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DISSERTATION

BACKTESTING AND PERFORMANCE OF PAIRS TRADING: COPULA VERSUS DISTANCE APPROACHES

ANTÓNIO MANUEL BARBOSA DE ARAÚJO PEREIRA ELIAS

OCTOBER - 2023

Lisbon School of Economics & Management
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SUPERVISION:

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GLOSSARY

CDO Collateralized Debt Obligation. i, 6

CDS Credit Default Swap. i, 6

CRRA Constant relative risk aversion. i, 6

CVaR Conditional Value at Risk. i, 28, 29

MDA Modified Distance Approach. i, 9, 13, 14, 21, 23–27, 29–32

PTS Pairs Trading Strategies. i, 1–3, 23

RI Return Index. i, 9, 10

TRI Total Return Index. i, 10

VaR Value at Risk. i, 23, 28, 29, 33

ABSTRACT AND KEYWORDS

In a universe of the historical constituents of the Euro STOXX 50, we implement two pairs trading strategies and compare their performance: in the first strategy, we use copula functions which were fitted to the logarithmic returns of each pair to emit trading signals, while in the second strategy the trading signals are emitted if the spread between the normalized prices of each pair has surpassed a certain threshold. Only the first strategy shows to be profitable with a relatively worse performance when compared to the benchmark, but at lower levels of volatility.

KEYWORDS: Pairs Trading; Copula; Distance approach; Algorithmic trading.

JEL CODES: G12, G15, G17

RESUMO E PALAVRAS-CHAVE

Implementamos num universo de constituintes históricos do Euro STOXX 50 duas estratégias de negociação de pares e comparamos o seu desempenho: na primeira estratégia, utilizamos funções cópula que foram ajustadas aos retornos logarítmicos de cada par para emitir sinais de negociação, enquanto na segunda estratégia os sinais de negociação são emitidos se o spread entre os preços normalizados de cada par ultrapassar um determinado limite. Apenas a primeira estratégia se mostrou rentável, embora tenha um desempenho relativamente pior em relação às marcas de referência do índice de mercado, mas com níveis de volatilidade mais baixos.

PALAVRAS-CHAVE: Negociação de pares; Funções cópula; Método da distância; Algoritmo de investimentos.

CÓDIGOS JEL: G12, G15, G17

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Acknowledgements

First and foremost, I want to express my gratitude to my father, João Elias, my mother, Paula Pereira, and my brother, Afonso Elias, for their unwavering support throughout my life and for their patience while I pursued this project.

I also want to extend my gratitude to all my friends who have been always on my side.

To Alice Chaves, Andreia Rebocho, Filipa Esteves, Gonçalo Almeida, Luís Gonçalo, Maria Ana Morais, Miguel Ferreira, Pedro Lima and Pedro Mendes: thank you for making my college years truly special. To more years to come with a lot of episodes of "Show do Américo".

To Diana Pereira, Karolina Azevedo, Leah Pereira, Luís Eduardo Graça, Marta Antunes and Mónica Gonçalves: thank you for helping me to grow as a person, and for putting up with me while I was undergoing that journey.

To Luis Aser Portela, Francisco Araújo and José Ascenção: thank you for being by my side for so long. Your friendship means a lot to me.

To Inês Vilela, João Neves, Letícia Azevedo, Maria João Rodrigues and Mariana Rocha, thank you for your support through harsh times.

To Catarina Marcão, João Pedro Roxo and Miguel Almeida: thank you for your companionship throughout the master's program, and for making the past two years more meaningful.

Lastly, I wish to thank Professor Raquel Gaspar for her support, advice, and availability throughout this process.

1 INTRODUCTION

The concept of Pairs Trading has been around Wall Street since the mid-1980's, when a group of Quants led by Nunzio Tartaglia over at Morgan Stanley group has reportedly made over 50 million USD in profit for the firm by, among other strategies, identifying pairs of securities that traded aligned with one another [Gatev et al. (2006)].

The premise behind Pairs Trading Strategies (PTS) is quite simple to understand: after finding two securities that move together at a "constant" spread, we simultaneously buy one security and sell the other once the spread widens or retracts and close the positions once the securities come back to their historical spread.

It is conjectured that this trading strategy is viable due to concepts of relative pricing: The law of one price states that "two investments with the same payoff in every state of nature must have the same current value" [Ingersoll and Ingersoll (1987)]. Hence, when securities that historically are priced identically, an arbitrage opportunity should appear when the spread between the two securities widens. Gatev et al. (2006) further conjectures that this arbitrage opportunity is what maintains a first order level of market efficiency.

However, this hypothesis seems to be challenged by the concept of second order level of market efficiency, which states that every security has already priced in all public information, including the prices of other securities. Hence, it should be unattainable to obtain abnormal or above average returns on investments on using any sort of pairs trading strategies.

This types of strategies are practiced mostly by hedge funds under their market neutral strategies; the hedge fund takes long positions in securities it has identified as undervalued and short positions in securities it has identified as overvalued. The hedge fund tries to target an overall beta to be approximately zero, meaning that the returns obtained are neutral with respect to market risk and other risk factors (size, industry, momentum, value, etc.) [Jacobs et al. (1999)].

The pairs trading literature has developed along several different strains since Gatev et al. (2006) published the first scientific paper regarding PTS: we find several authors using different forms of statistical tests to access if a certain pair is cointegrated (and hence a good selection for trading) or using either econometric models or stochastic processes to model each leg of a pair's price. More recently, there has been an increasing number of approaches which have used bivariate copula functions to model the dependence between the pairs and use their marginal distribution functions to define trading signals.

It is also known that human emotions also can play a negative influence in decision making, particularly in investment decisions [Loewenstein et al. (2001)]. Hence having an algorithm giving trading signals for under or overpriced securities might be helpful in avoiding personal biases.

In this dissertation, we implement and backtest two different pairs trading strategies using Python in a sample of stocks contained in the Euro STOXX 50 from 2013 to the end of 2022: a first one using the trading signals proposed by Gatev et al. (2006) and a Return-based copula approach, as proposed by Krauss and Stübinger (2017).

The remaining of this dissertation is structured as follows: The second section covers a brief literature review on pairs trading strategies and applications of copulas in finance, the third section describes the data in which our strategies are backtested, as well as a thorough explanation of the methodology to implement the strategies, the fourth section includes the results and the performance review of the strategies and a final fifth section in which we provide some concluding remarks.

2 Literature Review

2.1 Pairs trading

According to Krauss (2017), we can segment the study of pairs trading into five different streams of literature: the distance approach, the cointegration approach, the time series approach, the stochastic control approach and other alternative approaches.

Within the distance approach, we find the first piece of literature that emancipated the study of PTS, which is the already mentioned paper by Gatev et al. (2006). His strategy consists firstly in dividing the timeline in two parts: a 12 month formation period followed by a 6 month trading period. The formation period consists in finding pairs that historically have traded with a close spread, by choosing a fixed number of pairs which had the lowest Euclidean distance between the normalized prices of each pair. After recording the spread between normalized prices of each pair for the formation period, he uses its standard deviation to emit trading signals during the trading period: If a pairs' spread during a trading period has surpassed two historical standard deviations, the first leg of the pair is sold while the second leg is bought under the assumption that the spread is going to revert back to a smaller spread. The same reasoning is applied if the spread exceeds two negative standard deviations, but with opposite positions. Positions are only closed after the spread crosses back to zero.

With this simple technique, Gatev finds the 1.30% monthly returns for the top 5 pairs using this strategy to be statistically and economically significant,with them being seemingly uncorrelated to the returns of the S&P500. He considers that the generation of these abnormal returns are a compensation to investors for enforcing the “Law of One Price”, at the same time he admits that in the later years of its analysis the returns decrease due to "increased hedge fund activity".

More recent studies, like Do and Faff (2010), find that the abnormal profits obtained by the distance approach are declining, justifying it mainly with an increase in arbitrage risks as well as an overall increase in market efficiency. They also propose to add a restriction such that only pair from the same industry groups can be considered to be able for trading. In another paper, Do and Faff (2012) admit that the distance approach becomes unprofitable after taking in account time-varying transaction costs. Xie et al. (2016) further criticize by comparing the Distance approach to linear correlation analysis, while Huck and Afawubo (2015) mention that using the Euclidean distance as a method to select pairs has several deficiencies that impact its profitability, leading to selection of spreads with limited profit potential and higher divergence risk.

Advancing to other approaches, the cointegration approach was created by Vidyamurthy (2004) and then applied in a large scale by Rad et al. (2016). This approach combines the method used by Gatev et al. (2006) of using the Euclidean distance to find co-moving pairs with the Engle-Granger cointegration test, as they select for trading the top 20 pairs from the formation period with the lowest Euclidean distance which are also cointegrated. During the trading period, trading rules similar to Gatev et al. (2006) are emitted.

The time series approach and the stochastic control approach have the common ground that the formation period is often ignored. Elliott et al. (2005) proposes a framework which inspires the time series approach, where his idea is to model the spread between securities which is observed in Gaussian noise by using a mean reverting Gaussian Markov chain at a price level, while Do et al. (2006) do it a return level. The stochastic control approach is based around using stochastic control theory to find mispricing in securities, with Jurek and Yang (2007) and Liu and Timmermann (2013) being some of the most relevant papers in this approach.

