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Master's Final Work

Internship Report

Modeling Mortality Rates Under IFRS 17 And Solvency II

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ABSTRACT

Choosing appropriate biometric assumptions, such as mortality, is fundamental for life insurance companies under both IFRS 17 and Solvency II, as these frameworks demand a prudent and reliable valuation of insurance liabilities. This study aims to reassess the mortality modeling approach used by a life insurance company by comparing the current practice, consisting of the application of a uniform percentage adjustment to a standard mortality table across all ages, with more refined techniques based on mortality graduation. Two approaches were explored: graduation with reference to a standard table and graduation by parametric formula. The analysis focused exclusively on single-life policies for products covering mortality risk, excluding those related to longevity, such as annuities. Within this scope, the best-performing models based on relevant statistics, both applied without distinction by sex, were an exponential adjustment to the GKM80 table and the $GM(2,3)$ parametric model. The derived mortality rates were applied as actuarial assumptions and assessed under both the accounting and prudential regimes. In both cases, the revised rates provided a better fit to observed mortality, enhancing the accuracy of financial projections. These findings underscore the importance of regularly reviewing and updating actuarial assumptions to maintain the reliability of models and ensure compliance with regulatory standards. In addition, this application also led to a reduction in liabilities under the regulatory frameworks. Furthermore, recalculating the capital requirements under the life underwriting risk sub-module revealed an improvement in the solvency position, reflected in a higher solvency ratio. Although the parametric model demonstrated marginally better performance in reducing relative deviations between projected and actual outcomes, the standard table graduation was ultimately recommended for implementation due to its balanced combination of accuracy, operational simplicity, and alignment with current company practices. Adopting this approach will help ensure that mortality assumptions remain both practical and reliable, supporting consistency in actuarial valuations and sustained regulatory compliance.

KEYWORDS: Life Insurance; Mortality Graduation; IFRS 17; Solvency II; Capital Requirements.

RESUMO

A definição adequada de pressupostos biométricos, como a mortalidade, é fundamental para companhias de seguro do ramo vida tanto no âmbito do IFRS 17 como no de Solvência II, visto que estes quadros regulamentares exigem uma avaliação prudente e fiável das responsabilidades técnicas. Este estudo visa reavaliar a abordagem de modelação da mortalidade utilizada por uma seguradora do ramo vida, comparando a prática atual, onde é aplicado um ajuste percentual uniforme a uma tábua padrão de mortalidade para todas as idades, com técnicas mais sofisticadas baseadas na graduação da mortalidade. Foram exploradas duas abordagens: a graduação com referência a uma tábua padrão e graduação através de fórmula paramétrica. A análise incidiu exclusivamente sobre contratos de seguro de vida para produtos que cobrem o risco de mortalidade, excluindo aqueles com cobertura do risco de longevidade, como as rendas. Neste âmbito, os modelos que apresentaram melhor desempenho, com base em estatísticas relevantes e aplicados sem distinção por sexo, foram o ajuste exponencial à tabela GKM80 e o modelo paramétrico $GM(2,3)$. As taxas de mortalidade obtidas foram aplicadas como pressupostos atuariais e avaliadas nos regimes contabilístico e prudencial. Em ambos os casos, as taxas revistas proporcionaram um melhor ajuste à mortalidade observada, aumentado a precisão das projeções financeiras. Este resultado reforça a importância de rever e atualizar regularmente os pressupostos atuariais, garantindo a fiabilidade dos modelos e o cumprimento dos requisitos regulamentares. Importa salientar que esta aplicação resultou igualmente numa redução das responsabilidades nos regimes em questão. Adicionalmente, o recálculo dos requisitos de capital no submódulo de risco de subscrição vida evidenciou uma melhoria da posição de solvência, traduzida num aumento do rácio de solvência. Apesar de o modelo paramétrico ter superado marginalmente a alternativa em termos de redução dos desvios relativos entre os fluxos de caixa projetados e observados, a graduação com referência à tábua foi, em última análise, recomendada para implementação, devido ao seu equilíbrio entre precisão, simplicidade operacional e alinhamento com as práticas vigentes da empresa. A adoção desta abordagem permitirá assegurar que os pressupostos de mortalidade se mantenham práticos e fiáveis, promovendo avaliações atuariais fidedignas e garantindo o cumprimento dos requisitos regulatórios.

PALAVRAS-CHAVE: Ramo Vida; Graduação de Mortalidade; IFRS 17; Solvência II; Requisitos de Capital.

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Acronyms and Abbreviations

BEL Best Estimate Liabilities

BSCR Basic Solvency Capital Requirement

EIOPA European Insurance and Occupational Pensions Authority

EOF Eligible own funds

IFRS International Financial Reporting Standards

INE Instituto Nacional de Estatística

LIC Liability for incurred claims

LRC Liability for remaining coverage

LAC-DT Loss-absorbing capacity of deferred taxes

LAC-TP Loss-absorbing capacity of technical provisions

MSE Mean squared error

RJASR Regime jurídico de acesso e exercício da atividade seguradora e resseguradora

RM Risk Margin

SCR Solvency Capital Requirement

TVOG Time value of options and guarantees

WSS Weighted sum of squares

1 INTRODUCTION

This report is based on a six-month internship at Lusitania Vida within the actuarial function team and focuses on formulating biometric assumptions for life insurance, specifically mortality. The goal is to develop an actuarial evaluation process for determining reliable mortality assumptions that comply with both statutory and prudential standards.

In life insurance, the accurate valuation of liabilities is crucial. IFRS 17, the International Financial Reporting Standard that provides a framework for the financial reporting of insurance contracts, imposes requirements on their measurement. Under the General Measurement Model, the initial recognition of a group of insurance contracts involves calculating the fulfillment cash flows, which comprise unbiased estimates of future payments, an adjustment for the time value of money, and a risk adjustment for non-financial risks. This will apply only to contracts that expose the insurer to significant insurance risk.

In parallel, Solvency II, the prudential framework for insurers in the European Union, requires the valuation of technical provisions, the amount necessary to fulfill all the liabilities arising from the insurance policies, to be done in a prudent, reliable, and objective manner, as stated under paragraph 4 of Article 91 of *Regime jurídico de acesso e exercício da atividade seguradora e resseguradora* (RJASR), approved by Law 147/2015, 9th September. Under Solvency II, Article 92 of RJARS clarifies that technical provisions are obtained as the sum of the best estimate, which corresponds to the expected value of future cash flows weighted by their probability of occurrence, taking into account the time value of money, and a risk margin. According to Article 28 of the Commission Delegated Regulation (EU) 2015/35 of 10 October 2014, which outlines the cash flows to be included in the calculation of the best estimate, our focus is on the benefit payments to policyholders and beneficiaries.

Choosing appropriate mortality assumptions is essential under both regulatory frameworks. Accurate estimates are key for pricing and reserving, as well as for maintaining financial stability and ensuring compliance with capital requirements, as any overestimation or underestimation of mortality risk directly impacts the insurer's solvency ratio. The importance of selecting appropriate mortality assumptions is further reinforced by Article 272 of the Commission Delegated Regulation (EU) 35/2015, of 10 October 2014, which states that "the actuarial function shall assess whether the methodologies and assumptions used in the calculation of the technical provisions are appropriate for the specific lines of business of the undertaking".

A common practice in the life insurance industry to model mortality is the use of standard mortality tables, which are calculated with information from large populations and give estimates of the probabilities of death at certain ages. Keeping these tables up-to-date is crucial, as outdated models can lead to mispricing, insufficient reserves, and incorrect risk valuations. Many insurers use tables that are periodically updated by regulatory bodies or actuarial organizations, often applying adjustments to better reflect their own mortality experience. However, these tables may not fully capture the specific mortality dynamics of a company's insured population. Factors such as medical advancements, lifestyle changes, and portfolio-specific characteristics can lead to significant deviations from industry-wide assumptions. As a result, insurers must assess whether their current mortality assumptions remain suitable or if alternative models can provide more realistic ones.

This study aims to refine mortality assumptions based on the experience of a specific life insurance portfolio, with a primary focus on testing and comparing multiple models to evaluate their fit to observed data. To achieve this, the method of mortality graduation, a widely researched method in actuarial science (Dodd et al., 2017; da Rocha Neves and Migon, 2007), is employed, ensuring that the derived mortality rates progress smoothly while being consistent with actuarial principles. The study will also evaluate the implications of the chosen model under the accounting (IFRS 17) and prudential (Solvency II) frameworks. Ultimately, improving mortality estimates will support both financial stability and profitability by enabling more reliable reserve and premium calculations, ensuring compliance while strengthening the financial health of the company.

Several studies have applied mortality graduation techniques, though often in contexts different from the present one. Some focus on fitting parametric formulas to regional populations, emphasizing alternative methods for parameter estimation (Debón et al., 2005). Others concentrate on pension portfolios, either by modeling individual risk factors to obtain smooth mortality hazards (Richards et al., 2013) or by applying multiple parametric graduation approaches to Canadian pensioners (Society of Actuaries, n.d.). While the latter works primarily address longevity risk in annuity or pension settings, the present study examines a life insurance portfolio.

In this context, the analysis is restricted to products covering the risk of death. In principle, it would not be prudent to update assumptions for death benefits without also considering annuities, since policyholders may hold both types of contracts. However, two factors justify the narrower scope adopted here. First, the company's annuity portfolio is too limited to support a statistically robust graduation study. Second, even if annuities were included, combining them with death-benefit products would not be consistent with the company's practice, as Lusitania Vida determines biometric assumptions separately by product group and intends to maintain this separation. For these reasons, longevity assumptions are not reassessed in this report, although the interdependence between mortality and longevity risks is acknowledged.

This report is divided into five chapters, including the current one. Chapter 2 introduces basic concepts used in mortality theory, presents methods of graduation, explores parametric formulas, discusses techniques for model comparison and adequacy assessment, and provides a brief introduction to the statutory and prudential regimes. Chapter 3 provides an overview of the data, describes the data processing and exploratory analysis, and discusses the methodology used. Chapter 4 brings together the application of mortality graduation techniques and the evaluation of their financial impact under both regulatory frameworks, IFRS 17 and Solvency II. It begins with the selection of mortality models based on key performance statistics and practical considerations. This is followed by the projection and analysis of cash flows under different assumptions, a backtesting exercise comparing projections to actual outcomes, and an assessment of the impact of changes in mortality assumptions on the Solvency Capital Requirement and solvency ratio. Finally, Chapter 5 summarizes the key findings and offers recommendations for further research and model refinement.

2 KEY CONCEPTS

2.1 Basic concepts in mortality theory

In this section, some concepts commonly used in mortality modeling will be presented. The notions exposed below can be found in Dickson et al. (2019); Macdonald et al. (2018).

