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VECM APPROACH FOR DEFAULT RATE FORECASTING

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ABSTRACT

This research explores the application of a Vector Error Correction Model (VECM) in forecasting Default Rates, using key macroeconomic indicators such as Gross Domestic Product, inflation and unemployment rates. The VECM was selected due to its ability to deal with non-stationary cointegrated variables, allowing it to capture both the short-term dynamics and the long-term equilibrium relationships between the variables.

The forecasted Default Rate is a critical variable in the estimation of the Variable Scalar Approach. Under IFRS 9, this approach adjusts Through-the-Cycle Probabilities of Default into Point-in-Time Probabilities of Default, allowing the inclusion of forward-looking macroeconomic indicators in the Probability of Default estimate, thereby enhancing financial institutions' ability to estimate Expected Credit Losses and internal capital requirements.

The study finds that the VECM provided reasonably accurate forecasts for default rates during the period considered, with minimal deviations from the observed data, all within an acceptable range. While diagnostic tests confirmed the model's robustness and reliability, limitations were observed in its ability to predict extreme economic events, particularly during financial crises such as those of 2009 and 2020. To address this limitation, a worst-case scenario is incorporated into the scalar factor calculation. Despite these challenges, the model has proven to be a valuable tool for enhancing credit risk management.

Keywords: Default Rate; Probability of Default; Vector Error Correction Model; Variable Scalar Approach.

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ABBREVIATIONS

DR – Default Rate

ECL – Expected Credit Loss

GDP – Gross Domestic Product

IRF – Impulse Response Function

PD – Probability of Default

PIT – Point-in-Time

TTC – Through-the-cycle

VAR – Vector Autoregressive

VECM – Vector Error Correction Model

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1. INTRODUCTION

In response to the 2008 global financial crisis, banking sector regulators aimed to reinforce risk management policies and the accounting standards used, leading to more demanding requirements. The International Accounting Standards Board (IASB) and the European Banking Authority (EBA) have been fundamental in promoting the adoption of forward-looking approaches to estimate Probabilities of Default (PD). The IASB introduced IFRS 9[1], which requires financial institutions to implement expected credit loss (ECL) models that incorporate both historical data and forward-looking macroeconomic indicators, ensuring earlier and more accurate recognition of potential credit losses (International Accounting Standards Board, 2009). The EBA has supported this application in the European banking sector by issuing guidelines on credit risk management and the calculation of expected credit losses, ensuring compliance with IFRS 9 (European Banking Authority, 2017).

The Variable Scalar approach enables the inclusion of forward-looking macroeconomic indicators in PD estimation. A key aspect of this approach is the transformation of Through-the-Cycle (TTC) PD[2], which reflect long-term averages into Point-in-Time (PIT) PD[3] that adjust according to current macroeconomic conditions, allowing institutions to better anticipate and manage risks during economic downturns. This methodology promotes financial stability by ensuring that institutions proactively take future risks into account.

The Default Rate (DR) has a crucial role in estimating the scalar factor under the Variable Scalar Approach. By accurately predicting the default rate, institutions can compute the scalar factor, which adjusts the PD estimations to reflect current and future economic conditions, making it an essential tool for managing credit risk during periods of economic stress. For this reason, accurate forecasting of the DR is essential, as it

[1] International Financial Reporting Standard (IFRS) published by the International Accounting Standards Board. It addresses the accounting of financial instruments.

[2] Through the Cycle (TTC) PD predicts the average default rate for a particular rating over an economic cycle, disregarding short-term variations in economic conditions.

[3] Point-in-Time (PIT) PD assesses the likelihood of default at a point in time for a specific rating, factoring in current economic conditions.

directly impacts the precision of the scalar factor and, consequently, the institution's ability to estimate ECL and internal capital requirements.

In this research, the Vector Error Correction Model (VECM) was selected for forecasting default rates due to its ability to handle non-stationary cointegrated variables, allowing the capture of both short-term dynamics and long-term equilibrium relationships between macroeconomic variables. The VECM is particularly well-suited for this context since variables, such as Gross Domestic Product (GDP), inflation, and unemployment rates are expected to move together in the long term, exhibiting cointegration. These macroeconomic variables are anticipated to have a critical role in predicting future default rates, making the VECM an ideal choice for producing accurate and reliable forecasts. Throughout the study, the rationale behind the selection of the VECM, its estimation process, the diagnostic tests applied and the results obtained will be explored in detail.

2. LITERATURE REVIEW

In this chapter, reference will be made to papers that helped in the decision-making process regarding the model's development. Subsection 2.1 assesses the diverging views on the most appropriate statistical model for predicting the Default Rate. Subsection 2.2 covers the selection of the best approach for determining internal capital requirements for banks: using PD Point-in-Time or PD Through-the-Cycle models.

2.1 DEFAULT RATE FORECASTING METHODOLOGIES

After the 2008 global financial crisis, the prediction of default rates in stressed economic scenarios took on a new dimension, both in the academic and regulatory fields. Much due to this period, which exposed the weaknesses in traditional risk management systems, the importance of implementing robust models capable of predicting default probabilities was evident, especially in periods of economic instability. Over time, various methodologies have been developed to address these challenges, each offering distinct advantages and limitations. This literature review section summarizes the key contributions of existing models for predicting default rates, through macroeconomic drivers, econometric techniques and stress testing frameworks.

In response to the financial crisis, Basel III demanded stricter and higher capital requirements, encouraging the use of stress scenarios in assessing capital adequacy. Under the imposed framework, banks and financial institutions were incentivized to predict the ratings of their debtors, and consequently Probabilities of Default, including Default Rates, in stress scenarios, such as economic downturns (Basel Committee on Banking Supervision, 2017). This provided a strong foundation for many subsequent studies.

Credit rating models, which are based on transition matrices, are a common tool for predicting Probabilities of Default and are based on the historical rating transitions of borrowers, using the matrix generated to estimate future default probabilities (Hadad, et al., 2009; Malik & C., 2012). The cohort method is commonly used in this context, with the Basel Committee on Banking Supervision (BCBS) recommending a 1-year horizon for estimation (Basel Committee on Banking Supervision, 2004). However, this type of model is limited by the fact that it has linear assumptions and tends to struggle with capturing the non-linear dynamics that occur during volatile economic periods.

Due to its simplicity and easy interpretation, the Ordinary Least Squares (OLS) method is widely used to estimate the impact of macroeconomic variables such as Gross Domestic Product, interest rates, the change in real house prices and unemployment rates on Default Rates and Loan Loss Ratios. However, the omission of unobservable risk factors (such as firm-specific risks) in portfolio loss models leads to biased results of default risk. These variables are an additional motivation for using non-linear models, considering that all loans exposed to these types of risk factors are subject to an increase in default risk (Duffie, et al., 2009). Despite this, OLS has maintained its popularity, being applied in studies of Sveriges Riksbank (the Swedish central bank) due to its user-friendly nature and general applicability to aggregate-level data (Buncic, et al., 2019).

Recognizing the weaknesses of OLS, fixed effects models were developed with the purpose of controlling unobserved heterogeneity (Roesch & Scheule, 2012). Similar to those mentioned above, fixed effect models also estimate the relationship between PD and macroeconomic variables, although taking into account institution-specific effects that do not vary over time. An extension of this approach is the Least-Square Dummy Variable (LSDV) model, which also allows for industry-specific factors. Qu applied this type of model to predict PD at the industrial level, however, the author realized that the assumption of constant sectoral effects could be problematic over long-term horizons (Qu, 2006).

Logit models have been more widely used in the context of Default Rate forecasting due to their ability to deal with binary results (Default or Non-Default) (Bartual, et al., 2012; Tserng, et al., 2014). Simon and Rowles used this model to estimate Default Rates in the Netherlands using macroeconomic variables such as GDP growth and interest rates (Simons & Rolwes, 2009). Despite the flexibility and ease of interpretation, logit models assume that the relationship between explanatory variables and default rates remains constant over time, which may not necessarily be true, particularly in recessions.

Belotti and Crook applied a discrete survival model to analyze the impact of behavior and macroeconomic variables on credit card default probabilities. This type of dynamic model is more effective than static alternatives due to its ability to include time-varying covariates such as interest rates, unemployment and account balances. To further strengthen the robustness of stress testing, the authors used Monte Carlo simulations to map the distribution of default probabilities in several extreme economic scenarios. This simulation-based approach allows the generation of loss distributions and the prediction of potential scenarios under adverse conditions. By simulating thousands of potential macroeconomic outcomes, the Monte Carlo method provides a reliable way of stressing credit card portfolios and thus estimating default rates, whether at a portfolio or individual account level (Bellotti & Crook, 2013). Despite the robustness of the results obtained, applying this type of model to large datasets and estimating for several variables can become too computationally intensive.

Senior and Bailey estimated the impact of several macroeconomic variables on the Default Rate of Jamaica's banking sector using the Generalized Method of Moments (GMM). This estimation technique has proven to be highly predictive, providing reliable results for various economic scenarios. This method allowed policymakers to regulate systemic risk and financial fragility, providing valuable insights regarding this topic (Senior & Bailey, 2017). Despite its strong predictive capacity, GMM requires careful handling and, similarly to the dynamic model applied by Belotti and Crook, can be computationally intensive when applied to large datasets or very long horizons.

With the aim of addressing the limitations of traditional econometric models, researchers are turning to more complex econometric models, such as the Factor-Augmented Vector Autoregressive (FAVAR) model. FAVAR models are an “advanced” version of the traditional Vector Autoregressive (VAR), as they allow for the inclusion of latent factors that account for unobservable influences. Zsigraiova applied this model to evaluate the relationship between Non-Performing Loans (NPL) and economic shocks, showing that FAVAR is effective at capturing both observable and unobservable factors that impact default risk (Zsigraiová, 2014). However, this type of model is quite complex but also computationally intensive, making its application in real stress tests impractical.

Similarly, Global Vector Autoregressive (GVAR) models are an evolution of the VAR model, allowing several countries or regions to be incorporated into their analysis. Due to their ability to capture global interdependencies and spillover effects between economies, GVAR models gained recognition for analyzing the impact of global economies on DR, as these models are capable of stimulating risk factors in multiple countries (Pesaran, et al., 2006). Due to the usefulness of GVAR for assessing the spillover effects of economic shocks, Castrén, Dées and Zaher used it to simulate the effect on DR of shocks to GDP growth, interest rates and equity prices (Castrén, et al., 2010). Although powerful, these types of models are highly complex and can be difficult to implement, especially when data from several regions and industries is required.

In recent years, machine learning techniques such as the Least Absolute Shrinkage and Selection Operator (LASSO) have been applied. This model excels in high-dimensional environments, where there are many potential DR predictors. By reducing the coefficient of the least important variables to zero, LASSO simplifies the model and minimizes the risk of overfitting. Chan-Lau demonstrated the usefulness of LASSO in predicting 1-year PD in different industrial sectors, emphasizing its effectiveness in handling large datasets. However, the biggest criticism of this model is its reliance on data-driven selection, which could result in the inclusion of explanatory variables with no theoretical basis for their connection to default rates. As a result, models that perform well in-sample can fail in out-of-sample tests (Chan-Lau, 2017).

Each model has its strengths and weaknesses when it comes to predicting default rates using macroeconomic variables. Traditional approaches such as OLS or fixed effects models provide simplicity but can suffer from inconsistency when dynamic features or unobserved heterogeneity are involved. More advanced models such as LASSO, FAVAR or even GMM, offer more flexibility and accuracy in the results obtained but are more complex, computationally intensive and difficult to implement.

A recurring challenge of PD forecasting is capturing the natural non-linear dynamics, especially during economic downturns. The most advanced models adopt different strategies to mitigate this problem. GMM incorporates lagged variables and accounts for endogeneity, but this requires careful handling of the data to avoid bias. LASSO models mitigate the problem by operating on larger datasets but at the cost of failing theoretical grounding.

2.2 POINT-IN-TIME AND THROUGH-THE-CYCLE MODELS

There has been a debate about the most effective approach in credit risk modeling: Point-in-Time or Through-the-Cycle. In accordance with the frameworks imposed by Basel II and III, both play a crucial role in estimating credit risk and banks' internal capital requirements. PIT models respond dynamically and expeditiously to changing macroeconomic conditions, providing a better expectation of borrowers' creditworthiness. In contrast, TTC models generate more stability by smoothing out the fluctuations of economic cycles, thus focusing on long-term risks. This chapter explores the key contributions to understanding the differences, advantages and disadvantages of each approach, the regulatory framework that contributed to their development and the methodologies employed by different authors to estimate the Probabilities of Default.

To analyze the differences between PIT and TTC Probabilities of Default, Topp and Perl examined the TTC methodology used by some rating agencies, such as Standard & Poor's (S&P). Contrary to expectations, the analysts concluded that, despite their independence from economic cycles, PD nevertheless vary in response to cycles, particularly across different industries. This discrepancy creates potential problems for institutions that rely on external Through-the-Cycle models for their internal PIT models. In cases where the connection is misaligned, it can generate incorrect risk assessments, either underpricing or overpricing risk, depending on the economic cycle. The authors therefore highlighted the importance of calibrating external TTC models with due care to avoid this type of miscalculation when integrating them into internal risk assessment models (Topp & Perl, 2010).

Begin and Thomas evaluated the impact of the Basel II regulations on PD modeling for retail portfolios, highlighting the shift towards more complex credit risk assessment techniques. They assessed the differences between the main methodologies for Through-the-Cycle Probability of Default modeling: structural models and the variable scalar method. This last approach is based on adjusting PIT PD into TTC PD by applying a scalar factor reflecting long-term economic conditions, creating more stability in capital requirements. However, the authors noted that this approach encounters some challenges in incorporating portfolio changes over time. On the other hand, structural models, in addition to macroeconomic variables, take advantage of non-cyclical risk factors, such as loan-to-value ratios (LTV) and debt service ratios (DSR), to generate more accurate long-term Probabilities of Default. In addition, the Probabilities of Default changes as maturity approaches. Nevertheless, this methodology is not very easily implemented and requires large data samples. As expected, both methodologies have strengths and weaknesses, depending on the portfolio and market conditions (Begin & Thomas, 2012).