The other approaches section encompass the pairs trading frameworks which use alternative techniques, such as the paper by Huck (2009) where he uses machine learning and multi-criteria decision techniques to obtain multiple return forecasts, or Avellaneda and Lee (2010) proposes to use principal component analysis to model idiosyncratic returns as mean reverting processes. There is also strategies which propose the use of bivariate copula functions to emit trading signals. We will see those in more detail in the next section after setting the theoretical background for a Copula.

2.2 *Copulas and its applications in finance*

The word copula translates from latin to the word "link" or "tie".According to Nelsen (2006), a d-dimensional copula $C : [0,1]^d \rightarrow [0,1]$ is formally defined by being a cumulative distribution function with uniform marginals. The term is coined by Sklar (1959), who afterwards provides the theoretical foundation for the application of copulas through his theorem, as he states that a copula realizes a functional relationship between a multivariate distribution and its marginals. Let $F_{X_1,X_2,...,X_n}$ be a multi-dimensional distribution function with marginal distribution functions F_{X_1}, F_{X_2}... and F_{X_n}. Then, there exists a n-dimensional Copula C, which satisfies the following equation:

$$F_{X_1,X_2,...,X_n}(x_1, x_2, ..., x_n) = C(F_{X_1}(x_1), F_{X_2}(x_2), ..., F_{X_n}(x_n)). \tag{1}$$

There are numerous families of copula functions, both either parametric and non-parametric. To narrow the focus of our study, we only look to ellyptical copulas, which are models where the univariate margins are joined by an elliptical distribution. In the case that the distribution in question is the multivariate normal distribution, we call it a Gaussian copula:

$$C_\rho(x_1, x_2, ..., x_n) = \phi_\rho(\phi_\rho^{-1}(x_1), \phi_\rho^{-1}(x_2), ..., \phi_\rho^{-1}(x_n)), \tag{2}$$

where ϕ_ρ is a multivariate t-distribution with parameter ρ, and ϕ_ρ^{-1} is the inverse function the students'-t distribution. On the other hand, if the distribution joining the univariate margins is a multivariate student's, we call it a t-copula:

$$C_{\rho,\upsilon}(x_1, x_2, ..., x_n) = t_{\rho,\upsilon}(t_\upsilon^{-1}(x_1), t_\upsilon^{-1}(x_2), ..., t_\upsilon^{-1}(x_2)), \quad (3)$$

where $t_{\rho,\upsilon}$ is a bivariate t-distribution with parameters ρ and υ, and t_υ^{-1} is the inverse function students'-t distribution.

Following Genest et al. (2013), we find that there are 3 areas of finance where copulas are mostly used, given its ability to model non normal asset returns and the dependence of between between extreme values of said assets. Those three pillars are derivative pricing, portfolio management, risk management and measurement.

One of the most notorious application of copulas in financial models was brought forward by Li (1999), where he fits copula functions the marginal distributions of the prices of Credit Default Swap (CDS) of the assets contained in a Collateralized Debt Obligation (CDO) in order to determine the probability of default of the CDO. This factor led to a subsequent increase of the CDS and CDO markets with the emergence of a whole new array of financial securities, such as CDO-squared and the synthetic CDO, which now could be (or thought that could be) priced correctly by using Li's model; some consider that the model was one of the many factors that led to the 2008 financial crisis [Salmon (2009)].

There also has been other applications of copulas in option pricing, such as Cherubini and Luciano (2002) and Van den Goorbergh et al. (2005) who suggest to adopt copula functions in order to price bivariate contingent claims and to address the joint issues of non-normality of returns.

In portfolio management, Patton (2004) developed a model to assess the impact that skewness and asymmetric dependence have on portfolio decisions of a Constant relative risk aversion (CRRA) investor. Copulas functions were employed to the asset returns which allowed for greater dependence during tail events. They conclude that their model shows economic evidence to provide better portfolio decisions than the base model.

When it comes to pairs trading strategies, Krauss (2017) identified two different approaches in fitting bivariate copulas and use them to emmit trading signals; a level based approach, as proposed by authors such as Xie et al. (2016) or Rad et al. (2016), and the return based approach, which is represented by the works of Stander et al. (2013) or Krauss and Stübinger (2017).

Both of the approaches derive from the Skalar's theorem (Sklar (1959)) that given a two-dimensional distribution function, F_{X_i,X_j}, with marginal distribution functions F_{X_i} and F_{X_j}. Then, there exists a bivariate Copula C, which satisfies the following equation:

$$F_{X_i,X_j}(x_i, x_j) = C(F_{X_i}(x_j), F_{X_i}(x_j)). \tag{4}$$

The main idea is to find pairs that meet certain levels of dependence between their components and trade them based upon probabilities derived from their marginal distribution functions. To obtain them, we create uniform variables through parametric methods out of the logarithmic returns of the paired up securities, U_i and U_j. A copula is fitted to these variables so we may compute the said marginal distribution functions

$$h(u_i|u_j) = P(U_i \leq u_i|U_j = u_j) = \frac{\partial C(u_i, u_j)}{\partial u_j} \tag{5}$$

$$h(u_i|u_j) = P(U_j \leq u_j|U_i = u_i) = \frac{\partial C(u_i, u_j)}{\partial u_i}, \tag{6}$$

which will provide us with information regarding the relative pricing between the pairs' legs: if $h(u_i|u_j) > 0.5$ and/ or $h(u_j|u_i) < 0.5$, the first leg is considered to be overvalued relative to its peer. On the other hand, $h(u_i|u_j) < 0.5$ and/ or $h(u_j|u_i) > 0.5$, the first leg is considered to be undervalued relative to its peer.

This is the point where both approaches diverge: for the level based approach, it is constructed a mispricing index for new incoming logarithmic returns; if either $h(u_i|u_j)$ or $h(u_i|u_j)$ exceed 0.5, the difference between its value and 0.5 is added to the respective mispricing index, and if either $h(u_i|u_j)$ or $h(u_i|u_j)$ do not exceed 0.5, the difference between and 0.5 its value is subtracted to the respective mispricing index. Trading signals are then emitted if any of the index surpasses predefined thresholds and then positions are closed when the indexes come back to zero.

For the return based approach, instead of constructing a mispricing index with the accumulated values of the conditional distribution functions, we use directly the conditional distribution functions are used to emit trading signals: if during the trading period we find that $h(u_i|u_j) > 0.95$ and $h(u_j|u_i) < 0.05$, leg i is sold and leg j is bought, while if $h(u_i|u_j) < 0.05$ and $h(u_j|u_i) > 0.95$ leg i is bought and leg j is sold.

3 DATA AND METHODOLOGY

This chapter describes the data used and the methodology applied in constructing the pairs trading strategies using two different approaches: a copula approach and a Modified Distance Approach (MDA).

3.1 Data

For the application of the trading strategies, we opt to use the Euro Stoxx 50, which consists in 50 stocks of Eurozone's sector leaders blue-chip companies, making them the most liquid equities in Europe. The source to obtain the securities data is Refinitiv Eikon Datastream. We build a historical constituent list of the stocks in the Index, for which we download the Return Index (RI) for every equity security available in that list. The RI serves as a proxy for the daily stock prices as it takes in consideration management decisions and payout policy, making it a suitable metric to compute daily returns. The analysis focus in a total of 83 stocks, and from it is clear from Figure 1, the majority of them having France, Germany and the Netherlands as their country of domicile, while financials, consumer discretionary and industrials are the industries with more representation.

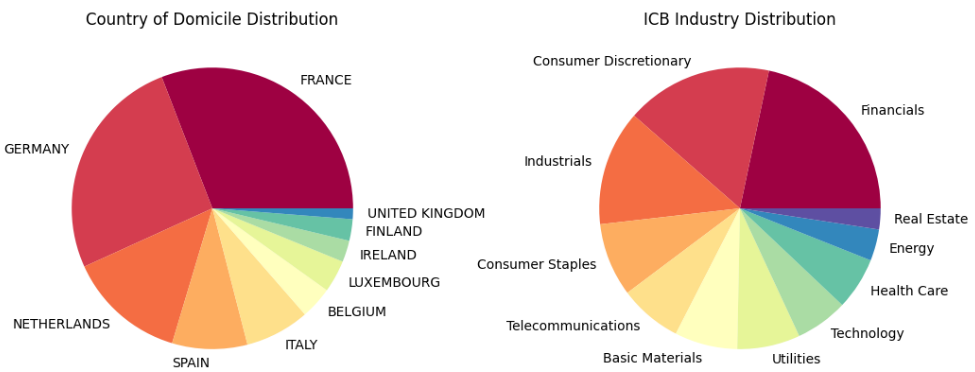

FIGURE 1: Historical constituent list's descriptive statistics

Since only 50 out of those 83 stocks are considered at any given time period, we construct a binary matrix which shows if a stock is available for trading in a specific date. This process allow us to mimic the historical evolution of the index composition, and thus eliminating survivorship bias from our sample. Finally, the Euro Stoxx 50 Total Return Index (TRI) is used as a proxy to the overall market performance and thus is the benchmark to "beat". For both the individual stocks RI and the Euro Stoxx 50 TRI, we have data ranging from January 2008 until December 2022, ending up with a total 3914 trading days and up to $\frac{50\times49}{2} = 1225$ pairs to consider for each time period.