2.1.1 The future lifetime random variable and related functions

Let (x) denote a life aged x, where $x \geq 0$. Let T_x be a continuous random variable denoted as the future lifetime of (x), in years. Thus, $x + T_x$ represents the age-at-death random variable.

The force of mortality at age x is defined as

$$\mu_x = \lim_{dx \to 0^+} \frac{1}{dx} \Pr(T_0 \leq x + dx \mid T_0 > x). \tag{1}$$

The probability that (x) does not survive beyond age $x + t$ is defined as

$$_tq_x = \Pr(T_x \leq t). \tag{2}$$

The relationship between $_tq_x$ and μ_x can be established through the equation

$$_tq_x = 1 - \exp(-\int_0^t \mu_{x+s}\, ds). \tag{3}$$

If $t = 1$, the probability of death is referred to as the mortality rate and the symbol q_x is used as a convention. Additionally, by assuming that for age intervals of one year, the force of mortality can be approximated at the midpoint, i.e. $\mu_{x+s} \approx \mu_{x+\frac{1}{2}}$, for $0 \leq$ s < 1, the following holds:

$$q_x \approx 1 - \exp(-\mu_{x+\frac{1}{2}}). \tag{4}$$

2.1.2 Central Exposure to Risk and Crude Mortality Rates

The central exposure to risk of death of an insured individual is defined as the total observed time at risk. When looking at *n* individuals, the central exposed-to-risk at age x, E^c_x, is given by the sum of the individual central exposures to risk between ages x and $x + 1$. For the exact calculation of this metric of exposure, it is necessary to have a record of all dates of birth, dates of entry into observation, and dates of exit from observation.

Next, let us define D_x as the random variable representing the number of deaths for individuals aged x and d_x as its realization. It is common to model this random variable as following a Poisson distribution with parameter $\lambda = E_x^c \mu_{x+\frac{1}{2}}$ (Brouhns et al., 2002). One of the shortcomings of this model is that it allows a non-zero probability of more deaths than the number of individuals at study. However, this is often a good approximation and is taken as a starting point for the actuarial analysis of mortality data.

Estimates of the force of mortality can be obtained using the maximum likelihood method and are often referred to as crude mortality rates. By using the same age definition to calculate d_x and E_x^c, the resulting estimate of μ corresponds to the midpoint of the rate interval, in this case $x + \frac{1}{2}$. It is shown by Institute and Faculty of Actuaries (2023) that

$$\hat{\mu}_{x+\frac{1}{2}} = \frac{d_x}{E_x^c}. \tag{5}$$

Then, using (4) and applying the invariance principle of maximum likelihood estimators, an estimate for the mortality rate at the beginning of the rate interval can be obtained as follows:

$$\hat{q}_x \approx 1 - \exp(-\hat{\mu}_{x+\frac{1}{2}}). \tag{6}$$

2.2 Graduation

In practice, an investigation includes a wide range of ages. The mortality rates obtained using (6) will not be the final ones published. This is because they still need to go through a process called graduation. By using statistical techniques, a set of rates that progress smoothly is produced, which is a particularly important characteristic for insurance companies, whose goal is to avoid sudden changes and inconsistencies in mortality when calculating premiums and reserves. Another important feature of the graduated rates is the adherence to the data, and a compromise between the two must be made.

As stated by Institute and Faculty of Actuaries (2023), graduation is commonly performed using one of three methods:

- graduation by reference to a standard table;
- graduation by parametric formula;
- graduation using spline functions.

The scope of this paper is limited to the first two methods, as they are the most established and also simpler to apply. Given the exploratory nature of this work and time constraints, the decision to start with straightforward approaches was made. More complex methods like splines can be considered later, if simpler models are proved inadequate.

2.2.1 *Graduation by reference to a standard table*

In actuarial science, when internal data is considered insufficient, it is common practice to rely on established life tables as a basis for estimating mortality rates. These tables, commonly known as standard tables, are derived from extensive and reliable population data. By applying appropriate transformations to such references, actuaries can approximate the mortality experience of a specific portfolio. This method follows the premise that the true underlying mortality rates can be closely modeled by adjusting a suitable external table. The process involves selecting a reference table that best reflects the demographic characteristics of the insured population, such as age, sex, and geographical region. Once an appropriate table is identified, the next step is to choose a function that links the mortality rates of this reference to the graduated rates derived from crude estimates:

$$\mathring{\mu}_{x+\frac{1}{2}} = f(\alpha_1, ..., \alpha_n, \mu^s_{x+\frac{1}{2}}), \tag{7}$$

where $\mathring{\mu}_{x+\frac{1}{2}}$ is the graduated crude mortality rate at age x, α=(α_1,...,α_{r+s}) is a vector of coefficients and $\mu^s_{x+\frac{1}{2}}$ is the force of mortality of a standard table at age $x + \frac{1}{2}$.

Once a possible relationship has been identified, the parameters are estimated by

- maximum likelihood: the goal is to minimize the logarithm of the total likelihood. If D_x follows a Poisson distribution, the likelihood function is

$$\prod_x \frac{(E^c_x \mathring{\mu}_{x+\frac{1}{2}})^{d_x} \exp(-E^c_x \mathring{\mu}_{x+\frac{1}{2}})}{d_x!}; \tag{8}$$

- least squares: the goal is to minimize

$$\sum_x w_x (\hat{\mu}_{x+\frac{1}{2}} - \mathring{\mu}_{x+\frac{1}{2}}), \tag{9}$$

where the w_x are suitable weights, for example, the exposures to risk (E^c_x).

2.2.2 *Graduation by parametric formula*

For large datasets, graduation typically involves fitting a parametric formula to the crude mortality estimates, with numerical methods used to estimate the unknown parameters. One should be careful when choosing a parametric formula, as including too many parameters will provide greater flexibility to the model at the cost of adherence to the data and vice versa. The search for a suitable function benefits from visual inspection of the data. Once selected, the parameters are estimated to fit the model, similarly as in 2.2.1, to derive graduated mortality rates. Finally, the model's fit is evaluated by applying statistical tests to assess how well the graduated rates align with the observed data.

2.3 Parametric formulas

This section reviews widely used mortality laws, recognized for their empirical relevance. The corresponding formulas are shown in Table I.

One of the first models was defined by Gompertz (1825), where mortality is assumed to increase exponentially with age, a simplification that has proven to be a reasonable model for human mortality, especially for adult lives.

In order to consider cases of accidental death, a fixed non-negative term θ unrelated to age was introduced by Makeham (1860). This extends Gompertz's law by adding a parameter that captures external risks.

A generalized Gompertz-Makeham family of laws, denoted as $GM(r, s)$, was proposed by Forfar et al. (1988) and incorporates as many parameters $r + s$ as are found to be significant, r and s being non-negative integers, not both zero. The Continuous Mortality Investigation (CMI) has used the $GM(r, s)$ formulae for a number of years to perform graduation. Note that the Gompertz's and Makeham's formulas of Table I can be obtained from this by setting ($r = 0, s = 2$) and ($r = 1, s = 2$), respectively.

The previous models assume that the force of mortality increases exponentially with age. However, several studies, including those by Perks (1932) and Beard (1959), have shown that mortality rates tend to decelerate at advanced ages, eventually stabilizing at a constant level. This phenomenon is commonly modeled using logistic-type functions, with Beard's model representing a simplified version of Perks' formulation.

TABLE I: MORTALITY LAWS.

Model	Force of mortality μ_x	Constraints
Gompertz	$\alpha \exp(\beta x)$	$\alpha, \beta > 0$
Makeham	$\theta + \alpha \exp(\beta x)$	$\alpha, \beta > 0, \theta \geq 0$
Perks	$\dfrac{\alpha \exp(\beta x) + \gamma}{\delta \exp(\beta x) + 1}$	$\alpha, \beta, \delta, \gamma \geq 0$
Beard	$\dfrac{\alpha \exp(\beta x)}{\delta \exp(\beta x) + 1}$	$\alpha, \beta, \delta \geq 0$
$GM(r, s)$	$\sum_{i=0}^{r-1} a_i x^{i-1} + \exp\left(\sum_{j=0}^{s-1} b_i x^j\right)$	

2.4 Model selection

2.4.1 Information Criteria

In Section 2.2, three approaches to graduation were introduced, of which two were explored in detail. After estimating the parameters using the maximum likelihood method, models can be compared using information criteria. These criteria, which depend on both the log-likelihood and the number of parameters, tend to reward models that adhere well to the data while penalizing those excessively complex. Generally, the most used information criteria are:

- Akaike's Information Criterion - proposed by Akaike (1987), it depends simply on the log-likelihood, *l*, and the number of parameters, *k*, as follows:

$$AIC = -2l + 2k. \tag{10}$$

  A lower AIC indicates a better model fit, accounting for both the goodness of fit and the risk of overfitting.

- Bayesian Information Criterion - proposed by Schwarz (1978), it is similar to AIC but adds a penalty based on the number of independent observations, *n*, as follows:

$$BIC = -2l + k \ log\, n. \tag{11}$$

  A lower BIC indicates a model with a better trade-off between fit and complexity.

The main drawback of relying solely on information criteria is that limited insight is provided about how well the model fits the data at specific ages.

2.4.2 Chi-squared Test

The Chi-squared test is a standard goodness-of-fit measure used in this context to evaluate the discrepancy between observed and expected deaths. It is based on the test statistic:

$$X = \sum_x z_x^2\,, \tag{12}$$

where z_x represents the standardized deviation of deaths at each age. Under the null hypothesis, according to which the fitted model provides an adequate representation of the mortality experience, this statistic follows a χ^2 distribution with degrees of freedom equal to the number of age groups minus the number of fitted parameters and constraints imposed.

To compute z_x, the Poisson distribution of the number of deaths D_x is first approximated by a Normal distribution, as follows:

$$D_x \overset{a}{\sim} N(E_x^c \mathring{\mu}_{x+\frac{1}{2}}, E_x^c \mathring{\mu}_{x+\frac{1}{2}}). \tag{13}$$

Based on this approximation, the standardized deviations are expressed as

$$z_x = \frac{D_x - E_x^c \mathring{\mu}_{x+\frac{1}{2}}}{\sqrt{E_x^c \mathring{\mu}_{x+\frac{1}{2}}}}. \tag{14}$$

As a result, z_x follows an approximate standard Normal distribution:

$$z_x \overset{a}{\sim} N(0, 1). \tag{15}$$

However, this approximation may not hold when the expected number of deaths is small. In such cases, Forfar et al. (1988) suggest grouping adjacent ages to ensure that each group contains at least five expected deaths, improving the validity of the test.

Goodness of fit is evaluated by comparing the observed statistic to the upper $\alpha\%$ point of the distribution. If the observed value is significantly large, there is strong evidence to reject the null hypothesis, indicating a poor fit.

