02 | 0.06 | 0.03 |
| Oil Prices | 0.02 | 0.15 | (0.1) | (0.3) | (0.15) |
| Exchange Rates | 0 | 0.03 | 0.02 | 0.08 | 0.02 |
| Industrial Production | (0.015) | (0.01) | (0.05) | (0.12) | (0.08) |
| Housing mkt | (0.05) | (0.08) | (0.1) | (0.15) | (0.05) |
| Consumer Confidence Index | (0.02) | (0.05) | (0.15) | (0.35) | (0.25) |
| Governament Spending | 0.08 | 0 | 0.12 | 0.2 | 0.15 |

#### *4.2.6 Quantile Regression*

A quantile regression analysis was performed for each portfolio to examine the impact of macroeconomic variables not only on the mean returns but also on specific points of the return distribution, particularly focusing on extreme outcomes (Koenker & Bassett, 1978). In this case, data on macroeconomic indicators was downloaded using the FREDAPI and Yahoo Finance libraries. The datasets were synchronized and standardized to ensure consistency across the analysis. Unlike linear regression, which focuses on the conditional mean of the dependent variable, quantile regression allows for an exploration of how independent variables affect different quantiles of the return distribution. In this analysis, attention was given to the lower (10th percentile) and upper (90th percentile) parts of the return distribution to investigate the influence of macroeconomic factors on more extreme outcomes. This approach provided a deeper understanding of how these variables affect both downside and upside risk in the portfolios.

#### *4.2.7 Arch-Garch Models*

The analysis was structured in phases to maintain a logical approach and address the different aspects of the study in a focused manner. The objective was to evaluate the volatility and risk of the portfolios. To ensure greater precision, logarithmic returns were used, and portfolio returns were scaled by 100% to stabilize ARCH and

GARCH models, improving numerical stability. The distribution of portfolio returns was analyzed using skewness, kurtosis, and the Shapiro-Wilk normality test, to better understand the characteristics of the returns. To confirm the presence of ARCH effects, the Engle test was conducted, which highlighted the need to apply GARCH models to model conditional volatility.

Subsequently, several distributions were evaluated, including the normal, Student's t, skewed Student's t, and the Generalized Error Distribution (GED), a model useful for capturing significant deviations from normality. The GED is particularly suitable for modeling distributions with heavy or light tails due to its flexibility in controlling the shape of the distribution through a specific shape parameter, making it ideal for analyzing financial data with atypical distributions. The selection of the optimal model was based on the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), which penalize model complexity and help identify the most appropriate model.

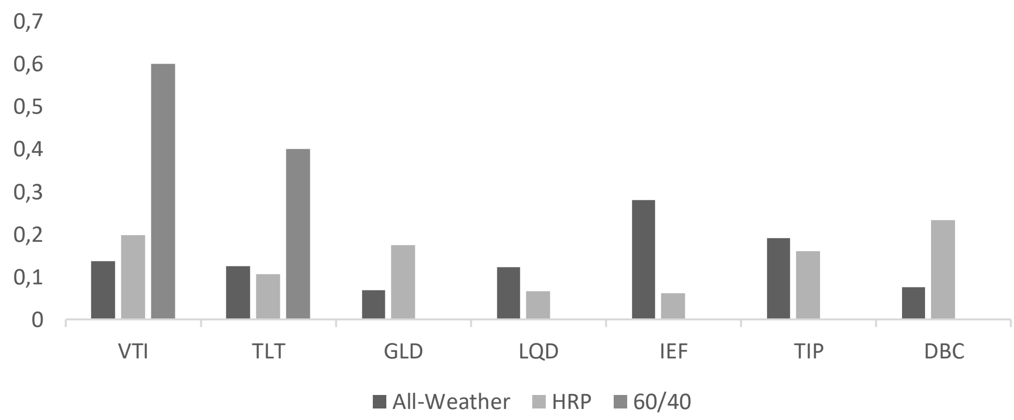
To capture the impact of external economic events, dummy variables were introduced. The Dummy_COVID variable took the value 1 between January 1, 2020, and December 31, 2021, to represent the impact of the pandemic, while the Dummy_Inflation variable took the value 1 between January 1, 2022, and December 31, 2022, to indicate the period of high inflation. Lagged versions of these variables were included to capture any delayed effects, and a GARCH (1,1) model with the best-fitting distribution was subsequently estimated, including the dummy variables as exogenous factors in both the mean and variance equations.

Finally, advanced models such as EGARCH, GJR-GARCH, and APARCH were analyzed, designed to capture asymmetric volatility dynamics or other specific variations. By comparing the AIC and BIC values, the model that best fit the portfolio in analysis was identified, ensuring an effective balance between accuracy and simplicity. An empirical Value-at-Risk (VaR) analysis was conducted during the COVID-19 period, as this approach is better suited for cases where an exact distribution is not known, effectively accounting for non-normality and heavy tails in the data. The Hit Ratio was used to assess the accuracy of the VaR model by measuring how often the model correctly predicted risk threshold breaches. The Hit Ratio is calculated as the ratio of the number of times losses exceed the predicted VaR to the total number of observations, providing a measure of the effectiveness of the forecasts.

## 5. RESULTS

The first analysis was to create generic portfolios and assessing the performance to determine how the portfolios behave under different market conditions. In the following image, we can see the weights assigned to each asset for each portfolio.

Figure V Weights Comparison




The All-Weather portfolio aims for maximum diversification by allocating higher weights to bonds, particularly during inflation and deflation, while the HRP portfolio allocates more to positively correlated assets like DBC and VTI, balancing risk with diversification. The 60/40 portfolio follows a traditional 60% equities and 40% bonds allocation, balancing growth with risk protection

The correlation between asset prices and macroeconomic indicators (Appendix VI) shows that DBC and VTI have positive correlations, utilized in HRP, while IEF's negative correlation with VTI supports All-Weather's strategy. Gold, with its low correlation, serves as a reliable diversifier across portfolios.

Performance comparisons revealed that the 60/40 portfolio had the highest return at 49.91% but also experienced the largest maximum drawdown (MDD) of -27.87%, with 12.97% annualized volatility. The All-Weather portfolio provided stability, with a lower return of 23.57% but reduced volatility (7.32%) and MDD (-18.32%), reflecting its focus on risk mitigation across economic cycles. The HRP portfolio achieved a return of 42.67%, a Sharpe Ratio of 0.45, and the lowest MDD (-16.47%), demonstrating a balanced approach to risk and return.

