Lisbon School of Economics & Management
Universidade de Lisboa

MASTER
APPLIED ECONOMETRICS AND FORECASTING

MASTER´S FINAL WORK
PROJECT

DOCUMENT SPECIFICALLY MADE FOR OBTAINING THE MASTER’S DEGREE

FINAL DOCUMENT

VOLATILITY SPILLOVERS FROM STOCK MARKET TO CRYPTO MARKET

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October-2022

ABSTRACT

This dissertation presents evidence that there is a relationship between the stock market and non-regulated crypto markets, particularly in times of uncertainty of the macroeconomic environment, as the year 2020. I exploit the relationship between the S&P500 and DAX stock indices and the cryptocurrency markets of Bitcoin and Ethereum, using a dataset that comprehends daily price variations between 2017 and 2022. A breaking point was created in 2020 in order to understand the relationship between the stock and cryptocurrency markets in two subsamples that shape different market environments. Before 2020, there is no evidence found in volatility spillovers from the S&P500 to Bitcoin market at a 5% significance level. However, the findings suggest that after the breaking point, there are volatility spillovers from the stock market (mainly S&P500) to Bitcoin and Ethereum markets, particularly in the year 2020, a critical period of the pandemic crisis.

Vector Autoregressive methods were used in order to model the time series, allowing for the study of Granger causality relations and perform Impulse Response Functions. A triangular VAR-GARCH model is also estimated to further incorporate heteroskedasticity in the series.

Keywords: Stock market; Cryptocurrencies; Covid-19 Crisis; *Volatility Spillovers*; VAR-GARCH Models.

Resumo

O objetivo deste trabalho é analisar a relação entre o mercado de ações e o mercado de criptomoedas, particularmente evidente em períodos de incerteza face ao ambiente macroeconómico, como é o caso do ano 2020. Para estudar esta relação, foram usados dados entre 2017 e 2022 dos preços de fecho dos índices de ações S&P500 e DAX e dos mercados das criptomoedas Bitcoin e Ethereum. Considerei um ponto de interrupção das séries temporais o início de 2020, para conseguir perceber a diferença dos volatility spillovers entre os mercados em análise em ambientes distintos de mercado. Na primeira subamostra (2017-2019), não há evidência estatística de volatility spillover de S&P500 para Bitcoin ao nível de significância de 5%. Contudo, na subamostra de 2020 a 2022, há evidência de volatility spillovers entre os mercados, principalmente entre o S&P500 e os mercados das criptomoedas, Bitcoin e Ethereum. Ainda assim, os rácios de volatilidade indicam que grande parte da volatilidade é transmitida no decorrer do ano de 2020, um período crítico de pandemia.

O método Vetor Autorregressivo (VAR) foi utilizado para modelar as séries temporais, permitindo estudar as relações de causalidade de Granger e realizar funções de resposta ao impulso. Para incorporar a heterocedasticidade exibida nas séries, é estimado um modelo triangular VAR-GARCH.

Palavras-chave: Mercado Acionista; Criptomoedas; Crise do COVID-19; *Volatility Spillovers*; Modelos VAR-GARCH.

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1. INTRODUCTION

As the globalization goes further, effects are no longer isolated in internal markets. From contagious respiratory diseases, until shocks affecting the main world economies, knowledge of international market linkages such as correlation in returns and volatility across different markets is essential. It contributes to expertise on diversification of a portfolio and reduces the risk of uncertainty for sudden price decline (hedging strategies).

One of the main goals from Central Banks is to conduct the monetary policy in such a way that promotes stability in prices, good functioning of the financial system. Such policy ingredients should try to guarantee some predictability to the economic actors and protection to the society against shocks. Suddenly, in 2020 the world witnessed a massive economic negative shock and, consequently, economic recession. The outcome was a dash for cash, a shock, and a freak out (Baron, et al., 2022). Moreover, the economic disruptions associated with the covid-19 pandemic seem to have accelerated new digital trends such as the increased adoption of the cryptocurrencies. This may have led to a new period of volatility spillovers from stock to crypto market with the increasing interconnectedness between conventional financial markets and the new trendy crypto market.

As regards to existing literature related with this work, Eun & Shim (1989) investigated the transmission mechanisms of international stock market movements by estimating a vector autoregressive model with data from 9 markets. Their evidence specifies that a considerable amount of interdependence occurs among different stock markets. For example, at the twenty day horizon, innovations in foreign markets jointly account for about twenty six percent of the error variance of a national stock market on the average.

Another related paper is Qarni, Gulzar, Fatima, Khan, & Shafi (2019) who studied the inter-markets volatilities in US Bitcoin and financial markets from 2010 to 2017. They found a decrease in integration of U.S. financial markets due to the presence of Bitcoin markets. Moreover, the volatility spillovers among the U.S. Bitcoin and financial markets depicted asymmetric behaviour and was found to be more dominated by short frequency connectedness.

Cobert, Hou, Hu, Xu, & Oxley (2021) studied pandemic-related financial market volatility spillovers from Chinese financial markets upon a broad number of traditional financial assets during the outbreak of the COVID-19 pandemic. They argue that the likelihood of recurrence of pandemics in the future similar to the COVID-19 outbreak, motivate the understanding of the behaviour of investors in the aftermath of such events.

In fact, the occurrence of new pandemics, lockdowns, demand shocks, and economic contractions may occur in the future. These situations require competence to respond to the difficulties that may arise. My research seeks to identify the effects not yet studied during these times in the western exchange market for digital currencies.

The objective of this work is to contribute to a greater understanding of the relationship that exists between the traditional stock market and the cryptocurrency market, through time series econometric methods. In particular, I study the Granger causality relationships between the two markets and the volatility spillover effects between markets, that is, volatility from non-idiosyncratic effects. In addition, I also intend to study whether there were any structural changes in the relationships between the two markets over the COVID-19 period.

For this purpose, I considered the methodology of Vector Autoregressive (VAR) models. This class of models has the advantage that it is easy to estimate, and the statistical analysis works more a less in the same way as multiple linear regressions and ARMA models, widely used in multivariate analysis and time series analysis, respectively. As it is well known, many times of this sort of data contains conditional heteroskedasticity. Consequently, I also analysed a VAR-GARCH model to study the interactions between both markets at the volatility level.

I defined as the vector of variables the returns of the S&P500 and DAX stock indices and the cryptocurrencies Bitcoin and Ethereum. The choice of these series is justified by the fact that Standard and Poor's 500 Index is a reference stock market index for the United States, which is one of the main world economies, and DAX will represent the European stock market dynamics. Bitcoin is incorporated in this study given its notability and value in the crypto market, and Ethereum is the second most valuable currency among the cryptocurrencies.

This research work has various findings. Before 2020, I find no evidence of volatility spillovers from the stock market to the well-known decentralized digital currency, Bitcoin. From the sovereign bond market functioning hit, in 2020, S&P500 demonstrates volatility spillovers in Bitcoin at 5% level. Yet, volatility ratio suggests that this result is predominantly in 2020 rather than onwards sample.

Additionally, we find that before 2020 Ethereum market could not reject the concept of market inefficiency since the lagged values of S&P500, DAX, Bitcoin and itself were statistically significant to model its returns. From 2020, the GARCH model could not identify statistical evidence of market inefficiency in this cryptocurrency at 5% significance level. Also, volatility ratio shows outstanding hight values of conditional variance coefficients from S&P500 to Ethereum specifically in the year of 2020. However, no volatility ratios are large comparing with the proportion of conditional variance to Ethereum caused by volatility spillover effects from Bitcoin.

This paper is organized in four main sections. The first section performs a descriptive statistics and exploratory analysis of data of the four main variables included in this study. Secondly, the VAR estimation results are split before and after 2020 to implement the statistical analysis in these two samples and check if there are any relevant differences.

Then, Structural Analysis integrated Granger and Instantaneous Causality, followed by Impulse Response Function. Finally, considering all the features detected in VAR, a GARCH model is performed and analysed. I used software R for descriptive statistics and VAR modelling and analysis, and E-VIEWS for the GARCH models.

2. DATA AND DESCRIPTIVE STATISTICS OF RETURNS ON INDICES

In order to study how the movements of the stock market may impact the crypto market I collected daily data of two stock indexes (the S&P 500 and the DAX) and two cryptocurrencies (the Bitcoin and the Ethereum). The data sources are Yahoo finance[1] and WSJ markets[2], and are publicly available. The sample period starts on 9 October

[1]Dax: https://finance.yahoo.com/quote/%5EGDAXI/history/
Bitcoin: https://finance.yahoo.com/quote/BTC-USD/history?p=BTC-USD
Ethereum: https://finance.yahoo.com/quote/ETH-USD/history?p=ETH-USD

[2] S&P500: https://www.wsj.com/market-data/quotes/index/SPX/historical-prices

2017, and ends on 18 May 2022, with a total of 1114 observations in each series. We then computed the returns series from the close price values available in these sources.

The choice of these time series may be justified as follows. S&P 500 Index, or Standard and Poor's 500 Index, is a market capitalisation weighted index of 500 leading publicly traded companies in the U.S. The list may not contain exactly 500 U.S. companies due to specific criteria and it is used as a proxy for the world market ((Baele, 2002)). DAX is a German stock index that gathers the 40 largest and most liquid German companies traded on the Frankfurt exchange. Some known corporations are Mercedes Benz, Allianz, and Siemens. Bitcoin (BTC) may be the most well-known cryptocurrency by the crypto market. It was developed in 2009 anonymously to act as money and a form of payment outside the control. Finally, Ethereum is the result of the success of the first cryptocurrency. It is a decentralized global software platform powered by blockchain technology, also identified as ETH.

Figure 9, in annex, shows plots the returns of the 4 series during the considered sample period. We observe that none of them shows an upward or downward trend, with frequent mean reversion and the values always remain around zero. Furthermore, the series display a somewhat erratic behaviour which poses some challenges for econometric modelling. In complement with the visual inspection of the plots, I performed ADF tests for the presence of a unit root, in which stationarity corresponds to the alternative hypothesis. The results of the ADF tests[3] have shown *p-values* below 0.01, which reinforces the fact observed from Figure 9 that the series seem stationary.

Considering the descriptive statistics in Table 1, the S&P 500 Index returns (S&P500) show a positive mean of 1.2 US Dollars, a very small value. However, its minimum and maximum are -324.89 and 230.38, respectively, and hence with no surprise I find a large variance value (1778.592). By its graph in Figure 9, we can suspect of ARCH effects since low (high) volatility periods are followed by periods with the same pattern of low (high) volatility. Moreover, a huge shock can be seen in the middle of the first semester of 2020.

[3] The results of the ADF tests are available upon request.

Following the same patterns of S&P 500 returns, the DAX returns (DAX) display, in EUR, a mean of 0.74, a minimum of -1277.55 and a maximum of 1016.42. Besides a positive almost null mean, these discrepancies lead to a variance of 27518.65. Furthermore, its median is considerably different from its mean, more precisely, 8.2 EUR bigger than the mean. Similarly, to S&P 500, this series exhibit a big event in the first months of 2020 since COVID-19 hit Europe and US more a less at the same time.