2.4.3 Mean Squared Error

Instead of testing the statistical significance of deviations, accuracy can be measured by the difference between crude estimates and graduated rates. By setting m as the number of age groups, the Mean Squared Error (MSE) can be defined as

$$\text{MSE} = \frac{1}{m} \sum_{x=1}^{m} (\hat{\mu}_{x+\frac{1}{2}} - \mathring{\mu}_{x+\frac{1}{2}})^2. \tag{16}$$

A lower MSE indicates a closer fit to observed mortality rates, serving as a useful measure of in-sample goodness of fit. However, because it assesses only the fit to the calibration dataset, it does not guarantee the model's predictive accuracy for future data. Thus, while a low MSE highlights fewer discrepancies within the observed data, it should be complemented with other evaluation methods to assess forecasting accuracy.

2.4.4 Weighted Sum of Squares

Small deviations in groups with higher exposure can greatly affect the overall model accuracy. To account for this, weighted measures like the Weighted Sum of Squares (WSS) assign greater importance to these groups, providing a more reliable assessment of model performance. Its interpretation is similar to that of the MSE, but with an added emphasis on high-exposure groups. It is defined as follows:

$$\text{WSS} = \sum_{x=1}^{m} E_x^c(\hat{\mu}_{x+\frac{1}{2}} - \mathring{\mu}_{x+\frac{1}{2}})^2. \tag{17}$$

2.4.5 *Actual-to-Expected Ratio*

While useful for assessing the adequacy of mortality models, previous measures rely on squared deviations and fail to indicate the direction of any potential bias. The Actual-to-Expected (A/E) ratio is sometimes used in the life insurance industry and offers a more intuitive accuracy check by directly comparing observed to expected outcomes, as follows:

$$\text{A/E} = \frac{1}{m}\sum_{x=1}^{m} \frac{\text{Actual number of deaths at age}\, x}{\text{Expected number of deaths at age}\, x}. \tag{18}$$

The denominator is obtained as the product of exposure and crude estimates, as defined in equation (5). This measure stands out for its simplicity and interpretability: values greater than 1 suggest that actual mortality exceeds expectations, meaning an underestimation that could lead to inadequate reserves and compromise the company's ability to meet future obligations.

2.5 *Basic concepts in IFRS 17*

IFRS 17 is the international accounting standard for insurance contracts, aiming to ensure transparency and comparability in financial reporting (IFRS Foundation, 2021). Under this framework, insurance liabilities are separated into two components: the liability for the remaining coverage (LRC), which reflects obligations related to future services, and the liability for incurred claims (LIC), which covers claims for events that have already occurred. Changes in mortality assumptions affect only the LRC, as they alter the fulfillment cash flows related to future death claims. In contrast, the LIC remains unchanged, since it is based on claims already incurred, for which the payout amounts are generally known.

For life insurance products, modifications in mortality assumptions primarily affect the expected value of death claims cash flows. These changes can also influence other components by altering the number of in-force policies. Under IFRS 17, there are three measurement models: the General Measurement Model, the Premium Allocation Approach, and the Variable Fee Approach. Only the first model is considered in this report, with the analysis limited to nominal LRC cash flows, excluding additional components such as discounting, the risk adjustment, and the contractual service margin.

2.6 Basic concepts in Solvency II

Solvency II is the prudential regulatory framework that applies to insurance and reinsurance undertakings in the European Union (European Commission, 2015). It establishes market-consistent principles for valuing liabilities and setting capital requirements to ensure that insurers remain solvent under adverse conditions.

Technical provisions, included in the liabilities, are defined as the sum of the Best Estimate Liabilities (BEL) and the Risk Margin (RM). The BEL reflects the expected present value of future cash flows, such as premiums, claims, and expenses, associated with insurance obligations, based on probability-weighted scenarios and discounted for the time value of money. The RM represents an estimate of the additional amount required by a third party to accept the transfer of the insurance portfolio and is calculated using a cost-of-capital approach, which involves projecting future capital requirements.

Unlike IFRS 17, which only covers contracts with significant insurance risk, Solvency II applies to all types of contracts, including those with minimal insurance components. As a result, projected values for claims, premiums, and expenses are generally higher under Solvency II. Nevertheless, while the absolute values may differ, the relative impact of changes in mortality assumptions remains broadly comparable between the two frameworks.

Changes in mortality assumptions directly affect the BEL, as they influence the projection of future cash flows associated with biometric risks. The RM will also be indirectly impacted, as it depends on projected solvency capital requirements, including shocks to mortality. Since the recalculation of the RM involves significant complexity and falls outside of the scope of this work, the focus will be exclusively on the direct impact on the best estimate.

Beyond the economic balance sheet, Solvency II requires insurers to hold sufficient capital to absorb unexpected losses, quantified through the Solvency Capital Requirement (SCR). The SCR is designed to ensure that insurers can withstand adverse events over a one-year horizon with a confidence level of 99.5%, and it is calculated as follows:

$$SCR = BSCR + SCR_{operational} + Adj, \tag{19}$$

where $BSCR$ is the Basic Solvency Capital Requirement, $SCR_{operational}$ represents the charge for operational risk and Adj corresponds to an adjustment for the loss-absorbing capacity of technical provisions and deferred taxes (LAC-TP and LAC-DT, respectively).

This adjustment accounts for the portion of losses that can be absorbed by reducing future discretionary benefits or by changes in the value of deferred tax assets and liabilities.

The BSCR aggregates the capital requirements from various risk modules. Using a predefined correlation matrix, it can be expressed as

$$BSCR = \sqrt{\sum_{i,j} Corr_{i,j} \cdot SCR_i \cdot SCR_j}, \tag{20}$$

with $i, j \in \{market, counterparty\_default, life, health, non\_life, intangibles\}$.

Specifically, for the life underwriting risk module, the capital charge is obtained as

$$SCR_{life} = \sqrt{\sum_{i,j} Corr_{i,j} \cdot SCR_i \cdot SCR_j}, \tag{21}$$

with $i, j \in \{mortality, longevity, disability, life_{expense}, revision, lapse, life_{catastrophe}\}$.

The focus in this report will be on the mortality risk sub-module, which assesses the increase in liabilities resulting from a permanent 15% rise in mortality rates, and the life catastrophe risk sub-module, which considers an immediate 0.15 percentage point increase in mortality rates, representing a sudden catastrophic event. The impact of both shocks is reflected as a change in Basic Own Funds (BOF), defined as the excess of assets over technical provisions and other liabilities. Assuming asset values remain unchanged, any increase in BEL, net of reinsurance, due to the mortality and catastrophe shocks, translates directly into a higher SCR.

A central indicator of an insurer's financial strength is the solvency ratio, given by

$$Solvency\ ratio = \frac{EOF}{SCR}, \tag{22}$$

where EOF represents the Eligible Own Funds to meet the SCR, subject to regulatory criteria regarding their availability and quality. A ratio above 100% implies that the company holds more capital than required to withstand adverse scenarios, with higher ratios reflecting a stronger solvency position.

3 Data and Methodology

In this section, all procedures were carried out using the R software, particularly with the package tidyverse (Wickham et al., 2019).

3.1 Data base

Since this report focuses on a mortality study, only policies that cover the event of death with a predefined sum assured were considered. A smaller portfolio in run-off linked to mortgages and comprising 3037 policies, from which 1083 were single-life, was acquired from another life insurance company at the end of 2016. As it is managed independently from the main portfolio, it was excluded from the scope of this analysis and will be considered separately.

The main dataset comprises 898,684 single-life policies, which were first aggregated into a single data frame. These policies were previously categorized in the following groups:

- Risk: policies primarily designed to cover the risk of mortality;
- Credit: life insurance linked to loan repayment in case of the policyholder's death;
- Annual Renewable Term (ART): similar to Credit, but are temporary life insurance policies that renew annually;
- Investment: policies designed primarily as financial instruments, offering a return on investment alongside life coverage;
- Savings: life insurance products that accumulate savings over time, sometimes with a guaranteed return;
- Retirement: policies aimed at providing financial security during retirement, often linked to pension schemes or annuities.

For a breakdown of policies by category, see Figure 1. Each policy contained the following information:

- Policy key - a unique identifier containing the modality, policy number, and certificate number;
- Issue date - the date the coverage begins, ranging from the year 1987 to 2024;

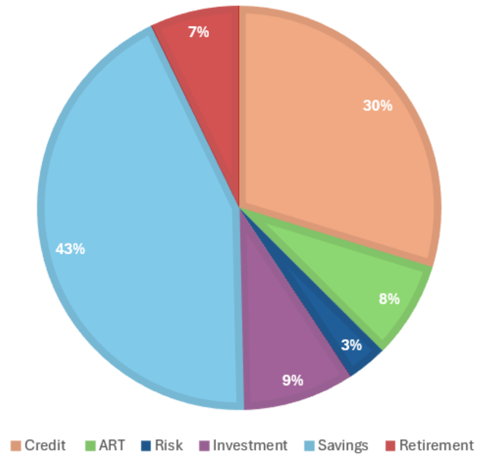

FIGURE 1: Policy categories.

Source: Own elaboration

- Term date - the date the policy ceases to be in force, provided no early termination occurs;
- Annulment date - the date at which the policy was annulled, if applicable;
- Maturity - the policy's duration in years;
- Client number - a unique identifier of each individual insured;
- Date of birth of the policyholder;
- Sex of policyholder: categorized as male, female, or unspecified.

The following exclusions and modifications were applied:

- Policies with missing entries for both issue and term dates were excluded. For policies missing only one of these dates, an inference was made using the maturity information (e.g., for a policy with an issue date of 01/01/2018, no term date, and a maturity of one year, the term date was set to 01/01/2019). If no inference was possible, the policy was excluded;
- Dates with a day greater than the number of days in the respective month (up to a maximum of 31) were corrected to the last valid day, accounting for leap years. If the day was recorded as 0, it was adjusted to 1;
- Policies with invalid dates were excluded (e.g., issue date prior to the company's establishment or after the term date);

- Policies with policyholders who were minors at the time of subscription were excluded, as determined by the difference between the date of birth and the issue date. These cases are likely due to subscription errors, since only adults can hold policies;
- Policies with a term or annulment date that occurred on or before the issue date were excluded;
- Policies with unspecified policyholder sex were classified as male as a precautionary measure, given that male mortality is generally higher than female mortality;
- A variable representing the death date, if applicable, was retrieved from another claims database and added to the policy data, with a correspondence of both the policy key and date of birth;
- Policies with death dates outside the coverage period or with invalid death dates were excluded.

A total of 778,050 policies remained after exclusions were made.