Table II Performance Metrics

| Metrics | 60/40 | All-Weather | HRP |
|---|---|---|---|
| Portf. Return % | 49.91 | 23.57 | 42.67 |
| MDD & | (27.87) | (18.32) | (16.47) |
| CAGR % | 6.99 | 3.78 | 6.39 |
| Sharpe Ratio | 0.36 | 0.2 | 0.45 |
| Ann. Volatiliy % | 12.97 | 7.32 | 9.07 |
| Daily VaR % | (1.29) | (0.72) | (0.86) |

### 5.1 *Systemic risk analysis: CAPM and Five-factor Fama-French model*

The Capital Asset Pricing Model (CAPM) and the Fama-French five-factor model are combined to give a complete systemic risk view across the portfolios. Beta of CAPM means the portfolio’s sensitivity to market movements. The strategy behind the All-Weather portfolio, with a beta of 0.28, low market risk exposure, is defensive and diversified. With a beta of 0.65, the 60/40 portfolio has greater market risk exposure in line with the portfolio’s equity-heavy structure. HRP portfolio shows a beta of 0.39, to strike a balance between the risk reduction and market participation.

Table III CAPM and FFM results

| Portfolios | Beta | MKT-Rf | SMB | HML | RMW | CMA |
|---|---|---|---|---|---|---|
| AW | 0.28 | 0.40<br>(0.002) | 0.13<br>(0.121) | (0.24)<br>(0.003) | 0.21<br>(0.051) | 0.10<br>(0.378) |
| HRP | 0.39 | 0.62<br>(0.000) | 0.25<br>(0.030) | (0.30)<br>(0.007) | 0.26<br>(0.090) | (0.10)<br>(0.531) |
| 60/40 | 0.65 | 0.53<br>(0.000) | 0.22<br>(0.015) | (0.03)<br>(0.651) | 0.21<br>(0.080) | (0.01)<br>(0.928) |

The Fama French model extends risk beyond market risk into variables such as size, value and profitability. The MKT-Rf (market minus risk-free rate) coefficient over the HRP portfolio is positive and significant, consistent with market risk; SMB (small minus big) and RMW (profitability) factors are also positive, indicating exposure to small-cap stocks and firms with high profitability respectively. The MKT-Rf factor is significant in the All-Weather portfolio, though it is lower than in the HRP portfolio, as this characterizes the defensive portfolio and the negative HML (high minus low) factor is consistent with the conservative diversification found in this portfolio. In the case of the 60/40 portfolio it has the highest MKT-Rf factor, showing its high sensitivity to market movements, accompanied by negative HML and high SMB factors, the latter meaning that small stocks tend to outperform when combined with large stocks, and this was what happened between 2014 and 2024.

### *5.2 Statistical Risk Assessment of Portfolios: Integrated Analysis with Monte Carlo, VaR and CvaR*

Additionally, the All-Weather, HRP, and 60/40 portfolios were evaluated using the Monte Carlo simulation with the traditional Value at Risk (VaR) and Conditional Value at Risk (CVaR) metrics. Monte Carlo simulations give insight into possible future outcomes, whereas VaR and CVaR are quantitative measures of possible losses based on past data. Results for both methods provide a consistent view on portfolio risk management.

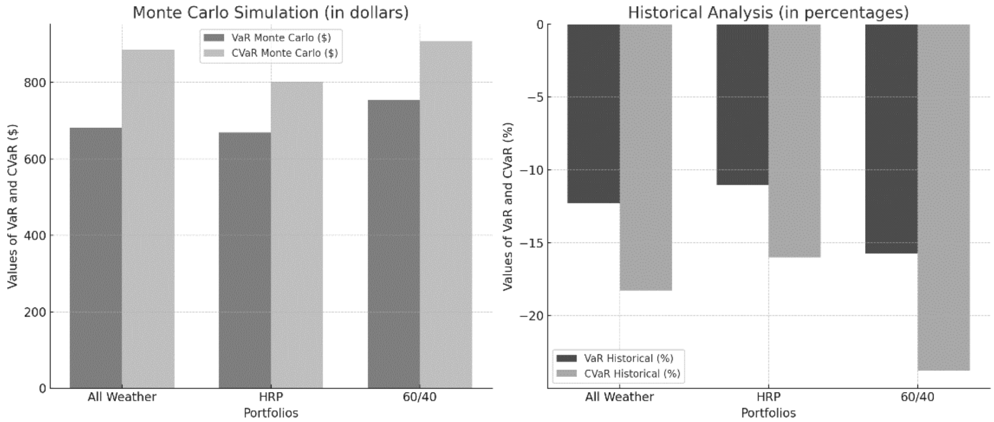
With the 60/40 portfolio being equity heavy, we see it has higher volatility as it has a VaR of $753.58 and CVaR of $907.04 in the Monte Carlo simulation. This is consistent with its historical annualized VaR (Appendix V) of -15.75% and suggests that it is more sensitive to market variations than it is to riskier growth opportunities.

All-Weather portfolio showed great stability with a VaR of $681.78 and a CVaR of $885.29 which is close to its historical annualized VaR of -12.26%. It is designed to handle systemic risks and works in any market conditions.

Monte Carlo simulation results showed that the HRP portfolio effectively balanced risk and return with a VaR of $668.75, CVaR of $ 801.02 and an annualized VaR of -11%. This shows a well thought out compromise between the reduction of risk and the pursuit of earnings.

By combining Monte Carlo results with historical VaR and CVaR data, the 60/40 portfolio sees the highest volatility and growth potential, the All-Weather portfolio offers stability, and the HRP portfolio provides a trade-off between risk and return.

Figure VI Monte Carlo and VaR Comparison




### *5.3 Integrated analysis of Stress Test and Quantile Regression*

A dual methodology, which involved stress tests under extreme economic scenarios but also quantile regression, was used to analyse the performance of the 60/40, HRP, and All-Weather portfolios. With this approach, portfolio performance and sensitivity to economic shock was uncovered, including a detailed view of resilience during volatile and crisis periods.

In stress tests, the 60/40 portfolio showed vulnerability to high inflation and other shocks, with a high Max Drawdown of -32.17% and a Sharpe Ratio of 0.47. While it performs well during periods of growth, particularly in the upper quantiles where GDP boosts returns, it suffers significantly in the lower quantiles, demonstrating sensitivity to macroeconomic shocks. This reflects its reliance on equity-heavy strategies, making it susceptible to severe market downturns.

The HRP portfolio, with its risk-parity strategy, managed risks more effectively. Its Max Drawdown was lower -17.47%, and its standard deviation of 7.06% was lower, suggesting it had better loss management under stress. The result is reinforced by quantile regression which indicates a weak sensitivity of HRP to shocks in the lower quantiles likely resulting from asset diversification protection. While not as responsive to growth in middle to upper quantiles as 60/40, it offers stable returns with lower risk and is appropriate for capital protection in moments of uncertainty.

