



LISBON  
SCHOOL OF  
ECONOMICS &  
MANAGEMENT  
UNIVERSIDADE DE LISBOA



## **MASTER**

MASTER'S IN MATHEMATICAL FINANCE

### **MASTER'S FINAL WORK**

INTERNSHIP REPORT

TRAINEESHIP WITHIN A BROKERAGE FIRM:  
ANALYSIS OF THE FUNCTIONING OF INSURANCE COMPANIES  
AND THEIR LINK TO EXTERNAL BROKERS

ANNE-SOPHIE MARGUERITE VIENNE

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#### **SUPERVISION :**

PROFESSOR TIAGO CARDÃO-PITO

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A big thank you to my mother and father for their advice, as well as for their unconditional support, both moral and economic.

## Introduction

As part of my Master's degree in Mathematical Finance, at ISEG Lisbon School of Economics & Management, I have developed an internship of three months and a half in the society Groupement Interprofessionnel Européen d'Assurances 974, henceforth referred to a GIEA. My internship was conducted in Reunion Island, a French island. This document contains my internship report.

The principle of insurance is based on the concept of risk, which means the exposure to a potential danger inherent in a situation or activity and the financial consequences of which could not be faced, whether related to property or persons (Assurance et mutuelle. 2021). In that way, insurance companies and brokerage firms work in collaboration to enable people to be covered in every situation. From its side, insurance companies will make different computations regarding each risk to create contracts for customers. It must have a balance between the probabilities of compensating the client and the goal of the insurance company that is of making money. (Frank Jones. 2019) Thereafter, the brokerage firm will select the best contracts according to the needs of the clients. Usually, a brokerage firm works with different insurance companies. It is the case of the GIEA which works with about fifty companies. Currently, the GIEA works in collaboration with the insurance company SwissLife to create the individual person contract. Indeed, the SwissLife company SwissLife has created a basic provident insurance contract and the GIEA had the opportunity to adjust guarantees. This contract is exclusively proposed by this brokerage firm, and it is called the SwissLife Prévoyance Indépendant or SLPI. Also, the GIEA proposes pure SwissLife contracts: it is usually the case for the complementary pension contract called PERin contract, that I decided to study in a part of this report.

Furthermore, the GIEA is not only a brokerage firm but also a French office of review and analysis. An experts group realizes studies of different insurance companies and their contracts before selection. For example, in the case of the mutual health contract, they selected another insurance company: the April company.

Finally, even if there is no common point between my studies and the missions I realized at the GIEA company, I choose to realize this internship because I wanted to gain experience and to develop skills. It has allowed me, not only to discover one profession but also different activities.

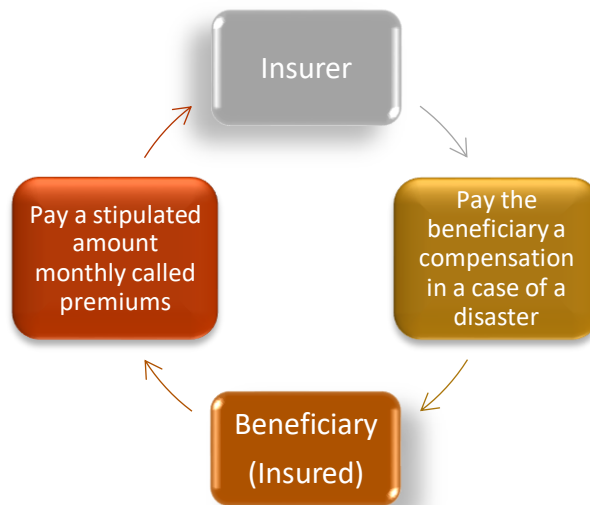
## 1. How insurance companies work

### 1. 1. Structure and fundamentals of an insurance company

AM Best (2019), in the *Best's Guide to Understanding The insurance company*, defines insurance as a financial protection against all types of risk that a person or entity may encounter during its existence. There are all kinds of insurance: for the car, for life, for everything related to professional life and so on. It is the operation whereby an insurer organizes on a mutual basis a multitude of insureds, exposed to the realization of certain risks, and compensates those of them who suffer a claim through the common mass of premiums collected. Those who do not have insurance must take full responsibility for their loss and can spend all their money there, which can lead to ruin.

Frank Fabozzi and Frank Jones (2019), in their book *"Foundations of Global Financial Markets and Institutions"*, state that insurance companies provide legal contracts between two parties: the insurer and the insured. According to the contract, the insurer promises to compensate the insured in the case of financial losses due to a specific event or disaster in return for a contribution called premiums paid by the insured periodically. In insurance industry, the risk is then the damaging event against the arrival of which one seeks to protect oneself and therefore corresponds to the insured event. For a risk to be insurable, it must necessarily meet three conditions: it must be future, uncertain, and independent of the will of the insured (Assurance et mutuelle. 2021). By realizing a contract, the policyholder underwrites the risk at the insurance company to protect what is important according to him. The insurer then carries out the pooling of risks using the law of large numbers and the laws

of statistics. In that respect, insurance is said to be a risk transfer mechanism. Figure 1 summarizes the basic operation of an insurance contract:



*Figure 1: general scheme of an insurance – Adapted from: Béhar Thomas (2011)*

To use less technical language and to quote a sentence of Queen Elizabeth I of England, it can be said that “insurance has been established so that the loss weighs slightly on many, rather than heavily on few”. (Assurance et mutuelle. 2021)

Sometimes, the contract can include four parties (Ménart Stéphane. 2011):

- the one who runs the risk: the insured,
- the one who signs the contract: the underwriter,
- the one who receives the benefit: the beneficiary,
- and the insurer, who is the legal person who accepts the risk, collects the contributions and settles the claims.

For example, in the case of death insurance, the underwriter will be different from the beneficiary who will receive the compensation. But for preventive healthcare, the underwriter is the one who is insured and who receives the allowances. Usually, the compensation is in form of capital or annuity.

Frank Fabozzi and Frank Jones (2019) describe that insurance companies, generally have at least three internal structures, namely,

- The home office is called the manufacturer and guarantor of the insurance policy: it conceives insurance contracts and provides finances to pay the compensation to the beneficiary of the contract. Some home offices hire external actuarial firms to do the computations of the contract.
- The investment company: it invests premiums collected in the investment portfolio by diversifying.
- The distribution component or sales force: they are the ones who sell the contracts. Insurance companies themselves have the possibility of selling their contracts through their general distribution agent named agent of the insurance company. The latter is in charge of finding customers and can possibly take care of the management of insurance operations. Then there are insurance brokers who operate as independent traders, such as the GIEA company which is an insurance brokerage firm. The broker is seen as the agent of the insured, of which he is the agent vis-à-vis the insurer. It is generally remunerated by commission and bonuses and sometimes by fees since it acts as a service provider. Banks and some websites also have insurance contracts (Jean-François Carlot. 2021).

Furthermore, there exist two categories of insurance (Jean-François Carlot. 2021):

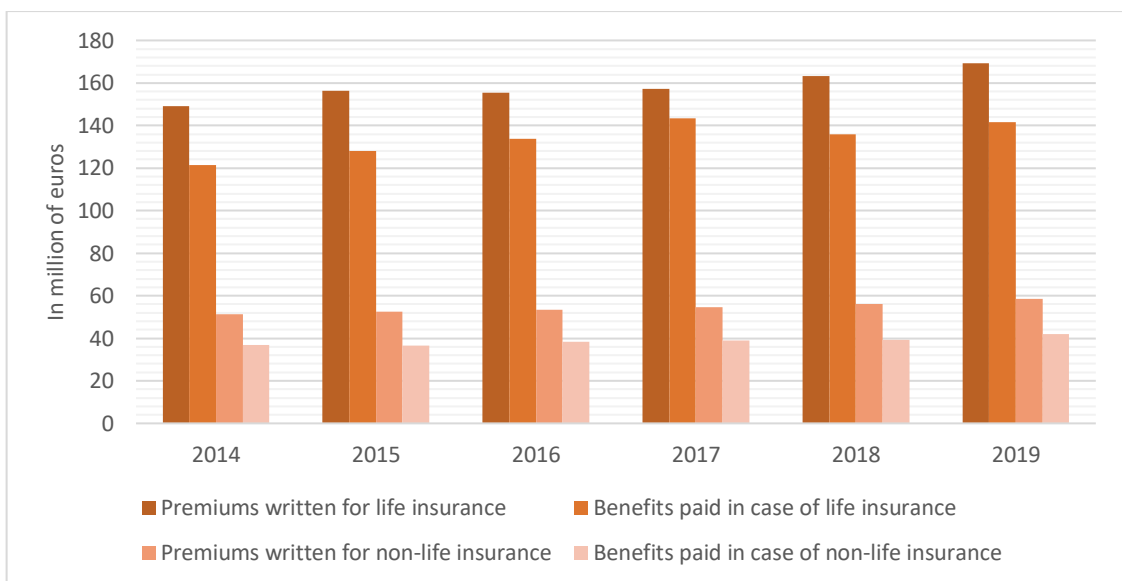
- Short-term business or non-life insurance insures property, casualty, personal accident, and health. It includes property insurance like property loss, fire, natural risks, company risks. It also involves liabilities insurances, namely responsibility for personal actions, responsibility for the things you have in your custody or for the people you must answer for, debts of contractual liabilities, and professional risks within the company. They usually are a pay-as-you-go system. In this method of managing contributions, the insurer uses the mass of premiums paid by all insureds during the same financial year to compensate those of them who will be affected. Usually, the insurer pays the claims of the year with the premiums of the year. Insurance is therefore the organisation of solidarity between insureds against the



