Lisbon School of Economics & Management
Universidade de Lisboa

MASTER
MATHEMATICAL FINANCE

MASTER´S FINAL WORK
DISSERTATION

A REAL OPTIONS APPROACH FOR RENEWABLE AUCTIONS

DANIEL OUTOR BARBOSA FERNANDES GOMES

JUNE - 2025

MASTER
MATHEMATICAL FINANCE

MASTER´S FINAL WORK
DISSERTATION

A REAL OPTIONS APPROACH FOR RENEWABLE AUCTIONS

DANIEL OUTOR BARBOSA FERNANDES GOMES

SUPERVISION:
CARLOS MIGUEL DOS SANTOS OLIVEIRA

JUNE - 2025

GLOSSARY

JEL Journal of Economic Literature. i, ii

MFW A Real Options Approach For Renewable Auctions. i

OLS Ordinary Least Squares. i

ABSTRACT

The model by Wozabal et al. (2021) combines auction theory and real options theory to represent renewable energy auctions, where the right to build subsidized renewable projects is valued as a European put option. In their framework, bidders are heterogeneous and hold private information on their initial costs and volatility signals. Two bidding strategies are considered: a net present cost (NPC) approach, based on net present value, and an option-based cost (OBC) approach, based on real option valuation.

This thesis builds on that model by introducing one key difference: investment opportunities are modeled as American put options. This extension captures the flexibility of investment timing within a predefined maturity, providing a richer representation of bidders' strategic behavior in renewable energy auctions for Contracts for Differences (CfDs). Using Least squares Monte Carlo (LSMC) method introduced by Longstaff & Schwartz (2001), we evaluate investment projects as American put options, capturing the option to defer under cost uncertainty to compute the OBC valuations. We show that higher cost projects exhibit higher exercise thresholds and delayed exercise, while exercise times tend to cluster near maturity. The model is applied to simulate outcomes of the german onshore wind auction (ONWA17) and compared to real auction outcomes. Incorporating American-style option valuation provides a more realistic understanding of bidder behavior and highlights the value of early exercise flexibility in project realization decisions.

KEYWORDS: Real Options; Investment under Uncertainty; Renewable Energy; Auctions; Option pricing; Stochastic Dynamic Programming.

JOURNAL OF ECONOMIC LITERATURE (JEL) CODES: C61; C63; D44; G13; Q42; H23.

Acknowledgements

I would like to express my sincere gratitude to my family for their unwavering support throughout my studies. I am also thankful to my friends and colleagues, whose discussions and companionship made this journey more enjoyable. I am especially indebted to my supervisor, Professor Carlos Miguel dos Santos Oliveira, for his invaluable guidance, encouragement, and constructive feedback. His continuous support and direction were fundamental to the development and completion of this work.

TABLE OF CONTENTS

LIST OF FIGURES

List of Tables

A Real Options Approach For Renewable Auctions

Daniel O. Gomes

1 Introduction

The accelerating transition from fossil fuels to renewable energy sources has become a central objective in global environmental policy, in response to the urgent challenges of climate change, biodiversity loss, environmental pollution, and resource scarcity (Greenpeace International 2024). Renewable energy deployment requires not only technological progress but also substantial financial investment, typically underpinned by supportive public policy frameworks designed to reduce risks for private investors. The European Union (EU), under its European Green Deal and legislated by the European Climate Law, aims to achieve climate neutrality (net-zero greenhouse gas emissions) by 2050, with interim targets of reducing emissions by at least 55% by 2030 and potentially 90% by 2040 European Parliament and Council of the European Union (2021).

Within this framework, the EU has established a comprehensive regulatory structure, most notably through its Renewable Energy Directive (RED), first introduced in 2009 and updated in 2018 (Directive 2018/2001/EU). The directive mandated that at least 32% of EU energy consumption must come from renewable sources by 2030, with the latest revision raising the binding target to 42.5%, alongside an indicative goal of 45%. Member States have developed national implementation plans to meet these ambitions. For example, Germany's Renewable Energy Sources Act (EEG) sets an 80% renewable electricity target for 2030; Spain aims to achieve 74% renewable electricity by the same year; Denmark targets a 70% reduction in greenhouse gas emissions compared to 1990 levels by 2030. Although the United Kingdom is no longer part of the EU, it maintains cooperative relations under the Trade and Cooperation Agreement and pursues similar goals through its *Clean Power 2030 Action Plan*, which targets 95% low-carbon electricity by 2030. These diverse national strategies reflect both the shared European commitment to the energy transition and the recognition of climate risks.

To promote renewable investment effectively, many countries have increasingly adopted competitive auctions combined with Contracts for Difference (CfDs) as the primary subsidy mechanism. A CfD is a long-term contract between a generator and a public counterparty, typically lasting fifteen years in which the generator is paid the difference between a fixed "strike price" and a reference market price—typically the day-ahead market price, though in principle a futures price could also be used—for each MWh of electricity produced. In these auctions, also known as renewable auctions, CfD contracts are competitively allocated to generators that submit the lowest bid prices. Generators awarded a CfD receive a fixed price per MWh, which reduces revenue volatility and mitigates investment risk. At the same time, the competitive nature of CfD auctions promotes price discovery and allocative efficiency, lowering the overall subsidy burden compared to alternative support schemes. Studies like del Río (2022), Fleck & Anatolitis (2022), Kreiss et al. (2017*b*), Matthäus (2020), Kreiss et al. (2017*a*), Szabó (2025), have extensively analyzed auction design, pricing rules, penalties, and pre-qualification criteria, focusing on project realization and cost efficiency. However, these studies often overlook project-level valuation under uncertainty. Similarly, Fleck & Anatolitis (2023) provides a theoretical assessment of the relationship between policy objectives and auction design, finding that many countries applying renewable auctions pursue inconsistent strategies and offers recommendations for aligning design with objectives. These contributions significantly advance our understanding of multi-unit procurement auctions in renewable energy, but they largely overlook project-level valuation under uncertainty.

Valuation of renewable energy investments can be enhanced by applying concepts from financial and real options. Financial options are derivative contracts whose value depends on an underlying asset, such as a stock, bond, or commodity. A call option grants the holder the right, but not the obligation, to purchase the asset at a predetermined strike price within a specified period, while a put option allows selling at the strike price. European options can be exercised only at maturity, whereas American options can be exercised at any time until maturity. The payoff for a put option at maturity T with strike price K and underlying asset price S_T is $\max(K - S_T, 0)$. Option contracts always involve two counterparties: the buyer pays a premium upfront to acquire the right, while the seller receives this premium in return for bearing the obligation to deliver if exercised. The profit or loss of the writer is therefore the exact negative of the holder's payoff.

Real options extend the financial option framework to investment decisions by treating the project's expected cash flows as the underlying asset, the required investment cost as the strike price, and the decision period as the option's maturity. In contrast to the net present value (NPV) approach, which assumes immediate and irreversible investment, real options explicitly incorporate managerial flexibility under uncertainty. This flexibility may include deferring investment, expanding or contracting operations, suspending activity temporarily, or abandoning the project altogether. In energy projects, where uncertainty about regulatory conditions, technological progress, and market prices is substantial, these options represent a significant source of value that NPV methods fail to capture as shown in Haahtela (2012), Dixit & Pindyck (1994). By embedding this flexibility in the valuation process, real options analysis provides a more realistic and forward-looking assessment of investment opportunities and is therefore considered superior to static NPV methods in contexts characterized by long horizons and high uncertainty Oliveira & Perkowski (2020), Hagspiel et al. (2021), Fernandes et al. (2011), Ceseña (2013).

Despite the theoretical development of real options, most studies do not account for auction-specific features, such as strike prices emerging from competitive bidding, grace periods, or penalties that influence investment timing. One of the few exceptions is Wozabal et al. (2021), who develop a model in which CfD contracts are valued as European put options, distinguishing between option-based cost (OBC) and NPV-based cost (NPC) bidders. This thesis extends that approach by modeling the OBC bidder's investment opportunity as an American-style put option, allowing continuous exercise over the investment window and capturing the flexibility to adapt to evolving market or policy conditions.

The main contributions of this work are fourfold. First, it introduces a real options framework that integrates CfD strike prices within an American-style option structure. Second, it applies the Least-Squares Monte Carlo (LSMC) method in this novel context. Third, it provides a systematic comparison of valuation methods, contrasting European- and American-style options. Finally, it conducts extensive sensitivity analyses using comparative statics to explore how investor preferences interact with auction outcomes, offering valuable insights for both investors and policymakers.

The remainder of the paper is organized as follows. Section 2 develops the auction model, detailing the auction setup and bidder valuation approaches, contrasting the NPV-based (NPC) framework with the option-based (OBC) American put option framework. Section 3 develops the theoretical framework for valuing American-style options as an optimal stopping problem and presents the Least-Squares Monte Carlo method as a numerical solution, outlining its implementation and convergence properties. Section 4 applies the model to a real-world setting, presenting an auction simulation that analyzes bidding behavior and auction equilibrium. Section 5 investigates bidder behavior under variations in key parameters and compares simulation results between settings with European-option bidders and American-option bidders. Section 6 discusses potential extensions to the model, and Section 7 concludes.

2 MODEL

In this thesis we use a Real Options Analysis approach to evaluate renewable energy projects supported by Contracts for Differences (CfDs). Unlike the typical formulation where the underlying is the value of the future cash flows, in this work the underlying variable is the levelized cost of energy (LCOE) of the project. The strike price, in turn, corresponds to the guaranteed revenue per MWh under the CfD contact. This framing reflects the central question faced by bidders: whether it is worthwhile to commit to the project given uncertainty in costs. The option-like nature arises because the investor has the right, but not the obligation, to proceed. If the realized cost of producing energy (L_t) falls below the strike price guaranteed in the CfD (K), then exercising the option yields a positive payoff. If the costs remain above the strike, the option expires worthless, since the project would not be undertaken. The option may be exercised at any time before the contract deadline, reflecting the investor's ability to decide strategically when to commit. The optimal exercise occurs when the immediate gain from exercising the option (invest in the project) exceeds the value of waiting for further cost uncertainty to resolve. This interpretation differs from conventional real option models, where the project is seen as an option to acquire an asset yielding uncertain revenues. In the CfD setting, the revenue side is effectively fixed by policy, while uncertainty lies on the cost side. The real option framework therefore captures the value of waiting to invest until costs evolve favorably.

