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TRADE-OFF BETWEEN ASSET ALLOCATION AND SOLVENCY II REQUIREMENTS – THE CASE OF A PORTUGUESE LIFE INSURER

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OCTOBER - 2024

ABBREVIATIONS AND ACRONYMS

BE – Best Estimate of Technical Provisions

BOF – Basic Own Funds

BSCR – Basic Solvency Capital Requirement

CQP – Convex Quadratic Programming

EEA – European Economic Area

EIOPA – European Insurance and Occupational Pension Authority

LAC – Adjustment for the loss-absorbing capacity of technical provisions and deferred taxes

OECD – Organisation for Economic Co-operation and Development

SCR – Solvency Capital Requirement

ABSTRACT

This study examines the trade-off between asset allocation and Solvency II requirements in the context of a specific Portuguese life insurer. Under the Solvency II regime, life insurers must maintain sufficient capital to cover risks, particularly market risk, which is significantly impacted by asset allocation decisions. The challenge lies in balancing profitability with the necessity to maintain a strong solvency position.

To address this, an optimization model was developed to derive optimized asset allocation strategies that aim to maximize the insurer's profitability while adequately accounting for the Solvency Capital Requirement for the Market Risk sub-module (SCR Market). The model also incorporates investment limits to ensure that the asset allocations align with the life insurer's investment strategy.

The results demonstrate that the life insurer's profitability can be increased while maintaining the same SCR Market value by reallocating toward more capital-efficient asset classes, such as corporate bonds and property. However, despite their higher return potential, equities were excluded from the optimized portfolio due to their significant impact on the SCR Market. An efficient frontier analysis further illustrates the trade-off between profitability and solvency, showing how asset allocation shifts to maximize profitability as different solvency positions are targeted.

This work provides valuable insights for life insurers, demonstrating how optimized asset allocation strategies focused on capital-efficient assets can improve profitability while still maintaining strong solvency positions.

KEYWORDS: Asset Allocation; Solvency II; Market Risk; Capital Efficiency

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ACKNOWLEDGMENTS

My journey at ISEG | Lisbon School of Economics & Management began with a bachelor's degree in Economics and has now come to an end with a master's degree in Actuarial Science. This has been a very happy and important chapter in my life, and I want to thank everyone who contributed to making it happen.

Firstly, I want to express my gratitude to my company for providing the useful data that made this study possible. A special thanks to Ana Brito for accepting to be my supervisor and for encouraging me throughout this process. I also want to extend my appreciation to my colleagues Gonçalo Mendes and Mariana Godinho, who shared this journey of the master's degree with me and made it much easier.

Additionally, I would like to extend a very special thanks to my professor and supervisor, Onofre Simões, for always encouraging me and being available to help. His dedication and effort in guiding me through this project were greatly appreciated.

Lastly, and most importantly, I would like to thank my parents for everything. Their education, support, and sacrifices in helping me pursue my goals have led me to where I am today.

1. INTRODUCTION

Solvency II has transformed the regulatory environment in the European insurance industry. Introduced in 2016, Solvency II is a regulatory framework aimed at ensuring financial stability and protecting policyholders. This framework is built around three pillars: quantitative requirements (Pillar I), qualitative requirements (Pillar II), and disclosure and transparency (Pillar III). Pillar I includes the Solvency Capital Requirement (SCR), a measure designed to ensure that insurers hold enough capital to fulfil their obligations with a 99.5% confidence level, over a one-year time horizon.

The SCR can be calculated using either a standard formula provided by the European Insurance and Occupational Pensions Authority (EIOPA) or through an internal model specific to the insurer's risk profile. The SCR standard formula is divided into several risk modules. For life insurers, the market risk module is particularly significant, and it determines the capital required to absorb losses arising from fluctuations in financial markets.

Asset allocation is a very important aspect for life insurers, particularly under the Solvency II regime, where it has a direct influence on the SCR for the market risk module. The way in which assets are allocated across the different classes – such as bonds, equities, and property – significantly affects the life insurer's solvency position, as each asset class has a different impact on the SCR Market's value. For instance, riskier assets like equities offer higher potential returns but also lead to larger capital requirements, increasing the SCR Market. Conversely, more stable assets like bonds have a lower impact on the SCR Market but may limit the portfolio's overall profitability.

In this framework, the interaction between asset allocation and Solvency II requirements is highly relevant. The composition of the portfolio not only determines the life insurer's profitability but also influences its capacity to meet the regulatory capital requirements. Therefore, an optimized asset allocation strategy must carefully consider its impact on the SCR Market, aiming to maximize returns while maintaining a strong solvency position. The challenge for life insurers is to balance profitability and capital efficiency, ensuring that the chosen asset allocation does not disproportionately increase the SCR Market, which could compromise their regulatory compliance.

This project aims to study the trade-off between asset allocation and Solvency II requirements within the context of a Portuguese life insurer. The two main objectives of this study are: (i) to analyse how the life insurer can optimize its asset allocation to improve capital efficiency and increase profitability while maintaining a strong solvency position, specifically focusing on the SCR Market; and (ii) to explore the trade-off between profitability and solvency, analysing the adjustments required in asset allocation to maximize profitability at different solvency levels.

To achieve these objectives an optimization model is constructed, inspired by the works of Kouwenberg (2017, 2018), who searches optimized asset allocation strategies under the Solvency II framework, by both maximizing the expected return on the insurer's own funds and assuring compliance with the SCR Market. Building on this foundation, the current study contributes to the literature by applying the optimization model to a real Portuguese life insurer, adapting it to the specific context of the company. In addition to following Kouwenberg's methodology, this study introduces an important extension by incorporating investment limits for the different asset classes into the model's constraints. This adjustment ensures that the optimized asset allocation aligns with the company's strategic investment objectives, making the model more applicable to real-world scenarios.

While Kouwenberg's work serves as the primary foundation for this project, three other studies have significantly contributed to a broader understanding of the topic, since they also explore the impact of Solvency II on asset allocation and capital efficiency for insurers: Höring (2013), who investigates how Solvency II might influence insurer's investment portfolios by comparing its market risk capital requirements with the Standard & Poor's rating model; Braun et al. (2015), who incorporate the SCR Market into a constrained portfolio optimization framework, applying classical portfolio theory to identify efficient frontiers for the asset allocation that are admissible under the Solvency II requirements, for an exogenously given amount of the insurer's own funds. Their study further includes investment limits within the optimization process; Escobar et al. (2018), who explore the implications of Solvency II on the investment strategies using a two-step approach to approximate optimal asset allocations within an expected utility framework focused on the SCR Market.

The remainder of this paper is organized as follows:

- Chapter 2 covers the Solvency II regulatory framework, with a particular focus on the SCR standard formula and the market risk submodule.

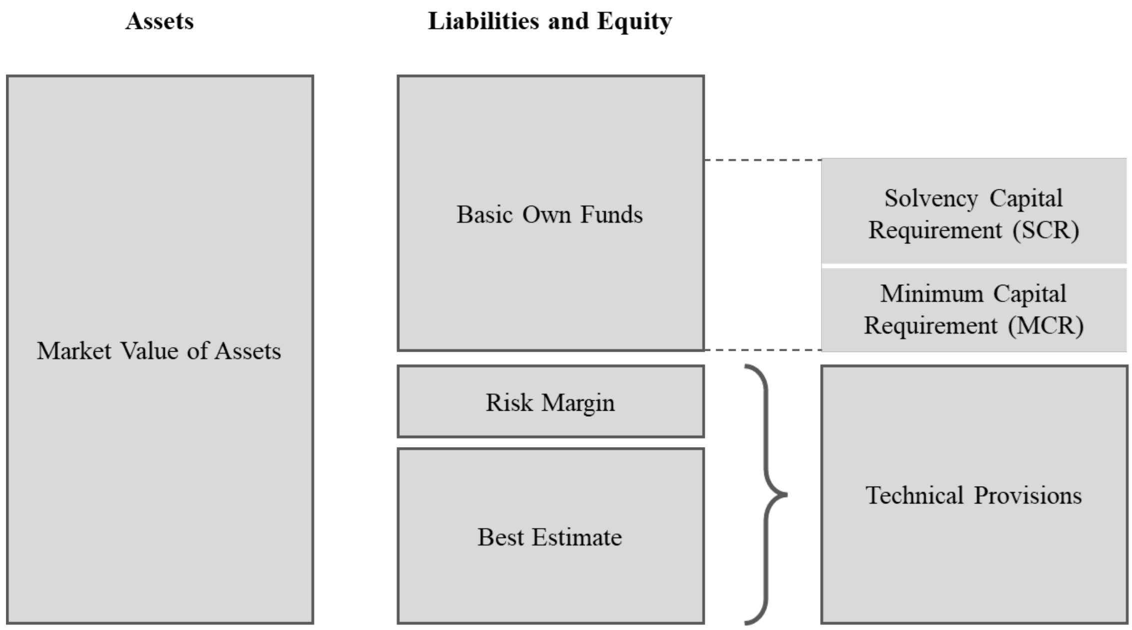
- Chapter 3 details the optimization problem and describes the optimization model.
- Chapter 4 outlines the data and methodology used to implement the optimization model.
- Chapter 5 presents the results and the respective analysis.
- Chapter 6 summarizes the main findings, discusses their practical implications, and identifies the limitations of the study, along with suggestions for future research.

2. SOLVENCY II

Solvency II is a comprehensive regulatory framework for European insurance companies that has been in effect since January 1, 2016. Its aim is to create a more unified and efficient regulatory environment, thereby improving policyholder protection and contributing to the financial system stability. This framework is structured around three pillars. The first pillar requires insurance companies to meet quantitative capital requirements based on a market-consistent valuation of their assets and liabilities. The second pillar focuses on qualitative requirements, such as the insurer's governance and risk management system. The third pillar addresses transparency and disclosure.