Focusing on the cyclicality of PD, driven by the Internal Ratings-Based (IRB) approach within the Basel II regulations, Nagy and Biró underlined the procyclicality of PIT-based PD, where capital requirements fluctuate within the economic cycle, generating risk underestimation during an economic expansion and overestimation during recessions. To mitigate this, the authors discussed various methodologies, such as a TTC ratings-based approach, which minimizes fluctuations by averaging Probabilities of Default over the cycle. They also debate calibration methods and the use of the Vasicek model. Calibration methods involve adjusting PD to ensure that they reflect the long-term and are less influenced by short-term economic conditions. These methods usually include adjustments such as the variable scalar approach. Vasicek's model, which forms the basis of the IRB's capital function, calculates the conditional Probability of Default, assuming that the PD is normally distributed and is influenced both by the borrower's individual (idiosyncratic) risk and by systematic economic factors. The main concept of the model is that the unconditional PD (TTC) is the average of the conditional Probability of Default in different economic scenarios. Similar to Begin and Thomas, Nagy and Biró also suggest mitigating these effects through the Prudential Regulation Authority's (PRA) variable scalar approach. In the end, their conclusion is that it is challenging to separate cyclical and non-cyclical components, especially in portfolios that require more active management (Nagy & Biró, 2018).

In 2021, Eder released a paper revisiting the dualism of PIT versus TTC models, criticizing the binary distinction made regarding the use of both models. As such, the author argued that credit risk models operate on a spectrum with elements of both Point in Time and Through-the-Cycle. The paper details the historical evolution of these concepts, as well as the impact of regulatory frameworks such as Basel II and III, and IFRS9, on the development of credit risk models. While Basel II and III tend to favor TTC estimation for capital requirements, IFRS9 pushes for a forward-looking Point-in-Time models approach to estimate expected credit losses. Eder concludes that an oversimplified view of PIT and TTC models as opposing models causes confusion in practice, and further concludes that the models should be seen as flexible tools that can be adapted depending on the regulatory context and the risk environment (Eder, 2021).

The literature on PIT and TTC credit risk models describes the complexities involved in balancing short-term and long-term risk assessment. As discussed, Point-in-Time models are a short-term approach, reacting instantly to economic conditions and introducing volatility into capital requirements. On the other hand, Through-the-Cycle models offer more stability by averaging PD over the cycle, while ignoring short-term risks. Various approaches, such as variable scalar models, structural models and calibration techniques, have been proposed in order to address the limitations of both models. However, as the literature reveals, no approach provides a definitive solution. Thus, credit risk modeling remains a dynamic field, with the need for ongoing adaptation of the methodologies applied by regulators and financial institutions, depending on the conditions of the economy, the markets, regulatory changes and the uniqueness of each portfolio.

3. METHODOLOGY

Despite not being an option explicitly considered by the researchers, the model selected to forecast the default rate was the Vector Error Correction Model. Similar to FAVAR, this model is a more “advanced version” of VAR. The advantage of the VECM over the VAR is that it can handle non-stationary cointegrated variables, allowing to capture both short-term fluctuations and long-term equilibrium relationships. Although computationally simpler, the VAR model was not considered since it is not suitable for non-stationary or cointegrated variables.

When forecasting the Default Rate using macroeconomic data, such as inflation, interest rates, unemployment rate and the Default Rate itself, it is expected that these variables will move together in the long term, exhibiting cointegration. So, by considering these relationships, the VECM provides more accurate and meaningful predictions of the Default Rate. This was the main reason for the model's selection.

While the VECM is not among the simplest models, it is also not excessively complex. Instead, it strikes a balance by effectively capturing long-term equilibrium relationships and short-term dynamics among variables, without being overly complex or computationally intensive. It provides the sophistication needed to obtain reliable and robust estimates, without having a high computational burden and without major implementation difficulties.

Throughout this chapter, the methodology used to develop the model will be described. Starting with the VECM modeling process (Subsection 3.1) and progressing to the application of the forecasted Default Rate in estimating future Probabilities of Default, and consequently, its impact on banks' internal capital requirements (Subsection 3.2).

3.1 VECTOR ERROR CORRECTION MODELING

3.1.1 Time Series

Time series analysis involves the study of a range of data points collected over a specific period of time, with the purpose of identifying underlying patterns, understanding the dynamics between variables over time, or forecasting future values (Hamilton, 1994). For the analysis to be reliable and consistent, it must be carried out on a sizable sample with a large number of data points. This assures the detection of reliable trends and patterns, the detection of possible seasonal variance and the minimization of noisy data.

If the model's output is completely determined by the parameter values of the model and its initial conditions, the time series is considered deterministic. If there is randomness present, and therefore the output is not driven by the parameter values but by probability distributions, the time series is considered stochastic.

Time series modeling is commonly used in fields such as finance, economics, engineering, and meteorology. Some practical examples of time series analysis are rainfall measurements, heart rate monitoring (ECG), automated stock trading, and interest rate forecasting.

One of the first and main steps in time series modeling, which heavily influences the modeling approach and the estimates achieved, is determining if the series is stationary or non-stationary.

A time series is considered stationary if its statistical properties, such as mean, variance and covariance, are constant over time. When performing a graphical analysis of this type of series, it is possible to note that they usually do not show trends, seasonality or cyclicality; these series fluctuate around a constant mean. The following graphic is a representation of a stationary series, per Juselius (Juselius, 2006):

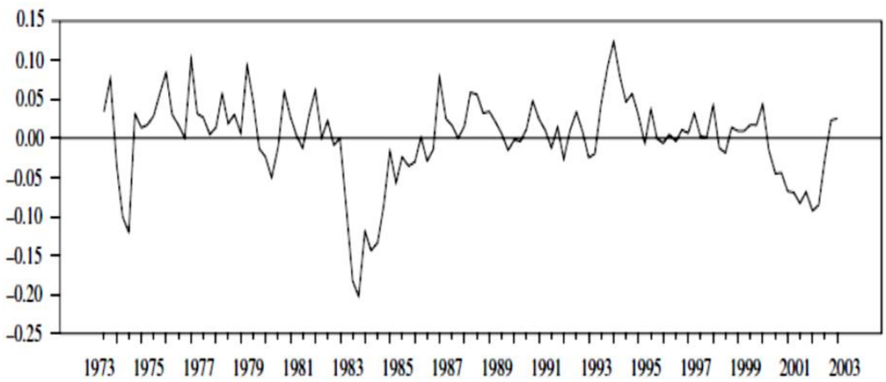

Figure 1: Stationary Series

If a time series' statistical properties change over time, it is non-stationary. Unlike stationary series, non-stationary series exhibit trends, seasonality, cyclicality and changing variances. The following is a graphical representation of a non-stationary series, per Juselius (Juselius, 2006):

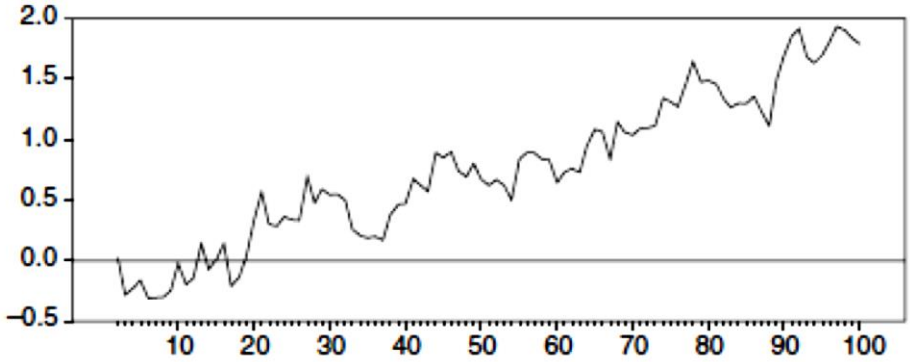

Figure 2: Non-stationary Series

In order to analyze the stationarity of a series, several methods are recommended. The first approach is visual inspection, where the graphical representation should show a constant mean and variance, with no trend or seasonality. Nevertheless, this technique is intended only as a first analysis, as it is not very reliable and requires some experience from the analyst. Another approach is the statistics summary method, which splitting the time series into several periods and compare the mean and variance of each period. If these differ considerably, the series is most probably non-stationary. Although it is more effective and accurate than visual inspection, as it is a more objective approach, it is still not 100% reliable. The last approach is to perform a statistical test. For instance, the Augmented Dickey-Fuller (ADF) test is one of the most widely used methods of stationarity testing. Briefly, ADF's null hypothesis is that the series has a unit root, meaning it is non-stationary. If the test statistic is less than the critical value, the null hypothesis is rejected, indicating that the series is stationary (Dickey & Fuller, 1979).

Determining whether the time series is stationary or not is a key factor for model selection, since non-stationary time series are not suitable for a considerable number of time series models. Applying non-stationary data inappropriately to a type of model designed for stationary data can lead to false results, i.e., relationships between variables may appear stronger than they are. This can be seen as a problem when estimating financial and economic data, which are mostly non-stationary. For instance, VAR models require stationary variables to perform accurate estimates. Meanwhile, Vector Error Correction Models can properly estimate using non-stationary time series. Nevertheless, there are techniques for transforming non-stationary variables into stationary, such as differencing.

3.1.2 Trend Analysis

In time series analysis, trend refers to long-term movements or the data's direction (upwards or downwards) (Hamilton, 1994). It can represent sustained increases or decreases, as well as gradual changes over time. Trends can be classified into several types, being the most common linear, quadratic, and exponential.

A linear trend is a constant movement upward or downward, over time. It is typically represented by a straight line that fits the data points, as shown below (Canela, et al., 2019):

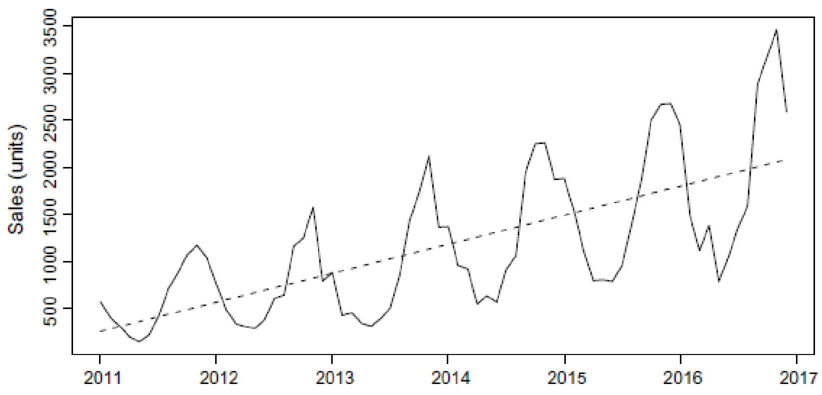

Figure 3: Linear Trend

A quadratic trend is when the time series exhibits a curvilinear movement. The direction of the trend changes over time, creating a parabola-like pattern. The series may start decreasing (increasing), reach a peak, and then increase (decrease). The figure below compares a quadratic trend (dashed line) with a linear trend (straight line) (Canela, et al., 2019).

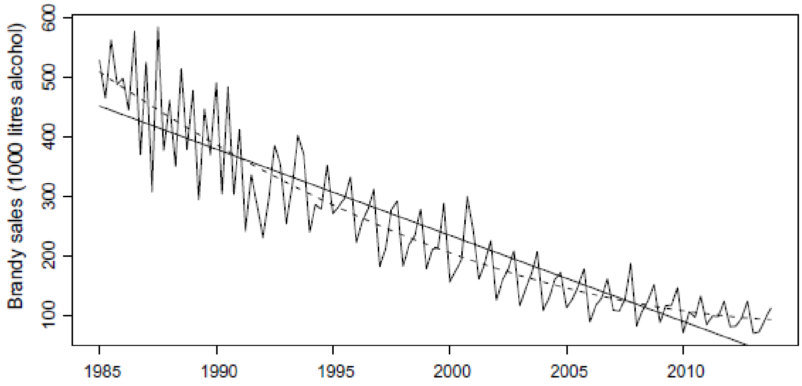

Figure 4: Quadratic Trend

An exponential trend is when the increase or decrease in the series' movement occurs at an increasing rate. For this case, Canela et al. (2019) demonstrate the application of a technique called exponential smoothing to trends (Canela, et al., 2019). This technique consists of applying exponentially decreasing weights to past observations, giving more weight to recent data while smoothing short-term fluctuations to reveal long-term trends.

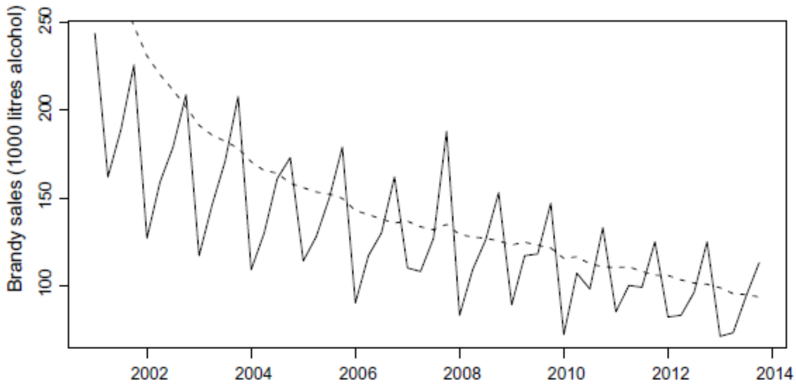

Figure 5: Exponential Trend

Identifying trend existence and its type is crucial in time series modeling, as it greatly impacts the model's ability to estimate long-term relationships between variables. Ignoring trends frequently leads to inaccurate models and biased results. For instance, if a linear trend is omitted in an increasing series, the model may falsely attribute this growth to short-term dynamics, leading to a spurious regression where relationships appear significant but are actually misleading. Identifying the existence and type of trend can even be essential in determining which model to apply.

Since this research aims to use a VECM to forecast the default rate, it is important to analyze how trends are incorporated into the model. The VECM allows for the inclusion of trends in both the cointegration equation and the short-term dynamics. The following trend specifications can be applied to VECM:

- There is no constant or trend included, the series is considered to oscillate around the constant mean;
- A constant is included in the cointegration equation and in the short-term dynamics. It allows the model to take into account the long-term mean or equilibrium level of the variables;
- A constant and a trend are included, as well as capturing the equilibrium level of the variables, it also captures the upward or downward movement over time.