2.1 Auction setup

This section outlines the auction environment in our analysis, drawing from and extending the framework of Wozabal et al. (2021). The model captures key characteristics of renewable energy auctions, including uncertainty in bidder participation, stochastic project costs, and penalties for failing to start energy generation by a pre-determined deadline (maturity T). To assess the impact of investment flexibility on bidding strategies, we introduce two bidder types. Net Present Cost (NPC) bidders evaluate their projects using a traditional net present value approach, thereby ignoring the value of flexibility embedded in the investment. In contrast, Option-Based Cost (OBC) bidders recognize the managerial flexibility inherent in the investment decision. They treat the CfD contract as a real option, valuing the right to invest or not under uncertain future costs. Our extension departs from Wozabal et al. (2021) by modeling OBC bidders' valuation as an American put option, whereas the original framework assumes a European put option. While the authors argue that the early exercise feature has negligible impact, we explicitly account for it to examine whether this holds in our setting.

Given that the investor holds both the right to invest and a contract with a fixed price under the CfD, the primary source of uncertainty we model is the cost of the project. The CfD effectively mitigates volatility and risk on the revenue side, so price uncertainty in the electricity market is not considered. Furthermore, since we do not allow for the possibility of investing without a CfD, we assume that holding such a contract is always preferable to being exposed to market risk. We also assume that the option value of participating in the auction is always greater than the costs of meeting the eligibility requirements, or equivalently, that entering the auction is costless. In theory, this implies that for private investors considering renewable energy projects, participating in an auction for a CfD contract is always preferable to investing without support. This assumption is not overly restrictive, since CfD policies are specifically designed to mitigate market risk and thereby accelerate the deployment of clean energy technologies.

We consider N risk-neutral bidders. Each bidder $i = 1, 2, \ldots, N$ may offer several projects, indexed by $h = 1, 2, \ldots, H^i$, each with different generation capacity. The auctioneer goal is to contract a fixed total capacity of $C \in \mathbb{N}$ megawatt (MW). Project sizes are modeled in increments of c kilowatt (kW), which defines the scale of the auction in terms of contract units. Each project bid consists of a price per megawatt hour and the associated project capacity. Bids consist of discrete price-quantity pairs, which together from a step function representing the bidders supply offer. The auction can follow either a uniform pricing rule, where all awarded projects are paid the clearing price, or a discriminatory pricing rule, where each project is paid its own bid price. The auctioneer receives the bids and selects the winning bidders, going from projects with the lowest bids to highest bids until the procurement target is reached. In most renewable auctions, the pre-qualification criterion is a deposit that becomes non-refundable if winning investors choose not to proceed with the project. Assuming the absence of credit or deposit risk, we interpret this deposit as a fee, effectively serving as a penalty for non-realization, i.e., for not exercising the option to invest. In our analysis, we focus on uniform pricing, where all awarded projects receive the same clearing price determined by the marginal bid required to fulfill the auctioned capacity. Project costs are summarized by their Levelized Cost of Electricity (LCOE), capturing all fixed and variable costs over the project's life cycle. We model LCOE dynamics using a geometric Brownian motion, reflecting uncertainty over input costs and market conditions

$$dL_t^{ih} = \mu^{ih} L_t^{ih}\, dt + \sigma^i L_t^{ih}\, dB_t^{ih}, \tag{1}$$

where L_t^{ih} is the LCOE of the project h from bidder i, μ^{ih} is its drift, σ^i is the volatility and dB_t^{ih} standard Brownian motion. While in reality cost processes may be correlated due to common input markets or supply chains, we assume uncorrelated Brownian motions, following Wozabal et al. (2021). For a more detailed justification, see the discussion in that paper. As argued there, this assumption does not materially affect the results, since bidding behavior and realization probabilities are determined by marginal rather than joint cost distributions. We later relax this assumption. Following the assumptions used in Wozabal et al. (2021) frame the auction setup

Assumption 1. (Uncertain Participation). Each bidder has a strictly positive probability of not participating in the auction. Participation decisions are independent across bidders. This introduces strategic uncertainty and avoids degenerate equilibria by preventing bidders from knowing the exact number of competitors ex-ante. Swinkels (2001), Jackson & Kremer (2006)

Assumption 2. (Independent Private Values) We assume that at the time of bidding $(t = 0)$, each firm observes L_0^{ih} and σ^i its firm-specific volatility. These signals are assumed to be independent private values.

Assumption 3. (Competitive pressure) Bidders anticipate the presence of competitors with similar private information, particularly when participation rates are high. This assumption ensures a sufficiently competitive bidding environment.

We adopt a risk-neutral approach to estimate the fair value of the projects. The transformation from the physical measure $\mathbb{P}$ to the risk-neutral measure $\mathbb{Q}$ is justified by arbitrage-free pricing theory and complete market assumption. Under $\mathbb{Q}$, the process L_t^{ih} must satisfy the condition that the discounted process $e^{-rt}L_t^{ih}$ is a $\mathbb{Q}$-martingale

$$\mathbb{E}_0^{\mathbb{Q}}\left[e^{-rt}L_t^{ih}\,\middle|\,\mathcal{F}_t\right] = L_0^{ih}, \quad \text{for all } t \leq T.$$

This requirement uniquely determines the drift of the process under $\mathbb{Q}$ to be equal to the risk-free rate r. Hence, the dynamics of L_t^{ih} under the equivalent martingale measure $\mathbb{Q}$ becomes

$$dL_t^{ih} = rL_t^{ih}\,dt + \sigma^i L_t^{ih}\,dB_t^{\mathbb{Q},ih}, \tag{2}$$

where $B_t^{\mathbb{Q},ih}$ is a standard Brownian motion under the risk-neutral measure $\mathbb{Q}$, and r is the constant risk-free interest rate. This ensures that the cost process is compatible with risk-neutral valuation principles, and allows option-based investment decisions to be priced using expected discounted payoffs under $\mathbb{Q}$.

2.2 *Bidder's Real Option Valuations*

This subsection details the valuation methods implemented for each bidder type under risk-neutral framework. We consider two types of bidders that use one of the following valuations: Net Present Value valuation, which treats projects as deterministic cash flows without flexibility. American option-based valuation, which captures the possibility of early exercise and is solved numerically via Least Squares Monte Carlo simulation.

Wozabal et al. (2021) argues that typically renewable auctions have a sufficient number of bidders to be reasonable to assume that bidders are truthful. A bid is considered *truthful* if it equals the bidder's reservation price that is, the price at which the bidder is indifferent between accepting the contract or not. In this context, submitting such a bid is referred to as truth-telling. We further assume that the decision to invest is instantaneous: if the option is exercised energy production starts immediately.

NPC bidders evaluate the investment opportunity using a standard net present value (NPV) approach which means they do not account for the *value of flexibility* or the *option to delay investment* that is embedded in the awarded project. To allow for a consistent comparison with OBC (option-based cost) bidders, we assume that NPV bidders develop their projects at the contract maturity date, $t = T$ and that commit to exercise the contract regardless of how project costs evolve. Hence, this valuation approach is equivalent to viewing the awarded CfD as a forward contract on electricity revenues. NPC bidders commit to invest at maturity T at the strike price that makes them indifferent between investing or not, i.e., the strike that satisfies the conditions in the following equations. Under a uniform pricing rule, this strike should be interpreted as the minimum acceptable price, since the clearing price may turn out to be higher than the bidder's submitted strike price.

$$\mathrm{NPV}(L_T^{ih}, K) = e^{-rT} \left(K - L_T^{ih} \right) \tag{3}$$

Where K is the awarded price at auction. So the expected project valuation per MWh corresponds to the discounted difference between the fixed remuneration price K and the expected levelized cost of energy (LCOE) at time T. Formally, the expected NPV is given by

$$\mathbb{E}_0^{\mathbb{Q}} \left[\text{NPV}(L_T^{ih}, K) \right] = e^{-rT} \left(K - \mathbb{E}_0^{\mathbb{Q}}[L_T^{ih}] \right) = e^{-rT} \left(K - e^{rT} L_0^{ih} \right) = K e^{-rT} - L_0^{ih} \tag{4}$$

where L_0^{ih} denotes the initial LCOE for each bidder and project. For Net Present Cost (NPC) bidders, the valuation of the investment opportunity is straightforward. The bid submitted by an NPC bidder corresponds to the strike price K that results to the net present value of the project to zero, conditional on investment occurring at T. This results in a simple valuation formula:

$$\begin{aligned}
&\mathbb{E}^{\mathbb{Q}} \left[e^{-rT} \left(K_{\text{NPC}}^{*,ih} - L_T^{ih} \right) \right] = 0, \\
&\Leftrightarrow e^{-rT} \left(K_{\text{NPC}}^{*,ih} - \mathbb{E}^{\mathbb{Q}}[L_T^{ih}] \right) = 0, \\
&\Leftrightarrow e^{-rT} \left(K_{\text{NPC}}^{*,ih} - e^{rT} L_0^{ih} \right) = 0, \\
&\Leftrightarrow K_{\text{NPC}}^{*,ih} = e^{rT} L_0^{ih}.
\end{aligned}$$

Now for OBC bidder's, as mentioned, this bidder's values their awarded project as a real option, explicitly accounting for the managerial flexibility embedded in the investment decision. To capture the full flexibility, we model the project as an *American-style put option*, which allows the bidder to decide at any time $t \leq T$ whether to invest. Exercising the option corresponds to committing to the project when it is economically favorable.