The Bitcoin returns series (BTC) shown in Figure 9 demonstrates a clear pattern of ARCH effects as 2021 shows a turning point for this latest popular payment system. Prior to that year, volatility is shown as low. Then, amounts in transactions raised in absolute value showing distinct new paths. Yet, the data returns do not demonstrate a new increasing or decreasing trend. Table 1 shows that its mean is near zero, 19.38607 USD, which is slightly bigger than in previous variables. Its maximum value is 8052.156 and a minimum of -7554.039, both in USD, which justifies a variance of 1529898. Large discrepancy in median and mean may be a reason to suspect of heavy tailed distribution. Besides a mean value close to 20 USD, the most common return in Bitcoin is 10.7 USD, which is a substantial difference.

As in BTC, the time series plot of Ethereum (ETH) returns in Figure 9 strongly suggests relevant ARCH effects. From 2021 volatility starts to boost, without changing its mean around zero. Table 1 reveals a mean value of 1.433758 USD, a minimum of -919.3909 and a maximum of 657.8792.

TABLE 1- DESCRIPTIVE STATISTICS

Variable	Median	Mean	Variance	Min	Max
SP500	3.23	1.2031	1778.592	-324.89	230.38
DAX	8.9795	0.74	27518.65	-1277.55	1016.42
Bitcoin	10.7461	19.38607	1529898	-7554.039	8052.156
Ethereum	0.2471	1.433758	9539.599	-919.3909	657.8792

To analyse the effect of the new market conditions triggered by COVID-19 on the results of this research, I decided to split the sample in two parts. The first subsample starts from 9 October 2017 and ends on 30 December 2019 the second subsample begins on 2 January 2020 and ends on 18 May 2022. Besides similar stationarity paths in both samples which may be suspected from figure 9, there are distinct results in Autocorrelation Function (ACF) plots. Figures 1 and 2 plot the Sample ACF before 2020 and from 2020 onwards, respectively. Contrary to figure 1, from 2020 stock market of US suggests some autocorrelation in the first lags of its returns. This is an outcome not expected by the literature as the returns may display correlation close to zero in different days (Lanza, Manera, & McAleer, 2006). In contrast, ACF plot from Ethereum returns shows large magnitude of the ACF values since lag 5 onwards before 2020. This is an outcome that disappear in the second subsample as the bands stop crossing confidence intervals.

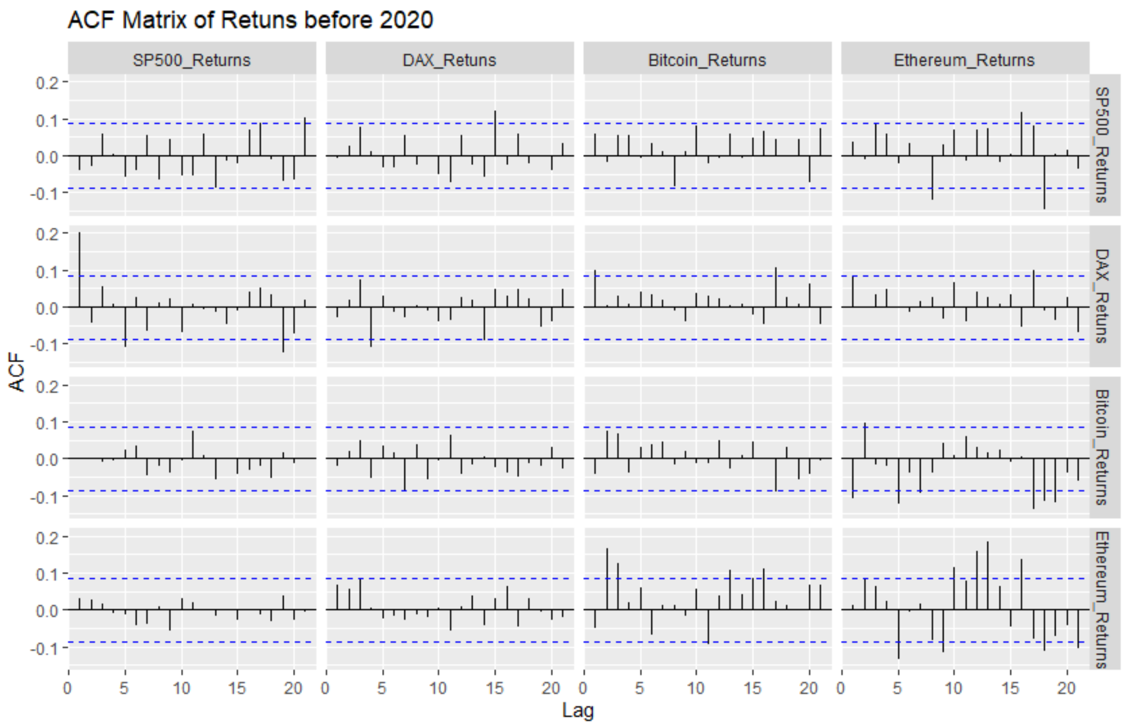

FIGURE 1 – Matrix of ACF from Returns Before 2020.

FIGURE 2 – Matrix of ACF from Returns From 2020

3.VAR ESTIMATION RESULTS

To study the dynamic relationships from S&P500 and DAX to Bitcoin and Ethereum, vector autoregressive (VAR) models are going to be performed. This is one of the most widely used multivariate time series models in the econometric literature. The model is relatively easy to estimate, and the statistical analysis works more a less in the same way as multiple linear regressions and ARMA models which may also justify its popularity. Additionally, the properties of VAR models have been studied extensively in the literature (Tsay, 2014).

The formula for the VAR model in the context of this dissertation has the following form:

$$\begin{bmatrix} \boldsymbol{SP500_t} \\ \boldsymbol{DAX_t} \\ \boldsymbol{BTC_t} \\ \boldsymbol{ETH_t} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} + \begin{bmatrix} \boldsymbol{a_{1,11}} & \boldsymbol{a_{1,12}} & \boldsymbol{a_{1,13}} & \boldsymbol{a_{1,14}} \\ \boldsymbol{a_{1,21}} & \boldsymbol{a_{1,22}} & \boldsymbol{a_{1,23}} & \boldsymbol{a_{1,24}} \\ \boldsymbol{a_{1.31}} & \boldsymbol{a_{1,32}} & \boldsymbol{a_{1,33}} & \boldsymbol{a_{1,34}} \\ \boldsymbol{a_{1,41}} & \boldsymbol{a_{1,42}} & \boldsymbol{a_{1,43}} & \boldsymbol{a_{1,44}} \end{bmatrix} \begin{bmatrix} \boldsymbol{SP500_{t-1}} \\ \boldsymbol{DAX_{t-1}} \\ \boldsymbol{BTC_{t-1}} \\ \boldsymbol{ETH_{t-1}} \end{bmatrix} +$$

$$\begin{bmatrix} \boldsymbol{a_{2,11}} & \boldsymbol{a_{2,12}} & \boldsymbol{a_{2,13}} & \boldsymbol{a_{2,14}} \\ \boldsymbol{a_{2,21}} & \boldsymbol{a_{2,22}} & \boldsymbol{a_{1223}} & \boldsymbol{a_{2,24}} \\ \boldsymbol{a_{2.31}} & \boldsymbol{a_{2,32}} & \boldsymbol{a_{2,33}} & \boldsymbol{a_{2,34}} \\ \boldsymbol{a_{2,41}} & \boldsymbol{a_{2,42}} & \boldsymbol{a_{2,43}} & \boldsymbol{a_{2,44}} \end{bmatrix} \begin{bmatrix} \boldsymbol{SP500_{t-2}} \\ \boldsymbol{DAX_{t-2}} \\ \boldsymbol{BTC_{t-2}} \\ \boldsymbol{ETH_{t-2}} \end{bmatrix} + \cdots + \begin{bmatrix} \boldsymbol{a_{p,11}} & \boldsymbol{a_{p,12}} & \boldsymbol{a_{p,13}} & \boldsymbol{a_{p,14}} \\ \boldsymbol{a_{p,21}} & \boldsymbol{a_{p,22}} & \boldsymbol{a_{p,23}} & \boldsymbol{a_{p,24}} \\ \boldsymbol{a_{p.31}} & \boldsymbol{a_{p,32}} & \boldsymbol{a_{p,33}} & \boldsymbol{a_{1p34}} \\ \boldsymbol{a_{p,41}} & \boldsymbol{a_{p,42}} & \boldsymbol{a_{p,43}} & \boldsymbol{a_{p,44}} \end{bmatrix} \begin{bmatrix} \boldsymbol{SP500_{t-p}} \\ \boldsymbol{DAX_{t-p}} \\ \boldsymbol{BTC_{t-p}} \\ \boldsymbol{ETH_{t-p}} \end{bmatrix} +$$

$$\begin{bmatrix} \boldsymbol{u_{SP500_t}} \\ \boldsymbol{u_{DAX_t}} \\ \boldsymbol{u_{BTC_t}} \\ \boldsymbol{u_{ETH_t}} \end{bmatrix}$$

Where the vector $\boldsymbol{u_t}$ follows a White Noise process with mean equal to zero and variance covariance matrix equal to $\sum_u$.

To choose the lag order of the VAR (p in the formula above) I am going to use the well-known model selection criteria Akaike Information Criterion (AIC), Hannan-Quinn Information Criterion (HQ), Bayesian or Schwarz Information Criterion (SC/BIC) and Final Prediction Error (FPE). These have shown to be effective in selecting the most appropriate model (see, for example, (Konishi & Kitagawa, 2008)).

The mathematical formulas for these criteria are presented below

$$AIC(p) = \ln \left|\widetilde{\textstyle\sum}_u(p)\right| + \frac{2}{T} 16p$$

$$HQ(p) = \ln \left|\widetilde{\textstyle\sum}_u(p)\right| + \frac{2ln[\ln (T)]}{T} 16p$$

$$SC(p) = \ln \left|\widetilde{\textstyle\sum}_u(p)\right| + \frac{\ln (T)}{T} 16p$$

$$FPE(p) = \ln \left|\widetilde{\textstyle\sum}_u(p)\right| \left[\frac{T + 4p + 1}{T - 4p - 1}\right]^k$$

where T is the number of observations, p is the order of a fitted VAR model, $\widetilde{\sum}_u(p)$is the maximum likelihood estimator of the variance covariance matrix of u. The formulas are motivated by two opposing components. The first component, $\ln \left|\widetilde{\sum}_u(p)\right|$, is linked with the goodness of fit of the model to the data unadjusted by the number of used parameters. The second component penalizes more heavily complex models with several parameters to be estimated. Different penalties result in different information criteria and so it can happen that information criteria deliver conflicting results.

3.1 First subsample before 2020

TABLE 2- INFORMATION CRITERIA BEFORE 2020

	AIC(n)	HQ(n)	SC(n)	FPE(n)
Selection	1	1	1	1

As it can be seen from Table 2, the model selection criteria suggests a VAR(1) to model the returns in the first subsample. Hence, using matrix notation the mathematical formula is represented below,

$$\boldsymbol{y_t} = \boldsymbol{C} + \boldsymbol{A_1}\boldsymbol{y_{t-1}} + \boldsymbol{u_t}$$

where $\boldsymbol{y_t}$ is the 4 dimensional vector with the returns of ***S&P500***, ***DAX***, ***BTC*** and ***ETH*** at time *t*, respectively, $\boldsymbol{C}$ is the vector for the constant terms, $\boldsymbol{A_1}$ is a matrix of unknown parameters, $\boldsymbol{y_{t-1}}$ the lagged values of $\boldsymbol{y_t}$ and $\boldsymbol{u_t}$ is a vector of the errors.