occurrence of the same type of event. According to this principle, if the risk increases, the contract tariff increases, and, if the risk decreases, the tariff decreases.

- On the other side, long-term business or life insurances depend on the human lifespan. This category includes providences like disease, disability, retirement, and/or company pension. It includes heritage management too such as life insurance, investment funds, or transfer of small and medium-sized enterprises (SMEs). They usually use the capitalization method: the insurer is committed to growing the premiums collected according to the technique of compound interest. In that sense, they are used to compensate a special sinister when this one occurs.

Insurances make money in two ways: from financial investment and by generating an underwriting profit (Frank Fabozzi and Frank Jones. 2019). According to the French insurance federation:



*Figure 2: life and non-life insurance direct premiums written in France compared to the entitlements from 2014 to 2019 – Adapted from: Fédération Française de l'Assurance (2020)*

On this figure, we can notice a few things: the premiums written between 2014 and 2019 were more important in the case of life insurance contracts. Secondly, during this period, the premiums written is always greater than the benefits paid in case of a sinister, so the result

of French insurance companies was always positive. We also note that the number of subscriptions increased between 2019 and 2014.

## 1. 2. Difference between short-term and long-term businesses

Following figure 1, if the insurance company insures one person, the compensation this one could receive can be zero if there is no sinister. Otherwise, it could be positive if the risk is realized. In that case, the insurer must pay the compensation to the beneficiaries. If the sum of premiums is lower than the amount of the allowance, the insurer goes down the drain. To avoid ruin, the insurance company must sign a large number of persons to obtain enough premiums to be able to provide financial compensation in a case of the realization of the considered risk by the contract. According to Thomas, Michel, and Stéphane (2011), this rule follows a binomial law.

Generally, we know the price of the base material, called purchase price, before the sale price. In the case of insurance, the amount of premiums is collected before knowing the amount of damage to be compensated: it is an inverted production cycle. To compute the income statement, we subtract the charges from the product

YIELD (=revenue: premiums collected)  
- FINANCE CHARGES (=expenditure: compensation to pay, running costs of the company)  
= RESULT

This calculation explains the past results and can be used to anticipate future results.

Now, let's have a new data to consider: time. The figure 1 becomes:

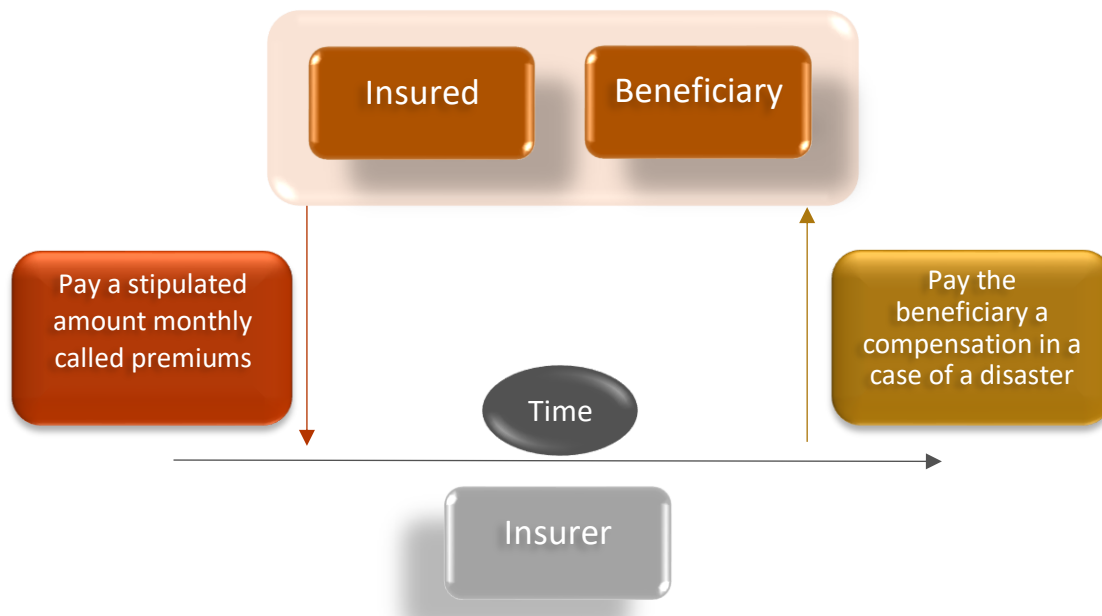


Figure 3: More detailed scheme of an insurance – Adapted from: Béhar Thomas (2011)

Depending on the nature of the contract, the result of the insurer will differ.

Béhar Thomas, Fromenteau Michel and Ménart Stéphane (2011) define:

- $n_a$  the number of insureds
- $\pi_i''$  the commercial premium with pure premium  $\pi_i$
- $c$  the capital to pay to policyholder following a disaster
- $p$  the probability of the sinister
- $R_{n_a}$  the result of the insurer

They take the case of a simple short-term risk with high volatility, like death insurance. For now, they do not consider the management fees. They suppose that the pure prime is the same for all insureds and that the company agrees to pay a capital if a death occurs during a short period (less than one year). The number of deaths  $N_d$  is unknown and we search to estimate the result of the insurer. To achieve this result, we must subtract the number of benefits paid by the insurer from the amount of premiums received and they obtain:

$$R_{n_a} = n_a \pi - c N_d$$

With expected return and volatility:

$$E(R_{n_a}) = n_a \pi - c n_a p$$

$$\sigma(R_{n_a}) = c \sigma(N_d) = c \sqrt{n_a p (1 - p)}$$

As we notice, the result of the insurer  $R_{n_a}$  follows a binomial law\* with parameters  $n_a$  and  $p$ .

If they take into consideration the management fees ( $MF$ ) necessary for the well being of the company and they add the financial products ( $FP$ ) obtained from financial investments, the result will be:

$$R_{n_a} = n_a \pi'' + FP - c N_d - MF$$

As the expected return and the volatility are independent of management fees and financial products, their computations do not change.

Now let's see the case of a simple long-term risk at low volatility: life insurance. From the time  $t=0$  to de  $k$  number of years, the company has the time to invest the premiums collected to make money. To compute the final value  $FV$  of the initial premium  $IP$ , we do:

$$FV = IP(1 + r)^k$$

With here,  $IP = n_a \pi$ .

Here they suppose that the company covenant to pay a capital to those who are still alive after  $k$  years. We define:

- $N_s$  = number of survivors
- $q$  = probability of being alive after  $k$  years, closed to 1

with  $E(N_s) = n_a q$

The difference with the short-term contract is that in the case of long-term insurance the result takes into consideration the financial investment realized over the  $k$  years:

$$R_{n_a} = n_a \pi \times (1 + r)^k - c N_s$$

With  $E(R_{n_a}) = n_a \pi \times (1 + r)^k - c n_a q$

Hence, we notice it is easier to do an estimation of the result in the case of non-life insurance insures property and casualty as the time does not interfere in the final computation compared to the long-term business.