Given the aim of this study, described in the Introduction, it is important to provide some more detail about Pillar 1, which specifically covers all the components of the Solvency II economic balance sheet, shown in Figure 1.

FIGURE 1 – Solvency II Economic Balance Sheet.



A crucial component of this pillar is the valuation of assets and liabilities that must be market-consistent, as mentioned above. This means that the assets should be valued 'at the amount for which they could be exchanged between knowledgeable willing parties' The European Parliament (2009), which corresponds to their market value. In addition, liabilities should be valued 'at the amount for which they could be transferred, or settled, between knowledgeable willing parties' The European Parliament (2009), which corresponds to the sum of a best estimate and a risk margin.

Another essential component of the first pillar is the Solvency Capital Requirement (SCR), that sets a lower bound for the basic own funds (BOF) of an insurance company, SCR is the amount of capital it must hold to cover its risks and ensure financial stability; If this threshold is breached, regulatory intervention may be triggered.

The SCR is defined as the Value-at-Risk of the basic own funds, subject to a confidence level of 99.5% over a one-year time horizon, according to The European Parliament (2009).

The SCR can be calculated using an internal model developed by the insurance company or a standard formula provided by the regulator, EIOPA. This study adopts the standard formula due to its regulatory consistency, diminished complexity, and lesser resource demands relative to internal models, coupled with a more straightforward regulatory approval process and enhanced transparency and simplicity.

2.1.Solvency Capital Requirement – Standard Formula

The SCR under the standard formula is calculated by adding together three key components: the Basic Solvency Capital Requirement (BSCR), the Capital Requirement for Operational Risk ($SCR_{Operational}$) and the Adjustment for the loss-absorbing capacity of technical provisions and deferred taxes (LAC):

$$SCR = BSCR + SCR_{Operational} + LAC \tag{1}$$

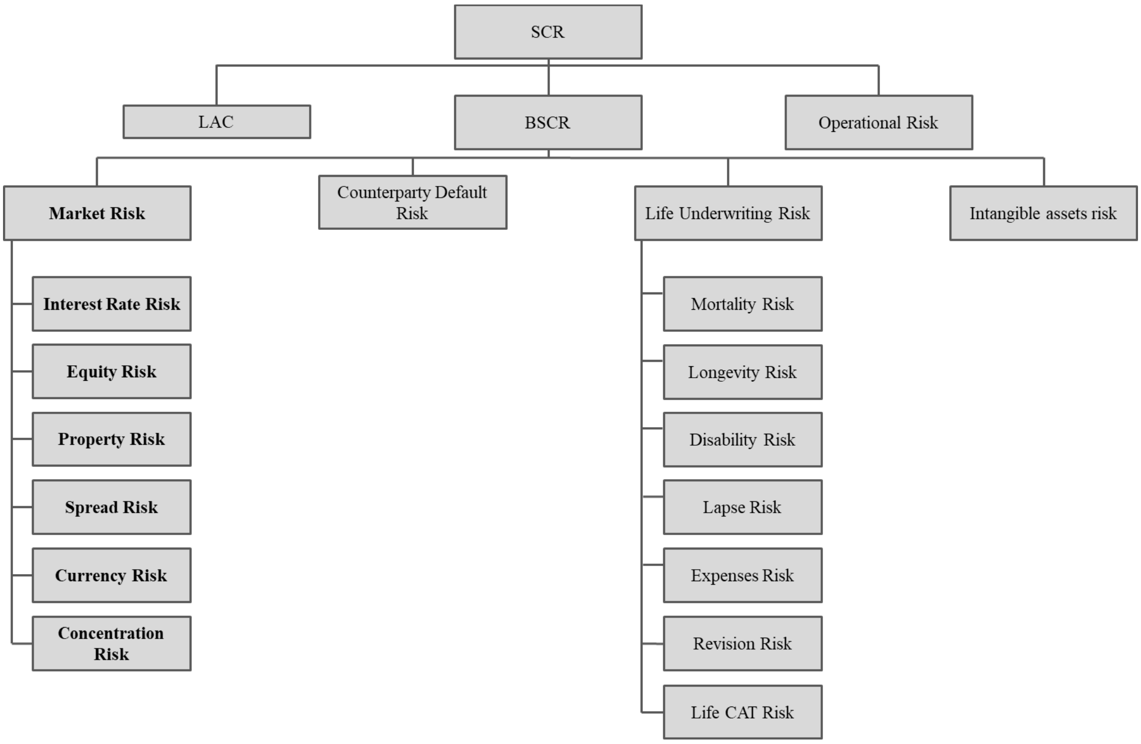
The BSCR itself is derived through a modular approach. It is divided into various risk modules, each composed of sub-modules. For each risk module a capital requirement is determined as the aggregation of its sub-modules' capital requirements. The capital requirements for each risk module are then aggregated to form the BSCR. The formula for BSCR is as follows:

$$BSCR = \sqrt{\sum_{i,j} \rho_{i,j} \times SCR_i \times SCR_j} + SCR_{intangibles} \tag{2}$$

In equation (2), SCR_i and SCR_j denote the capital requirements for risk module i and risk module j, respectively. The term $\rho_{i,j}$ is the correlation parameter between risk modules i and j, and can be found in the BSCR's correlation matrix provided in the appendices as Table 7. Additionally, the sum covers all possible combinations of the risk modules i and j.

For life insurance companies specifically, the following risk modules are considered: Market Risk module, Counterparty Default Risk module, Life Underwriting Risk module, as well as the Intangible Assets Risk, as it is illustrated in Figure 2.

FIGURE 2 - SCR Standard Formula Structure for a Life Insurance Undertaking.



For the purpose of this study, only the market risk module will be under consideration. This exclusive focus on market risk is justified both by its predominant impact – accounting for around 70% of the overall BSCR for European life insurance companies, according to EIOPA (2023) – and by practical constraints, such as the time available to conduct the research and the maximum length allowed for the thesis. By concentrating solely on market risk and excluding other risk modules, such as life underwriting risk, the study enables a detailed analysis of the direct influences of market risk on asset allocation decisions.

2.1.1. Market Risk module

Market risk, a critical factor in the SCR for life insurance undertakings as noted above, have a fundamental role in this study. Hence, a more detailed explanation of this topic is required.

The market risk module addresses the risks that life insurance undertakings face due to fluctuations in the financial markets. This risk module is composed of several sub-modules:

- Interest rate risk: reflects the risk of loss due to the impact of changes in the term structure of interest rates in the value of assets and liabilities.
- Equity risk: reflects the risk of loss due to changes in the market prices of equities.
- Property risk: reflects the risk of loss due to changes in the market prices of properties.
- Spread risk: reflects the risk of loss due to changes in the creditworthiness of the issuers of securities held in the insurer's investment portfolio, reflected in changes on the underlying credit spreads.
- Currency risk: reflects the risk of loss due to the impact of changes of currency exchange rates in the value of assets and liabilities.
- Concentration risk: reflects the risk of loss due to the reduced level of diversification of the asset portfolio, which means an increased exposure to individual counterparties.

These risk sub-modules collectively impact the market-consistent valuation of assets and liabilities, ultimately affecting the SCR. Understanding and managing market risk is therefore crucial for life insurers to ensure compliance with the Solvency II regulations and maintain financial stability.

Before exploring how capital requirements for the market risk module and its sub-modules are calculated, it is necessary to first establish the notational framework that will be utilized throughout this study.

Notation

For asset allocation purposes, the assets in the portfolio of the life insurer are categorized into six distinct classes. The rationale behind selecting these particular asset classes will be elaborated further, in the 'Optimization Problem' chapter. The asset classes, along with their corresponding notations, are as follows:

- A_{gov}: represents the market value of the government bonds issued by European Economic Area (EEA) countries, which have maturities of more than one year.
- A_{corp}: represents the market value of the corporate bonds.
- A_{eq}^1: represents the market value of type 1 equities, i.e. equities listed in regulated markets of the EEA and Organisation for Economic Co-operation and Development (OECD) members.
- A_{eq}^2: represents the market value of type 2 equities, i.e. equities listed in markets of other countries, not listed equities, and investment funds.
- A_{prop}: represents the market value of properties, which include listed real estate, direct property investments, and the value of office buildings owned by the insurance undertaking for its own use.
- $A_{t-bills}$: represents the market value of treasury bills issued by EAA countries, which have maturities of up to one year.

The total market value of assets, denoted as A, is given by:

$$A = \sum_i A_i \, , \tag{3}$$

where $i \in \{gov, corp, eq^1, eq^2, prop, t-bills\}$.

It is important to note that the categorization of asset classes not only relates to asset allocation strategies but is also aligned with the capital charge methodologies outlined in the standard formula, following Kouwenberg (2017). All asset classes are charged in the Market Risk module.

Regarding the liabilities, in this thesis only the best estimate of technical provisions (L_{BE}) is considered. Once again, this assumption will be further explained in the 'Optimization Problem' chapter.

Finally, regarding the basic own funds, the concept will be simplified to represent only the difference between assets and liabilities. This simplification aligns with the study's focus on asset allocation. Remark that BOF under Solvency II is a more complex concept, encompassing classifications into tiers (Tier 1, Tier 2, and Tier 3), and consisting of both excess of assets over liabilities and subordinated debt. Nevertheless, for our purpose is appropriate that:

$$BOF = A - L_{BE}. \tag{4}$$

With the notations for assets, liabilities and the basic own funds now clearly defined, the next step is to explain how the capital requirements for each sub-modules are calculated and, finally, how to aggregate them to obtain the capital requirement for the Market Risk module.