After fitting the VAR(1) selected unanimously the the model selection criteria, I examine some diagnostic checking tools. In particular, I start by testing the whiteness of the residuals. To test this assumption, the Portmanteau test was performed where the hypotheses are specified as:

$$H_0: R_u(1) = R_u(2) = \cdots = R_u(h) = 0$$

$$H_1: R_u(1) \neq 0 \vee R_u(2) \neq 0 \vee \ldots \vee R_u(h) \neq 0$$

With $R_u(i) = Corr(u_t, u_{t-i}) = 0, i = 1, 2, \ldots$

A high value of *h* indicates that a sensible statistical power of the test may be missed, and a low value of *h* may ignore crucial information at higher lags (Hyndman & Athanasopoulos, 2021). Hence, as returns are daily, there are no weekends on the data and the critical value of the first lag does not exist, Q test is performed with *h=2* to *h=5* as can be observed in the output, figure 12, in the appendices. Thus, $i = 2, 3, 4, 5.$ At both tests, asymptotic and adjusted, the null hypothesis of no residual (auto and cross) correlation from lag 2 to lag 5 is rejected. When testing for serial correlation in the VAR disturbances with a LM test, conclusions do not alter. Using the following auxiliary regression:

$$\hat{u}_t = v + A_1 Y_{t-1} + D_1 \hat{u}_{t-1} + D_2 \hat{u}_{t-2} + D_3 \hat{u}_{t-3} + D_4 \hat{u}_{t-4} + D_5 \hat{u}_{t-5} + error_t$$

The null and alternative hypotheses are

$$H_0: D_1 = D_2 = D_3 = D_4 = D_5 = 0$$

$$H_1: D_1 \neq 0 \vee D_2 \neq 0 \vee D_3 \neq 0 \vee D_4 \neq 0 \vee D_5 \neq 0$$

At 5% significance level, there is evidence of serial correlation in the disturbances of this vector autoregressive model of order 1.

However, some p-values of the tests for serial correlation are close to the threshold 0.05. Hence, conscious about the restraint of the rejection of no autocorrelation hypothesis, I will continue to use the VAR(1) model in the subsequent analysis. This is a decision based in the agreement of the whole selection criteria in order 1 for the Vector Autoregressive model and considering parsimony principle. Furthermore, table 16 shows inverse roots inside the unit circle.

The estimation output for the VAR(1) model fitted to $\boldsymbol{y_t}$ is provided in Table 3

TABLE 3- VAR(1) MODEL BEFORE 2020

Parameters	***SP500***	***DAX***	***BTC***	***ETH***
SP500(-1)	-0.052	0.003***	0.003	0.011
DAX (-1)	1.341	-0.187***	0.021	0.045
BTC (-1)	0.526	-0.131	0.012	-1.837
ETH (-1)	-0.012	0.019	-0.005*	0.046
constant	1.294	-1.387	-0.014	-0.292

*, ** and *** is a variable statistically significant at 10, 5 and 1% level, respectively

From table 3 I find that the ***DAX*** regression reveals the unique two variables statistically significant at 5% significance level. They are the lagged variables of ***SP500*** and ***DAX***. Furthermore, figure 11, in annex, exposes the VAR(1) estimation results of this equation where the F-statistic of the test for global significance shows that this regression has global statistical significance at the usual levels. The same outcome is not disclosed in the remaining three regressions. The greater coefficient of determination clarifies that 7.5% of the variation in ***DAX*** returns is explained by the model. This is an expected small value, considering the studied series.

Following the theory of stylized facts, distribution of returns is negatively skewed, because market exhibits very large drops but not equally large up moves. However, negative skewness is mainly related to the way how firm-specific (bad and good) news is disclosed. Good news is fast announced by firms, while the bad news is shown slowly over time. Hence, a normality test was performed in figure 13. With an alternative hypothesis being asymmetry of the residuals, that is, skew is different than zero, the residuals of this model reveal a p-value very small, close to zero. The series shows evidence of skewness.

Comparing with Normal Distribution, heavy-tailed distributions have hight probability to observe extreme values, in other words, significant deviations from the mean value. A distribution has heavy tails if Kurtosis is bigger than 3, that is a Distribution Leptokurtic. In this test, the null hypothesis of Kurtosis equals three is rejected in favour of alternative, at 5% significance level.

3.2 Second subsample after 2020

TABLE 4- INFORMATION CRITERIA FROM 2020

	AIC(n)	HQ(n)	SC(n)	FPE(n)
Selection	2	1	1	2

We now analyse the data from the second subsample. As it can be seen from Table 4, the AIC and FPE model selection criteria suggest a VAR(2), whereas HQ and SC prefer a VAR(1). Estimation output is presented in tables 5 and 6:

TABLE 5- VAR(1) MODEL FROM 2020

Parameters	SP500	DAX	Bitcoin	Ethereum
SP500(-1)	-0.219***	0.004***	3.822	0.024
DAX (-1)	0.739	-0.125*	-0.001	0.039
Bitcoin (-1)	0.649	-0.277	0.11	-1.957**
Ethereum (-1)	0.078	-0.032	0.014*	-0.222***
Constant	1.39e+00	0.385	37.894	3.161

*, ** and *** is a variable statistically significant at 10, 5 and 1 per cent level, respectively.

TABLE 6- VAR(2) MODEL FROM 2020

Parameters	***SP500***	***DAX***	***Bitcoin***	***Ethereum***
SP500(-1)	0.191***	0.001***	-0.00007	0.023
DAX (-1)	0.786	-0.135*	-0.001	0.037
Bitcoin (-1)	0.598	-0.273	0.1099	-1.885**
Ethereum (-1)	0.081	-0.038	0.014*	-0.207***
SP500(-2)	0.002	0.049	0.002	-0.027
DAX (-2)	0.054***	0.042	0.005	-0.025
Bitcoin (-2)	-0.211	0.169	-0.03	0.56*
Ethereum (-2)	0.072	-0.02	-0.01	0.147*
Constant	1.305e+00	0.291	36.645	2.948

*, ** and *** is a variable statistically significant at 10, 5 and 1 per cent level, respectively.

The individual significance of the coefficients lagged one period from each equation reveals the same conclusions in both models (VAR(1) and VAR(2)) at 5% level. Moreover, the explanatory variable ***DAX*** lagged two periods in ***SP500*** equation indicates individual significance at 5% in VAR (2).

Contrarily to the first subsample, complemented results are displayed in Overall Significance test, found in appendices (figures 16 & 21). ***Bitcoin*** exhibits the unique equation where the null hypothesis of no global significance of all variables is not rejected at 5% level. The conclusion is identical in both VARs from 2020. Hence, this outcome reverses in the remaining equations of the VAR, that is, the regressions of ***SP500***, ***DAX*** and ***Ethereum*** provide a better fit than a model that contains no independent variables.

The *R squared* of the equations remains low. In fact, none of variation of the equations is explained by more than 8% by the model, an undesirable small value. Considering all inverse roots inside the unit circle in both models and same conclusions in the serial correlation of the residuals, VAR(2) is the chosen model as it seems to slightly perform better given the individual significance of the variables.

$$\boldsymbol{y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + u_t}$$

$$\equiv \begin{bmatrix} SP500_t \\ DAX_t \\ BTC_t \\ ETH_t \end{bmatrix} = \begin{bmatrix} 1.305e+00 \\ 0.291 \\ 36.645 \\ 2.948 \end{bmatrix} + \begin{bmatrix} 0.191 & 0.001 & -0.00007 & 0.023 \\ 0.786 & -0.135 & -0.001 & 0.037 \\ 0.598 & -0.273 & 0.1099 & -1.885 \\ 0.081 & -0.038 & 0.014 & -0.207 \end{bmatrix} \begin{bmatrix} SP500_{t-1} \\ DAX_{t-1} \\ BTC_{t-1} \\ ETH_{t-1} \end{bmatrix}$$
$$+ \begin{bmatrix} 0.002 & 0.049 & 0.002 & -0.027 \\ 0.054 & 0.042 & 0.005 & -0.025 \\ -0.211 & 0.169 & -0.03 & 0.56 \\ 0.072 & -0.02 & -0.01 & 0.147 \end{bmatrix} \begin{bmatrix} SP500_{t-2} \\ DAX_{t-2} \\ BTC_{t-2} \\ ETH_{t-2} \end{bmatrix}$$

Consequently, stylized facts of financial analysis were tested as the series may follow the same path of the first subsample. Jarque-Bera is a test statistic for joint hypothesis that the residual skewness and Kurtosis are equal to the reference values of a Normal distribution (0 and 3, respectively) (Bera & Jaque, 1981). A *p-value* of 2.2e-16 rejects the null hypothesis at 5% significance level. Considering the statistical evidence performed by the tests and the visual form of returns plots, it is expected data to exhibit ARCH effects. This possibility is confirmed by tests in figure 25 in annex.

To sum-up, the selection criteria of the first subsample agreed in a VAR(1) to describe the joint dynamics of the series. Unfortunately, the null hypothesis of no autocorrelation was rejected, although sometimes by a small margin. Assuming this as a model limitation, the inference was done taking into account the agreement of selection criteria and the parsimonious principle. This decision was made for understanding purposes of the relationships from stock markets to crypto markets in this class of processes. Later, model diagnostics tested normality of the residuals. Skewness indicated that the distribution in the residuals is not symmetric, and kurtosis weighted mass probability in the tails of the distribution of the residuals. Also, ARCH effects are presented in the residual series which lead this investigation to the last section of the report, named GARCH models.

The second half of the sample does not demonstrate significant changes in the results comparing with the first half which does not bring this paper to the structural change that is being analysed. Tests reveal *H0* hypothesis of no serial correlation have been rejected in VAR(2) of the second subsample. Observing figure 22, it can be seen that this test is marginally rejected. Breusch-Godfrey test, figure 23 in the annex, shows a p-value of 0.0000 when degrees of freedom equal 80. Hence, considering drawbacks that a Vector Autoregressive model with a very high order would have, I chose a VAR(2) model giving the selection criteria conclusions, for the lake of simplicity employing parsimonious principle, and to decently deduce the structures among markets. Furthermore, this procedure allows me to better visualize the results before and after the split of the sample. Later, Jaque-Bera test proves that the residuals do not follow a Normal Distribution. When testing if the residuals present ARCH dynamics figure 25 reveals conditional heteroskedasticity effects exhibited by the data.

In 1960, Fama stated that current prices incorporate all expectations and relevant available information, that is, in an efficient market all information is already incorporated in prices (Malkiel, 1989). Hence, the models analysed in this study do not contradict the theory of Efficient Markets Hypothesis (EMH). Yet, although it serves as a useful benchmark for measuring relative efficiency, MacKinlay & Lo (1999) acknowledged the EMH is an idealization that is economically unachievable. To solve the problems presented in the models would be favourable to consider a VAR with more lags. Nevertheless, the analysis of a VAR with high order may also present plenty of constraints. An overparameterized VAR may produce overfitting, creation of considerable uncertainty and hamper of data analysis.