## 1. 3. General studies of a contract

### 1. 3. 1) Probabilities used to approximate the compensation

Now let's have (Béhar Thomas. 2011):

- $i$  the  $i$ -th insured
- $i \in [1; n_a]$
- The  $i$ -th insured pays a commercial premium  $\pi_i''$  with pure premium  $\pi_i$
- $X_i$  the financial compensation the  $i$ -th insured could receive

To recapitulate, the insurer subscribes  $n_a$  contracts and so, his yield is the sum of the  $n_a$  premiums  $\sum_{i=1}^{n_a} \pi_i''$  and it is known in advance. Moreover, he must take into consideration the possible compensation to pay to the beneficiaries of the contracts  $\sum_{i=1}^{n_a} X_i$  which are unknown, and the net management fees (NMF). To find the insurer's result, we must subtract the total amount of benefits payable and management fees from the amount of premiums received (Béhar Thomas. 2011):

$$R_{n_a} = \sum_{i=1}^{n_a} \pi_i'' - \sum_{i=1}^{n_a} X_i - NMF$$

By the way, by taking  $n_a$  identically and independently distributed, they find the expectation of the amount of compensation to pay and its volatility:

$$\forall i = 1, \dots, n_a, E(X_i) = E(X) \text{ and } \sigma(X_i) = \sigma(X)$$

And so (Annex 1; 1):

$$E\left(\sum_{i=1}^{n_a} X_i\right) = n_a E(X)$$

$$\sigma\left(\sum_{i=1}^{n_a} X_i\right) = \sqrt{n_a}\sigma(X)$$

Disasters' annual charges have as expectation  $n_a E(X)$  and standard deviation  $\sqrt{n_a}\sigma(X)$ .

Then, to be able of computing probabilities and estimation of the result, Béhar Thomas, Michel Fromenteau And Stéphane Ménart (2011) use the central limit theorem\*, and so the uniform central distribution in the book “Assurance: Comptabilité- Réglementation – Actuariat”:

$$U_{\sum X_i} = \frac{\sum X_i - E(\sum X_i)}{\sigma(\sum X_i)} = \frac{\sum X_i - n_a E(X)}{\sqrt{n_a}\sigma(X)}$$

We can notice that the total load centered and reduced  $U_{\sum X_i}$  follows the standard Gaussian distribution with the parameters  $n_a E(X) = 0$  and  $\sqrt{n_a}\sigma(X) = 1$  if the number of insureds is large enough.

$$U_{\sum X_i} \xrightarrow[n_a \rightarrow \infty]{} N(0,1)$$

By the way, following the table of Normal distribution, we can find the probability  $P(|U_{\sum X_i}| < 3.3) = 0.999$ . They conclude there exists a 99.9% of chance that:

$$|U_{\sum X_i}| < 3.3$$

$$\left| \frac{\sum X_i - n_a E(X)}{\sqrt{n_a}\sigma(X)} \right| < 3.3$$

$$\left| \sum X_i - n_a E(X) \right| < 3.3\sqrt{n_a}\sigma(X)$$

If we know the expectation  $E(X)$  and  $\sigma(X)$ , and if the approximation from the central limit theorem is justified, we can approximate a future amount of compensation  $\sum X_i$  the insurer may have to pay. By example, we have 99,9% chance of having the interval of the compensation:

$$E(X) - \frac{3.3\sigma(X)}{\sqrt{n_a}} < \sum X_i < E(X) + \frac{3.3\sigma(X)}{\sqrt{n_a}}$$

### 1. 3. 2) Notation of the result with decomposition of premium

Jean-François Cabot (2021) defines the premium as the contribution which the insured pays to the insurer in exchange for the guarantee which is granted to him to be indemnified (according to the contractual conditions) in the event of the realization of the risk for which he insured. It is payable periodically. If it is fixed, the contribution cannot be modified during the validity of the contract without the consent of the subscriber and this, regardless of the results of the insurer.

We have seen that insurance companies collect the commercial premiums  $\sum_{i=1}^{n_a} \pi_i'' = n_a \pi_i''$ . For now, we suppose that each underwriter pays the same commercial premium  $\pi_i'' = \pi''$  and we have the result (Béhar Thomas. 2011):

$$R_{n_a} = \sum_{i=1}^{n_a} \pi_i'' - \sum_{i=1}^{n_a} X_i - NMF$$

Furthermore, the commercial premium is composed of the pure prime used to finance the losses of the insured, the management fees, and the profit of the insurer (Béhar Thomas, Fromenteau Michel, Ménart Stéphane. 2011) It is composed of:

- the pure prime  $\pi = E(X)$ , it is an estimation of the amount used to pay the financial compensations
- the management fees  $m\pi$  proportional to the commercial premium
- the security fees  $s\pi$  proportional to the pure premium used in the case of a price difference of the estimate allowance to pay: it is a safety margin

It is therefore the product of a complex calculation based above all on the forecast assessment of the loss. We can write it like this:

$$\pi'' = \pi + m\pi'' + s\pi$$

By using the two previous equations, we replace  $\pi_i''$  in the first equation by the equation found previously and we obtain the result (*Annex 1; 2*):

$$R_{n_a} = \sum_{i=1}^{n_a} \pi_i + m\pi_i'' + s\pi_i - \sum_{i=1}^{n_a} X_i - NMF$$

As  $m\pi''$  are the managements fees, we take  $\sum m\pi''_i = NMF$  and we have:

$$R_{n_a} = \sum_{i=1}^{n_a} \pi_i + s\pi_i - \sum_{i=1}^{n_a} X_i$$

With  $n_a$  identically and independently distributed and by supposing the premium is the same for each underwriter, we obtain the random variable as the result of the insurer:

$$R_{n_a} = n_a\pi + n_a s\pi - \sum_{i=1}^{n_a} X_i = n_a\pi(1 - s) - \sum_{i=1}^{n_a} X_i$$

Moreover, as  $\pi = E(X)$  (Annex 1; 3):  $E(R_{n_a}) = n_a s\pi$

And the volatility is equal to:  $\sigma(R_{n_a}) = \sqrt{n_a}\sigma(X)$

## 1. 4. Risk of loss and reinsurance

Let us see the balance sheet of the insurance company (Béhar Thomas. 2011):

ASSETS	LIABILITIES
Real asset= financial investment	Equity net
	Real Liabilities = commitments made to policyholders = technical provisions

The real assets are composed of the total volume of assets and financial investments. The real liabilities are compound of all debts contracted, meaning, the commitments to insureds. As the amount of premium is collected first, the commitments to the policyholder, also called technical provisions are insured by financial investments. To determine the net equity of the insurance company, real liabilities are subtracted from real assets :

$$\text{Net EQUITY} = \text{real ASSETS} - \text{real LIABILITIES}$$



If the net equity increases, the return is put in reserve (Béhar Thomas. 2011). However, if it decreases, the insurance company faces a loss. Moreover, the net equity should be greater than zero, meaning real assets must be greater than real liabilities, otherwise, it can lead to bankruptcy. Let us use the central limit theorem to study this.

If the insurer's result becomes lower than zero, the insurance company can face the risk of loss. To understand this risk, we can use the central limit theorem:

$$U_{R_{n_a}} = \frac{R_{n_a} - E(R_{n_a})}{\sigma(R_{n_a})} \xrightarrow{n_a \rightarrow \infty} N(0,1)$$

Béhar Thomas, Fromenteau Michel, Ménart Stéphane (2011) notice that  $U_{R_{n_a}}$  a refined function of the random variable  $\sum X_i$ ,

$$U_{R_{n_a}} = \frac{R_{n_a} - E(R_{n_a})}{\sigma(R_{n_a})} = \frac{R_{n_a} - n_a s E(X)}{\sqrt{n_a} \sigma(X)}$$

follows an approximately normal law centered- reduced.

Furthermore, the insurer can face a risk of loss if the probability of having a result lower than zero:

$$\begin{aligned} P(R_{n_a} < 0) &= P\left(U_{R_{n_a}} \sqrt{n_a} \sigma(X) + n_a s E(X) < 0\right) = P\left(U_{R_{n_a}} < -\frac{n_a s E(X)}{\sqrt{n_a} \sigma(X)}\right) \\ &= P\left(U_{R_{n_a}} < -s \sqrt{n_a} \frac{E(X)}{\sigma(X)}\right) \end{aligned}$$

According to the law of large numbers\*, if the financial compensations  $X_i$  are identically and independently distributed, the greater the number of insureds  $n_a$ , the more we can predict the amount of compensation  $\sum_{i=1}^{n_a} X_i$ . In that sense, to avoid the risk of loss, the insurer should sign a large number of insureds  $n_a$  or increase the commercial premium.

