Lisbon School of Economics & Management
Universidade de Lisboa

Master in
Actuarial Science

Master's Final Work
Project

The Term Structure of Discount Rates under IFRS 17

A Top-Down Approach

Author:
Soraia Filipa Pereira Francisco

June 2024

Lisbon School of Economics & Management
Universidade de Lisboa

Master in

Actuarial Science

Master's Final Work

Project

The Term Structure of Discount Rates under IFRS 17

A Top-Down Approach

Author:

Soraia Filipa Pereira Francisco

Supervisors:

Professor Raquel M. Gaspar

Professor Paulo M. Silva

June 2024

Acknowledgements

I would like to express my gratitude to Professors Raquel M. Gaspar and Paulo M. Silva for their guidance and support. My sincere thanks also go to all the professors who have taught me during my master's and bachelor's degrees at this institution.
To everyone at Lusitania and Lusitania Vida for their warm welcome. A special mention goes to Luis Lança for his gracious reception and willingness to help, and to Nuno Matos for his constant support, patience, availability, belief, and encouragement.
To my parents and sister for providing me with these opportunities and for their endless belief in me. Their love, support, and encouraging words have been determinant. I am also grateful to my grandparents for their love and understanding regarding my lower availability to spend time with them.
To João, for always being there with his patience, love, and faith in me.
Lastly, I extend my gratitude to all my friends who have encouraged and believed in me, and who have always been available to help.

DISCLAIMER

This master project was developed with strict adherence to the academic integrity policies and guidelines set forth by ISEG, Universidade de Lisboa. The work presented herein is the result of my own research, analysis, and writing, unless otherwise cited. In the interest of transparency, I provide the following disclosure regarding the use of artificial intelligence (AI) tools in the creation of this project:

I disclose that generative AI tools were consulted for brainstorming and outlining purposes to help select R packages, as well as for resolving any code errors and translating expressions. However, all final writing, synthesis, and critical analysis are my own work. Nonetheless, I have ensured that the use of AI tools did not compromise the originality and integrity of my work. All sources of information, whether traditional or AI-assisted, have been appropriately cited in accordance with academic standards. The ethical use of AI in research and writing has been a guiding principle throughout the preparation of this thesis.

I understand the importance of maintaining academic integrity and take full responsibility for the content and originality of this work.

Soraia Francisco, 05 June 2024

Resumo

Em 2017, a norma contabilística aplicável a contratos de seguro IFRS 17 foi oficialmente emitida e entrou em vigor em 1 de janeiro de 2023. Esta norma descreve princípios para reconhecer, mensurar e divulgar contratos de seguros com o objetivo de aumentar a sua comparabilidade a nível internacional, ajudando os investidores a identificar oportunidades e riscos e melhorando a alocação de capital. Para esta medição, é necessário estimar taxas de desconto para atualizar os fluxos de caixa de entrada e saída esperados. Uma vez que o IFRS 17 estabelece princípios, mas não especifica nem impõe nenhuma metodologia, um certo nível de interpretação e subjetividade da norma é necessário. Para derivar as taxas de desconto, são permitidas duas abordagens: abordagem top-down e abordagem bottom-up. Este projeto explora a aplicação da abordagem top-down, com o objetivo de desenvolver uma metodologia que cumpra com os requisitos do IFRS 17. Os resultados demonstram que a curva de taxas de desconto obtida cumpre com o objetivo, embora limitações e oportunidades de melhoria também sejam apresentadas.

Palavras-chave: IFRS 17; contratos de seguro; abordagem top-down; risco de crédito; modelo de Svensson.

Classificação JEL: G12; G22; M41.

Abstract

In 2017, the accounting standard applicable to insurance contracts, IFRS 17, was officially issued with an effective implementation date of 1 January 2023. This standard outlines principles for recognizing, measuring and disclosing insurance contracts with the aim of increasing comparability between insurance contracts internationally, helping investors to identify opportunities and risks, and improving capital allocation. For this measurement, it is necessary to estimate rates to discount the expected cash inflows and outflows. Since IFRS 17 establishes principles but does not specify or impose any particular methodology, a certain level of interpretation and subjectivity is allowed. To derive the discount rates, two approaches are proposed: the top-down approach and the bottom-up approach. This project explores the application of the top-down approach, with the objective of developing a methodology that complies with the IFRS 17 requirements. The results demonstrate that the discount rate curve obtained fulfills the objective, although limitations and opportunities for improvement are also presented.

Keywords: IFRS 17; insurance contracts; top-down approach; credit risk; Svensson model.

JEL Classification: G12; G22; M41.

Contents

List of Tables

List of Figures

Glossary

ALM	Asset and Liability Management
BHHH algorithm	Berndt–Hall–Hall–Hausman algorithm
CDS	Credit Default Swap
CIA	Canadian Institute of Actuaries
CF	Cash Flow
CSM	Contractual Service Margin
D	Default
ECB	European Central Bank
ECL	Expected Credit Losses
EIOPA	European Insurance and Occupational Pensions Authority
EU	European Union
EV	Expected Value
FCF	Future Cash Flows
GMM	General Measurement Model
IASB	International Accounting Standards Board
IASC	International Accounting Standards Committee
IFRS	International Financial Reporting Standard
ISIN	International Securities Identification Number
LGD	Loss-Given-Default
ND	No Default
OECD	Organisation for Economic Co-operation and Development
ORSA	Own Risk and Solvency Asssessment
PAA	Premium Allocation Approach
PD	Probability of Default
PV	Present Value
RA	Risk Adjustment
VFA	Variable Fee Approach
UCL	Unexpected Credit Losses
UFR	Ultimate Forward Rate
YTM	Yield-to-Maturity

1 Introduction

The International Accounting Standards Board (IASB) issued IFRS 17 in 2017, an International Financial Reporting Standard that provides a comprehensive approach to the accounting for insurance contracts. This standard is effective for annual reporting periods beginning on or after 1 January 2023. IFRS 17 replaces IFRS 4 by enhancing transparency and comparability within financial statements, consequently enabling investors to make more informed decisions. The standard requires insurers to recognise, measure and disclose detailed information about the timing, amount, and uncertainty of insurance cash flows, including the related risks and how they are managed.

For measuring insurance contracts, there are three models: (1) the General Measurement Model (GMM), which can be applied to all insurance contracts, unless they have direct participation features; (2) the Premium Allocation Approach (PAA), a simplified version of the GMM that is optionally applicable to insurance contracts with maturities less than 1 year; and (3) the Variable Fee Approach (VFA), which applies only to insurance contracts with direct participation features. The GMM is implemented using a building block approach, where future cash flows are discounted to obtain their present value (PV), a risk adjustment (RA) is then added to account for non-financial risks and, if the contracts are non-onerous, the Contractual Service Margin (CSM) is added to spread the profit as the services are provided. If the contracts are non-onerous at inception, the CSM ensures that no profit is recognized initially, resulting in a disclosed zero profit. If the contracts are onerous at inception, the CSM is zero, and a loss is recognized immediately.

The discount rates applied to the estimates of Future Cash Flows (FCF) should account for the time value of money, the characteristics of the cash flows and the liquidity characteristics of the insurance contracts. However, these rates are not predetermined or disclosed by an entity, and each insurer needs to derive them. IFRS 17 proposes two different approaches to estimate the discount rates: the bottom-up and the top-down approaches. In theory, the results should be the same. However, in practice, this may not always be the case due to limitations in the calculations and the potential lack of adjustments for different liquidity characteristics in the top-down approach. Each insurance company may choose the approach to use and does not need to compare the rates it would obtain using the other approach.

Under the top-down approach, the yield curve of a reference portfolio must be adjusted to eliminate factors that are not relevant to insurance contracts, such as the credit risk of the reference portfolio. As a principles-based standard, IFRS

17 calls for a certain level of subjectivity of its concepts, allowing flexibility in both the selection of the reference portfolio and adjustment methods. Furthermore, it does not impose specific restrictions on these selections or the methods. This absence of specific restrictions or methods makes the derivation of discount rates a greater challenge. Given the principles-based nature and recent adoption of IFRS 17, practical examples of its application are limited, and several areas still require improvement. Consequently, the main challenge resides in identifying a suitable methodology for implementation. Therefore, the purpose of this project is to find an appropriate methodology to derive the interest rate term structure applicable to future cash flows under IFRS 17 for Lusitania Vida, a Portuguese life insurance company, using the top-down approach. This analysis is conducted with reference to the date as at 29 December 2023.

As anticipated, this project reveals that developing an IFRS 17 discount rate methodology is challenging due to the lack of specific guidance or restrictions and the principle-based nature of IFRS 17. This requires interpretation and expert judgement, particularly in the removal of credit risk. While finding a methodology that complies with IFRS 17 requirements may seem straightforward in principle, ensuring its accuracy is difficult. The subjective nature of the process represents a significant shortfall of IFRS 17.

Before embarking on the derivation of the discount rates, a comprehensive overview of the standard IFRS 17, including the measurement models and approaches, is presented in Section 2. Following this, Section 3 presents the literature review, describing available examples of the derivation of discount rates using the top-down approach, along with an extensive review of methodologies applicable at each phase of discount rate estimation. Section 4 describes the input data used and the chosen methodology, including the underlying assumptions and their justifications. The analysis and discussion of the results obtained through the aforementioned methodology are provided in Section 5. Finally, Section 6 provides concluding remarks, discusses research limitations, and offers recommendations for future improvement.

2 IFRS 17

2.1 Background

IFRS 17 is an International Financial Reporting Standard issued by the IASB, which provides a comprehensive approach to the accounting for insurance contracts (IFRS Foundation, 2021b). Figure 1 represents the main phases of IFRS 17 development.

Figure 1: A Chronological Overview of IFRS 17

In 1973, the International Accounting Standards Committee (IASC) was created to develop comprehensive accounting standards and promote their worldwide acceptance. In 1997, the IASC, the predecessor to the IASB, started the development of a standard to address the accounting for insurance contracts and in 2000, IASC was restructured into a full-time IASB and IFRS Foundation was created. IFRS 4 - Insurance Contracts was issued in 2004 by IASB. It is the accounting standard that preceded IFRS 17 and served as an interim standard for the accounting of insurance contracts. IFRS 4 permitted companies to maintain their existing accounting policies for insurance contracts, with modifications to enhance comparability and transparency. For example, it required insurers to retain insurance liabilities on their statements of financial position until they were discharged, canceled, or expired, and to present insurance liabilities without offsetting them against related reinsurance assets. It provided some basic guidelines for recognition, measurement, and disclosure but did not address all aspects of insurance accounting comprehensively. In 2017, IFRS 17 is officially issued by IASB and in 2020 IASB issued amendments to IFRS 17 to address implementation challenges and improve the quality of information provided. The effective implementation date was set for 1 January 2023, and from that date onwards, all companies in scope are required to adopt the standard.

2.2 Scope

All listed companies in the European Union (EU) must prepare their consolidated financial statements in accordance with IFRS as adopted by the EU. The use of IFRS can also be extended to non-listed companies, if the country opts so. The main advantages of implementing IFRS 17 include the comparability of the insurers and reinsurers' financial reporting on a worldwide basis, by establishing a standardized approach for depicting a company's financial position and performance. This reduces information asymmetry among market participants and aids investors in identifying global opportunities and risks, thereby improving capital allocation.

An entity shall apply IFRS 17 to issued insurance and reinsurance contracts, to held reinsurance contracts and to issued investment contracts with discretionary participation features, provided it also issues insurance contracts.

An insurance contract is defined as "a contract under which one party (the issuer) accepts significant insurance risk from another party (the policyholder) by agreeing to compensate the policyholder if a specified uncertain future event (the insured event) adversely affects the policyholder" (IFRS Foundation, 2021b).

A reinsurance contract is an "insurance contract issued by an entity (the reinsurer) to compensate another entity for claims arising from one or more insurance contracts issued by that other entity (underlying insurance contracts)" (IFRS Foundation, 2021b).

An insurance contract with direct participation features is an insurance contract in which the investor participates in a share of a clearly identified pool of underlying items and expects to receive an additional amount equal to a share of the fair value returns of the underlying items.

An insurance contract may have components that should be separated and excluded from the scope of IFRS 17, such as the investment component (under the scope of IFRS 9 - Financial Instruments) and goods or non-insurance services (under the scope of IFRS 15 - Revenue from contracts with customers). However, the separation is not required if there is a high interrelation between both components or if the component could not be provided separately in a non-insurer contract in the same market or jurisdiction.