4. STRUCTURAL ANALYSIS

Granger (1969) introduced the concept of causality. In the context of a VAR(1), it implies that if off-diagonal elements of $\boldsymbol{A_1}$ are zero, then the variables are not dynamically correlated. As the data under analysis may exhibit ARCH behaviour, Granger causality tests were performed under bootstrap method.

TABLE 7- GRANGER CAUSALITY TESTS IN VAR(1) BEFORE 2020

Cause	F-Stat[a]	p-value
SP500	11.894	2.2e-16
DAX	1.3273	0.3406
Bitcoin	2.1589	0.2978
ETH	1.7848	0.4584
SP500, DAX	1.0911	0.3035
SP500, Bitcoin	10.049	0.0062
DAX, ETH	1.4294	0.5652
DAX, Bitcoin	1.4181	0.5187
Bitcoin, ETH	1.4236	0.2245

(a) Bootstrap runs: 10000.

TABLE 8- GRANGER CAUSALITY TESTS IN VAR(2) FROM 2020

Cause	F-Stat[a]	p-value
SP500	2.7294	0.0784
DAX	3.2555	0.0327
Bitcoin	2.6434	0.0523
ETH	2.1675	0.2537
SP500, DAX	0.37903	0.8173
SP500, Bitcoin	3.6018	0.0267
DAX, ETH	3.5052	0.0699
DAX, Bitcoin	4.389	0.0071
Bitcoin, ETH	0.58455	0.6644

(a) Bootstrap runs: 10000.

Observing table 7, ***SP500*** reveals a *F-statistic* of 11.894 (*p-value*= 2.2e-16) demonstrating evidence that this variable Granger causes ***DAX***, ***Bitcoin*** and ***Ethereum*** returns. This implies that the forecast of these returns may be improved if the information of past values of ***SP500*** is incorporated in the model. This was an expected result as US uses to be considered as a proxy for the world market (Baele, 2002). Hence, ***SP500*** may be the most exogenous variable presented within the set of variables. This is an outcome uncovered by the granger causality tests corroborated before and after the split in 2020.

Surprisingly, in VAR (1) before 2020, the null hypothesis of no granger-cause nor from ***DAX*** returns or ***SP500*** and ***DAX*** returns to the remaining variables, cannot be rejected at all usual levels. Therefore, before covid-19 crisis hits western countries, the information about one of the biggest stock markets in Europe seems to be unnecessary to forecast future values of the new emerging decentralized ledger system, identified as blockchain. This conclusion changes in the second subsample. At 5% level, ***DAX*** Granger causes ***SP500***, ***Bitcoin*** and ***Ethereum*** implying that to forecast these variables containing information of past values of ***DAX*** is more accurate than without using its data. Additionally, it worth to mention the change in the conclusion from the two subsamples of Granger-causality from ***DAX*** and ***Bitcoin*** to the remaining variables.

Crypto market demonstrates new paths in the second time interval. From 2020, with a *F-statistic* of 2.6434 *(p-value*= 0.0523) ***BTC*** granger-cause the ***ETH*** and the stock market variables analysed at 10% significance level. Hence, from 2020 the Bitcoin market starts to demonstrate its individual importance to forecast returns of the stock market. Additionally, there is no evidence that the second most valuable cryptocurrency Granger cause the three variables studied in this thesis in both subsamples. Thus, Ethereum does not demonstrate statistical evidence that it is required to calculate future returns in the stock market.

Later, to explore the relation between variables a different approach is explored. In order to understand the effect of changes in one variable on other variables in multivariate time series analysis, Impulse Response Function (IRF) is performed. Figures 3 and 4 are shown below where the solid lines represent the impulse responses, and the dashed lines represent a 95% confidence interval. The methodology incorporates orthogonal errors, bootstrap version with ten thousand runs and both cumulative and non-cumulative analysis.

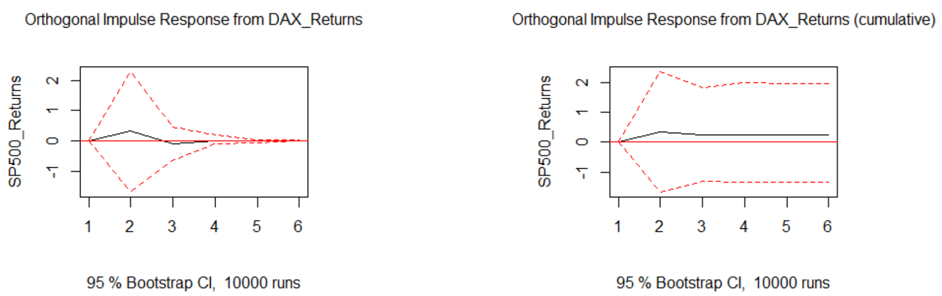

FIGURE 3 – Orthogonal IRF from ***DAX*** to ***SP500*** returns before 2020.

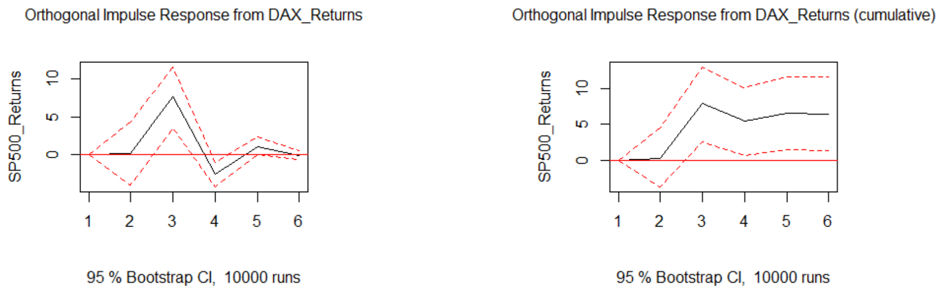

FIGURE 4 – Orthogonal IRF from ***DAX*** to ***SP500*** returns from 2020.

In figure 3 the deviations in the representative European stock market on the Standard and Poor's 500 Index are never significant before the covid-19 crisis. However, considering the changes in stock market from 2020, figure 4 shows the individual IRF for the first *p(k-1)=6* estimates, from ***DAX*** to ***SP500*** where it is revealed to be statistically significant three and four periods ahead using individual significance of 5%. That is, a change in ***DAX*** returns at time *t* affects the returns from ***SP500*** positively at time *t+3* and negatively at time *t+4*. It does not remain persistent for further periods, yet the cumulative orthogonal impulse response function of ***DAX*** seems to demonstrate an impact on variable ***SP500***. This conclusion may be related with the responses to the covid-19 from central banks, state policies and public health services between Europe and America.

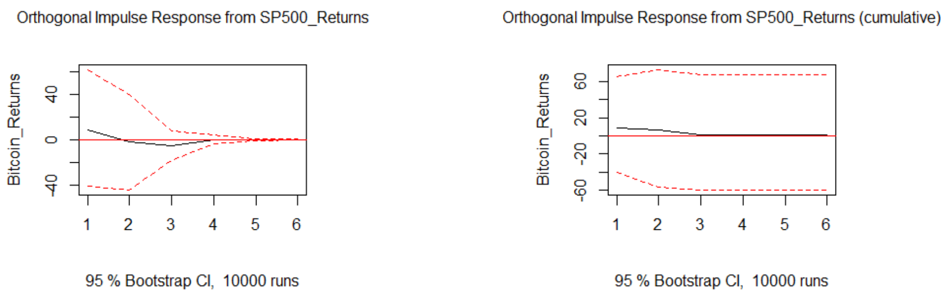

FIGURE 5 – Orthogonal IRF from ***SP500*** to ***Bitcoin*** returns before 2020.

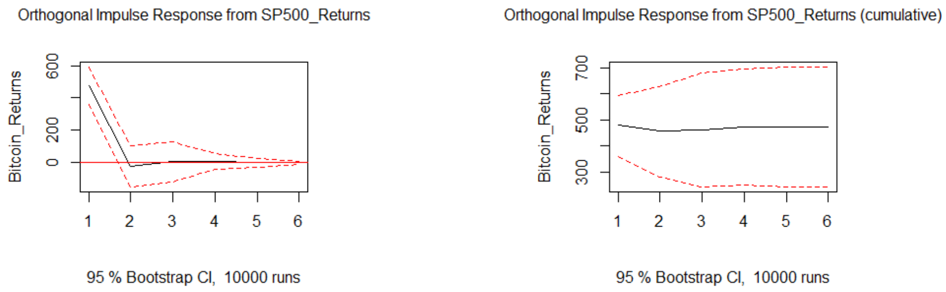

FIGURE 6 – Orthogonal IRF from ***SP500*** to ***Bitcoin*** returns from 2020.

In figures 5 and 6 are represented the *null hypothesis* of "no effect" between ***SP500*** and ***Bitcoin***. Before 2020, in both cumulative and non-cumulative, the *null hypothesis* cannot be rejected at 5% level, hence there is no evidence in this period that the US stock market representative variable of this study significantly affect returns from the most traded cryptocurrency. Oppositely, this result change in the second subsample. The period-to-period IRF for the effects of ***SP500*** on ***Bitcoin*** uncovers dynamic relationship between both. Therefore, with 95% confidence interval, there is statistical evidence that an impulse on *Standard and Poor's 500* have impact on the most expensive digital currency one period ahead. Consequently, the cumulative Impulse Response Function exhibits significant effects on this time interval as the hypothesis of jointly insignificance of all coefficients is rejected at 5% level. Therefore, from the beginning of the exceptional period of 2020, there is evidence of cumulative effect from the ***SP500*** on ***Bitcoin***.

5.GARCH MODELS

In previous sections the variance covariance matrix of the conditional distribution was assumed to be time invariant. Yet, this assumption may be problematic in the analysis of financial time series. Unsurprisingly, the statistical tests employed in Section 3 suggest serial correlation and ARCH effects among the residuals of the calculated VARs.

In many financial series, such as stock returns, there are co-movements of volatility present in the series. In other words, when volatility of a series expands (declines), the same pattern is observed in other interconnected financial series which also tend, in general to increase (decrease) (Nicolau, 2012). These co-movements may be estimated in a multivariate framework of equations such as a VAR but allowing for conditional heteroskedasticity.