Furthermore, the equity of a company is composed of the shareholders' capital and the benefits realized. If the firm does not have enough funds, it will not be able to face the risks incurred like unpredictable compensation to an insured. If the annual loss is greater than the equity, the company may be bankrupted, meaning the result becomes lower than the equity of the insurer (Béhar Thomas. 2011).

$$P(R_{na} < -Equity) = P\left(U_{R_{na}} < -\frac{Equity + E(R_{na})}{\sigma(R_{na})}\right)$$

According to the book *Assurance: Comptabilité-Réglementation-Actuariat (2011)*, the coefficient  $\frac{Equity + E(R_{na})}{\sigma(R_{na})}$  is called the safety factor.

To reduce the risk of loss, insurance companies have the possibility or reinsure their liabilities. By example, the company MACSF or Mutuelle d'Assurances du Corps de Santé Français is reinsured by the SwissLife French company.

Reinsurances are stop-loss companies. They could be presented as insurance of insurance companies, since it allows insurers to transfer to another establishment authorized to carry out insurance operations (the reinsurer) all or part of the risks that it has agreed to assume from its customers, the insured (Frank Fabozzi and Frank Jones. 2019). The German Commercial Code considers the reinsurer as “the insurer’s insurer”. In that sense, an agreement between the parties is written: the ceding party will transfer portions of their risk portfolios to the reinsurer. This one accepts to handle the potential risk in exchange for shares of the insurance premiums.

## 1. 5. A new type of insurance: “insurtech”

The term “insurtech” appeared, acronym of “insurance” and “technology”. According to the Insurance Information Institute (2020), insurtech defines all small and medium-sized businesses using new technologies to improve insurers’ operations by saving costs and simplifying processes. It is a subdivision of fintech which describes new technologies introduced in the bank and financial sector. It appeared around 2010.

The main tasks of insurtech are to make insurance simpler and more accessible and to facilitate and secure the work of the actors. It allows to dematerialize insurance contracts and to manage all contract processes online, like sinister or individualization of customer relationship management via conversation agents and other chatbots. Moreover, the use of

artificial intelligence, machine learning, and big data provides customization of offers and automation of processes while achieving cost-cutting or cost-killing.

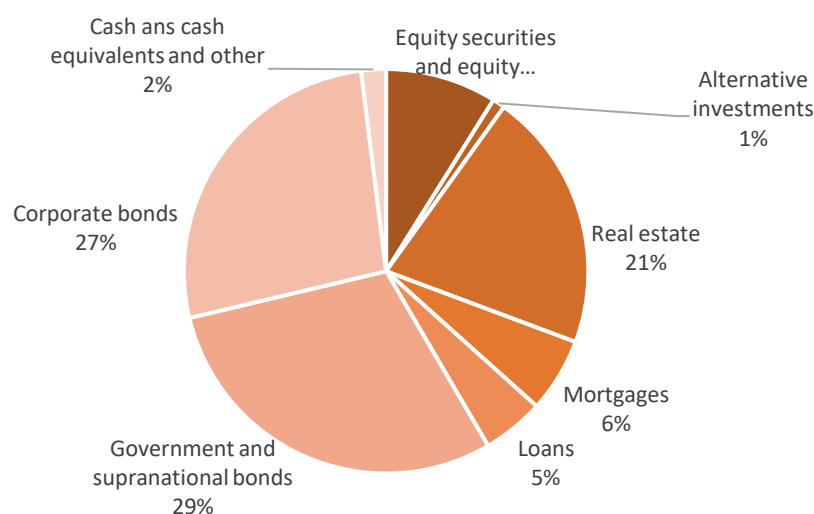
Now that we have learned about the operation of insurance, we will study in a second part an insurance with which the GIEA company works: SwissLife insurance.

## 2. The Swisslife company and its pension contract

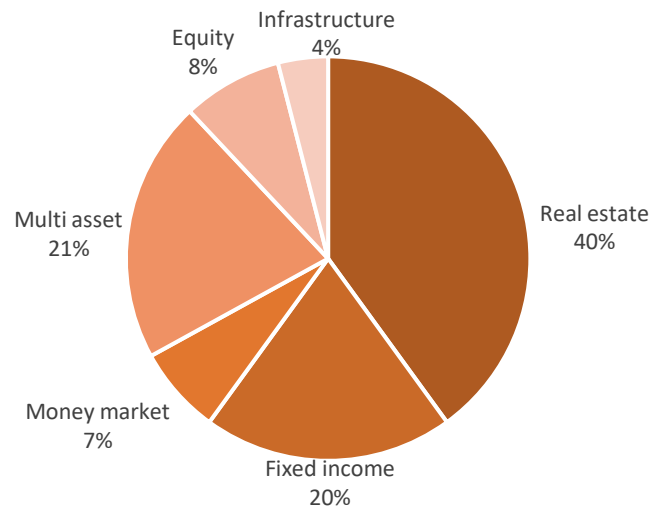
### 2. 1. Introduction about the SwissLife company

The Groupement Interprofessionel Européen d'Assurance is one of the first insurance brokerage firms to work with the SwissLife French insurance company. The SwissLife company is the largest group in Switzerland. It was created on the 28<sup>th</sup> of September of 1857 by Conrad Widmer with the assistance of Credit Suisse. The SwissLife is composed of 4 sub-brands.

The first one is the SwissLife Asset Managers which manage the assets of the company. Usually, this sub-brand invests in government and supranational bonds as shown in figure 4. But the investment will be different for customers. Here are two diagrams showing the repartition of the financial investments:



*Figure 4: assets under management for Swisslife business; breakdown by asset class on the 31.12.2019 – Adapted from: Annual report (2020)*



*Figure 5: assets under management for customers; breakdown by asset class on the 31.12.2019 – Adapted from: Annual report (2020)*

The figure 5 highlights the investments realized by the two other sub-brands: the SwissLife Banque Privée, a private bank managing financial investments for high-net-worth individuals, and the SwissLife Select which selects plans financial investments for private households and brokering of financial products. Finally, SwissLife Global Solutions helps clients to build wealth and secure their future by combining a wide range of investment opportunities. It focuses on innovative approaches and digital channels to extend the Swisslife company and win new business.

Furthermore, Swisslife created the Swiss Climate Foundation in 2008 in partnership with others Swiss companies to make an active contribution to climate protection. The purpose is to help SMEs to improve their energy effectiveness and reduce their CO2 emissions.

As we can see from the next diagram, SwissLife is implanted principally in three countries: Switzerland, France, and Germany.

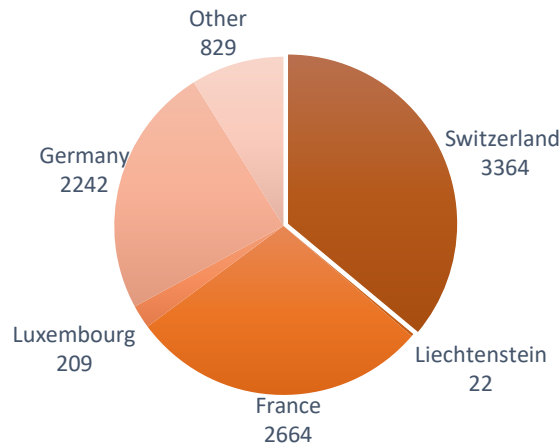


Figure 6: Full-time equivalents by country on the 31.12.2019 – Adapted from: Annual report (2020)

SwissLife France is one of the leading players on the market of life and pension wealth insurance and health and pension insurance, meaning personal insurance. Indeed, as we can notice in figure 7, most of their contracts are individual life contracts like life insurance or complementary pension.