Under the risk-neutral measure $\mathbb{Q}$, the value of this option at time t, denoted V_t, is:

$$V(t, L_t^{ih}) = \sup_{\tau \in \mathcal{T}_{[t,T]}} \mathbb{E}^{\mathbb{Q}}\Big[e^{-r(\tau-t)} \max\big(K - L_\tau^{ih}, -P\big) \mid L_t^{ih}\Big]. \tag{5}$$

where $\mathcal{T}$ is the set of admissible stopping times, L_t^{ih} is the stochastic LCOE process defined in (2) and P the fee. Since this optimal stopping problem does not admit a closed-form analytical solution for general stochastic processes, we rely on numerical approximation methods to estimate the option value. In particular, we employ the Longstaff–Schwartz Monte Carlo (LSMC) algorithm, which approximates the continuation value through regression on simulated paths. So the payoff function is

$$\text{Payoff}_t = \max(K - L_t^{ih}, -P), \tag{6}$$

which accounts for both the gain from investing when costs are favorable and the potential cost associated with the fee. The continuation value at each step is then estimated via regression on a chosen set of basis functions. In our implementation, we choose the first four Laguerre polynomials as basis functions, which are commonly used in LSMC due to their orthogonality and numerical stability. We consider only paths where the payoff exceeds -P. Comparing this adapted payoff to the estimated continuation value determines the optimal exercise decision for each path and time step.

The optimal bid $K^{*\mathbb{Q}}_{\text{OBC,Am}}$ is the strike price such that the American option value equals zero:

$$K^{*\mathbb{Q}}_{\text{OBC,Am}}: \quad V_0^{\mathbb{Q}}(K^{*,ih}_{\text{OBC,Am}}; T, L_t^{ih}, \sigma, P, r) = 0.$$

A detailed description of how we compute and solve the optimal stopping problem, as well as the American-style option valuation, is provided in Section 3.

3 LEAST-SQUARES MONTE CARLO AND OPTIMAL STOPPING PROBLEM

The valuation of American-style options, which permit exercise at any time before maturity, is essential for modeling managerial flexibility in renewable energy investments under CfDs. This valuation problem is formulated as an optimal stopping problem, where the investor seeks the optimal time to commit to a project based on the stochastic evolution of project costs relative to the fixed strike price guaranteed by the CfD.

This optimal stopping problem is governed by the Hamilton-Jacobi-Bellman (HJB) variational inequality:

$$\max\left\{max(K-L,-P)-V,\ \frac{\partial V}{\partial t}+rL\frac{\partial V}{\partial L}+\frac{1}{2}\sigma^2L^2\frac{\partial^2V}{\partial L^2}-rV\right\}=0, \tag{7}$$

where the first term corresponds to immediate exercise and the second to continuation. The solution must satisfy the boundary condition $V(T,L)=(K-L,-P)$.

Analytical solutions are generally intractable, especially in multi-factor or path-dependent settings. The Least-Squares Monte Carlo (LSMC) method, introduced by Longstaff & Schwartz (2001), provides a flexible numerical approach by approximating the continuation value using regression on simulated paths.

Let N denote the number of discrete time steps and M the number of simulated paths. Define the time grid $\{t_0,t_1,\ldots,t_N=T\}$ with $t_n=n\Delta t$ and $\Delta t=T/N$. For each path $m=1,\ldots,M$, simulate the project cost $\{L^m_{t_n}\}^N_{n=0}$ under $\mathbb{Q}$. The payoff at each time is:

$$H^{ih,m}_n=max(K-L^{ih,m}_{t_n},-P). \tag{8}$$

The value process $\{V^{ih,m}_n\}$, which represents the value of the option at time step n, is computed backward using dynamic programming:

$$\begin{cases} V^{ih,m}_N=H^{ih,m}_N, \\ V^{ih,m}_n=\max\left(H^{ih,m}_n,\widehat{C}_n(L^{ih,m}_{t_n})\right), \quad n=N-1,\ldots,0, \end{cases} \tag{9}$$

where $\widehat{C}^{ih,m}_n(L^{ih,m}_{t_n})$ is the estimated continuation value. The true continuation value at time t_n can be written as the conditional expectation of the discounted future cash flows (realized when following the optimal stopping rule from t_{n+1} onward):

$$C^{ih,m}_n(L^{ih,m}_{t_n}) \;=\; \mathbb{E}^{\mathbb{Q}}\big[e^{-r\Delta t}\,Y_{n+1}\;\big|\;L^{ih,m}_{t_n}\big], \tag{10}$$

where Y_{n+1} denotes the path of cash flows from t_{n+1} onward conditional on the option not being exercised at or prior to t_n and on the option holder following the optimal stopping strategy for all t_s, $t_n<t_s\le T$. The LSMC fitted continuation value $\widehat{C}_n(\cdot)$ is the projection of the true continuation value onto the span of the chosen basis functions $\{\phi_j\}^J_{j=1}$. Explicitly,

$$\widehat{C}^{ih,m}_n(L^{ih,m}_{t_n})=\sum_{j=1}^{J}\widehat{\alpha}_j\phi_j(L^{ih,m}_{t_n}), \tag{11}$$

obtained by regressing the discounted future cash flows generated by the option

$$Y_{n+1}^{ih,m} = e^{-r\Delta t} V_{n+1}^{ih,m} \tag{12}$$

onto a set of basis functions $\{\phi_j\}_{j=1}^J$ for paths that follow $\mathcal{I}_n = \{m \mid H_n^m > -P\}$:

$$\min_{\alpha \in \mathbb{R}^J} \sum_{m \in \mathcal{I}_n} \left(Y_{n+1}^{ih,m} - \sum_{j=1}^{J} \alpha_j \phi_j (L_{t_n}^{ih,m}) \right)^2 . \tag{13}$$

The stopping rule for each path and time step is:

$$\tau^m = \min \left\{ t_n \mid H_n^{ih,m} \geq \widehat{C}_n (L_{t_n}^{ih,m}) \right\}, \tag{14}$$

and the estimated option value is:

$$\widehat{V}_0^{ih,m} = \frac{1}{M} \sum_{m=1}^{M} e^{-r(\tau^m - t_0)} H_{\tau^m}^{ih,m}. \tag{15}$$

The procedure is summarized as:

1. Simulate M paths $\{L_{t_n}^{ih,m}\}$ under $\mathbb{Q}$, over the discrete time grid $0 = t_0 < t_1 < \cdots < t_N = T.$.
2. Set terminal condition $V_N^{ih,m} = H_N^{ih,m}$ for all m.
3. For $n = N - 1$ down to 0:
   (a) Regress $Y_{n+1}^{ih,m}$ onto $\{\phi_j(L_{t_n}^{ih,m})\}_{j=1}^J$ for $m \in \mathcal{I}_n$.
   (b) Compute $\widehat{C}_n^{ih,m}(L_{t_n}^{ih,m})$ and update $V_n^{ih,m}$.
4. Compute $\widehat{V}_0^{ih,m}$ as the average discounted payoff.

The convergence properties of the Least-Squares Monte Carlo (LSMC) method have been extensively studied and are well established within the literature on numerical optimal stopping. The method can be understood as a layered approximation procedure involving (i) discretization of the continuous-time stopping problem, (ii) approximation of conditional expectations through regression onto a finite set of basis functions, and (iii) estimation via Monte Carlo sampling. Each layer introduces an approximation error, but under suitable conditions these errors can be controlled.

Rigorous results show that the LSMC estimator converges to the true option value as the number of simulated paths increases, the set of basis functions is enriched, and the time discretization becomes smaller. In particular, Clément et al. (2002) provide a detailed convergence analysis under fixed basis functions, while Stentoft (2004) extends the results to settings where the number of basis functions grows with the sample size.

4 CASE STUDY

In this section, we apply the theoretical framework and modeling approaches developed to a practical example. Specifically, we analyze a renewable auction scenario where different types of bidders submit projects with varying cost structures and risk profiles. This case study shows how the valuation methods and auction mechanisms perform in a realistic setting.

We simulate the outcomes of the German onshore wind energy auction held in August 2017. Following the auction settings in Wozabal et al. (2021), we consider: a benchmark scenario with NPC and OBC bidders using European-style valuation, and a second setting where OBC bidders adopt American-style valuation. This allows us to isolate and evaluate the impact of early exercise flexibility on bidding strategies and auction results.

4.1 Auction simulation

To simulate auction outcomes, we require information on the auction's regulatory framework (such as pricing format, bid caps, and contract duration), bidder characteristics (number of participants, project costs, and cost uncertainty), and a financial parameter (the risk-free interest rate).

The ONWA17 auction procured 1000 MW of onshore wind capacity, with a bid cap of €70/MWh. Two bidder categories were eligible: commercial entities and non-commercial bidding groups (NCBGs). NCBGs benefitted from longer grace periods (4.5 vs. 2.5 years), lower financial pre-qualification thresholds (€15,000/MW vs. €30,000/MW), and uniform pricing (award equal to the clearing price), whereas commercial bidders faced discriminatory pricing (award equal to their submitted bid). Since NCBGs constituted 81% of participants and 99% of awarded capacity, we focus our simulation on this group, applying their parameters. Also, Wozabal et al. (2021) calibrated the ratio of NPC to OBC bidders using the real auction results, finding that the simulations matched most closely with the real outcomes when 65% of bidders were OBC. Instead of performing a similar calibration, we directly adopt a 60/40 ratio of OBC to NPC bidders in our simulations.

Financial pre-qualification requirements were converted into a per-MWh charge by assuming a project lifetime of 25 years, an annual full-load hour figure of 2,721 hours, and discounting using a 1.17% risk-free rate (German 30-year bond yield in 2017). This yields a pre-qualification cost equivalent to €0.2443/MWh.

The auction received 281 bids with project sizes ranging from 750 kW to 23.8 MW. Our simulation replicates bid sizes through a two-stage process: first, we draw the capacity bracket for each project based on empirical probabilities Bundesnetzagentur (2025); second, we sample the exact capacity from a uniform distribution within that bracket. We simulate 150 bidders, each submitting one to three projects with equal probability, which results in an expected total of roughly 300 bids, close to the observed figure.

Levelized cost of energy (LCOE) values for German onshore wind in 2017 ranged from €39.9 to €82.3/MWh, depending on location and site conditions. Consistent with Wozabal et al. (2021), we assign one volatility estimate per bidder rather than per project, capturing firm-specific uncertainty in costs.

Overall, the parameters and setup build on the renewable auction literature and closely follow the ONWA17 benchmark. Our main contribution lies in extending this framework to analyze the role of American-style bidders.