Considering the purpose of this study, this approach will permit to assess the risk associated with the crypto market and the direct effect from the expectations of the stock market. The innovation of *Autoregressive Conditional Heteroskedasticity (*ARCH) boosted by Engle (1982) lead to difficult estimation parameter and proneness to convergence issues. Consequently, the GARCH class was proposed to have a model that is able to describe the strong persistence of volatility in a more parsimonious manner (Lutkepohl & Kratzig, 2004). Therefore, I study a "Triangular" GARCH model, *Generalized Autoregressive Conditional Heteroskedasticity* model, to verify the existence of empirical evidence of volatility spillover effects from the stock market on the crypto market. This enables me to analyse if the volatility shocks to one variable affect the volatility of other related variables. In fact, it may happen that the volatility of one financial time series increases as a result of a similar movement in another financial time series, if their volatilities are interrelated (Enders, 2015). The mathematical formulas for the model are shown below

$$\begin{bmatrix} SP500_t \\ DAX_t \\ BTC_t \\ ETH_t \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} + \begin{bmatrix} \emptyset_{11} & 0 & 0 & 0 \\ \emptyset_{21} & \emptyset_{22} & 0 & 0 \\ \emptyset_{31} & \emptyset_{32} & \emptyset_{33} & 0 \\ \emptyset_{41} & \emptyset_{42} & \emptyset_{34} & \emptyset_{44} \end{bmatrix} \begin{bmatrix} SP500_{t-1} \\ DAX_{t-1} \\ BTC_{t-1} \\ ETH_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \end{bmatrix}$$

, where

$$\begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \vartheta_{21} & 1 & 0 & 0 \\ \vartheta_{31} & \vartheta_{32} & 1 & 0 \\ \vartheta_{41} & \vartheta_{42} & \vartheta_{34} & 1 \end{bmatrix} \begin{bmatrix} e_{SP500,t} \\ e_{DAX,t} \\ e_{BTC,t} \\ e_{ETH,t} \end{bmatrix}$$

, and

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

This model was fitted using the EViews software. Before analysing results, I performed model diagnostics. A basic assumption to correctly interpret statistical tests in a VAR-

GARCH model is the absence of serial correlation in the errors. Figure 26 in the annex shows a Q test performed to assess autocorrelation in the errors for the four equations and considering the two subsamples. The same conclusion is displayed by all equations except for DAX equation before 2020.The null hypothesis of no autocorrelation in the errors from lag 2 to lag 5 cannot be rejected, at all usual levels, in all equations except in Dax equation before 2020. Subsequently, I analyse separately equations in this dissertation. The estimation output is presented in Tables 9 until 14:

TABLE 9- TRIANGULAR GARCH MODEL BEFORE 2020

Parameters	***SP500***	***DAX***	***Bitcoin***	***Ethereum***
constant	3.6085***	5.1379	-12.7228	-0.5388
SP500(-1)	-0.0472	1.2129***	1.0597	0.0295**
DAX (-1)	0	-0.1627***	0.0926	0.0062***
Bitcoin (-1)	0	0	0.0312	-0.0033***
Ethereum (-1)	0	0	0	0.1115***
errors	$\boldsymbol{u_{1,t}}$	$\boldsymbol{u_{2,t}}$	$\boldsymbol{u_{3,t}}$	$\boldsymbol{u_{4,t}}$

*, ** and *** is a variable statistically significant at 10, 5 and 1 per cent level, respectively

TABLE 10- TRIANGULAR GARCH MODEL BEFORE 2020- THE ERRORS

Parameters	$\boldsymbol{u_{1,t}}$	$\boldsymbol{u_{2,t}}$	$\boldsymbol{u_{3,t}}$	$\boldsymbol{u_{4,t}}$
$\boldsymbol{e_{S\&P500,t}}$	1	2.6719***	0.14304	0.0271**
$\boldsymbol{e_{DAX,t}}$	0	1	0.1238	0.0030
$\boldsymbol{e_{Bitcoin,t}}$	0	0	1	0.0248***
$\boldsymbol{e_{Ethereum,t}}$	0	0	0	1

*, ** and *** is a variable statistically significant at 10, 5 and 1 per cent level, respectively

TABLE 11- VARIANCE EQUATION- TRIANGULAR GARCH MODEL BEFORE 2020

Parameters	$\sigma^2_{t\,S\&P500}$	$\sigma^2_{t\,DAX}$	$\sigma^2_{t\,Bitcoin}$	$\sigma^2_{t\,Ethereum}$
constant	31.7986***	1507.523*	3058.421***	0.6865**
e^2_{t-1}	0.2176***	0.1012**	0.1399***	0.1443***
σ^2_{t-1}	0.7507***	0.7235***	0.8507***	0.8601***

*, ** and *** is a variable statistically significant at 10, 5 and 1 per cent level, respectively

TABLE 12- TRIANGULAR GARCH MODEL AFTER 2020

Parameters	***SP500***	***DAX***	***Bitcoin***	***Ethereum***
constant	3.9938**	5.756	45.5658	2.2505***
SP500(-1)	-0.0867*	1.1549***	0.2834	0.0014
DAX (-1)	0	-0.1903***	-0.155874	-0.0031
Bitcoin (-1)	0	0	-0.0002	0.0019
Ethereum (-1)	0	0	0	0.0203
errors	u_{1t}	u_{2t}	u_{3t}	u_{4t}

*, ** and *** is a variable statistically significant at 10, 5 and 1 per cent level, respectively

TABLE 13- TRIANGULAR GARCH MODEL AFTER 2020- THE ERRORS

Parameters	$u_{1,t}$	$u_{2,t}$	$u_{3,t}$	$u_{4,t}$
$e_{S\&P500,t}$	1	2.2808***	3.6689***	0.1287***
$e_{DAX,t}$	0	1	-0.0584	-0.0039*
$e_{Bitcoin,t}$	0	0	1	0.033***
$e_{Ethereum,t}$	0	0	0	1

*, ** and *** is a variable statistically significant at 10, 5 and 1 per cent level, respectively

TABLE 14- VARIANCE EQUATION- TRIANGULAR GARCH MODEL AFTER 2020

Parameters	$\sigma^2_{t\ S\&P500}$	$\sigma^2_{t\ DAX}$	$\sigma^2_{t\ Bitcoin}$	$\sigma^2_{t\ Ethereum}$
constant	125.1421***	1812.383***	3553.680***	0.8747**
e^2_{t-1}	0.2038***	0.1574***	0.0900***	0.2402***
σ^2_{t-1}	0.7546***	0.7773***	0.9185***	0.8412***

*, ** and *** is a variable statistically significant at 10, 5 and 1 per cent level, respectively

In each subsample, SP500 equation was firstly performed due to its exogeneity as it is assumed to be as a proxy for the world market (Baele, 2002). Under Generalized ARCH model the equation is represented below

$$SP500_t = c_1 + \emptyset_{11} SP500_{t-1} + e_{SP500,t}$$

With $u_{1t} = e_{SP500t}$ and $\sigma^2_{SP500,t} = \alpha_0 + \alpha_1 e^2_{SP500,t-1} + \beta_1 \sigma^2_{t-1}$

SP500 equation works as a univariate GARCH model. Considering table 11, variance equation has its coefficients statistically significant at 5% level in both subsamples. Hence, absolute returns of this stock market may display strong positive decaying autocorrelation which is in line with the stylized fact of volatility clustering.

Then, as this research intends to study the volatility spillovers from the stock market to the crypto market, a European stock market equation was performed under the same method as previous adding the error series of the previous equation. Hence, to model this series it was incorporated the lagged values of ***SP500***, and its residual series, previously saved. Thus,

$$DAX_t = c_2 + \emptyset_{21} SP500_{t-1} + \emptyset_{22} DAX_{t-1} + \vartheta_{21} e_{SP500,t} + e_{DAX,t}$$

With $\sigma^2_{DAX,t} = \alpha_0 + \alpha_1 e^2_{DAX,t-1} + \beta_1 \sigma^2_{DAX,t-1}$

As mentioned before, Q test of DAX equation before 2020 shows evidence of serial correlation in the residuals, found in figure 26. Hence, it is not appropriate to do inference in this equation in the first subsample. Looking to the output from the second subsample in tables 12 to 14, the lagged values of both stock markets are statistically significant in the DAX equation. This is not an expected outcome by the theory of efficient markets where all information is already incorporated in the prices. Furthermore, a *p-value* smaller than 0.05 rejects the null hypothesis of $\boldsymbol{\vartheta_{21} = 0}$ (see table 13) showing statistical evidence of volatility spillover effects from US market to the EU.

Finally, to understand the influence that crypto market is receiving from American and European stocks, Bitcoin equation was performed considering the residuals from ***SP500*** and ***DAX*** as well as its lagged values

$$\boldsymbol{BTC_t = c_3 + \emptyset_{31} SP500_{t-1} + \emptyset_{32} DAX_{t-1} + \emptyset_{33} BTC_{t-1} + \vartheta_{31} e_{SP500,t} + \vartheta_{32} e_{DAX,t} + e_{BTC,t}}$$

$$\text{With} \qquad \boldsymbol{\sigma^2_{BTC,t} = \alpha_0 + \alpha_1 e^2_{BTC,t-1} + \beta_1 \sigma^2_{BTC,t-1}}$$

Following the same reasoning of previous models, in ***BTC*** equation we reject the null hypothesis of no ARCH errors at 5% level considering table 12. Hence, there are statistical evidence of volatility clustering proving that large changes tend to be followed by large changes, of eighter sign, and small changes tend to be followed by small changes (Nicolau, 2012).

Nevertheless, in the previous equations it was not possible to observe different statistical evidence across the two periods. In the third equation of this triangular-GARCH model, there are significant differences. Before 2020, there are no statistical evidence of volatility spillovers from stock market (***SP500*** and ***DAX***) to ***BTC*** as we can see in table 11. Table 14 shows that from 2020, the index composed by 500 assets listed in the American stock market demonstrates volatility spillovers in the digital currency created in 2009. Therefore, the concept of co-movements among American stocks and ***BTC*** cannot be disregarded from the beginning of covid-19 crisis, implying that big movements in one market tend to be matched by big movements in another, in terms of volatility. Nonetheless, European stocks reveals absence of correlation in their absolute returns among these markets.

To understand the proportion of conditional variance of ***BTC*** caused by volatility spillover effects from the American stock market, I computed the Volatility Ratio which is obtained as (see, for example, Nicolau, 2012, for a similar exercise):

$$\boldsymbol{VR_t^{SP500,BTC}} = \left(\boldsymbol{\vartheta_{31}}^2\boldsymbol{\sigma_{SP500,t}^2}\right)/\left(\boldsymbol{\vartheta_{31}}^2\boldsymbol{\sigma_{SP500,t}^2} + \boldsymbol{\vartheta_{32}}^2\boldsymbol{\sigma_{DAX,t}^2} + \boldsymbol{\sigma_{BTC,t}^2}\right)$$

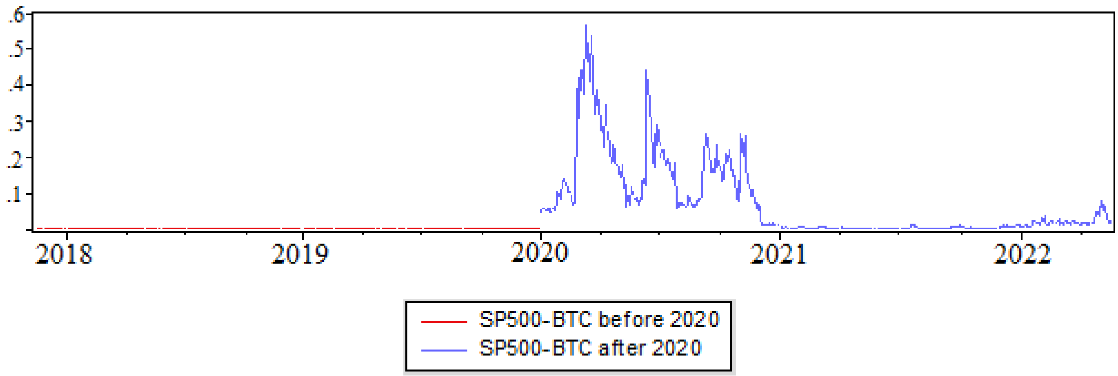

FIGURE 7- Conditional correlation coefficients in ***BTC***.