2.3 Level of Aggregation

The groups of insurance contracts are the unit of account for measurement purposes. The contracts should be grouped by: (1) the type of contract, comprising portfolios of insurance contracts subject to similar risks and managed together; (2) 12 months' cohorts, as the group of contracts should comprise only contracts issued no more than one year apart, but not necessarily by calendar years; and (3) profitability buckets, with one group for onerous contracts at initial recognition, other for contracts that at initial recognition have no significant possibility of becoming onerous subsequently and other group including the remaining contracts.

Figure 2 provides an example of the levels of aggregation into six groups for a specific type of contract, life annuities.

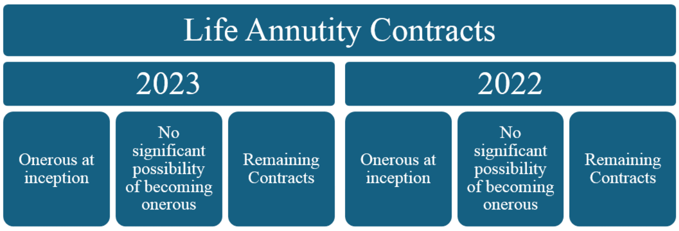

Figure 2: Level of Aggregation Example

2.4 Measurement Models

The information provided by following this standard is disclosed in the financial statements, which includes the level of technical provisions in the statement of financial position and (expected) profit in the income statement. For that, insurance companies need to calculate insurance contract liabilities and assets. Under this standard, there are three measurement approaches for different types of insurance contracts which may influence the pattern of recognition of insurers' IFRS 17 profits:

- The General Measurement model (GMM) may be applied to all insurance contracts, unless they have direct participation features.
- The Premium Allocation Approach (PAA) is a simplification of the GMM available only for short duration contracts (one year or less).
- The Variable Fee Approach is applied to insurance contracts with direct participation features.

For the purpose of this project, a more detailed analysis of the general measurement model is undertaken.

2.4.1 General Measurement Model

Under the general measurement model, the measurement is made with the building block approach, with the following blocks:

- The expected value of future insurance cash flows, which is equal to the difference between expected cash inflows and expected cash outflows based on current estimates. They should include all information on amount, timing and uncertainty of cash flows and use market consistent information where relevant, and the entity's perspective otherwise. Examples of cash flows in life insurance include premiums received by policyholders, claims and benefits paid to beneficiaries, policy administration and maintenance cost.

- Discount rate, to take into account the time value of money and the financial risks attached to the cash flows, save for the credit risk. It shall also reflect the characteristics of the cash flows and the liquidity characteristics of the insurance contracts, be consistent with observable current market prices for financial instruments whose characteristics are consistent with those of insurance contracts, exclude factors that influence such observable market prices but do not affect the future cash flows of insurance contracts.
- Risk adjustment (RA), to reflect the compensation an entity requires for bearing uncertainty about the amount and timing of cash flows arising from non-financial risks. Examples of risks covered in life insurance are claim occurrence, lapse, surrender and claim and expense inflation risk, excluding inflation index linked risk. It does not cover asset-liability mismatch risk and operational risk.

The first three blocks constitute the fulfillment cash flows, in Equation (1),

$$\text{Fulfillment Cash Flows} = -\text{PV Cash Inflows} + \text{PV Cash Outflows} + \text{RA}. \qquad (1)$$

- Contractual service margin (CSM), in Equation (2), represents the unearned profit to be recognized over time into Profit or Loss as services are provided. It applies solely to unexpired risk, encompassing future service. In case of a non-onerous contract at initial recognition no profit is recognized at inception. However, in case of an onerous contract, the CSM is zero and a loss is recognized at initial recognition.

$$\text{CSM} = \max\{-\text{Fulfillment Cash Flows}; 0\} \qquad (2)$$

Figure 3 depicts the components of the building block approach of a non-onerous contract at initial recognition, which may be denoted by the positive CSM.

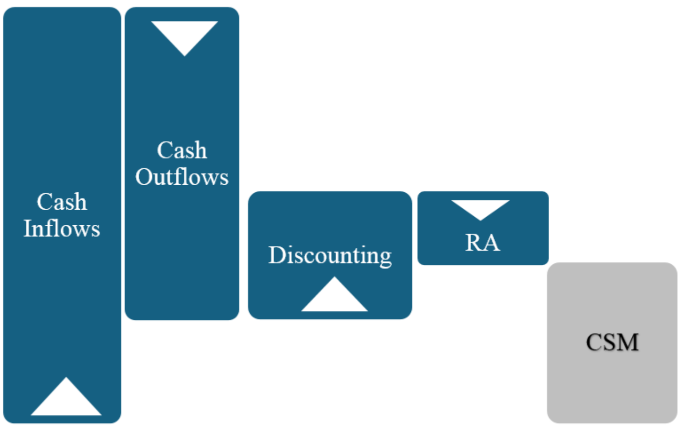

Figure 3: Non-Onerous Contract - Building Block Approach

Current discount rates are used to measure fulfillment cash flows in subsequent periods, ensuring values are based on prevailing market conditions. The contractual service margin (CSM) accumulates interest based on discount rates set at inception, thereby reflecting initial expectations throughout the contract's duration.

There are two general approaches to determine the discount rates:

- The bottom-up approach starts with a risk-free yield curve and an illiquidity premium is added to account for the liquidity differences between the insurance contracts to which the discount rates apply and the financial instruments on which the credit risk-free yield curve is based.
- The top-down approach begins with observable market rates, such as those from the actual portfolio held by the entity or a reference portfolio. Adjustments are then made to remove factors that are not relevant to measure insurance contracts under IFRS 17, such as credit risk and duration mismatches between the portfolio's assets and the insurance contracts being discounted. The reference portfolio refers to a set of financial assets used as a basis for determining discount rates for insurance contracts. It is chosen to reflect the nature, currency, and duration of the insurance liabilities being measured when the actual portfolio is unavailable or unsuitable for calculating discount rates.

The entity can choose the approach used to determine the discount rates with the only requirement to disclose the chosen methodology. Both approaches are represented in Figure 4. In principle, both approaches should result in the same yield curve, but in practice, this is very unlikely due to the limitations involved in credit and liquidity risk estimation techniques.

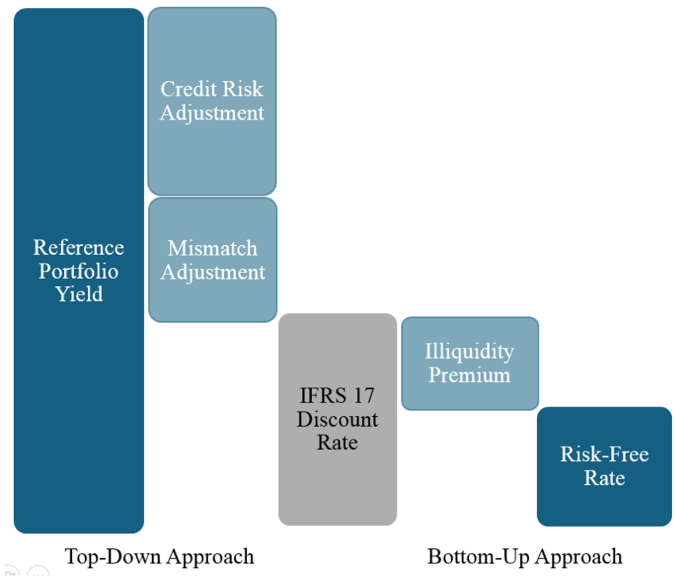

Figure 4: Discount Rate Approaches

3 Literature Review

The standard IFRS 17 issued in 2017 introduces significant changes to the recognition, measurement and disclosure of insurance contracts (IFRS Foundation, 2021). Before IFRS 17, the interim standard IFRS 4 did not establish a specific measurement model for insurance contracts. Consequently, companies had the flexibility to use local accounting practices, resulting in a wide variety of accounting principles worldwide. A critical change in the measurement of insurance liabilities under IFRS 17 is the use of streamlined discount rates, which play a crucial role in determining the present expected value of future cash flows associated with insurance contracts. These rates are used to measure the fulfillment cash flows, allowing companies to assess their insurance contracts assets or liabilities more accurately. By discounting future cash flows to present value, companies can account for the time value of money. Proper application of discount rates is essential for providing a realistic representation of insurance contracts and ensuring that financial statements accurately reflect the economic reality of the company's financial position and performance enabling stakeholders to make more informed decisions.

There are two main approaches for determining discount rates under IFRS 17: the top-down and the bottom-up approaches. The top-down method begins with observable financial asset market rates, and adjusts them to remove risk inherent to those assets that are not relevant for insurance liabilities, such as credit risk. This approach offers the advantage of being aligned with market conditions, promoting comparability and consistency across different entities. Besides, when companies construct their asset portfolios to match liabilities, those assets market rates could be used as a starting point. However, its reliance on market data may lead to challenges in isolating specific risk components accurately. In contrast, the bottom-up approach starts with risk-free rates and an adjustment is added to account for the risks associated with insurance contracts, such as illiquidity risk. This method allows for a more granular assessment of the risks but can be more complex to implement due to the need for precise risk estimation. Both methods aim to provide a current value of insurance contracts, yet each has its trade-offs in terms of complexity, data availability, and sensitivity to market conditions.

Practical case studies that have implemented the top-down approach demonstrate its effectiveness in providing market-consistent discount rates, promoting transparency and comparability in financial reporting. However, challenges may arise from accurately adjusting market rates to reflect the specific risk profile of the insurance contracts. Nonetheless, the approach offers the perceived benefit of leveraging market information to establish a realistic and reliable measurement of insurance contract assets and liabilities. The main challenges in determining discount rates under the top-down approach are the selection of the reference portfolio of assets, the methodology for removing credit risk, and the approach to extrapolating discount rates beyond the last liquid point to the ultimate forward rate. As IFRS 17 does not prescribe a specific methodology, this allows for several possibilities.

First, researchers emphasize the importance of selecting an appropriate reference portfolio that aligns with the risk profile, liquidity, and time structure of insurance contracts (IFoA, 2018). This selection process is critical as it directly influences the accuracy and relevance of the estimated discount rates. Some examples of application include a government bonds portfolio (IFoA, 2020) and a portfolio made up by government and corporate bonds from a single country (CIA, 2020). However, each practical application involving a different portfolio, particularly the one held by each company, encounters its own unique challenges and variations.

Following the selection of the reference portfolio, the top-down approach requires the removal of risks not relevant to insurance contracts, such as credit risk, from the observable yield curve (IFRS Foundation, 2021). Credit risk estimation is a central concern in finance, particularly in the context of banking and insurance. The credit risk removal may be done in two steps: estimating expected credit losses (ECL) due to default and then adjusting to account for unexpected credit losses (UCL), as stated by Jesop in a paper disclosed by Moody's Analytics (Jesop, 2018). This paper describes a number of permitted market-based approaches under IFRS 17, entailing structural credit models, market-based techniques and historical analysis.

Structural credit models analyse the probability of default and the loss-given-default (LGD) of a given company using market conditions, associated sector risks, capital structure and financial condition of the company. This class of credit models started with Merton (1974), who suggests that the credit risk of an insurance company can be reached by modelling the firm's equity as a call option on its assets. This model assumes that the company's assets follow a geometric Brownian motion and that its capital structure is composed by a zero-coupon debt and common equity. Additionally, it considers that the firm defaults if the value of its assets falls below the value of its liabilities. Under the framework of Black-Scholes (1973), it determines the probability of the call being exercised, which corresponds to the probability of the firm not defaulting. Although these models can provide issuer-specific estimates of credit risk, they are more complex to implement.

Market-based techniques include the use of credit derivative prices and reduced credit models. Credit Default Swaps (CDS) are financial derivatives that act as insurance against a default event (Giglio, 2016), with the spread compensating investors for assuming the associated credit risk. This spread encompasses both ECL and UCL, and can be deducted from the yield of the reference portfolio to obtain an adjusted credit risk-free rate (CIA, 2022), being a straightforward approach. However, while not all bond issuers have an associated CDS, those that do may exhibit different characteristics between bonds and CDS, such as timing, which can result in an inaccurate credit risk adjustment. Additionally, CDS prices are often subject to high volatility and can be influenced by market conditions and speculation. Moreover, CDS carry inherent liquidity risk and may not serve as pure measures of credit risk. Although there are potential methodologies to account for liquidity risk, none have gained universal acceptance (Brigo, 2010).