Interest Rate Risk

The capital requirement for interest rate risk (SCR_{IR}) is equal to the change in the value of the BOF resulting from the most severe of two scenarios:

- IR_{up}: results from the revaluation of the whole balance sheet using an interest rate term structure subject to a stipulated upward shock.
- IR_{down}: results from the revaluation of the whole balance sheet using an interest rate term structure subject to a stipulated downward shock.

For the sake of simplicity and applicability, this study adopts a duration-based approach to approximate the effects of the upward and downward interest rate shocks, in line with methodologies used by Höring (2013) and Kouwenberg (2017).

These interest rate fluctuations impact both the liabilities, in this study specifically the best estimate of technical provisions (L_{BE}), and the assets side, affecting the values of government bonds (A_{gov}), corporate bonds (A_{corp}), and treasury bills ($A_{t-bills}$).

In this context, Dur_{BE} represents the duration of the best estimate of technical provisions, while Dur_{gov}, Dur_{corp} and $Dur_{t-bills}$ denote the duration of government bonds, corporate bonds, and treasury bills, respectively. Furthermore, the parameters Δ_{up}^{IR} and Δ_{down}^{IR} are the parallel upward and downward shocks to the interest rate term structure.

Therefore, the SCR_{IR} calculation is as follows:

$$SCR_{IR} = max\{IR_{up}, IR_{down}\} \Leftrightarrow$$

$$\Leftrightarrow SCR_{IR} = \max\{\Delta_{up}^{IR}\big(Dur_{gov}A_{gov} + Dur_{corp}A_{corp} + Dur_{t-bills}A_{t-bills} - Dur_{BE}L_{BE}\big),$$

$$\Delta_{down}^{IR}(Dur_{BE}L_{BE} - Dur_{gov}A_{gov} - Dur_{corp}A_{corp} - Dur_{t-bills}A_{t-bills})\}. \tag{5}$$

The determination of whether the upward or downward shock constitutes the most severe scenario is contingent upon the side of the balance sheet that exhibits greater interest rate sensitivity, as indicated by duration. In general, if liabilities have a longer (shorter) duration than assets, then it is the downward (upward) shock scenario that determines the capital requirement.

Equity Risk

The capital requirement for equity risk (SCR_{eq}) is calculated through the aggregation of the capital charges for the two different types of equities (SCR_{eq}^1 and SCR_{eq}^2).

The SCR_{eq}^1 is given by the change in the value of the BOF resulting from an instantaneous decrease in the value of type 1 equities equal to 39%[1], which can be calculated directly by applying this shock to the value of the type 1 equity investments (A_{eq}^1):

$$SCR_{eq}^1 = 39\% \times A_{eq}^1 \,. \tag{6}$$

Additionally, the SCR_{eq}^2 is computed in the same way, but the shock that is applied to type 2 equities is equal to 49%[2]. Thus:

$$SCR_{eq}^2 = 49\% \times A_{eq}^2 \,. \tag{7}$$

It is worth mentioning that the shock values of 39% and 49% are not static, as they are subject to a monthly adjustment known as the Symmetric Adjustment. This adjustment, which can increase or decrease these shocks by up to 10 percentage points, is based on the relative position of equity indices compared to their historical averages and primarily reflects short-term market sentiments. This study will not consider the Symmetric Adjustment; this decision is driven by the aim to assess asset allocation strategies in the context of more stable and enduring market trends, rather than focusing on the volatility indicated by short-term monthly fluctuations in equity indices.

Finally, the SCR_{eq} is calculated as follows, according to the European Commission (2015):

$$SCR_{eq} = \sqrt{\left(SCR_{eq}^1\right)^2 + 2 \times 0{,}75 \times SCR_{eq}^1 \times SCR_{eq}^2 + \left(SCR_{eq}^2\right)^2} \,. \tag{8}$$

Property Risk

The capital requirement for property risk (SCR_{prop}) is equal to the change in the value of the BOF resulting from an instantaneous decrease in the value of property investments (A_{prop}) equal to 25%[3]. Thus:

[1] According to Article 169(1)b of the European Commission (2015)
[2] According to Article 169(2)b of the European Commission (2015)
[3] According to Article 174 of the European Commission (2015)

$$SCR_{prop} = 25\% \times A_{prop} \ . \tag{9}$$

Spread Risk

The capital requirement for spread risk (SCR_{spread}) is calculated as the sum of the capital charges for three different types of assets: bonds and loans, securitisations, and credit derivatives. The capital charges for securitisations and credit derivatives are not considered as these asset classes are not included in this study.

The capital charge for bonds and loans is given by the sum of the capital requirements for each individual bond or loan. Additionally, the individual capital requirements are calculated by applying a specific risk factor (stress_i) to the value of each bond or loan i. The $stress_i$ depends on the modified duration and credit rating of the bond or loan.

In this study, only the corporate bonds (A_{corp}) are charged in the spread risk sub-module, as government bonds issued by EEA governments (which includes treasury bills) have no capital charge according to Article 180(2) of the European Commission (2015).

Following the methodology used by Kouwenberg (2017), the SCR_{spread} is given by:

$$SCR_{spread} = \Delta_{corp} \times A_{corp} \ , \tag{10}$$

where Δ_{corp} represents the weighted average risk factor of the corporate bond portfolio.

Currency Risk

The capital requirement for currency risk (SCR_{curr}) is assumed to be zero in this study. Currency risk is mainly relevant for portfolios with significant foreign currency exposure. This study focuses on a Portuguese life insurer whose assets and liabilities are primarily denominated in euros, thereby minimizing any exposure to currency risk. Moreover, any residual currency risk can be managed through standard hedging strategies widely used in the insurance industry, which further diminishes its influence on the overall SCR_{Market}. Therefore, incorporating currency risk into this study would introduce unnecessary complexity without providing substantial additional insights, supporting its exclusion.

Concentration Risk

In this study, the capital requirement for concentration risk (SCR_{conc}) is assumed to be zero. This assumption is supported by Article 187(3) of the European Commission (2015), which states that government bonds issued by EEA governments are exempt from the concentration risk capital charge. Additionally, it is assumed that the asset allocation strategy and investment mix are designed to follow broadly diversified benchmarks for equities, corporate bonds, and property investments, ensuring that exposure to any single issuer remains below the threshold that would trigger a capital charge.

Given this context, delving into the calculation of SCR_{conc} is deemed unnecessary for this study, thereby simplifying the overall calculation of the capital requirement for the Market Risk module without diminishing its accuracy or relevance.

Aggregation of Market Risk

As mentioned above, the calculation of the capital requirement for Market Risk (SCR_{Market}) is achieved by aggregating the capital requirements of its sub-modules: interest rate, equity, property, spread, currency, and concentration risks. This aggregation is mathematically represented by the formula:

$$SCR_{Market} = \sqrt{\sum_{i,j} \rho_{i,j} \times SCR_i \times SCR_j} \ , \tag{11}$$

where SCR_i and SCR_j are the capital requirements for the sub-module i and sub-module j, respectively. The factor $\rho_{i,j}$ denotes the correlation coefficient between these sub-modules and can be found in the market risk correlation matrices presented in the appendices as Table 8 and Table 9, depending on whether the interest rate risk is determined by the downward or upward shock, respectively. Additionally, the sum covers all possible combinations of the sub-modules i and j.

3. OPTIMIZATION PROBLEM

3.1.Introduction

The implementation of Solvency II has transformed the landscape for insurance companies, compelling them to balance regulatory compliance with profitability. Strongly inspired by Kouwenberg (2017, 2018), who examined strategic asset allocations within this regulatory framework, our study builds upon these critical insights to derive an asset allocation strategy for a concrete life insurance company, that seeks to maximize profitability on one hand and is well-suited to the demands of Solvency II, on the other.

3.2.Problem Definition

The core problem this study addresses emerges directly from the inherent trade-off in asset allocation under Solvency II, which balances the pursuit of higher returns against the need to manage capital to sufficiently cover the SCR. Life insurers want to enhance the profitability of their BOF, but they are simultaneously constrained by the necessity to maintain these funds at levels sufficient to meet the SCR, and these are (to a certain extent) two conflicting goals.

Furthermore, the SCR's value is highly dependent on the asset allocation chosen, particularly through the market risk module. As detailed in Chapter 2, this module evaluates the risk inherent in each asset class and assigns capital charges accordingly, which in turn influences the overall SCR. Assets with higher volatility and risk, such as equity or real estate, have higher capital charges contributing to a higher SCR. Conversely, more stable investments like government bonds carry lower risk and, therefore, lower capital charges, affecting less the SCR. Thus, it is crucial to also consider the impact that asset allocation will have on the SCR.

Therefore, our optimization problem consists in determining an asset allocation that increases the BOF as much as possible, on one side, and sets levels of the SCR for the different market risk submodules that fulfil Solvency II constraints, on the other.

A possible way to address this two-way impact - the influence of the SCR on the asset allocation and vice-versa - is to incorporate the SCR for the market risk module (SCR_{Market}) into the optimization problem with an associated penalty term, following the approach outlined by Kouwenberg (2017, 2018). This approach not only aligns with the objective of maximizing the expected return on the life insurer's assets but also ensures that the SCR for the market risk module is adequately considered.

Having established the dual impacts of SCR and asset allocation on each other, the next step involves quantitatively modelling these interactions to find the best possible balance. This modelling underpins the development of the optimization model in this study, which is structured to align with regulatory constraints while seeking to maximize financial returns.

3.3. Optimization Model

As mentioned above, we follow the same approach as Kouwenberg (2017, 2018) in order to preserve the convexity and solvability of the problem and to ensure computational efficiency and clarity in the derivation of the solution.