Looking to figure 7, is evident the abrupt change around 2020. The conditional correlation from ***SP500*** to ***BTC*** is revealed as almost null from 2017 until the end of 2019. In the beginning of the covid-19 crisis, there is an extraordinary volatility spillover effect from the American stock market to the Bitcoin. Yet, the year of 2020 transmitted significantly more volatility comparing with the onwards sample. This graph raises questions about volatility spillovers among the two markets in crisis periods. The sudden changes over the studied period may suggest a closed cycle rather than a new trend since the proportion of volatility spillovers to ***BTC*** caused by ***SP500*** tend to vanish from the end of 2020.

Lastly, Ethereum equation is performed as it is considered as the most endogenous variable. In fact, our sample only starts from 2017 due to availability of data of this variable. Hence, all the residuals of the previous equations, as well as the lagged values of the previous variables, are considered to model the second most valuable cryptocurrency, currently.

$$\boldsymbol{ETH_t = c_4 + \emptyset_{41} SP500_{t-1} + \emptyset_{42} DAX_{t-1} + \emptyset_{43} BTC_{t-1} + \emptyset_{44} ETH_{t-1} +}$$

$$\boldsymbol{\vartheta_{41} e_{SP500,t} + \vartheta_{42} e_{DAX,t} + \vartheta_{43} e_{BTC,t} + e_{ETC,t}}$$

$$\text{With} \quad \boldsymbol{\sigma^2_{ETH,t} = \alpha_0 + \alpha_1 e^2_{ETH,t-1} + \beta_1 \sigma^2_{ETH,t-1}}$$

Before 2020, Ethereum equation shows statistical significance of ***SP500(-1)***, ***DAX(-1)***, ***BTC(-1)***, and ***ETH(-1)*** at 5% level, leading us to question again market efficiency hypothesis. If current prices do not incorporate all expectations and relevant available information, then it is not proper to assume ***ETH*** as an efficient market. This conclusion reverses in the second subsample. From 2020, there is no statistical evidence that returns at time *t-1* helps to model ***ETH*** returns at time *t*. Therefore, from the beginning of covid-19 crisis, there is no evidence that this crypto coin reject the hypothesis of market efficiency which did not occurred in the first subsample.

Regarding non idiosyncratic shocks, in both subsamples ***SP500*** and ***BTC*** demonstrate, individually, volatility spillover effects over ***ETH***, at 5% significance level. Moreover, at 1% level it is possible to see that ***DAX*** returns demonstrate volatility spillovers over ***ETH*** in the second subsample. To observe graphically the evolution of the conditional variance proportion on ***ETH*** market caused by volatility spillovers of the other markets, I computed Volatility Ratios as:

$$\boldsymbol{VR_t^{SP500,ETH} = \frac{\left({\vartheta_{41}}^2 \sigma^2_{SP500,t}\right)}{{\vartheta_{41}}^2 \sigma^2_{SP500,t} + {\vartheta_{42}}^2 \sigma^2_{DAX,t} + {\vartheta_{43}}^2 \sigma^2_{BTC,t} + \sigma^2_{ETH,t}}}$$

$$\boldsymbol{VR_t^{BTC,ETH} = \frac{\left({\vartheta_{43}}^2 \sigma^2_{BTC,t}\right)}{{\vartheta_{41}}^2 \sigma^2_{SP500,t} + {\vartheta_{42}}^2 \sigma^2_{DAX,t} + {\vartheta_{43}}^2 \sigma^2_{BTC,t} + \sigma^2_{ETH,t}}}$$

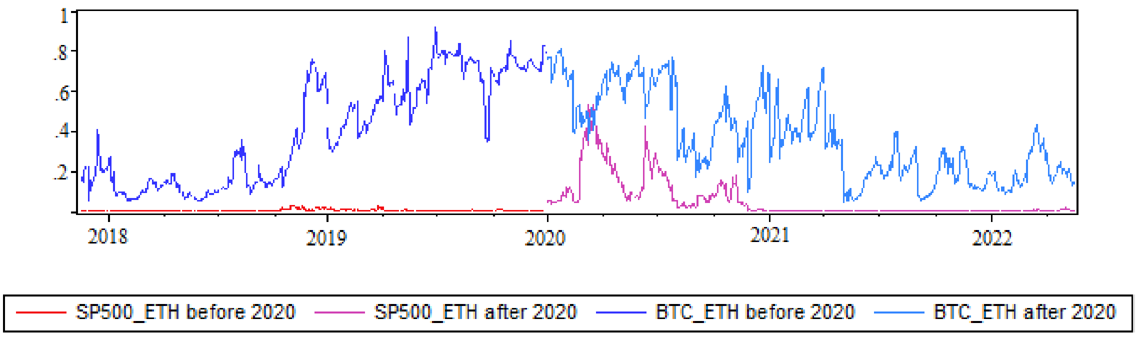

FIGURE 8- Conditional correlation coefficients in ***ETH***.

Looking to the red and pink, figure 8, conditional correlation coefficients from American stock market to ***ETH***, the conclusions are very similar as revealed in the previous equation in ***BTC***. There is an instant shift in the split of this sample. Furthermore, the year of 2020 presents remarkable high values of volatility spillovers in contrast with the sample afterwards. Hence, in this research I conclude that the ***SP500*** demonstrates exceptional volatility spillovers over the two studied crypto coins during most of the covid-19 crisis.

Nevertheless, the dark and light blue reveal the proportion of conditional variance to ***ETH*** triggered by the effect of volatility spillover from ***BTC*** as considerably greater. This volatility ratio does not show a consistent upward on downward trend. Yet, besides in the first subsample the very erratic series being in an ascending cycle, in the second subsample this pattern is not clearly seen, as it seems to be descending.

In conclusion, the triangular VAR-GARCH model was essential to interpret paths among stock and crypto markets. In addition to the fact of heteroskedasticity presented by the series being contemplated in the model, the process did not reject absence of serial correlation in the errors of almost all equations at all usual levels. This method uncovered that before 2020, ***SP500*** did not displayed volatility spillovers to ***BTC***. In the second half of the sample, the biggest world index of stock market transmitted volatility to both studied series of crypto market, at 5% significance level.

However, from volatility ratios was possible to observe evolution of volatility transmission throughout the sample. It is concluded that in the year of 2020 itself the proxy of the world market displays outstandingly massive values of volatility spillovers in crypto markets. Hence, this may imply that such effects do not represent a new trend, but simply part of a cycle. Therefore, I question if this an isolated effect of covid-19 crisis and all its particularities or if we shall expect more such episodes in future crisis. In fact, looking to figure 6, although not comparable with 2020 in scaling terms, volatility ratio of ***SP500*** in ***BTC*** shows spikes among 2022, a period marked by the invasion of Russia in Ukraine. Nonetheless, the conditional correlation coefficients in the volatility ratios from ***BTC*** to ***ETH*** are significantly higher throughout the whole sample.

6. CONCLUSION

Cryptocurrencies have been developing growing social and economic importance. Its aggravated volatility gives rise to strong price increases which attracts particular attention from investors. Besides the fact that there are no unanimous opinions about how to model this distinct financial series, as Lucas' critique mention, any change in policy will systematically alter the structure of econometric models, followed by changes in the structure of the series (Ericsson & Irons, 1995). Hence, the conclusions in this paper emerged.

In 2020, the covid-19 pandemic in late February, and in response to the economic repercussions of impending lockdown measures, investors began to demand higher-quality, safe assets (Baron, et al., 2022). This paper studies the unique features of data presented in a performed VAR. In both samples, the same conclusion was obtained: serial correlation in the errors followed by non-normality in their distribution, and ARCH effects. Then, it advocates an integrated approach to a triangular VAR-GARCH model. From here it is visible a new path characterized by new volatility spillovers from the stock market to the crypto market. However, volatility ratios may suggest that this new path of effects caused by volatility spillovers was exceptionally displayed by the year of 2020.

Nevertheless, my study is not free of limitations. To analyse properly the data, a VAR analysis was performed. Yet, in order to obtain a more robust model with no serial correlation presented in the residuals a VAR with more lags should be considered in future analysis. To solve this problem GARCH model was performed to be able to capture conditional heteroskedasticity where autocorrelation was no longer presented in the residues. Future work, should incorporate more GARCH features such as market expectations, change in the price of crude oil or the difference between the highest and lowest market prices over a fixed sampling interval.

A question remains: "Was this merely a stochastic cycle driven from the covid-19 crisis over 2020 or will stock market display volatility spillovers in crypto market in upcoming crisis?". Considering the extraordinary volatility ratios in 2020, it would be hard to achieve such values, nonetheless, this could be new research for a new report.