3.2 Methodology

For both of our trading strategies, we follow Krauss and Stübinger (2017) timeline framework by dividing its implementation into two stages: firstly, a Formation period of 60 months where pairs are formed and selected followed by 12-month Trading period where the strategies are performed. The formation period is composed by a total of 48 12-month estimation period to form all available pairs followed by a 1-month pseudo-trading period to test the pairs previously formed and the performance of the recorded trading signals. These periods have a 1-month overlap, resulting in 48 months worth of pseudo-trading periods where the trading signals recorded are always adjusted to a recent estimation period, in an attempt to avoid recording spurious trading signals. The last 12 months of the formation period there is a final estimation period to make the final selection of suitable pairs to be used in the trading period, where we select the top 5 pairs which performed better during the pseudo-trading periods. Figure 2 illustrates this timeline framework.

The first strategy, developed by Krauss and Stübinger (2017), is a return based copula method while the second strategy is inspired by the distance approach developed by Gatev et al. (2006). Even though the distance approach was originally composed by a 12-month formation period followed by a 6-month trading period, we decide to modify it to the 60-month formation period followed by the 12-month trading period mentioned

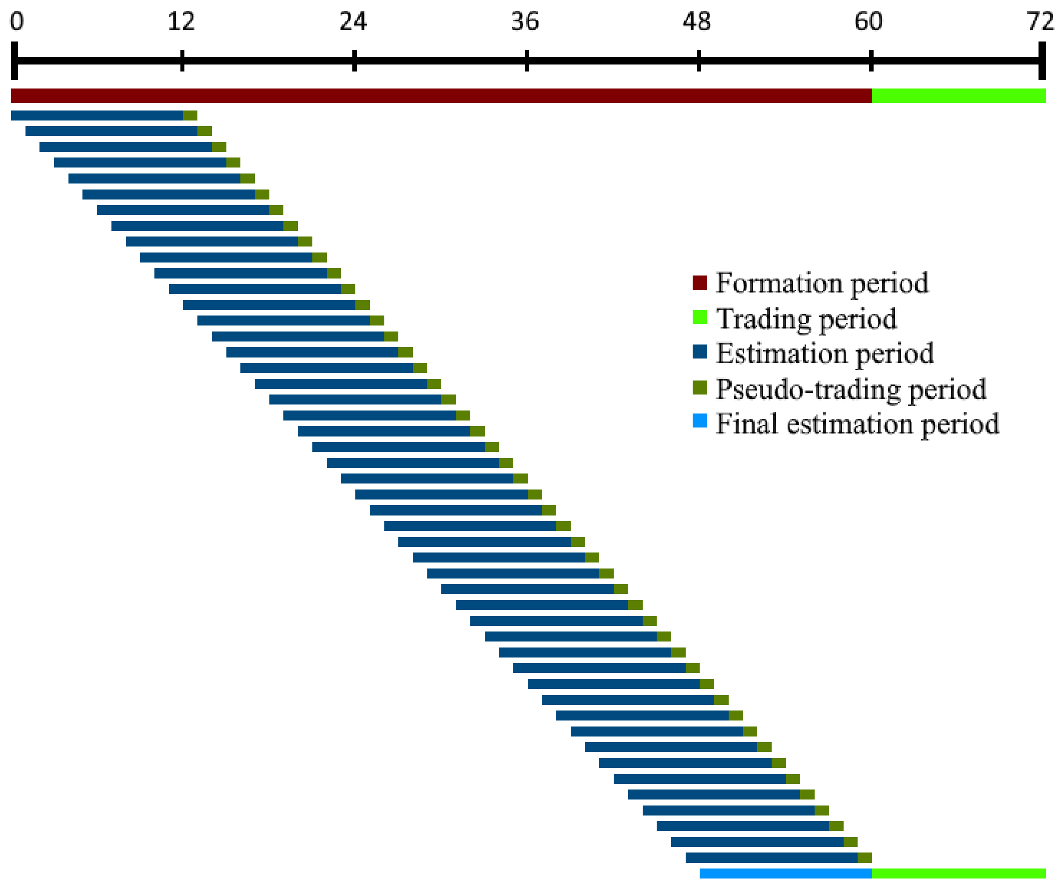

FIGURE 2: Timeline framework for the strategies

beforehand, allowing the comparability between the two strategies. The trading signals remain unaltered from the original paper, even though we add take profit and stop-loss levels using the framework from the Copula approach.

3.2.1 Estimation Period

For each estimation period, we pair up all the securities contained in the index during the period. For the copula approach, we diverge slightly from Krauss and Stübinger (2017) in the process of creating uniform variables: instead of using a non-parametric approach, we follow Stander et al. (2013) by fitting parametric marginal distributions through maximum likelihood estimation for the logarithmic return time series of both legs in each pair. Then, probability integral transformation is used to convert the logarithmic returns into uniform variables $U_i = F_{R_i}(R_i)$ and $U_j = F_{R_j}(R_j)$, where R_i and R_j are the logarithmic returns of leg i and j of a given pair. We fit these variables to a bivariate Student's t- copula:

$$C_{\rho,\upsilon}(u_i, u_j) = t_{\rho,\upsilon}(t_\upsilon^{-1}(u_i), t_\upsilon^{-1}(u_j)), \tag{7}$$

where $t_{\rho,\upsilon}$ is a bivariate t-distribution with parameters ρ and υ, and t_υ^{-1} is the inverse function the students'-t distribution. Estimates for all parameters are determined by maximum likelihood.

There are several out of sample analysis using statistical tests and information criteria which conclude that the t-copula is a better choice to model financial information data instead of other elliptical copulas, such as the Gaussian copula. This phenomenon is explained by the ability of the t-copula to capture the phenomenon of dependent extreme values, which is typically observed in financial return data. For further reading in that topic, see Lourme and Maurer (2017).

We then define the partial derivative of the copulas as the conditional distribution function of u_i conditional to u_j, and vice-versa.

$$h(u_i|u_j) = P(U_i \leq u_i|U_j = u_j) = \frac{\partial t_{\rho,\upsilon}(t_\upsilon^{-1}(u_i), t_\upsilon^{-1}(u_j))}{\partial u_j} \tag{8}$$

$$h(u_j|u_i) = P(U_j \leq u_j|U_i = u_i) = \frac{\partial t_{\rho,\upsilon}(t_\upsilon^{-1}(u_i), t_\upsilon^{-1}(u_j))}{\partial u_i}. \tag{9}$$

We use the conditional distributions to define confidence bands at a significance level of $\alpha = 0.05$ for all $(u_i, u_j) \in [0, 1]^2$ such as:

$$A = [0, 1]^2 \cap \{(u_i, u_j) : h(u_i|u_j) \leq 0.05 \wedge h(u_j|u_i) \geq 0.95\} \tag{10}$$

$$B = [0, 1]^2 \cap \{(u_i, u_j) : h(u_i|u_j) \geq 0.95 \wedge h(u_j|u_i) \leq 0.05\} \tag{11}$$

$$C = [0, 1]^2 \setminus \{A \cup B\}. \tag{12}$$

A depiction of the confidence bands can be found in Figure 3.

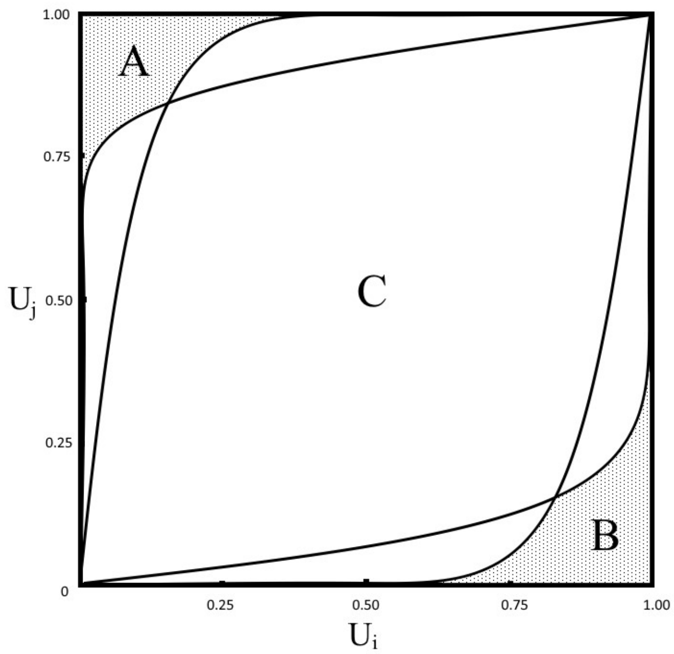

FIGURE 3: Confidence bands built using a copula fitted in the estimation period

Now, given new upcoming returns R_i and R_j computed in the pseudo-trading period (and afterwards in the trading period) are in one of the areas A, B or C, and we will be able to assess the relative pricing according to the fitted copula, with a 95% confidence level: if they belong to C, no considerations can be done about the pair; if they belong to A, the first leg is considered to be undervalued relative to the second leg while the second leg is considered to be overvalued relative to the first leg; if they belong to B, the first leg is considered to be overvalued relative to the second leg while the second leg is considered to be undervalued relative to the first leg.

For the MDA, we proceed by normalizing the securities' prices using the following formula:

$$P^i_{Normalized,t} = \frac{P^i_t - P^i_{Min}}{P^i_{Max} - P^i_{Min}}, \tag{13}$$

where P^i_{Min} and P^i_{Max} are respectively, the minimum and maximum prices of the security during the estimation period. Furthermore, we measure the spread between the prices of each pairs of securities and its respective standard deviation:

$$Spread_t^{ij} = P_t^i - P_t^j \tag{14}$$

$$\sigma_{Spread}^{ij} = \sqrt{\frac{\sum_{n=1}^{T}(Spread_t^{ij} - \overline{Spread^{ij}})^2}{T-1}}, \tag{15}$$

where $Spread_t^{ij}$ is the spread between the normalized prices of the pairs' two legs, and $\overline{Spread_t^{ij}}$ is the average spread across of the estimation period.