3.3.1. Assumptions

To simplify the complex relationship between assets and liabilities, while ensuring that the optimization model remains both practical and compliant with regulatory standards, certain assumptions were made. These assumptions will be justified and explained to facilitate a deeper understanding of their impact on the outcomes and applicability of the study.

Asset Class Selection

The asset classes chosen for this study - government bonds issued by EEA countries, corporate bonds, equities, property, and treasury bills issued by EEA countries - are selected to align with the typical investment portfolio composition of insurers operating under the Solvency II regime. According to EIOPA (2023), the government bonds, corporate bonds, equities, and property collectively account for almost 90% of insurer's investments, highlighting their predominance in the industry. Furthermore, the well-defined capital requirements under the Solvency II standard formula for each of these asset classes further ensure that the model aligns with the regulatory framework, enhancing its applicability to real-world scenarios.

These asset classes include both liquid and illiquid assets, providing a balanced overview of the typical investments found in life insurer's investment portfolios. Liquid assets such as government bonds and treasury bills, contribute to stability and quick access to funds, which are essential for managing short-term liabilities and cash flow requirements. Treasury bills, in particular, are chosen over typical cash instruments because they are among the safest and most liquid assets available. Backed by government guarantees, they carry no default risk and have short maturity periods, ensuring immediate liquidity. This makes treasury bills an excellent choice for covering immediate liabilities while minimizing the opportunity cost of holding cash. In contrast, corporate bonds, equities, and property, while associated with higher returns, entail greater liquidity risks and generally require longer investment horizons, which are important for long-term solvency and profitability. Additionally, the extensive availability of historical data on returns for these asset classes facilitates more accurate and reliable modelling of expected returns.

Furthermore, the decision to exclusively consider government bonds and treasury bills issued by EEA countries is due to their advantageous treatment under the Solvency II standard formula. As described in Chapter 2, these bonds are exempt from spread risk, making them particularly attractive for life insurers' investment portfolios. This strategic choice simplifies the investment landscape and aligns with the objective of efficiently managing the capital requirement.

Liabilities

This study makes two assumptions about the liabilities: first, that they are represented solely by the best estimate of technical provisions (BE), excluding the risk margin and other liabilities; and, second, that the BE is considered to be constant. These assumptions are necessary to focusing the analysis on the pivotal interplay between asset allocation and the SCR_{Market}.

The rationale for limiting the scope of liabilities to the BE lies in its predominant role under Solvency II. This simplification makes way to a clearer analysis by focusing on the BE, which comprises almost 85% of the total liabilities of life insurers under Solvency II, according to EIOPA (2024). This assumption enhances the model's clarity and the relevance to its purpose, since the risk margin and remaining liabilities do not have an impact on the SCR_{Market}.

Furthermore, assuming the BE remains constant is a strategic choice that stabilizes the model's parameters. This approach simplifies the assessment, while enables a focus on asset management and a direct evaluation of its effects on regulatory capital needs and vice-versa, without the "noise" of varying liability estimates.

Despite these assumptions, the model adequately addresses the interest rate sensitivity of the BE and its contribution to the SCR_{Market}, through the interest rate risk sub-module. This inclusion is important as it ensures that the model captures the dynamics of how interest rate fluctuations influence the insurer's financial stability, providing a simplified view of the strategic interactions between assets and liabilities, within the regulatory framework. This balanced approach maintains the practical utility of the study while aligning closely with Solvency II regulatory framework, ensuring that the insights derived are both actionable and compliant.

3.3.2. Decision Variables

The ten decision variables listed next are defined to reflect the allocation of capital across the different asset classes considered within the life insurer's investment portfolio, as well as the associated capital requirements for the different sub-modules of market risk. They are expressed in million euros.

- A_{gov}: The market value of the assets allocated to government bonds issued by EEA governments, excluding treasury bills.
- A_{corp}: The market value of the assets allocated to corporate bonds.
- A_{eq}^{1}: The market value of the assets allocated to equity type 1, representing equities listed on regulated markets of EEA and OECD members.
- A_{eq}^{2}: The market value of the assets allocated to equity type 2, primarily consisting of not listed equities, equities listed on markets outside the EEA and OECD, and investment funds where a look-through approach is not applied – according to Article 168(3) of European Commission (2015).
- A_{prop}: The market value of the assets allocated to property investments.
- $A_{t-bills}$: The market value of the assets allocated to treasury bills issued by EEA governments.
- s_{IR}: The solvency capital requirement for the interest rate risk sub-module.
- s_{eq}: The solvency capital requirement for the equity risk sub-module.
- s_{prop}: The solvency capital requirement for the property risk sub-module.
- s_{spread}: The solvency capital requirement for the spread risk sub-module.

3.3.3. Objective Function

The objective function of the model aims to maximize the expected value of a function $f(\boldsymbol{a}, \boldsymbol{s})$, where $\boldsymbol{a}$ represents the asset allocation decision variables, and $\boldsymbol{s}$ represents the SCR decision variables for the market risk sub-modules. The objective is to maximize the expected return on the assets while adequately managing the SCR. The inclusion of a penalty term based on the SCR_{Market} ensures that the model appropriately weighs the impact of the SCR_{Market} on the insurer's portfolio.

$$\max_{\boldsymbol{a},\boldsymbol{s}} E[f(\boldsymbol{a},\boldsymbol{s})] = E\left[\boldsymbol{r}_{\boldsymbol{A}}^{\boldsymbol{T}}\,\boldsymbol{a}\right] - \gamma \boldsymbol{s}^{\boldsymbol{T}}\boldsymbol{R}\boldsymbol{s} \tag{12}$$

Where:

- $E\left[\boldsymbol{r}_{\boldsymbol{A}}^{\boldsymbol{T}}\,\boldsymbol{a}\right]$ is the expected return on BOF, with $\boldsymbol{r}_{\boldsymbol{A}} = \left(r_{gov}, r_{corp}, r_{eq}^{1}, r_{eq}^{2}, r_{prop}, r_{t-bills}\right)^{T}$ and $\boldsymbol{a} = \left(A_{gov}, A_{corp}, A_{eq}^{1}, A_{eq}^{2}, A_{prop}, A_{t-bills}\right)^{T}$.
- $\gamma \boldsymbol{s}^{T}\boldsymbol{R}\boldsymbol{s}$ is the penalty term that accounts for the SCR_{Market}. Here, γ is a penalty parameter that adjusts the trade-off between maximizing returns and managing the SCR, $\boldsymbol{s} = \left(s_{IR}, s_{eq}, s_{prop}, s_{spread}\right)^{T}$ and $\boldsymbol{R}$ is the correlation matrix.

3.3.4. Constraints

The objective in (12) is subject to a number of constraints related to both Solvency II requirements and the investment strategy of the life insurer. These constraints can be broadly categorised into three groups: SCR constraints, investment constraints and non-negativity constraints.

SCR Constraints:

These constraints ensure that the SCR for each market risk sub-modules is appropriately inserted in the model. While some SCR constraints are directly aligned with the formulas presented in Chapter 2, other have been reformulated to preserve linearity.

- Interest Rate Risk Constraints:

$$s_{IR} \geq \Delta_{up}^{IR}\left(Dur_{gov}A_{gov} + Dur_{corp}A_{corp} + Dur_{t-bills}A_{t-bills} - Dur_{BE}L_{BE}\right) \tag{13}$$

$$s_{IR} \geq \Delta_{down}^{IR}\left(Dur_{BE}L_{BE} - Dur_{gov}A_{gov} - Dur_{corp}A_{corp} - Dur_{t-bills}A_{t-bills}\right) \tag{14}$$

- Equity Risk Constraint:

$$s_{eq} \geq \Delta_{eq}^{1}A_{eq}^{1} + \Delta_{eq}^{2}A_{eq}^{2} \tag{15}$$

- Property Risk Constraint:

$$s_{prop} \geq \Delta_{prop}A_{prop} \tag{16}$$

- Spread Risk Constraint:

$$s_{spread} \geq \Delta_{corp}A_{corp} \tag{17}$$

Investment Constraints:

These constraints ensure that the asset allocation adheres to the real life insurer's internal investment guidelines.

- Budget Constraint: This constraint ensures that the total allocation across all asset classes equals the total available funds (A).

$$\sum_{i \in \{gov, corp, eq^1, eq^2, prop, t-bills\}} A_i = A \tag{18}$$

- Investment Limits (Illiquid Assets and Corporate Bonds): These constraints cap the total allocation to illiquid assets (equities and property), and to corporate bonds.

$$A_{eq}^1 + A_{eq}^2 + A_{prop} \leq Max_{Illiquid} \times A \tag{19}$$

$$A_{corp} \leq Max_{corp} \times A \tag{20}$$

- Investment Limits (Government Bonds and T-bills): These constraints impose both upper and lower bounds to government bonds and treasury bills.

$$Min_{gov} \times A \leq A_{gov} \leq Max_{gov} \times A \tag{21}$$

$$Min_{t-bills} \times A \leq A_{t-bills} \leq Max_{t-bills} \times A \tag{22}$$

Non-Negativity Constraints:

These constraints ensure that the optimization model adheres to realistic investment practices, particularly in the insurance industry where the preference is for safer investments over speculative strategies. By enforcing non-negativity, the model prohibits short-selling and prevents negative SCR allocations.

$$\boldsymbol{a} \geq 0 \tag{23}$$

$$\boldsymbol{s} \geq 0 \tag{24}$$

3.3.5. Parameters

The model uses several parameters, most of which are explained in the 'Data and Methodology' chapter, along with the methods used to calculate their values. These include the durations and expected returns of each asset class, the value and duration of the BE, the shocks from the SCR Market Risk sub-modules, and the investment limits imposed on the asset classes. Additionally, the correlation matrix used in the SCR_{Market} penalty term can be found in the appendices as Table 8 or Table 9, depending on whether the interest rate risk is determined by the downward or upward shock, respectively.