3.2 Data analysis

Table II shows descriptive statistics of policyholders' ages at subscription. The close median and mean indicate a nearly symmetrical distribution. The 1st and 3rd quartiles reveal that half the policies were subscribed between ages 32 and 52, suggesting most policies are taken out during mid-life. The presence of policyholders as old as 100 can be explained by the demand for investment-focused policies, which typically have more flexible underwriting criteria (e.g., no age limits) than traditional life insurance products.

TABLE II: SUMMARY STATISTICS OF SUBSCRIPTION AGES.

Minimum	1st Quartile	Median	Mean	3rd Quartile	Maximum
18.00	32.00	42.00	42.79	52.00	100.00

Regarding the distribution by sex, males account for 55.44% of the policies, while females represent 44.56%, expressing a slight predominance of male policyholders.

Out of all policies under study, only 6,318 had an associated death claim, accounting for just 0.81% of the total.

Linking client numbers to policy keys allowed identification of the number of policies held by each policyholder, as shown in Figure 2.

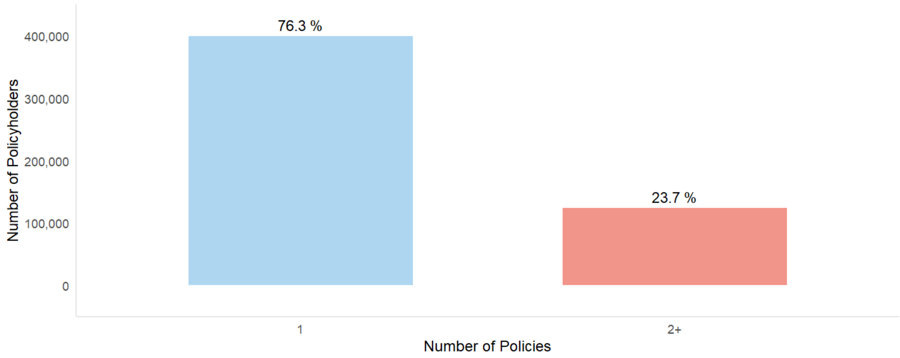

FIGURE 2: Number of policies per policyholder.

Source: Own elaboration

When individuals hold multiple policies within the same life insurance portfolio, dependence arises between observations, as two distinct death claims may originate from the death of a single policyholder. Forfar et al. (1988) have proven that the presence of duplicate policies does not alter the crude rates of mortality, but increases the variance of the number of death claims, which can affect the calculation of confidence intervals. If the number of policies held by each policyholder is known, they propose an adjustment by using the variance ratio at age x, r_x (Forfar et al., 1988). Let π_i^x be the proportion of the total lives aged x that own *i* insurance policies. Then, r_x is given by:

$$r_x = \frac{\sum_i i^2 \pi_i^x}{\sum_i i \pi_i^x}. \tag{23}$$

This ratio quantifies the increase in variance of death claims at age x due to multiple policies held by individual policyholders. To adjust for this, both the observed deaths and exposure to risk at each age x are divided by r_x before further analysis, as follows:

$$\hat{\mu}_{x+\frac{1}{2}} = \frac{\frac{d_x}{r_x}}{\frac{E_x^c}{r_x}} \tag{24}$$

Additionally, the variance of the random variable D_x is multiplied by r_x to account for this adjustment.

3.3 Central exposure to risk

As discussed in Section 2.1, central exposure to risk can be exactly calculated using the policyholder's date of birth, policy issue date, and exit date (the earliest of term, annulment, or death). For a policy issued at time t_0, with t_e denoting the exit date, the exposure in calendar year y corresponds to the number of days during which the policy provides risk coverage within that year. Assuming that coverage starts at the beginning of the issue day and ends at the end of the exit day, individual exposure is calculated as:

- $t_e - t_0 + 1$, if start and exit both fall within year y;
- $t_e - 01/01/y + 1$, if the policy had already been issued and ends in year y;
- $31/12/y - t_0 + 1$, if start falls within year y and exit afterwards;
- the total number of days in year y, if in-force throughout the entire year y;
- 0, in any other case.

The central exposure to risk of an individual aged *x* in year y is given by:

$$E^c_{x,y} = \frac{\text{Number of days of exposure for age } x \text{ in year } y}{\text{Total number of days in year } y}, \tag{25}$$

where the denominator is 365 or 366, depending on whether year y is a leap year.

Correct allocation of exposure by age is achieved by defining $z_{x,y}$ as the proportion of an individual's exposure at age *x* during year y, with the remainder, 1- $z_{x,y}$, assigned to age *x*+1. As an illustration, consider a policyholder who holds a policy from January 1^{st} till June 30^{th} of 2000 and whose 21^{st} birthday is on February 1^{st} of that year. In this case,

$$\begin{aligned} z_{20,2000} &= \frac{31}{182} \implies E_{20,2000} = z_{20,2000} \times \frac{182}{366} = \frac{31}{366}, \\ z_{21,2000} &= \frac{151}{182} \implies E_{21,2000} = z_{21,2000} \times \frac{182}{366} = \frac{151}{366}. \end{aligned}$$

The total central exposure to risk by age and year results from iteratively computing these formulas for each policyholder and summing across all policies.

3.4 Mortality rates estimates

Before applying mathematical graduation, the raw data may be examined graphically to reveal underlying patterns. Using the aggregated death and exposure data by age and year, mortality rate estimates $\hat{q}_x$ were obtained using equations (5) and (6) and are shown in Figure 3. As expected, male mortality rates are consistently higher than female rates, with the gap widening as age increases. The graph reflects the typical exponential increase in mortality from around age 40 onward, while fluctuations at older ages likely result from data sparsity or increased variance due to fewer exposures.

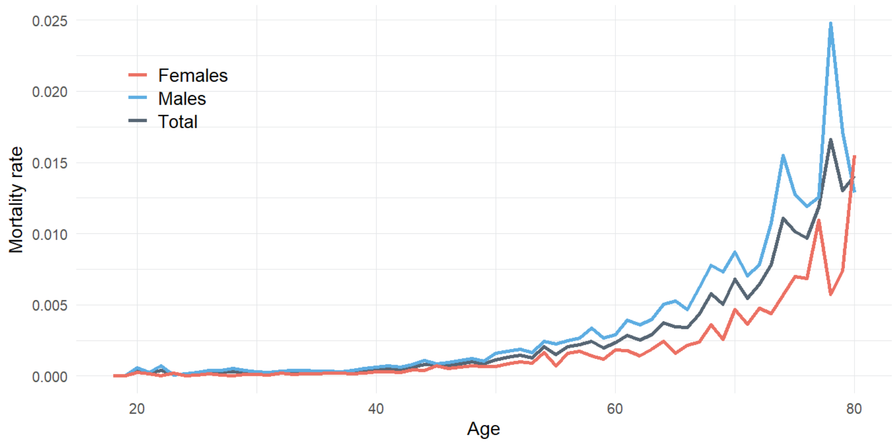

FIGURE 3: Mortality rate estimates.

Source: Own elaboration

In Portugal, the average life expectancy at birth for the 2021-2023 triennium was 81.17 (Instituto Nacional de Estatística, 2024). Given this, and considering the reduced amount of exposure at older ages, the aggregated death and exposure data were truncated at age 81. Additionally, ages 18 and 19 were excluded from the graduation process due to the absence of observed deaths in these age groups.

3.5 Two methodological issues

3.5.1 Policyholders with more than one policy

Only single-life policies are included in the analysis due to the absence of widely accepted methods for performing mortality graduation on joint-life policies. Furthermore, the joint-life dataset is insufficiently large to ensure statistical robustness, making it unsuitable for meaningful graduation. While this limitation narrows the study's scope, it ensures that the graduation methods remain practical and robust within the context of available techniques.

3.5.2 Incorporating Chebyshev polynomials

For improved fit, scaling, and parameter stability, the $GM(r,s)$ models can be restructured to incorporate Chebyshev polynomials of the first kind, as demonstrated in the methodology used by Ramonat and Kaufhold (2018). Using the age transformation $T(x) = \frac{x-70}{50}$, the force of mortality is formulated as follows:

$$\mu_x = \sum_{i=0}^{r-1} a_i C(i, T(x)) + \exp\left(\sum_{j=0}^{s-1} b_i C(j, T(x))\right), \tag{26}$$

where the Chebyshev polynomials $C(N, X)$ are defined recursively as:

$$C(N,X) = \begin{cases} 1 & \text{if } N = 0 \\ X & \text{if } N = 1 \\ 2XC(N-1,X) - C(N-2,X) & \text{if } N \geq 2. \end{cases} \tag{27}$$

3.6 Company's current practice

For comparison, it is useful to summarize Lusitania Vida's current approach to deriving mortality assumptions. The company aggregates policy and claim data at the product-group level, applies a basic cleaning process, and calculates annual exposures based on the number of days a policy is in-force, aggregated across first and second insured lives, when applicable. Using these exposures together with the standard mortality table GKM80, the expected number of deaths is estimated for each age and year and compared with observed deaths to obtain actual-to-expected ratios. The final percentage adjustment applied to the standard table is derived from the average of these ratios, typically using only the most recent years. In some product groups, an additional margin or an IBNR factor is also incorporated to account for deaths occurred but not yet reported.

4 Application on Mortality Graduation and Financial Impact

4.1 Mortality Graduation

Before beginning the graduation process, it is important to note that the smaller run-off portfolio was excluded from the analysis due to its unique characteristics and size. A separate evaluation of this exclusion revealed negligible differences in the final conclusions. Given the portfolio's limited impact on the overall technical provisions, applying the same graduated mortality assumptions across all portfolios is justified. With this clarification, the graduation approaches in this section consistently use maximum likelihood estimation for parameter fitting, ensuring comparability across models and supporting the robustness of the results.

4.1.1 Graduation with reference to standard tables and no distinction by sex

The first step involves identifying appropriate mortality tables believed to represent the portfolio's mortality experience, subject to minor transformations. At this stage, only the male versions of the tables were used to adopt a more conservative assumption, given that male mortality rates are typically higher. The distinction by sex is addressed in a later section of this report. For this study, the Swiss tables GKM80 and GKM95 (Society of Actuaries, n.d.) were selected due to their established use in life insurance. Additionally, Portugal's most recent mortality tables, INE 2020/2022 and INE 2021/2023, were also included in the analysis (Instituto Nacional de Estatística, n.d.). The forces of mortality for each table were derived from the mortality rates using approximation (4). To help identify an appropriate model, Figure 4 compares the estimated forces of mortality, $\hat{\mu}_{x+\frac{1}{2}}$, with those from the standard tables, $\mu^s_{x+\frac{1}{2}}$.

To ensure smoothness, the relationship used must be simple and involve few parameters. Three functional forms, referred to as links (1), (2), and (3), respectively, were tested:

- $\overset{\circ}{\mu}_x = a + b\mu^s_x$ - a linear adjustment allowing direct scaling of the standard mortality;
- $\overset{\circ}{\mu}_x = (a + bx)\mu^s_x$ - extends the previous model by letting the scaling factor vary linearly with age, addressing the observed differences between age groups;
- $\overset{\circ}{\mu}_x = \mu^s_x * \exp(a + bx)$ - introduces non-linearity through an exponential term, capturing increasing deviations at older ages.