Before proceeding to the pseudo-trading period, we just want to make a reference to what happens in the final estimation period; as we repeat the procedures made in the previous estimation periods for both strategies, we need to consider that all the computations have impact in the actual trading period: for example, in the first strategy, the fitted copula's confidence bands are used to determine real trading signals; in the second strategy, the maximum and minimum prices in the final estimation period are used to normalise the prices in the trading period.

Additionally, for the copula approach, we compute Pearson's correlation coefficient during the final estimation period between the daily logarithmic returns of each pair component:

$$\rho_{ij} = \frac{\sum_{t=1}^{T}(r_i^t - \overline{r_i^t})(r_j^t - \overline{r_j^t})}{\sqrt{\sum_{t=1}^{T}(r_i^t - \overline{r_i^t})^2 \sum_{t=1}^{T}(r_j^t - \overline{r_j^t})^2}} \tag{16}$$

The correlation sets the level of dependence (or relationship) between the legs of a pair. Hence, we need to set a minimum value of correlation to ensure dependence. After some backtesting to ensure the availability of pairs, we filter out pairs with a Pearson's correlation coefficient of less than $\rho_{max} = 0.6$ from the trading period.

For the MDA, we compute the Euclidean between each normalized pair during the T

days that encompass the final estimation period:

$$d(P_i, P_j) = \sqrt{\sum_{t=1}^{T}(P_i^t - P_j^t)^2} \tag{17}$$

The Euclidean distance works as a measure of the price and spread's volatility, as low values of $d(P_i, P_j)$ imply that the normalized prices tend to move together at a low spread, a condition to guarantees the mean reversion. After some back testing to ensure the availability of pairs, we make the decision to filter out pairs with an Euclidean distance of more than $d(P_i, P_j)_{max} = 2.5$ from the trading period.

Simply put, we make the decision to look first at the final estimation period to see which pairs are going to be filtered out, meaning that in all estimation periods we do not fit copulas nor compute standard deviation of spreads for these pairs, as they are not going to be considered in the trading period. This allow us to improve significantly the computational performance of the backtesting, as we reduce from the 1225 pairs available at the start down to around a tenth of that. Table 1 shows the pairs' Euclidean distance ranked from lowest to highest for a formation period between January 2014 and December 2018 using the second strategy. A total of 72 pairs are considered to be eligible to go into the formation and consequently the pseudo-trading period.

3.2.2 Pseudo-trading period

We simulate trading signals between all the pairs formed in the previous estimation period under the condition that both securities are still constituents of the index during the pseudo-trading period.

For the copula approach, We compute the logarithmic returns of the available pairs in the index and construct the pairs (u_{it}, u_{jt}) using the fitted distributions in the estimation period and compute the conditional probabilities $h(u_i|u_j)$ and $h(u_j|u_i)$. Those points are in one of the areas defined in the estimation period (A, B or C), hence we can define the following trading signals:

TABLE 1: Securities selection in the final estimation period

Rank	Pairs	Euclidean distance
65	E:IND-B:ABI	2.449709
66	F:SGM-F:AIR	2.456952
67	F:SGM-F:QT@F	2.457659
68	F:LVMH-D:MUV2X	2.463461
69	F:AIR-D:MUV2X	2.466961
70	E:BBVA-B:ABI	2.484772
71	I:ISP-E:SAN	2.487208
72	D:SIEX-I:ISP	2.494835
73	D:ADSX-F:SGM	2.523495
74	E:SAN-D:MBGX	2.534324
75	D:MUV2X-F:BSN	2.543013
76	H:INGA-I:ENI	2.545524
77	F:AIRS-F:QT@F	2.545684
78	H:INGA-F:BNP	2.545829
79	D:SAPX-D:ALVX	2.558284

- If $h(u_{it}|u_{jt}) \leq 0.05$ and $h(u_{jt}|u_{it}) \geq 0.95$ or, in other words, if $(u_{it}, u_{jt}) \in A$, we consider the leg i (leg j) of the pair to be undervalued (overvalued) when compared to its peer, under the assumption that the returns are correlated. we make a 1 EUR long position in the first leg i and a 1 EUR short position in the second leg j. We name the combination of these two positions as making a long position in the pair.

- If $h(u_{it}|u_{jt}) \geq 0.95$ and $h(u_{jt}|u_{it}) \leq 0.05$, or, in other words, if $(u_{it}, u_{jt}) \in B$, we consider the leg i (leg j) to be overvalued (undervalued) when compared to its peer, under the assumption that the returns are correlated. Hence, we make a 1 EUR short position in the first leg i and a 1 EUR long position in the second leg j. We name the combination of these two positions as making a short position in the pair.

- In any other case, no trading signal is recorded.

For the second strategy, after normalizing the securities' prices using the previous estimation period's maximum and minimum prices, we recall the trading rules proposed by Gatev et al. (2006):

- If the spread drifts positively more than two historical standard deviations from zero, we consider the leg i (leg j) to be overvalued (undervalued) when compared to its peer, under the assumption that the pairs spread should revert back to a lower spread. Hence, we make a short position in the pair.

- If the spread drifts negatively more than two historical standard deviations from zero, we consider the leg i (leg j) of the pair to be undervalued (overvalued) when compared to its peer, under the assumption that the pairs spread should revert back to a higher spread. Hence, we make a long position in the pair.

- In any other case, no trading signal is recorded.

It is important to note that, for both strategies, we do not actually make any trading decision, but we rather record the profitability for a pair for the 120 days after each trading signal. Hence, for every s trading signal emitted, a cumulative return time series CR_{ij}^t is built for the 120 days after the signal:

$$CR_{ij}^t = \ln PI_{ij}^t - \ln PI_{ij}^0, t \in \{1, ..., 120\}, \tag{18}$$

where $PI_{ij;\,t}$ is a price index of the pair normalized to 1 Euro:

$$PI_{ij}^t = PI_{ij}^{t-1} \times e^{(r_i^t \times s) - (r_j^t \times s)}, s \in \{1, -1\}, \tag{19}$$

where r_i^t and r_j^t are the daily logarithmic returns for the first and second legs of the pair, respectively. The trading signal s equals to 1 if we enter a long position, or equals -1 if we enter a short position.

We also aggregate the cumulative return time series in ACR_{ij}^t for every trading signal S emitted for a given a pair ij:

$$ACR_{ij}^t = \sum_{s=1}^{S} CR_{ij}^t. \tag{20}$$

The utility in computing the aforementioned time series is twofold: on the one hand, they identify the suitable pairs for trading through the aggregate cumulative return time series ACR_{ij}^t of a given pair; on the other hand, they'll give us individualized exit rules for every trade through every single cumulative return time series CR_{ij}^t for a given pair, as we explain below.

In order to find the suitable pairs for trading, we need to identify the profitable pairs, which are those that revert back to their equilibrium relationship. Those pairs naturally have a positive mean aggregate cumulative return $\overline{ACR_{ij}^t}$, showing a trend for those pairs to be profitable. Hence, we create a rank where all the eligible pairs for trading are sorted from the highest mean aggregate cumulative return $\overline{ACR_{ij}^t}$ to the lowest and select only the top 5 pairs on that rank for the trading period.

On the other hand, we set the closing positions for each pair based upon the historical profitability of the pseudo-trading period signals, by taking the average of a percentile of the cumulative return time series CR_{ij}^t of all s trading signals. We use the average of the 95^{th} percentile of the S cumulative return time series as sugested by Krauss and Stübinger (2017) to be the take profit level for the pair, as, on average, 95 percent of the cumulative returns "obtained" with trading signals emitted during the pseudo-trading period do not surpass that level. Using the same line of thought, we use the average of the 10^{th} percentile of the S cumulative return time series to be the the stop loss level for the pair, as, on average, 90 percent of the cumulative returns "obtained" with trading signals emitted during the pseudo-trading period do not go below that level.

To show an example for the pair selection criteria, we come back to formation period between January 2014 to December 2018, where we take a closer look at the pair between the stock of BBVA (E:BBVA) and ING Group (H:INGA). Their logarithmic returns had a Pearson's correlation coefficient of around 0.656, making it eligible for the first strategy. A copula was fitted for every estimation period which were then used to calculate the conditional probabilities which are then used to calculate obtain the trading signals during

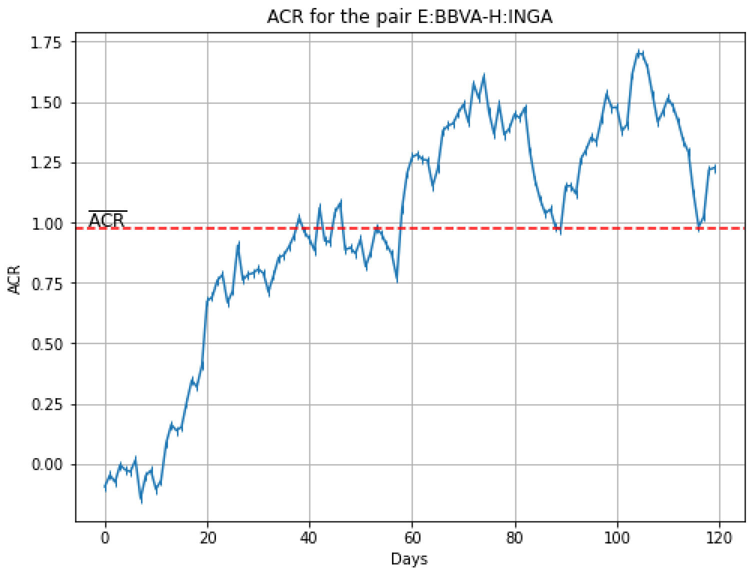

FIGURE 4: Aggregate cumulative return for a pair

the pseudo-trading period. A total of 68 trading signals were emitted, hence generating 68 cumulative return time series which show the pair profitability for the 120 days after the trading signal. We aggregate them into the aggregate cumulative return time series, which is shown in Figure 4.