To analyze the existence of a trend in a series, the main approaches are as follows:

- Visual inspection: the graphical representation of the time series should show one of the trends mentioned above clearly. However, the analysis should be supplemented with statistical tests.
- Statistical Test: One approach to validating the usefulness of a trend is to perform an ordinary least squares (OLS) regression on the time series, considering the linear and quadratic terms of the trend. This approach allows for modeling a quadratic trend, helping to assess how the variables change over time. The statistical significance of the trend is then evaluated using an F-test. The regression coefficients are estimated using the OLS model. The p-values are then calculated to test the null hypothesis: the coefficient is zero, which means the trend in the series is not statistically significant. If the p-value is below a chosen significance level (e.g., 0.05), the null hypothesis is rejected, indicating that the trend is statistically significant.

3.1.3 Lag-order selection criteria

In time series analysis, a lag is the amount of time that a variable is shifted backwards in order to analyze the impact of those past values on its current or future values (Hamilton, 1994). From the standpoint of developing a VAR/VECM model, selecting the optimal lag is a critical step, as it guarantees the accurate modeling of short and long-term dynamics between time series variables. The lag length choice determines the model's ability to capture historical dependencies and cointegration relationships among variables. The main criteria used for lag selection are the Akaike Information Criterion (AIC), the Schwarz Bayesian Criterion (SBIC), and the Hannan-Quinn Criterion (HQC), each having its particular way of assessing the balance of model complexity *vis-à-vis* goodness-of-fit. The selection of the optimal lag is not merely a technical requirement, but a decisive factor in the model's accuracy and effectiveness in forecasting. Underfitting the lag length can generate biased estimates and overfitting can introduce unnecessary complexity, reducing the accuracy of forecasts, as research has shown (DeSerres & Guay, 1995).

In order to calculate each criterion, the Lütkepohl approach was applied. Although very similar to the standard method, and while maintaining its core structure, Lütkepohl focuses on the residual behavior of each lag evaluated, rather than the log-likelihood, as it provides a better assessment of suitability, especially for VAR and VECM. In addition, this approach is able to accommodate larger data samples for these models (Lütkepohl, 2005).

The final formulas obtained by Lütkepohl for calculating each criterion were as follows (Lütkepohl, 2005):

$$\mathrm{AIC} = \frac{2pK^2}{T} + \ln(|\Sigma_u|) \tag{1}$$

$$\mathrm{SBIC} = \frac{\ln(T)}{T} pK^2 + \ln(|\Sigma_u|) \tag{2}$$

$$\mathrm{HQC} = \frac{2ln\{\ln(T)\}}{T} pK^2 + \ln(|\Sigma_u|) \tag{3}$$

Where p is the number of lagged terms, T the number of observations, K the number of endogenous variables, and $\hat{\Sigma}_u$ is the maximum likelihood estimate for the error covariance matrix, with u_t being the $K \times 1$ vector of disturbances.

When selecting the optimal lag length, by balancing model fitness and complexity, AIC tends to be more tolerant of adding more parameters than the other criteria, which can lead to more complex models. To the contrary, SBIC tends to have higher penalties for complex models and is therefore more likely to prefer simpler models, especially with large data samples. HQC finds a balance between AIC and SBIC, having a penalty for model complexity that grows more quickly than AIC but slower than SBIC. Often the criteria will not indicate the same lag as the optimal one, requiring an extra analysis, such as a robustness check. In conclusion, AIC tends to overestimate lag length, SBIC underestimates it and HQC tends to identify the optimal lag length.

3.1.4 Vector Autoregression Model

The VAR model is a statistical model designed to capture the linear interdependencies between different variables, each represented by a time series (Stock & Watson, 2001). A VAR(p) model, where p is the lag length selected after performing the AIC, SBIC and HQC criteria, is a seemingly unrelated regression model containing the same explanatory variables in each equation, in case of no constraints placed on the coefficients. Applying a linear regression on each equation generates the maximum likelihood estimates of the coefficients. Once estimated, these coefficients are used to evaluate the residuals that, subsequently, are applied to obtain the cross-equation error variance-covariance matrix $\hat{\Sigma}_u$.

One advantage of the VAR model, compared to univariate autoregressive models, is that it allows studying several time series variables and the relationships between them, rather than a single time series. In addition, VAR models are particularly useful for predicting future values based on historical data and analyzing how a shock to one variable affects the other variables in the system, often measured using impulse response functions (IRFs), which trace the impact of such shocks over time.

The following formula is considered for the VAR(p) model with exogenous variables, as per Lütkepohl (Lütkepohl, 2005):

$$\mathrm{y}_t = c + \mathrm{AY}_{t-1} + \mathrm{B}_0\mathrm{x}_t + \epsilon_t \tag{4}$$

Where:

- y_t is the $K \times 1$ vector of endogenous variables;
- c is a $K \times 1$ vector of parameters (constants and/or trends);
- A is a $K \times K_p$ matrix of coefficients. With K_p being the total number of lagged endogenous variables in the system;
- B_0 is a $K \times m$ matrix of coefficients;

- x_t is the $m \times 1$ vector of exogenous variables;
- ϵ_t is the $K \times 1$ vector of white noise innovations;
- Y_t is the $K_p \times 1$ matrix given by $Y_t = \begin{pmatrix} y_t \\ \vdots \\ y_{t-p+1} \end{pmatrix}$

Estimating a VAR model plays an important role in Vector Error Correction modeling. Firstly, it is fundamental for conducting a Granger Causality Test, which is essential for understanding whether the variables contribute to predicting each other by analyzing the lagged relationships between them (Granger, 1969). Additionally, a VAR model provides a basis for understanding the structure of a VECM, as the VECM can be seen as a restricted form of VAR. By visualizing the structure of the VAR, it is possible to better understand short-term dynamics and prepare for transitioning to a VECM, which incorporates both short-term dynamics and long-term equilibrium through the inclusion of error correction terms.

3.1.5 Granger Causality Test

Granger causality is a statistical test used to determine whether variables help predict each other (Granger, 1969). Specifically, it examines whether the past values of one variable contain information that helps predict the future values of another variable, in addition to the information provided by its own past values. This test is essential in the identification of the causality direction between variables, which is important for understanding the interactions and dependencies in the system.

By determining whether the past values of a variable help to predict another variable, the test clarifies which variables contribute significantly to the model's predictive power. If a variable does not Granger-cause another variable, it can be considered unnecessary for the model. This process helps to refine the process by eliminating variables that do not provide predictive value, thus improving the efficiency and interpretability of the model.

The test is based on the following definition, by Granger: "Definition 1: Causality. We say that Y_t is causing Z_t if we are better able to predict Z_t using all the available information than if the information apart from Y_t had been used." (Granger, 1969).

To demonstrate the application of the Granger Causality test, a simple VAR model is considered with p lags, m exogenous variables and two variables, represented by time series y_{1t} and y_{2t}. The model consists of the following equations:

$$\mathrm{y}_t = \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \sum_{j=1}^{p} \begin{bmatrix} a_{11,j} & a_{12,j} \\ a_{21,j} & a_{22,j} \end{bmatrix} \begin{bmatrix} y_{1,t-j} \\ y_{2,t-j} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \end{bmatrix} \begin{bmatrix} x_{1t} \\ x_{2t} \\ \vdots \\ x_{mt} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} \quad (5)$$

In order to assess whether y_{2t} Granger-causes y_{1t}, the null hypothesis is that $a_{12,j} = 0$ for all lags ($j = 1, \dots, p$), while the alternative hypothesis states that at least one $a_{12,j} \neq 0$ for all lags ($j = 1, \dots, p$). The null hypothesis claims that the past values of y_{2t} do not contribute to predicting y_{1t}. Thus, if the null hypothesis is rejected, it shows that the past values of y_{2t} Granger-cause y_{1t}.

For small to medium sample sizes, the approach to test the hypotheses is typically the F-test, as it is a more suitable method for these sample sizes, considering that it is easier to interpret and computationally simpler to obtain. For such cases, the test statistic is calculated as follows:

$$F = \frac{(RSS_{restricted} - RSS_{unrestriced})/q}{RSS_{unrestriced}/(T - k - 1)} \sim F(q, T - k - 1) \quad (6)$$

Where:

- $RSS_{restricted}$ and $RSS_{unrestriced}$ are the Residual Sum of Squares from the restricted (where $a_{12,i} = 0$ for all lags ($i = 1, \dots, p$)) and unrestricted models, respectively;
- q is the number of restrictions, which represents the number of coefficients being tested (in this case, it is equal to the number of lagged terms p);
- k is the total number of parameters in the unrestricted model, meaning that $k = K \times p + K \times m$.

One crucial point is to always test in both directions: $y_{2t} \Rightarrow y_{1t}$ and $y_{2t} \Leftarrow y_{1t}$. Therefore, to assess bidirectional causality, the same procedure is applied to test whether y_{1t} has predictive power for y_{2t}, but adapting the hypotheses to $a_{21,i} = 0$ for all lags ($i = 1, \dots, p$).

3.1.6 Johansen cointegration test

Testing for cointegration is a crucial step in VECM modeling, as it ensures the presence of cointegration among the multiple time series variables, which is a prerequisite for VEC models. Although the individual time series are non-stationary, cointegration implies the existence of a linear combination of these variables that is stationary, indicating a long-run equilibrium relationship. The approach chosen to test for cointegration was the Johansen test, which employs the maximum likelihood estimation method to determine the number of cointegrating vectors in the system (Johansen, 1995). This test was preferred over other options such as the Engle-Granger test (Engle & Granger, 1987), as it has advantages such as testing multiple cointegrating relationships simultaneously, and is particularly effective in multivariable contexts where these variables influence each other dynamically. By correctly identifying cointegration relationships, the Johansen test ensures that the VECM captures both short-term and long-term dynamics, guaranteeing the accuracy of the model.

The Johansen cointegration test is based on a VECM and its matrix Π, also known as the long-run impact matrix, which plays a central role in identifying cointegration relationships. A more detailed description of the importance and role of this matrix in a VECM will be presented in the upcoming section. The Johansen test is a fundamental factor in determining whether there are cointegration vectors and how many there are.

The rank of the matrix Π, denoted as r, represents the number of cointegration relationships. This rank can vary from 0 to $K - 1$. The results of a Johansen test and their meaning are as follows:

i. If $r = 0$, there is no cointegration;
ii. If $0 < r < K$, there is one or more cointegrating relationships;
iii. If $r = K$, the system is stationary.

The eigenvalues of the matrix Π, given as λ_i, are essential for evaluating its rank, since the matrix's rank is equivalent to the number of non-zero eigenvalues.

The Johansen cointegration test uses the following two main test statistics:

1. **Trace Statistic ($\boldsymbol{\lambda_{\text{trace}}}$)**

The trace statistic tests the null hypothesis that the number of cointegrating vectors is less than or equal to r, against the alternative hypothesis that there are more than r cointegrating vectors. The formula given by Johansen for the Trace Statistic is (Johansen, 1995):

$$\lambda_{\text{trace}}(r) = -T \sum_{i=r+1}^{K} \ln\left(1 - \hat{\lambda}_i\right) \tag{7}$$

2. **Maximum Eigenvalue Statistic ($\boldsymbol{\lambda}_{\mathbf{max}}$)**

The maximal eigenvalue statistic tests the null hypothesis that the number of cointegrating vectors is r, against the alternative hypothesis that there are $r+1$ cointegrating vectors. The formula is the following:

$$\lambda_{\max}(r, r+1) = -T \ln\left(1 - \hat{\lambda}_{r+1}\right) \tag{8}$$

This test is based on looking at the next largest eigenvalue to determine if adding another cointegration vector significantly improves the model.

The results of both test statistics are then compared with the critical values provided by the Johansen test tables. These tables offer critical values needed to evaluate the test statistics.

3.1.7 Vector Error Correction Model

The Vector Error Correction Model is a restricted form of the VAR model designed to analyze systems of non-stationary series that are cointegrated, i.e., although the individual variables may be non-stationary, they share a long-term equilibrium relationship (Johansen, 1995). The main objective of the VECM is to capture both the short-term dynamics and the long-term equilibrium relationships between the variables.

The role of the VECM in long-term equilibrium is to model how deviations from long-term equilibrium (cointegration relationships) are corrected. This is captured by the error correction term, which ensures that deviations are gradually adjusted over time. On the other hand, the function of the VECM in short-term dynamics is, while considering long-term relationships, to account for short-term fluctuations or interactions between variables, by including lagged differences of the variables. In the model, these short-term dynamics are captured through the Γ matrices.

Following confirmation of cointegration via the Johansen test, the modeling of the VECM is carried out. The Vector Error Correction Model is parameterized as follows:

$$\Delta \mathrm{y}_t = \Pi \mathrm{y}_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta \mathrm{y}_{t-i} + \mathbf{v} + \delta t + \theta_1 s_1 + \cdots + \theta_d s_d + \epsilon_t \qquad (10)$$

Where:

- y_t is a $K \times 1$ vector of endogenous variables;
- Π is the $K \times K$ long-run impact matrix;
- $\Gamma_1, \ldots, \Gamma_{p-1}$ are $K \times K$ matrices of parameters;
- $\mathbf{v}$ is a $K \times 1$ vector of parameters;
- δ is a $K \times 1$ vector of coefficients;
- t is a linear time trend;
- $s_1, \ldots, s_d$ are orthogonalized seasonal indicators;
- $\theta_1, \ldots, \theta_d$ are $K \times 1$ vectors of coefficients on the orthogonalized seasonal indicators.

In a VECM, the long-run impact matrix is crucial in understanding the long-term dynamics between the time series variables of the system and is key to determining the number and nature of these dynamics. The matrix is defined as follows:

$$\Pi = \alpha \beta' \qquad (9)$$

α is a $K \times r$ adjustment matrix, which indicates how each variable adjusts in response to deviations from the long-term equilibrium. β is a cointegration matrix that contains the cointegration vectors representing the long-term relationships between the variables. Therefore, β' is a $r \times K$ matrix corresponding to the transpose of the β cointegration matrix.

In the VECM equation described, there are two types of deterministic elements:

- The trend component which varies over time, denoted as $\mathbf{v} + \delta t$;
- The Seasonal component, shown as $\theta_1 s_1 + \ldots + \theta_d s_d$, which repeats over specific periods.

However, Johansen demonstrated that the inclusion of these deterministic elements introduces certain constraints to the model. Essentially, the estimated number of cointegration equations is based on non-standard distributions, and adding any term that generalizes the deterministic specification, such as event indicators, to the equation changes these distributions. As a result, the inclusion of these indicators is not feasible in the presented version of the VECM (Johansen, 1995).