The first two were suggested by Institute and Faculty of Actuaries (2023), while the third,

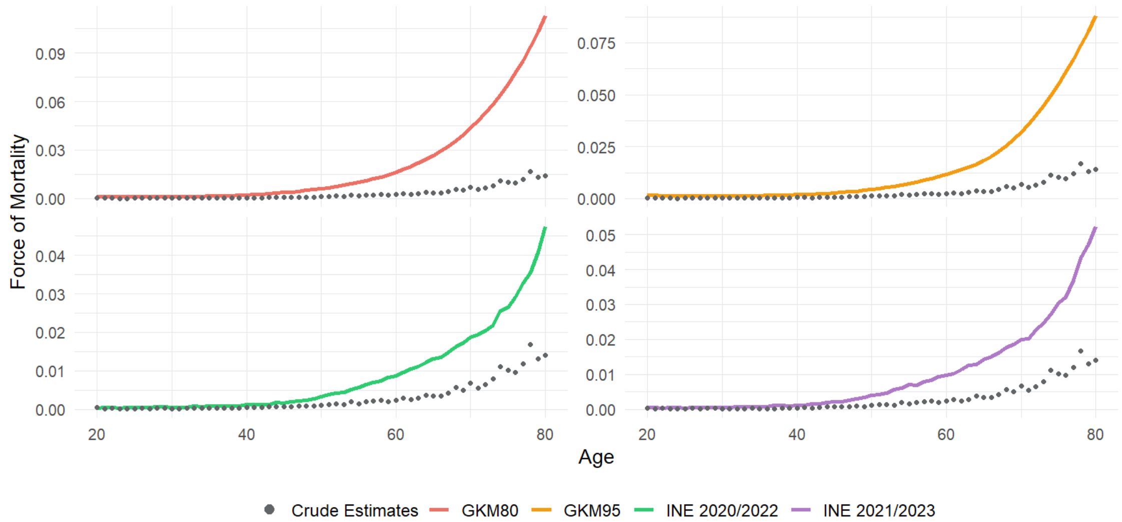

FIGURE 4: Crude Mortality Estimates vs. Male Reference Tables.

Source: Own elaboration

proposed here, addresses the increasing discrepancies in mortality rates at older ages by providing a more flexible modeling approach.

Table III presents the fitting results without separating by sex. Since all models include only two parameters, information criteria such as AIC and BIC, which penalize model complexity, offer little additional insight; thus, model selection is based on log-likelihood values. Similarly, Chi-square goodness-of-fit test p-values are omitted, as they were consistently high across models, indicating an adequate overall fit. However, this likely reflects the test's limited discriminatory power, especially given data aggregation and sparse observations at certain ages.

Based on the evaluation metrics, the GKM80 table with link (3) stands out as the most appropriate model. It achieves a strong overall fit, evidenced by the highest log-likelihood value and the lowest error measures, WSS and MSE, indicating superior accuracy. Additionally, the A/E ratio of 1.0008 indicates an almost perfect alignment between expected and observed deaths. These results support the selection of this model as the best fit for the data. The final graduated mortality function, shown in Figure 5 alongside the crude mortality estimates, is given by:

$$\hat{\mu}_x = \mu_x^s * \exp(-1.261251083 - 0.009860279x).$$

The graduated mortality curve closely follows the crude estimates, capturing the over-

TABLE III: MODEL SELECTION USING MALE REFERENCE TABLES.

Reference table	Link	Log-Likelihood	MSE	WSS	A/E
GKM80	1	-171.16	6.01e-07	0.1720	1.0021
	2	-165.34	6.00e-07	0.1498	1.0071
	3	-165.62	5.46e-07	0.1448	1.0008
GKM95	1	-179.84	1.11e-06	0.2477	0.9870
	2	-177.09	6.16e-07	0.1701	0.9570
	3	-177.49	6.83e-07	0.1805	0.9606
INE 2020/2022	1	-168.57	1.03e-06	0.2118	1.0677
	2	-172.29	1.18e-06	0.2387	1.1327
	3	-172.04	1.21e-06	0.2422	1.1255
INE 2021/2023	1	-170.35	9.14e-07	0.2141	1.0593
	2	-175.85	9.66e-07	0.2299	1.1377
	3	-175.65	1.00e-06	0.2349	1.1308

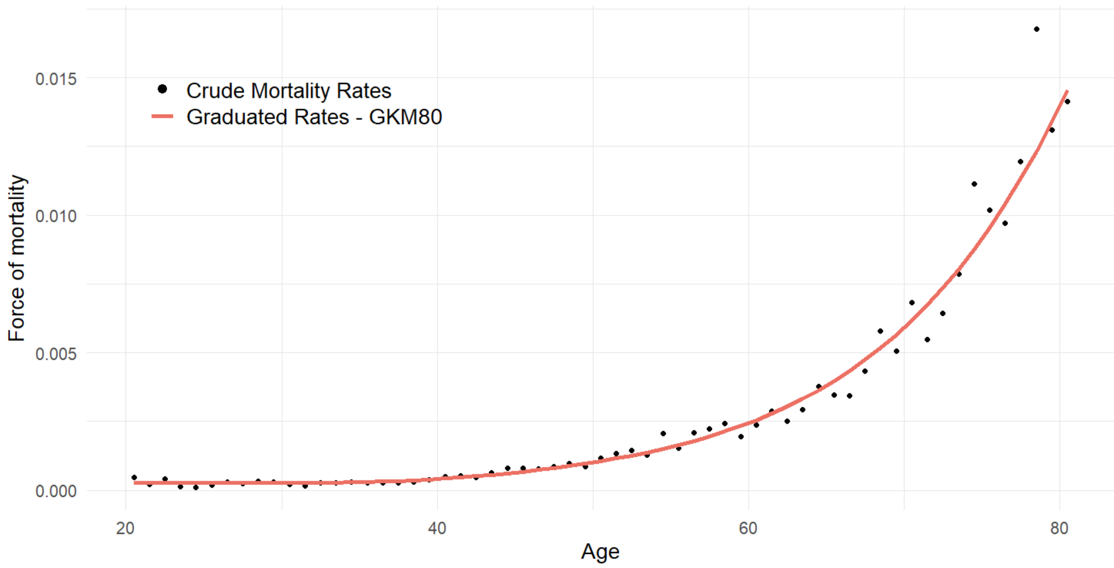

FIGURE 5: Graduated Mortality under GKM80 vs. Crude Estimates.

Source: Own elaboration

all trend with a smooth progression. At younger ages, the fit is strong, with minimal deviations that indicate the model accurately reflects observed mortality patterns. At advanced ages, however, increased variability in the crude rates results in more noticeable discrepancies, primarily due to lower exposure. While the graduation process effectively reduces noise, additional statistical testing could help assess potential biases, especially at higher ages. However, due to limited data in those age ranges, such testing was deemed beyond the scope of this study. The curve was therefore extended beyond the graduation age range to ensure a complete mortality table for all relevant ages.

4.1.2 Graduation with reference to standard tables and distinction by sex

The selected mortality tables were fitted separately for male and female policyholders using the corresponding gender-specific versions. All other aspects of the analysis followed the previously outlined process. The results for male and female models are presented in Tables IV and V, respectively.

TABLE IV: MODEL SELECTION - MALES (REFERENCE TABLE).

Reference table	Link	Log-Likelihood	MSE	WSS	A/E
GKM80	1	-66.25	3.53e-06	0.00751	0.9864
	2	-66.06	2.50e-06	0.00579	1.0070
	3	-66.07	2.51e-06	0.00581	1.0004
GKM95	1	-66.58	6.00e-06	0.01103	0.9680
	2	-66.60	5.26e-06	0.00987	0.9651
	3	-66.60	5.27e-06	0.00988	0.9651
INE 2020/2022	1	-66.14	3.35e-06	0.00689	1.0375
	2	-66.33	4.43e-06	0.00933	1.0861
	3	-66.27	4.45e-06	0.00910	1.0767
INE 2021/2023	1	-66.47	3.11e-06	0.00787	1.0435
	2	-66.95	4.20e-06	0.01069	1.1004
	3	-66.87	4.24e-06	0.01068	1.0908

Graduation results for male policyholders confirm that the GKM80 model remains satisfactory, with links 2 and 3 as the best-fitting options, indicating some benefit from modeling by sex. However, for female policyholders, the models with the lowest log-likelihood values, MSE, and WSS, namely the INE 2020/2022 table with links 2 and 3, produce unsatisfactory A/E ratios. None of the female-specific models achieves A/E ratios as close to one as those from the no-distinction approach. As a result, distinguishing by sex does not offer significant improvement and will not be pursued further in this analysis.

4.1.3 Parametric graduation with no distinction by sex

This section presents the parameter estimates for the parametric mortality models described in Section 2.3, fitted without sex differentiation. The Perks and Beard models are excluded, as their parameters δ converged to zero, effectively reducing them to the Makeham and Gompertz laws. Table VI summarizes the performance measures discussed in Section 2.4 for the fitted models.

TABLE V: MODEL SELECTION - FEMALES (REFERENCE TABLE).

Reference table	Link	Log-Likelihood	MSE	WSS	A/E
GKF80	1	-69.21	9.24e-07	0.02573	0.8908
	2	-68.83	1.02e-06	0.02052	0.8768
	3	-68.93	9.61e-07	0.02186	0.8803
GKF95	1	-68.97	1.94e-06	0.04402	0.9335
	2	-68.12	9.61e-07	0.02257	0.8817
	3	-68.34	1.12e-06	0.02728	0.8884
INE 2020/2022	1	-65.14	1.05e-06	0.02100	0.9583
	2	-64.43	8.01e-07	0.01322	0.9372
	3	-64.52	7.94e-07	0.01410	0.9369
INE 2021/2023	1	-65.25	1.03e-06	0.02208	0.9762
	2	-64.71	8.45e-07	0.01474	0.9370
	3	-64.78	8.38e-07	0.01559	0.9384

TABLE VI: OVERALL MODEL SELECTION (PARAMETRIC).