This pair is going to be at the top of the pair selection algorithm, as it is the pair with the highest $\overline{ACR_{ij}^t}$ out of the eligible pairs. Besides that, the 10^{th} and the 95^{th} percentiles of each one of the 68 CR_{ij}^t are recorded and then averaged out to obtain the stop loss and take profit levels. Table 2 stores the ranked pairs according to the $\overline{ACR_{ij}^t}$ criteria, as well as the information regarding the stop loss and take profit exit rules.

3.2.3 Trading period

For both strategies, we follow the same trading rules to open pairs as we did for the pseudo-trading period, and only close positions once take-profit or stop-loss levels are reached unless the trading period comes to an end. Each pair can only have one active position at a time, i.e. if a pair has an open position, no further trading order is placed on that pair until that position is closed. We allocate 1 Euro to each pair at the beginning

TABLE 2: Pair selection for the trading period

Rank	Pairs	Mean ACR	Stop-loss level	Take-profit level
1	E:BBVA-H:INGA	0.974757	-0.034847	0.073638
2	F:AIR-F:QT@F	0.926007	-0.046481	0.082341
3	H:INGA-F:SGE	0.881206	-0.039911	0.088645
4	I:ISP-E:BBVA	0.725744	-0.050588	0.084134
5	F:LVMH-D:BASX	0.597799	-0.073012	0.103857
6	E:SAN-D:ALVX	0.5913	-0.060444	0.097622
7	E:SAN-F:SGE	0.571469	-0.052213	0.080223
8	F:FP-I:ENI	0.52767	-0.01957	0.046533
9	D:MBGX-F:BNP	0.521955	-0.061064	0.090003
10	H:INGA-F:BNP	0.465722	-0.036171	0.059347
11	F:QT@F-D:BASX	0.454448	-0.045676	0.077919
12	E:SAN-H:INGA	0.452367	-0.04562	0.071304
13	F:SGE-F:BNP	0.331577	-0.03125	0.050154
14	F:AIR-D:SIEX	0.311842	-0.043924	0.064382
15	F:LVMH-F:OR@F	0.304652	-0.056572	0.077958
16	F:DG@F-D:ALVX	0.303735	-0.047166	0.07098
17	F:AIR-D:SAPX	0.303065	-0.046511	0.068281
18	F:AIR-D:ALVX	0.301689	-0.045629	0.065883
19	I:ISP-H:INGA	0.290374	-0.062248	0.083629
20	D:DTEX-E:TEF	0.269831	-0.055505	0.080345
21	E:SAN-E:BBVA	0.253661	-0.036445	0.05057

of each trading period, and the returns obtained from previous positions are reinvested in future positions.

Coming back to our example of the formation period between January 2014 to December 2018, we have seen that the pairs between the stock of BBVA (E:BBVA) and ING Group (F:QT@F) were ranked first in the pairs selection criteria under the first strategy. For the same formation period, the pair between the stock of Société Générale (F:SGE) and BASF (D:BASX) was the top ranked pair for the second strategy. The trading period for each pair can be visualized in figures 5 and 6.

For the copula approach, we transform the logarithmic return of each leg of the pair into uniform variables using the fitted distributions in the final estimation period, which are then used to compute the conditional probabilities $h(u_i|u_j)$ and $h(u_j|u_i)$. In the 8th of

January of 2019, the value of $h(u_i|u_j)$ exceeded 0.95 while the value of $h(u_j|u_i)$ didn't surpass 0.05, making us open a short position in the pair. that position remained open until it reached the take profit level in the end of February. A few days latter, the value of $h(u_i|u_j)$ didn't surpass 0.05, while the value of $h(u_j|u_i)$ exceeded 0.95, making us open a long position in the pair.

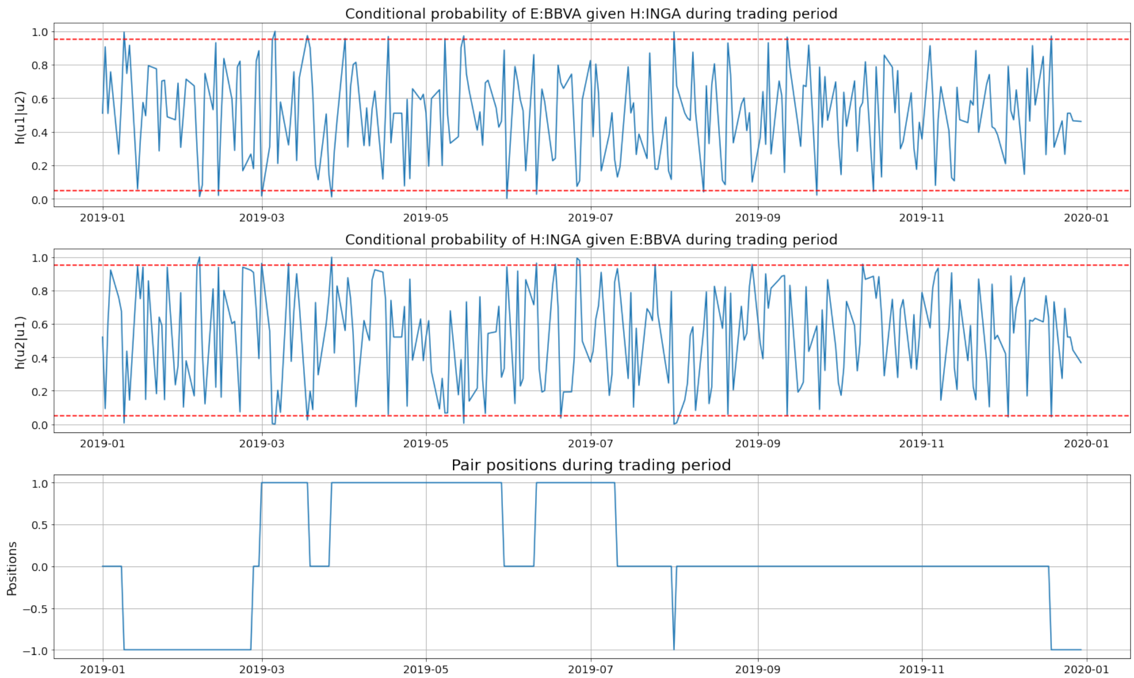

FIGURE 5: Trading period for the top ranked pair for the first strategy

For the MDA, the prices of each leg of the pair is normalized using the maximum and minimum prices in the final estimation period. Once the spread between each leg has exceed 2 negative standard deviations, we open a long position in the pair until it reached the stop loss level, where the position was closed. However, in the following day, since the spread still exceeded (even by a larger margin) the 2 negative standard deviations, another long position is opened.

We follow Gatev et al. (2006) in defining the return measures, as we define the return on employed capital as the total sum of payoffs obtained during the trading period by the number of pairs that have been selected, which in our case will always be 5. We opt to use this measure rather than other less conservative measures (such as the return on employed

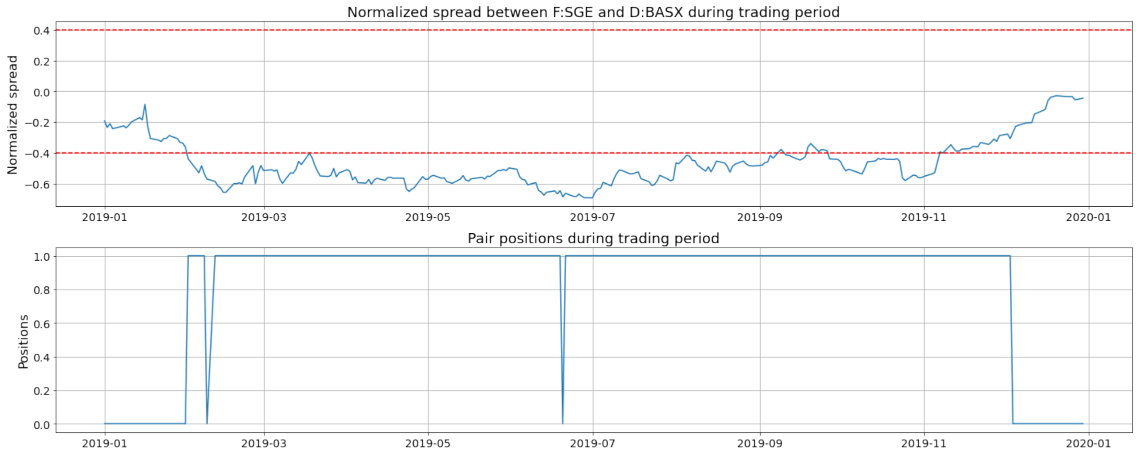

FIGURE 6: Trading period for the top ranked pair for the second strategy

capital, which divides payoffs by the number of pairs that had open positions during the trading period) since we will be assuming that there is not trading costs nor short selling costs, and we don't intend to overestimate our results.