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APPENDIX

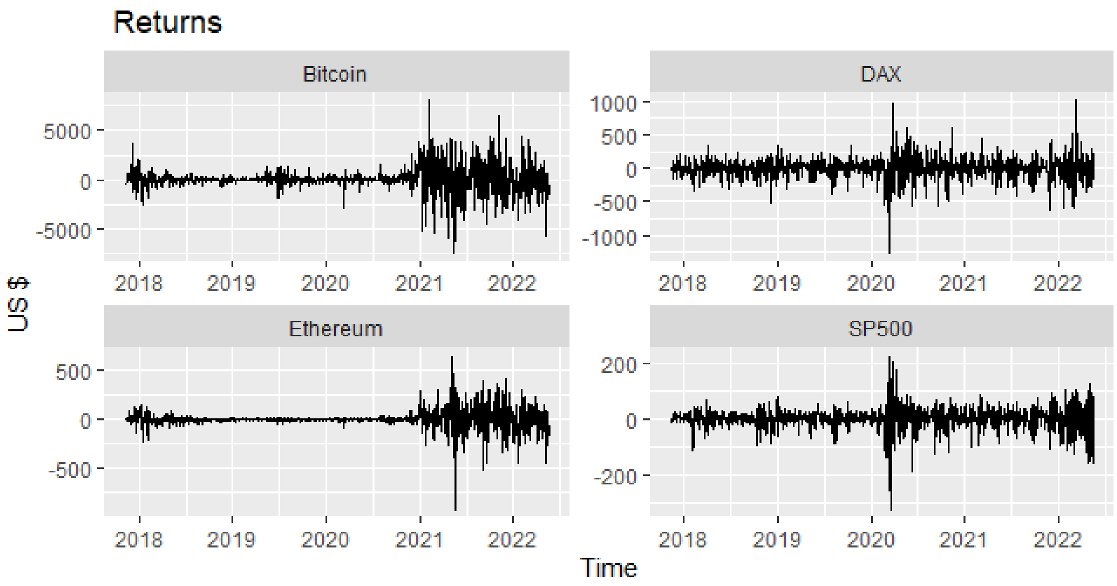

FIGURE 9– Returns Plots

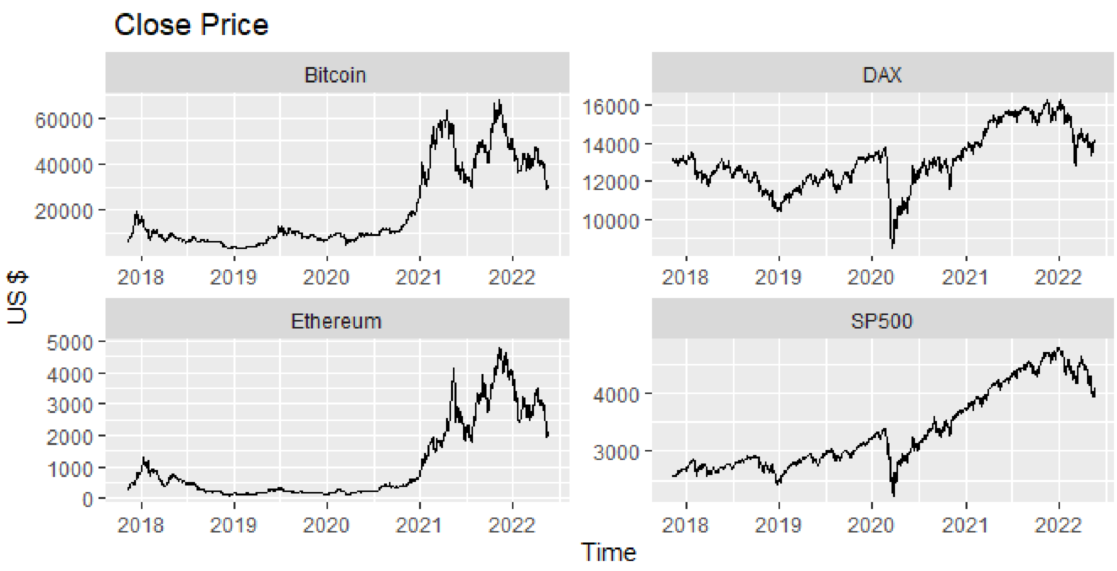

FIGURE 10– Close Price Plots

TABLE 15- INFORMATION CRITERIA BEFORE 2020

Criteria	1	2	3	4
AIC(n)	3.454479e+01	3.456004e+01	3.457544e+01	3.458400e+01
HQ(n)	3.460919e+01	3.467595e+01	3.474286e+01	3.480294e+01
SC(n)	3.470913e+01	3.485584e+01	3.500271e+01	3.514274e+01
FPE(n)	1.006037e+15	1.021506e+15	1.037388e+15	1.046366e+15

SP500	Residual standard error: 24.88 on 516 degrees of freedom Multiple R-Squared: 0.005857, Adjusted R-squared: -0.00185 F-statistic: 0.7599 on 4 and 516 DF, p-value: 0.5517
DAX	Residual standard error: 109.3 on 516 degrees of freedom Multiple R-Squared: 0.07501, Adjusted R-squared: 0.06784 F-statistic: 10.46 on 4 and 516 DF, p-value: 3.729e-08
BTC	Residual standard error: 499.9 on 516 degrees of freedom Multiple R-Squared: 0.01262, Adjusted R-squared: 0.004966 F-statistic: 1.649 on 4 and 516 DF, p-value: 0.1607
ETH	Residual standard error: 31.58 on 516 degrees of freedom Multiple R-Squared: 0.008786, Adjusted R-squared: 0.001102 F-statistic: 1.143 on 4 and 516 DF, p-value: 0.3352

FIGURE 11 – Estimation results before 2020 of the VAR(1) Equations

TABLE 16- INVERSE ROOTS OF THE MODELS BEFORE 2020

Cause	***SP500***(-1)	***DAX*** (-1)	***BTC*** (-1)	***ETH***(-1)
VAR(1)	0.21057871	0.10522646	0.08190203	0.05301545

```
> for (j in 2:5){
+   show(vars::serial.test(VAR1, lags.pt = j,
+                  type = "PT.asymptotic"))}

        Portmanteau Test (asymptotic)

data:  Residuals of VAR object VAR1
Chi-squared = 26.841, df = 16, p-value = 0.04328

        Portmanteau Test (asymptotic)

data:  Residuals of VAR object VAR1
Chi-squared = 47.363, df = 32, p-value = 0.03933

        Portmanteau Test (asymptotic)

data:  Residuals of VAR object VAR1
Chi-squared = 67.58, df = 48, p-value = 0.03264

        Portmanteau Test (asymptotic)

data:  Residuals of VAR object VAR1
Chi-squared = 103.8, df = 64, p-value = 0.001217
```

```
> for (j in 2:5){
+   show(vars::serial.test(VAR1, lags.pt = j,
+                  type = "PT.adjusted"))}

        Portmanteau Test (adjusted)

data:  Residuals of VAR object VAR1
Chi-squared = 26.944, df = 16, p-value = 0.04211

        Portmanteau Test (adjusted)

data:  Residuals of VAR object VAR1
Chi-squared = 47.584, df = 32, p-value = 0.03755

        Portmanteau Test (adjusted)

data:  Residuals of VAR object VAR1
Chi-squared = 67.958, df = 48, p-value = 0.03046

        Portmanteau Test (adjusted)

data:  Residuals of VAR object VAR1
Chi-squared = 104.53, df = 64, p-value = 0.001041
```

FIGURE 12 – Portemanteau test of residuals from VAR(1) before 2020

```
> for (j in 2:5){
+   show(vars::serial.test(VAR1, lags.bg = j,
+                  type = "BG"))}

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR1
Chi-squared = 39.144, df = 32, p-value = 0.1798

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR1
Chi-squared = 62.941, df = 48, p-value = 0.07257

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR1
Chi-squared = 87.744, df = 64, p-value = 0.02611

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR1
Chi-squared = 115.68, df = 80, p-value = 0.005608
```

```
> for (j in 2:5){
+   show(vars::serial.test(VAR1, lags.bg = j,
+                  type = "ES"))}

        Edgerton-Shukur F test

data:  Residuals of VAR object VAR1
F statistic = 1.2196, df1 = 32, df2 = 1863, p-value = 0.1859

        Edgerton-Shukur F test

data:  Residuals of VAR object VAR1
F statistic = 1.3111, df1 = 48, df2 = 1931, p-value = 0.07591

        Edgerton-Shukur F test

data:  Residuals of VAR object VAR1
F statistic = 1.3739, df1 = 64, df2 = 1947, p-value = 0.02782

        Edgerton-Shukur F test

data:  Residuals of VAR object VAR1
F statistic = 1.4676, df1 = 80, df2 = 1947, p-value = 0.005046
```

FIGURE 13– Breusch-Godfrey test of residuals from VAR(1) before 2020

```
> vars::normality.test(VAR1, multivariate.only = TRUE)
$JB

        JB-Test (multivariate)

data:  Residuals of VAR object VAR1
Chi-squared = 8386, df = 8, p-value < 2.2e-16


$Skewness

        Skewness only (multivariate)

data:  Residuals of VAR object VAR1
Chi-squared = 235.33, df = 4, p-value < 2.2e-16


$Kurtosis

        Kurtosis only (multivariate)

data:  Residuals of VAR object VAR1
Chi-squared = 8150.7, df = 4, p-value < 2.2e-16
```

FIGURE 14– Normality test of residuals from VAR(1) before 2020

```
> for (j in 1:5){
+   show(vars::arch.test(VAR1, lags.multi = j, multivariate.only = TRUE))
+ }

        ARCH (multivariate)

data:  Residuals of VAR object VAR1
Chi-squared = 551.11, df = 100, p-value < 2.2e-16


        ARCH (multivariate)

data:  Residuals of VAR object VAR1
Chi-squared = 746.53, df = 200, p-value < 2.2e-16


        ARCH (multivariate)

data:  Residuals of VAR object VAR1
Chi-squared = 1013, df = 300, p-value < 2.2e-16


        ARCH (multivariate)

data:  Residuals of VAR object VAR1
Chi-squared = 1317.2, df = 400, p-value < 2.2e-16


        ARCH (multivariate)

data:  Residuals of VAR object VAR1
Chi-squared = 1539.7, df = 500, p-value < 2.2e-16
```

FIGURE 15 – ARCH effects of residuals from VAR(1) before 2020

TABLE 17- INFORMATION CRITERIA FROM 2020

Criteria	1	2	3	4
AIC(n)	4.174908e+01	4.173154e+01	4.173328e+01	4.174873e+01
HQ(n)	4.180732e+01	4.183638e+01	4.188472e+01	4.194677e+01
SC(n)	4.189853e+01	4.200056e+01	4.212187e+01	4.225689e+01
FPE(n)	1.353300e+18	1.329786e+18	1.332132e+18	1.352925e+18

SP500	Residual standard error: 52.14 on 584 degrees of freedom Multiple R-Squared: 0.0403, Adjusted R-squared: 0.03373 F-statistic: 6.131 on 4 and 584 DF, p-value: 7.756e-05
DAX	Residual standard error: 199.5 on 584 degrees of freedom Multiple R-Squared: 0.02661, Adjusted R-squared: 0.01994 F-statistic: 3.991 on 4 and 584 DF, p-value: 0.003332
BTC	Residual standard error: 1629 on 584 degrees of freedom Multiple R-Squared: 0.0122, Adjusted R-squared: 0.005433 F-statistic: 1.803 on 4 and 584 DF, p-value: 0.1267
ETH	Residual standard error: 129.8 on 584 degrees of freedom Multiple R-Squared: 0.02451, Adjusted R-squared: 0.01783 F-statistic: 3.668 on 4 and 584 DF, p-value: 0.00582

FIGURE 16 – Estimation results after 2020 of the VAR(1) equations.

```
> for (j in 2:5){
+   show(vars::serial.test(VAR1, lags.pt = j,
+             type = "PT.asymptotic"))}

        Portmanteau Test (asymptotic)

data:  Residuals of VAR object VAR1
Chi-squared = 41.757, df = 16, p-value = 0.0004288

        Portmanteau Test (asymptotic)

data:  Residuals of VAR object VAR1
Chi-squared = 74.712, df = 32, p-value = 2.865e-05

        Portmanteau Test (asymptotic)

data:  Residuals of VAR object VAR1
Chi-squared = 90.928, df = 48, p-value = 0.0001814

        Portmanteau Test (asymptotic)

data:  Residuals of VAR object VAR1
Chi-squared = 123.8, df = 64, p-value = 1.082e-05
```

```
> for (j in 2:5){
+   show(vars::serial.test(VAR1, lags.pt = j,
+             type = "PT.adjusted"))}

        Portmanteau Test (adjusted)

data:  Residuals of VAR object VAR1
Chi-squared = 41.899, df = 16, p-value = 0.0004086

        Portmanteau Test (adjusted)

data:  Residuals of VAR object VAR1
Chi-squared = 75.021, df = 32, p-value = 2.604e-05

        Portmanteau Test (adjusted)

data:  Residuals of VAR object VAR1
Chi-squared = 91.349, df = 48, p-value = 0.0001627

        Portmanteau Test (adjusted)

data:  Residuals of VAR object VAR1
Chi-squared = 124.5, df = 64, p-value = 9.034e-06
```

FIGURE 17- Portemanteau test of residuals from VAR(1) after 2020

```
> for (j in 2:5){
+   show(vars::serial.test(VAR1, lags.bg = j,
+             type = "BG"))}

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR1
Chi-squared = 64.328, df = 32, p-value = 0.0006025

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR1
Chi-squared = 85.268, df = 48, p-value = 0.0007441

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR1
Chi-squared = 102.18, df = 64, p-value = 0.001712

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR1
Chi-squared = 149.2, df = 80, p-value = 4.352e-06
```

```
> for (j in 2:5){
+   show(vars::serial.test(VAR1, lags.bg = j,
+             type = "ES"))}

        Edgerton-Shukur F test

data:  Residuals of VAR object VAR1
F statistic = 2.0435, df1 = 32, df2 = 2114, p-value = 0.00052

        Edgerton-Shukur F test

data:  Residuals of VAR object VAR1
F statistic = 1.8016, df1 = 48, df2 = 2193, p-value = 0.0006701

        Edgerton-Shukur F test

data:  Residuals of VAR object VAR1
F statistic = 1.6175, df1 = 64, df2 = 2214, p-value = 0.001576

        Edgerton-Shukur F test

data:  Residuals of VAR object VAR1
F statistic = 1.9151, df1 = 80, df2 = 2215, p-value = 2.982e-06
```