In terms of resilience, the All-Weather portfolio was very resilient, its capital was preserved, thanks to its broad diversification. It had relatively low Max Drawdown of -18.93%, showed good performance between quantiles indicating consistency in returns across both lower and upper quantiles, and was capable to adjust according to the different types of economic conditions, especially in the situation of managing inflation and commodity related variables. It's, therefore, a good option for long term investors on capital preservation. Stress test results, when combined with quantile regression, show the diversified portfolios have different resilience. A major weakness of the 60/40 portfolio is its sensitivity to economic shocks, particularly while the portfolio performs well during growth periods. The stability and capital preservation over the long term mean that the All-Weather portfolio is the most reliable for conservative, long term investors and the balanced risk management makes the HRP portfolio a better solution than traditional asset allocation.

Table IV Stress Test results

| All- Weather | **Earthquake** | **High Inflat.** | **Deflation** | **Fin. Crisis** | **Global Rec.** |
|---|---|---|---|---|---|
| Exp. Return % | 2.36 | 3.41 | 1.72 | 1.39 | 1.7 |
| MDD% | (12.98) | (18.93) | (9.47) | (7.6) | (9.36) |
| Sharpe Ratio | 0.011 | 0.14 | (0.15) | (0.3) | (0.16) |
| Stand. Dev. % | 5.26 | 7.85 | 3.78 | 3.02 | 3.74 |

| HRP | **Earthquake** | **High Inflat.** | **Deflation** | **Fin. Crisis** | **Global Rec.** |
|---|---|---|---|---|---|
| Exp. Return % | 2.91 | 4.26 | 2.12 | 1.7 | 2.09 |
| MDD% | (11.92) | (17.47) | (8.66) | (6.95) | (8.56) |
| Sharpe Ratio | 0.13 | 0.28 | (0.05) | 0.022 | (0.0597) |
| Stand. Dev. % | 4.73 | 7.06 | 3.4 | 2.71 | 3.36 |

| 60/40 | **Earthquake** | **High Inflat.** | **Deflation** | **Fin. Crisis** | **Global Rec.** |
|---|---|---|---|---|---|
| Exp. Return % | 7.33 | 7.39 | 2.66 | 3.04 | 4.47 |
| MDD% | (31.97) | (32.17) | (12.81) | (14.51) | (20.7) |
| Sharpe Ratio | 0.47 | 0.47 | 0.1 | 0.17 | 0.33 |
| Stand. Dev. % | 10.67 | 10.75 | 3.86 | 4.42 | 6.5 |

### *5.4 Arch-Garch results*

The first analysis carried out was to understand the distribution of the portfolio returns, which showed a non-normal distribution confirmed by the Shapiro-Wilk Test with a statistic of 0.9427 and an extremely low p-value of 3.4581e-30. Subsequently, the Engle's ARCH Test was applied, with a statistic of 464.51 and a p-value of 1.6734e-93, indicating the presence of significant ARCH effects in the portfolio returns and confirming the need to use GARCH models. To determine the best-fitting distribution for the portfolio returns, various return distributions were analysed, including normal, t-student, skew-t, and GED, to ensure the model accurately captured the portfolio's dynamics. As shown in Appendix 3, the skew-t distribution provided the best fit, effectively capturing the heavy tails and skewness common in financial returns, as confirmed by the lowest AIC and BIC values compared to other distributions.

The analysis then continued with the GARCH(1,1) model. For the 60/40 portfolio, the model revealed a quick response of volatility to shocks, with an alpha coefficient of 0.0788 and a beta coefficient of 0.9071, indicating strong volatility persistence typical of financial time series, and confirming the presence of clustering, as shown below.

Table V 60/40 GARCH(1,1) results

| GARCH (1,1) | | |
|---|---|---|
| Model Parameter | Coefficient (coef) | p-value (P>\|t\|) |
| Mean Model (mu) | 0.042 | 0.00001163 |
| Omega | 0.00521 | 0.00487 |
| Alpha[1] | 0.0788 | 1.045E-07 |
| Beta[1] | 0.9071 | 0 |
| Eta | 10.3704 | 1.014E-07 |
| Lambda | (0.1721) | 3.288E-09 |

Subsequently, the GARCH model was extended by introducing dummy variables to capture the effects of external economic events such as the Covid-19 pandemic and inflation. The Covid dummy initially increased returns, which then quickly diminished, while the inflation dummy had a consistent negative impact, aligning with the theory that inflation erodes real returns by reducing purchasing power. The results of this analysis are presented in Appendix VIII, demonstrating the model's effectiveness in adapting to external shocks.

Next, GJR-GARCH, EGARCH, and APARCH models were examined. The GJR-GARCH and EGARCH models proved effective in capturing the leverage effect, where negative shocks amplify volatility more than positive ones, explaining the increased risk during market downturns. The APARCH model, as detailed in Appendix IX, showed high flexibility in modelling volatility with power dynamics; the delta coefficient (1.8095) highlighted the sensitivity of volatility to market movements, confirming the model's ability to accurately capture variations in the return distribution.

Finally, an analysis of the empirical VaR was conducted during the Covid period, confirming elevated risks and validating the accuracy of the VaR model through the Kupiec test, demonstrating that the model effectively captures risk levels, although some extreme events may have been underestimated, as indicated in Appendix II.

The initial analysis of the All Weather portfolio revealed a non-normal distribution of returns, as confirmed by the Shapiro-Wilk Test with a test statistic of 0.9484 and a p-value of 7.2290e-2. The Engle's ARCH Test then showed a test statistic of 609.66 and an extremely low p-value of 1.5018e-124, indicating significant ARCH effects and thus confirming the use of GARCH models to model conditional volatility.

Several return distributions were evaluated, including normal, t-student, skew-t, and GED, to determine which model best fit the portfolio's returns. The GARCH(1,1) model with a skew-t distribution provided the best fit, with an alpha coefficient of 0.0636 and a beta coefficient of 0.9195, indicating strong volatility persistence typical of clustering, and achieving the lowest AIC and BIC values. This suggests that skewed distributions better capture the behavior of the portfolio's returns, especially when modeling volatility over time.

Table VI All-Weather GARCH(1,1) results

| GARCH (1,1) | | |
|---|---|---|
| Model Parameter | Coefficient (coef) | p-value (P>\|t\|) |
| Mean Model (mu) | 0.0162 | 5.09E-02 |
| Omega | 3.93E-03 | 3.08E-02 |
| Alpha[1] | 0.0636 | 2.71E-04 |
| Beta[1] | 0.9195 | 0 |
| Eta | 7.2486 | 2.50E-13 |
| Lambda | (0.0958) | 6.05E-04 |

Dummy variables for Covid and inflation were included, and the results are presented in Appendix VIII. During the Covid period, the dummy variable showed an initial increase in returns, followed by a rapid attenuation, while the inflation dummy had a consistently negative impact.