In the case of having seasonal indicators in the model, these must not be collinear with a constant term, otherwise one of the indicator variables will be omitted.

If the inclusion of event indicators, such as constant and trend, in the equation is required, the model can be reparametrized as follows:

$$\Delta \mathrm{y}_t = \alpha(\beta \mathrm{y}_{t-1} + \mu + \rho t) + \sum_{i=1}^{p-1} \Gamma_i \Delta \mathrm{y}_{t-i} + \gamma + \tau t + \epsilon_t \tag{11}$$

Where:

- μ is a $K \times 1$ vector of constant terms in the long-term relationship;
- ρ is a $K \times 1$ vector of trend coefficients in the long-term relationship;
- γ is a $K \times 1$ vector of constant terms in the short-term relationship;
- τ is a $K \times 1$ vector of trend coefficients in the short-term relationship.

The specific characteristics exhibited by the variables in the time series and trend analysis will determine the most appropriate VECM equation to apply.

Time Series and trend Analysis, lag-order selection criteria, the Granger causality test, and the Johansen cointegration test are essential steps that precede the VECM estimation. These steps are crucial for identifying the correct model parameters and ensuring that the VECM is as accurate as possible. After estimating the VECM, diagnostic tests are conducted to validate its robustness. These tests include the Lagrange multiplier test, the stability test, the Jarque-Bera test, and the impulse response function, all of which will be discussed in the following sections.

3.1.8 Lagrange-Multiplier Test

The first diagnostic test performed is the Lagrange-multiplier (LM) test, which is essential for detecting the presence of autocorrelation in the residuals of a VECM (Johansen, 1995). Confirming that the residuals are uncorrelated is crucial to ensure valid statistical inference. This test is particularly useful in systems with a multivariable nature, such as the VECM.

The autocorrelation is verified up to a specified lag. The detection of autocorrelation may suggest that the model is misspecified, meaning that additional lags or a different model structure is required. In contrast, non-correlation indicates that the model is well specified and that the model is capturing all relevant dynamics.

To perform the Lagrange-multiplier test for autocorrelation, a VECM without any trend is considered:

$$\Delta \mathrm{y}_t = \alpha \hat{\mathbf{E}}_t + \sum_{i=1}^{p-1} \Gamma_i \Delta \mathrm{y}_{t-i} + \epsilon_t \tag{12}$$

With $\hat{\mathbf{E}}_t = \hat{\beta} \mathrm{y}_t$.

$\hat{\mathbf{E}}_t$ represents the exogenous variables in the equation presented, which is in essence a VAR model with $p-1$ lags.

In order to perform the test statistic, an augmented VAR must be computed. This VAR is obtained by adding a $K \times 1$ vector of residuals. This vector, denoted as $\mathbf{e}_t$, is composed by the residuals $\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_K$ of the K equations in the system.

The LM test statistic at lag j is calculated as:

$$\mathrm{LM}_j = (T - d - 0.5) \ln \left(\frac{|\hat{\Sigma}|}{|\tilde{\Sigma}_j|} \right) \sim \chi^2(K^2) \tag{13}$$

Where:

- d is the number of coefficients estimated in the Augmented VAR;
- $\hat{\Sigma}$ is the maximum likelihood estimate of the variance-covariance matrix of the residuals from the estimated VAR;

- $\tilde{\Sigma}_j$ is the maximum likelihood estimate of the variance-covariance matrix of the residuals from the augmented VAR.

For each lag j, an augmented regression is formed in which the new residual variables are lagged j times. If there are missing values for these j lags, they are replaced by zero, as proposed by R. Davidson and J. G. MacKinnon (Davidson & MacKinnon, 1993).

The test statistic is performed from lag 1 until the lag defined by the researcher. The null hypothesis of the test, for each lag j, is that there is no autocorrelation at the evaluated lag length.

3.1.9 Stability Test

The stability test ensures that the system is dynamically well behaved and that the relationships between the variables will remain within reasonable bounds over time, which implies that after a shock, the system will return to equilibrium quickly (Lütkepohl, 2005). The stability test is conducted by analyzing the companion matrix derived from the VECM. If the model is proven to be unstable, the estimates and forecasts are considered unreliable.

In order to perform the stability test, the estimates of the VECM parameters must first be converted into the corresponding VAR model estimates.

The estimated long-run impact matrix $\widehat{\Pi}$ can be defined as (Johansen, 1995):

$$\widehat{\Pi} = \sum_{i=1}^{p} \mathrm{A}_i - \mathbf{I}_K \tag{14}$$

Where $\mathbf{I}_K$ is the K-dimensional identity matrix.

And the estimated short-term dynamics matrix $\hat{\Gamma}$ is equivalent to:

$$\hat{\Gamma}_i = -\sum_{j=i+1}^{p} \mathrm{A}_j \tag{15}$$

Defining

$$\Gamma = \mathbf{I}_K - \sum_{i=1}^{p-1} \Gamma_i \tag{16}$$

Solving the equations for A, we obtain:

$$\mathrm{A}_1 = \Pi + \Gamma_1 + \mathbf{I}_K \tag{17}$$

$$\mathrm{A}_i = \Gamma_i - \Gamma_{i-1} \quad \text{for } i = \{2, \dots, p-1\} \tag{18}$$

$$\mathrm{A}_p = \Gamma_{p-1} \tag{19}$$

Thus, A is the companion matrix that captures the dynamics of the system, and it can be constructed as:

$$\mathrm{A} = \begin{pmatrix} \mathrm{A}_1 & \mathrm{A}_2 & \cdots & \mathrm{A}_{p-1} & \mathrm{A}_p \\ \mathbf{I} & 0 & \cdots & 0 & 0 \\ 0 & \mathbf{I} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \mathbf{I} & 0 \end{pmatrix} \tag{20}$$

If all the eigenvalues of the companion matrix A, denoted as λ_i, lie within the unit circle, i.e., the module of each eigenvalue is strictly less than 1, the VECM is stable, which implies that the system will return to equilibrium after any shock. These eigenvalues are obtained by solving the characteristic equation $\det(\mathrm{A} - \lambda\mathbf{I}) = 0$. If any eigenvalue lies outside the unit circle, the VECM is considered unstable, its results are considered unreliable, and the model may display "explosive" behavior, never reverting to equilibrium.

An additional validation step is that since there are K endogenous variables and r cointegrating vectors in the VECM, the companion matrix must have $K - r$ unit moduli. If any of the remaining moduli approach the unit value, it suggests that the system is on the verge of instability. This could indicate that trends or seasonal patterns are present but not modeled properly or that the cointegration equations estimated by the Johansen test are too high or that they are not stationary.

3.1.10 Normality of residuals

Testing for normality verifies whether the error terms follow a Gaussian distribution, a condition that supports valid statistical inference (Johansen, 1995). Normality ensures that asymptotic distributions can be achieved without large sample sizes. This test helps to assess the robustness and reliability of the estimated VECM, even in smaller samples.

The Jarque-Bera, skewness, and kurtosis tests are essential diagnostic tools for assessing the normality of residuals.

The Jarque-Bera, Skewness and Kurtosis tests are essential diagnostic tools for assessing the normality of the residuals. These test statistics must be computed using the orthogonalized residuals. A method for obtaining the orthogonalized residuals, denoted as $\hat{\mathbf{w}}_t$, is to premultiply the vector of residuals from the K equations of the model with the Cholesky decomposition of $\hat{\Sigma}$ (as defined in (13)):

$$\hat{\mathbf{w}}_t = (\hat{w}_{1t}, \dots, \hat{w}_{Kt})' = \hat{\mathbf{P}}^{-1}\hat{\epsilon}_t \tag{21}$$

Where $\hat{\mathbf{P}}$ is the Cholesky decomposition of $\hat{\Sigma}$.

Skewness Test

The Skewness test evaluates the asymmetry of the residual distribution. For example, a skewness value of 0 indicates a symmetric distribution, while a significant skewness value indicates model misspecification or the presence of outliers.

The skewness coefficients are then calculated using the residuals orthogonalized as follows:

$$\hat{b}_{k1} = \frac{1}{T}\sum_{i=1}^{T} \hat{w}_{kt}^3 \tag{22}$$

Resulting in the $K \times 1$ vector,

$$\hat{\mathbf{b}}_1 = \left(\hat{b}_{11}, \dots, \hat{b}_{K1}\right)';$$

Based on the null hypothesis of Gaussian disturbances, the skewness statistic is

$$\hat{\lambda}_1 = \frac{T\hat{\mathbf{b}}_1'\hat{\mathbf{b}}_1}{6} \sim \chi^2(K) \tag{23}$$

Kurtosis Test

Kurtosis is a fundamental diagnostic test for Vector Error Correction models as it assesses the “tailedness” of the residual distribution, indicating the likelihood of outliers. In a VECM, excessive kurtosis can indicate that extreme values are more common than expected.

The kurtosis coefficients are then calculated using the residuals orthogonalized as follows:

$$\hat{b}_{k2} = \frac{1}{T}\sum_{i=1}^{T} \hat{w}_{kt}^{4} \tag{24}$$

Resulting in the $K \times 1$ vector,

$$\hat{\mathbf{b}}_2 = \left(\hat{b}_{12}, \dots, \hat{b}_{K2}\right)';$$

Based on the null hypothesis of Gaussian disturbances, the kurtosis statistic is

$$\hat{\lambda}_2 = \frac{T\left(\hat{\mathbf{b}}_2 - 3\right)'\left(\hat{\mathbf{b}}_2 - 3\right)}{24} \sim \chi^2(K) \tag{25}$$

Jarque-bera Test

The Jarque-bera is a combination of the skewness and kurtosis tests, assessing the normality of the residuals. If the test indicates that the distribution of the residuals deviates from the normal distribution, this suggests that asymptotic distributions are not attainable and that a larger data sample is required.

The Jarque-Bera test statistic is calculated as:

$$\text{JB} = \text{Skewness} + \text{Kurtosis} \sim \chi^2(2K) \tag{26}$$

3.1.11 Impulse Response Function

The Impulse Response Function is an important tool for assessing the impact of a one-time shock to a variable on the current and future values of the model's endogenous variables (Lütkepohl, 2005). Specifically, the IRF helps to understand how a shock to a variable would propagate through the system over time, affecting the variable itself and the remaining variables. In the context of a VECM, where the variables are cointegrated, the IRF highlights both short-term fluctuations and long-term adjustments towards equilibrium between the system's variables. So, the main advantage of applying the IRF to the VECM, lies in its ability to provide insights into the interaction dynamics and causal relationships between variables. By analyzing the IRF, it is possible to analyze how quickly and to what extent variables return to equilibrium after a shock. In addition, IRF contributes to the next step in modeling a VECM: Forecasting, as it shows the possibilities of future variable values under different scenarios. By simulating shocks and analyzing the resulting impulse responses, researchers can predict how variables might evolve in the future.

Then, given the companion matrix $\widehat{\mathrm{A}}_j$ from (20), the estimates of the simple IRFs are obtained by:

$$\widehat{\Phi}_i = \sum_{j=1}^{i} \widehat{\Phi}_{i-j}\widehat{\mathrm{A}}_j \tag{27}$$

Where $\widehat{\mathrm{A}}_j = 0_K$ for $j > p$, with 0_K being the $K \times K$ null matrix.

3.2 FORWARD-LOOKING ADJUSTMENT

Under IFRS 9, financial institutions are encouraged to forecast single or multiple economic scenarios in order to enhance the accuracy of the Expected Credit Losses calculation, while being required to maintain robust impairment models (European Banking Authority, 2017). Incorporating the macroeconomic component into Probabilities of Default is one method of capturing the requirements of the accounting standard. It is possible to include macroeconomic conditions in Probabilities of Default by adjusting Through-the-Cycle PD, based on the macroeconomic cycle, into Point-in-Time PD, which reflect current macroeconomic conditions, being more accurate and timely adjusted. In order to make the adjustment, the Financial Services Authority (FSA) suggests using a scalar factor corresponding to a coefficient that converts the TTC PD into a PIT PD, denominated the Variable Scalar Approach.

Estimating "forward-looking" economic conditions is the first step in the adjustment process. The purpose of using the Vector Error Correction Model was to forecast these future economic conditions, specifically the Default Rate. Then, the scalar factor is calculated by dividing the forecasted Default Rate by the average Default Rate across all T periods (where the forecasted estimates are already included), as shown in the following formula:

$$\mathrm{SF}_T = \mathrm{DR}_T \times T \times \left[\sum_{t=1}^{T} \mathrm{DR}_t\right]^{-1} \tag{28}$$

Where SF_T is the scalar factor for the last observation t, and DR_T is the Forecasted Default Rate.

Remembering that the estimated Probabilities of Default have a direct impact on banks' internal capital requirements, a calculation based entirely on PIT PD would introduce significant volatility, due to the PD's sensitivity to short-term economic fluctuations (Nagy & Biró, 2018).

In order to create more stability in capital requirements, the minimum scalar factor assumed is 1 (100%), implying that in economic expansion periods, where the capital requirements are minimal, the expected credit losses will be calculated using TTC PD, as recommended by the Financial Conduct Authority (Financial Conduct Authority, 2021). This means that, in reality, neither a PIT PD nor a TTC PD approach is being applied, but rather a hybrid between the two, referred to as the Forward-looking adjusted PD. The following graph is presented to better understand the approach applied:

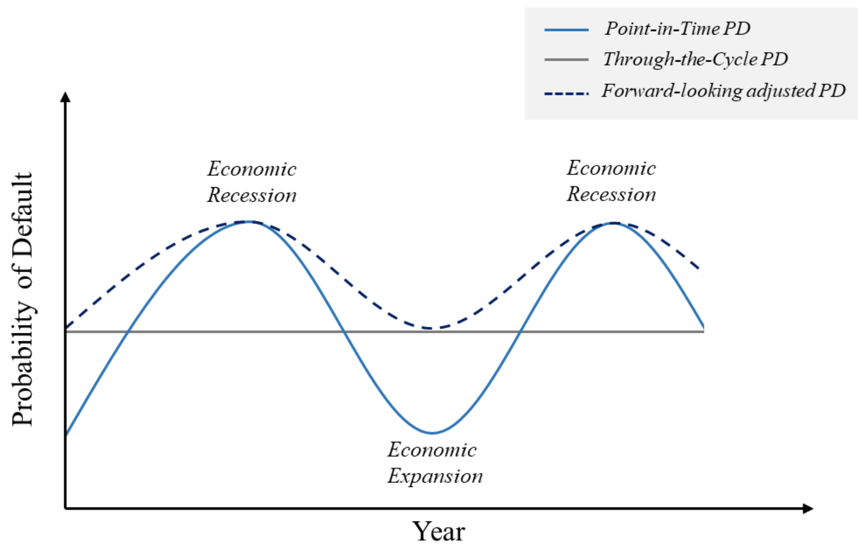

Figure 6: Forward-looking Adjusted PD

The scalar factor's minimum value of 1 is not the only factor that helps to reduce volatility. As the European Banking Authority suggests, incorporating multiple scenarios, such as the base case and the worst-case, also prevents large PD fluctuations (European Banking Authority, 2023).