FIGURE 18 – Breusch-Godfrey test of residuals from VAR(1) after 2020

```
> vars::normality.test(VAR1, multivariate.only = TRUE)
$JB

        JB-Test (multivariate)

data:  Residuals of VAR object VAR1
Chi-squared = 5054.5, df = 8, p-value < 2.2e-16


$Skewness

        Skewness only (multivariate)

data:  Residuals of VAR object VAR1
Chi-squared = 119.32, df = 4, p-value < 2.2e-16


$Kurtosis

        Kurtosis only (multivariate)

data:  Residuals of VAR object VAR1
Chi-squared = 4935.2, df = 4, p-value < 2.2e-16
```

FIGURE 19 – Normality test of residuals from VAR(1) after 2020

```
> for (j in 1:5){
+   show(vars::arch.test(VAR1, lags.multi = j, multivariate.only = TRUE))
+ }

        ARCH (multivariate)

data:  Residuals of VAR object VAR1
Chi-squared = 259.62, df = 100, p-value = 4.441e-16

        ARCH (multivariate)

data:  Residuals of VAR object VAR1
Chi-squared = 686.03, df = 200, p-value < 2.2e-16

        ARCH (multivariate)

data:  Residuals of VAR object VAR1
Chi-squared = 981.02, df = 300, p-value < 2.2e-16

        ARCH (multivariate)

data:  Residuals of VAR object VAR1
Chi-squared = 1155.4, df = 400, p-value < 2.2e-16

        ARCH (multivariate)

data:  Residuals of VAR object VAR1
Chi-squared = 1356.8, df = 500, p-value < 2.2e-16
```

FIGURE 20 – ARCH effects of residuals from VAR(1) after 2020

SP500	Residual standard error: 51.32 on 579 degrees of freedom Multiple R-Squared: 0.07815, Adjusted R-squared: 0.06541 F-statistic: 6.135 on 8 and 579 DF, p-value: 1.311e-07
DAX	Residual standard error: 199.9 on 579 degrees of freedom Multiple R-Squared: 0.03054, Adjusted R-squared: 0.01715 F-statistic: 2.28 on 8 and 579 DF, p-value: 0.02084
BTC	Residual standard error: 1635 on 579 degrees of freedom Multiple R-Squared: 0.01344, Adjusted R-squared: -0.0001912 F-statistic: 0.986 on 8 and 579 DF, p-value: 0.4458
ETH	Residual standard error: 129.6 on 579 degrees of freedom Multiple R-Squared: 0.03521, Adjusted R-squared: 0.02188 F-statistic: 2.641 on 8 and 579 DF, p-value: 0.007525

FIGURE 21– Estimation results after 2020 of the VAR(2) equations.

```
> for (j in 3:5){
+   show(vars::serial.test(VAR2, lags.pt = j,
+                  type = "PT.asymptotic"))}

        Portmanteau Test (asymptotic)

data:  Residuals of VAR object VAR2
Chi-squared = 39.265, df = 16, p-value = 0.0009959

        Portmanteau Test (asymptotic)

data:  Residuals of VAR object VAR2
Chi-squared = 51.44, df = 32, p-value = 0.01612

        Portmanteau Test (asymptotic)

data:  Residuals of VAR object VAR2
Chi-squared = 80.788, df = 48, p-value = 0.002136
```

```
> for (j in 3:5){
+   show(vars::serial.test(VAR2, lags.pt = j,
+                  type = "PT.adjusted"))}

        Portmanteau Test (adjusted)

data:  Residuals of VAR object VAR2
Chi-squared = 39.461, df = 16, p-value = 0.0009327

        Portmanteau Test (adjusted)

data:  Residuals of VAR object VAR2
Chi-squared = 51.72, df = 32, p-value = 0.01511

        Portmanteau Test (adjusted)

data:  Residuals of VAR object VAR2
Chi-squared = 81.319, df = 48, p-value = 0.001891
```

FIGURE 22– Portemanteau test of residuals from VAR(2) after 2020

```
> for (j in 3:5){
+   show(vars::serial.test(VAR2, lags.bg = j,
+                  type = "BG"))}

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR2
Chi-squared = 71.355, df = 48, p-value = 0.01596

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR2
Chi-squared = 97.007, df = 64, p-value = 0.004876

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR2
Chi-squared = 137.16, df = 80, p-value = 7.362e-05
```

```
> for (j in 3:5){
+   show(vars::serial.test(VAR2, lags.bg = j,
+                  type = "ES"))}

        Edgerton-Shukur F test

data:  Residuals of VAR object VAR2
F statistic = 1.4792, df1 = 48, df2 = 2174, p-value = 0.01849

        Edgerton-Shukur F test

data:  Residuals of VAR object VAR2
F statistic = 1.5162, df1 = 64, df2 = 2194, p-value = 0.005606

        Edgerton-Shukur F test

data:  Residuals of VAR object VAR2
F statistic = 1.7459, df1 = 80, df2 = 2195, p-value = 6.381e-05
```

FIGURE 23 – Breusch-Godfrey test of residuals from VAR(2) after 2020

```
> vars::normality.test(VAR2, multivariate.only = TRUE)
$JB

        JB-Test (multivariate)

data:  Residuals of VAR object VAR2
Chi-squared = 4174.8, df = 8, p-value < 2.2e-16


$Skewness

        Skewness only (multivariate)

data:  Residuals of VAR object VAR2
Chi-squared = 91.573, df = 4, p-value < 2.2e-16


$Kurtosis

        Kurtosis only (multivariate)

data:  Residuals of VAR object VAR2
Chi-squared = 4083.2, df = 4, p-value < 2.2e-16
```

FIGURE 24 – Normality test of residuals from VAR(2) after 2020

```
> for (j in 1:5){
+   show(vars::arch.test(VAR2, lags.multi = j, multivariate.only = TRUE))
+ }

        ARCH (multivariate)

data:  Residuals of VAR object VAR2
Chi-squared = 278.02, df = 100, p-value < 2.2e-16


        ARCH (multivariate)

data:  Residuals of VAR object VAR2
Chi-squared = 626.8, df = 200, p-value < 2.2e-16


        ARCH (multivariate)

data:  Residuals of VAR object VAR2
Chi-squared = 918.67, df = 300, p-value < 2.2e-16


        ARCH (multivariate)

data:  Residuals of VAR object VAR2
Chi-squared = 1089, df = 400, p-value < 2.2e-16


        ARCH (multivariate)

data:  Residuals of VAR object VAR2
Chi-squared = 1279.1, df = 500, p-value < 2.2e-16
```

FIGURE 25– ARCH effects of residuals from VAR(2) after 2020

TABLE 18- INVERSE ROOTS OF THE MODELS FROM 2020

Cause	VAR(1)	VAR(2)
SP500(-1)	0.27247653	0.3947078
DAX (-1)	0.10270387	0.3947078
BTC (-1)	0.10270387	0.3250250
ETH (-1)	0.01972924	0.3227011
SP500 (-2)	N\A	0.2745544
DAX (-2)	N\A	0.1571209
BTC (-2)	N\A	0.1571209
ETH (-2)	N\A	0.1126041

TABLE 19- INSTANTANEOUS CAUSALITY TESTS IN VAR(1) BEFORE 2020

Cause	Chi-Squared[a]	p-value
SP500	122.79	2.2e-16
DAX	119.76	2.2e-16
Bitcoin	105.14	2.2e-16
ETH	109.08	2.2e-16
SP500, DAX	10.369	0.03465
SP500, Bitcoin	224.58	2.2e-16
DAX, ETH	224.24	2.2e-16
DAX, Bitcoin	224.58	2.2e-16
Bitcoin, ETH	10.369	0.03465

(a) Bootstrap runs: 10000.

TABLE 20- INSTANTANEOUS CAUSALITY TESTS IN VAR(2) FROM 2020

Cause	Chi-Squared[a]	p-value
SP500	123.05	2.2e-16
DAX	119.85	2.2e-16
Bitcoin	104.78	2.2e-16
ETH	108.87	2.2e-16
SP500, DAX	10.685	8.217e-11
SP500, Bitcoin	224.09	2.2e-16
DAX, ETH	224.09	2.2e-16
DAX, Bitcoin	224.36	2.2e-16
Bitcoin, ETH	10.685	8.217e-11

(a) Bootstrap runs: 10000.

<table>
<tr><th></th><th>Before 2020</th><th>From 2020</th></tr>
<tr><td>SP500</td>
<td>Date: 10/05/22 Time: 17:36
Sample: 11/09/2017 12/30/2019
Included observations: 521
Q-statistic probabilities adjusted for 1 dynamic regressor
<table>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob*</th></tr>
<tr><td></td><td></td><td>1</td><td>0.054</td><td>0.054</td><td>1.5232</td><td>0.217</td></tr>
<tr><td></td><td></td><td>2</td><td>-0.007</td><td>-0.009</td><td>1.5455</td><td>0.462</td></tr>
<tr><td></td><td></td><td>3</td><td>0.032</td><td>0.033</td><td>2.0941</td><td>0.553</td></tr>
<tr><td></td><td></td><td>4</td><td>0.012</td><td>0.009</td><td>2.1713</td><td>0.704</td></tr>
<tr><td></td><td></td><td>5</td><td>-0.036</td><td>-0.036</td><td>2.8380</td><td>0.725</td></tr>
</table>
*Probabilities may not be valid for this equation specification.</td>
<td>Date: 10/05/22 Time: 17:32
Sample: 1/02/2020 5/18/2022
Included observations: 591
Q-statistic probabilities adjusted for 1 dynamic regressor
<table>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob*</th></tr>
<tr><td></td><td></td><td>1</td><td>0.025</td><td>0.025</td><td>0.3792</td><td>0.538</td></tr>
<tr><td></td><td></td><td>2</td><td>0.042</td><td>0.041</td><td>1.4283</td><td>0.490</td></tr>
<tr><td></td><td></td><td>3</td><td>-0.043</td><td>-0.046</td><td>2.5513</td><td>0.466</td></tr>
<tr><td></td><td></td><td>4</td><td>-0.034</td><td>-0.034</td><td>3.2374</td><td>0.519</td></tr>
<tr><td></td><td></td><td>5</td><td>-0.011</td><td>-0.006</td><td>3.3105</td><td>0.652</td></tr>
</table>
*Probabilities may not be valid for this equation specification.</td></tr>
<tr><td>DAX</td>
<td>Date: 10/08/22 Time: 16:00
Sample: 11/09/2017 12/30/2019
Included observations: 521
Q-statistic probabilities adjusted for 1 dynamic regressor
<table>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob*</th></tr>
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*Probabilities may not be valid for this equation specification.</td>
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Sample: 1/02/2020 5/18/2022
Included observations: 591
Q-statistic probabilities adjusted for 1 dynamic regressor
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Included observations: 521
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FIGURE 26- Q tests of equations from VAR-GARCH model