An important element of the model is the penalty parameter (γ), which penalizes asset allocations that significantly increase the SCR_{Market}, ensuring that the model does not favour high-return asset allocations at the expense of excessive capital charges. The value of γ is calibrated to reflect the life insurer's risk appetite, ensuring a balance between profitability and the Solvency II requirements. By changing the value of this parameter, an efficient frontier of the Expected Return on Assets versus the Market Risk Solvency Ratio can be derived, as mentioned by Kouwenberg (2017).

4. DATA AND METHODOLOGY

4.1. Data Collection

4.1.1. Initial Asset Allocation

The data used in this study reflects the real asset allocation of a Portuguese life insurance company as of December 31, 2023. The asset classes included in this allocation – government bonds, corporate bonds, equity type 1, equity type 2, property and treasury bills – are the same used in the optimization model. Their market values, like the BE, are expressed in millions of euros.

4.1.2. Parameters Computation

To implement the model, several parameters must be computed, each derived from specific aspects of the financial environment as of December 31, 2023. While some parameters, such as durations and the spread shock, are derived from the initial asset allocation, others, like the interest rate shocks, depend solely on the prevailing market conditions of the period.

- Duration of Asset Classes:

The duration of each asset class – specifically government bonds, corporate bonds, and treasury bills – is a parameter associated to the interest rate risk sub-module. Our study uses the modified duration to measure the sensitivity of the asset class's value to changes in interest rates. This modified duration is determined by calculating the weighted average of the individual securities' modified durations within each asset class – for instance, see Broverman (2017).

- Best Estimate and Duration:

The BE is determined by summing the present value (PV) of the liabilities' cash flows over maturities from $t = 1$ to $t = 30$. These PVs are discounted using the EIOPA risk-free curve with a volatility adjustment, from December 2023. The duration of these cash flows, specifically their modified duration, is also calculated and used as a parameter associated to the interest rate risk sub-module.

- Interest Rate Shocks:

The interest rate shocks, both upward and downward, inserted in the optimization model are calculated following the methodology outlined in Articles 165 to 167 of the European Commission (2015). This approach applies to the EIOPA risk-free interest rate curve, including a volatility adjustment, as of December 2023. The data for these interest rate shocks is obtained from EIOPA's monthly publications, providing the required upward and downward shocked curves for a wide range of maturities, specifically from 1 to 30 years. These selected maturities correspond to the time frame of the liability cash flows of the BE, ensuring consistency across the model's parameters. The final shock parameters used in the optimization are the average of these individual shocks, offering a robust measure of interest rate risk in alignment with regulatory standards.

- Spread Shock for Corporate Bonds:

The spread shock for the corporate bonds used in the spread risk sub-module is calculated based on the methodology outlined in the European Commission (2015). This involves computing individual shocks for each corporate bond and then determining a weighted average shock for the entire portfolio.

- Investment Limits of the Asset Classes:

The investment limits are set according to the life insurer's internal investment guidelines, which are designed to ensure compliance with regulatory requirements and to manage risk exposure. These limits cap allocations to higher-risk asset classes, such as equities and real estate (illiquid assets), and impose constraints on corporate bond holdings. Additionally, limits are placed on government bonds and treasury bills to ensure sufficient liquidity and prevent over-reliance on low-return assets. The primary objectives of these limits are to control risk, maintain adequate diversification, and prevent excessive concentration in any single asset class. By adhering to these limits, the model ensures that the optimized asset allocation aligns with the insurer's risk management strategy.

4.1.3. Expected Returns

The expected returns for each asset class in the model are derived from historical data, ensuring that it reflects realistic investment outcomes. This process involves selecting appropriate benchmarks that align with the characteristics of each asset class within the portfolio. Given the importance of accurately capturing the risk-return profile of the insurer's portfolio, the historical performance of selected benchmarks is used to compute the expected returns, focusing on a long-term horizon (20 years) to mitigate short-term market volatility. The computation utilizes the geometric average of annual returns, which provides a more accurate measure of compounded growth over time, making it particularly suitable for the long-term perspective inherent in life insurance portfolios.

Follows the explanations of the rationale for each benchmark selection and the methodology used to compute the expected returns for each asset class:

- Government Bonds (r_{gov})

The benchmark chosen to represent the government bonds issued by EEA countries, excluding treasury bills, is the Bloomberg Euro Aggregate Treasury Index (LEATTREU Index). According to Bloomberg (2023), this index contains euro-denominated sovereign debt with remaining maturity of at least one year from 19 European Union (EU) countries, making it a suitable representative for this asset class. The annualized expected return (r_{gov}) was derived by applying a geometric average to the annual returns from 2003 to 2023, calculated based on the index's price data.

- Corporate Bonds (r_{corp})

The corporate bonds are represented by two benchmarks: the Bloomberg Euro-Aggregate: Corporates Index (LECPTREU Index) for investment-grade (IG) corporate bonds and the Bloomberg Pan-European High Yield Index (LP01TREU Index) for high-yield (HY) corporate bonds. According to Bloomberg (2024a, 2024b), the LECPTREU Index contains euro-denominated IG corporate bonds, while the LP01TREU Index includes HY corporate bonds issued in a range of currencies, including the euro, British pound, Danish krone, and others. The annualized expected return (r_{corp}) was calculated using a weighted geometric average, based on the proportion of IG and HY corporate bonds in the initial allocation. These weights were applied to the annual returns calculated from 2003 to 2023, using the indices' price data.

- Equity (r_{eq}^1, r_{eq}^2)

For equity type 1, the MSCI World Index (MXWO Index) was selected as the benchmark, as it represents equities listed on regulated markets across 23 developed markets countries, according to MSCI (2024), making it an appropriate representative. The annualized expected return of equity type 1 (r_{eq}^1) was computed by applying a geometric average to the annual returns from 2003 to 2023, calculated based on the index's price data.

Regarding equity type 2, which primarily consists of investment funds where a look-through approach is not applied, not listed equities and equities listed on markets outside the EEA and OECD, it is difficult to define a clear benchmark due to the diverse nature of these assets. According to Bodie et al. (2024), higher-risk assets should offer higher expected returns than lower-risk assets to compensate for their additional risk, which is often referred to as the risk-return trade-off. Given this, it is reasonable to assume that the annualized expected return of equity type 2 (r_{eq}^{2}) should not be lower than that of equity type 1, given the higher risk reflected in its greater capital charge. Therefore, in the absence of reliable quantifiable data for equity type 2, a conservative assumption is made that its annualized expected return is equal to that of equity type 1.

- Property (r_{prop})

The benchmark selected to represent property investments is the FTSE EPRA/Nareit Developed Europe Index (RPRA Index). According to FTSE Russell (2024), this index tracks the performance of listed real estate companies and REITs across developed European markets, making it an appropriate proxy for property investments within the optimization model. To compute the annualized expected return (r_{prop}), a geometric average was applied to the annual returns from 2003 to 2023, based on the index's price data.

- Treasury Bills ($r_{t-bills}$)

The benchmark selected to represent treasury bills issued by EEA government is the 3-month German Treasury Bills, which are known for their high liquidity and low-risk profile. Due to the short-term nature of these investments, it may be more appropriate to use the arithmetic average rather than the geometric average when calculating the annualized expected return ($r_{t-bills}$), see Broverman (2017). The arithmetic average is applied to the yields to maturity from the past 20 years, covering the period from 2003 to 2023. These yields reflect the return an investor would receive if holding these treasury bills until maturity, and by averaging them, a realistic proxy of the annualized expected return for this asset class is obtained.

4.1.4. Data Summary

The initial asset allocation, as of December 31, 2023, is presented in Table 1. This allocation reflects the market value of the company's assets across the different asset classes. The parameters, such as modified duration and annualized expected returns for the respective asset classes are also provided in this table.

TABLE 1 – Initial Asset Allocation.

	Market Value	Weight	Modified Duration	Annualized Expected Return
Government Bonds	782.6	47.4%	5.2	2.9%
Corporate Bonds	586.0	35.5%	5.0	4.1%
Equity Type 1	0.0	0.0%		6.4%
Equity Type 2	102.5	6.2%		6.4%
Property	42.0	2.5%		5.6%
Treasury Bills	139.6	8.4%	0.1	0.6%
Total	1652.7	100.0%	4.7	3.4%

The value of the BE, also as of December 31, 2023, and the respective modified duration are presented in Table 2.

TABLE 2 – Initial Liabilities.

	Value	Modified Duration
Best Estimate of Liabilities	1424.2	6.6

The parameters regarding the SCR Market Risk inserted in the optimization model are in Table 3.

TABLE 3 – SCR Market Risk parameters.

	Shock
Interest Rate upward	1.1%
Interest Rate downward	0.9%
Equity Type 1	39.0%
Equity Type 2	49.0%
Property	25.0%
Spread	10.3%

Finally, the investment limits in the constraints are in Table 4.