The graph shows an ideal application of these approaches: for prudential purposes, the adjusted PD is always higher than the PIT PD, which is possible by incorporating the worst-case scenario in its calculation. This inclusion is crucial to ensure that actual capital requirements are never higher than estimated capital requirements. And, in the event of the worst-case scenario, such as an unexpected economic recession, the values converge, as shown in the graph.

Therefore, with the inclusion of the worst-case scenario and the minimum value of 1 assumed for the scalar factor, the forward-looking adjusted PD formula would be:

$$\text{Forward} - \text{looking adjusted PD}_t = max\left[\left(\text{SF}_{\text{BC},t} \times W_{\text{BC}} + \text{SF}_{\text{WC},t} \times W_{\text{WC}}\right); 1\right] \times \text{PD}_{\text{TTC}} \quad (29)$$

Where $\text{SF}_{\text{BC},t}$ and $\text{SF}_{\text{WC},t}$ are the scalar factors of the base and worst-case scenarios, respectively; And W_{BC} and W_{WC} are the weights of the base and worst-case scenarios, respectively. The term $max\left[\left(\text{SF}_{\text{BC},t} \times W_{\text{BC}} + \text{SF}_{\text{WC},t} \times W_{\text{WC}}\right); 1\right]$ in the formula is referred to as the Forward-Looking Adjustment, representing the scalar factor that incorporates both base and worst-case scenarios while maintaining a minimum value of 1.

4.DATA & RESULTS

This chapter presents and discusses the results obtained with the model, which was developed according to the steps described in the Methodology Section. Subsection 4.1 provides a detailed description and analysis of the data used, along with the VECM specification tests. In Subsection 4.2, the VECM is estimated and the diagnostic test results are presented. Subsection 4.3 focuses on the model's forecasts, and finally, in Subsection 4.4, the application of the forward-looking adjustment is discussed.

4.1 DATA ANALYSIS

This study is based on the Default Rate reported annually by S&P. This Default Rate was chosen due to the agency's reputation, prestige and reliability, as well as due to the size and global nature of the study sample (with approximately 23288 issuers from over one hundred different countries). The data comprises the period between 1981 and 2023, consists of forty-three observations and has been extracted from the official S&P Global website.

The impact of the following macroeconomic variables on the Default Rate was analyzed in the model:

- Unemployment Rate is the percentage of the labor force that is unemployed. Unemployment refers to individuals capable of working and of working age, but who are not currently employed;
- Inflation is a rate that measures how much more expensive a set of goods and services has become in a certain period, reflecting a loss of purchasing power;
- Gross Domestic Product (GDP) at constant prices is the total monetary value of all goods and services produced during a specific period, adjusted for inflation. Using constant prices ensures that the values reflect real economic growth by eliminating the effects of rising price levels. The values are presented as year-on-year rates of change, expressed in percentages, to illustrate the pace of economic growth over time.

In order to align the macroeconomic data sample with the one from the S&P Default Rate report, annual figures for the Unemployment Rate, Inflation, and GDP of Advanced

Economies[4] from 1981 to 2023 were used. These figures were extracted from the World Economic Outlook database, available on the International Monetary Fund's website.

The initial step in the analysis is verifying whether the variables are stationary. While the Augmented Dickey-Fuller test is commonly used to assess stationarity, it is essential to first evaluate the presence of trends and determine the optimal lag length for each variable, as both factors can significantly influence the accuracy and reliability of the stationarity test results.

To begin this process, the presence of trends in the data is assessed. The graphical representations (Figure 7) suggest that the Default Rate and the GDP have no trend. The presence of a trend in the remaining variables is unclear, but it appears that the Unemployment Rate shows a downward linear trend and Inflation exhibits a quadratic convex trend. This potential trend in the Unemployment Rate graph suggests the possibility of non-stationarity. Moreover, the visible cycles and lack of clear mean reversion further highlight its non-stationary nature, as the series does not maintain a stable mean or variance over time.

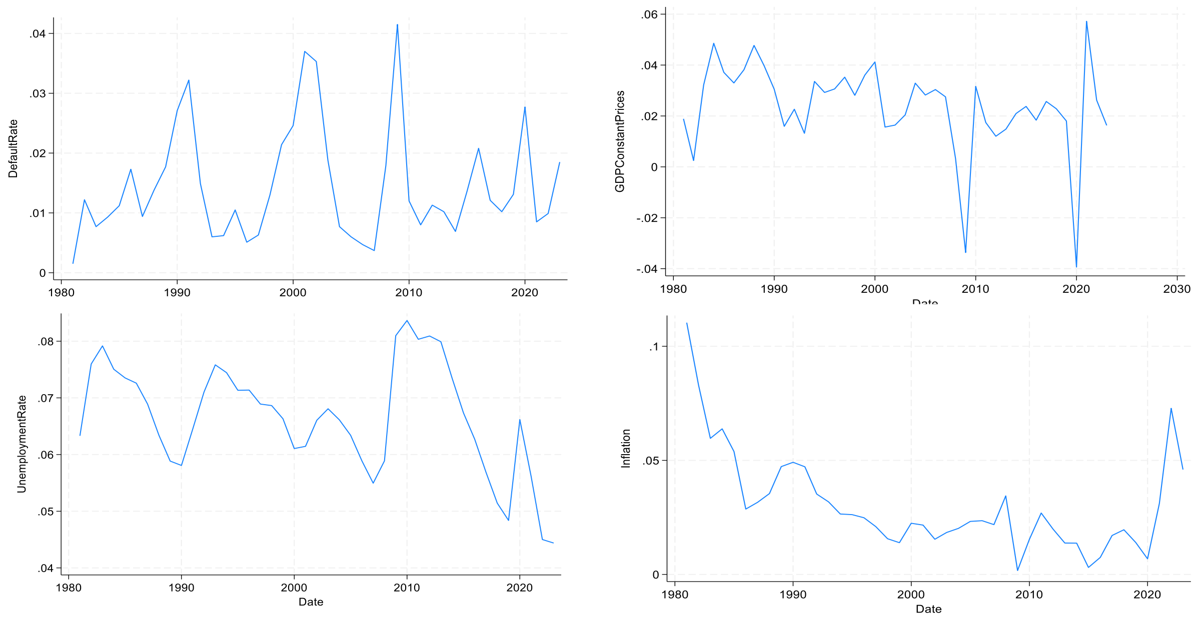

Figure 7: Time Series

[4] Countries with a high level of economic and social development, evidenced by criteria such as GDP per capita and the Human Development Index.

To complement the analysis, a graphical representation of the time series in first differences was also produced, where no variable appears to display a trend (Appendix Figure 11). Additionally, the statistical test described in the methodology was carried out, and the following results were obtained (complete analysis in Table 9 of the Appendix):

Variable	DefaultRate	GDPConstant~s	Unemploymen~e	Inflation
Trend	.00060228	-.00037432	.00049765	-.00522951***
c.Trend# c.Trend	-.00001247	-1.310e-06	-.00001818*	.00009734***
_cons	.00918997**	.03282035***	.06711282***	.08354757***

Legend: * p<.1; ** p<.05; *** p<.01

Table 1: Trend Statistical Tests Results Summary

In conclusion, no single trend is representative of all the variables in the system. There is no evidence of a linear or quadratic trend in the target variable - Default Rate -, in the GDP or in the Unemployment Rate. As not all variables exhibit a trend, and to avoid unnecessarily increasing model complexity, the trend will be omitted. Including a trend could risk over-parameterizing the model without providing significant improvements in predictive power. However, a constant should be included.

After assessing the presence of trends and determining the optimal lag length for each variable (Appendix Table 9), the Augmented Dickey-Fuller test was applied. The results (Appendix Table 10) indicate that the Default Rate and GDP are stationary at the 5% significance level, as their test statistics are lower than the critical values and their p-values are below 0.05. On the other hand, the Unemployment Rate and Inflation are not stationary, as its test statistics are higher than the critical values and p-values of 0.5034 and 0.1420, respectively – both exceeding the 5% threshold. This indicates that further differencing or transformation is required to achieve stationarity. Having non-stationary data further justifies the use of the VECM, as this methodology is specifically designed to handle non-stationary time series.

At first glance, the Augmented Dickey-Fuller test result indicating non-stationarity in the Unemployment Rate may seem counterintuitive. Typically, unemployment rates are expected to revert to their natural levels over time, exhibiting mean-reverting behavior characteristic of a stationary process. However, the observed non-stationarity can be attributed to the phenomenon of hysteresis in unemployment. Hysteresis occurs when temporary economic shocks, such as recessions, have long-lasting effects on the unemployment rate, preventing it from returning to pre-shock levels even after the economy recovers. This form of long-term unemployment arises from the persistence of high unemployment rates over extended periods, often leading to structural unemployment. During prolonged economic downturns, unemployed workers may experience skill degradation, labor market detachment, or stigmatization, making increasingly difficult to re-enter the workforce. These dynamics contribute to a permanent shift in the natural rate of unemployment. The existence of hysteresis has been widely studied and empirically confirmed in various economies, supporting the observation of non-stationary behavior in unemployment rates (Khraief, et al., 2020; Yilanci, 2008).

Next, the lag selection is considered. Computing the Lütkepohl approach for the lag-order selection criteria, the following results were obtained:

Lutkepohl's lag-order selection criteria

Sample: **1985** thru **2023** Number of obs = **39**

Lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	476.186				3.6e-16	-35.7713	-35.7713	-35.7713
1	558.421	164.47	16	0.000	1.2e-17	-39.1679	-38.9231	-38.4855
2	616.065	115.29*	16	0.000	1.5e-18*	-41.3036*	-40.8138*	-39.9386*
3	623.502	14.873	16	0.534	2.4e-18	-40.8644	-40.1298	-38.817
4	634.526	22.049	16	0.142	3.6e-18	-40.6093	-39.6298	-37.8793

* optimal lag
Endogenous: **DefaultRate GDPConstantPrices UnemploymentRate Inflation**
Exogenous: **_cons**

Table 2: Lütkepohl's Lag-order Selection Criteria Results

The results indicate that all the selection criteria - AIC, SBIC and HQC - unanimously recommend a lag length of two. This agreement across all criteria removes any ambiguity in determining the optimal lag. Selecting two lags ensures an appropriate balance between model adequacy and complexity, making it the most reasonable choice for the analysis.

With the specifications established, the underlying VAR model can now be estimated to perform the Granger Causality Test. Based on the results obtained (Appendix Table 10), the following diagram was created:

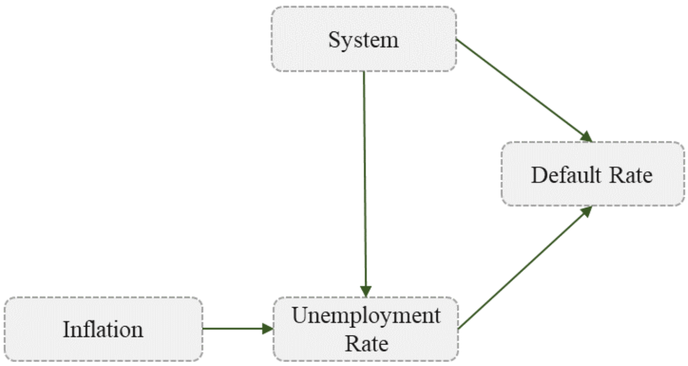

Figure 8: Granger Causality Relationships Diagram

The diagram illustrates the four Granger causality relationships between the time series, confirming that all variables, apart from GDP, are actively involved in the model. The overall system, which incorporates all variables, demonstrates predictive power over the Default Rate and the Unemployment Rate. Additionally, it should be noted that the other variable that granger-causes the DR is the Unemployment Rate. The Granger Causality Test confirms the presence of short-term dynamics between the variables. However, no pair of variables exhibits mutual causality.

In order to assess the existence of cointegration between the variables, the Johansen cointegration test is performed.

Johansen tests for cointegration
Trend: Constant — Number of obs = 41
Sample: 1983 thru 2023 — Number of lags = 2

Maximum rank	Params	LL	Eigenvalue	Trace statistic	Critical value 5%
0	20	610.74253	.	62.6924	47.21
1	27	628.78666	0.58530	26.6041*	29.68
2	32	638.02041	0.36264	8.1366	15.41
3	35	640.83188	0.12816	2.5137	3.76
4	36	642.08872	0.05947		

* selected rank

Table 3: Johansen Cointegration Test Results

The results displayed in Table 3 confirm the existence of one cointegration relationship, thus justifying the use of the VECM for this research. The estimated value of the test statistic is greater than the critical value at rank 0, leading to the rejection of the null hypothesis and indicating the presence of one or more cointegration vectors.

However, as signaled in rank 1, the estimated statistic falls below the critical value, which implies the existence of only one long-term equilibrium relationship.

4.2 VECM & DIAGNOSTIC TESTS RESULTS

The model's output, shown in Table 11 of the Appendix, reveals a statistically significant long-term relationship between the Default Rate and both the Unemployment Rate and Inflation, in the cointegration equation. The coefficient reveals a positive correlation between these variables, meaning that as Unemployment Rate and Inflation increase, the Default Rate also tends to increase in the long run. Despite the existence of long-term equilibrium relationships between the macroeconomic variables and the Default Rate, these relationships do not exhibit the expected predictive power for short-term changes in the Default Rate. This is evidenced by the lack of statistical significance in the error correction term within the Default Rate equation, which suggests that deviations from the long-term equilibrium do not significantly influence short-term adjustments in the Default Rate. As a result, while the model captures meaningful long-term dynamics, its utility for short-term forecasting of the Default Rate is limited.