Reduced form models are statistical models which use statistics disclosed by credit rating agencies and data on the default-free market, rather than specific data relating to the issuing corporate entity, to model bond credit rating movements. The Jarrow-Lando-Turnbull (1997) model is the pioneer reduced credit model which considers there are n states ($n - 1$ credit ratings plus default) and uses transition intensities to estimate the probability of default. These models, relying on market-observable data instead of the firm's capital structure, are simpler and more flexible to implement. However, they are sensitive to market volatility and do not incorporate detailed, firm-specific information. Additionally, market data can be limited or of lower quality, especially for less liquid or infrequently traded instruments, leading to potentially inaccurate results. An example of calibration of IFRS 17 discount rates using this model is disclosed by the Institut des Actuaires (El Bekri, 2020).

Historical analysis uses historical available databases with default rates, LGD and transition rates disclosed by rating agencies to estimate the credit risk. One example is the ECL model, commonly applied in financial reporting, which estimates potential losses based on factors such as probability of default (PD) and LGD (Volarevi and Varovi, 2018). This model is used for estimating the impairment value of financial assets under IFRS 9 (IFRS Foundation, 2021a), offering another approach to isolate and adjust for credit risk in yield curve modeling. To account for UCL, an Educational Note prepared by the Canadian Institute of Actuaries (CIA) presents two different approaches: instead of using the default probability at the expected value, uses a greater probability level (90^{th} percentile of default); or use a multiple (twice) of the expected default risk. The main disadvantage of this model is the dependence on available market data and the lack of theoretical support when choosing the multiple of the expected default risk (CIA, 2022).

Other credit risk estimation approaches are disclosed in the Saunders and Allen

(2002). This book describes a set of possible approaches to measure the credit risk. One of the approaches which may be easily applied in the context of IFRS 17 discount rates determination is the Default Model which assumes that recovery rates are fixed and independent of the distribution of default probabilities and assumes the default is binomial distributed. In this case, the UCL are simply equal to the standard deviation of the PD times the LGD times the exposure at default. This model is easy to implement but very sensitive to the accuracy of the available market data. Besides this, the model output is an amount expressed in the same currency of the bond and, in this context, it needs to be adapted in order to obtain the credit-adjusted risk-free rate.

Finally, IFoA (Sharp, 2021) published a note using the Belkin and Suchower model (1998) disclosed by J.P. Morgan. This model uses historical transition matrices and allows the computation of expected and unexpected default risk by the generation of transition matrices at different percentiles. However, it assumes the credit risk follows a Normal distribution, which may lead to inaccurate results when using a higher percentile and the transition matrix is summarized in one single number, which leads to loss of information.

Once credit risk is removed, the final objective is nearly achieved; however, a method allowing for the extension of discount rates to periods not directly observed in the market is required first.

In the context of fitting the term structure, various methods for constructing yield curves have been introduced, such as linear interpolation, spline interpolation and the Nelson-Siegel model. Linear interpolation offers simplicity but may not accurately capture the yield curve's curvature. Spline interpolation methods, such as quadratic spline (McCulloch, 1971) and cubic spline (McCulloch, 1975), provide a smoother fit but introduce complexity due to the system of equations involved. Additionally, too many knots may fit the noise in the data rather than the underlying trend. They can also be sensitive to the choice of knots and boundary conditions.

Nelson and Siegel (1987) proposed a parametric model using a combination of exponential and polynomial functions to model the term structure of interest rates. Svensson (1994) extended this model to increase flexibility and improve goodness-of-fit through the use of two extra parameters. These parsimonious models are popular for fitting yield curves with a limited number of parameters, making them computationally efficient. The model coefficients have economic interpretations, and besides the level, slope and curvature factors present in the Nelson-Siegel model, the Svensson model contains a second hump/trough factor, allowing for an even broader and more complex range of term structure shapes.

The Svensson model is used by many European institutions, such as the European Central Bank (Nymand-Andersen, 2018) for estimating daily euro zone yield curves, and the Bank of International Settlements (BIS) (BIS Monetary and Economic Department, 2005). Estimation of the Svensson model parameters can be done using maximum likelihood, nonlinear least squares or the generalized method of moments (Svensson, 1994). BIS published technical documentation of zero-coupon yield curve estimation for thirteen national banks (BIS Monetary and Economic Department, 2005), five of which use the Svensson model. Some banks' methodologies are detailed in BIS papers. For example, the Norway bank (Eitrheim, 2005) maximises the likelihood function assuming yield errors follow a Normal distribution with zero mean and variance equal to the sample error variance, while the Swiss bank (Müller, 2005) first uses the Simplex algorithm and then the Berndt–Hall–Hall–Hausman (BHHH) algorithm (Berndt et al., 1974) until the convergence condition is met. Various algorithms can be used to perform the estimation. The BHHH is a gradient method that assures convergence when maximising the likelihood function. The Swiss bank assumes the estimated forward rates follow a normal distribution.

Manousopoulos and Michalopoulos (2009) compares non-linear optimization algorithms and suggests a two-step optimization process for the yield curve estimation problem using the Svensson model. First, solve the problem using a direct search algorithm such as the Nelder-Mead algorithm (Nelder and Mead, 1965) or a global optimization algorithm such as the Simulated Annealing algorithm (Cerny, 1985). These algorithms do not use gradient information. To refine the solution, a gradient based algorithm, such as the generalized reduced gradient (GRG) algorithm (Landson et al., 1978), is then used, reducing the average error.

Finally, the Smith-Wilson method (FINANSTILSYNET, 2010) fits a zero coupon rate curve with the ultimate forward rate (UFR) as a fixed input. Another input is a parameter alpha that determines how fast the estimated forward rates converge to the UFR. The pricing function is thus set up as the sum of an exponential term dependent on the UFR and N kernel functions dependent on alpha, with N being the number of observed yields. EIOPA uses this method to extrapolate interest rates. This method is easy to implement, provides a perfect fit of the estimated term structure to the liquid part data, can be analytically solved, and may be applied directly to raw data. However, the inputs require expert judgement to be chosen.

This project presents an example of a methodology and its implementation, starting with the actual portfolio of bonds held by the insurance company, which includes corporate and government bonds from various countries, addressing gaps in existing research and possible limitations of the regulatory framework.

4 Data and Methodology

The top-down approach under IFRS 17 is defined by the IASB as an approach "based on a yield curve that reflects the current market rates of return implicit in a fair value measurement of a reference portfolio of assets" (IFRS Foundation, 2021b). All reference data is provided by Lusitania Vida.

4.1 Reference Portfolio

Under IFRS 17, there are no specific restrictions on the reference portfolio and the actual bond portfolio held by the company is used. This study is based upon a portfolio constructed as at 29 December 2023 and the estimation of the term structure of interest rates with respect to the specified date is the focus. This choice is motivated by the bond portfolio's construction, which aligns with the assets or liabilities of insurance contracts through a strategy known as immunization. By matching the assets or liabilities of insurance contracts with the one from the backed financial assets, the portfolio minimizes interest rate risk and ensures stable measurement of insurance contracts over time which also enables balance sheet volatility reduction.

The portfolio is composed by 165 fixed-income bonds with 42 being sovereign bonds and the remaining 123 being corporate bonds. Six government bonds are zero-coupon bonds and eight bonds pay semi-annual coupons, while all others pay annual coupons. All government bonds, except one from Mexico, are from EU countries. There is only one corporate bond within this portfolio originating from a country that is not a member of either the OECD or the EU (India). Importantly, despite its non-member status, this bond maintain an investment-grade rating, and the analysis is not adversely affected by this. This bond can be considered without any issue. Figure 5 presents the portfolio composition by continent of origin of the bonds, measured in terms of principal. It demonstrates that nearly 90% of the bonds are sourced from Europe[1], with approximately 85% originating from EU member countries. All American bonds, except the one issued by the government of Mexico, are from the United States of America (USA).

[1] The "Europe (other)" category comprises bonds originating from European countries other than European Union countries.

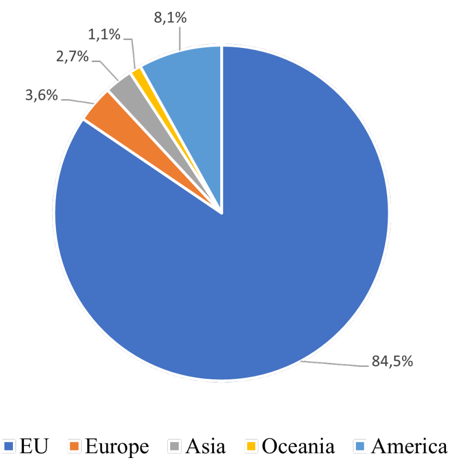

Figure 5: Portfolio Composition by Continent of Origin

The portfolio data disclosed by the company comprises the International Securities Identification Number (ISIN), serving as a unique identifier for each security within the portfolio, the type of bond, whether it is a government bond or a corporate bond, the coupon rate, the maturity date, the credit rating assigned, and the quantity in euro of each bond. The credit rating is the second best, when available, between S&P, Moody's and Fitch ratings. They are all converted to the equivalent Moody's credit rating.

The yield-to-maturity (YTM) of each bond is calculated using the dirty ask price disclosed in Bloomberg as of 29 December 2023, and then confirmed with the YTM also disclosed in Bloomberg. This confirmation is required to verify the accuracy of calculations, including coupon payment dates and amounts, which is essential for the subsequent credit risk adjustment. All bond prices are in euro. The YTM was calculated to satisfy the equation for the bond price, which is equivalent to the sum of discounted cash flows using the YTM, as expressed in Equation (3) for a bond with maturity at t_n and making coupon payments at times t_1, t_2, ... , t_n,

$$P_t = \sum_{t=t_1}^{t_n} \frac{CF_t}{(1+YTM)^t} \tag{3}$$

where P_t represents the bond price as at 29 December 2023 and CF_t represents the bond cash flow at time t, which includes the coupon payment and, if it is the maturity date, the face value as well. This was computed with the internal rate of return formula.

The bond portfolio is represented in Figure 6 by the type of bond. Since only one government bond is not from the EU, the Mexican government bond is represented differently by the orange triangle.

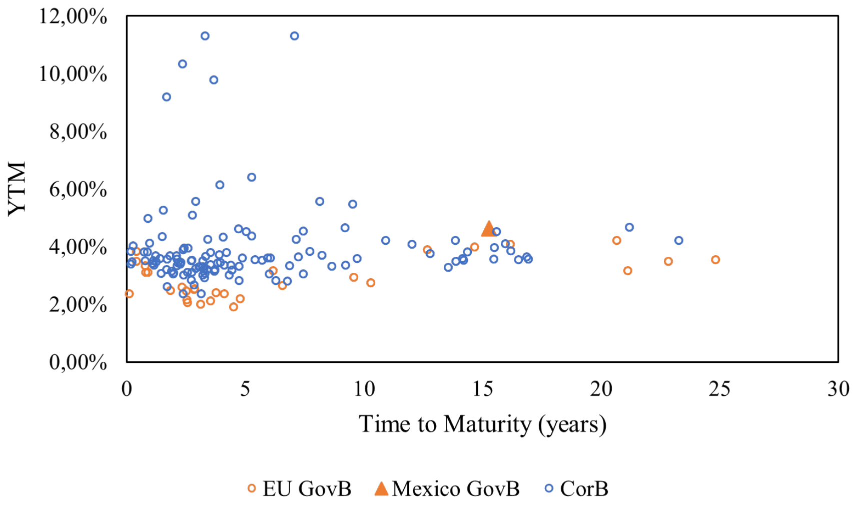

Figure 6: Bond Portfolio

In general, the government bonds' YTM are lower than the similar maturity corporate bonds' YTM. There are some exceptions, for example, between 12 years and 17 years, there are three government bonds whose YTM is greater when compared with those of similar maturities corporate bonds. There are also some corporate bonds whose credit rating is higher than from government bonds. However, a higher YTM is justified with a multiple of factors beyond credit risk, including liquidity, market risk, inflation expectations and currency risk. The corporations correspondent to those corporate bonds are from other countries than EU or they are large corporations whose country's government hold a large share which increases the safety investor perception.

Table I provides the range, average[2] and median of some key bond metrics, including time to maturity[3], bond price as percentage of the face value and the YTM, as well as Macaulay duration, from Bloomberg.

Table I: Descriptive Statistics of the Bond Portfolio

	Minimum	Mean	Median	Maximum
YTM	1.91%	3.65%	3.51%	11.33%
Time To Maturity	0.13	6.81	3.43	24.84
Coupon Rate	0.00%	2.39%	2.38%	7.30%
Duration	0.13	5.64	3.27	17.59
Price	53.89	94.51	97.43	126.62

[2]The relative quantity was used as weights.
[3]The time to maturity is measured in years with an Actual/Actual base.

Table II presents a summary of the portfolio categorized by credit rating and bond type, including the proportion relative to the quantity of each bond, as well as the Macaulay duration and YTM weighted averages, with weights based on the quantity of each bond.