An overview of all parameter values and distributions is provided in Table I. Following

Parameter	**Value / Distribution**
Auctioned capacity	1000 MW
Bid cap	70 €/MWh
Project life time	25 years
Grace period	4.5 yrs (NCBGs); 2.5 yrs (commercial)
Financial pre-qualification	15,000 €/MW (NCBGs); 30,000 €/MW (commercial)
Pre-qualification payment	0.2443 €/MWh (converted, 25 yrs, 2721 full load hours, r=1.17%)
Pricing format	Uniform (NCBGs)
Risk-free rate	1.17% (German 30-yr bond, 2017–2018)
Number of participants	150 bidders
Projects per bidder	1–3, equal probability
Share of bidder types	60% OBC, 40% NPC
Bid capacities	750 kW–23.8 MW
LCOE range	39.9–82.3 €/MWh
Volatility σ	0–15%, triangular distribution
Capital expenditure (Cc) [€/kW]	Normal, 1500–2000 €/kW (mean 1750, SD 350)
WACC [%]	3.5% (constant)
Loan period	25 years
Fixed O&M	Triangular, 12–48 €/kW (mode 30)
Variable O&M	0.005 €/kWh
Capacity factor [%]	Normal, mean 31.06, SD 7 (range 28.53–41.10%)
Monte Carlo runs	200,000

TABLE I: Parameters used in the auction simulations.

Heck et al. (2016), the initial levelized cost of electricity (LCOE) for each project is computed in two steps. First, the annualized capital repayment P is derived from capital expenditure C_c using the annuity formula

$$P = C_c \cdot \left[\frac{w(1+w)^n}{(1+w)^n - 1} \right], \tag{16}$$

where w denotes the weighted average cost of capital (WACC) and n the project lifetime in years. Second, total LCOE per MWh is calculated as

$$\text{LCOE} = \frac{P + O\&M_F}{8760 \cdot C_f} + O\&M_V, \tag{17}$$

where $O\&M_F$ are fixed operation and maintenance costs, $O\&M_V$ variable operation and maintenance costs, and C_f the capacity factor of the plant. This yields a distribution of project-specific initial costs L_0^{ih}, which are then used as the starting values in the stochastic cost process.

5 RESULTS AND DISCUSSION

This section analyzes bidder behavior under variations in key parameters and compares auction simulation results between settings with European-option bidders and American-option bidders. In particular, it examines how bidder valuations vary with auction settings and economic factors fee, maturity, and volatility. The comparison highlights the differences in option valuation approaches and their implications for bidding strategies and auction outcomes. The analysis is supported by numerical simulations and illustrated through a case study on the ONWA17 auction.

5.1 Option values, exercise boundary and time distribution

To illustrate the effect of the fee parameter P on option values and exercise behavior, three representative bidders were selected from the auction simulation. These bidders were chosen directly from one auction simulation, and thus face the real option contract under their own initial project costs and volatilities. All bidders face the same strike price, given by the auction clearing price ($K = 42.54$), but differ in their underlying cost dynamics:

- Bidder 1: $L_0 = 41.20$, $\sigma = 0.13$ (lowest accepted strike, winning project)
- Bidder 2: $L_0 = 48.87$, $\sigma = 0.11$ (intermediate winning bidder)
- Bidder 3: $L_0 = 56.88$, $\sigma = 0.10$ (marginal winning bidder, sets the clearing price)

These bidders were taken from one simulation of the auction. This setup allows interpreting the auction as allocating the same American put option (with strike K and fee P) to heterogeneous projects, each with distinct initial costs and risk profiles. Understanding option exercise behavior requires examining not only the value of the option over time, but also when exercise occurs and how frequently bidders exercise under different fee levels.

Figures 1–3 display, for each representative bidder, the estimated density of exercise times (left panel) and the cumulative probability of exercise before expiry (right panel) for varying fees P.

From Bidder 1 to Bidder 3, we observe a clear decrease in both the overall probability of investing in the project and the frequency of early exercise. For example, Bidder 1 has several paths with optimal exercise between years 0 and 2, whereas Bidder 3 has almost no paths for which early exercise is optimal. This behavior is largely explained by the higher initial cost of Bidder 3. The same pattern is evident in the exercise boundary plots 8–10: since the exercise frontier is computed from the simulated paths, time intervals with almost no exercisable paths make the boundary estimation unreliable or infeasible. Therefore, we discard these intervals from the analysis, as they are difficult to estimate and have limited relevance.

Figure 1 shows that Bidder 1's exercise behavior is concentrated at the end of the horizon, but not as extremely delayed as the other bidders. The mean exercise times (2.82, 2.80, 2.77 for $P = 0.2443,\ 0.4886,\ 0.7329$ respectively) are relatively earlier. The fee parameter P has virtually no effect on the timing distribution: all density curves nearly overlap. On the right panel, the cumulative probability of exercise increases modestly with P: from 57.61% to 59.35%. Thus, Bidder 1 exhibits the highest overall propensity to exercise among the three bidders.

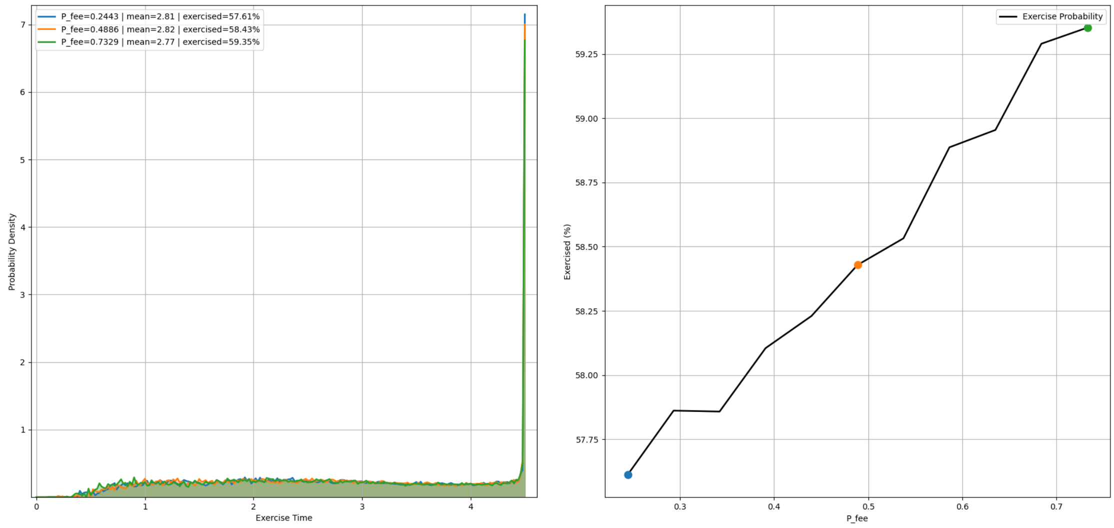

FIGURE 1: Bidder 1: distribution of exercise time (left) and probability of exercising before T (right).

For Bidder 2 (Figure 2), the exercise time distribution is shifted further toward the end of the horizon, with mean exercise times around 3.5. The fee parameter P again does not materially change the timing profile, but the overall exercise probability is much lower: rising from 27.03% to 28.71% as P increases. This suggests that, compared to Bidder 1, Bidder 2 is considerably more reluctant to exercise, despite showing the same positive relationship between P and exercise likelihood.

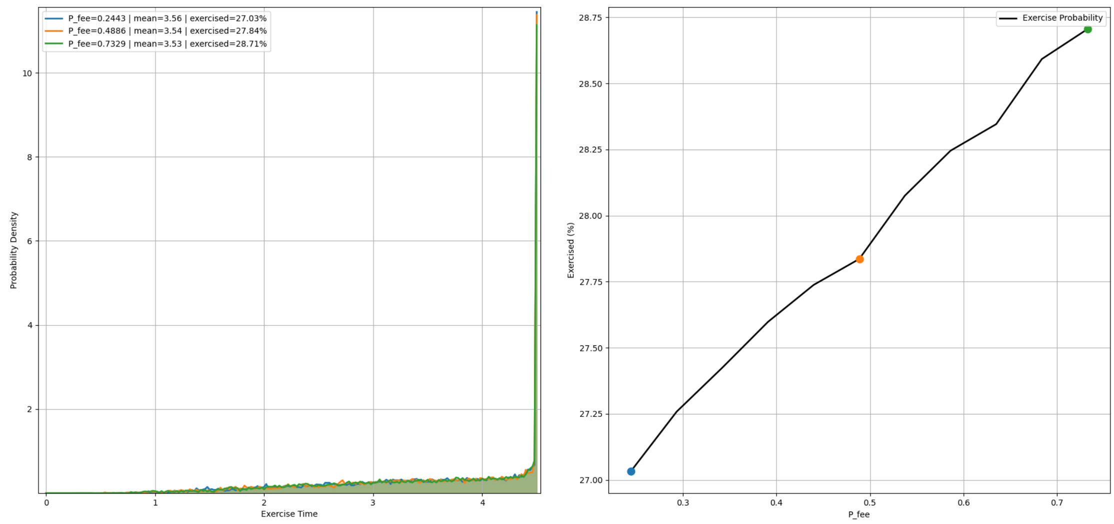

FIGURE 2: Bidder 2: distribution of exercise time (left) and probability of exercising before T (right).

Finally, Bidder 3 (Figure 3) exhibits the most extreme behavior. The density of exercise time shows a very sharp spike at maturity, with mean exercise times essentially at T (3.96–3.99). This bidder almost always delays until the very last moment, and exercises with very low frequency: only 6.97% at $P = 0.2443$, increasing slightly to 7.78% at $P = 0.7329$. Compared to the others, Bidder 3 displays both the lowest overall exercise probability and the strongest preference for waiting.