Regarding advanced models, GJR-GARCH, EGARCH, and APARCH all demonstrated a significant capability to capture asymmetric volatility dynamics. The GJR-GARCH model exhibited a leverage effect through a positive gamma coefficient (0.0499), while the EGARCH model showed a similar sensitivity, confirming that negative shocks amplify volatility more than positive ones. The APARCH model, as indicated in Appendix X, introduced a delta parameter (2.9981) that indicates significant asymmetry and non-linearity in the portfolio returns. This result suggests that the All-Weather portfolio's returns react more strongly to market shocks, with volatility responding differently depending on the intensity and direction of the shocks.

Finally, an empirical VaR analysis was conducted during the Covid period, demonstrating the All-Weather portfolio's vulnerability in the presence of high volatility. The accuracy of the VaR model was validated through the Kupiec test, confirming that the model effectively captures risks, although some extreme events may have been underestimated, as detailed in Appendix II.

The initial analysis of the HRP portfolio revealed a non-normal distribution of returns, confirmed by the Shapiro-Wilk Test with a test statistic of 0.9484 and a p-value of 7.2301e-29. The Engle's ARCH Test further confirmed the presence of significant ARCH effects, with a test statistic of 417.27 and an extremely low p-value of 1.9760e-83, justifying the use of GARCH models to model conditional volatility.

Several return distributions were evaluated, including the normal, t-student, skew-t, and GED, to determine which model best fit the portfolio returns. As shown in Appendix III, the GARCH(1,1) model with a skew-t distribution provided the best fit, with an alpha coefficient of 0.060 and a beta coefficient of 0.9317, highlighting significant volatility persistence typical of clustering. The relatively low omega value (0.00235) suggests that in the absence of market shocks, the baseline volatility remains contained.

Table VII HRP GARCH(1,1) results

| GARCH (1,1) | | |
|---|---|---|
| Model Parameter | Coefficient (coef) | p-value (P>\|t\|) |
| Mean Model (mu) | 0.0152 | 3.92E-02 |
| Omega | 2.35E-0,3 | 1.11E-02 |
| Alpha[1] | 0.0559 | 3.11E-06 |
| Beta[1] | 0.9317 | 0 |
| Eta | 9.1867 | 1.36E-10 |
| Lambda | (0.084) | 5.02E-03 |

Dummy variables for COVID and inflation were included, as evidenced in Appendix VIII. The COVID dummy variable showed a significant initial increase in returns, followed by an attenuation, while the inflation dummy had a consistent negative impact, aligning with theoretical expectations.

Regarding the advanced models, the GJR-GARCH, EGARCH, and APARCH were applied to study the leverage effect and the asymmetry in the response of volatility to positive and negative shocks (appendix XI). The GJR-GARCH model displayed an insignificant leverage effect, with a positive gamma of 0.0035 and a p-value of 0.784, suggesting that the HRP portfolio's volatility response to market shocks is similar regardless of the shock's direction. The EGARCH model confirmed high volatility persistence with a beta coefficient of 0.9897 but did not detect a significant leverage effect. Lastly, the APARCH model introduced a significant delta parameter of 2.376, indicating a degree of asymmetry and nonlinearity in the returns. This result suggests that the HRP portfolio's volatility adapts variably depending on the intensity of shocks, effectively capturing irregular fluctuations compared to other models.

The empirical VaR analysis during the COVID period confirmed the model's ability to forecast risk in unstable market conditions. However, the Kupiec test highlighted a potential underestimation of extreme events, with a p-value of 0.0724 for the 1% VaR and 0.4361 for the 5% VaR, suggesting that some extreme risks may have been overlooked, as detailed in Appendix II.

In conclusion, the HRP model stands out as the superior choice for long-term investing due to its effective risk management and adaptability. Compared to the 60/40 and All-Weather portfolios, HRP better mitigates volatility and absorbs market shocks, as evidenced by lower volatility persistence in the GARCH models. Its ability to dynamically adjust risk based on asset correlations ensures better protection during turbulent times, making it more resilient in the face of market crises. This makes HRP the preferable strategy for long-term investors seeking both stability and growth.

## 6. Conclusion

Three different portfolios, 60/40 - All-Weather - Hierarchical Risk Parity (HRP), were examined in this study using a passive investment framework in an effort to determine which approach provides the best return and risk balance for passive investors working in unpredictable economic conditions. Using advanced statistical and econometric models, the study evaluated the robustness of each portfolio under both normal market conditions and stress scenarios. The results constantly showed that when it comes to managing risk and return, the HRP portfolio performs better than its peers. In almost every analysis, it showed reduced volatility and a smaller maximum drawdown, demonstrating its ability to provide better resilience during times of market instability. HRP offers a complicated yet dependable mechanism that makes it a great place for young investors with a long-term perspective to start building their portfolios.

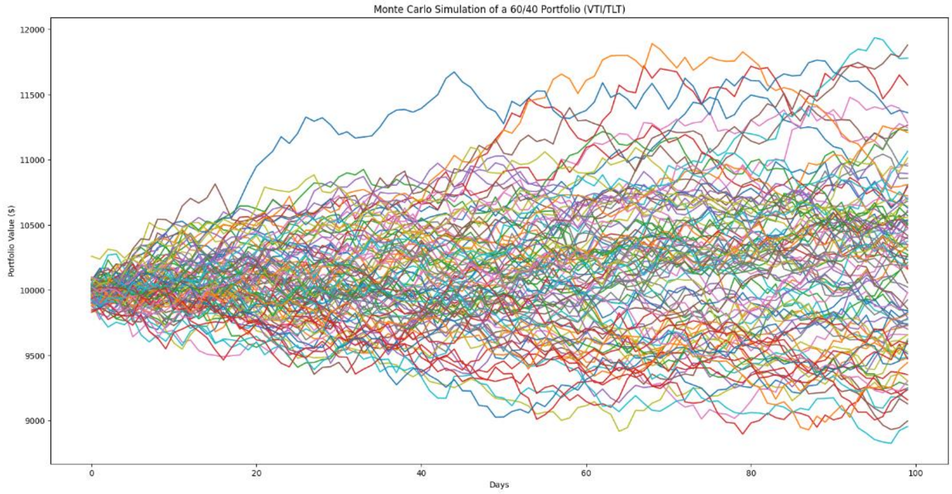
The All-Weather portfolio is still a good choice, but it works best for people who are extremely risk averse and value defensiveness over striking a balance between risk and return. Diversification among cyclical and countercyclical assets contributes to its intrinsic stability, which guarantees protection in a range of market scenarios. However, investors looking for long-term capital growth find it less tempting because of the reduced returns associated with this defense-focused approach.