TABLE 4 – Investment Limits

	Min	Max
Government Bonds	25%	75%
Corporate Bonds	0%	50%
Illiquid Assets	0%	20%
Treasury Bills	1%	5%

4.2.Methodology

Given the specific structure of the problem, convex quadratic programming (CQP) was selected as the appropriate approach to solve it. This is a well-suited way for handling problems with a quadratic objective function alongside linear equality and inequality constraints while guaranteeing a global, rather than local, solution, see for instance Boyd & Vandenberghe (2004) and Nocedal & Wright (2006). After some calculations it was possible to write our problem according to the following general form of a convex quadratic programming problem:

$$\min \frac{1}{2}\boldsymbol{x}^{\boldsymbol{T}}\boldsymbol{P}\boldsymbol{x} + \boldsymbol{q}^{\boldsymbol{T}}\boldsymbol{x} \tag{25}$$

Subject to:

$$\boldsymbol{G}\boldsymbol{x} \leq \boldsymbol{h} \tag{26}$$

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \tag{27}$$

Where $\boldsymbol{P}$ is a positive semidefinite matrix, $\boldsymbol{G}$ represents the inequality constraints matrix, and $\boldsymbol{A}$ represents the equality constraints matrix. The vectors $\boldsymbol{q}, \boldsymbol{h},$ and $\boldsymbol{b}$ contain coefficients of the objective function and constraints.

The tool to finally solve the problem is the CVXOPT package in Python. CVXOPT is designed to handle CQP, allowing a clear formulation of the problem and its solution using methods such as the interior-points method. For further technical insights into the package, see Andersen et al. (2023).

5. RESULTS

This chapter presents a comprehensive analysis of the results obtained with the model. Throughout the analysis, two important metrics are used: (i) the marginal SCR with respect to each asset class (mSCR), which gives the approximate increase in the SCR_{Market} when the allocation in the respective asset class increases by 1 unit (for example, €1M), see Kouwenberg (2017) for further details; (ii) the expected return per unit of marginal SCR (E[r]/mSCR ratio), which serves as an indicator of capital efficiency, also inspired by a measure presented by Kouwenberg (2017). These metrics help to assess how different asset classes contribute to the overall risk and return of the asset allocation under Solvency II constraints.

The chapter begins by evaluating the initial asset allocation, examining this asset allocation and its impact on the insurer's profitability and solvency position. Next we show how the optimized asset allocation can enhance capital efficiency while maintaining the same SCR_{Market}. Finally, the efficient frontier is analyzed to illustrate the trade-off between maximizing returns and managing risk, under the Solvency II framework.

5.1. Initial Asset Allocation Results

Table 5 presents the results of the initial asset allocation of the life insurer, reflecting a conservative approach with a strong emphasis on bonds. Specifically, 82.9% of the portfolio is allocated to bonds, with 47.4% of that in government bonds and 35.5% in corporate bonds. In addition, illiquid assets (equities and property) make up only 8.7% of the portfolio, a relatively small portion. Notably, the amount held in liquidity (treasury bills), at 8.4%, is almost the same as the allocation to illiquid assets, ensuring immediate liquidity to manage short-term liabilities.

In this case, government bonds benefit from a negative mSCR (-0.03), as do treasury bills, because the SCR_{IR} is driven by the downward shock scenario. There is a significant gap between the modified durations of assets (4.7) and the liabilities (6.6), indicating that the portfolio is more exposed to a downward interest rate shock. In such a scenario, the increase in the value of the liabilities exceeds the rise in the market value of the assets. While the market value of the assets that influence the SCR_{IR} positively (government bonds, corporate bonds, and treasury bills) is higher than the value of the liabilities, the large duration gap leads to an SCR_{IR} of €20.6M. Given their negative mSCR, the E[r]/mSCR ratio of the government bonds and treasury bills is not displayed, as it is non-interpretable.

Corporate bonds offer an expected return of 4.1% and have a positive mSCR due to spread risk, which is the market risk's most significant sub-module in the initial allocation, with a value of €60.3M. Despite the positive mSCR, corporate bonds still exhibit a favourable E[r]/mSCR ratio of 0.61, the highest of all asset classes, indicating that this asset class provides a good balance between risk and return under Solvency II constraints.

In contrast, the allocation to equity type 2, which accounts for 6.2% of the portfolio, reflects the life insurer's attempt to capture higher returns, as this asset class has an expected return of 6.4%. However, this class (consisting of investment funds for which a look-through approach could not be applied), is subject to a 49% capital charge under the Solvency II standard formula, resulting in a significant equity risk of €50.2M. The resulting E[r]/mSCR ratio for equity type 2 is 0.14, the lowest in the portfolio, indicating a less efficient use of capital compared to other asset classes. Although equity type 2 offers attractive returns, the high mSCR (0.45) reduces its appeal, highlighting the trade-off between profitability and capital efficiency under Solvency II within the portfolio.

Property, which represents 2.5% of the portfolio, offers an expected return of 5.6% and has a much more favourable E[r]/mSCR ratio of 0.31, making it the second most capital-efficient asset in the portfolio. Despite being subject to a high capital charge of 25%, resulting in a property risk of €10.5M, this asset class still stands out as a valuable component of the life insurer's portfolio. It provides better capital efficiency than equities while still contributing to the portfolio's long-term return.

In terms of capital efficiency, the E[r]/mSCR ratios help to reveal the strengths and weaknesses of the portfolio. Corporate bonds and property stand out for their favourable ratios, indicating that they generate returns more efficiently relative to their contribution to the SCR_{Market}. By contrast, equity type 2, despite its high return, is less capital-efficient due to the higher capital charge. Government bonds and treasury bills in this case do not negatively affect the value of the SCR_{Market}, allowing more capital to be allocated toward higher-return assets without compromising the insurer's solvency position. The portfolio's strong market risk solvency ratio of 186% reflects the insurer's ability to meet the Solvency II requirements while maintaining a profitable asset allocation, with an expected return on assets of 3.40%.

Overall, the initial asset allocation demonstrates a conservative yet profitable approach, with a substantial portion of the portfolio in low-risk assets. The use of illiquid assets is balanced with a reasonable focus on capital efficiency, and the favourable E[r]/mSCR ratios for corporate bonds and property highlight their importance in achieving profitability within the constraints of Solvency II.

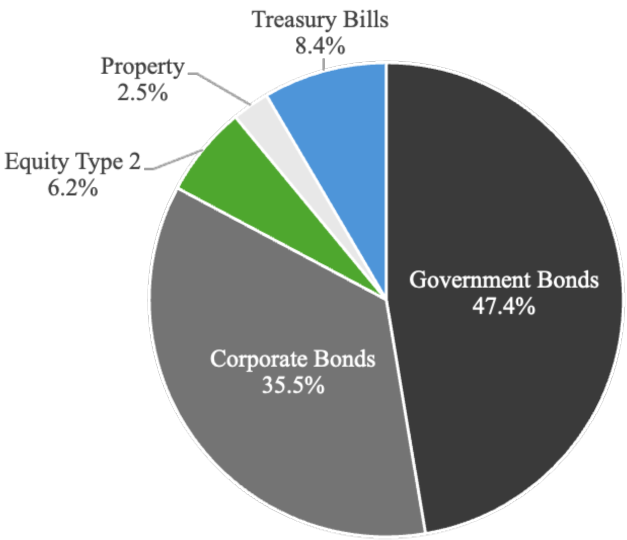
TABLE 5 - Initial Asset Allocation Results.

	Market Value	Weight	Modified Duration	E[r]	mSCR	E[r]/mSCR
Assets	**1652.7**	**100.0%**	**4.7**	**3.4%**		
Government Bonds	782.6	47.4%	5.2	2.9%	-0.03	
Corporate Bonds	586.0	35.5%	5.0	4.1%	0.07	0.61
Equity Type 1	0.0	0.0%		6.4%	0.27	0.24
Equity Type 2	102.5	6.2%		6.4%	0.45	0.14
Property	42.0	2.5%		5.6%	0.18	0.31
Treasury Bills	139.6	8.4%	0.1	0.6%	-0.00	
Liabilities	**1424.2**	**100.0%**	**6.6**			
Basic Own Funds	**228.5**					

	Value
Interest Rate Risk	20.6
Equity Risk	50.2
Property Risk	10.5
Spread Risk	60.3
Diversification	-18.5
SCR Market	**123.1**

$E[\Delta BOF]$	**56.2**
Solvency Ratio	**186%**
Expected Return on Assets	**3.40%**

FIGURE 3 - Initial Asset Allocation.



5.2. Optimized Asset Allocation Results

The optimized asset allocation, as shown in Table 6, reflects a strategic shift towards assets that improve capital efficiency while maintaining the same SCR_{Market} as the initial asset allocation, which allows for a direct comparison. The primary objective of this optimization is to enhance the expected return on assets while preserving compliance with Solvency II requirements, resulting in an expected return of 3.74%, up from 3.40% in the initial asset allocation.

The overall allocation to bonds remains substantial at 88.6%, slightly increasing from the initial allocation of 82.9%. The allocation to government bonds has been reduced to 38.7% from the original 47.4%, while corporate bonds now constitute 50.0% of the portfolio, reaching the maximum investment limit for this asset class. This strategic shift towards corporate bonds is driven by their favourable E[r]/mSCR ratio of 0.60, the highest among all asset classes. Corporate bonds offer an expected return of 4.1% and provide the best balance between risk and return. The reallocation has also contributed to a decrease in the SCR_{IR}, which is now €16.9M, down from €20.6M in the initial asset allocation. Despite the reduction in government bonds and treasury bills, both of which have a negative mSCR, the overall duration gap has narrowed, and the increased allocation to corporate bonds has further mitigated the interest rate risk.

In the optimized asset allocation, illiquid assets, specifically property, have increased to 10.4% of the portfolio, up from 2.5%. This increase is driven by property's favourable E[r]/mSCR ratio of 0.29, which, although lower than that of corporate bonds, represents the second-best capital efficiency among asset classes. With an expected return of 5.6%, property continues to provide a strong return relative to its capital charge under the property risk sub-module, now valued at €42.9M. The optimization suggests that increasing exposure to property would enhance long-term returns while maintaining compliance with Solvency II requirements. Furthermore, the optimization eliminates the holdings in equity type 2, which accounted for 6.2% of the initial asset allocation. Equity type 2 have the lowest E[r]/mSCR ratio (0.15) and the highest mSCR (0.42), making it the least efficient use of capital in the portfolio.