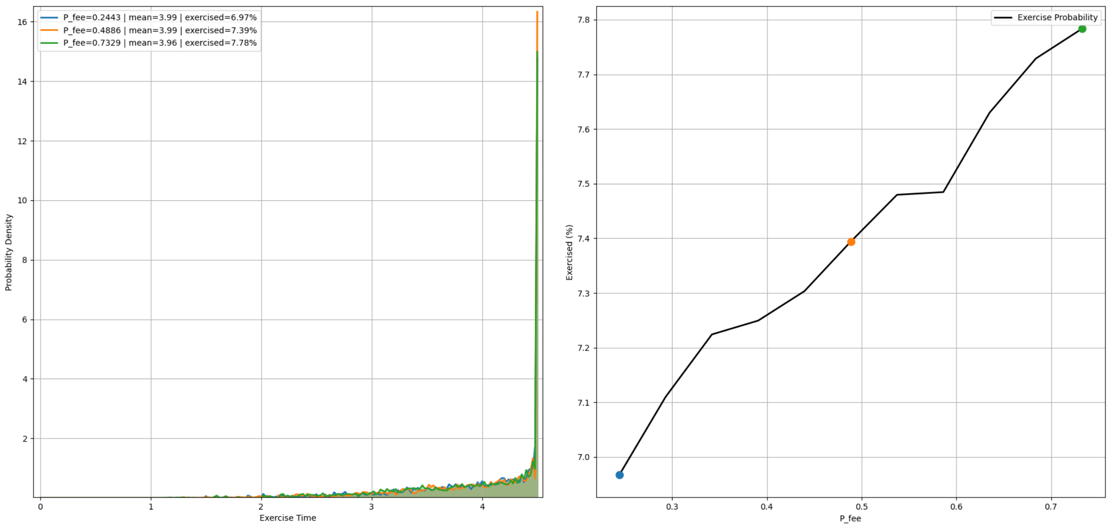

FIGURE 3: Bidder 3: distribution of exercise time (left) and probability of exercising before T (right).

Overall, these results highlight how the uniform auction mechanism implicitly allocates heterogeneous option values: the same contract is worth more to bidders with lower initial costs, while being closer to fairly priced for the marginal bidder. This valuation heterogeneity is directly reflected in the subsequent exercise behavior. Across all three bidders, two consistent patterns emerge. First, the mean exercise time distribution decreases slightly as the fee parameter P increases, indicating that higher fees induce somewhat earlier exercise. Second, the fee also raises the cumulative probability of exercise before expiry, although to different degrees across bidders. Bidder 1 exercises most often (about 58–59%), Bidder 2 less so (about 27–28%), and Bidder 3 very rarely (about 7%). This heterogeneity reflects underlying differences in initial cost levels and risk, and highlights how the same auction contract produces very different incentives across bidders.

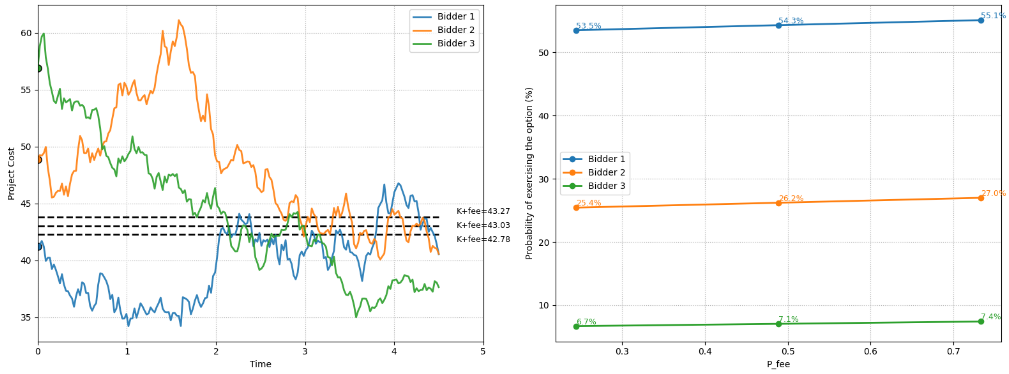

FIGURE 4: Cost path simulations and exercise probabilities of bidders using European put option for comparison

On the left, we present three examples of simulated cost paths for each bidder that are in the money at maturity (T), using an European put Option valuation for comparison. While on the right we show their corresponding exercise probabilities. We observe a consistent pattern: as the fee P increases, the exercise probabilities also increase, but at a decreasing rate. This indicates that bidders become less sensitive to changes in the fee as it grows larger than American-type bidders.

Figures 5–7 show the evolution of option values over time for each bidder under three levels of the fee parameter P. In all cases, option values decline as maturity approaches, reflecting the shrinking value of waiting. A higher P systematically reduces option values at every point in time, capturing the effect of fees as a "penalty" for postponement. The decline is sharper for Bidders 2 and 3, who begin with higher cost levels ($L_0 > K$), leaving their options closer to being out of the money.

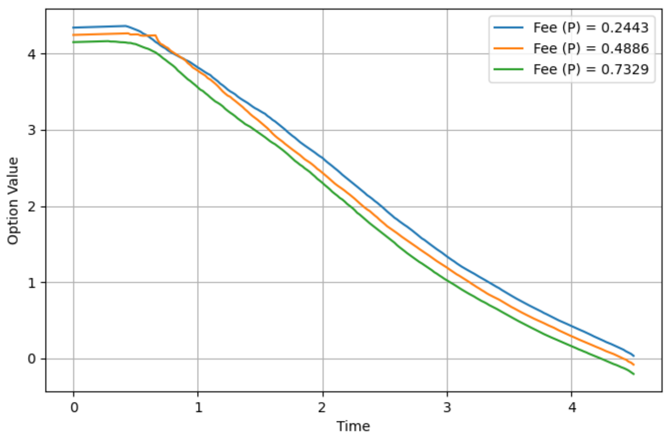

FIGURE 5: Bidder 1 option values over time for three different fee levels.

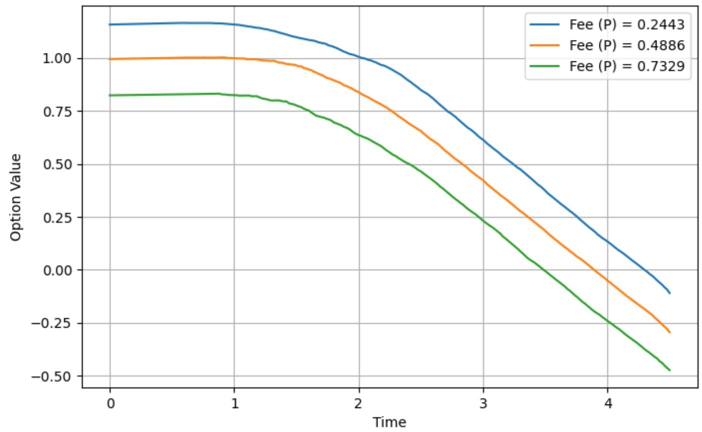

FIGURE 6: Bidder 2 option values over time for three different fee levels.

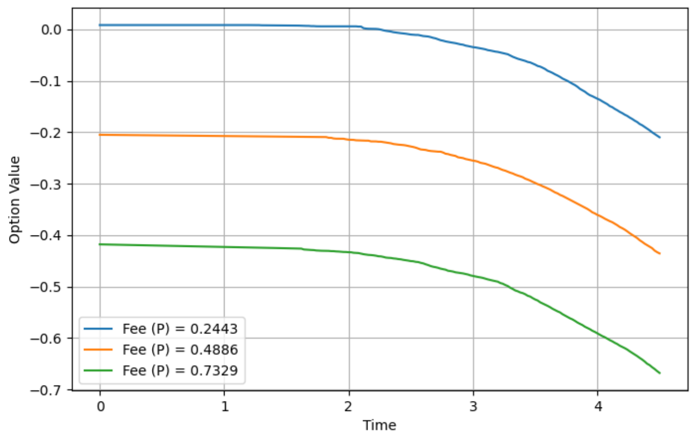

FIGURE 7: Bidder 3 option values over time for three different fee levels.

Note that, as expected, in Figure 7 the option values for Bidder 3 are negative. This bidder corresponds to the marginal bidder whose strike price satisfies the optimal bidding condition, that is, the option value at $t = 0$ equals zero ($V_0 = 0$). For this reason, as time progresses, the option value converges to the fee P. In contrast, Bidders 1 and 2 have strike prices above their respective optimal bidding strategies, which results in positive option values. Hence, while Bidders 1 and 2 exhibit positive option values throughout, Bidder 3—being the marginal, optimally bidding participant—shows negative option values that converge toward the fee as time passes.

In addition to option values, Figures 8–10 report the estimated exercise boundary for each bidder under varying fee levels. The comparative static is clear: higher P shifts the exercise threshold upward at nearly all maturities, making earlier exercise more attractive.

This pattern is consistent with interpreting P as a cost of waiting. At the very beginning of the horizon, some irregularities appear in the estimated boundary, which are due to numerical noise in the LSMC procedure (few paths exercise early, weakening the regression fit). We therefore focus on the mid-to-late horizon, where the results are more reliable: as maturity approaches, all boundaries converge upward toward the strike price.

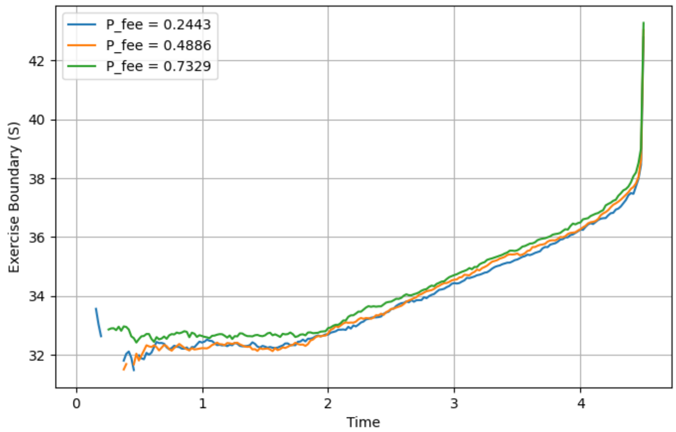

FIGURE 8: Bidder 1 exercise boundary for three different fee levels.