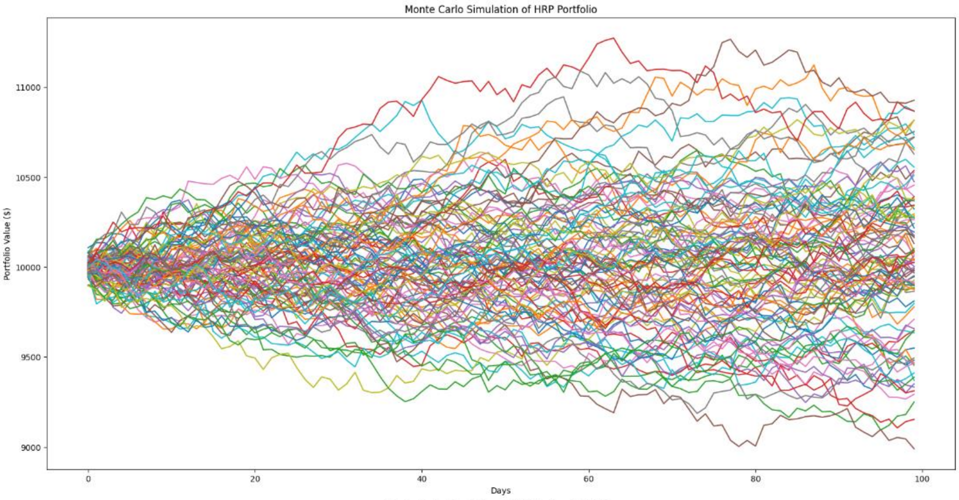
Similar to this, the 60/40 portfolio, which has long been considered a standard for passive investment, had significant weaknesses during times of high inflation and recession, underscoring its inability to adjust to the demands of the modern market.

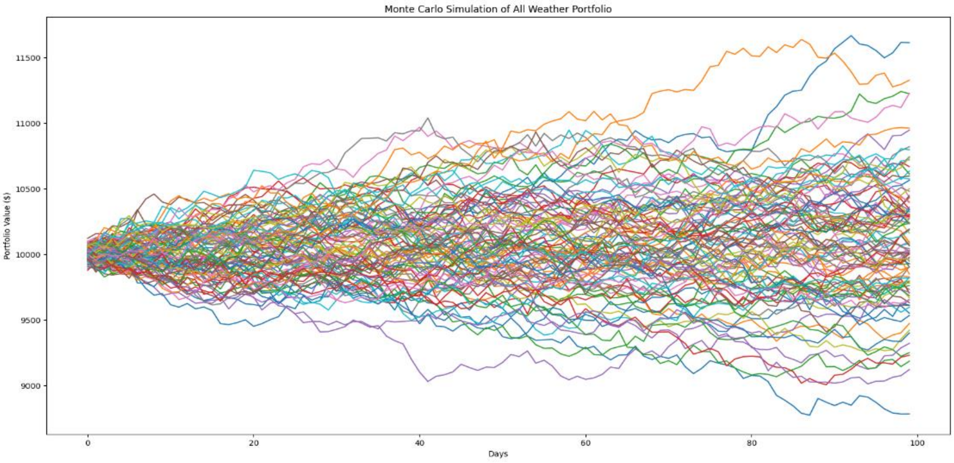
Although the HRP portfolio's dependence on sophisticated machine learning algorithms is beneficial in providing better risk management, it also emphasizes the necessity of having sufficient technical expertise. This suggests that young investors may need to have a firm grasp of machine learning methods and how they are used in financial settings in order to embrace HRP. Furthermore, if semiannual rebalancing is necessary to preserve portfolio efficiency, investors must have the necessary technical know-how to carry out such modifications successfully, guaranteeing that the strategy stays in line with their investment goals. By extending the time horizon beyond the 10-year period examined in this study, future research could further improve our understanding of HRP and capture a wider range of market situations and economic cycles. The flexibility and resilience of HRP may also be better understood by broadening the asset universe to include alternative investments like cryptocurrency, real estate, or emerging markets. Furthermore, incorporating transaction cost studies and investigating real-time rebalancing techniques would offer a more thorough understanding of this strategy's viability in practice.

For youthful, long-term passive investors trying to strike a balance between capital growth and efficient risk avoidance, the HRP portfolio stands out as a creative and forward-thinking option. Its proven capacity to adjust to a variety of market conditions makes it a useful tool for navigating an increasingly complicated and turbulent global economy, even though its application calls substantial technical competence. However, because it serves as a protective tactic for risk-averse people, the All-Weather portfolio is still a good option for people who value security and consistency, albeit at the price of reduced returns. This study emphasizes how crucial it is to adopt diversified and technologically sophisticated investing techniques in order to guarantee long-term success and durability in passive portfolio management.