While the error correction term within the Default Rate equation is not statistically significant, the model still identifies statistically significant short-term relationships. In particular, the Unemployment Rate emerges as the macroeconomic variable with a statistically significant short-term impact on the Default Rate. This is consistent with the results of the Granger causality test, which showed that the past values of the Unemployment Rate are a strong predictor of the Default Rate. The coefficient presented for this short-term relationship suggests that rising unemployment is a strong indicator of deteriorating economic conditions, leading to higher default rates. Thus, as theoretically expected, the default and unemployment rates are positively correlated.

The comparison with the VAR model (Appendix Table 12), estimated with variables in first differences due to the non-stationarity of the Unemployment Rate, highlights differences in short-term results. Unlike the VAR, the VECM incorporates an error correction term, which allows it to account for long-term equilibrium relationships. While the error correction term in the VECM does not play a significant role in predicting short-term changes in the Default Rate, the model still captures meaningful short-term dynamics through significant relationships, such as that with the Unemployment Rate. In contrast, the VAR model shows limited explanatory power for short-term changes, with none of the lagged variables being statistically significant in explaining changes in the Default Rate and generally low R-squared values. This makes the VECM a more comprehensive tool for analyzing short-term relationships alongside its long-term insights.

The first diagnostic test applied to the model was the Lagrange-multiplier, ensuring that there is no autocorrelation present in the residuals. Computing the LM Test, the following results were obtained:

Lagrange-multiplier test

lag	chi2	df	Prob > chi2
1	8.7846	16	0.92202
2	12.8971	16	0.68027
3	8.7957	16	0.92159
4	7.8584	16	0.95297

H0: no autocorrelation at lag order

Table 4: Lagrange-multiplier Test Results

The test statistic was performed from lag 1 until lag 4, and the null hypothesis cannot be rejected for any of these, indicating that there is no autocorrelation in the residuals up until order 4. The LM results suggest that the model is robust and reliable.

Moving on to the stability test, the aim is to assess whether all the eigenvalues of the estimated VECM companion matrix lie within the unit circle.

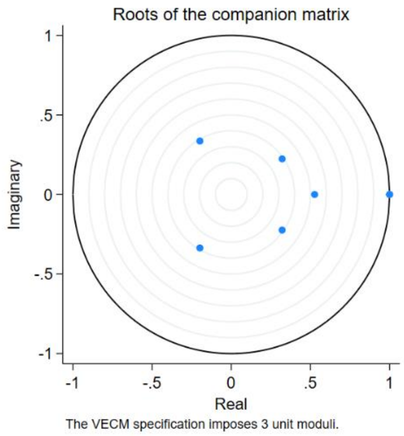

Figure 9: Companion Matrix Roots

The graphical representation shows that the estimated eigenvalues are all within the unit circle, ensuring the stability of the model. Hence, after a shock, the system will return to equilibrium quickly.

Since there are four endogenous variables and one cointegration vector in the VECM, the companion matrix must have $4 - 1$ unit moduli, which is the case, as proven in Table 13 of the Appendix.

To assess the normality of the residuals, the Jarque-Bera test was performed and the following p-values were obtained:

Jarque-Bera test

Equation	chi2	df	Prob > chi2
D_DefaultRate	3.317	2	0.19047
D_GDPConstantPrices	0.817	2	0.66455
D_UnemploymentRate	1.106	2	0.57518
D_Inflation	2.337	2	0.31088
ALL	7.577	8	0.47588

Table 5: Jacque-Bera Test Results

Examining the results, it can be noted that the residuals of all the variables, including the Default Rate, follow a normal distribution. The null hypothesis is not rejected for any variable, indicating that their residuals do not deviate from normality. To confirm this, the results of the Kurtosis and Skewness tests were analyzed. As the kurtosis test shows (Appendix Table 14), no residual distribution exhibits significant “tailedness”, indicating that outliers are not expected. Similarly, the Skewness test (Appendix Table 15) reveals no significant asymmetry in the residuals' distribution, confirming that they are symmetrically distributed around the mean.

An analysis of the variable's historical data, along with the graphical representation of its residuals (Appendix Figure 12), indicates that there are some extreme positive values. This implies that the model tends to underestimate cases of extreme upward movements in the Default Rate. The assessment of these extreme values reveals that there is a theoretical and economic justification for the periods in which the model underestimates the Default Rate values - specifically in 2009 and 2020. Both years are well known for their extremely adverse macroeconomic conditions, caused by two unpredictable occurrences: in 2009, the consequences of the 2008 global financial crisis severely impacted the world's economies, leading to an increase in defaults and financial instability (International Monetary Fund, 2009). Similarly, in 2020, the COVID-19 pandemic caused unprecedented global economic stoppages, leading to serious financial disruptions (International Monetary Fund, 2020).

It can be concluded that, although the residuals in the model do not exhibit significant skewness or deviations from normality, some extreme values are observed. These extreme values can be attributed to the impact of years of extreme macroeconomic distress, such as 2009 and 2020. However, these isolated shocks do not invalidate the model, as the residuals remain symmetrically distributed around the mean overall, confirming the robustness and reliability of the results.

Moving on to the last diagnostic test, the results from the Impulse Response Function illustrate how a one-time shock to each of the system's variables affects the future values of the default rate. In accordance with the findings of the Granger causality test and the statistically significant relationships identified in VECM, the IRF test (Annex Table 16 and Figure 13) indicates that the variable with the most significant short-term impact on the DR is the Unemployment Rate. Additionally, the results confirm that Inflation does not have a notable influence on short-term fluctuations in the DR but contributes to the long-term equilibrium relationship.

4.3 FORECASTED DEFAULT RATE

Before proceeding with the forecasting of the Default Rate for future years, a VECM was estimated with data until 2021 (inclusive), allowing for a comparison between forecasted and observed values. This comparison helps determine whether the predicted data deviates significantly from the actual observed data.

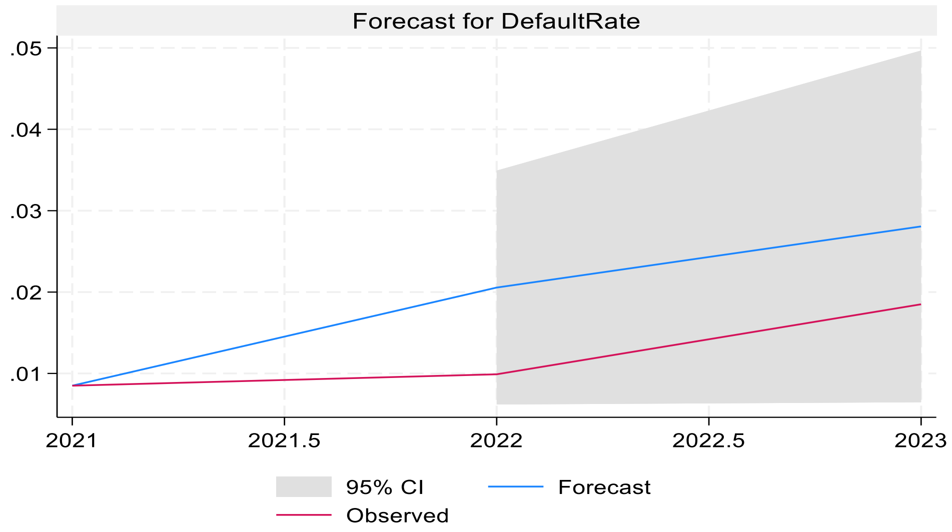

Figure 10: Observed and Forecasted Default Rates

The model predicted a DR of 2.06% for 2022 and 2.81% for 2023. When compared to the actual observed rates of 0.99% for 2022 and 1.85% for 2023, there is a deviation, although it cannot be considered too significant, especially when considering the inherent unpredictability of macroeconomic data. These differences result in a root mean square error (RMSE) of 1.01%. To put this into context, since 1981, the maximum and minimum default rates observed have been 4.15% and 0.15%, respectively. Given that the predicted rates are higher than the observed rates, the deviation is more tolerable, as it demonstrates a conservative approach designed to take preventive measures into account.

Having obtained acceptable estimates, the forecasting can be pursued for future years. In accordance with IFRS 9 guidelines, financial institutions typically forecast macroeconomic variables over a three-year horizon, as estimates for longer periods may no longer be considered reliable. Thus, the VECM obtained the following forecasts for 2024, 2025 and 2026:

Year	Default Rate	GDP	GDP (IMF)	Unemployment Rate	Unemployment Rate (IMF)	Inflation	Inflation (IMF)
2024	1,66%	1,77%	1,74%	4,35%	4,64%	5,01%	2,62%
2025	1,55%	1,48%	1,77%	4,31%	4,67%	5,44%	2,05%
2026	1,45%	1,44%	1,77%	4,30%	4,65%	5,63%	2,01%

Table 6: Forecasted Default Rate, GDP, Unemployment Rate and Inflation

The model predicted a gradual decline in the Default and Unemployment rates over the next three years. Regarding GDP, the forecast indicates a slow but steady growth. And, for Inflation, the VECM predicted a gradual increase, which, considering that inflation has already fallen in 2023, signaling a turning point, makes the forecast seem slightly unrealistic. To support this analysis, Figure 14 in the appendix provides a graphical representation of the variables' evolution over time.

In addition to the estimates obtained, the IMF forecasts for GDP, unemployment rates and inflation were also included for comparison with the VECM results. Comparing the results further supports the previous conclusion that the Inflation forecasted by VECM is likely to diverge from the actual results, as these inflation estimates differ significantly from those of the IMF, representing an RMSE of 3.18%. In contrast, the forecasts for GDP and the Unemployment Rate are aligned with the IMF's projections, with RMSE values of 0.25% and 0.33%, respectively.

4.4 FORWARD-LOOKING ADJUSTMENT

Once the forecast has been completed, the following step is to calculate the Scalar Factor, beginning with its estimation for the base scenario as described in the methodology. Then, as VECM is not capable of generating a worst-case scenario, this scenario was derived from historical data. The historical analysis determined that in 11.63% of the years, a scalar factor exceeding 200% was required to accurately estimate the Default Rate. This percentage will serve as the weighting for the worst-case scenario. For the worst-case value, 238.53% was used, which corresponds to the average value of the scalar factors exceeding 200%. The inclusion of a worst-case scenario is crucial when estimating the scalar factor and, consequently, the forward-looking adjusted PD, as it enables the integration of extremely adverse macroeconomic conditions into the model, which are always unpredictable.

Thus, the values achieved for the Scalar Factor that incorporates the worst and base case, referred to as the Forward-looking Adjustment, are the following:

Year	Default Rate	Average Default Rate	Base Case Scalar Factor	Worst Case Scalar Factor	Worst Case Weight	Forward-looking Adjustment
2024	1,66%	1,45%	114,18%	238,53%	11,63%	128,64%
2025	1,55%	1,46%	106,64%			121,98%
2026	1,45%	1,46%	99,80%			115,93%

Table 7: Forward-looking Adjustment Results

The default rate forecast for 2024 and 2025 alone exceeds the average of past years, which is sufficient for the Scalar Factor to exceed the minimum threshold of 100%. When the worst-case scenario is included, the scalar factor surpasses the 100% threshold for all three years. Consequently, the Forward-Looking Adjustment parameter has been adapted to reflect these forecasted macroeconomic conditions, ensuring compliance with prudential requirements.

In conclusion, the estimated forward-looking adjusted PD corresponds a PD PIT. For instance, in 2024, the PD in place corresponds to the PD TTC provided by S&P, multiplied by the forward-looking adjustment factor of 1.2864. This adjustment has resulted in a modest increase in internal capital requirements and expected credit losses to account for potential risks.

5. CONCLUSION

Although the VECM is a suitable model for predicting the default rate, there is limited evidence of its application in this specific context. However, through the development of the model in this research, it has proven to be a valid method for forecasting the default rate. The diagnostic tests that were carried out confirm that the model is reliable and robust, capturing both short-term dynamics and long-term equilibrium relationships between the default rate and key macroeconomic variables, such as GDP, inflation, and the Unemployment Rate.

In addition, the model was able to provide reasonably accurate predictions of default rates for the period considered, since the deviations from the observed data were minimal and within an acceptable range. This evidence highlights the practical applicability of the model for financial institutions. As demonstrated in this study, the Default Rate can be used as a basis for estimating a scalar factor and, consequently, a forward-looking adjusted Probability of Default, with the ultimate goal of predicting expected credit losses and forecasting internal capital requirements.

Nevertheless, the estimated model has certain limitations. The relatively small sample size of forty-three observations may hinder its ability to accurately capture the relationships between variables. Furthermore, the model's reliance on historical data presents challenges in forecasting extreme economic events, as evidenced by the extreme positive values in the residuals during major financial crises (e.g., 2009 and 2020). Nevertheless, the underestimation of default rates in these cases is believed to be attributable to the unpredictable nature of these economic shocks, rather than a model misspecification. On the other hand, the use of annual data may have further constrained the model's capacity to anticipate severe economic conditions, as key indicators of economic distress may be seen over shorter intervals, such as quarterly or monthly periods, and may not have been fully captured.

The fact that the study sample for the default rate and macroeconomic variables is not perfectly aligned in terms of the countries included in the sample can also introduce inconsistencies. This misalignment can lead to inaccuracies in capturing the true relationships between the variables and can affect the reliability of the model's predictions.

Another point worth noting is that the discrepancies between the model's inflation forecasts and those of the IMF may indicate that the model did not fully capture certain economic dynamics.

In conclusion, even though the VECM demonstrates robustness and reliability in forecasting default rates in relatively stable economic conditions, its results should be interpreted cautiously during periods of anticipated severe economic stress. Future research could mitigate this limitation by expanding the data set, incorporating quarterly data, exploring additional variables or considering alternative modeling approaches that take extreme economic events into account.