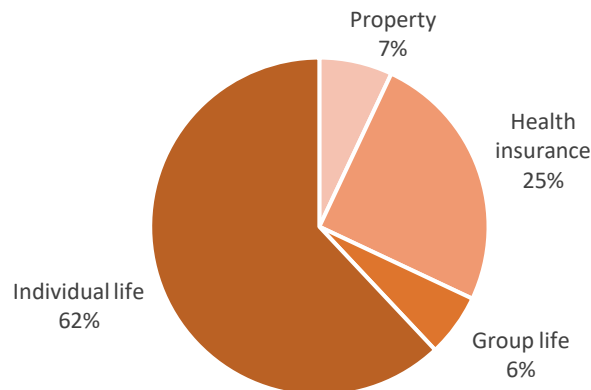


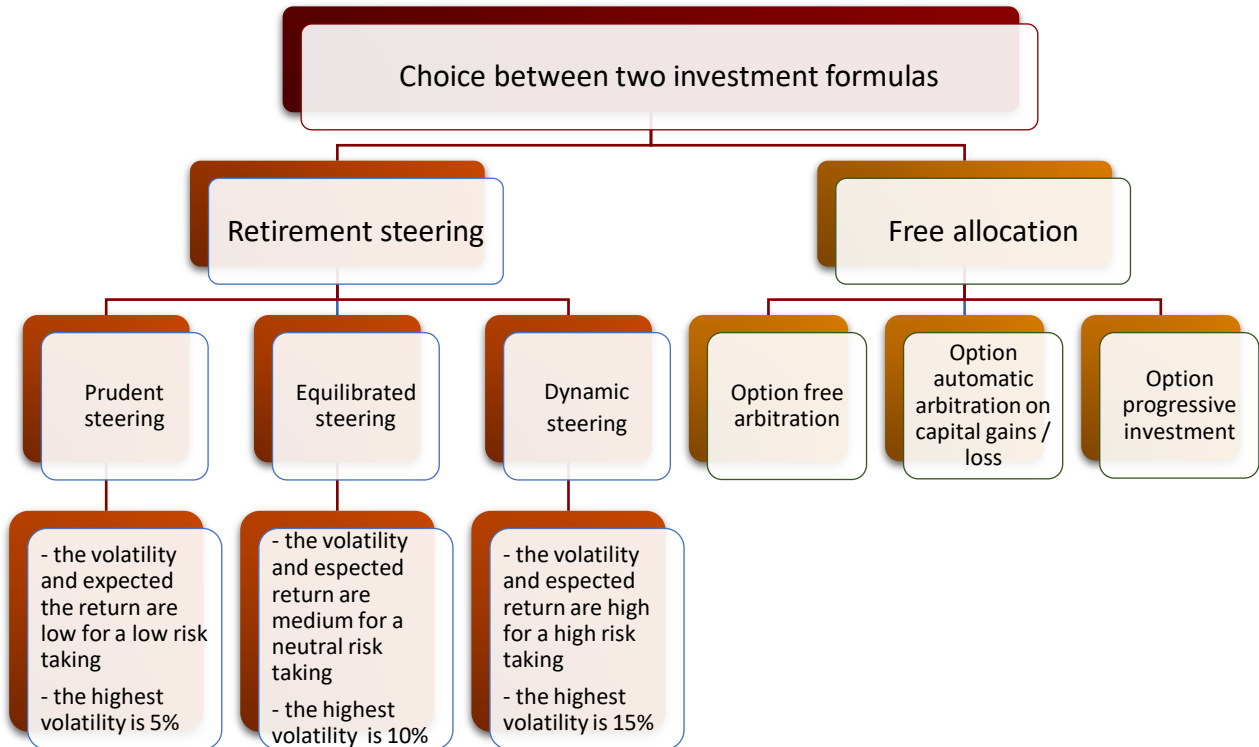
Figure 7: Premiums for France, by type of insurance – Adapted from: Annual report (2020)

The GIEA proposes a large number of contracts from SwissLife. The most used is the pension insurance called PERin or individual Retirement Savings Plan. Let us see a brief explanation of this contract.

## 2. 2. A SwissLife contract proposed at the GIEA: the PERin contract

At the retirement age, which is currently 62 years nowadays, we could need to compensate for any losses of income. In that sense, insurance companies created retirement savings plans to constitute an extra pension. The SwissLife pension fund contract is called Individual PER (PERin) or Individual Retirement Savings Plan. We find two kinds of PERin contracts. The PERin TNS is specific to the self-employed person. They can make payments from their professional bank account; it is usually used to reduce the charges. The employees can have access to the PERin private individual. Everyone can contract it and it is paid from the private bank account; it also allows to reduce the income taxes.

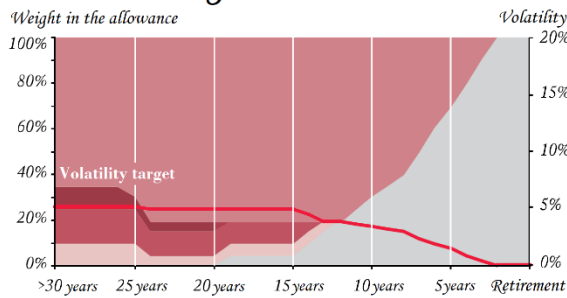
At the time of the insurance contract signing, the underwriter can choose between two investment formulas. He has the possibility of deciding himself the Swisslife investment vehicles and the percentage of the repartition of the investment between the euro fund and each unit of account. In another hand, he can opt for retirement steering where the percentage in each asset is scheduled by Swisslife according to risk level and the remaining time before the underwriter's retirement. This is an overview of this system:



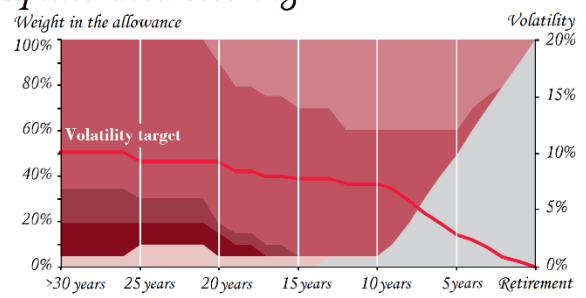
*Figure 8: Investment formulas for the Swisslife PERin contract – Adapted from:  
 SwissLife PER Individuel Notice. 2019*

Concerning retirement steering, there exists a grid for each investment profile defining the distribution of the payments invested and the individual rights constituted between the different financial support. This method allows us to gradually reduce financial risks as we approach the projected retirement age. The figure 9 is showing the repartition of the payment in the principal Swisslife assets regarding the volatility of each support according to the time remaining before the retirement date:

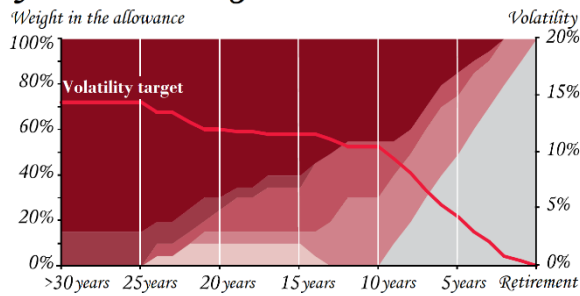
### Prudent steering



### Equilibrated steering



### Dynamic steering



- Euro funds
- SLF (F) Global Inflation P
- SLF (Lux) Prudent R
- SLF (Lux) Harmony R
- SLF (F) Equity Euro Zone Minimum Volatility P
- SLF (Lux) Vitality R

Figure 9: Investment profiles for SwissLife PERin contract – Source from: SwissLife  
Retraite Madelin (2018)

The euro fund of SwissLife PER Individual is the general asset of SwissLife Assurance et Patrimoine.

In the free allocation base, the underwriter can choose his supports, between the fund in euros and the supports in units of the accounts referenced within SwissLife, on which the payments and return on investment will be invested.

Now that we have studied the basics of an insurance contract and we have introduced SwissLife, we can discuss the GIEA brokerage firm and the internship I did within this company.



## 3. My internship at GIEA

### 3. 1. The GIEA company

The GIEA, or Groupement Interprofessionel Européen d'Assurance is a society with limited responsibilities. It means that it is a private company with the purpose of conducting commercial activity. The GIEA is an office for review and analysis specialized in insurance brokerage. They compare multiple numbers of insurance contracts from numerous companies to select the products they will propose. Nowadays, they work with around fifty European companies. The GIEA is composed of 28 agencies in France and French overseas departments including 3 in Reunion Island. It was created by Mr. Henry Dery on the 8<sup>th</sup> of April of 1992. The head office takes place at Enghien-Les-Bains in France. Henry Dery is considered as the GIEA company's CEO. The last annual revenue known was 244 060 euros in 2017.

On the 19<sup>th</sup> of June of 2007, the GIEA R, the first agency in Reunion Island, was created by the co-managers, Mr. Sébastien Couffinhal and Mr. Dery at Saint-Pierre. Then, on the 1<sup>st</sup> of September on 2014, Mr. Couffinhal opened his agency, GIEA 974, at the Tampon. In 2017 he realized a turnover of 109 300 euros. I completed my internship there.