# APPENDIX I Monte Carlo Analysis

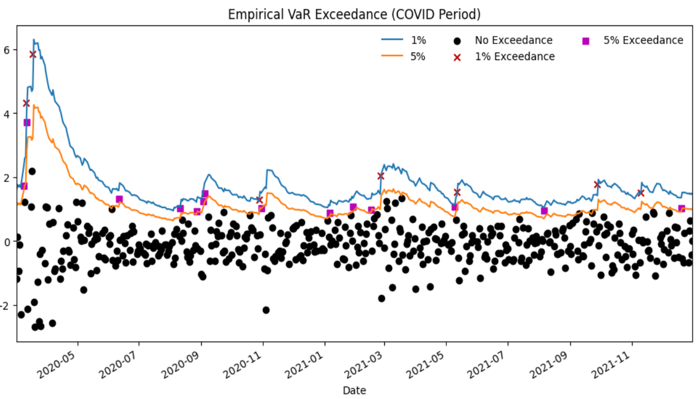


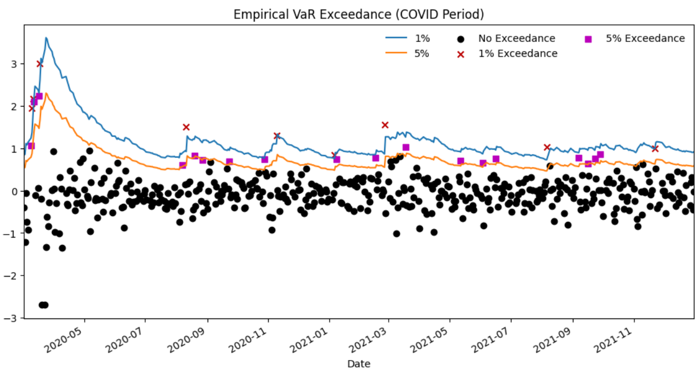



# APPENDIX II Empyrical VaR with GARCH model

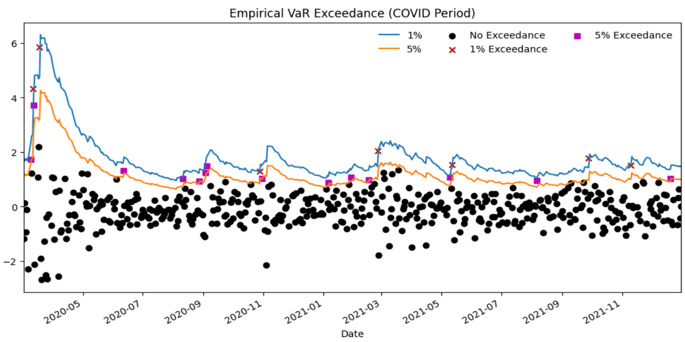
60/40




HRP




All-Weather




# APPENDIX III AIC and BIC Comparison

**60/40**

| Model | AIC | BIC |
|---|---|---|
| GARCH(1,1) Normal | 4115.10 | 4138.42 |
| GARCH(1,1) t-Student | 4065.52 | 4094.67 |
| GARCH(1,1) Skew-t | 4032.76 | 4067.74 |
| GARCH(1,1) GED | 4077.88 | 4107.03 |

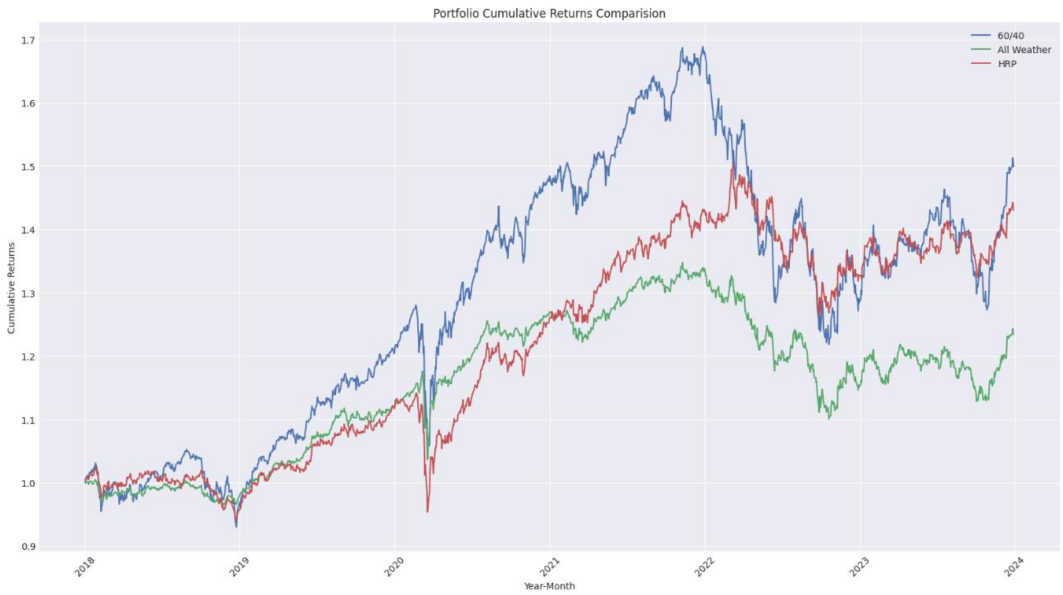
## All Weather

| Model | AIC | BIC |
|---|---|---|
| GARCH(1,1) Normal | 3041.82 | 3065.14 |
| GARCH(1,1) t-Student | 2948.11 | 2977.26 |
| GARCH(1,1) Skew-t | 2938.46 | 2973.44 |
| GARCH(1,1) GED | 2962.89 | 2992.04 |

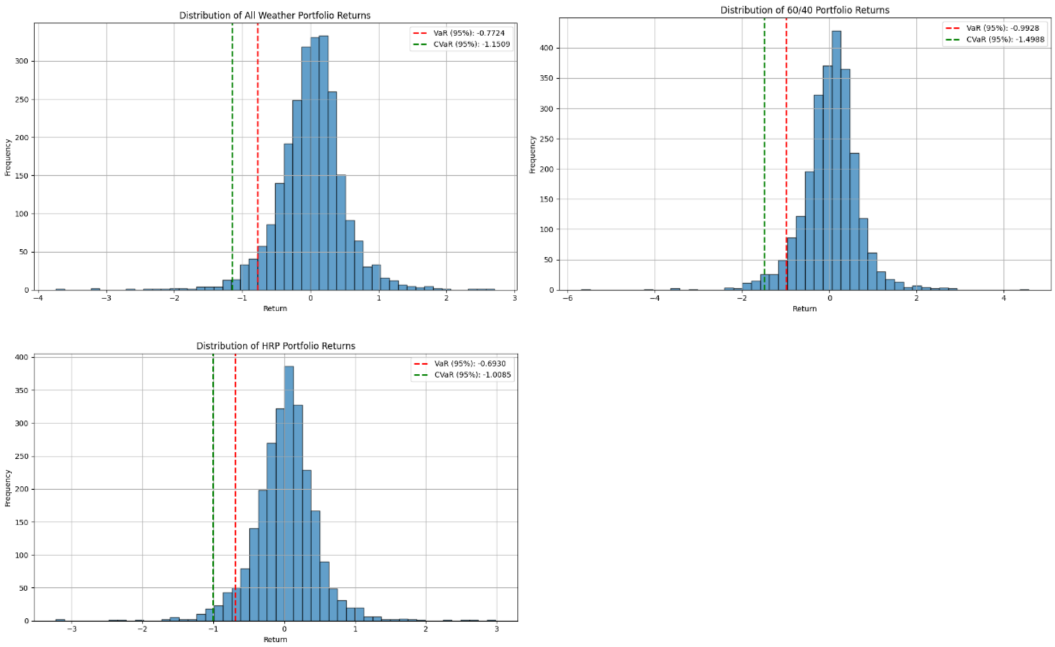
## HRP

| Model | AIC | BIC |
|---|---|---|
| GARCH(1,1) Normal | 2560.6 | 2583.92 |
| GARCH(1,1) t-Student | 2507.21 | 2536.36 |
| GARCH(1,1) Skew-t | 2500.88 | 2535.86 |
| GARCH(1,1) GED | 2521.62 | 2550.77 |