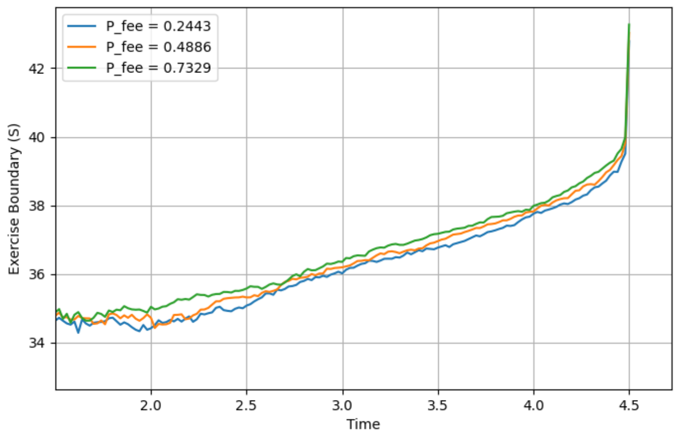

FIGURE 9: Bidder 2 exercise boundary for three different fee levels.

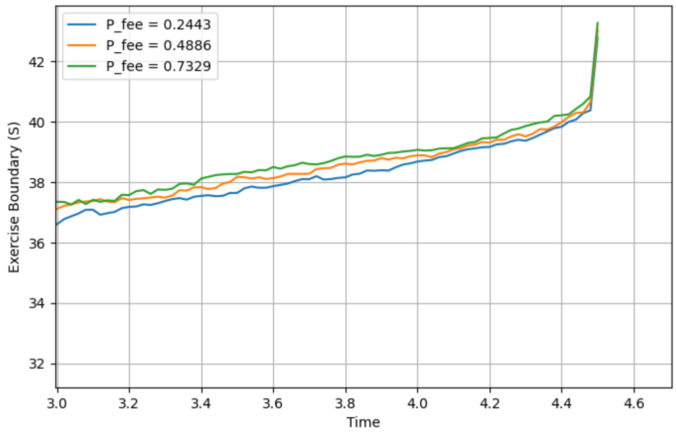

FIGURE 10: Bidder 3 exercise boundary for three different fee levels.

Figures 11–13 compare exercise boundaries across maturities ($T = 3.5, 4.5, 5.5$). As maturity increases, the exercise frontier shifts downward, reflecting the larger value of waiting when more time remains. Crucially, the frontier itself is determined by option parameters (volatility, discounting, strike, and exercise-related costs) and does not depend on the initial cost level L_0; L_0 only determines a bidder's position relative to that frontier. In our simulations bidders differ both in L_0 and in volatility. Importantly, differences in the exercise frontier across bidders arise from variation in volatility (and, where applicable, discount rates or fees), not from the initial cost level L_0. Lower volatility raises the put's exercise frontier, meaning investors are willing to exercise at relatively higher cost realizations, while higher volatility lowers the frontier because the option value of waiting increases. Accordingly, the observed decline in early-exercise frequency from Bidder 1 to Bidder 3 reflects the effect of their volatility profiles: greater uncertainty delays investment, whereas lower uncertainty accelerates it.

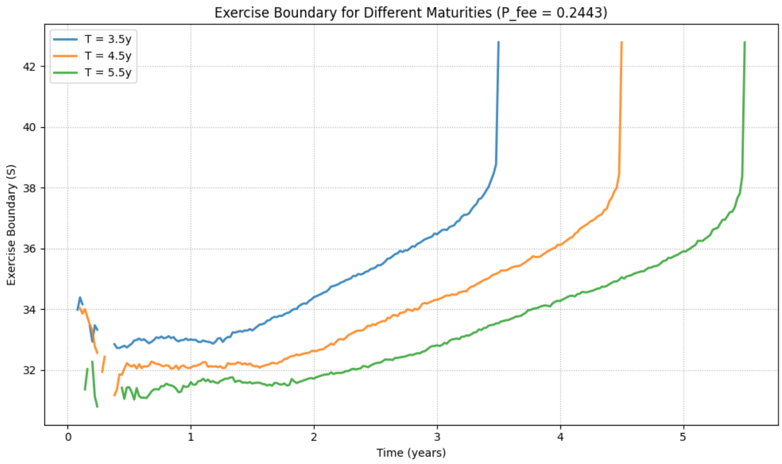

FIGURE 11: Bidder 1 exercise boundary for different maturities.

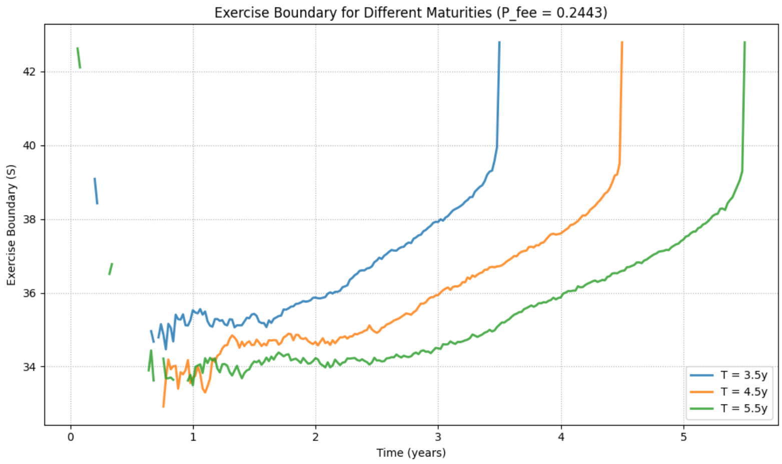

FIGURE 12: Bidder 2 exercise boundary for different maturities.

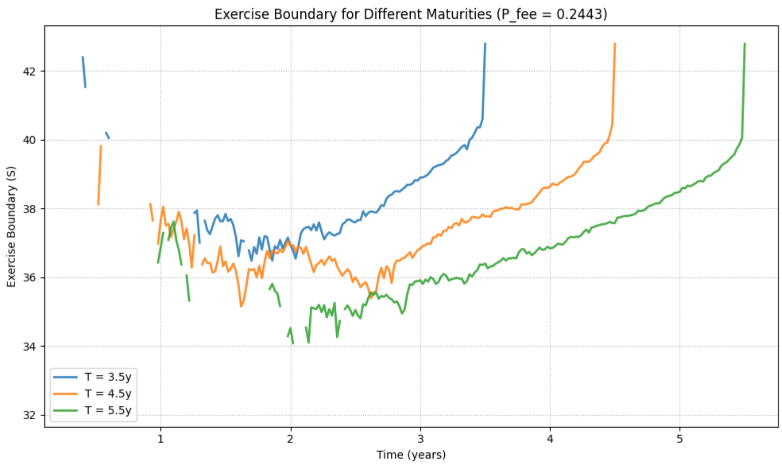

FIGURE 13: Bidder 3 exercise boundary for different maturities.

5.2 Varying Volatility

Figure 14 presents two key relationships: Bidding Strike Price (K^*) vs. Volatility and OBC American-style option value at fixed K vs. Volatility. The plots are constructed by considering a representative bidder with an initial cost L_0. For the left panel, the optimal bidding strike price $K^{*,ih}$ is recalculated for each level of volatility, while for the right panel, the bidding strike $K^{*,ih}$ is fixed and the option value is evaluated across different volatilities.

The left panel shows that as volatility increases, the optimal bidding strike price decreases, indicating that greater uncertainty leads bidders to lower their bids. The right panel illustrates that the American option value at a fixed strike generally rises with volatility, consistent with option theory, since higher volatility increases the likelihood of favorable outcomes (the option ending in-the-money).

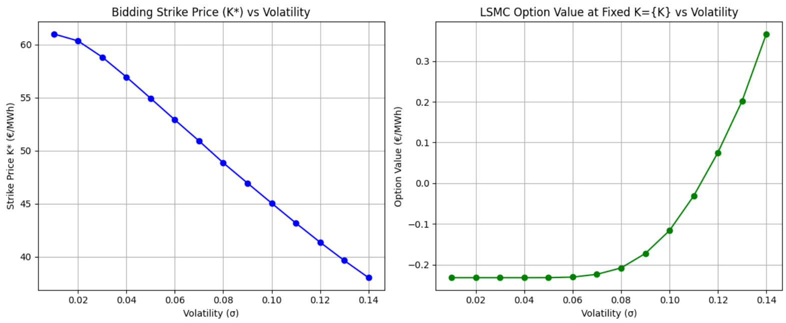

FIGURE 14: American option value varying volatility

Figure 15 shows the OBC American Option Value vs L_0 at a $K = 37.4828$€/MWh for three different volatility levels (σ= 0, 0.075, 0.15). Demonstrates how the option value changes with the initial underlying cost (L_0) and volatility. As expected, when volatility is zero, the option value is $-P$ unless the costs significantly go below the strike price. As volatility increases, the option value becomes positive and increases as L_t decreases, reflecting the higher probability of the option being in-the-money. The higher the volatility, the greater the option value for a given L_t, reinforcing the role of uncertainty in real options valuation. These figures collectively provide an analysis of the model's behavior under varying market conditions.

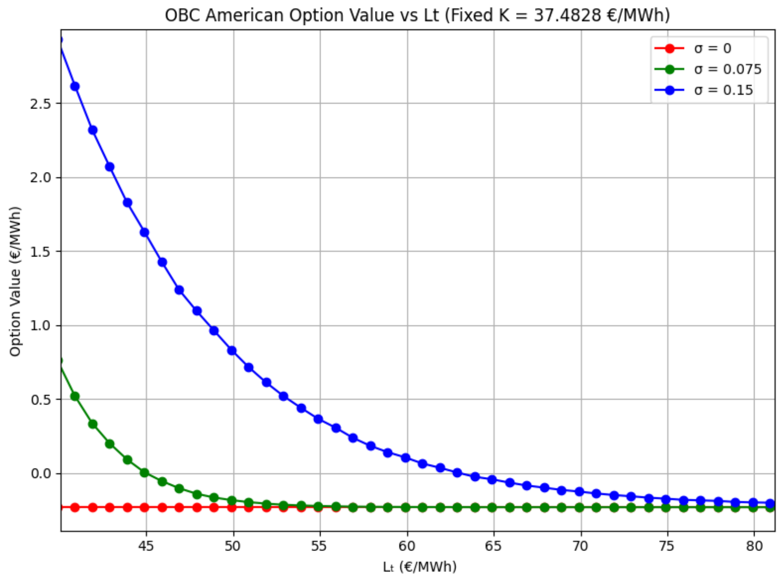

FIGURE 15: American option value vs Lt for different volatilities

Figure 16 compares the OBC American Put option value with the European Put Option value across different strike prices K, with an initial LCOE of 55 €/MWh. The graph show that the American option consistently holds a higher value than the European, particularly at higher strikes. This difference in value is attributed to the American option's flexibility, allowing for early exercise when optimal. The vertical dashed lines indicate the optimal bidding strike prices (K^*) for both valuation methods. The slightly lower K^* for the American option further supports the idea that the added flexibility leads to more competitive bids.