Table II: Portfolio Summary by Credit Rating and Bond Type

Rating	N. of Bonds	Proportion	Av. YTM	Av. Duration
Aaa	10	9.50%	3.12%	3.70
Aa	9	7.16%	3.65%	5.27
A	41	24.13%	3.41%	5.78
Baa	96	54.65%	3.92%	6.44
Ba	6	2.65%	2.69%	1.73
B	3	0.92%	2.85%	1.15
Government	42	40.85%	3.09%	6.07
Corporate	123	59.15%	3.99%	5.34
All bonds	165	100%	3.62%	5.64

The corporate bonds are represented in Figure 7 by credit rating.

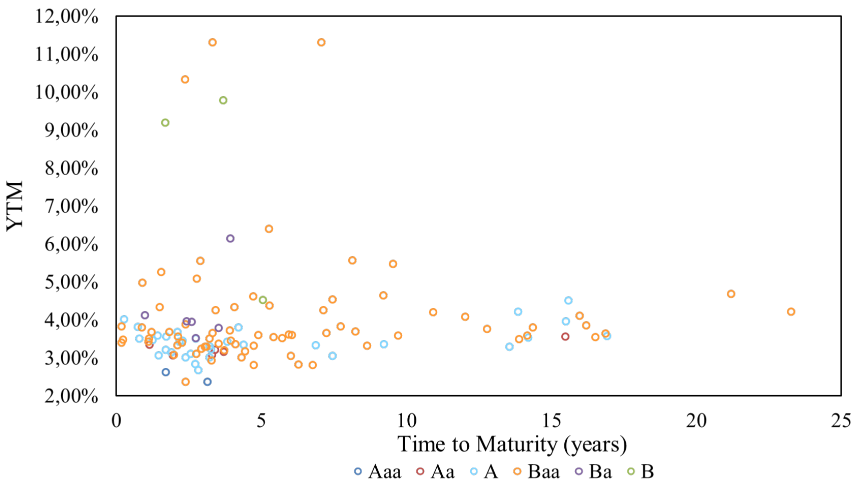

Figure 7: Corporate Bonds by Rating Category

There are three Baa bonds which can be denoted with YTM greater than 10%, being the maximum 11.30%. These YTMs are even higher than YTM of rated B-bonds. These three bonds are from the same issuer and the higher YTMs are motivated by an announcement made in November 2023 of repurchase shares, which was negatively perceived by investors. Since the reference date 29 December 2023, the YTM has decreased and coming back to normal values not justifying the use of a lower credit rating in the credit risk removal phase in order to account more credit risk in the calculations. Based on the plot, it can be inferred that credit rating does not fully account for all variations in credit risk, and credit risk is not the sole factor influencing YTMs.

4.2 Credit Risk Removal

According to IFRS 17, factors from the reference portfolio that are not relevant to insurance contracts should be removed. Given that the reference portfolio is a bond portfolio which has been constructed to achieve immunization, there is no need to make a mismatch adjustment and the factors which are not relevant to insurance contracts are credit risk and liquidity risk. Under the top-down approach (IFRS Foundation, 2021b), there is no requirement to adjust the yield curve for differences in liquidity characteristics of the insurance contracts and the asset portfolio. Thus, for the bond portfolio only the effect of credit risk must be eliminated from the total bond yield. The use of credit derivative prices to estimate the effect of credit risk is given as an example. However, it does not impose a specific methodology.

The chosen methodology is based on the approach proposed by the Canadian Institute of Actuaries (CIA, 2020), where credit-adjusted cash flows are estimated and the YTM is recalculated to obtain the credit risk-free yield. The credit-adjusted cash flows are calculated based on the theory underlying the expected credit loss (ECL) model, which is a financial framework used to estimate potential credit losses on financial assets over their lifetime. The model calculates the weighted average of possible credit losses, taking into account the probability of default (PD), loss given default (LGD), and exposure at default (EAD).
Let ECL_t, in Equation (4), represents the expected credit loss of a given bond at time t with $0 < t \leq T$ and T the time to maturity,

$$ECL_t = PD_t \times LGD_t \times EAD_t, \tag{4}$$

where LGD_t is the loss given default at time t, EAD_t is the exposure at default (nominal value of the bond plus the accrued interest) at time t and PD_t is the lifetime probability of default at time t.

According to the ECL model, the portfolio expected credit loss, in Equation (5), is the sum of the expected credit losses of the bonds at a given time t,

$$ECL = \sum_{i=1}^{n} ECL_i, \tag{5}$$

where n is the number of bonds in the portfolio and ECL_i is the expected credit loss of the i^{th} bond.

The credit risk adjustment presented here is not conducted directly using the expected credit loss model, as a single amount is not being estimated. Instead, each bond yield in the portfolio is adjusted as mentioned before.

There are three possible scenarios: (i) the bond either defaults from the time of purchase or the most recent payment, whichever is later; (ii) the bond does not default; or (iii) the bond has already defaulted at the time of the most recent payment, if any. Considering a bond with cash flow payments at times t_1, t_2, ..., t_n, where t_n is the time to maturity, the cash flow at time t_i for $1 \leq i \leq n$ corresponds to one of the three scenarios:

(i) In the scenario where the bond defaults the cash flow is equal to (Coupon Payment$_{t_i}$ + Face Value) * Recovery Rate and has a probability equal to $PD_{]t_{i-1},t_i]}$, where $PD_{]t_{i-1},t_i]}$ represents the incremental PD from time t_{i-1}, the time of purchase or the most recent payment, whichever is later, to time t_i;

(ii) In the scenario where the bond does not default the cash flow is equal to the coupon payment corresponding to time t_i and, if t_i equals t_n, the face value as well. This scenario is associated with a probability of $1 - PD_{\leq t_i}$, where $PD_{\leq t_i}$ represents the cumulative PD from the time of purchase to time t_i;

(iii) In the scenario where the bond has already defaulted at the time of the most recent previous payment t_{i-1} there is no cash flow at time t_i. This scenario is associated with a probability of $PD_{\leq t_{i-1}}$, where $PD_{\leq t_{i-1}}$ represents the cumulative PD from the time of purchase to time t_{i-1}

The adjusted cash flow at time t_i, in Equation (6), with $0 \leq t \leq T$, where T is the time to maturity, denoted as $\overline{CF}_{t_i}$ is the expected value of the cash flow, which is equal to a weighted average of the three scenarios,

$$\overline{CF}_{t_i} = x_1 \times \text{PD}_{]t_{i-1},t_i]} + x_2 \times (1 - \text{PD}_{\leq t_i}) + x_3 \times \text{PD}_{\leq t_{i-1}}, \tag{6}$$

where x_j, with $j = 1, 2, 3$ is the cash flow corresponding to the j-scenario.

Given there is no cash flow in the third scenario, x_3 is equal to zero and Equation (6) can be simplified to Equation (7),

$$\overline{CF}_{t_i} = x_1 \times \text{PD}_{]t_{i-1},t_i]} + x_2 \times (1 - \text{PD}_{\leq t_i}). \tag{7}$$

The recovery rate is the proportion of the principal amount that is reclaimed by the bondholder in the event of a default and it is equal to $1 - LGD$. The LGD values were disclosed by Moody's. Table III illustrates the four distinct LGD values depending on whether the bond is senior secured, senior unsecured (only corporate bonds), subordinated, or sovereign (including government senior unsecured bonds), as each type presents different levels of risk and the proportion with the relative quantity used as weights. Senior secured debt provides creditors with the lowest LGD due to the backing of specific assets that can be liquidated in the event of default. In contrast, senior unsecured debt issued by corporations carries a higher LGD since it is not secured by specific assets and relies solely on the issuer's creditworthiness. Subordinated debt carries even higher LGD as it is ranked below senior debt in terms of claim priority, thus exposing creditors to greater risk during insolvency proceedings. Sovereign bonds and senior unsecured government bonds are issued by governments and typically provide assurance to investors due to the issuing country's taxing authority and control over monetary policy.

Table III: Portfolio Loss-Given-Default

	LGD	Count	Proportion
Senior Secured	38.80%	2	0.57%
Senior Unsecured	52.87%	109	53.78%
Sovereign	61.00%	42	41.13%
Subordinated	72.17%	12	4.51%

Under IFRS 17 besides ECL, unexpected credit losses (UCL) should also be accounted for. UCL refer to the portion of credit losses that exceed the best estimate based on historical data. There are two sources of UCL: an unexpected credit event and mis-estimation of long term expected default. CIA proposes two approaches for the derivation of expected and unexpected credit losses: (1) use a multiple of the expected default probability (such as twice the default probability); and (2) use a probability level greater than the mean (use the 90^{th} percentile of default). These approaches allow to use the default probability with a margin to also account for unexpected credit losses. The Moody's default report of 2023 presents cumulative issuer-weighted default rates by annual cohort and rating category from 1970 to 2022. This data presents cumulative default rates for different time horizons, ranging from 1 to 20 years and is used to calculate the 90^{th} percentile of default rates, in Equation (8), for each rating category from 1 to 20 years. The selection of the 90^{th} percentile as a measure of default rates provides a threshold below which 90% of the observed default rates fall. This measure is thus indicative of the value above which the top 10% of default rates are situated,

$$P_{90} = (1 - d) \times D[k - 1] + d \times D[k] \tag{8}$$

where $k = 0.9 * (n + 1)$, with n the total number of observations; D represents the default rate vector sorted in ascending order; $[k]$ represents the integer part of k; and d represents the decimal part of k, which is equal to $k - [k]$.

Based on the default rates reported by Moody's from 1970 to 2022, $n = 53$, $k = 48.6$ and $d = 0.6$. From this part of the analysis on, the probability notation used represents the 90^{th} percentile of default instead of the default probability only. The t-year cumulative default probability $PD_{\leq t}$ denotes the cumulative 90^{th} percentile of the t-year default rates, $0 \leq t \leq 20$. The incremental default probability ($PD_{]t-1,t]}$), in Equation (9), denotes the difference between the cumulative 90^{th} percentile of the t-year default rates and the cumulative 90^{th} percentile of the $(t - 1)$-year default rates, with $1 \leq t \leq 20$,

$$PD_{]t-1,t]} = PD_{\leq t} - PD_{\leq t-1}. \tag{9}$$

The probability applied to cash flows during intra-annual periods was calculated using linear interpolation. For example, if a payment for an annual coupon bond occurs on 31 March 2025, corresponding to the end of the first quarter of the year 2025, the probability of default for the payment is calculated by considering 0.25 of the current year's incremental probability and 0.75 of the previous year's incremental probability,

$$\mathrm{PD}_{]0.25,1.25]} = \mathrm{PD}_{]1,2]} \times 0.25 + \mathrm{PD}_{]0,1]} \times 0.75$$

There are six bonds with time to maturity higher than 20 years, being 24.84 years the maximum. For these cases, a zero incremental default probability from 20 years until the maturity deemed reasonable. Thus, the cumulative probability of default of 24.8 years is equal to the cumulative probability of 20 years.

From the total forty-two sovereign bonds, only one was from outside the EU (Mexico). All EU bonds were considered credit risk-free for three main reasons: (1) under Solvency II, the EU Member States' government bonds shall be assigned a zero capital charge for spread risk [4]; (2) since the beginning of the EU only one case of default has been registered, Greece in 2012; and (3) Moody's default report only relies on corporate data. The Mexico bond was treated as a corporate bond.

The recalculation of YTMs is performed using the credit-adjusted cash flows, resulting in new YTMs that represent the credit-adjusted risk-free yields. The price

[4] A detailed explanation of this assumption is provided in the Appendix A

used to calculate these adjusted YTMs is the original price as at 29 December 2023, which is paid at time 0 regardless of whether a default occurs in the future.

4.3 Curve Extrapolation

Having the adjusted credit-risk free yields derived, the following step is the extrapolation of the yield curve to estimate interest rates for maturities beyond those available, and interpolation for intermediate terms. From this point of the analysis on, only the credit-adjusted risk-free yields and cash flows are considered. Thus, the credit-adjusted risk-free YTM is denoted as YTM and the credit-adjusted cash flows are denoted by CF.

4.3.1 Svensson Model: Theoretical Framework

The curve extrapolation was performed using the parametric Svensson (1994) model, which provides a mathematical representation of the term structure of interest rates, describing the relationship between interest rates and different maturities of debt securities. The use of this model is justified by its widespread adoption by government entities, allowing for comparability. Additionally, all the adjusted cash flows obtained through the chosen credit risk estimation technique, which are necessary to implement the model, are available.