Model	AIC	BIC	χ^2-test p-value	MSE	WSS	A/E
Gompertz / $GM(0,2)$	345.17	349.39	0.8683	8.39e-07	0.1724	1.1186
Makeham / $GM(1,2)$	338.69	345.02	0.9904	5.48e-07	0.1401	1.0297
$GM(0,3)$	339.80	346.14	0.9799	5.14e-07	0.1332	1.0382
$GM(1,3)$	340.64	349.08	0.9884	5.48e-07	0.1401	1.0296
$GM(2,2)$	338.66	347.11	0.9967	6.75e-07	0.1567	1.0210
$GM(2,3)$	335.58	346.13	0.9986	5.32e-07	0.1288	0.9859
$GM(2,4)$	335.77	348.44	0.9992	6.81e-07	0.1383	1.0044
$GM(3,3)$	336.62	349.28	0.9986	5.77e-07	0.1300	0.9877
$GM(3,4)$	337.26	352.04	0.9988	6.11e-07	0.1308	1.0108

All models provide a reasonable fit to the data, as none are rejected by the goodness-of-fit test, which was applied after grouping ages to ensure at least five expected deaths per group. Effective model comparison requires considering both MSE and WSS, as they offer complementary insights. In this context, the $GM(2,3)$ stands out, achieving the lowest AIC and WSS. Its A/E ratio of 0.9859, very close to 1, indicates a close alignment between expected and observed deaths, with a slight conservative bias preferred in life insurance applications. While other models show marginally better performance in isolated metrics, they often compromise the overall performance. In contrast, $GM(2,3)$ consistently delivers strong results across all major metrics, without any significant weaknesses, making it a robust choice for developing mortality assumptions. The resulting parametric formula, represented in Figure 6 alongside the crude mortality estimates, is given by:

$$\hat{\mu}_x = -0.0001797365 + 0.0012220382 \left(\frac{x-70}{50}\right) + \exp\left(-3.6456783380 + \right.$$
$$\left. 4.1991014003 \left(\frac{x-70}{50}\right) + 1.4979136630 \left(2\left(\frac{x-70}{50}\right)^2 - 1\right)\right).$$

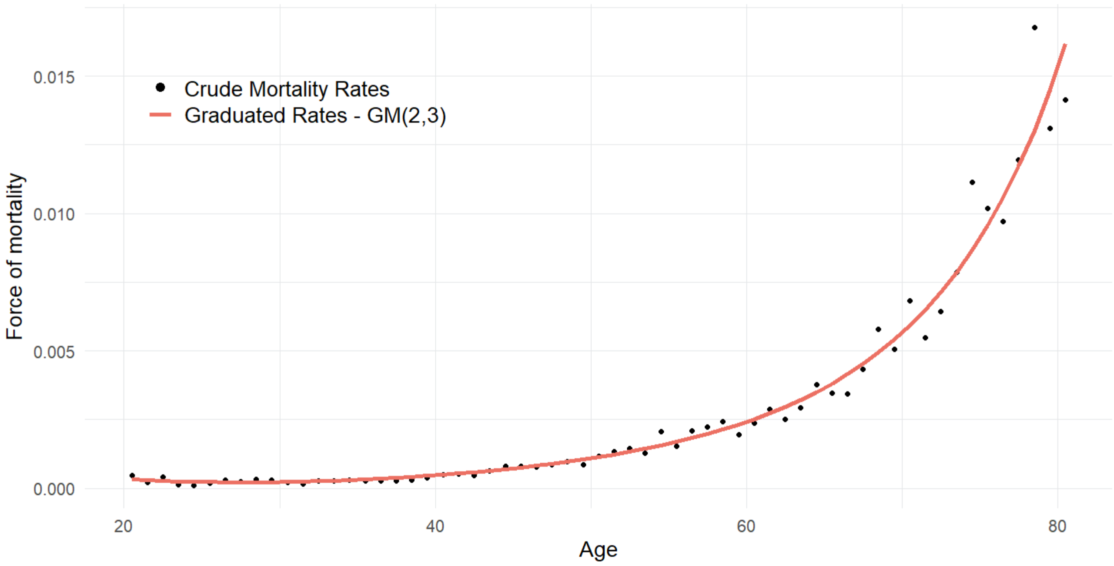

FIGURE 6: Graduated Mortality under $GM(2,3)$ vs. Crude Estimates.

Source: Own elaboration

The graduated mortality curve from the parametric model closely mirrors that of the standard table for the ages presented, effectively following the crude estimates. As before, fit quality is strong at younger ages with minimal deviations, while slight discrepancies emerge at older ages due to data variability and lower exposure. Overall, the results are consistent with those from the previous analysis, reinforcing the model's consistency with observed mortality trends.

4.1.4 Parametric graduation with distinction by sex

The effect of distinguishing by sex was also examined in the parametric graduation, using the same methodology as previously described. The results for male and female policyholders are presented in Tables VII and VIII, respectively.

The sex-specific models do not yield substantial improvements over the combined model. For the male subgroup, although the A/E ratio is generally good, the gains rel-

TABLE VII: MODEL SELECTION - MALES (PARAMETRIC).

Model	AIC	BIC	χ^2-test p-value	MSE	WSS	A/E
Gompertz / $GM(0,2)$	138.80	143.02	0.9994	3.45e-06	0.0082	1.1273
Makeham / $GM(1,2)$	138.94	145.27	1.0000	3.12e-06	0.0067	1.0066
$GM(0,3)$	139.43	145.76	1.0000	4.69e-06	0.0078	1.0078
$GM(1,3)$	140.52	148.97	1.0000	3.13e-06	0.0068	1.0331
$GM(2,2)$	140.17	148.61	1.0000	2.77e-06	0.0062	1.0172
$GM(2,3)$	142.15	152.70	1.0000	2.58e-06	0.0059	1.0092
$GM(2,4)$	144.10	156.76	1.0000	2.92e-06	0.0058	1.0023
$GM(3,3)$	144.14	156.81	1.0000	2.57e-06	0.0058	1.0061
$GM(3,4)$	146.08	160.86	1.0000	3.53e-06	0.0060	1.0007

TABLE VIII: MODEL SELECTION - FEMALES (PARAMETRIC).

Model	AIC	BIC	χ^2-test p-value	MSE	WSS	A/E
Gompertz / $GM(0,2)$	133.01	137.23	0.9978	1.39e-06	0.0158	1.0032
Makeham / $GM(1,2)$	134.13	140.46	0.9988	1.72e-06	0.0162	1.5464
$GM(0,3)$	134.25	140.58	0.9987	2.07e-06	0.0182	1.1388
$GM(1,3)$	136.11	144.55	0.9970	1.86e-06	0.0168	1.4118
$GM(2,2)$	136.12	144.56	0.9967	1.73e-06	0.0162	1.4881
$GM(2,3)$	138.11	148.66	0.9924	1.86e-06	0.0168	1.4118
$GM(2,4)$	139.43	152.10	0.9891	1.23e-06	0.0129	1.4759
$GM(3,3)$	140.11	152.77	0.9816	1.86e-06	0.0168	1.4118
$GM(3,4)$	141.43	156.21	0.9730	1.23e-06	0.0129	1.4759

ative to the full-portfolio model are minimal. For the female subgroup, the best-fitting model in terms of MSE and WSS, namely the $GM(2,4)$, has an A/E ratio well above 1, indicating underprediction of deaths. Meanwhile, the only model with a satisfactory A/E ratio, the $GM(0,2)$, exhibits higher WSS, reflecting weaker performance in age groups with greater exposure. Since the unified model applied to the entire dataset achieves an A/E ratio of 0.9859 along with balanced MSE and WSS values, indicating a robust fit across all policyholders, and the sex-specific models either underestimate mortality or perform worse overall, the combined model provides a better balance across key metrics. Therefore, distinguishing by sex does not produce meaningful improvements in mortality graduation, and the analysis proceeds using the combined model.

4.2 *Financial Impact of Changes in Mortality Assumptions*

4.2.1 *Effects on nominal cash flows*

After selecting the final mortality models through graduation, their impact on future cash flow projections and resulting changes under IFRS 17 and Solvency II were evaluated. The projections were generated using the RAFM (RiskAgility Financial Modeller) software, incorporating the newly derived mortality rates. This analysis focuses exclusively on the direct insurance component, excluding any reinsurance arrangements, and considers nominal cash flows only, without discounting. By holding all other factors constant, this approach facilitates a clear assessment of the effect of updated biometric assumptions on reserves and financial statements.

In the projections performed using the software, joint-life policies are modeled on a first death basis. This means the probability of a death claim corresponds to the likelihood that at least one of the two lives dies during the projection period. As outlined in 2.1.1, T_{xy} is defined as the future lifetime random variable of two individuals aged x and y. It represents the time until the first death between (x) and (y) occurs, defined as:

$$T_{x,y} = \min(T_x, T_y). \tag{28}$$

The associated mortality rate, representing the one-year death probability for a joint-life policy, is calculated as the probability that at least one of the two policyholders dies within the year, expressed as:

$$q_{xy} = 1 - (1 - q_x)(1 - q_y) \tag{29}$$

where q_x and q_y denote the individual mortality rates of each policyholder at the given age. This formulation ensures the valuation accurately reflects the product's benefit structure and appropriately captures the joint-life risk.

Once all assumptions are established, the cash flows of products covering mortality risk are projected until run-off. Table IX presents the aggregated values under IFRS 17 as of 31/12/2024. This includes premiums, claims (death, disability, surrenders, maturities, complementary coverages linked to a main benefit, guaranteed amounts, and scheduled surrenders), and expenses (acquisition costs, administrative expenses, initial and renewal commissions). The company's values are compared with results calculated using the mortality rates derived from the graduation process without distinction by sex. Additionally, the total nominal cash flow is provided for each set of assumptions, calculated as the sum of claims and expenses minus premiums.

The use of mortality graduations results in a reduction of total claims compared to the

TABLE IX: NOMINAL CASH FLOWS UNDER IFRS 17 (IN € MILLION)

	Current Assumptions	Standard Table Graduation	Parametric Graduation
Claims	430.43	425.03	425.55
from which death	49.64	46.33	49.59
Expenses	42.23	42.27	42.26
Premiums	279.58	280.48	280.54
Total cash flows	193.08	186.82	187.27

company's original assumptions, although the mechanisms vary between models. The standard table graduation achieves this mainly through a larger decrease in death claims, which leads to more policies remaining in-force and a slight increase in premiums, while expenses remain nearly unchanged. This results in an overall decrease of approximately 6.3 million in total nominal cash flows. In contrast, the parametric graduation produces death claims that remain close to the original projections, with the total claims reduction driven primarily by decreased payouts for maturities, surrenders, and other non-death-related benefits. Under this model, the premiums and expenses follow a similar pattern to that of the standard table graduation, producing a slightly smaller reduction in total cash flow of about 5.8 million.

Although both graduations reduce total nominal cash flows, their underlying drivers differ. Total claim amounts depend not only on the number of deaths but also on the characteristics and benefits of the affected policies. Consequently, two models with seemingly similar mortality rates can have notably different financial impacts. While these results appear favorable from a reserving perspective, it is important to note that Lusitania Vida's current assumptions are calculated by product group, whereas the new assumptions are obtained using only single-life policies but applied across all product groups. Without further analysis, such as assessing statistical robustness with separation by product group, these findings alone cannot confirm the superiority of the graduated mortality models.