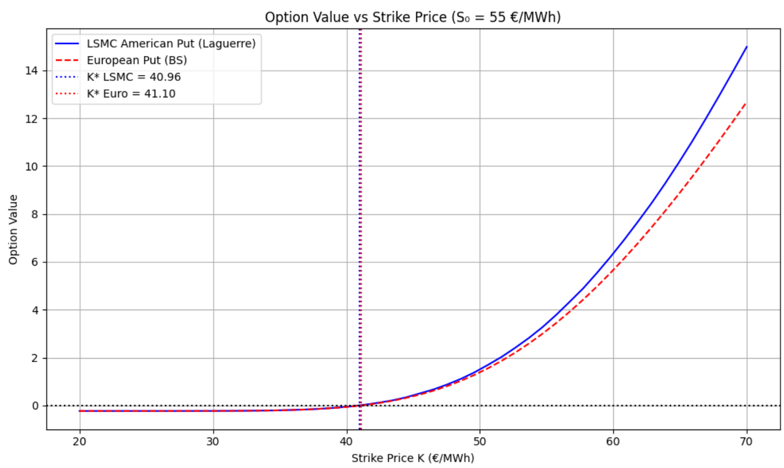

FIGURE 16: American option value vs Lt

These results collectively demonstrate the impact of incorporating American-style options and real-world valuation into the analysis of renewable energy auctions. The differences observed in bidding functions and exercise behavior provide crucial insights for understanding investor strategies and designing more effective auction mechanisms.

5.3 Auction Outcomes Comparison

Table II compares the simulated auction outcomes against the real auction outcomes for the German ONWA17. These results come from a different run of the simulation than the one used in Sections 5.1 and 5.2. Here, the American auction refers to our model using the American put, while the European auction uses the European put.

TABLE II: Simulated outcomes vs. Real Auction Outcomes (Germany, Onshore Wind, August 2017)

Outcome	**European Auction**	**American Auction**	**ONWA17 (Real)**
Maximal Awarded Price (€/MWh)	41.27	40.53	42.90
Weighted Avg. Award Price (€/MWh)	41.27 (uniform)	40.53 (uniform)	42.80*
Minimal Bid (€/MWh)	27.67	27.54	35.00
Maximal Bid (€/MWh)	69.50	69.50	64.50

*The real auction average is slightly below the maximal awarded price because only 99% of winning bidders were NCBGs, who benefit from the uniform price rule. The remaining 1% were commercial entities, as explained above.

The point of this comparison is not the closeness of the simulated results to the real ONWA17 outcomes, but rather to highlight how accounting for investment flexibility alters the results. At first glance, the differences between the European and American models may seem small. However, when viewed in terms of their impact on subsidy payments, the effect becomes significant.

By itself, this comparison is not particularly informative. What matters is whether variations in auction settings, such as changes in fees, lead to different outcomes under European put versus American put valuation. This distinction is crucial, as it directly affects the policy goals behind adjusting auction design. For example, ONWA17 auctioned 1,000 MW with a clearing price of 42.9 €/MWh (under the uniform price rule). Assuming a mean capacity factor of 31.06%, 2,721 full load hours per year, and a 50% realization rate, this corresponds to a subsidy burden of approximately €18.13 million per year. If instead the auction clearing price were 40.53 €/MWh, as in the American option simulation, the subsidy burden would fall to about €17.10 million annually. Hence, a seemingly small reduction of 2.44 €/MWh in the clearing price translates into nearly

€1.03 million less in subsidies per year for just one auction round. Figure 17 illustrates the bidding function comparison between the NPC + OBC European-style model and the NPC + OBC American-style model, highlighting the effect of early exercise flexibility on bidding strategies.

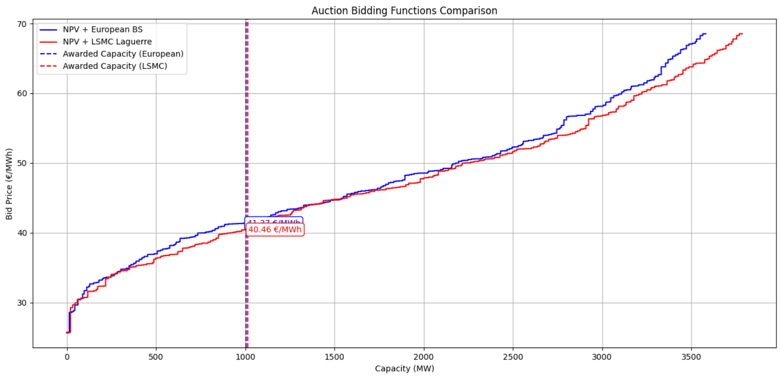

FIGURE 17: Auction bidding functions for ONWA17: NPC + OBC European vs. American model

6 EXTENSIONS

Here we relax some assumptions. First the no correlation assumption between projects of the same bidder. Then we talk about the risk-neutral assumption and discuss about the theory and approaches that exist that relax this assumption.

6.1 Correlated Project Costs

We relax the previous assumption of independent project costs and introduce correlation between the costs of projects belonging to the same investor. Using the same fixed random seed, we examine how this correlation affects auction outcomes and the resulting equilibrium bids. To do this, we specify a correlation matrix for each bidder's set of projects and apply a Cholesky decomposition Pourahmadi (2007). Independent standard normal shocks are generated at each time step and transformed using Cholesky factor to produce correlated shocks. These correlated shocks are then used in the stochastic process governing project costs.

In the baseline model, project cost dynamics for project h of bidder i were modeled as independent geometric Brownian motions (GBM).

$$dX^{ih}(t) = \left(r - \tfrac{1}{2}\sigma_{ih}^2\right) dt + \sigma_{ih}\, dB^{ih}(t), \quad h = 1, \dots, n_i, \tag{18}$$

where $X^{ih}(t) = \ln L_{ih}(t)$ denotes the log-cost process and $B_{i,h}(t)$ are independent Brownian motions. To relax the independence assumption, we introduce correlation within the bidder's portfolio. Let Σ_i denote the $n_i \times n_i$ correlation matrix of bidder i's projects. We compute its Cholesky decomposition C_i such that:

$$\Sigma_i = C_i C_i^\top . \tag{19}$$

At each time step, we draw a vector of i.i.d. standard normals $\mathbf{z}_t \sim hcalN(0, I)$ and construct correlated shocks as:

$$\boldsymbol{\varepsilon}_t = C_i \mathbf{z}_t . \tag{20}$$

The discretized log-cost dynamics for project j are then:

$$X_{i,h}(t + \Delta t) = X_{i,h}(t) + \left(\mu - \tfrac{1}{2}\sigma_{i,h}^2\right) \Delta t + \sigma_{i,h}\sqrt{\Delta t}\, \varepsilon_{t,h} . \tag{21}$$

Finally, the cost level can be recovered via:

$$L_{i,h}(t) = \exp(X_{i,h}(t)) . \tag{22}$$

Thus, correlation is introduced only between projects of the same bidder, while projects of different bidders remain independent. Economically, this reflects the idea that multiple projects of the same investor may share common risk drivers, such as financing conditions, technological dependencies, or geographical factors. Figure 18 presents the same results as Figure 17, but now accounting for correlation between projects of the same bidder, which we set at $\rho = 0.7$ (70%). We observe a general increase in bidding prices. However, to determine whether this increase is truly due to the introduction of correlation or simply a result of simulation randomness, it would be necessary to repeat the simulation multiple times and compute the average outcomes.

Auction Type	**Minimum Bid (€/MWh)**	**Maximum Bid (€/MWh)**	**Maximal Awarded Price (€/MWh)**
NPC + European option	28.33	73.72	41.83
NPC + American option	27.51	73.23	40.87

TABLE III: Auction Results

Total runtime: 36,173.38 seconds

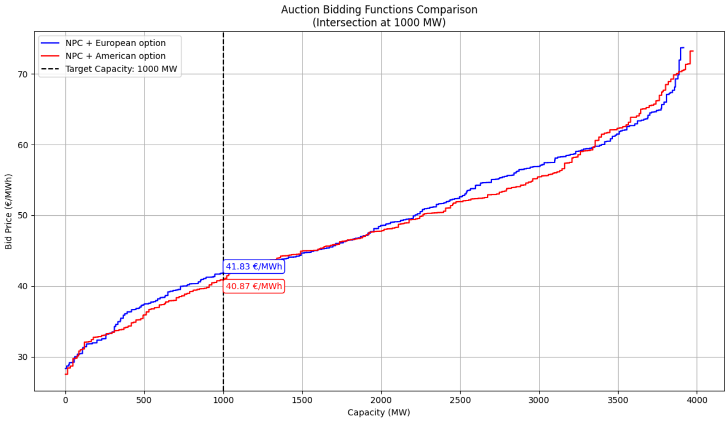

FIGURE 18: Auction simulation results of the bidding function for ONWA17 with intra-bidder project correlation fixed at $\rho = 0.7$ (70%).