Under IFRS 17, the liabilities are discounted using the spot rates. Let s_t represent the spot rate function, in Equation (10), for a given time to maturity t, with $0 \leq t$ derived from the Svensson model,

$$s_t = \beta_0 + (\beta_1 + \beta_2)\frac{1 - \exp^{-\frac{t}{\tau_1}}}{\frac{t}{\tau_1}} - \beta_2 \exp^{-\frac{t}{\tau_1}} + \beta_3 \left(\frac{1 - \exp^{-\frac{t}{\tau_2}}}{\frac{t}{\tau_2}} - \exp^{-\frac{t}{\tau_2}} \right) \tag{10}$$

where β_0, β_1, β_2, β_3, τ_1 and τ_2 specify the parameters to be estimated.
The fourth term, in Equation (11), is the extension to the Nelson-Siegel model proposed by Svensson,

$$\beta_3 \left(\frac{1 - \exp^{-\frac{t}{\tau_2}}}{\frac{t}{\tau_2}} - \exp^{-\frac{t}{\tau_2}} \right). \tag{11}$$

One of the advantages of the model is that the parameters can be interpreted (Garcia and Carvalho, 2018) as follows: β_0 as the very long-term interest rate (as the time to maturity approaches infinity, the yield rate converges to this value); $\beta_0 + \beta_1$ as an instantaneous interest rate (as the time to maturity goes to zero, the yield rate converges to this value); $-\beta_1$ turns to be the average slope of the term structure (the difference between the very long-term interest rate and the instantaneous interest rate); β_2 as the rate at which the slope and the curvature component decay to zero (it is responsible for the hump if positive or the trough if negative and for how the short and long interest rates interchange); β_3 as an independent decay that introduces a new hump or a new trough to better fit the model; and although τ_1 and τ_2 do not have an economic interpretation, they are decay parameters that control how inclination and curvature behave.

Estimated prices (P^*), in Equation (12), are the present value of the future credit-adjusted cash flows using the spot rates (s_t),

$$P^* = \sum_{t=0}^{T} \frac{CF_t}{(1+s_t)^t} \tag{12}$$

where CF_t represents the future credit-adjusted cash flow at time t, $0 \leq t \leq T$, with T the bond's time to maturity.

Estimated yields (YTM^*), in Equation (13), are then obtained from estimated prices, since the YTM can be seen as the internal rate of return of an investment in a bond if held until maturity,

$$P^* = \sum_{t=0}^{T} \frac{CF_t}{(1+YTM^*)^t}. \tag{13}$$

The model parameters may be estimated through the minimization between yield squared errors, in Equation (14), or price squared errors, in Equation (15), with maximum likelihood, non-linear least squares or using the generalized method of moments,

$$\sum_{i=1}^{n} (YTM_i^* - YTM_i)^2 \tag{14}$$

or

$$\sum_{i=1}^{n} (P_i^* - P_i)^2, \tag{15}$$

where n represents the number of bonds in the portfolio and P the actual price of the bond.

4.3.2 Svensson Model: Implementation Process

Portfolio yield squared errors, in Equation (14), are minimized through a non-linear optimization process. The decision is based on Svensson's suggestion that prices demonstrate reduced sensitivity to yield changes for shorter maturities and, consequently, minimizing price errors might result in relatively large yield errors for bonds with short maturities.

Three constraints are imposed in the minimization problem: firstly, the sum of β_0 and β_1 must be equal to the overnight rate, in Equation (16); and secondly, both decay parameters (τ_1 and τ_2) should be positive, in Equation (17) and Equation (18). The overnight rate used is the euro short-term rate (€STR) published by the European Central Bank (ECB) as at 29 December 2023. Formally, the problem is stated as:

$$\min \sum_{i=1}^{165} (YTM_i - YTM_i{}^*)^2$$

$$\text{s.t. } \beta_0 + \beta_1 = 3.882\% \tag{16}$$

$$\tau_1 > 0 \tag{17}$$

$$\tau_2 > 0. \tag{18}$$

The first step entails determining the initial values of the parameters[5]. It is considered that $\beta_0 + \beta_1$ equals the overnight rate. Since β_0 can be interpreted as the very long-term interest rate, β_0 is set as the average of the yields of the two longest maturity bonds. As a result, β_1 is set as the difference between the overnight rate and β_0. Both β_2 and β_3 are set to zero, while the decay parameters τ_1 and τ_2 are each set to 1.

Spot rates, in Equation (10), are computed with these initial values and then estimated prices, in Equation (12), are computed. With these prices, each bond's YTM^*, in Equation (13), is estimated using a Newton-Raphson method in R.

The optimisation problem is then solved with a two-step optimisation process. Firstly, a direct search or a global optimization algorithm is used to solve the problem, and then the solution is refined using one of the gradient-based algorithms (Manousopoulos and Michalopoulos, 2009). This process is justified by the fact that direct search or global optimization algorithms can explore a larger region of the search space and utilize additional method parameters more efficiently. In contrast, gradient-based algorithms depend heavily on the starting point. Consequently, the first group is expected to achieve a smaller error than a gradient-based algorithm for the Extended Nelson–Siegel method. However, the solution obtained by the first group of algorithms may not be strictly a local optimum, as no gradient information is used. Therefore, it is reasonable to refine the solution with a gradient-based

[5]The starting values are based on an article published in the European Journal of Operational Research (Manousopoulos and Michalopoulos, 2009).

algorithm.

Direct search algorithms are numerical optimization methods that do not require a gradient function. These algorithms search a set of points around the current point, looking for one where the value of the objective function is lower than the value at the current point. The direct search algorithm used is the Nelder-Mead (1965). This direct search algorithm is also known as simplex-search algorithm. The algorithm operates on a simplex with $(n+1)$ vertices when the objective function has n variables. The objective function is then evaluated at each vertex of the simplex and iteratively updates the simplex converging to the minimum. It is a direct search method since no gradient function is required. It is easy to implement, since it is a derivative-free method, and it is one of the most popular optimization algorithms.

The refinement of the solution is performed with gradient-based algorithms. These algorithms use the gradient of the objective function to iteratively find the function's minima or maxima. They utilize the gradient vector, which indicates the direction of the greatest increase, to guide the search process. By moving in the opposite direction of the gradient, these methods aim to minimize the objective function efficiently. The Generalized Reduced Gradient (GRG) method (Landson et al., 1978) is applied. This gradient based algorithm iteratively adjusts the decision variables to minimize the objective function while ensuring the given constraints are met using gradient information to guide the search for the optimal solution, making it more efficient and accurate for smooth problems. It is also easy to implement.

4.4 Sensitivity Analysis

Given the new inter dependencies at the balance sheet and income statements it is important to assess the portfolio behaviour against interest rates changes to incorporate into an Asset and Liability Management (ALM) approach. Therefore, a scenario analysis is conducted to evaluate the sensitivity of the portfolio market value. Four scenarios are computed: (1) a parallel shift of the spot rates of plus 50 basis points (bps); (2) a parallel shift of the spot rates of minus 50 bps; (3) a rotation of the spot curve with its center considered as the 10-year spot rate, decreasing by 50 bps at time 0 and increasing by 50 bps at time 20. This analyses a short-term rates fall and long-term rates rise; and (4) a rotation of the spot curve with its center considered as the 10-year spot rate, increasing by 50 bps at time 0 and decreasing by 50 bps at time 20. This analyses a short-term rates rise and long-term rates fall.

4.5 Comparison with EIOPA risk-free curve

As previously mentioned, the top-down approach is one of the two approaches allowed under IFRS 17 for estimating discount rates. The other approach, the bottom-up approach, begins with risk-free rates, to which an illiquidity premium should be added. This premium should reflect the differences between the liquidity characteristics of the financial instruments that underlie the rates observed in the market and the liquidity characteristics of the insurance contracts. Some implementations of this approach start with the euro EIOPA risk-free curve. The EIOPA's basic term structure is derived from the observed swap rates, adjusted for credit risk. It is not expected that the derived curve and the derived from EIOPA will coincide as a liquidity premium is not computed. However, a comparison between the derived curve and the EIOPA risk-free curve disclosed at the end of 2023 is made to explore more reasons that explain the differences between them.

4.6 Comparison with ECB yield curve

The derived curve considers the government bonds as risk-free and does not include any credit adjustment for these bonds. The ECB curve uses all euro zone government bonds. In addition to yearly spot rates, rates for 3 months, 6 months, and 9 months are also disclosed. Another comparison is made with the ECB curve as at 29 December 2023, given that both use the same extrapolation method, the Svensson model.

5 Results

5.1 Credit Risk Estimation

The credit risk was removed from the reference bond portfolio as described in Chapter 4. First, the 90^{th} percentile of default, which was used instead of the default probability to also account for UCL, was calculated. Table IV presents the values of the computed 90^{th} percentile of default rates. Based on these values, the interpretation which can be made is that the likelihood of an Aaa bond defaulting within the first 7 years is 0.47%. As time progresses, the cumulative default percentile increases, given that the initial group of observed companies in each year remains consistent, while the number of defaults either increases or remains constant over time. This is because, once a bond defaults, it does not experience recovery. As a rule of thumb, the lower the credit rating, the higher the default rate, which is consistent with the concept of credit rating. An exception occurs from 11 to 15 years, the Aaa 90^{th} percentile is greater than that for Aa bonds. This illustrates that credit ratings, even those assigned by considerable credible rating agencies, may fail, as they are determined based on models with limitations and may overlook certain factors.

Table IV: 90^{th} Percentile of Default Rates (in %)

Year	Aaa	Aa	A	Baa	Ba	B	**Year**	Aaa	Aa	A	Baa	Ba	B
1	0.00	0.00	0.14	0.54	2.70	7.62	**11**	2.27	1.88	4.06	7.97	31.60	52.77
2	0.00	0.14	0.34	0.98	5.87	15.58	**12**	2.48	1.94	4.60	8.75	32.49	54.93
3	0.00	0.25	0.83	1.79	8.74	24.10	**13**	2.56	2.28	4.78	9.68	34,75	55.67
4	0.00	0.68	1.16	2.46	12.25	29.20	**14**	2.58	2.51	5.13	10.39	35.92	57.81
5	0.00	0.83	1.81	3.17	17.91	33.56	**15**	2.64	2.60	5.15	11.02	37.82	59.73
6	0.00	1.01	2.22	3.42	21.54	39.11	**16**	2.65	2.69	5.53	11.12	39.56	60.10
7	0.47	1.41	2.42	4.45	23.94	43.22	**17**	2.70	2.99	5.63	11.42	40.92	60.47
8	1.20	1.64	2.73	5.25	25.61	44.73	**18**	2.70	3.04	5.89	12.12	42.25	60.85
9	1.31	1.86	3.17	5.78	28.16	46.51	**19**	2.70	3.41	6.08	13.69	43.37	61.98
10	1.53	1.87	3.62	6.99	30.73	49.5	**20**	2.70	3.81	6.75	14.05	44.62	65.46

With these percentiles, the credit adjustment is carried out. A detailed example of computing the credit-adjusted risk-free YTM is provided in the Appendix B. The credit-adjusted risk-free YTMs are represented in Figure 8. It is noted that the three Baa bonds still have a higher YTM compared to the remaining ones, as anticipated. The other two B-rated bonds which initially had YTMs close to 10%, experienced a higher adjustment and are now more comparable to the remaining bonds.

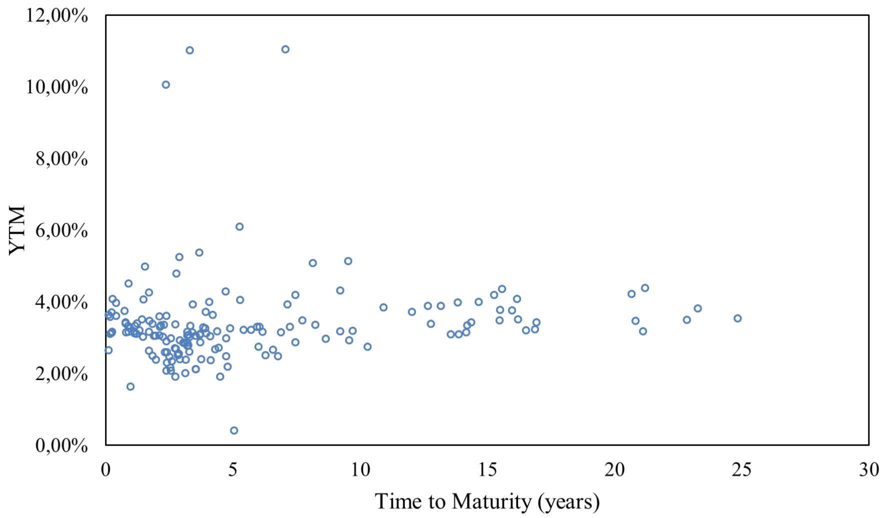

Figure 8: Credit-Adjusted Risk-Free YTM

A deeper analysis of the yields and the adjustments may be done with the values presented in Table V. This table presents the average of the time to maturity, the YTM before adjustment and the credit-adjusted risk-free YTM (YTM*) and the minimum, the average and the maximum credit risk adjustments of government bonds and per rating category for corporate bonds.