A breakdown of projected death claims by product reveals that 41.83% correspond to products calibrated by the company at 16.65% of GKM80, with an additional 34.95% associated with products using even higher percentages of the standard table as the mortality assumption. As illustrated in Figure 7, the graduated mortality rates $\hat{q}_x$ obtained in this study are consistently lower from around age 55, precisely where most deaths occur. This helps explain the observed reduction in death claims under the new assumptions, highlighting the need to compare model outcomes with the company's assumptions to fully understand the financial implications.

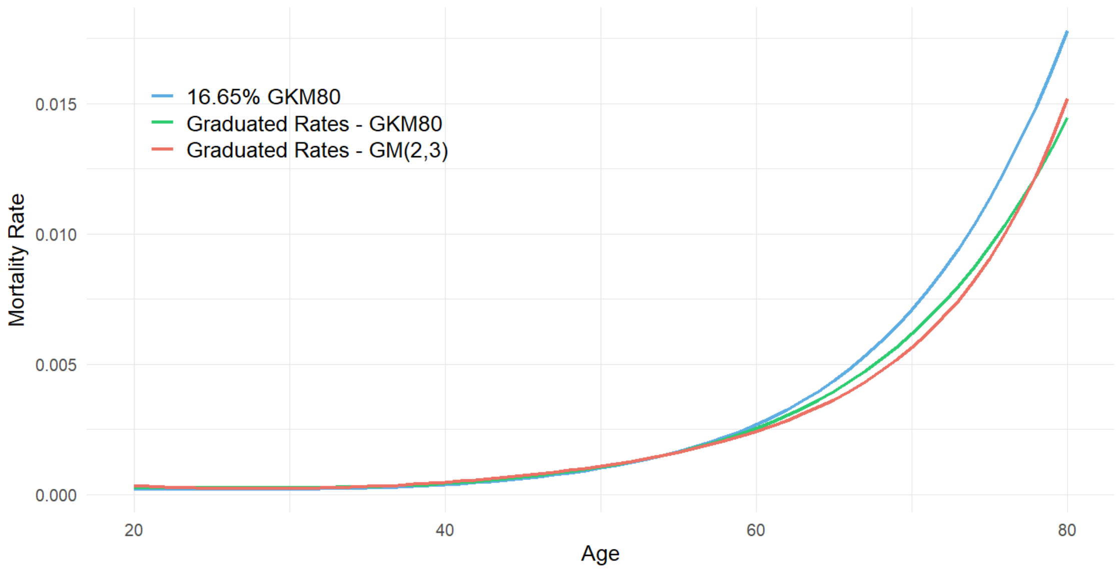

FIGURE 7: Graduated mortality rates and 16.65% of GKM80.

Source: Own elaboration

A similar cash flow analysis was conducted under the Solvency II framework, which includes a broader range of products than IFRS 17. These additional products contribute approximately €454 million in mathematical provisions, leading to higher projected figures. Table X presents the undiscounted nominal cash flows as of 31/12/2024 for all products covering mortality risk.

TABLE X: NOMINAL CASH FLOWS UNDER SOLVENCY II (IN € MILLION)

	Current Assumptions	Standard Table Graduation	Parametric Graduation
Claims	940.15	934.58	935.00
from which death	57.90	54.51	59.38
Expenses	74.51	74.51	74.47
Premiums	316.29	317.21	317.28
Total cash flows	698.36	691.88	692.19

Both the standard table and parametric graduations lead to lower total nominal cash flows compared to the company's original assumptions, amounting to approximately 6.5 million and 6.2 million, respectively. The components influencing the total reductions are distinct for each approach. The standard table graduation achieves this mainly through a 3.4 million drop in death claims, while the parametric graduation results in a 1.5 million increase in these claims. This increase is more than offset by reduced payouts for maturities, surrenders, and other non-death-related events, ultimately leading to an overall reduction in total claims. In both cases, fewer claims result in a marginal increase in premiums, as more policies remain in force, while expenses remain largely unchanged.

The overall improvement in total nominal cash flows is proportionally greater under IFRS 17, since this framework focuses exclusively on contracts with significant insurance risk, most of which are designed specifically to cover death events, making mortality assumptions a key driver of projections. In contrast, under the prudential regime, the majority of products are investment contracts, which diminishes the relative impact of changes in mortality assumptions.

Once again, a closer examination at the breakdown of projected death claims by mortality assumption group provides additional insight. Approximately 65.82% of death claims are linked to products calibrated at 16.65% of GKM80 or higher, a smaller proportion compared to the statutory regime. This reduction reflects the inclusion of additional contracts under Solvency II, many calibrated at even lower percentages of the standard table, which helps explain the decrease in death claims seen with the standard table graduation. However, this shift does not explain the increase in death claims observed under the parametric model. The explanation lies in the behavior of the parametric model at higher ages: since the graduation was performed up to age 80 and then extended beyond that, mortality rates diverge at higher ages. Figure 8 compares these estimates for both graduations.

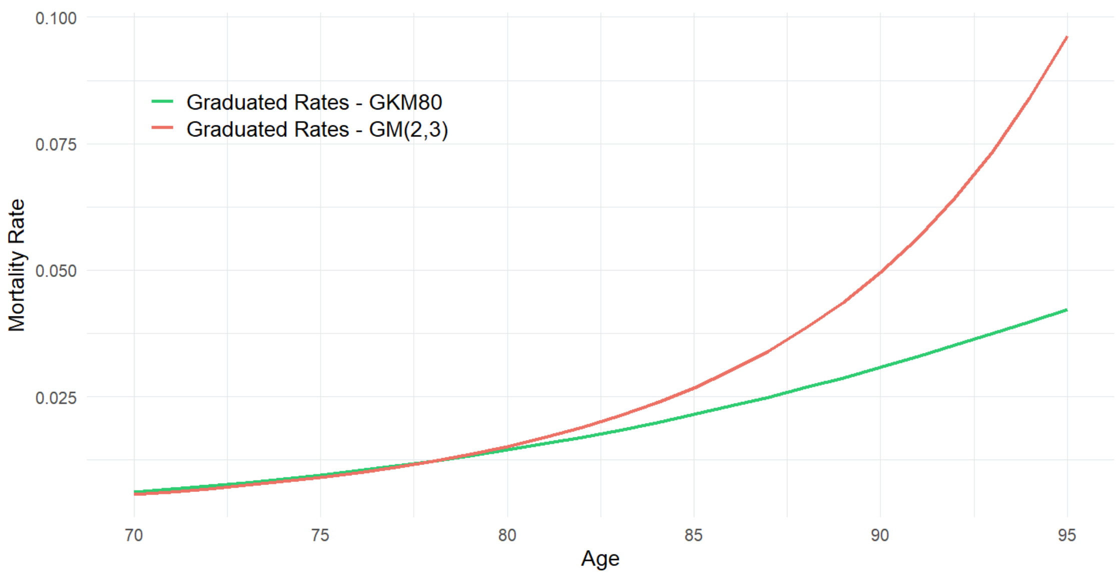

FIGURE 8: Graduated mortality rates at older ages.

Source: Own elaboration

Because the parametric model projects sharply higher mortality rates at older ages, policies held by older individuals result in greater claim estimates under this model. Since the Solvency II framework covers a broader portfolio with a larger share of older policyholders, this increase in mortality has a more pronounced impact on total death claims. This underscores the need for model evaluation to extend beyond simply comparing mortality rates, as projections are influenced by the specific characteristics of policies in-force, such as the age profile of policyholders, exposure levels, and sums assured. Even minor variations in mortality assumptions can have substantial financial consequences in large and heterogeneous portfolios. Therefore, thorough review and careful calibration of assumptions are essential to ensure the models faithfully reflect the portfolio's unique composition.

4.2.2 Backtesting

The accuracy of the mortality rates used under both the prudential and statutory frameworks can be assessed through backtesting, which compares projected outcomes based on our assumptions with actual observed data. This exercise enables assessment of the predictive performance of the models and verification of whether the adjustments accurately reflect mortality trends and policyholder experience. In this report, the graduation process was performed using policies as of the reference date 31/12/2024. However, since the complete data for 2025 is not yet available, backtesting will be conducted by comparing projections made at the end of 2023 with the actual cash flows recorded during 2024. While not ideal, this approach still provides a valuable means of evaluating the predictive performance of the graduated mortality rates and their alignment with observed mortality trends and policyholder experience.

Table XI summarizes the relative deviations between projected and actual values for 2024 under both IFRS 17 and Solvency II frameworks, focusing on death claims and total liabilities (claims plus expenses minus premiums). Unlike the earlier analysis, which assessed the impact of assumption changes over the entire projection horizon, this backtesting exercise considers only cash flows over a one-year horizon. Actual claims and premiums data were provided by Lusitania Vida. For expenses, IFRS 17 includes only those directly attributed to products, while Solvency II also incorporates expenses not directly attributed, excluding non-technical components such as pension fund costs and subordinated liability interest. To ensure consistency with projections, the remaining expenses were allocated proportionally across products based on their mathematical provisions as of 31/12/2023. A more detailed breakdown of these figures is presented in Tables XV and XVI in the Appendix.

Under IFRS 17, all models yield relatively small deviations. The company's original assumptions underestimate death claims by 8.73%, while the graduated models improve alignment, with the parametric model achieving the lowest deviation of 5.22%. For to-

TABLE XI: RELATIVE DEVIATION OF ACTUAL CASH FLOWS TO PROJECTIONS

	Death Claims		Total Liabilities	
	IFRS 17	Solvency II	IFRS 17	Solvency II
Current Assumptions	8.73%	28.26%	6.92%	17.12%
Standard Table Graduation	7.70%	16.58%	6.61%	16.68%
Parametric Graduation	5.22%	11.15%	6.61%	14.60%

tal liabilities, deviations decrease modestly from 6.92% under the original assumptions to 6.61% with both graduated models. In contrast, discrepancies under Solvency II are substantially larger. The company's assumptions underestimate death claims by 28.26%, likely reflecting the broader and more heterogeneous portfolio within this framework. The graduated models reduce this deviation significantly, with the parametric approach lowering it to 11.15%. Improvements in total liabilities are also evident but less pronounced.

Overall, the analysis supports the effectiveness of graduated mortality models, particularly the parametric one, in enhancing short-term projection accuracy and more closely reflecting observed experience across both frameworks. While improvements are clear in death claims projections, the effect is less pronounced when considering total liabilities, which also include premiums, expenses, and non-death-related benefits. This underscores the need to review and refine all key actuarial assumptions beyond mortality to achieve more reliable liability estimates.