Their direct competitors are all the other insurance brokerage firms, insurance companies, and even some banks. Indeed, nowadays banks propose insurance contracts.

The GIEA company proposes insurance of person contracts, life insurances, supplementary pensions, and health mutual. The GIEA is the first insurance brokerage working exclusively with Swisslife France. Together, they create unique contracts with exclusive guarantees.

Let me explain a little bit how it works in France. Here we must join and pay the mandatory scheme. There exist three mandatory funds:

The sickness insurance scheme which pays 70% of the standard charges concerning the medical appointments and some drugs

The pension fund which pays pensions to retirees, from 250 to 300 euros per month

The provident fund which pays allowances in the case of a work stoppage greater than 4 days

Usually, the mandatory regime is not enough, and people decide to subscribe to complementary funds. To complete the sickness fund, they can take out a health mutual which will pay at least 30% of the standard charges and a part of the excess medical fees. Also, they may contract a complementary pension and/or complementary insurance of persons to benefit from daily allowances during a work stoppage.

The GIEA hires independents to negotiate and to market contracts. Those are formed by the branch manager and obtain the status of fourth-class underwriting agent for insurance companies.

### 3. 2. My formation and missions

#### 3. 2. 1) The hands-on training of new agents

I had the opportunity to attend the training of new agents. Mr. Couffinhal made a presentation about the GIEA company as well as the contracts negotiated. He explained to us the link between the brokerage company and the insurance companies, like SwissLife. He introduced the department Cegema: it is a SwissLife's department dedicated exclusively to the GIEA's clients having signed a provident insurance contract. This foresight contract was created in collaboration by GIEA and Swisslife especially for the self-employed person: the craftsmen, the merchants, and the liberal profession like paramedics.

Thereafter he described to us how managing to make an appointment with prospective customers by phoning and through the client's recommendations. He showed us how Cegema website works. This site is used to make quotes for prospects and customers.

Then he realized an argument to give us an example and we trained ourselves to make phoning, to find the best approach to get a meeting. We realized some role-playing, an agent and a prospect. The aim of this game was to practise negotiating contracts in order to achieve one-shot signatures of provident insurance contracts.

Once we understood and we manage to make an appointment, we went to the appointments with Mr. Couffinhal. For the two first appointments, the manager went with us to realize the argumentation and negotiation of the provident contracts. When we arrive at a meeting, the first step is to create a link with the client. Usually, at this stage, we speak about the customer's work. Then, we realize a cost assumption to compute the amount needed in the case of a work stoppage: he should have enough allowances to pay his bills and his basic needs. Once all the costs are clarified, we enter the client's data in Cegema, and we manipulate the different amounts:

- the death grants the beneficiaries of the contract would receive after the death by accident or by disease of the insured,
- the daily allowances related to the net remuneration when there is a work stoppage following an accident or a sick absence,
- the overhead costs linked to the professional expenses,
- and the hospital comfort added to the daily allowances in the case of a hospitalization.

After this, we add guarantees according to what the customer needs: the hospital comfort in the case of his children hospitalization and/or marriage partner hospitalization, and/or the disability capital from 1% to 32 % of invalidity. Then, using the SwissLife Prévoyance Indépendant or SLPI document, we expose to the client the different guarantees of the contract and we explain each amount we added in Cegema and so, the daily allowances he would receive in each case of work stoppage. Indeed, the contract we propose to the client ensures them in the case of an accident or a disease. To make a sale, we need to be fully aware of our contracts. We must be able to explain the different clauses and answer any question from the prospect. Then we tell them the costs of the monthly contributions. If the client needs to think about it, we set up another appointment. If he is ready to sign, we ask him to fill in a duty of advice to be sure he became aware of what he is getting himself into and we scan it to join to Cegema website file. He must fill in a health questionnaire afterward. This survey will be sent to the SwissLife company to be examined and to decide if the client file is accepted or if there are more information to ask for, like the medical file. Then we ask for the administrative fees and the bank transfer information for the automatic monthly payment.

If the Swisslife company needs more medical information from the client, the files can remain pending. After examination of the complementary documents, the folder can be

accepted with or without exclusion of the concerned pathology and/or an increased pension contribution. It can also be rejected by the insurance company. Usually, if the SLPI contract is not accepted, it is because of a recent work stoppage greater than three weeks or long-term diseases (ALD). For this kind of person, SwissLife has created Special Response Plan (PPI).

The PPI is a special contract allowing all people who have had a recent pathology, a work stoppage, or having an ALD be able to benefit from personal insurance even if the proposal of the classic GIEA insurance contract was met with a flat refusal by the insurance company. To request a PPI, we use an excel spreadsheet with specific computation. This has another health questionnaire to fill in with more detailed questions. Depending on his answers and on the medical file, the calculation will select a PPI's level. There exist 3 levels according to the ALD of the client and the premium will be proportional to the diseases. For the two firsts PPI, the biggest difference with the SLPI contract is that the professional fees are not taken into consideration and there is no death capital. Regarding the PPI 3, the client is insured only for accidents.

In addition, if the underwriter is already insured by another company, one step is added to the contract process. Indeed, the other insurance must be made aware of the termination of their contract. As a result, we are in charge of sending the necessary letters to inform them of the change in the situation and of the fact that they must close the file of our new customer. This must be done two months before the anniversary date of the contract or before October 31<sup>st</sup>.

For the next meetings, until the end of my internship, I went to appointments with the collaborators. It allowed me to see diverse approaches, different ways of arguing and comparing them to improve ourselves.

Also, we had a formation on the supplementary pension contracts and the life insurance. In October 2020, the Pacte law was signed. Indeed, at the legal age of retirement, 62 years, instead of retrieving the pension in the form of a life annuity, nowadays we can choose to retrieve it as a capital. Before de Pacte law, so during the Madelin law, we could not retrieve all our savings in capital. We have 2 kinds of contracts for the additional pension:

- the PERin TNS: we can pay with the professional bank account,

- the PERin private individual: everyone can contract it, even the employees and it is paid from the private bank account.

Moreover, these contracts can be used as a tax exemption.

SwissLife also proposes life insurance. The biggest difference with the PERin private individual is that the money is not locked until 62 years, we can get it back at any time. We suggest the client to invest their money for at least four years because of the management fees to pay from the first to the third year and we usually make profit from the fourth year. Moreover, we cannot use the life insurance contract to reduce the taxes.

Furthermore, the GIEA proposes health mutual. For this contract, they selected the April company. This insurance is specialized in real estate loan insurance, complementary health, car insurance, and welfare contract. As we do not have a paper document for this agreement, all is done by the internet. Usually, we perform a different estimate and we send it by email to the customer. If a client agrees with one of the estimates, the signature can be made online in a dematerialized way and April will directly manage it.

### 3. 2. 2) Typical week

A typical week at the GIEA is divided into two parts.

The Monday and Tuesday we must work at the office to make phoning to manage to get appointments with prospects. During these two days, we have the possibility of working on contracts with the secretary, Sabrina Valentin, and we should be careful about the replenishment of the papers we would need during our appointments.

On Monday we work from 9 am to 6 pm with a lunch break of one hour and on Tuesday we can leave the office at 5 pm.

Some Mondays, we have a big meeting by videoconference with the CEO of the company, Mr. Dery. Usually, he provides an update on the different agencies, on the turnover of everyone. Then he or a collaborator makes a sales pitch, and we detail what is wrong and how to improve it. Finally, he concludes the speech we heard and he realizes a summary of what we should say to be a better seller.

From Wednesday to Friday, we should have meetings with clients. I went to the meetings with my supervisor or with the collaborators. If I did not have any meetings, sometimes, I went to the office to help the secretary to manage the clients' files.

### 3. 4. On the administrative level

#### 3. 4. 1) The secretary

The secretary of the GIEA R and The GIEA 974 is handled by Mrs. Sabrina Valentin. She is the link between the GIEA's clients and the company (Swisslife France, Cegema and GIEA Enghien-Les-Bains and April). She showed me her tasks and made me participate in some of them.