APPENDIX IV Cumulative Returns



## APPENDIX V VaR & CVaR Comparison




| Portfolios | **VaR(Annual.)%** | **CVaR(Annual.)%** |
|---|---|---|
| 60/40 | (15.75) | (23.79) |
| HRP | (11.00) | (16.00) |
| All-Weather | (12.26) | (18.27) |

## APPENDIX VI Correlation Matrix

| Variables | I | II | III | IV | V | VI | VII | VIII | IX |
|---|---|---|---|---|---|---|---|---|---|
| **GDP** | 1 | | | | | | | | |
| **CPI** | 0.6 | | | | | | | | |
| **DBC** | 0.78 | 0.8 | | | | | | | |
| **GLD** | (0.15) | (0.12) | (0.076) | | | | | | |
| **IEF** | (0.54) | (0.66) | (0.65) | 0.51 | | | | | |
| **LQD** | (0.24) | (0.6) | (0.39) | 0.52 | 0.87 | | | | |
| **TIP** | (0.017) | (0.23) | (0.02) | 0.63 | 0.73 | 0.82 | | | |
| **TLT** | (0.54) | (0.61) | (0.61) | 0.46 | 0.96 | 0.84 | 0.7 | | |
| **VTI** | 0,.1 | (0.018) | 0.39 | 0.013 | 0.028 | 0.43 | 0.49 | 0.023 | 1 |
| | **GDP** | **CPI** | **DBC** | **GLD** | **IEF** | **LQD** | **TIP** | **TLT** | **VTI** |

## APPENDIX VII Quantile regression for Q0.1 and Q0.9

| **60/40 Q0.1** | Coeff | Std. Error | **60/40 Q0.9** | Coeff | Std. Error |
|---|---|---|---|---|---|
| Intercept | (1.0941)*** | 0.032 | Intercept | 1.0104*** | 0.032 |
| GDP | (0.072) | 0.057 | GDP | 0.1957*** | 0.057 |
| Interest Rates | (0.0832)** | 0.032 | Interest Rates | 0.0146 | 0.032 |
| CPI | (0.1923)*** | 0.041 | CPI | 0.0908*** | 0.041 |
| Unem. Rate | (0.3451)*** | 0.043 | Unem. Rate | 0.0946** | 0.043 |
| Oil Prices | (0.0075) | 0.061 | Oil Prices | -0.0509 | 0.061 |
| Exchange Rates | 0.1058*** | 0.040 | Exchange Rates | 0.0803** | 0.040 |
| Ind. Production | 0.0816 | 0.057 | Ind. Production | (0.2791)*** | 0.057 |
| Housing Mkt Inde. | (0.0443) | 0.037 | Housing Mkt Inde. | 1.77E-05 | 0.037 |
| Cons. Confidence | (0.0204) | 0.039 | Cons. Confidence | 0.1192*** | 0.039 |
| Gov. Spending | 0.002 | 0.042 | Gov. Spending | 0.1519*** | 0.042 |
| VIX | (0.4164)*** | 0.032 | VIX | (0.2362)*** | 0.032 |
| ***p<0.01 | **p<0.05 | *p<0.1 | | | |

| **HRP Q0.1** | Coeff | Std. Error | **HRP Q0.9** | Coeff | Std. Error |
|---|---|---|---|---|---|
| Intercept | (1.1324)*** | 0.032 | Intercept | 1.0596*** | 0.029 |
| GDP | (0.1041)** | 0.053 | GDP | 0.0768 | 0.049 |
| Interest Rates | (0.0860)*** | 0.030 | Interest Rates | 0.0241 | 0.029 |
| CPI | (0.1550)*** | 0.042 | CPI | 0.0798** | 0.037 |
| Unem. Rate | (0.3422)*** | 0.045 | Unem. Rate | 0.0720** | 0.035 |
| Oil Prices | 0.0496 | 0.055 | Oil Prices | 0.0578* | 0.056 |
| Exchange Rates | 0.2429*** | 0.038 | Exchange Rates | 0.1928** | 0.037 |
| Ind. Production | 0.1199* | 0.065 | Ind. Production | (0.1795)* | 0.059 |
| Housing Mkt Inde. | (0.0434)** | 0.036 | Housing Mkt Inde. | 0.0338** | 0.035 |
| Cons. Confidence | (0.0472)** | 0.038 | Cons. Confidence | 0.1104** | 0.036 |
| Gov. Spending | (0.0219) | 0.042 | Gov. Spending | 0.0465** | 0.040 |
| VIX | (0.1220)*** | 0.035 | VIX | (0.0073)** | 0.034 |
| ***p<0.01 | **p<0.05 | *p<0.1 | | | |

| **AW Q0.1** | Coeff | Std. Error | **AW Q0.9** | Coeff | Std. Error |
|---|---|---|---|---|---|
| Intercept | (0.9794)*** | 0.031 | Intercept | 0.9150*** | 0.025 |
| GDP | (0.1650)*** | 0.056 | GDP | 0.1574*** | 0.044 |
| Interest Rates | (0.1193)*** | 0.023 | Interest Rates | 0.0265 | 0.047 |
| CPI | (0.0810)* | 0.041 | CPI | 0.1360*** | 0.032 |
| Unem. Rate | (0.3043)*** | 0.048 | Unem. Rate | 0.2186*** | 0.033 |
| Oil Prices | 0.5091*** | 0.059 | Oil Prices | 0.5321*** | 0.049 |
| Exchange Rates | 0.2043*** | 0.038 | Exchange Rates | 0.1521*** | 0.033 |
| Ind. Production | 0.1472** | 0.061 | Ind. Production | (0.1962)** | 0.060 |
| Housing Mkt Inde. | (0.0307) | 0.035 | Housing Mkt Inde. | 0.0375 | 0.031 |
| Cons. Confidence | (0.0621)* | 0.036 | Cons. Confidence | 0.0805*** | 0.031 |
| Gov. Spending | (0.0615) | 0.043 | Gov. Spending | 0.0776** | 0.035 |
| VIX | (0.2803)*** | 0.034 | VIX | (0.2008)*** | 0.034 |
| ***p<0.01 | **p<0.05 | *p<0.1 | | | |