Table V: Credit Risk Adjustment per Type of Bond

				Credit Risk		
	Time	YTM	YTM*	Min.	Average	Max.
Corporate						
Aaa	2.43	2.50%	2.50%	0.00%	0.00%	0.00%
Aa	4.68	3.24%	3.17%	0.03%	0.06%	0.08%
A	5.21	3.45%	3.31%	0.07%	0.14%	0.23%
Baa	6.07	4.11%	3.74%	0.26%	0.36%	0.56%
Ba	2.71	4.26%	2.33%	1.61%	1.93%	2.54%
B	3.48	7.87%	3.35%	4.12%	4.52%	5.00%
Government						
EU	6.32	2.99%	2.99%	0.00%	0.00%	0.00%
Mexico (Baa)	15.28	4.63%	4.19%	0.44%	0.44%	0.44%

In Table IV, it is denoted that the cumulative 90^{th} percentile of default within a 6-year period is zero. Given that the longest maturity Aaa bond extends to 3.15 years, this explains the zero risk adjustment for this type of bonds. As anticipated, there exists a positive correlation between credit risk and rating, with lower ratings incurring greater credit risk adjustments. The adjustment also depends on the time to maturity of the bond, longer maturities result in a higher 90^{th} percentile of default, thereby leading to a greater adjustment. Moreover, it is important to highlight the average credit-adjusted risk-free YTM for Baa bonds, which surpasses the remaining values. This discrepancy is attributed to the influence of the three aforementioned bonds. Additionally, the average YTM* for Ba bonds is lower than that of Aaa bonds and EU government bonds. On one hand, the average time to maturity of Ba bonds is 2.71 years, whereas for EU bonds, it is 6.32 years, and this disparity partially justifies the difference in yields. On the other hand, with the average time to maturity of Aaa bonds at 2.43 years, it raises questions as to whether the zero-credit adjustment of Aaa bonds was adequate or if the credit adjustment for Ba bonds was excessively high.

5.2 Curve Extrapolation

The Svensson model was applied to the credit-adjusted risk-free yields in order to obtain a smooth and continuous spot curve. The first step in the methodology is the initialization of the model parameters. β_0 is equal to the average of the two bond rates with longer maturity, 3.54% and 3.81% with time to maturity 24.84 and 23.28 years, respectively. β_1 is the difference between the overnight rate 3.882% and β_0. The Nelder-Mead algorithm was then implemented to minimise the sum of the yield squared errors with three penalties equal to 10^6 in the objective function: one if the sum of β_0 and β_1 is not equal to 3.882%, the overnight rate; and two penalties one for each one of the decay parameters τ_1 and τ_2, if they are lower or equal to zero. The refinement of the Nelder-Mead solution is achieved by implementing the GRG algorithm. The method was implemented in Excel Solver, and the constraints were explicitly modeled. The initial values and estimated parameters by each algorithm, the respective Mean Squared Error (MSE) and standard deviation (Sigma) are presented in Table VI.

Table VI: Initial Values

Parameter	Initial Value	Nelder-Mead	GRG
β_0	0.03675	0.03674	0.03789
β_1	0.00207	0.00208	0.00093
β_2	0	-0.04122	-0.04090
β_3	0	0.01964	0.01622
τ_1	1	1.00568	1.00578
τ_2	1	1.06011	1.06146
MSE (bps)	1.740	1.509	1.507
Sigma (%)	1.245	1.228	1.227

The estimated parameters with both Nelder-Mead and GRG algorithms respect the three imposed restrictions. Since both β_2 and β_3 parameters are different from zero we have two curvatures. With the application of this model, a smooth curve of spot rates is obtained, which can be used for discounting the FCF. The final curve starts at the overnight rate 3.882%, decreases to 3.089% at 1.83 years and then increases converging to the very long-term spot rate equal to 3.789%. Figure 9 presents the spot rate curve with these final estimated parameters.

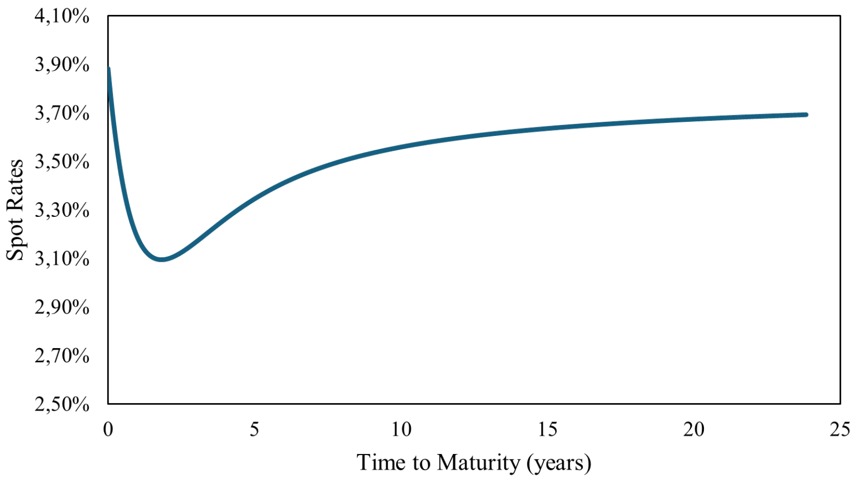

Figure 9: Fitted Curve

The MSE is equal to 1.509 bps with the Nelder-Mead algorithm, which represents an average error reduction from the model with the initialised values of 13.244%. The MSE with the application of the second algorithm also decreases to 1.507 bps, representing an average error reduction of 0.126% from the Nelder-Mead optimisation and 13.351% from the initial values. This lower MSE indicates a more accurate curve that better reflects the reality of the market bond portfolio. The standard error deviation with these parameters is equal to 0.01227, which is also lower than the

0.01228 Nelder-Mead standard deviation and than the 0.01245 initial standard deviation. The lower standard error deviation with these parameters indicates higher precision and robustness in the model. As anticipated, the three aforementioned Baa bonds have a significant influence on the squared error, representing 66.45% of the total squared error.

5.3 Sensitivity Analysis

From the estimated spot curve, several scenarios were tested to analyze the sensitivity of the portfolio value. The first two scenarios, represented in Figure 10, entail parallel shifts of 50 basis points in magnitude. The portfolio value using the estimated spot curves is equal to 667,267,434 euro. Given an increase of the market rates in 0.5% for all the maturities, the portfolio value decreases, due to the inverse relation between yields and price. The decrease represents 3.06% of the market value. However, in the inverse scenario, when all rates decrease by 50 basis points, the value of the portfolio does not increase as much, experiencing an increase of 2.08% only.

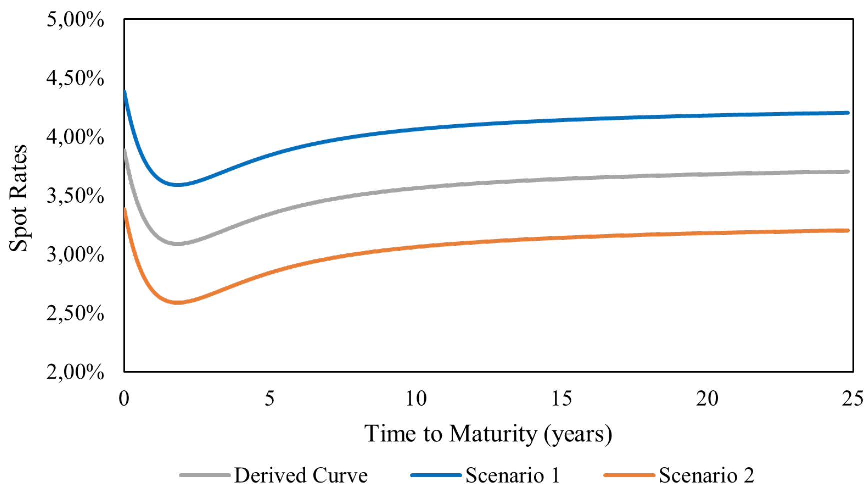

Figure 10: Parallel Shifts Scenarios

The two other computed scenarios are shifts centered around the 10-year mark with a decrease or increase of 0.5% in the shorter maturities and an increase or decrease in the longer maturities, respectively. They are graphically represented in Figure 11.

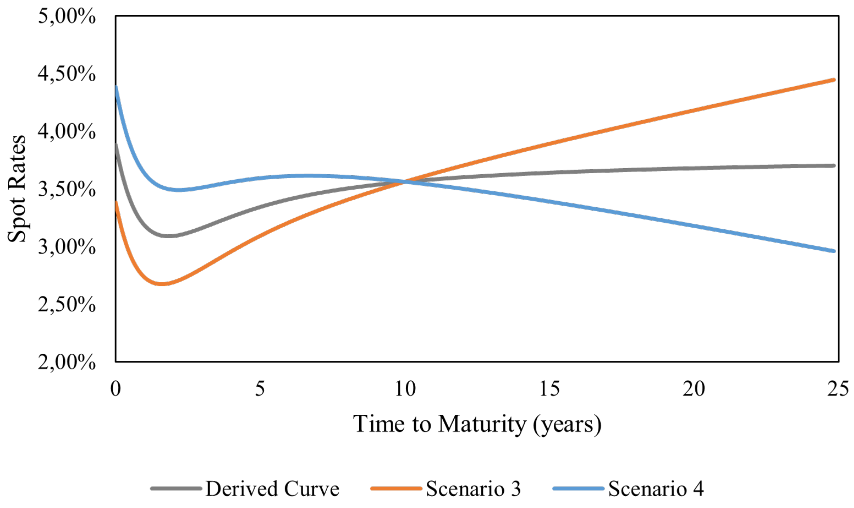

Figure 11: Rotation Shifts Scenarios

Under the third scenario, the market portfolio experiences a decrease of 0.27% in value. This outcome underscores the nuanced impact of the shifting yield curve on asset valuation within the portfolio, attributed to the differing effects across various maturity segments. Conversely, in the fourth scenario, where the rotation similarly centers around the 10-year mark but with an inverse adjustment, the market portfolio demonstrates an increase of 0.37%. The growth rate of the portfolio value in each scenario is represented in table VII.

Table VII: Sensitivity Analysis

Scenario	Growth Value
1	-3.06%
2	2.08%
3	-0.27%
4	0.37%

This information is critical to perform an ALM analysis integrating assets and liabilities and define portfolio asset class allocation and define, if needed, a derisking plan for each macroecomic scenario in the Own Risk and Solvency Assessment (ORSA).

5.4 Comparison with EIOPA risk-free curve

A visual comparison between EIOPA risk-free curve from 1 to 30 years and the derived spot curve can be depicted in Figure 12.

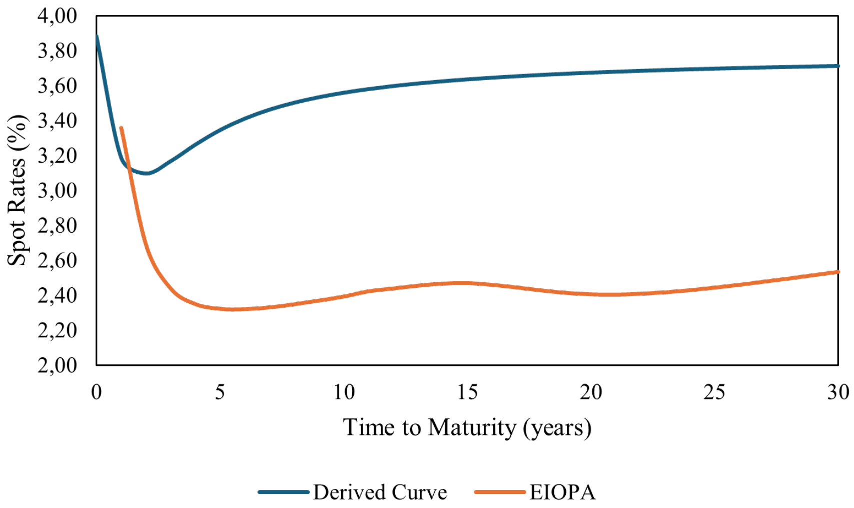

Figure 12: EIOPA risk-free and Derived Curve: Comparison

In the first year, the EIOPA risk-free rate (3.36%) is higher than the estimated rate (3.18%). Subsequently, from the second year onwards, all derived spot rates surpass those estimated by EIOPA, as expected. The maximum difference occurs at the 22-year mark (a difference equal to 1.280%).