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APPENDIX

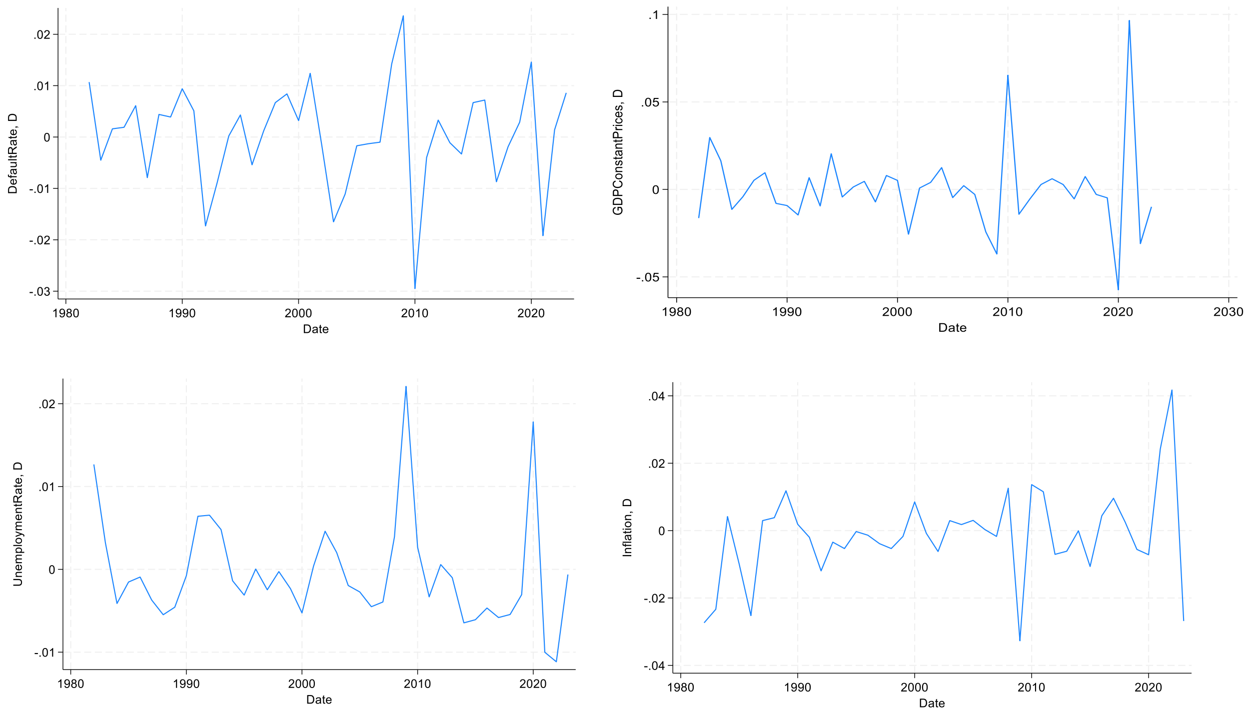


Figure 11: Time Series in First Differences

Source	SS	df	MS			
				Number of obs	=	43
				F(2, 40)	=	0.82
Model	.000145671	2	.000072835	Prob > F	=	0.4496
Residual	.003572388	40	.00008931	R-squared	=	0.0392
				Adj R-squared	=	-0.0089
Total	.003718059	42	.000088525	Root MSE	=	.00945

DefaultRate	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
Trend	.0006023	.0004751	1.27	0.212	-.000358	.0015626
c.Trend#c.Trend	-.0000125	.0000105	-1.19	0.241	-.0000336	8.69e-06
_cons	.00919	.0045327	2.03	0.049	.000029	.0183509

Source	SS	df	MS			
				Number of obs	=	43
				F(2, 40)	=	2.10
Model	.00123709	2	.000618545	Prob > F	=	0.1361
Residual	.011798284	40	.000294957	R-squared	=	0.0949
				Adj R-squared	=	0.0496
Total	.013035373	42	.000310366	Root MSE	=	.01717

GDPConstantPr~s	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
Trend	-.0003743	.0008635	-0.43	0.667	-.0021195	.0013709
c.Trend#c.Trend	-1.31e-06	.000019	-0.07	0.945	-.0000398	.0000372
_cons	.0328204	.0082373	3.98	0.000	.0161721	.0494686

Source	SS	df	MS			
				Number of obs	=	43
				F(2, 40)	=	5.65
Model	.000873552	2	.000436776	Prob > F	=	0.0069
Residual	.003093446	40	.000077336	R-squared	=	0.2202
				Adj R-squared	=	0.1812
Total	.003966998	42	.000094452	Root MSE	=	.00879

UnemploymentR~e	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
Trend	.0004977	.0004422	1.13	0.267	-.000396	.0013913
c.Trend#c.Trend	-.0000182	9.74e-06	-1.87	0.069	-.0000379	1.52e-06
_cons	.0671128	.0042179	15.91	0.000	.0585881	.0756376

Source	SS	df	MS			
				Number of obs	=	43
				F(2, 40)	=	41.32
Model	.013651314	2	.006825657	Prob > F	=	0.0000
Residual	.006607936	40	.000165198	R-squared	=	0.6738
				Adj R-squared	=	0.6575
Total	.02025925	42	.000482363	Root MSE	=	.01285

Inflation	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
Trend	-.0052295	.0006462	-8.09	0.000	-.0065356	-.0039234
c.Trend#c.Trend	.0000973	.0000142	6.83	0.000	.0000686	.0001261
_cons	.0835476	.0061647	13.55	0.000	.0710883	.0960069

Table 8: Deterministic Components of the Time Series

Lag-order selection criteria

Sample: **1991** thru **2023** Number of obs = **33**

Lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	105.411				.000105	-6.32792	-6.31266	-6.28257
1	109.145	7.469	1	0.006	.000089	-6.49365	-6.46313	-6.40295
2	111.238	4.1859	1	0.041	.000083	-6.55989	-6.51411	-6.42384*
3	111.248	.01886	1	0.891	.000088	-6.49985	-6.43882	-6.31846
4	113.063	3.6299	1	0.057	.000084	-6.54924	-6.47295	-6.3225
5	115.491	4.8561*	1	0.028	.000077*	-6.63579*	-6.54424*	-6.3637
6	115.51	.03936	1	0.843	.000082	-6.57638	-6.46957	-6.25894
7	115.556	.09245	1	0.761	.000087	-6.51857	-6.3965	-6.15578
8	117.389	3.6652	1	0.056	.000083	-6.56903	-6.43171	-6.16089
9	117.499	.21984	1	0.639	.000088	-6.51509	-6.3625	-6.0616
10	117.501	.00356	1	0.952	.000095	-6.45459	-6.28675	-5.95575

* optimal lag
Endogenous: **DefaultRate**
Exogenous: **_cons**

Lag-order selection criteria

Sample: **1991** thru **2023** Number of obs = **33**

Lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	86.4069				.000331*	-5.17617*	-5.16091*	-5.13082*
1	86.4925	.17128	1	0.679	.00035	-5.12076	-5.09024	-5.03006
2	86.601	.21698	1	0.641	.000369	-5.06673	-5.02095	-4.93068
3	86.6044	.00688	1	0.934	.000392	-5.00633	-4.9453	-4.82493
4	86.6785	.14814	1	0.700	.000416	-4.95021	-4.87392	-4.72347
5	86.7365	.11601	1	0.733	.000441	-4.89312	-4.80157	-4.62103
6	86.93	.38695	1	0.534	.000464	-4.84424	-4.73743	-4.5268
7	87.3609	.8618	1	0.353	.000482	-4.80975	-4.68768	-4.44696
8	87.4638	.20592	1	0.650	.000511	-4.75538	-4.61806	-4.34725
9	87.5671	.20657	1	0.649	.000543	-4.70104	-4.54845	-4.24755
10	87.6587	.18317	1	0.669	.000577	-4.64598	-4.47814	-4.14715

* optimal lag
Endogenous: **GDPConstantPrices**
Exogenous: **_cons**

Lag-order selection criteria

Sample: **1991** thru **2023** Number of obs = **33**

Lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	107.997				.000095	-6.42409	-6.39357	-6.33339
1	121.564	27.133*	1	0.000	.000044	-7.18568	-7.1399	-7.04963
2	123.414	3.7002	1	0.054	.000042*	-7.2372*	-7.17617*	-7.05581*
3	123.705	.58243	1	0.445	.000044	-7.19424	-7.11795	-6.9675
4	123.952	.49325	1	0.482	.000046	-7.14858	-7.05703	-6.87649
5	124.077	.25051	1	0.617	.000049	-7.09557	-6.98876	-6.77813
6	124.107	.05938	1	0.807	.000052	-7.03676	-6.9147	-6.67397
7	124.421	.62851	1	0.428	.000054	-6.9952	-6.85788	-6.58706
8	125.271	1.7004	1	0.192	.000055	-6.98612	-6.83354	-6.53264
9	125.284	.02654	1	0.871	.000059	-6.92632	-6.75848	-6.42749
10	125.296	.02264	1	0.880	.000063	-6.8664	-6.6833	-6.32222

* optimal lag
Endogenous: **UnemploymentRate**
Exogenous: **Trend _cons**

Lag-order selection criteria

Sample: **1991** thru **2023** Number of obs = **33**

Lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	95.3056				.000205	-5.65488	-5.62437	-5.56419
1	101.616	12.62	1	0.000	.000149*	-5.97671*	-5.93094*	-5.84067*
2	101.889	.54664	1	0.460	.000155	-5.93267	-5.87164	-5.75128
3	102.916	2.0544	1	0.152	.000155	-5.93432	-5.85803	-5.70758
4	103.08	.32662	1	0.568	.000164	-5.88361	-5.79206	-5.61152
5	103.08	.00111	1	0.973	.000174	-5.82304	-5.71623	-5.5056
6	103.086	.01098	1	0.917	.000186	-5.76277	-5.6407	-5.39998
7	103.168	.16534	1	0.684	.000197	-5.70717	-5.56984	-5.29903
8	103.194	.05232	1	0.819	.00021	-5.64815	-5.49557	-5.19466
9	103.351	.31375	1	0.575	.000223	-5.59705	-5.42921	-5.09822
10	106.431	6.1589*	1	0.013	.000198	-5.72308	-5.53998	-5.1789

* optimal lag
Endogenous: **Inflation**
Exogenous: **Trend _cons**

Table 9: Lag-order Selection for the Augmented Dickey-Fuller Test

Augmented Dickey-Fuller test for unit root

Variable: **DefaultRate** Number of obs = **40**
Number of lags = **2**

H0: Random walk without drift, d = 0

	Test statistic	Dickey-Fuller critical value 1%	5%	10%
Z(t)	**-3.779**	**-3.648**	**-2.958**	**-2.612**

MacKinnon approximate *p*-value for Z(t) = **0.0031**.

Dickey-Fuller test for unit root Number of obs = **42**
Variable: **GDPConstantPri~s** Number of lags = **0**

H0: Random walk without drift, d = 0

	Test statistic	Dickey-Fuller critical value 1%	5%	10%
Z(t)	**-5.834**	**-3.634**	**-2.952**	**-2.610**

MacKinnon approximate *p*-value for Z(t) = **0.0000**.

Augmented Dickey-Fuller test for unit root

Variable: **UnemploymentRate** Number of obs = **40**
Number of lags = **2**

H0: Random walk with or without drift

	Test statistic	Dickey-Fuller critical value 1%	5%	10%
Z(t)	**-2.176**	**-4.242**	**-3.540**	**-3.204**

MacKinnon approximate *p*-value for Z(t) = **0.5034**.

Augmented Dickey-Fuller test for unit root

Variable: **Inflation** Number of obs = **41**
Number of lags = **1**

H0: Random walk with or without drift

	Test statistic	Dickey-Fuller critical value 1%	5%	10%
Z(t)	**-2.965**	**-4.233**	**-3.536**	**-3.202**

MacKinnon approximate *p*-value for Z(t) = **0.1420**.

Table 10: Augmented Dickey-Fuller Tests Results

Granger causality Wald tests

Equation	Excluded	F	df	df_r	Prob > F
DefaultRate	GDPConstantPrices	**5.1536**	**1**	**36**	**0.0293**
DefaultRate	UnemploymentRate	**3.9512**	**1**	**36**	**0.0545**
DefaultRate	Inflation	**.97514**	**1**	**36**	**0.3300**
DefaultRate	ALL	**4.2942**	**3**	**36**	**0.0109**
GDPConstantPrices	DefaultRate	**.60988**	**1**	**36**	**0.4399**
GDPConstantPrices	UnemploymentRate	**4.0507**	**1**	**36**	**0.0517**
GDPConstantPrices	Inflation	**2.2816**	**1**	**36**	**0.1396**
GDPConstantPrices	ALL	**2.2353**	**3**	**36**	**0.1008**
UnemploymentRate	DefaultRate	**.38444**	**1**	**36**	**0.5391**
UnemploymentRate	GDPConstantPrices	**.59554**	**1**	**36**	**0.4453**
UnemploymentRate	Inflation	**9.1281**	**1**	**36**	**0.0046**
UnemploymentRate	ALL	**3.1493**	**3**	**36**	**0.0367**
Inflation	DefaultRate	**.67017**	**1**	**36**	**0.4184**
Inflation	GDPConstantPrices	**1.9094**	**1**	**36**	**0.1755**
Inflation	UnemploymentRate	**.3053**	**1**	**36**	**0.5840**
Inflation	ALL	**1.5638**	**3**	**36**	**0.2150**

Table 11: Granger Causality Results

Vector error-correction model

Sample: 1983 thru 2023		Number of obs =	41
		AIC =	-29.35545
Log likelihood =	628.7867	HQIC =	-28.94453
Det(Sigma_ml) =	5.61e-19	SBIC =	-28.227

Equation	Parms	RMSE	R-sq	chi2	P>chi2
D_DefaultRate	6	.007624	0.4779	32.03414	0.0000
D_GDPConstantP~s	6	.017562	0.5438	41.72514	0.0000
D_Unemployment~e	6	.006118	0.1626	6.79393	0.3403
D_Inflation	6	.009757	0.5124	36.78147	0.0000