Following Jegadeesh and Titman (1993), we start a trading strategy in the beginning of each month, resulting in 12 overlapping portfolios with excess returns calculations. The correlation induced by this overlapping is corrected by averaging out the returns obtained for the corresponding months.

4 Empirical results

We implement both of the PTS for our sample of selected stocks, having our results encompass returns obtained from January 2014 to December 2021 and compare the results to a buy-and-hold Euro STOXX 50 strategy, which from now on is taken as "benchmark". We start by comparing the pair selection between each strategy for the trading periods starting in January and ending in December of the same year. We continue the performance review by analysing the return distribution characteristics as well as the annualized risk and return characteristics. In combination with a sub period and Value at Risk (VaR) analysis, we also analyse the exposure of the strategy to common systematic risks, by regressing the returns against the Fama and French (1992) three and Fama and French (2015) five research factors.

4.1 Pair selection characteristics

The following Tables 3 and 4 represent the selected pairs for the trading periods starting and ending in each year for each strategy, respectively.

Table 3: Selected pairs through the copula approach

Year	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5
2013	I:UCG-F:BNP	D:ALVX-H:INGA	E:SAN-F:BNP	I:UCG-F:SGE	D:DBKX-I:ENI
2014	I:UCG-F:BNP	I:UCG-F:SGE	I:UCG-I:ENEL	F:BNP-D:ALVX	F:BNP-F:MIDI
2015	I:UCG-F:BNP	I:UCG-F:MIDI	I:UCG-F:SGE	I:UCG-H:INGA	E:BBVA-E:IBE
2016	F:SGE-F:BNP	I:ISP-I:G	F:MIDI-D:ALVX	F:EI-F:OR@F	D:BAYNX-D:ALVX
2017	D:ALVX-D:DBKX	D:DBKX-I:ISP	H:UNIL-B:ABI	H:INGA-F:SGO	E:BBVA-E:TEF
2018	F:MIDI-H:INGA	E:SAN-H:INGA	H:INGA-E:BBVA	F:BNP-F:SGE	F:SGE-E:BBVA
2019	E:BBVA-H:INGA	F:AIR-F:QT@F	H:INGA-F:SGE	I:ISP-E:BBVA	F:LVMH-D:BASX
2020	E:BBVA-H:INGA	F:SGE-H:INGA	E:BBVA-I:ISP	D:BMWX-F:BNP	I:ENI-F:FP
2021	F:MIDI-E:SAN	D:VOW3X-D:SIEX	D:SIEX-D:BMWX	D:BASX-H:INGA	I:ISP-D:BMWX
2022	H:INGA-I:ISP	F:BNP-F:MIDI	D:MUV2X-F:MIDI	I:ISP-F:MIDI	F:BNP-I:ISP

We find that the large majority of the selected pairs only is selected once across all trading periods in both strategies. In the copula approach, there are only two pairs that are selected 3 times (I:UCG-F:BNP and I:UCG-F:SGE) and also two pairs that are selected twice (E:BBVA-H:INGA and F:BNP-F:MIDI). In the MDA, only three pairs actually are selected twice (F:BNP-H:PHIL, D:DPWX-F:BNP and I:ISP-F:MIDI). We can carefully

Table 4: Selected pairs through the modified distance approach

Year	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5
2013	I:ENI-F:MIDI	H:UNIL-D:BAYNX	I:G-H:INGA	E:BBVA-E:SAN	F:OR@F-D:MUV2X
2014	F:BNP-H:PHIL	D:MBGX-E:REP	F:FP-F:SGO	I:ISP-F:ENGI	F:EX@F-F:SGE
2015	F:ORA-E:IBE	F:QT@F-F:FP	F:ENGI-I:ISP	F:SGE-F:BNP	H:UBL-F:BSN
2016	F:SGE-D:DTEX	I:UCG-E:BBVA	H:INGA-F:SQ@F	D:SIEX-I:G	E:SAN-D:VOW3X
2017	H:INGA-D:ALVX	F:BNP-F:SGE	D:VOW3X-I:ENI	D:BASX-D:SIEX	E:SAN-D:BMWX
2018	F:BSN-F:MIDI	D:MUV2X-F:AIRS	F:EX@F-F:AIR	D:ALVX-H:ASML	F:BNP-H:PHIL
2019	D:BASX-F:SGE	D:DPWX-F:BNP	F:LVMH-I:ENI	H:UBL-B:ABI	D:BAYNX-D:MBGX
2020	F:OR@F-F:MIDI	F:AIRS-F:EX@F	I:ENEL-H:ASML	D:ALVX-D:SAPX	D:DPWX-F:BNP
2021	H:INGA-E:SAN	D:DTEX-IE:CRG	D:ALVX-D:ADSX	I:ISP-F:MIDI	F:BNP-B:ABI
2022	I:ENI-F:EI	IE:CRG-F:BNP	I:ISP-F:MIDI	D:SAPX-F:AIR	H:INGA-D:MBGX

conclude that the selection algorithm does not select constantly the same set of pairs for different trading periods, as it changes the selection when there is an adjustment in the formation period time frame.

When comparing the selected pairs across the two strategies, we find that there is only one pair that is selected at the same time in both strategies (I:ISP-F:MIDI during the trading period of 2022). This leads us to the conclusion that the pairs selection algorithm may be profoundly different in each case.

4.2 Return distribution characteristics

Table 5 and Figure 7 present the return distribution characteristics information for each strategy and a visualization for each distribution, respectively.

During our trading window, the Copula approach has outperformed by far the MDA, demonstrating a much higher mean monthly return (0.3354 %) than the one obtained using the MDA (0.0538 %). The lack of performance of the of the MDA seem to agree with the literature that declares that simple pairs trading is not profitable anymore (see Do and Faff (2012), as t-statistic value of 0.625 implies that we cannot declare the monthly returns to be statistically nor economically significant.

On the other hand, the returns obtained using the copula approach are statistically significant t-statistic of 3.18.

Looking over to the benchmark, we find that the copula strategy is under perform-

TABLE 5: Monthly return distribution characteristics

Strategy	Copula	MDA	Benchmark
Mean monthly return	0.003354	0.000538	0.006915
Standard error	0.001055	0.000861	0.00474
t-statistic	3.178901	0.625254	1.458948
Standard deviation	0.010444	0.008519	0.046921
Skewness	1.01494	2.491094	-0.072037
Kurtosis	3.149696	13.229121	1.993712
Minimum	-0.020134	-0.014351	-0.161841
Maximum	0.050415	0.053555	0.180902
Observations with return < 0	38.775 %	53.0612 %	40.8163 %

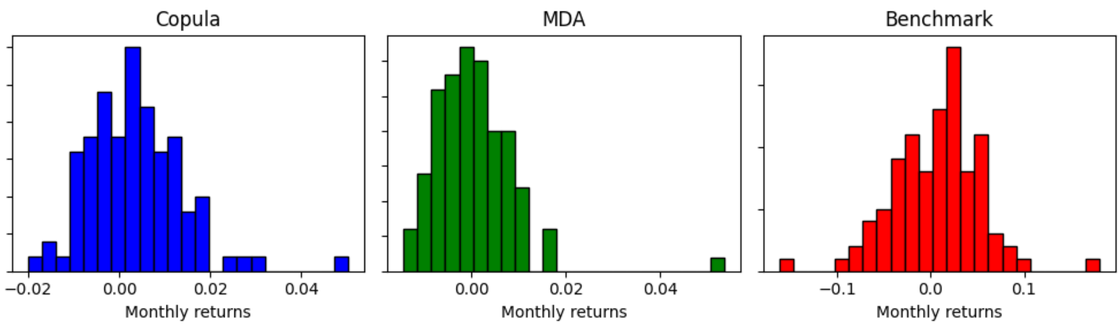

FIGURE 7: Return distribution visualization

ing the market, as the benchmark has two times more mean monthly return (0.6915 %). However, this higher level of return also comes with increased levels of volatility, as the standard deviation of the benchmark is more than four times greater than the one obtained using the copula approach and the MDA, which implies that the benchmark also bears a more significant level of risk.

Both strategies have positively skewed distributions, but they should be interpreted differently: while in the MDA the investors should expect frequent small losses (as the observations with negative returns account for the majority of the monthly returns), the investors using the copula approach have a few number of opportunities to make higher returns, while not expecting that many frequent losses (the observations with negative returns are even less than the observation in the benchmark)

It is not surprising finding that the MDA is the most leptokurtic by a big margin, indicating the highest exposure to extreme values. Meanwhile, the copula approach has a distribution with mesokurtic characteristics, aproximating to the normal distribution in terms of the heaviness of its tails.

4.3 Annualized risk and return characteristics

We annualize the monthly returns time series based on the Mean monthly return obtained in the previous section. Table 6 resumes this information alongside the annualized risk measures.

TABLE 6: Annualized Return Characteristics

Strategy	Copula	MDA	Benchmark
Return	0.040995	0.006476	0.08621
Excess return	0.022485	-0.012034	0.0677
Standard deviation	0.036178	0.029509	0.162539
Downside deviation	0.015774	0.011668	0.103998
Sharpe ratio	0.621497	-0.40782	0.416517
Sortino ratio	1.425416	-1.031417	0.650976

Once again, we observe the two main factors already described in the previous section: the poor performance of the MDA manages only to get a very modest 0.65 % annual returns and negative excess returns when it is considered the yield of a 10-year German bond. The copula approach has achieved approximately 4.1 % annual returns, being heavily outperformed by the benchmark, achieving 8.6%. However, after taking in consideration standard and downside deviation, we come to the conclusion that the Copula approach actually provides higher excess return per unit of volatility, as it is shown by the Sharpe ratio figures. When the excess returns are scaled by their downside deviation (shown in the Sortino ratio), the Copula approach actually obtain an even better ratio of 1.42 when compared to the benchmark, which only reached 0.65.