The secretary is responsible for managing the contracts once the customer has signed them. The first step is to record the contract on the internal GIEA's software. We enter the client's data to create a new file. From there, we have been doing client follow-up, changing some points in their contracts, increasing, or decreasing their monthly contribution. When a client reports a work stoppage, he transmits the stoppage and the complementary papers to the secretary, and we transfer them to SwissLife. The file goes forward to the commission to be examined. If the work stoppage is validated by the insurance company's investigation, the client receives his allowances. When the clients have a reclamation about their contract, they call the GIEA's secretary. If needed, we call to Swisslife company to make a point about this client, and then we keep the client informed about his file's progress. From October 2020, Mrs. Valentin manages the transformation of the previous pension contracts, called pension Madelin, to the PERin contracts. Indeed, as the pension regulation changed in France, we must call the clients to inform them about this and to rearrange the contracts according to the law PERin. I took care of a table to a transfer from the Madelin to the PER. I had to compare both contracts to find the different fees, mostly, management fees. Thanks to this, I have been able to study the PER contract and its computation, as we have seen in the third part. Furthermore, at the end of June, the GIEA changed some guarantees in their contracts. In that condition, we

had to take care of updating the client and organizing appointments to sign the new contract. She also manages unpaid payments, that is, customers who do not pay their monthly contributions. These must be referred to SwissLife for a decision: if the customer is no longer able to make the payments, the contract can then be closed.

Furthermore, when arrives the time to pay the collaborators, the secretary realizes the monthly slip commission at the end of each month. All the contracts signed by the different collaborators are registered on the GIEA intern software. She prints the summary document and uses it to fill in the excel spreadsheet: the commission amount, the management fees, the recovery on commission in the case of contract cancellation, and the bonus amount. Meanwhile, the collaborator realizes his computations to compare with the secretary's result. Once they find the same amount, the slip commission is sent to Mr. Couffinhal and he realizes the bank transfer.

### 3. 4. 2) The recruitment

In addition, Mr. Couffinhal also realizes the role of the recruiter. Recruitment is done at the office, often face to face. The candidates present themselves and their experience and then explain why they want to enter the trade. Afterwards Mr. Couffinhal explains how the GIEA works. A discussion followed with questions from the recruiter and candidate. The recruiter seeks to determine whether the candidate has the character and motivation necessary for the trade. According to him, "the journey can say anything. For me, the person must be motivated, and I am for the second chance." On his side, the candidate inquiries about the missions that he could carry out in the context of the insurance brokerage.

Once the manager has made his choice, he will conduct a training like the one I was able to attend at the beginning of my internship to train future agents. The duration of this training depends on the progress of each and can range from two weeks to two months. Thereafter, the representatives will be considered as junior representatives for 6 months. They may be supervised throughout this period by the Director.

## 4. Contribution of this internship and skills developed

This internship was very informative and very interesting in the sense that I was able to develop my skills not just for working as an insurance broker. As expected, the world of work has many differences from the programs of the master's degree. Indeed, financial mathematics is more about analysis, except that insurance brokerage requires a more commercial spirit. I was also able to discover the world of insurance thanks to the research carried out. This report was helpful to connect my internship and my studies.

In general, even if I have not been able to put into practice every theoretical insight gained at the master's degree, this internship has been a real opportunity for me. It allowed me to discover a profession and gain experience. Also, I had the opportunity to participate in the theoretical training phase of the new brokerage agents conducted by Mr. Couffinhal. Thus, I had the pleasure of creating positive links of mutual support with the team.

As an Intern, my biggest difficulty at the firm was to find customers interested in the products we were proposing. Moreover, since I am more of an analyst than a merchant, selling a contract was a real challenge and I learned to be resilient.

By attending the meeting with the customers and working as part of a team with the collaborators or the manager, I have observed the different ways they argue to sell the contracts and I have developed my arguments as well as my commercial skills. In that sense, this internship allowed me to overcome my fears, to get out of my comfort zone and to have more confidence in myself. It brought me more assurance and it taught me to assert myself, especially in meetings with company managers who are often very picky people. I was also able to develop my professional network thanks to the various customer appointments.

Furthermore, the profession of insurer broker can be difficult. At the GIEA, agents are paid as a percentage of the commissions on the contracts they have managed to sign and as application fees. There are no fixed salaries. This may be seen as a double-edged sword. The agent can sign many contracts and get very good remuneration at the end of the month, in addition to a bonus. However, if no contract is signed within a month, the agent may end up with no cash inflow. This work is therefore not suitable for everyone, skimming is quickly



carried out naturally. At the beginning of my internship, when we started the training with Mr. Couffinhal, there were six of us. After two weeks, one person stopped and by the end of the second month, there were only two of us.

## Conclusion

At the beginning, I felt confused about the lack of connection between my missions in the GIEA and the tasks I expected to complete following my Master's degree. Agents of an insurance brokerage company are considered to be independent traders commissioned on signed contracts. Their most sensitive mission is therefore to succeed in sufficiently arousing the interest of a prospect in order to land an appointment. Then, they must be able to identify the needs of their customers in order to achieve the best possible contract and obtain an agreement and signature. I thus had the opportunity to participate in customer meetings as well as in the negotiation of exclusive contracts.

Subsequently, I became interested in the operation of insurance and the relationship between it and the brokerage firm. Having no knowledge of the insurance world, I then wondered how a company could make a profit by collecting premiums while using its premiums to pay benefits to their clients. After having asked my company supervisor about this, I studied the PERin contract which brought me an overview of the different manners of investing. Studying a pension contract has allowed me to understand how insurance generates profit through financial investments. Then, I began to research the functioning of insurance. Thus, I noticed some common points between my research and my courses. As the insurance brokerage firms and the insurance companies are linked by the contracts, it provided me a better understanding of the products the GIEA proposes. Moreover, my research has allowed me to complete my knowledge and to see how probabilities can be concretely used in finance. Also, I was able to discover how insurance and a brokerage firm work and the link that can unite two companies.

Furthermore, this internship, and especially the client appointments, have brought me a lot of personal enrichment in terms of psychology and human relations. Thanks to this

internship in a brokerage firm, I have developed new skills which will be certainly useful in my future work. As I have studied insurance contracts, the functioning of an insurance company, and how to sell an insurance to a client, I currently understand the operations of insurance companies much better. Thus, for my next step, I would like to work in an insurance firm to make into practice what I have discovered and learned during my internship and to deepen my knowledge. Moreover, I am thinking about following a training course about psychology to have a better understanding of the way of thinking of the customers I would meet during my professional appointments or simply understand the colleagues around me.

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## Abbreviations

ALD : Affection Longue Duré (long-term diseases)

GIEA : Groupement Interprofessionnel Européen d'Assurances (literally : European Interprofessional Insurance Group)

MACSF : Mutuelle d'Assurances du Corps de Santé Français (literally : Mutual Insurance of the French Health Corps)

PERin: Plan Épargne Retraite individuel (individual Retirement Savings Plan)

PPI : Plan Particulier d'Intervention (literally : Special Response Plan)

SARL : Société À Responsabilité Limité (literally : society with limited responsibilities)

SLPI : SwissLife Prévoyance Indépendant

TNS : Travailleurs Non Salarié (self-employed person)

## Rating index

- $i$  the  $i$ -th insured,  $i \in [1; n_a]$
- $n_a$  the number of insured people
- $\pi_i''$  the commercial premium paid by the  $i$ -th insured  $\pi_i''$
- $\pi_i$  the pure premium
- $X_i$  the financial compensation the  $i$ -th insured could receive
- $c$  the capital to pay to policyholder following a disaster
- $p$  the probability of the sinister
- $R_{n_a}$  the hazard result of the insurer
- $MF, m\pi$  the management fees
- $FP$  the financial products
- $FV$  the final value
- $IP$  the initial premium
- $N_s$  the number of survivors with  $E(N_s) = n_a q$
- $q$  the probability of being alive
- $k$  the number of years
- $NMF$  the net management fees
- $s\pi$  the security fees

## Annexes

### Annex 1 : Computations details

(Béhar Thomas, Fromenteau Michel, Ménart Stéphane, 2011)

#### **Annex 1; 1**

Computation of expectation and volatility of the financial compensation.

$n_a$  identically and independently distributed:

$$E\left(\sum_{i=1}^{n_a} X_i\right) = \sum_{i=1}^{n_a} E(X_i) = n_a E(X_i) = n_a E(X)$$

$$\sigma^2\left(\sum_{i=1}^{n_a} X_i\right) = \sum_{i=1}^{n_a} \sigma^2(X_i) = n_a \sigma^2(X)$$

⇔

$$\sigma\left(\sum_{i=1}^{n_a} X_i\right) = \sqrt{n_a} \sigma(X)$$

#### **Annex 1; 2**

Writing of the insurer's result, taking into account the breakdown of a contribution.