	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
D_DefaultRate						
_ce1 L1.	-.0740362	.0502652	-1.47	0.141	-.1725541	.0244817
DefaultRate LD.	.2574418	.1908845	1.35	0.177	-.1166849	.6315686
GDPConstantPrices LD.	.006455	.0774037	0.08	0.934	-.1452534	.1581634
UnemploymentRate LD.	-1.067548	.2643488	-4.04	0.000	-1.585663	-.5494343
Inflation LD.	.0935616	.1295655	0.72	0.470	-.1603821	.3475053
_cons	-.0005358	.0012191	-0.44	0.660	-.0029252	.0018536
D_GDPConstantPrices						
_ce1 L1.	-.0772006	.1157916	-0.67	0.505	-.3041479	.1497466
DefaultRate LD.	-.0847502	.4397244	-0.19	0.847	-.9465942	.7770937
GDPConstantPrices LD.	-.2295453	.1783082	-1.29	0.198	-.579023	.1199324
UnemploymentRate LD.	2.124294	.6089579	3.49	0.000	.9307588	3.31783
Inflation LD.	-.0690214	.298469	-0.23	0.817	-.6540098	.5159671
_cons	.0010248	.0028083	0.36	0.715	-.0044794	.006529
D_UnemploymentRate						
_ce1 L1.	.0169224	.0403373	0.42	0.675	-.0621372	.095982
DefaultRate LD.	.0942956	.1531829	0.62	0.538	-.2059374	.3945286
GDPConstantPrices LD.	-.0469871	.0621157	-0.76	0.449	-.1687316	.0747574
UnemploymentRate LD.	.1928028	.2121373	0.91	0.363	-.2229787	.6085843
Inflation LD.	.0704141	.103975	0.68	0.498	-.1333732	.2742014
_cons	-.0005749	.0009783	-0.59	0.557	-.0024924	.0013426

D_Inflation						
_ce1						
L1.	.2550777	.0643346	3.96	0.000	.1289842	.3811712
DefaultRate						
LD.	.0715644	.2443139	0.29	0.770	-.4072822	.5504109
GDPConstantPrices						
LD.	.3691054	.0990693	3.73	0.000	.1749331	.5632776
UnemploymentRate						
LD.	.8779025	.3383413	2.59	0.009	.2147658	1.541039
Inflation						
LD.	-.0638242	.1658315	-0.38	0.700	-.3888479	.2611994
_cons	.0001928	.0015603	0.12	0.902	-.0028654	.003251

Cointegrating equations

Equation	Parms	chi2	P>chi2
_ce1	3	36.08101	0.0000

Identification: beta is exactly identified

Johansen normalization restriction imposed

beta	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
_ce1						
DefaultRate	1	.	.	.	.	.
GDPConstantPrices	.4765423	.5111263	0.93	0.351	-.5252468	1.478331
UnemploymentRate	-1.877492	.4874496	-3.85	0.000	-2.832876	-.9221086
Inflation	-.9492098	.229664	-4.13	0.000	-1.399343	-.4990767
_cons	.1233151	.	.	.	.	.

Table 12: VECM

```
Vector autoregression

Sample: 1983 thru 2023                          Number of obs     =         41
Log likelihood =  538.8607                      AIC               =  -25.31028
FPE            =  1.20e-16                      HQIC              =  -25.00589
Det(Sigma_ml)  =  4.51e-17                      SBIC              =  -24.47439

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
DefaultRate           5     .008504   0.2642   14.72512   0.0053
GDPConstantPri~s      5     .017138   0.1570   7.638197   0.1058
UnemploymentRate      5     .008345   0.3514   22.21207   0.0002
Inflation             5     .013453   0.3924   26.48278   0.0000
----------------------------------------------------------------
```

	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
DefaultRate						
DefaultRate L2.	.0849847	.1534724	0.55	0.580	-.2158158	.3857851
GDPConstantPrices L2.	.1991626	.0822078	2.42	0.015	.0380382	.360287
UnemploymentRate L2.	-.3217289	.1516645	-2.12	0.034	-.6189858	-.0244719
Inflation L2.	-.0649334	.061616	-1.05	0.292	-.1856985	.0558318
_cons	.0324908	.011802	2.75	0.006	.0093593	.0556222
GDPConstantPrices						
DefaultRate L2.	.2577656	.3092871	0.83	0.405	-.3484261	.8639572
GDPConstantPrices L2.	.0701485	.1656703	0.42	0.672	-.2545593	.3948562
UnemploymentRate L2.	.656477	.3056436	2.15	0.032	.0574265	1.255528
Inflation L2.	.2001637	.1241724	1.61	0.107	-.0432097	.4435371
_cons	-.0311982	.0237841	-1.31	0.190	-.0778142	.0154177
UnemploymentRate						
DefaultRate L2.	.0996558	.1506069	0.66	0.508	-.1955284	.39484
GDPConstantPrices L2.	-.0664389	.0806729	-0.82	0.410	-.2245549	.0916771
UnemploymentRate L2.	.4453077	.1488328	2.99	0.003	.1536008	.7370145
Inflation L2.	.1949574	.0604656	3.22	0.001	.0764471	.3134678
_cons	.0306887	.0115816	2.65	0.008	.0079891	.0533883

Inflation						
DefaultRate						
L2.	.2120982	.2427751	0.87	0.382	-.2637322	.6879287
GDPConstantPrices						
L2.	-.1917683	.130043	-1.47	0.140	-.4466478	.0631112
UnemploymentRate						
L2.	-.1414684	.2399151	-0.59	0.555	-.6116935	.3287566
Inflation						
L2.	.4907697	.0974692	5.04	0.000	.2997337	.6818058
_cons	.0241035	.0186693	1.29	0.197	-.0124877	.0606948

Table 13: VAR

Eigenvalue stability condition

Eigenvalue	Modulus
1	1
1	1
1	1
.5262563	.526256
.3210579 + .2239265*i*	.391435
.3210579 - .2239265*i*	.391435
-.1981083 + .3360918*i*	.390134
-.1981083 - .3360918*i*	.390134

The VECM specification imposes 3 unit moduli.

Table 14: Stability Test Results

Kurtosis test

Equation	Kurtosis	chi2	df	Prob > chi2
D_DefaultRate	3.3838	0.252	1	0.61588
D_GDPConstantPrices	3.3916	0.262	1	0.60875
D_UnemploymentRate	2.2427	0.980	1	0.32224
D_Inflation	2.477	0.467	1	0.49428
ALL		1.961	4	0.74298

Table 15: Kurtosis Test Results

Skewness test

Equation	Skewness	chi2	df	Prob > chi2
D_DefaultRate	.66971	3.065	1	0.08000
D_GDPConstantPrices	.28507	0.555	1	0.45616
D_UnemploymentRate	.13596	0.126	1	0.72228
D_Inflation	.52306	1.870	1	0.17153
ALL		5.616	4	0.22973

Table 16: Skewness Test Results

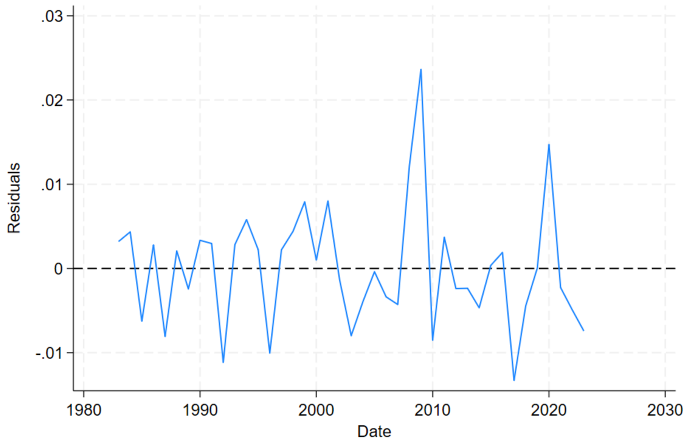


Figure 12: VECM Residuals

Results from **IRF**

Step	(1) irf	(2) irf	(3) irf	(4) irf
0	**1**	**0**	**0**	**0**
1	**1.18341**	**-.028826**	**-.928546**	**.163837**
2	**1.09792**	**.054813**	**-1.10941**	**.163446**
3	**.989142**	**.013585**	**-.758163**	**.196385**
4	**.963093**	**.00036**	**-.563478**	**.246821**
5	**.961567**	**.004162**	**-.448327**	**.272785**
6	**.964051**	**.002916**	**-.375757**	**.287865**
7	**.968187**	**.003216**	**-.345317**	**.296631**
8	**.970616**	**.003974**	**-.331574**	**.300583**
9	**.971649**	**.004101**	**-.324359**	**.302524**
10	**.972148**	**.004182**	**-.321016**	**.303542**

(1) irfname = **IRF**, impulse = **DefaultRate**, and response = **DefaultRate**.
(2) irfname = **IRF**, impulse = **GDPConstantPrices**, and response = **DefaultRate**.
(3) irfname = **IRF**, impulse = **UnemploymentRate**, and response = **DefaultRate**.
(4) irfname = **IRF**, impulse = **Inflation**, and response = **DefaultRate**.

Table 17: Impulse Response Functions on Default Rate

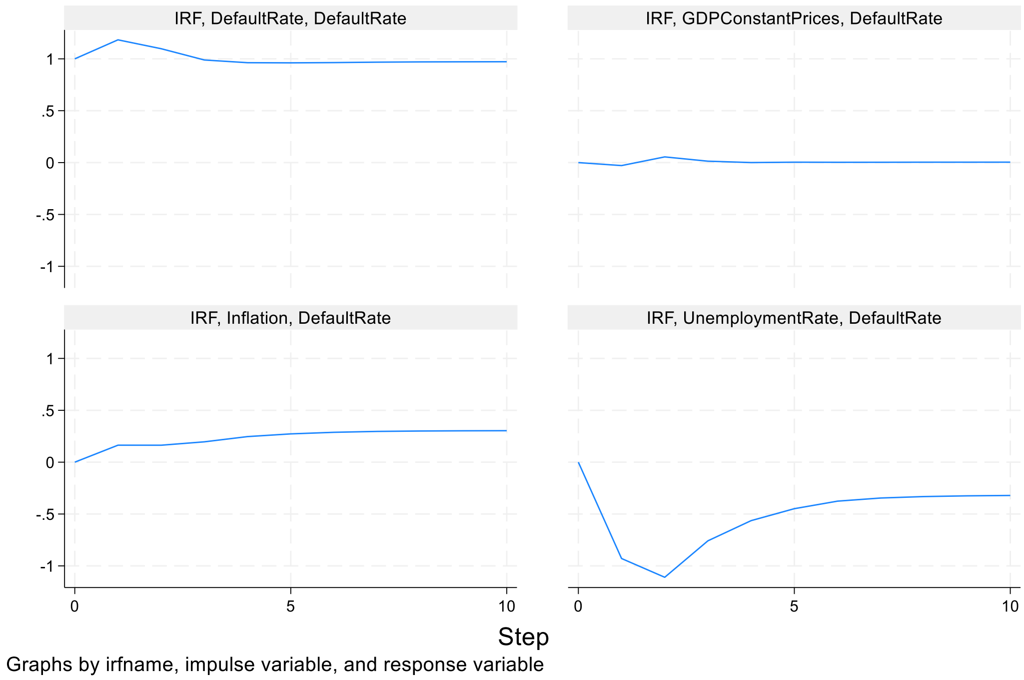
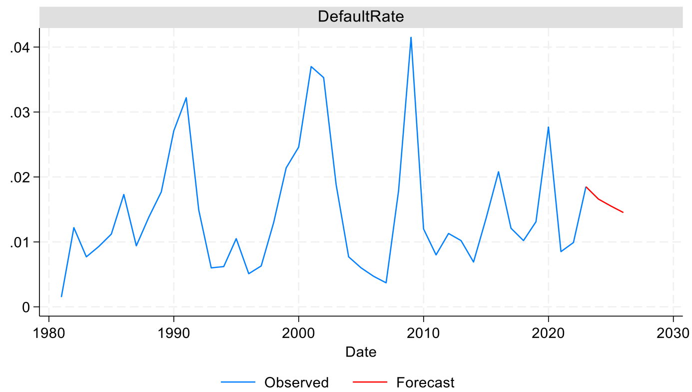
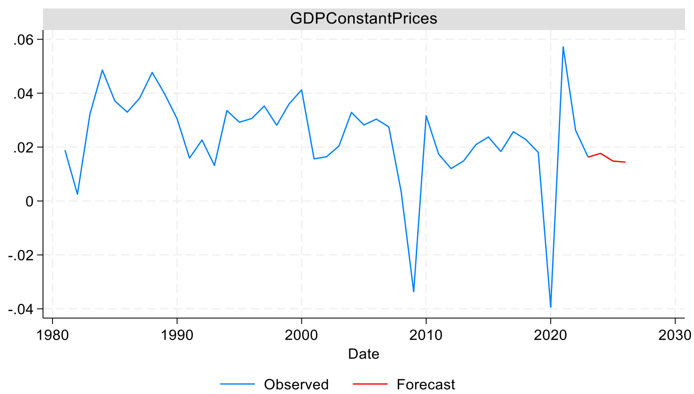
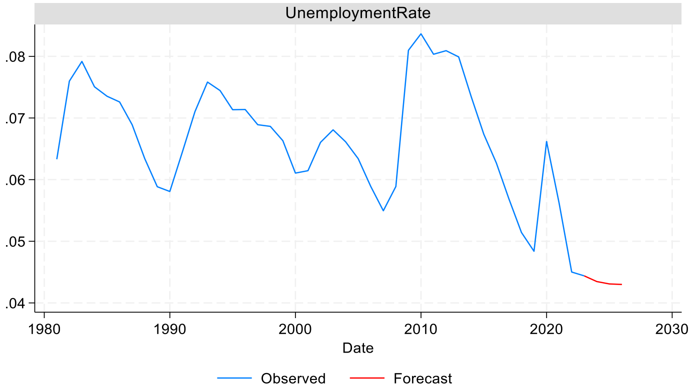


Figure 13: Impulse Response Functions on Default Rate







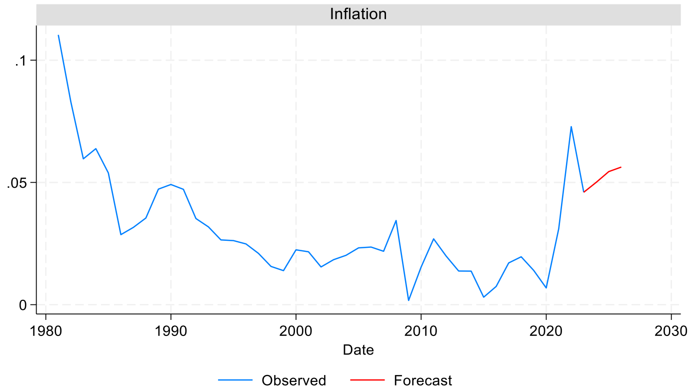


Figure 14: Time Series including Forecasted Values