EIOPA extrapolates its risk-free curve using a model other than Svensson, namely the Smith-Wilson method, and only discloses yearly rates. The differences in the shape of the curves are explained by the different extrapolation methods. The Svensson model accommodates two humps or troughs, whereas the Smith-Wilson method uses the actual observed data to exactly fit bond prices where data are available. EIOPA considers the 20-year threshold as the last liquid point, extrapolating from that point onwards. The UFR is fixed at 3.45%[6] and is reached at 60 years.

The derived 1-year spot rate is lower than EIOPA's, which is justified by EIOPA's exclusion of yields with maturities below 1 year to derive its curve. Other deviations arise from portfolio distinctions and assumptions made in the credit adjustment. One difference is the assumption that government bonds are risk-free in the derivation of the discount rates[7]. Moreover, debt securities issued by financial institutions, perceived as riskier than those issued by non-financial institutions by the market[8], constitute 30% of the total portfolio market value and 52% of the total corporate bonds' market value. From the sixty-one bonds issued by financial institutions, twelve are also subordinated heightening higher credit risk exposure. The default data utilized for deriving the yield curve is based on global corporate default rates

[6]Figure 12 represents the spot rates, and the fixed rate 3.45% is a forward rate.

[7]The difference between some government yields and EIOPA rates is seen with more detail in Appendix C.

[8]The difference between debt securities issued by financial and by non-financial institutions' YTM is seen with more detail in Appendix D.

and does not account for the differences in bond issuers or bond types. This partially explains the observed discrepancy between the two curves. Such discordance underscores the limitations of default rate time series in capturing the whole credit risk. Even after adjustments, the residual credit risk inherent in the derived curve may deviate from that of EIOPA's.

5.5 Comparison with ECB yield curve

Another comparison that can be made is with the euro zone yield curve published daily, as euro zone government bonds were considered risk-free in the derivation of the spot curves in this project. The curve disclosed by the ECB is also derived using the Svensson model and encompasses all euro zone central government bonds. Figure 13 represents the two spot curves. Once again, there is a difference between the two curves, with the maximum disparity being 0.9288% at 5 years. This reinforces the previous suggestion that credit risk may not be entirely removed, potentially due to corporate bonds. To mitigate these differences, finding more sector default rate time series is necessary.

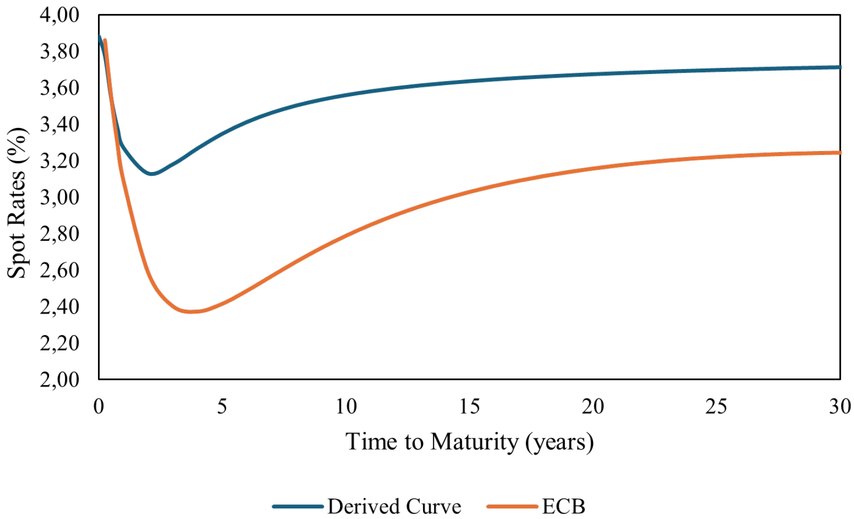

Figure 13: Euro Zone and Derived Curve: Comparison

There are some derived rates that are lower than ECB rates, namely the rates for 3 and 6 months. This can also be explained by the specific characteristics of the bonds included in the derivation, namely Aaa corporate bonds.

Thus, the extrapolation method, the risk-free assumptions, the different dataset and its quality, the lack of liquidity premium adjustment and the differences performing the credit risk adjustment contribute to the discrepancy between the curves.

6 Conclusion

The purpose of this work was to construct a discount rate curve using the top-down approach, in accordance with the principles outlined in IFRS 17. The top-down approach can be succinctly described as the process of deriving discount rates from a reference portfolio, with adjustments made to eliminate factors that are irrelevant to insurance contracts.

IFRS 17 is a principles-based standard that requires a certain level of subjectivity and judgement in interpreting its concepts, which can present a challenge when selecting the methodology for deriving the discount rate term structure.

This project methodology may be divided into three main steps: (1) the selection of the reference portfolio; (2) the necessary adjustments; and (3) the extrapolation of the curve to obtain rates for periods both within and beyond the available maturities. The selection of the reference portfolio was straightforward, as the company has a portfolio designed to match the durations of its insurance contracts, which provides an advantage. Since the portfolio is composed solely of bonds, the only adjustment made was for credit risk. IFRS 17 suggests the use of CDS for this purpose but does not impose so. Given the illiquidity and the unavailability of some CDS, this option was discarded. Instead, a model based on the ECL approach was used, motivated by the availability of all expected cash flows, allowing for the application of the chosen extrapolation method. This ECL-based model employs default rate and LGD data to assess credit risk. To adjust for unexpected credit losses in addition to expected credit losses, the 90^{th} percentile of default rates was used instead of the probability of default. This model recalculates the bonds' yield with credit-adjusted cash flows. These credit-adjusted cash flows are equal to a weighted average between the cash flow paid in the event of default and the cash flow paid in the event of no default. The credit-adjusted risk-free yields were then used to extrapolate the yield curve. The extrapolation model used was the Svensson model, due to its widespread use by government entities. The application was carried out through a two-step nonlinear optimization process to minimize the yield squared errors. Initially, a direct search algorithm, the Nelder-Mead algorithm, was implemented. Subsequently, the solution was refined using a gradient-based algorithm, the Generalized Reduced Gradient. This approach effectively reduced the MSE of the minimization problem.

The derived smooth spot yield curve was finally compared with other reference curves, motivating questions regarding the effective removal of credit risk. While the principles delineated by IFRS 17 were fulfilled, the comparative analysis with EIOPA risk-free curve and ECB yield curve led to contemplation. These questions underscore the need for continuous refinement and optimization of the methodology employed. For instance, the possibility of distinguishing between sector bonds, such as bonds issued by financial and non-financial institutions, or adjusting the assessment of credit risk associated with government bonds could improve the accuracy of the curve.

To conclude, the new accounting standard IFRS 17 established by the IASB aims to improve comparability among insurance and reinsurance companies. Nevertheless, there is no universally accepted and defined methodology in place, which allows for business-specific judgement from each company. Given the recent implementation of the standard, there is considerable scope for improvement. Over time, as the standard matures, it is expected that new and more refined methodologies will emerge, leading to further advancements in the field.

References

Bank of International Settlements. (2005). *Zero-coupon yield curves: technical documentation Monetary and Economic Department.* Available at: https://www.bis.org/publ/bppdf/bispap25.pdf

Belkin, B., & Suchower, S. (1998). *A one-parameter representation of credit risk and transition matrices.* Available at: https://www.z-riskengine.com/media/hqtnwlmb/a-one-parameter-representation-of-credit-risk-and-transition-matrices.pdf

Berndt, E. K., Hall, B. H., Hall, R. E., & Hausman, J. A. (1974). Estimation and Inference in Nonlinear Structural Models. In *NBER* (p. p. 653-665). Available at NBER website: https://www.nber.org/system/files/chapters/c10206/c10206.pdf

BIS Monetary and Economic Department (2005). *Zero-coupon yield curves: technical documentation.* Available at: https://www.bis.org/publ/bppdf/bispap25.pdf

Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81(3), 637–654. https://doi.org/10.1086/260062

Brigo, D., Predescu, M., & Capponi, A. (2010). *Credit Default Swaps Liquidity Modeling: A Survey.* Derivatives eJournal. Available at: https://www.semanticscholar.org/reader/008e08c3315d48703cb8766de8d0b2cfcd8ddcd5

Cerny, V. (1985). Thermodynamical Approach to the Traveling Salesman Problem: An Efficient Simulation Algorithm I. *Journal of Optimization Theory and Applications*, 45(1), 41–51. Available at: https://mk.bcgsc.ca/papers/cerny-travelingsalesman.pdf

CIA (2020). *IFRS 17 Discount Rates and Cash Flow Considerations for Property and Casualty Insurance Contracts.* Available at: https://www.casact.org/sites/default/files/2023-05/6C_CIA_Revised_Educational_Note_IFRS_17_Discount_Rates_and_Cash_Flow_C.pdf

CIA (2022). *Revised Educational Note: IFRS 17 Discount Rates and Cash Flow Considerations for Property and Casualty Insurance Contracts.* Available at Canadian Institute of Actuaries website: https://www.cia-ica.ca/publications/222159e/

Commission Delegated Regulation (EU). (2015). Commission Delegated Regulation (EU) 2015/35 of 10 October 2014 supplementing Directive 2009/138/EC of the European Parliament and of the Council on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II). Available at Official Journal of the European Union, L 12, 17.1.2015, p. 1 website:https://publications.europa.eu/resource/cellar/e0c803af-9e0f-11e4-872e-01aa75ed71a1.0006.03/DOC_477

El Bekri, H. (2020). *Calibration de la courbe de taux en normes IFRS 17.* Available at: https://www.institutdesactuaires.com/docs/mem/3abc273a2bf55cf2a77df4248e054c95.pdf

Eitrheim, Ø. (2005). Estimation of Spot and Forward Rates from Daily Observations.

Bank of International Settlements. Available at Bank of International Settlements website: https://www.bis.org/publ/bppdf/bispap25i.pdf

FINANSTILSYNET (2010). *A Technical Note on the Smith-Wilson Method.* Available at: https://www.ressources-actuarielles.net/EXT/ISFA/fp-isfa.nsf/2b0481298458b3d1c1256f8a0024c478/bd689cce9bb2aeb5c1257998001ede2b/

Garcia, M. T., & Carvalho, V. H. F. (2018). *A Static Approach to the Nelson-Siegel-Svensson Model: an Application for Several Negative Yield Cases.* Available at: https://www.repository.utl.pt/handle/10400.5/16016

Giglio, S. (2016). Working Paper Series Credit default swap spreads and systemic financial risk. *European Systemic Risk Board.* Available at European Systemic Risk Board website: https://www.esrb.europa.eu/pub/pdf/wp/esrbwp15.en.pdf

IFoA (2018). *IFRS17 Discount Rate considerations.* Available at: https://www.actuaries.org.uk/system/files/field/document/IFRS%2017%20discount%20rate%20considerations_20190925.pdf

IFoA (2020). *IFRS 17 - Future of Discount Rates Working Party Case study on the "top-down" approach 1.* Available at: https://www.actuaries.org.uk/system/files/field/document/Case%20study%20on%20the%20top-down%20approach%20%20-%20Final_20200106_0.pdf

IFRS Foundation (2021a). *Financial Instruments.* Available at: https://www.ifrs.org/content/dam/ifrs/publications/pdf-standards/english/2021/issued/part-a/ifrs-9-financial-instruments.pdf

IFRS Foundation (2021b). *Insurance Contracts.* Available at: https://www.ifrs.org/content/dam/ifrs/publications/pdf-standards/english/2021/issued/part-a/ifrs-17-insurance-contracts.pdf

Jarrow, R. A., Lando, D., & Turnbull, S. M. (1997). A Markov Model for the Term Structure of Credit Risk Spreads. *Review of Financial Studies*, 10(2), 481–523. Available at: https://doi.org/10.1093/rfs/10.2.481

Jessop, N. (2018). *Permitted approaches for constructing IFRS 17 Discount Rates.* Available at: https://www.moodysanalytics.com/-/media/article/2019/permitted-approaches-for-constructing-ifrs17-discount-rates.pdf

Lasdon, L. S., Waren, A. D., Jain, A., & Ratner, M. (1978). Design and Testing of a Generalized Reduced Gradient Code for Nonlinear Programming. *ACM Transactions on Mathematical Software*, 4(1), 34–50. Available at: https://doi.org/10.1145/355769.355773

Manousopoulos, P., & Michalopoulos, M. (2009). Comparison of non-linear optimization algorithms for yield curve estimation. *European Journal of Operational Research*, 192(2), 594–602. Available at:https://doi.org/10.1016/j.ejor.2007.09.017

McCulloch, J. H. (1971). The Term Structure of Interest Rates. *The Journal of Business*, 44(1):19-31.