4.2.3 Solvency ratio sensitivity

To further assess the impact of mortality risk on the insurer's solvency position, this section examines the sensitivity of the SCR and solvency ratio to shocks in mortality. Cash flows, including claims, premiums, expenses, and profit sharing, are projected until run-off under both base and stressed scenarios. Profit sharing refers to payments to policyholders that depend on the insurer's financial performance and are sensitive to changes in mortality assumptions. The projections are performed monthly, aggregated annually, and discounted to the valuation date using the European Insurance and Occupational Pensions Authority (EIOPA) risk-free spot rates without volatility adjustment. The analysis focuses on the insurer's net mortality risk exposure after reinsurance recoverables, requiring the projection of both direct insurance and reinsurance cash flows. Certain components are assumed to remain unaffected by the mortality shock and thus remain constant in both scenarios, not contributing to liability differences between scenarios. These include: the claims provision, considered net of reinsurance, for claims incurred but not yet paid; the profit sharing provision, representing the amount earmarked for policyholder profit sharing, pending distribution; and the Time Value of Options and Guarantees (TVOG), which reflects the cost of embedded financial guarantees within contracts.

The capital charges for the risk sub-modules are calculated by summing positive increases in the BEL across all products; products that do not exhibit an increase in liabilities under the stress scenario are excluded from this aggregation, as they are not adversely affected by the shock. Considering each product i, then:

$$Capital\ charge = \sum_{i\,:\,BEL_{stressed,i} > BEL_{base,i}} \left(BEL_{stressed,i} - BEL_{base,i}\right). \tag{30}$$

However, because the insurer's obligation to pay profit sharing may decrease under stressed mortality, thereby reducing the overall loss, this loss-absorbing effect must be incorporated to determine the net capital requirement accurately. For each mortality model, three BEL values are computed: before the shock, the stressed BEL including the impact of the shock on profit sharing, and the stressed BEL with profit sharing at the base scenario level (i.e., excluding LAC-TP). The gross SCR is calculated as the difference between the base BEL and the stressed BEL without adjusting for LAC-TP, while the net SCR reflects the adjustment for changes in profit sharing. The capital requirements for the mortality risk and life catastrophe risk sub-modules under each model are presented in Table XII. Detailed liability values for products exposed to these shocks are provided in Tables XVII and XVIII.

TABLE XII: MORTALITY AND LIFE CATASTROPHE RISK SCR (IN € MILLION)

	Mortality risk		Life catastrophe risk	
	Net SCR	Gross SCR	Net SCR	Gross SCR
Current Assumptions	5.01	5.04	4.25	4.58
Standard Table Graduation	4.43	4.94	4.30	4.89
Parametric Graduation	4.54	5.06	4.23	4.81

For both mortality and life catastrophe risks, the graduation models affect capital requirements differently. Mortality risk models lead to notable reductions in net SCR, while changes in gross SCR are less marked, with one model even producing a slight increase. In contrast, life catastrophe risk generally sees an increase in capital charges across models, except for a slight decrease in net SCR under the parametric graduation. These differences between net and gross SCR highlight the role of profit sharing adjustments in shaping final capital requirements. However, the impact of these variations diminishes once diversification effects are incorporated within the overall life underwriting risk module and even more so in the calculation of the BSCR. Table XIII illustrates this attenuation by showing the corresponding percentage variations in capital charges under the graduated models relative to the company's current assumptions. The effect of diversification is evident, as it moderates the impact of changes in individual risk sub-modules, resulting in a more balanced change in the overall capital requirement.

TABLE XIII: CAPITAL CHARGES VARIATIONS

	Life Underwriting Risk SCR		BSCR	
	Net	Gross	Net	Gross
Standard Table Graduation	0.37%	-0.29%	0.15%	-0.11%
Parametric Graduation	0.38%	-0.28%	0.16%	-0.11%

To fully assess the impact of the revised methodology for setting mortality assumptions, both the SCR and its effect on the EOF must be calculated. The total SCR is recalculated by adding other components to the net BSCR: namely, the capital requirement for operational risk, the LAC-TP, and the LAC-DT. The operational risk charge is assumed unchanged across all models. The LAC-TP is calculated as the difference between the net and gross BSCR. For simplicity, the LAC-DT is held constant across models due to the complexity of its recalculation and its exclusion from the scope of this study. Nevertheless, it is recognized that changes in mortality assumptions would likely impact deferred taxes. In parallel, the EOF are recalculated to capture the effect of the revised mortality assumptions. For each model, these were derived from the company's values, with increases estimated as

$$\Delta EOF \approx -\Delta BEL \times (1 - tax\ rate), \tag{31}$$

meaning the total reduction in liabilities under the base scenario, net of reinsurance, which was further adjusted for taxes, applying a 24.5% effective tax rate for consistency with the company's approach. Table XIV presents the resulting changes in the solvency ratio under each set of assumptions.

TABLE XIV: SOLVENCY RATIO CALCULATION (IN € MILLION)

	EOF	SCR	Solvency Ratio
Current Assumptions	89.92	54.64	164.58%
Standard Table Graduation	93.27	54.55	170.98%
Parametric Graduation	92.77	54.55	170.07%

The solvency ratio improves across both graduation models compared to that reported at the reference date of 31/12/2024, with the standard table producing the largest increase at 6.40%. A higher solvency ratio highlights the company's financial resilience and the existence of a stronger buffer against adverse events. This improvement enhances compliance with Solvency II requirements, reinforcing trust with regulators. For investors, the increased ratio signals financial stability and sound risk management, which may boost confidence and potentially reduce the cost of capital. Overall, a stronger solvency position elevates the company's reputation and supports sustainable growth.

5 CONCLUSION

This study aimed to develop an actuarial evaluation process for refining mortality assumptions in a way that is both technically sound and aligned with statutory (IFRS 17) and prudential (Solvency II) requirements. To this end, mortality graduation techniques were applied to test the fit of multiple models to the experience of a specific life insurance portfolio. The primary objective was to enhance the adequacy of the company's mortality assumptions and improve the accuracy of projected cash flows, using methods selected for their strong theoretical basis and practical relevance, and evaluated through statistical and adequacy-based criteria. After reviewing the theoretical and regulatory background, the analysis proceeded with the treatment and structuring of portfolio data, including cleaning, validation, and exploratory analysis. New sets of mortality rates were then derived using the selected models, tested through projections and backtesting exercises, and their financial impact was assessed through the recalculation of liabilities and capital requirements.

Mortality graduation was carried out using two techniques: graduation with reference to a standard table and parametric graduation. The reference-table approach produced the best results with an exponential adjustment to the GKM80 table, while the parametric method identified the $GM(2,3)$ model as the best statistical fit. Both models showed strong alignment with actual experience.

The high quality of the adjustments was reflected in the reduced deviations between projected and observed cash flows for 2024. When applied as mortality assumptions, these rates led to notable liability reductions under the Solvency II framework, contributing to a solvency ratio improvement of up to 6.40%. Although the parametric model achieved slightly lower deviations under both statutory and prudential regimes, the standard table approach offers a better balance between accuracy and operational simplicity. It improves the adequacy of the mortality assumptions while maintaining consistency with the company's current methodology, which also involves adjustments to the GKM80 table using a different method that applies a uniform percentage adjustment across all ages.

Unlike the company's current approach, the new exponential adjustment likely provides a more accurate reflection of recent advances in healthcare and longevity, recognizing that the underlying structure of the mortality table itself has evolved and better captures the changing age-specific mortality patterns. Due to its lower complexity and operational demands, the standard table graduation is recommended for adoption, while the parametric graduation may serve as a valuable alternative for more advanced analyses in the future.

Further development should address several areas to improve the methodology and expand its applicability. First, joint-life policies, left out of the graduation process due to the absence of reliable methods for their inclusion, should be explored in more detail. Although the graduated rates were later applied to these policies with some adjustments, developing a tailored approach, either by extending the current framework or constructing a separate model, would allow a more accurate representation of these contracts.

Incorporating an IBNR factor is another relevant consideration: while it was not included in this analysis due to the annual aggregation of deaths and exposure, doing so in future work could help account for delays in the reporting of death claims.

It would also be valuable to reassess the use of the entire historical dataset, as limiting the modeling to more recent years may result in assumptions that better reflect current mortality trends.

Additionally, evaluating whether mortality assumptions should vary by product group could improve the model fit and provide insights into the underlying risk structure. This segmentation was not pursued in the current analysis to ensure statistical robustness, but it is worth investigating whether such differentiation is meaningful, especially considering that investment products are typically offered without risk selection criteria, resulting in a more heterogeneous insured population, whereas products that exclusively cover mortality generally involve stricter underwriting standards that influence the risk profile of policyholders.

Finally, as additional data become available, the backtesting methodology can be enhanced by comparing mortality projections at the end of 2024 with the actual outcomes observed in 2025. This approach would offer a more objective assessment of the models' predictive accuracy, thereby complementing the current analysis.

This study successfully demonstrates the benefits of applying mortality graduation to improve the accuracy and consistency of actuarial assumptions. The methods examined contribute to more reliable financial projections and strengthen the company's solvency position. By establishing a solid foundation, this work supports ongoing refinement and application of mortality modeling techniques, ultimately enhancing risk assessment and decision-making processes in life insurance.

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Appendix

TABLE XV: BACKTESTING UNDER IFRS 17 (IN € MILLION)

	Current Assumptions	Standard Table Graduation	Parametric Graduation	Actual
Claims	36.02	36.07	36.07	57.38
from which death	5.71	5.78	5.93	6.26
Expenses	3.87	3.87	3.87	0.96
Premiums	24.46	24.45	24.45	41.77
Nominal cash flows	15.43	15.48	15.48	16.58

TABLE XVI: BACKTESTING UNDER SOLVENCY II (IN € MILLION)

	Current Assumptions	Standard Table Graduation	Parametric Graduation	Actual
Claims	204.02	204.99	204.99	261.08
from which death	7.34	8.53	9.09	10.23
Expenses	9.23	9.23	9.23	8.76
Premiums	31.31	31.31	31.31	50.33
Nominal cash flows	181.94	182.91	182.91	219.52

TABLE XVII: LIABILITIES FOR MORTALITY RISK SUB-MODULE (IN € MILLION)

	Before shock	After shock and LAC-TP	After shock, before LAC-TP
Current Assumptions	216.42	221.43	221.46
Standard Table Graduation	211.99	216.42	216.93
Parametric Graduation	212.72	217.27	217.78

TABLE XVIII: LIABILITIES FOR LIFE CATASTROPHE RISK SUB-MODULE (IN € MILLION)

	Before shock	After shock and LAC-TP	After shock, before LAC-TP
Current Assumptions	218.46	222.71	223.03
Standard Table Graduation	214.02	218.32	218.91
Parametric Graduation	214.76	218.98	219.57