Interestingly, the optimization also continues not to invest in equity type 1. Despite the potential for higher returns, equity type 1 is subject to a 39% capital charge under the Solvency II standard formula, which makes the optimization prefer other asset classes, such as property and corporate bonds, that provide better capital efficiency, thereby avoiding the increased capital burden that would come from equity investments.

The allocation to treasury bills has decreased from 8.5% to 1.0%. This reduced allocation reflects the optimization's focus on shifting capital towards higher-return assets. Notably, the 1.0% allocated to treasury bills represents the minimum investment limit for this asset class, ensuring that only the minimum level of liquidity is maintained within the portfolio. While treasury bills continue to provide immediate liquidity, their role has been minimized in favour of more capital-efficient and higher-return investments, like corporate bonds and property. Maximization of returns also comes at the cost of reduced liquidity.

The optimized asset allocation maintains the same SCR_{Market} of €123.1M. However, there have been changes within the SCR Market sub-modules. Spread risk remains the most significant sub-module, increasing to €85.0M due to the larger allocation to corporate bonds. The SCR_{IR} has decreased to €16.9M, due to the shift in the portfolio's composition and a narrower duration gap between assets and liabilities. Additionally, the elimination of equity type 2 from the portfolio has removed the equity risk entirely. Finally, the diversification benefits have increased.

Overall, the optimized asset allocation demonstrates an improvement in capital efficiency while maintaining the same SCR_{Market}. The shifts towards corporate bonds and property highlight the optimization's focus on maximizing returns through assets with higher E[r]/mSCR ratios. However, this comes at the cost of reduced liquidity. The strong market risk solvency ratio of 186% has been preserved, ensuring that the insurer continues to meet the Solvency II requirements while improving the overall profitability, with the expected return on assets increasing to 3.74%.

TABLE 6 - Optimized Asset Allocation Results.

	Market Value	Weight	Modified Duration	E[r]	mSCR	E[r]/mSCR
Assets	**1652.7**	**100.0%**	**5.0**	**3.7%**		
Government Bonds	639.1	38.7%	5.2	2.9%	-0.03	
Corporate Bonds	825.7	50.0%	5.0	4.1%	0.07	0.60
Equity Type 1	0.0	0.0%		6.4%	0.25	0.26
Equity Type 2	0.0	0.0%		6.4%	0.42	0.15
Property	171.4	10.4%		5.6%	0.19	0.29
Treasury Bills	16.5	1.0%	0.1	0.6%	-0.00	
Liabilities	**1424.2**	**100.0%**	**6.6**			
Basic Own Funds	**228.5**					

	Value
Interest Rate Risk	16.9
Equity Risk	0.0
Property Risk	42.9
Spread Risk	85.0
Diversification	-21.7
SCR Market	**123.1**

$E[\Delta BOF]$	**61.7**
Solvency Ratio	**186%**
Expected Return on Assets	**3.74%**

5.3. Efficient Frontier

The efficient frontier shown in Figure 3 illustrates the optimized relationship between return on assets and the market risk solvency ratio within the life insurer's portfolio, constrained by investment limits. As mentioned previously, this efficient frontier is obtained by changing the penalty parameter in the model, inspired by Kouwenberg (2017), which allows for exploring different asset allocations that maximize returns while adhering to Solvency II and investment limits constraints. Figure 3 also shows the initial asset allocation, and the optimized asset allocation analysed above.

As we move along the efficient frontier, the different levels of solvency ratio are obtained through shifts in the asset allocation to balance risk and return. At lower solvency ratios, we observe a greater allocation to higher-return assets such as corporate bonds and property. These assets, while offering favourable expected returns, also increase the SCR_{Market}, leading to a lower solvency ratio. This strategy is evident in the more aggressive asset allocations at the lower end of the solvency ratio spectrum, where the focus is on maximizing returns.

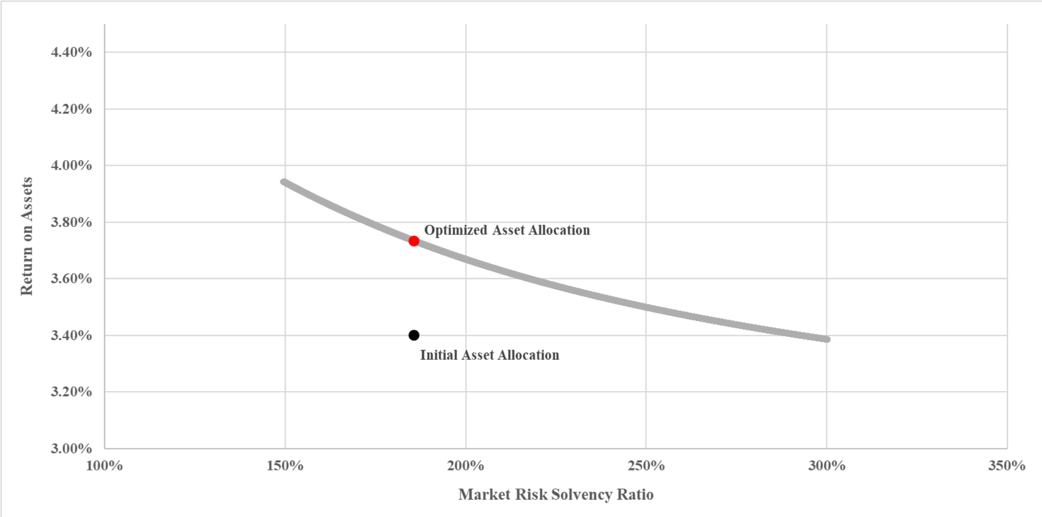
At moderate solvency ratios (around 200%), the asset allocation becomes more balanced. Government bonds, which offer lower returns but also lower risk, start to make up a larger portion of the portfolio. The allocation to corporate bonds remains substantial due to their capital efficiency, but slightly decreases to reduce risk. This asset allocation clearly reflects the aim to maximize returns while maintaining a strong solvency position.

As we move toward higher solvency ratios, the portfolio becomes increasingly conservative, as expected. The allocations to corporate bonds and property are reduced in favour of a higher allocation to government bonds, which rises significantly, reflecting a shift toward safer, lower-risk assets. This strategy, which prioritizes the solvency position over profitability, leads to a portfolio with a higher solvency ratio (lower SCR_{Market}) but lower overall returns.

Interestingly, equities do not receive any allocation at any of the solvency ratio levels along the efficient frontier. Both equity type 1 and equity type 2 are excluded from the optimized portfolios, proving that their high capital charges make them less capital-efficient compared to other asset classes. Additionally, the allocation to treasury bills remains at the minimum investment limit of 1% across all solvency ratio levels. It is important to note that if more liquidity were required, the minimum investment limit for treasury bills would need to be adjusted accordingly, to accommodate this need.

The efficient frontier clearly highlights the trade-off between maximizing returns and ensuring regulatory compliance under Solvency II. While the insurer can achieve higher returns through more risky asset allocations, this increased exposure to market risk comes with the cost of a lower solvency ratio, reflecting the life insurer's reduced ability to cover unexpected market fluctuations. Conversely, more conservative asset allocations, which involve reduced market risk and a stronger solvency position, protect against market downturns but also limits the potential for higher returns. This represents the inherent trade-off between risk and return that the life insurers must manage under the Solvency II framework.

FIGURE 4 – Efficient Frontier.



5.4. Sensitivity Analysis

The sensitivity analysis evaluates how the optimization model responds to changes in important parameters, specifically the impact of lower bond expected returns, higher equity expected returns, and increased liquidity needs. These adjustments provide a clear view of how different assumptions impact the life insurer's asset allocation and expected returns, ensuring that the portfolio remains compliant with Solvency II requirements while striving to maximize returns. Therefore, three different scenarios are analysed, and their efficient frontiers are shown in Figure 4, as well as the Base Scenario to provide comparability.

In Scenario 1, the expected returns for government bonds and corporate bonds were decreased by 0.5 percentage points and 1 percentage point, respectively. In response, property has a stronger weight in the new allocations but there is also an increase in the class of government bonds, which act as a stabilizing force to compensate for the increased allocation to property – an asset class with higher return but also a significant impact on the SCR_{Market}, due to its high capital charge. This adjustment allows the portfolio to pursue higher returns from property while maintaining overall stability through the increased allocation to government bonds. Meanwhile, the allocation to corporate bonds decreases, as their lower expected returns make them less attractive. Overall, the expected return on assets declines in this scenario, which is clearly reflected in Figure 4, where the entire frontier shifts downward, indicating that for any given solvency ratio, the portfolio in this scenario delivers lower returns.