7 CONCLUSION

This work has explored the application of real options theory to investment decisions in renewable energy projects under Contracts for Difference. By framing the Levelized Cost of Energy as the underlying variable and the guaranteed CfD strike price as the exercise price, the model naturally captures the managerial flexibility available to investors, which is crucial for understanding bidding dynamics in renewable energy auctions where cost uncertainty and the ability to delay investment are key drivers of strategy. Two types of bidders were considered: Net Present Cost (NPC) bidders, who rely on a traditional discounted cash flow approach without valuing flexibility, and Option-Based (OBC) bidders, who explicitly recognize the value of flexibility. A key extension of this work was to model OBC bidders using American-style put options, allowing for early exercise in contrast to the more common European formulations. The Least-Squares Monte Carlo (LSMC) method was employed to estimate both option values and optimal exercise boundaries under uncertainty. The results highlight three central implications. First, incorporating American-style flexibility systematically leads to lower bids, reflecting the added value of deferral. Second, the uniform auction mechanism implicitly allocates heterogeneous option values: projects with lower initial costs benefit relatively more, while the marginal bidder receives a contract closer to fair value. Third, exercise behavior exhibits consistent patterns: all bidders tend to postpone investment until close to maturity, but the penalty parameter (P) reduces the mean exercise time and increases the cumulative probability of exercise, although to different degrees across bidder types. From a policy perspective, these findings emphasize the importance of accounting for bidder heterogeneity in the design of CfDs. Because projects differ in initial costs, risk profiles, and flexibility, a uniform CfD or changes in the economic environment (such as adjustments to the fee or market conditions) will not generate uniform responses. Instead, each bidder may react differently, resulting in varied investment incentives and behaviors across the set of awarded projects. For regulators, acknowledging the option-like nature of CfDs helps explain observed behaviors such as aggressive bidding, delayed investments, and heterogeneous strategies among winners. Real options analysis thus bridges financial valuation and policy design, offering insights into how auction rules, penalty schemes, and contract maturities shape investment timing and project viability in renewable energy. In this model, we assume that the duration of the CfD contract coincides with the project lifetime of 25 years. This assumption is not entirely realistic, since most CfDs typically have a maturity of around fifteen years. A natural extension would be to incorporate policy uncertainty, whereby the investor accounts for the possibility that the CfD may expire before the end of the project lifetime. In such a case, the project would be exposed to market prices after contract expiration, introducing an additional source of uncertainty through the selling price. This can be represented either by assigning a cumulative probability distribution to early contract termination or by explicitly modeling shorter contract durations relative to the project lifetime. A suitable framework for capturing this type of risk is to use regime-switching models, as in Hagspiel et al. (2021).

Several avenues remain open for future research. For instance, the assumption that it is always preferable to enter an investment with a CfD could be relaxed by introducing participation costs. In this case, the investor's problem could be modeled as holding a compound where $V_{option,CfD}$ is the value of the real option under a CfD contract, $C_{auction}$ represents the cost of entering the auction, and p_{win} denotes the probability of winning a CfD. Recognizing that investors may always retain the possibility of entering the energy market directly, even if they do not win a CfD contract. Then the investor's decision problem is framed as a compound option with multiple branches: by not participating, the investor preserves the option to invest directly in the market, with value $V_{option,Market}$; by participating, the investor incurs a cost $C_{auction}$ and faces a probabilistic outcome. With probability p_{win}, the investor gains access to the CfD-backed option with value $V_{option,CfD}$; with probability $1 - p_{win}$, the investor falls back to the market option $V_{option,Market}$. Heuristically, the investor's payoff from participating in the auction can be seen as the maximum between two possibilities: either winning the CfD and gaining the associated option value net of the auction cost, or not winning and retaining the value of investing directly in the market. This can be written as

$$\max\Big(p_{win} \cdot V_{option,CfD} + (1 - p_{win}) \cdot V_{option,Market} - C_{auction},\ V_{option,Market}\Big),$$

highlighting the idea that participation preserves the option to invest while accounting for the probabilistic outcome of winning a CfD. Considering also the option to enter the energy market directly and the option to enter a CfD if awarded in auction depends on two stochastic processes: L_t, representing the project's cost evolution, and S_t, representing the market electricity price. In this case, the market option payoff becomes contingent on the joint dynamics of these two variables, for example $\max(S_\tau - L_\tau, 0)$, where τ denotes the optimal investment time. Incorporating this structure transforms the participation decision into a compound, multi-dimensional real option, as the investor must weigh the probabilistic outcome of winning a CfD against the stochastic value of entering the market.

The Least-Squares Monte Carlo (LSMC) method is well suited to handle such an extension, as it can simultaneously simulate joint paths for (L_t, S_t) and estimate continuation values across multiple underlying variables, allowing the optimal exercise strategy and option value to be approximated even in this higher-dimensional setting. This nested structure can be interpreted as a generalization of compound options, consistent with the "options on options" framework of Geske (1979), but tailored to the context of renewable energy auctions. Future extensions could also examine the role of risk aversion, correlated project risks between bidder's, secondary CfD markets, and consider the construction time of the project.

In sum, by integrating real options theory with auction analysis, this study provides insights into how uncertainty and flexibility may shape renewable energy investment strategies, offering a perspective that could be useful for researchers and for informing the design of auction mechanisms and regulatory frameworks in support of the energy transition.

REFERENCES

Bundesnetzagentur (2025), 'Statistiken zum ausschreibungsverfahren für windenergieanlagen an land', Excel file. Accessed: 2025-06-29.
URL: *https://www.bundesnetzagentur.de/DE/Sachgebiete/ErneuerbareEnergien*

Ceseña, E. A. M. (2013), 'Real options theory applied to electricity generation projects: A review', *Renewable and Sustainable Energy Reviews* **19**, 573–581.
URL: *https://doi.org/10.1016/j.rser.2012.11.059*

Clément, E., Lamberton, D. & Protter, P. (2002), 'Ananalysis of a least squares regression method for american option pricing', *Finance and Stochastics* **6**(4), 449–471.

del Río, P. (2022), 'Which policy instruments promote innovation in renewable electricity technologies? a critical review of the literature with a focus on auctions', *Energy Research Social Science* **89**, 102501.

Dixit, A. K. & Pindyck, R. S. (1994), *Investment Under Uncertainty*, Princeton University Press, Princeton, NJ.

European Parliament and Council of the European Union (2021), 'Regulation (EU) 2021/1119 of the European Parliament and of the Council of 30 June 2021 establishing the framework for achieving climate neutrality and amending Regulations (EC) No 401/2009 and (EU) 2018/1999 ('European Climate Law')', Official Journal of the European Union. Regulation (EU) 2021/1119.
URL: *https://eur-lex.europa.eu/eli/reg/2021/1119/oj/eng*

Fernandes, B., Cunha, J. & Ferreira, P. (2011), 'The use of real options approach in energy sector investments', *Renewable and Sustainable Energy Reviews* **15**(9), 4491–4497.
URL: *https://doi.org/10.1016/j.rser.2011.07.102*

Fleck, A.-K. & Anatolitis, V. (2022), 'How to design efficient renewable energy auctions? empirical insights from europe', *Energy Policy* **166**, 112982. Open access under CC BY license.
URL: *https://doi.org/10.1016/j.enpol.2022.112982*

Fleck, A.-K. & Anatolitis, V. (2023), 'Achieving the objectives of renewable energy policy – insights from renewable energy auction design in europe', *Energy Policy* **173**, 113357. Open access under CC BY license.
URL: *https://doi.org/10.1016/j.enpol.2022.113357*

Geske, R. (1979), 'The valuation of compound options', *Journal of Financial Economics* **7**(1), 63–81.

Greenpeace International (2024), 'Annual report 2023'. Accessed: 2025-06-26.
URL: *https://www.greenpeace.org/international/publication/67950/annual-report-2023/*

Haahtela, T. (2012), 'Differences between financial options and real options', *Lecture Notes in Management Science* **4**, 169–178.

Hagspiel, V., Nunes, C., Oliveira, C. & Portela, M. (2021), 'Green investment under time-dependent subsidy retraction risk', *Journal of Economic Dynamics and Control* **126**, 103936. Received 31 October 2019; Revised 13 May 2020; Accepted 16 May 2020; Available online 28 May 2020; Version of Record 8 May 2021.
URL: *https://doi.org/10.1016/j.jedc.2020.103936*

Heck, N., Smith, C. & Hittinger, E. (2016), 'A monte carlo approach to integrating uncertainty into the levelized cost of electricity', *Energy Policy* **91**, 341–350.
URL: *https://doi.org/10.1016/j.enpol.2016.01.031*

Jackson, M. O. & Kremer, I. (2006), 'The relevance of a choice of auction format in a competitive environment', *The Review of Economic Studies* **73**(4), 961–981.

Kreiss, J., Ehrhart, K.-M. & Haufe, M.-C. (2017*a*), 'Appropriate design of auctions for renewable energy support – prequalifications and penalties', *Energy Policy* **101**, 512–520. Received 4 July 2016, Revised 25 October 2016, Accepted 3 November 2016, Available online 10 November 2016, Version of Record 30 December 2016.
URL: *https://doi.org/10.1016/j.enpol.2016.11.007*

Kreiss, J., Ehrhart, K.-M. & Haufe, M.-C. (2017*b*), 'Appropriate design of auctions for renewable energy support–prequalifications and penalties', *Energy Policy* **101**, 512–520.

Longstaff, F. A. & Schwartz, E. S. (2001), 'Valuing american options by simulation: A simple least-squares approach', *The Review of Financial Studies* **14**(1), 113–147.

Matthäus, D. (2020), 'Designing effective auctions for renewable energy support', *Energy Policy* **142**.

Oliveira, C. & Perkowski, N. (2020), 'Optimal investment decision under switching regimes of subsidy support', *European Journal of Operational Research* **285**(1), 120–132. Received 1 March 2018; Revised 25 January 2019; Accepted 7 February 2019;

Available online 11 February 2019; Version of Record 9 April 2020.
URL: *https://doi.org/10.1016/j.ejor.2019.02.019*

Pourahmadi, M. (2007), 'Cholesky decompositions and estimation of a covariance matrix: Orthogonality of variance–correlation parameters', *Biometrika* **94**(4), 1006–1013.

Stentoft, L. (2004), 'Convergence of the least squares monte carlo approach to american option valuation', *Management Science* **50**(9), 1193–1203.

Swinkels, J. M. (2001), 'Efficiency of large private value auctions', *Econometrica* **69**(1), 37–68.

Szabó, L. (2025), 'Policy relevant lessons from research on renewable energy auctions', *Energy Policy* **203**, 114624. Open access under CC BY license.
URL: *https://doi.org/10.1016/j.enpol.2025.114624*

Wozabal, D., Schwene, S. & Matthäus, D. (2021), 'Renewable auctions: Bidding for real options', *European Journal of Operational Research* **291**(3), 1091–1105.