McCulloch, J. H. (1975). The tax-adjusted yield curve. *The Journal of Finance*, 30(3), 811–830. Available at:https://doi.org/10.1111/j.1540-6261.1975.tb01852.x

Merton, R. C. (1974). On the Pricing of Corporate Debt: the Risk Structure of Interest Rates. *The Journal of Finance*, 29(2), 449–470. Available at: https://doi.org/10.1111/j.1540-6261.1974.tb03058.x

Müller, R. & Bank for International Settlements. (2005). *A Technical Note on the Svensson Model as Applied to the Swiss Term Structure*. Available at: https://www.bis.org/publ/bppdf/bispap25l.pdf

Nelder, J. A., & Mead, R. (1965). A Simplex Method for Function Minimization. *The Computer Journal*, 7(4), 308–313. Available at:https://doi.org/10.1093/comjnl/7.4.308

Nelson, C. R., & Siegel, A. F. (1987). Parsimonious Modeling of Yield Curves. *The Journal of Business*, 60(4), 473–489. Available at: https://www.jstor.org/stable/2352957

Nymand-Andersen, P. (2018). *Statistics Paper Series. In European Central Bank.* Available at: https://www.ecb.europa.eu/pub/pdf/scpsps/ecb.sps27.en.pdf

Saunders, A., & Allen, L. (2002). *Credit risk measurement: new approaches to value at risk and other paradigms.* In Google Books. John Wiley Sons. Available at: https://books.google.pt/books?hl=pt-PT&lr=&id=pGMLAd2WvasC&oi=fnd&pg=PR5&dq=credit+risk+measurement+saunders+allen&ots=4KcynV_NkD&sig=mmVvOrfOX4bPoW5YffrRQBKZPqk&redir_esc=y#v=onepage&q=credit%20risk%20measurement%20saunders%20allen&f=false

Sharp, J. (2021). *IFRS 17 - Default Model - Historic Calibration 1. Executive Summary.* Available at: https://www.actuaries.org.uk/system/files/field/document/IFRS%2017%20default%20allowance%20-%20v0.5.pdf

Svensson, L. E. O. (1994). *Estimating and Interpreting Forward Interest Rates: Sweden 1992 - 1994.* Available at National Bureau of Economic Research website: https://www.nber.org/papers/w4871

Volarevi, H., & Varovi, M. (2018). Internal Model for IFRS 9 -Expected Credit Losses Calculation. *EKONOMSKI PREGLED*, 69(3), 269–297. Available at: https://hrcak.srce.hr/en/file/298130

A Solvency II

Solvency II constitutes the regulatory and supervisory framework for the EU insurance sector. In force since 1 January 2016, it sets out capital requirements and risk management standards for (re)insurance companies. Its implementation aims to protect policyholders and beneficiaries and to increase the sensitivity of the capital measures to the insurer risks, improving transparency and market discipline.

To comply with Solvency II requirements, insurance and reinsurance companies must calculate capital requirements based on the risks inherent in their operations. EU government bonds carry zero credit risk charges (Commission Delegated Regulation (EU), 2015), exempting insurers from holding additional capital against these bonds due to their perceived safety and stability. This assumption, employed in the top-down approach, aligns with the regulatory framework, ensuring consistency with established standards.

B Credit Risk Adjustment: Example

Table B.I illustrates the computation of credit-risk adjusted YTM for a subordinated Baa bond with annual coupons, maturing on 4 April 2026. The principal is 4,000 thousands of euro, and the coupon rate is 3.25%. As it is a subordinated bond, the recovery rate is 27.83% (1-72.17%, with 72.17% representing the LGD). The "CF" (Cash Flow) column depicts the payments for each date. At maturity (time 2.25), the payment includes the redemption of the principal. The "ND" (No Default) column shows the cash flows in the absence of default. The "D" (Default) column equals 4,130 × 27.83%, representing the recovered amount (sum of the coupon and principal payments multiplied by the recovery rate). The "EV" (Expected Value) column contains the weighted average cash flows. For example, on 4 April 2025, 135 results from the sum of 130 multiplied by (1-0.65%) (ND cash flow times the probability of non-default until that moment), and 4,130 times 0.51% (recovered amount times the probability of defaulting at that time). The bond's price is 4 083 paid at time 0.0. All monetary amounts (CF, ND, D and EV columns) are expressed in thousands of euro. The column $PD_{\leq t}$ contains the cumulative 90^{th} percentile of default rates until the time expressed in column t. The 0.14% is the result of the product of 0.25 and 0.54% (the 1-year 90^{th} percentile of default rates for Baa bonds). The bond's YTM is 3.41%, and the credit-adjusted risk-free YTM is 3.02%. In this example, the credit adjustment equals 0.39%, i.e., the difference between the two yields.

Table B.I: Credit-Adjusted Risk-Free YTM - Example of Computation

Date	t	CF	$PD_{]t-1,t]}$	$PD_{\leq t}$	ND	D	EV
03/01/2024	0.0				-4,083	-4,083	-4,083
04/04/2024	0.25	130	0.14%	0.14%	130	1,149	131
04/04/2025	1.25	130	0.51%	0.65%	130	1,149	135
04/04/2026	2.25	4 130	0.53%	1.18%	4,130	1,149	4,087
Recovery Rate:	27.83%			YTM:	3.41%		3.02%

The bond prices are as at 29 December 2023. However, errors arose when applying the XIRR formula in Excel with this date. The shorter the bond maturity, the greater the difference between the computed YTM and that disclosed in Bloomberg. The maximum difference recorded was 0.24% for a bond with a YTM of 3.63% maturing on 14 February 2024. To align with the Bloomberg's disclosed YTM, the YTM calculation starts from 3 January 2024. This is because 29 December 2023, serves as the trade date, while the settlement date, when the actual exchange of funds and securities occur, is two business days later. As a result, although bond prices are based on 29 December 2023, the YTM calculation accounts for the settlement date. Having taken this detail into account in the calculation, the YTMs disclosed in Bloomberg matches the computed YTMs.

C EIOPA vs. European government bonds

The consideration of government bonds as risk-free assets is one explanation for the disparity between the derived yield curve and the EIOPA risk-free curve. EIOPA adjusts government bonds, and the disparities between rates of Italy, Spain, and Portugal government bonds (source: Bloomberg), EIOPA's risk-free rates at 29 December 2023 and the derived curve are depicted in Figure C.1.

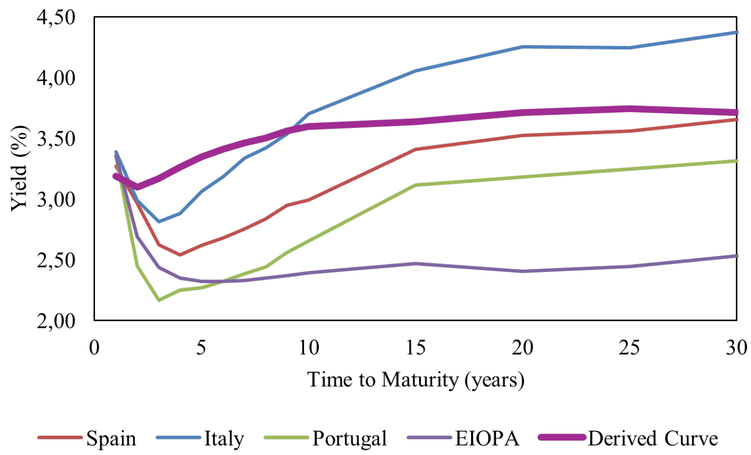

Figure C.1: EIOPA vs. Government Rates

With the exception of Italy for maturities exceeding 10 years, the derived rates are higher than those of Spain, Italy, and Portugal's government and EIOPA rates. As a result, the assumption of government risk-free rates fails to account for the entire discrepancy. Nevertheless, yields for Spain and Italy exceed EIOPA's risk-free rates across all maturities, while Portuguese yields consistently surpass them beyond the initial five years, partially explaining the discrepancy. The difference between 30-year Italian yields and EIOPA's yields is 1.164%. The assumption of government bonds as risk-free in the derivation of discount rates is justified by various factors, including limitations in the available data, the historical stability - with only one default recorded to date - suggesting that such defaults can be disregarded as pursuant to Solvency II requirements.

D Sector Bond Comparison

Bonds categorized by sector, particularly distinguishing between bonds issued by financial and non-financial institutions, may enhance the credit risk adjustment process. This is justified by the perception among investors that financial corporate bonds carry higher credit risk compared to bonds issued by non-financial institutions. This perception is reflected in the higher YTM observed for financial bonds, as depicted in Figures D.1, D.2 and D.3. These plots depict the yield curve constructed as at 29 December 2023, for euro-denominated senior unsecured fixed Aa, A and Baa-rated bonds, respectively, issued by European companies, as disclosed in Bloomberg, categorized into financial, non-financial, and composite (both financial and non-financial issuer) bonds. The maximum yield discrepancy between financial and composite yields is 0.381% for Baa bonds with a 30-year maturity.

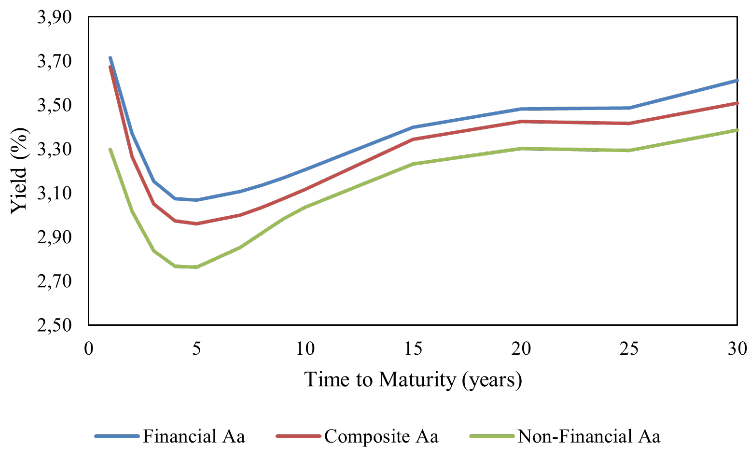

Figure D.1: Sector Comparison between Aa bonds

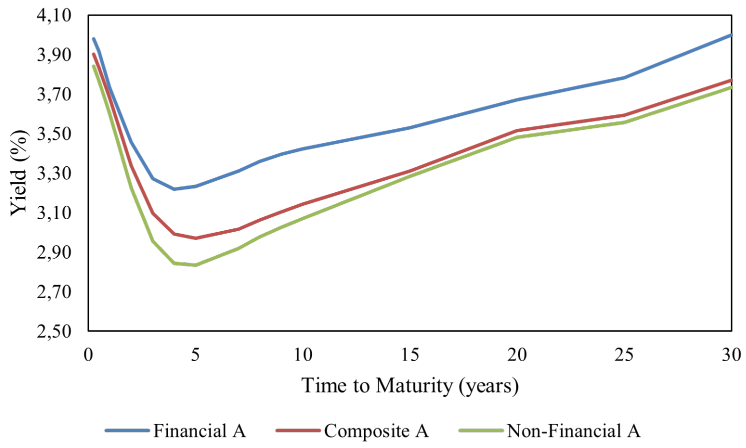

Figure D.2: Sector Comparison between A bonds

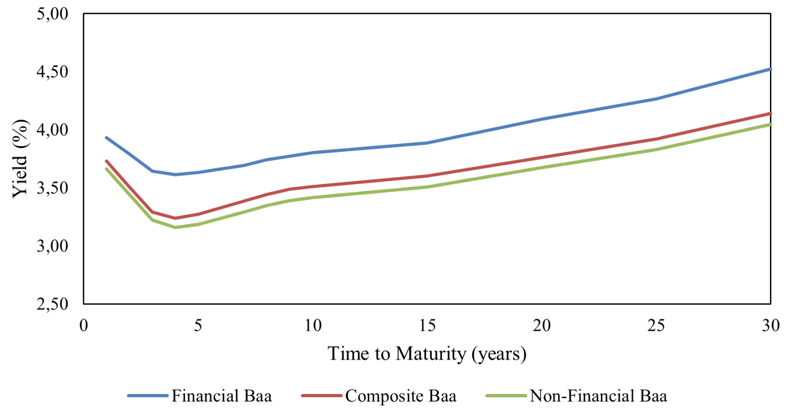

Figure D.3: Sector Comparison between Baa bonds