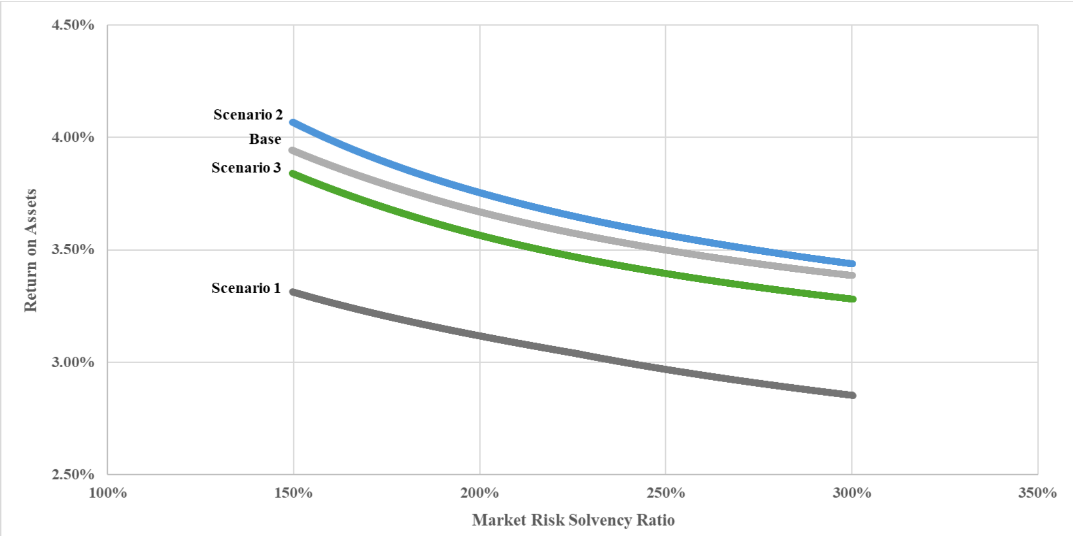
In Scenario 2, the expected returns for equity type 1 and equity type 2 increase by 2 percentage points. In response, the model introduces equity type 1 into the portfolio, significantly reallocating capital toward these higher-returning assets. Despite the high capital charges associated with equity type 1 under Solvency II, the increased returns justify its inclusion, making this asset class attractive in this scenario. Equity type 2 continues to not have any allocation, since equity type 1 is preferred due to its superior capital efficiency, under the assumption of equal returns. At the same time, the allocation to government bonds also increases, again to maintain portfolio stability, following the rationale discussed in Scenario 1, while corporate bonds decrease. The effects of these changes are visible in Figure 4, where the efficient frontier shifts upward, indicating that the life insurer can achieve higher returns for the same levels of solvency ratio compared to the base scenario. This upward shift underscores the potential for increased profitability when equities are included under favourable return assumptions.

Scenario 3 introduces higher liquidity requirements, where the minimum investment limit to treasury bills is set at 5%. In this case, the model reduces the allocation to property, corporate bonds, and government bonds to meet the increased demand for liquidity, with treasury bills now accounting for 5% of the portfolio. As a result, the overall expected return on assets drops slightly, which is reflected in Figure 4 by a moderate downward shift in the efficient frontier. This shift highlights the trade-off between maintaining liquidity, to meet short-term obligations, and achieving higher returns, as a larger share of the life insurer’s portfolio is allocated to low-return treasury bills, reducing the portfolio’s ability to generate significant returns.

Figure 4 effectively visualizes these trade-offs between return on assets and the market risk solvency ratio. As expected, Scenario 2 – where equity returns are higher – presents the most favourable risk-return trade-off, pushing the efficient frontier upward and offering higher potential returns. In contrast, Scenario 1, with lower bond returns, shows the most conservative perspective, with the frontier shifting downward, reflecting the reduced potential for high returns. Scenario 3, which increases the allocation to treasury bills, also shows a modest downward shift, illustrating how higher liquidity requirements can restrict the life insurer’s ability to generate higher returns.

Together, these scenarios provide valuable insights into the model’s flexibility in adjusting asset allocations to balance profitability, Solvency II compliance, and liquidity, but also its sensitivity to the parameters, especially the expected returns assumed for the asset classes. The detailed asset allocations and specific expected returns on assets for each scenario (including the base scenario), for the same level of SCR_{Market} as the initial asset allocation, to ensure direct comparability, are provided in the appendices as Table 10. While many other scenarios could be explored, these three offer a good overview of how asset allocations might respond under different conditions.

FIGURE 5 – Base Scenario, Scenario 1, Scenario 2, and Scenario 3 Efficient Frontiers.



6. CONCLUSION

The ability to manage asset allocation effectively, while complying with Solvency II requirements, is now an important factor in assessing the performance of life insurance companies. In an industry where both profitability and solvency are constantly under scrutiny, insurers face the difficult task of balancing return generation with the need to maintain a robust solvency position.

Therefore, this study aimed to explore the trade-off between asset allocation and Solvency II requirements in the context of a Portuguese life insurer. The primary objectives were twofold: (i) to analyse how this insurer could improve its asset allocation to enhance profitability while maintaining a strong solvency position; and (ii) to examine the trade-off between the profitability of the portfolio and the solvency position of the insurer, particularly focusing on the changes in asset allocation required to achieve different levels of the solvency ratio while maximizing expected returns. To achieve these objectives, an optimization model was employed to derive asset allocation strategies that maximize the expected return on the insurer's assets while adequately accounting for the SCR_{Market}.

The findings of this study demonstrated that it is possible to enhance profitability while maintaining regulatory compliance under Solvency II. By optimizing the insurer's asset allocation, the expected return on assets increased from 3.40% to 3.74%, all while keeping the same SCR_{Market} value. The analysis of this optimized asset allocation provided valuable insights into the balance between return generation and capital efficiency, showing that by reallocating towards more capital-efficient asset classes – particularly corporate bonds and property – the insurer can increase expected returns without compromising its solvency position. In contrast, equities were excluded from the optimized portfolio despite their high expected return, due to their significant capital charge under the Solvency II framework, highlighting the importance of considering an asset's impact on the SCR_{Market} alongside its return potential.

This project also explored the trade-off between profitability and solvency by analysing the efficient frontier of expected return on assets versus the market risk solvency ratio. The efficient frontier provided a visual representation of how changes in asset allocation affect the insurer's solvency ratio and potential returns. As the solvency ratio decreased, the asset allocation shifted toward higher-return but capital-efficient assets such as corporate bonds and property, reflecting a more aggressive strategy to maximize returns. Conversely, at higher solvency ratios, the strategy became more conservative, with increasing allocations to government bonds. This analysis underscores the inherent tension between pursuing higher returns and maintaining a robust solvency position, demonstrating how asset allocation strategies must be carefully considered to balance these competing objectives.

The sensitivity analysis provided further insights into the adaptability of the model under different scenarios. By adjusting important parameters such as expected returns and liquidity requirements, the analysis demonstrated how changes in these factors influenced asset allocation decisions and the potential portfolio returns. For instance, when expected returns on bonds were reduced, the model responded by increasing allocations to property and government bonds to maintain a stable solvency position while still pursuing higher returns. In contrast, when the expected returns on equities increased, the model introduced equity type 1 into the asset allocation, reflecting the flexibility to adjust to favourable market conditions despite the high capital charges associated with this asset class. Finally, this analysis showed how increased liquidity needs can limit the potential for maximizing returns.

Despite the insights provided by this study, some limitations should be noted. First, the analysis focused only on the market risk module, leaving out other important risk modules, especially the life underwriting risk, that could affect the life insurer's solvency position. Additionally, the optimization model relied on assumptions regarding expected returns of the asset classes, which have a significant influence on the results. The assumption of constant liabilities also presents a limitation, as it does not capture their dynamics, and the profit-sharing mechanisms typically found in life insurance contracts. Future research could address these limitations by incorporating the life underwriting risk to provide a more comprehensive view of the insurer's overall SCR. Moreover, the development of more robust methods for estimating expected returns would enhance the accuracy of the model. Finally, incorporating dynamic liabilities and profit-sharing mechanisms into the model would better capture the complexities of life insurance portfolios, leading to more precise insights into asset allocation strategies under the Solvency II regulatory framework.

To conclude, it is worth mentioning that although this study focused on a specific Portuguese life insurer, the methodology developed here can easily be extended to other life insurance companies operating under the Solvency II regime.

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APPENDICES

TABLE 7 – BSCR correlation matrix.

	Market	Counterparty Default	Life Underwriting
Market	1	0.25	0.25
Counterparty Default	0.25	1	0.25
Life Underwriting	0.25	0.25	1

TABLE 8 – SCR Market correlation matrix for the interest rate downward scenario.

	Interest Rate	Equity	Property	Spread
Interest Rate	1	0.5	0.5	0.5
Equity	0.5	1	0.75	0.75
Property	0.5	0.75	1	0.5
Spread	0.5	0.75	0.5	1

TABLE 9 – SCR Market correlation matrix for the interest rate upward scenario.

	Interest Rate	Equity	Property	Spread
Interest Rate	1	0	0	0
Equity	0	1	0.75	0.75
Property	0	0.75	1	0.5
Spread	0	0.75	0.5	1

TABLE 10 – Asset Allocations of the Base Scenario, Scenario 1, Scenario 2, and Scenario 3 for a 186% Solvency Ratio.

	Base		Scenario 1		Scenario 2		Scenario 3	
	Market Value	Weight	Market Value	Weight	Market Value	Weight	Market Value	Weight
Assets	**1652.7**	**100.0%**	**1652.7**	**100.0%**	**1652.7**	**100.0%**	**1652.7**	**100.0%**
Government Bonds	639.1	38.7%	915.4	55.4%	1143.4	69.2%	584.1	35.3%
Corporate Bonds	825.7	50.0%	390.2	23.6%	259.5	15.7%	821.4	49.7%
Equity Type 1	0.0	0.0%	0.4	0.0%	233.3	14.1%	0.0	0.0%
Equity Type 2	0.0	0.0%	0.0	0.0%	0.0	0.0%	0.0	0.0%
Property	171.4	10.4%	330.1	20.0%	0.0	0.0%	164.6	10.0%
Treasury Bills	16.5	1.0%	16.5	1.0%	16.5	1.0%	82.6	5.0%
Solvency Ratio	**186%**		**186%**		**186%**		**186%**	
Expected Return on Assets	**3.74%**		**3.17%**		**3.83%**		**3.63%**	