4.4 Sub-period analysis

Figures 8 and 9 represent the cumulative returns per one Euro invested in each strategy and the yearly returns obtained, respectively. During our time period analysis, for each Euro committed in each strategy grew up to 1.38 Euros in the Copula Approach, whereas the benchmark would have increase to 1.76 Euros. However, these figures do not tell the full story; there are several periods in which the Copula approach is outperforming the benchmark; however, the benchmark's better performance in 2019 and in 2021 justify the difference in cumulative returns. The MDA remained relatively stable during the period, oscillating between making profits and losses. There is a notable difference in 2020 where it performed quite well, having remained once again stable until the end of 2022, where it rendered a total of 1.05 Euros.

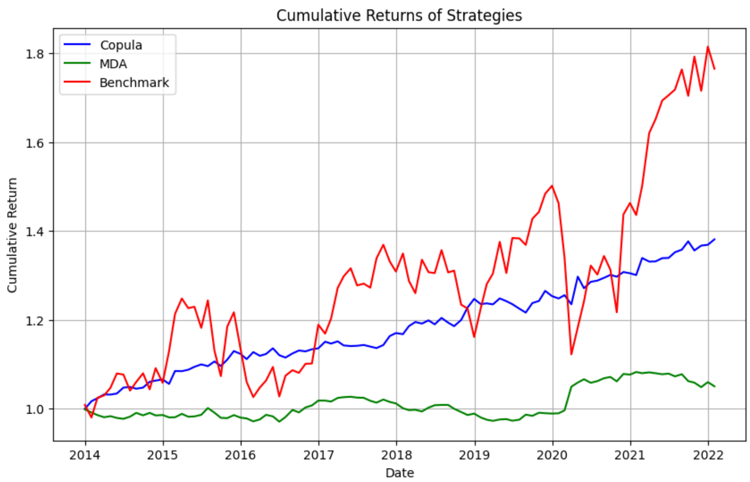

FIGURE 8: Cumulative returns on a 1 Euro investment across all strategies

When it comes to annual returns, we can access that the Copula approach actual outperforms in 3 out of the 8 years in analysis. However, when the benchmark outperforms the strategies, it tends to do it in a much larger margin. The MDA only outperforms the benchmark in 2020, where it was the best performing strategy. Comparing the two strategies, the copula approach has clearly outperformed the MDA in the majority of the years, obtaining greater returns in 6 out of the 8 years.

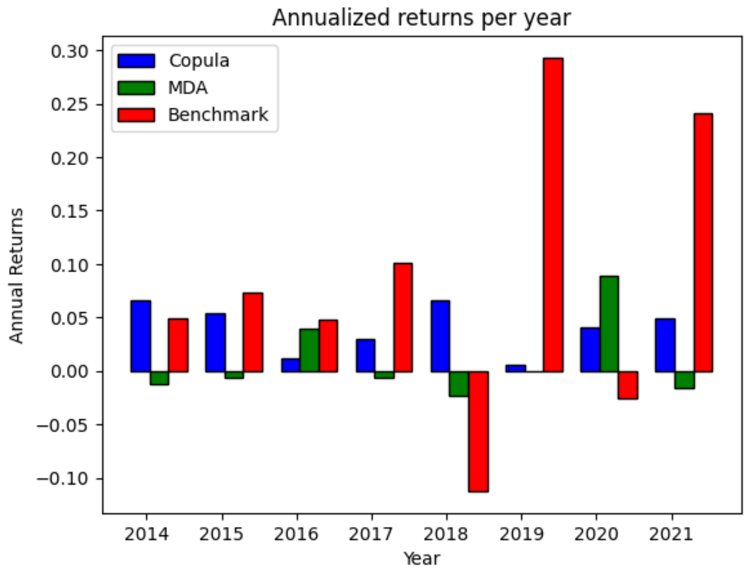

FIGURE 9: Annual returns across all strategies

4.5 Value at Risk analysis

We perform a VaR and Conditional Value at Risk (CVaR) analysis for the monthly returns in each strategy. We opt to use the simple Variance Covariance approach and the historical simulation approach for significance levels of 1%, 5% and 10%. Table 7 summarizes all this information.

TABLE 7: Monthly Value at Risk

Strategy	Copula	MDA	Benchmark
Variance Covariance VaR 1 %	-0.020942	-0.019279	-0.10224
Historical Simulation VaR 1 %	-0.016483	-0.012146	-0.093233
Historical Simulation CVaR 1 %	-0.020134	-0.014351	-0.161841
Variance Covariance VaR 5 %	-0.013825	-0.013474	-0.070263
Historical Simulation VaR 5 %	-0.009639	-0.009662	-0.066345
Historical Simulation CVaR 5 %	-0.015134	-0.011861	-0.095449
Variance Covariance VaR 10 %	-0.01003	-0.010379	-0.053217
Historical Simulation VaR 10 %	-0.00845	-0.007788	-0.050685
Historical Simulation CVaR 10 %	-0.012195	-0.010422	-0.076611

We can conclude that both strategies present much lower levels of tail risk when compared to the benchmark strategy. This relationship holds true when looking at both variance covariance and historical simulation VaR and for all significance levels. For example, in the according to the historical simulation approach, we can say with 99% confidence that the one month loss does not exceed 1.2146 % in the MDA and 1.6483 % in the Copula approach, while the benchmark excepts to not exceed a much higher 9.3233 % loss in the same month period.

When comparing the strategies themselves, the VaR figures remain relatively similar across higher significance levels; at the 1 % significance level the MDA seems to have lower tail risk than the copula approach, as the both approaches give lower values of VaR to the distance strategy. However, the increased tail risk in the copula approach becomes more clear when looking to the CVaR figures, as this strategy shows to consistently have higher average losses that occurs when the losses exceed the VaR threshold for all the significance levels considered.

4.6 Common risk factors

We regress the monthly returns obtained in each strategy against the three factor and the five factor Fama French models to access the statistical and economical significance of the strategies' constant term, which in this context translates to the excess returns not which are not explained by the factors, and/ or to evaluate if the excess returns are a compensation for risk implied in the factor. The specification of both the three-factor and the five-factor models are as follows:

$$R_i - R_f = \alpha + \beta_1(R_M - R_f) + \beta_2 SMB + \beta_3 HML + \epsilon$$

$$R_i - R_f = \alpha + \beta_1(R_M - R_f) + \beta_2 SMB + \beta_3 HML + \beta_4 RMW + \beta_3 CMA + \epsilon,$$

where $R_i - R_f$ is the monthly excess returns obtained with the strategy, α is the constant term, $(R_M - R_f)$ is the market risk premium, SMB is the size factor, which is the average return on the small market capitalization stock portfolios minus the average return on the big market capitalization stock portfolios, HML is the value factor, which is the average return on the value stocks portfolios minus the average return on the growth stocks portfolios, RMW is the profitability factor, which is the average return on the higher operating profitability portfolios minus the average return on the lower operating profitability portfolios and lastly CMA is the investment factor, which is the average return on the low investment portfolios minus the average return on the high investment portfolios.

All of the data from the factors is from the Kenneth R. French's website from the Fama/French European 3 Factors and the Fama/French European 5 Factors sections. Table 8 presents the regressions' output.

TABLE 8: Exposure to systematic risk sources across both strategies

	Copula		MDA	
	FF3	FF5	FF3	FF5
cons	0.0028^{***}	0.0024^{**}	-0.0001	0.0004
	(0.001)	(0.001)	(0.001)	(0.001)
$R_M - R_f$	-0.0070	-0.0246	0.0101	0.0145
	(0.025)	(0.028)	(0.021)	(0.021)
SMB	0.1674^{***}	0.1522^{**}	-0.0695	-0.1357^{***}
	(0.061)	(0.067)	(0.051)	(0.051)
HML	0.0993^{***}	0.1889^{**}	-0.0761^{**}	0.1445^{**}
	(0.037)	(0.080)	(0.031)	(0.061)
RMW		0.0968		-0.3352^{***}
		(0.107)		(0.082)
CMA		-0.1364		-0.1888
		(0.125)		(0.095)
F-test	4.55^{***}	3.14^{**}	2.42^{*}	6.05^{***}
R^2	0.127	0.146	0.072	0.247
Adjusted R^2	0.099	0.099	0.042	0.206
N	98	98	98	98

$^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

The first relevant conclusion we can arrive is that the copula approach manages to obtain statistical significant alphas in both of the regressions, while the MDA did not reach those thresholds. The market risk premium also did not managed to achieve statistical significance in any of the the regressions, which is a natural consequence of the market neutrality of pairs trading strategies.

An interesting result of the regressions is the statistical significance of the size and the value factors, specifically in the size factor, since the Eurostoxx 50 is built only from high market capitalization stocks. However, Krauss and Stübinger (2017) also obtained similar results in its regressions, and he devalues those results indicating that other studies find similar anomalies with respect to large market capitalization mutual funds. When we look the five factor model, we conclude that the profitability and the investment factor do not provide any more explanatory power in the Copula approach, even though in the MDA the profitability factor is statistically significant. It is also important to note that throughout all the regression the R-square and the adjusted R-Square remain low, meaning that a significant portion of the variance in the dependent variable is unexplained by the independent variables in the models. A possibility to try and improve these figures may be to try and add factors that explore other market anomalies, such as mean reversion or momentum.