APPENDIX VIII Dummy Variables Comparison

| 60/40 | | |
|---|---|---|
| Dummy | Coeff | p-value(P>\|t\|) |
| Const | 0.0401 | 9.54E-02 |
| COVID | 0.8459 | 0 |
| COVID_lag1 | (0.9579) | 1.76E-05 |
| COVID_lag5 | 0.1452 | 0.501 |
| INFLATION | (0.7398) | 3.11E-43 |
| INFLATION_lag1 | 1.33E-03 | 0.997 |
| INFLATION_lag5 | 0.6131 | 5.03E-02 |

| AW | | |
|---|---|---|
| Dummy | Coeff | p-value(P>\|t\|) |
| Const | 8.87E-03 | 0.319 |
| COVID | 0.5571 | 3.73E-08 |
| COVID_lag1 | (0.6489) | 2.0E-03 |
| COVID_lag5 | 0.1468 | 0.443 |
| INFLATION | (0.1355) | 0.497 |
| INFLATION_lag1 | (0.1880) | 0.548 |
| INFLATION_lag5 | 0.2963 | 0.236 |

| HRP | | |
|---|---|---|
| Dummy | Coeff | p-value(P>\|t\|) |
| Const | 0.0129 | 0.107 |
| COVID | 0.5526 | 1.42E-271 |
| COVID_lag1 | (0.5810) | 4.89E-03 |
| COVID_lag5 | 0.0553 | 0.785 |
| INFLATION | (0.5998) | 5.99E-108 |
| INFLATION_lag1 | 0.2047 | 0.384 |
| INFLATION_lag5 | 0.3367 | 0.153 |

## APPENDIX IX 60/40 GARCH Models Comparison

| 60/40 GRJ-GARCH | | |
|---|---|---|
| AIC | 4026.53 | |
| BIC | 4067.34 | |
| Model Parameter | Coeff | p-value(P>\|t\|) |
| mu | 0.0367 | 1.72E-04 |
| Omega | 5.86E-03 | 1.80E-03 |
| Alpha[1] | 0.0466 | 8.44E-04 |
| Gamma[1] | 0.0472 | 4.99E-03 |
| Beta[1] | 0.9108 | 0 |
| eta | 10.6183 | 2.59E-07 |
| lamba | (0.1741) | 2.93E-09 |

| 60/40 EGARCH | | |
|---|---|---|
| AIC | 4029.66 | |
| BIC | 4070.47 | |
| Model Parameter | Coeff | p-value(P>\|t\|) |
| mu | 0.0343 | 4.33E-04 |
| Omega | (0.0169 | 1.09E-02 |
| Alpha[1] | 0.1414 | 1.73E-09 |
| Gamma[1] | (0.0398) | 9.64E-04 |
| Beta[1] | 0.9836 | 0 |
| eta | 10.4284 | 1.52E-07 |
| lamba | (0.1745) | 2.33E-09 |

| 60/40 APARCH | | |
|---|---|---|
| AIC | 4034.43 | |
| BIC | 4075.24 | |
| Model Parameter | Coeff | p-value(P>\|t\|) |
| mu | 0.0419 | 1.22E-05 |
| Omega | 5.68E-03 | 4.92E-03 |
| Alpha[1] | 0.0819 | 3.97E-08 |
| Gamma[1] | 0.9096 | 0 |
| Beta[1] | 1.8095 | 7.48E-10 |
| eta | 10.3813 | 1.041E-07 |
| lamba | (0.1719) | 3.484E-09 |

APPENDIX X AW GARCH Models Comparison

| AW GRJ-GARCH | | |
|---|---|---|
| AIC | 2929.84 | |
| BIC | 2970.65 | |
| Model Parameter | Coeff | p-value(P>\|t\|) |
| mu | 0.0128 | 0.119 |
| Omega | 4.11E-03 | 2.46E-02 |
| Alpha[1] | 0.0321 | 2.46E-02 |
| Gamma[1] | 0.0499 | 6.94E-03 |
| Beta[1] | 0.9237 | 0 |
| eta | 7.4119 | 1.45E-12 |
| lamba | (0.0977) | 4.27E-04 |

| AW EGARCH | | |
|---|---|---|
| AIC | 2950.33 | |
| BIC | 2985.31 | |
| Model Parameter | Coeff | p-value(P>\|t\|) |
| mu | 0.0160 | 4.86E-02 |
| Omega | (0.0227) | 8.70E-02 |
| Alpha[1] | 0.1534 | 4.01E-05 |
| Beta[1] | 0.9823 | 0 |
| eta | 7.1911 | 2.22E-13 |
| lamba | (0.0974) | 3.67E-04 |

| AW APARCH | | |
|---|---|---|
| AIC | 2937.32 | |
| BIC | 2978.13 | |
| Model Parameter | Coeff | p-value(P>\|t\|) |
| mu | 0.0163 | 5.02E-02 |
| Omega | 1.76E-03 | 0.105 |
| Alpha[1] | 0.0403 | 1.34E-02 |
| Beta[1] | 0.9157 | 0 |
| delta | 2.9981 | 2.38E-06 |
| eta | 7.1769 | 1.24E-13 |
| lamba | (0.0954) | 6.98E-04 |

APPENDIX XI HRP GARCH Models Comparison

| HRP GRJ-GARCH | | |
|---|---|---|
| AIC | 2502.81 | |
| BIC | 2543.62 | |
| Model Parameter | Coeff | p-value(P>\|t\|) |
| mu | 0.0149 | 4.49E-02 |
| Omega | 2.38E-03 | 1.215E-02 |
| Alpha[1] | 0.0538 | 3.86E-05 |
| Gamma[1] | 3.51E-03 | 0.784 |
| Beta[1] | 0.9318 | 0 |
| eta | 9.2126 | 1.67E-10 |
| lamba | (0.0839) | 5.18E-03 |

| HRP EGARCH | | |
|---|---|---|
| AIC | 2503.32 | |
| BIC | 2538.30 | |
| Model Parameter | Coeff | p-value(P>\|t\|) |
| mu | 0.0152 | 3.76E-02 |
| Omega | (0.0156) | 7.29E-02 |
| Alpha[1] | 0.1198 | 2.18E-05 |
| Beta[1] | 9.0885 | 0 |
| eta | (0.0858) | 9.50E-11 |
| lamba | (0.0839) | 3.98E-03 |

| HRP APARCH | | |
|---|---|---|
| AIC | 2502.59 | |
| BIC | 2543.40 | |
| Model Parameter | Coeff | p-value(P>\|t\|) |
| mu | 0.0152 | 3.87E-02 |
| Omega | 2.74E-04 | 1.70E-02 |
| Alpha[1] | 0.0590 | 4.88E-07 |
| Beta[1] | 0.9343 | 0 |
| Delta | 1.7603 | 6.37E-04 |
| eta | 9.2058 | 1.45E-10 |
| lamba | (0.0842) | 4.86E-03 |

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Diego Nicola, 15-10-2024.