Formula of the result:

$$R_{n_a} = \sum_{i=1}^{n_a} \pi_i'' - \sum_{i=1}^{n_a} X_i - NMF$$

By isolating the sum of the commercial premiums:

$$\sum_{i=1}^{n_a} \pi_i'' = R_{n_a} + \sum_{i=1}^{n_a} X_i + NMF$$

And we have the position of  $\pi''$ :

$$\pi'' = \pi + m\pi'' + s\pi$$

⇔

$$\pi'' = \frac{1-s}{1-m}\pi$$

By using the 2<sup>nd</sup> equation, we replace  $\pi_i''$  by the equation found previously and we obtain the result:

$$R_{n_a} = \sum_{i=1}^{n_a} \pi_i + m\pi_i'' + s\pi_i - \sum_{i=1}^{n_a} X_i - NMF$$

### Annex 1; 3

Computation of the expectation and volatility of the insurer's result.

We have:

$$\pi = E(X)$$

So:

$$\begin{aligned} E(R_{n_a}) &= E\left(n_a\pi + n_a s\pi - \sum_{i=1}^{n_a} X_i\right) = n_a\pi + n_a s\pi - E\left(\sum_{i=1}^{n_a} X_i\right) \\ &= n_a\pi + n_a s\pi - n_a E(X) = n_a E(X) + n_a s\pi - n_a E(X) = n_a s\pi \end{aligned}$$

And:

$$\sigma^2(R_{n_a}) = \sigma^2\left(n_a\pi(1-s) - \sum_{i=1}^{n_a} X_i\right) = \sigma^2(n_a\pi(1-s)) + \sigma^2\left(-\sum_{i=1}^{n_a} X_i\right)$$

Furthermore, a variance is always positive or zero, so  $\sigma^2(-\sum_{i=1}^{n_a} X_i) = \sigma^2(\sum_{i=1}^{n_a} X_i)$  and we obtain:

$$\sigma^2(R_{n_a}) = 0 + n_a\sigma^2(X) = n_a\sigma^2(X)$$

Finally, the volatility is equal to:

$$\sigma(R_{n_a}) = \sqrt{n_a} \sigma(X)$$

## **Annex 2: probability distributions and theroms**

### **Binomial law**

(Stephanie Glen. 2021)

A Bernoulli (success-failure) experiment is performed  $n$  times, and the trials are independent. The probability of success on each trial is a constant  $p$ ; the probability of failure is  $q=1-p$ . The random variable  $X \hookrightarrow B(p)$  counts the number of successes in the  $n$  trials.

If  $X \hookrightarrow B(p)$ , then

- $E(X) = p$ ,
- $\sigma^2 = p(1 - p)$  and
- $\sigma(X) = \sqrt{p(1 - p)}$

The binomial distribution  $X \sim B(n;p)$  of parameters  $n$  and  $p$  is the discrete probability distribution of the random variable  $X$  which counts the number of success in a Bernoulli scheme of parameters  $n$  and  $p$ .

If  $X \hookrightarrow B(n; p)$ , then :

- $E(X) = np$
- $\sigma^2 = np(1 - p)$  and
- $\sigma(X) = \sqrt{np(1 - p)}$

### **Uniform law**

(Uniform Distribution / Rectangular Distribution: What is it? . 2021)

The uniform law is characterized by a uniform distribution of values taken by a continuous random variable over an interval  $[a;b]$ . Therefore, its density function is constant on  $[a;b]$  and zero outside this interval since  $X$  does not take any values. The value of this



constant cannot be chosen randomly since the density function  $f(x)$  must check  $\int_a^b f(x)dx = 1$ .

Let  $a$  and  $b$  be two real ones such as  $a < b$ . To say that a continuous random variable  $X$  follows the uniform law on  $[a;b]$  means that its density  $f$  is defined on  $\mathbb{R}$  by:

$$\begin{cases} \frac{1}{b-a} & \text{if } x \in [a; b] \\ 0 & \text{otherwise} \end{cases}$$

If  $X$  follows the uniform law over  $[a;b]$ , we note  $X \hookrightarrow \mathcal{U}[a; b]$ .

With  $E(X) = \frac{b+a}{2}$

### Central limit theorem

(Stephanie Glen. 2021)

Let a sequence of  $n$  random variables  $X_1, X_2, \dots, X_n$  independent and of the same distribution (thus of the same expectation  $\mu$  and of the same standard deviation  $\sigma$ ). We have:

$$\mu = E(X_1), \sigma^2 = Var(X_1)$$

$$S_n = X_1 + X_2 + \dots + X_n \text{ with } E(S_n) = n\mu \text{ and } \sigma(S_n) = \sigma\sqrt{n}$$

Either the random variable  $S_n$  resulting from the sum of the  $n$  independent random variables and the same law, we construct the reduced centered variable as:

$$U_{S_n} = \frac{S_n - E(S_n)}{\sigma(S_n)} = \frac{S_n - n\mu}{\sigma\sqrt{n}}$$

So for all  $t \in \mathbb{R}$ , the distribution function  $F_n(t) = P(U_{S_n} < t)$  is such that

$$F_n(t) \xrightarrow{n \rightarrow \infty} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^t e^{-\frac{z^2}{2}} dz$$

In other words, the sequence  $(U_{S_n})$  converges in law to a random variable of distribution  $\mathcal{N}(0,1)$  (the normal centered reduced distribution).

## Strong law of large numbers

(Ben Eastaugh and Chris Sternal-Johnson. 2008)

Consider  $n$  independent random variables  $X_1, \dots, X_n$  that follow the same probability law, integrable (i.e.  $E(X) < \infty$ ). Taking up the above notations, the strong law of large numbers specifies that the empirical averages of this sequence  $Y_n = \frac{X_1 + \dots + X_n}{n}$ , converges to  $E(X)$  "almost surely":

$$P\left(\lim_{n \rightarrow \infty} Y_n = E(X)\right) = 1$$

## Bibliography

AM Best Company (2019). *Best's Guide to Understanding The Insurance Industry*. New Jersey: AM Best

Assurance et mutuelle (2021). *Définition de l'assurance* [online]. Available on <https://www.assurance-et-mutuelle.com/assurance/definition-assurance.html> [updated the September 23, 2021].

Béhar, T. Fromenteau, M. and Ménart S. (2011). *Assurance: comptabilité-réglementation-actuariat*. Paris : Economica.

Carlot, J.F. (2021). *Droit des assurances 2021-22*. Hachette Education. Vanves : Top'Actuel

Eastaugh, B. and Sternal-Johnson, C. (2008). *The strong law of large numbers* [online]. Available on: <https://terrytao.wordpress.com/2008/06/18/the-strong-law-of-large-numbers/> [Updated on July 2020].

Fabozzi, F.J. and Jones, F.J (2019). "Insurance companies." in *Foundations of global financial markets and institutions*. Fifth Edition. Cambridge, Massachusetts: MIT Press, pp. 153-184.

Fédération Française de l'Assurance. (2020) *Rapport économique et statistique 2019*. Paris : Fédération Française de l'Assurance

Glen, S. (2021) *Binomial Distribution: Formula, What it is, How to use it* [online] From StatisticsHowTo.com: Elementary Statistics for the rest of us!. Available on <https://www.statisticshowto.com/probability-and-statistics/binomial-theorem/binomial-distribution-formula/>

Glen, S. (2021). *Central Limit Theorem: Definition and Examples* [online] Available on <https://www.statisticshowto.com/probability-and-statistics/normal-distributions/central-limit-theorem-definition-examples/>

Glen, S. (2021). *Uniform Distribution / Rectangular Distribution: What is it?* [online]. Available on <https://www.statisticshowto.com/probability-and-statistics/statistics-definitions/uniform-distribution/>

Insurance Information Institute (2020). *Background on: Insurtech* [online]. Insurance Information Institute, January 6, 2020. Available on <https://www.iii.org/article/background-on-insurtech>

SwissLife Assurance et Patrimoine (2018). *SwissLife retraite Madelin*. Levallois-Perret : Direction de la Communication SwissLife.

SwissLife Assurance et Patrimoine. (2019) *SwissLife PER Individuel Notice*. Levallois-Perret : Direction de la Communication SwissLife.

Swiss Life Holding Ltd (2020). *Annual Report 2020*. Swiss Life, Group Communications and Strategic Marketing. Zurich: Swiss Life Holding Ltd