5 Conclusion

We backtested two pairs trading strategies with different trading rules using the same pair selection algorithm and individualized exit rules methods in order to allow their comparability. The strategies are implemented to the historical constituents of the Euro STOXX 50 between 2008 and 2022, while being careful to eliminate survivorship bias from our sample.

Both strategies follow the same timeline framework: a 60 month formation period in order to select the pairs available for trading followed by a 12 month trading period where we actually trade the pairs. The formation period is a actually composed by 48 overlapping 12 months estimation periods all followed by a pseudo-trading period: the pseudo-trading period serves to test the profitability of the trading rules for the pairs found in the estimation period (no actual trades happen in the pseudo-trading period).

The first strategy, coined the copula approach, fits t-copula functions to the logarithmic returns to each pair in the estimation period, and then afterwards in the pseudo-trading period or the trading period we use the two conditional distribution functions to emit trading signals and to open positions. In the second strategy, coined MDA, the spread between the pairs legs' normalized prices during the pseudo-trading period is used to emit trading signals or to open positions, from the point the point that the spread exceeds 2 historical deviations.

We conclude that the pair selection criteria does not select constantly the same set of pairs for different trading periods for both strategies, and most importantly, the selection is widely different when looking across the same trading period.

Only the copula approach managed to obtain statistically significant monthly returns, as it obtained a mean monthly return of 0.3354 %, much higher than the 0.0538 % mean monthly returns obtained by obtained using the MDA strategy, which leads us to to the conclusion that the MDA IS largely unprofitable in the period of analysis. Both the strategies have been outperformed by the benchmark.

The annualized returns shows us that neither of the strategy does not have average excess returns superior to the benchmark strategy, hence we conclude that we find no evidence against the market efficiency in the semi-strong form. However, the higher levels of returns obtained by the benchmark came with higher levels of volatility, as the copula approach presents better Sharpe and Sortino ratios than the benchmark.

Looking more closely to the year-by-year performance, we conclude that both strategies have a more constant growth in its committed capital, as the cumulative returns on the copula approach exceed the Benchmark during certain periods of market downturn.

The lower levels in volatility of both strategies is once again present in the VaR analysis, which the benchmark presents at all levels of significance and in both methods used have much higher VaR as well as expected shortfalls than the studied strategies.

When looking at the common risk factors, we confirmed the market neutrality of the strategy, as the results of the regressions do not show signs of statistical significance for the market factor. However, we also arrived to the conclusion that only the copula approach managed to gather a statistically significant alpha returns.

Regardless, future research could continue by assessing if the timeline strategy framework brought by Krauss and Stübinger (2017) is actually more profitable than a the classical 12 month formation period followed by the 6 month trading period. On the other hand, it may also be interesting to explore other ways to fit this timeline strategy framework in a shorter time period, analysing strategies to find intraday trading opportunities (for example, instead of having the 60 month formation period followed by a 12 month formation period, have a 15 month formation period followed by a 3 month trading period, where each day we have the prices for 4 specific times of the day). There might also be an interest in fitting the timeline framework to other pairs trading strategies involving other approaches, such as the time series or more interestingly, apply it to a level-based copula approach.

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A Appendices

TABLE 9: Historical constituent list of Eurostoxx 50

Company Name	Mnemonic	Country of Domicile	ICB Industry
ADIDAS	D:ADSX	Germany	Consumer Discretionary
ADYEN	H:ADYE	NETHERLANDS	Industrials
AEGON	H:AGN	NETHERLANDS	Financials
AGEASz	B:AGS	BELGIUM	Financials
AIRBUS	F:AIRS	NETHERLANDS	Industrials
ALCATEL-LUCENT	F:ALU	NA	Telecommunications
ALLIANZ	D:ALVX	GERMANY	Financials
ALSTOM	F:ALOT	FRANCE	Industrials
AMADEUS IT GROUP	E:AMS	SPAIN	Technology
ANHEUSER-BUSCH INBEV	B:ABI	BELGIUM	Consumer Staples
APERAM	H:APAM	LUXEMBOURG	Basic Materials
ARCELORMITTAL	H:MT	LUXEMBOURG	Basic Materials
ARCELORMITTAL	F:MTP	LUXEMBOURG	Basic Materials
ASML HOLDING	H:ASML	NETHERLANDS	Technology
ASSICURAZIONI GENERALI	I:G	ITALY	Financials
AXA	F:MIDI	FRANCE	Financials
BANCO SANTANDER	E:SAN	SPAIN	Financials
BASF	D:BASX	GERMANY	Basic Materials
BAYER	D:BAYNX	GERMANY	Health Care
BBV.ARGENTARIA	E:BBVA	SPAIN	Financials
BMW	D:BMWX	GERMANY	Consumer Discretionary
BNP PARIBAS	F:BNP	FRANCE	Financials

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Company Name	Mnemonic	Country of Domicile	ICB Industry
CARREFOUR	F:CRFR	FRANCE	Consumer Staples
CREDIT AGRICOLE	F:CRDA	FRANCE	Financials
CRH	IE:CRG	IRELAND	Industrials
DANONE	F:BSN	FRANCE	Consumer Staples
DEUTSCHE BANK	D:DBKX	GERMANY	Financials
DEUTSCHE BOERSE	D:DB1X	GERMANY	Financials
DEUTSCHE BOERSE (XET)	D:63UX	GERMANY	Financials
DEUTSCHE POST	D:DPWX	GERMANY	Industrials
DEUTSCHE TELEKOM	D:DTEX	GERMANY	Telecommunications
E ON N	D:EOANX	GERMANY	Utilities
ENEL	I:ENEL	ITALY	Utilities
ENGIE	F:ENGI	FRANCE	Utilities
ENI	I:ENI	ITALY	Energy
ESSILORLUXOTTICA	F:EI	FRANCE	Health Care
FLUTTER	IE:FLTR	IRELAND	Consumer Discretionary
FORTIS	H:AMEV	BELGIUM	Financials
FRESENIUS	D:FREX	GERMANY	Health Care
HERMES INTL.	F:RMS	FRANCE	Consumer Discretionary
IBERDROLA	E:IBE	SPAIN	Utilities
INDITEX	E:IND	SPAIN	Consumer Discretionary
INFINEON TECHS.	D:IFXX	GERMANY	Technology
ING GROEP	H:INGA	NETHERLANDS	Financials
INTESA SANPAOLO	I:ISP	ITALY	Financials
KERING	F:KER	FRANCE	Consumer Discretionary
KONE B	M:KNEBV	FINLAND	Industrials
AHOLD DELHAIZE	H:AD	NETHERLANDS	Consumer Staples

Continued on next page

Company Name	Mnemonic	Country of Domicile	ICB Industry
L AIR LQE.SC.ANYME.	F:AIR	FRANCE	Basic Materials
LINDE (XET)	D:LINX	UNITED KINGDOM	Basic Materials
L'OREAL	F:OR@F	FRANCE	Consumer Discretionary
LVMH	F:LVMH	FRANCE	Consumer Discretionary
MERCEDES-BENZ GROUP	D:MBGX	GERMANY	Consumer Discretionary
MUENCHENER RUCK	D:MUV2X	GERMANY	Financials
NOKIA	M:NOK1	FINLAND	Telecommunications
ORANGE	F:ORA	FRANCE	Telecommunications
PERNOD-RICARD	F:RCD	FRANCE	Consumer Staples
PHILIPS ELTN.KONINKLIJKE	H:PHIL	NETHERLANDS	Health Care
PROSUS	H:PROS	NETHERLANDS	Technology
RENAULT	F:RENU	FRANCE	Consumer Discretionary
REPSOL YPF	E:REP	SPAIN	Energy
RWE	D:RWEX	GERMANY	Utilities
SAFRAN	F:SGM	FRANCE	Industrials
SAINT GOBAIN	F:SGO	FRANCE	Industrials
SANOFI	F:SQ@F	FRANCE	Health Care
SAP	D:SAPX	GERMANY	Technology
SCHNEIDER ELECTRIC	F:QT@F	FRANCE	Industrials
SIEMENS	D:SIEX	GERMANY	Industrials
SOCIETE GENERALE	F:SGE	FRANCE	Financials
STELLANTIS	I:STL	NETHERLANDS	Consumer Discretionary
SUEZ(ROMPUS)	F:LE	NA	Utilities
TELECOM ITALIA	I:TIT	ITALY	Telecommunications
TELEFONICA	E:TEF	SPAIN	Telecommunications
TOTALENERGIES	F:FP	FRANCE	Energy

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Company Name	Mnemonic	Country of Domicile	ICB Industry
UNIBAIL	F:UBL	FRANCE	Real Estate
UNICREDIT	I:UCG	ITALY	Financials
UNILEVER	H:UNIL	NETHERLANDS	Consumer Staples
UNILEVER	H:UNI	NETHERLANDS	Consumer Staples
VINCI	F:DG@F	FRANCE	Industrials
VIVENDI	F:EX@F	FRANCE	Consumer Discretionary
VOLKSWAGEN	D:VOWX	GERMANY	Consumer Discretionary
VOLKSWAGEN PREF.	D:VOW3X	GERMANY	Consumer Discretionary
VONOVIA	D:VNAX	GERMANY	Real Estate